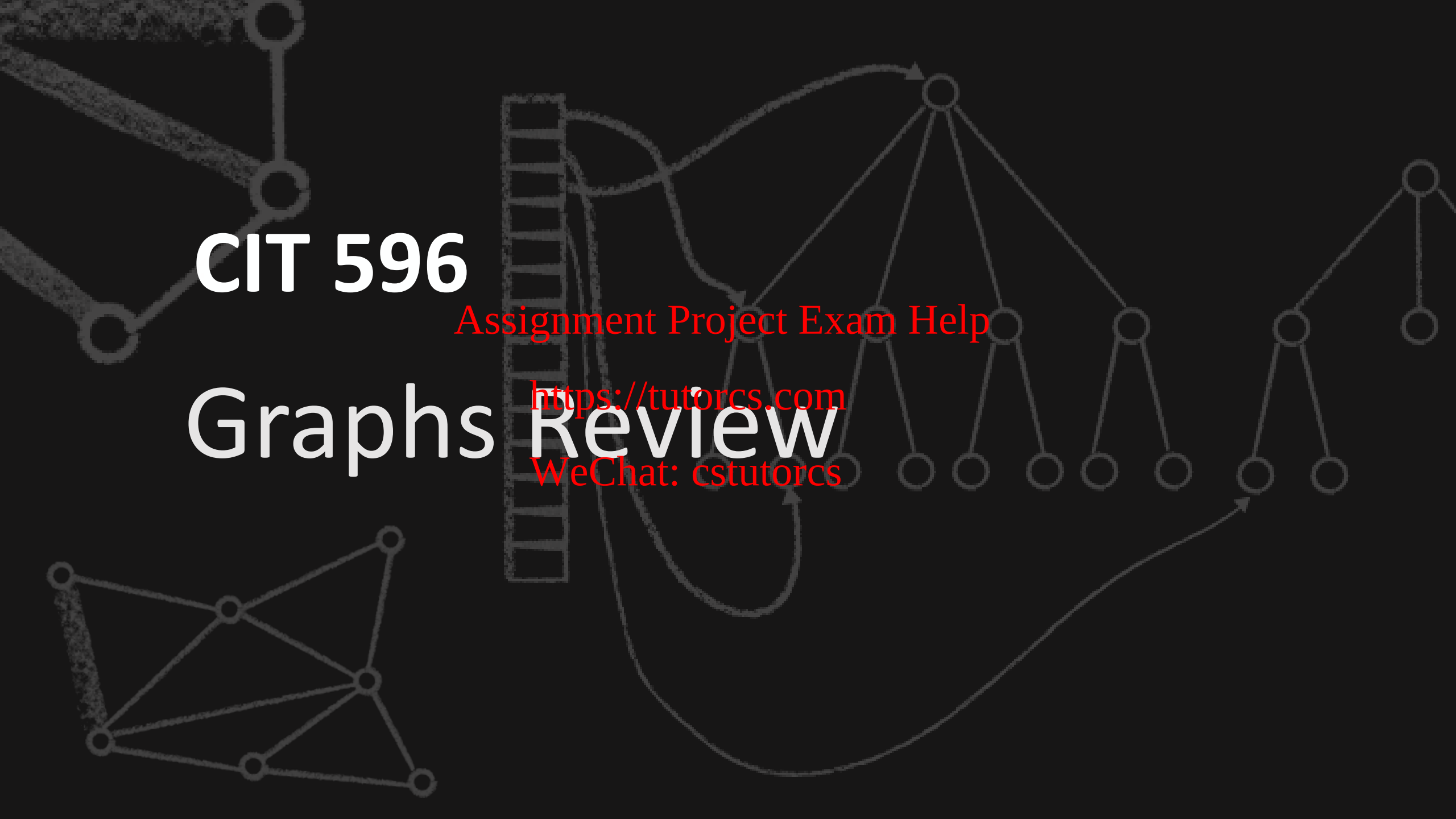


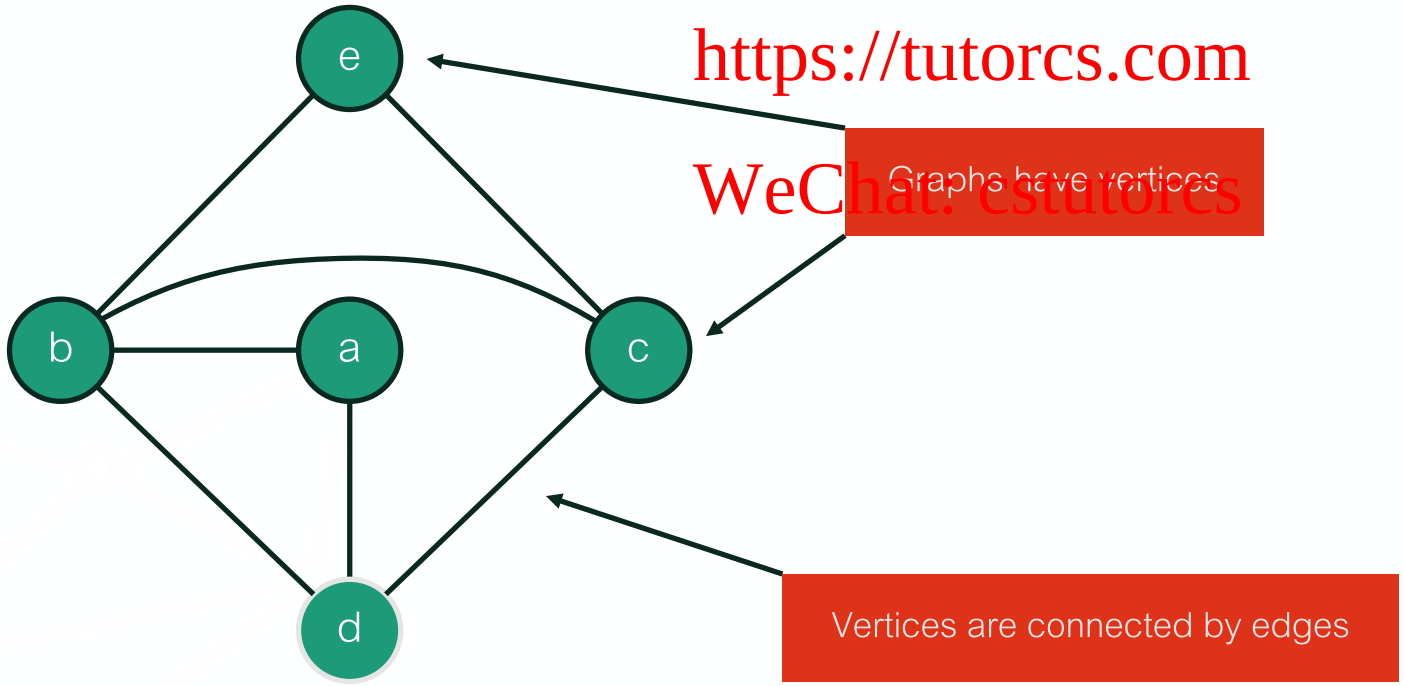
CIT 596

Graphs Review

GRAPHS

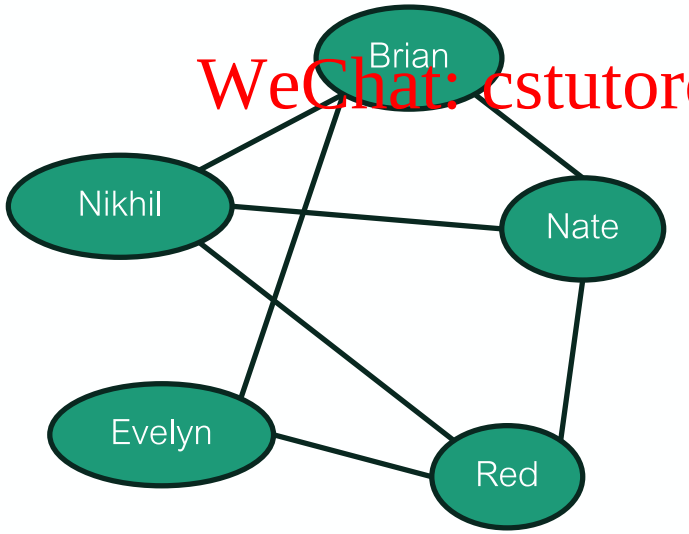
Graphs are represented as a set of vertices and a set of edges.

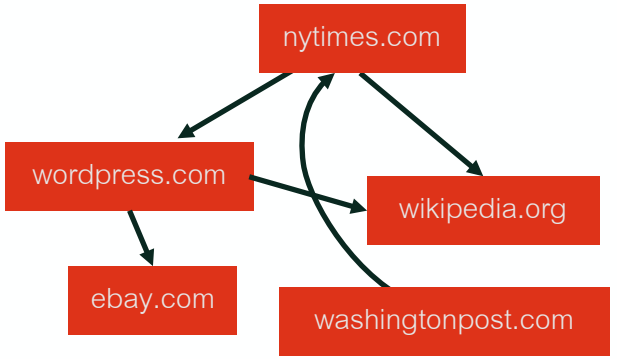
Sometimes they concretely model a network (roads, communication) but usually they represent abstract relationships (people/friendships or documents/similarity).



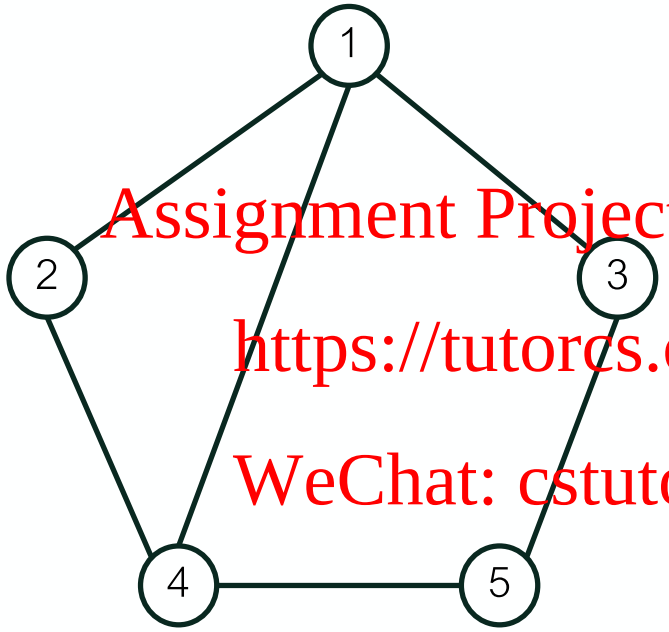
GRAPHS

- Graphs model relationships.
- We often model problems as graphs and then design algorithms leveraging theory about graphs.
- Examples:
  - Friend Networks
  - Links on the web





EXAMPLE



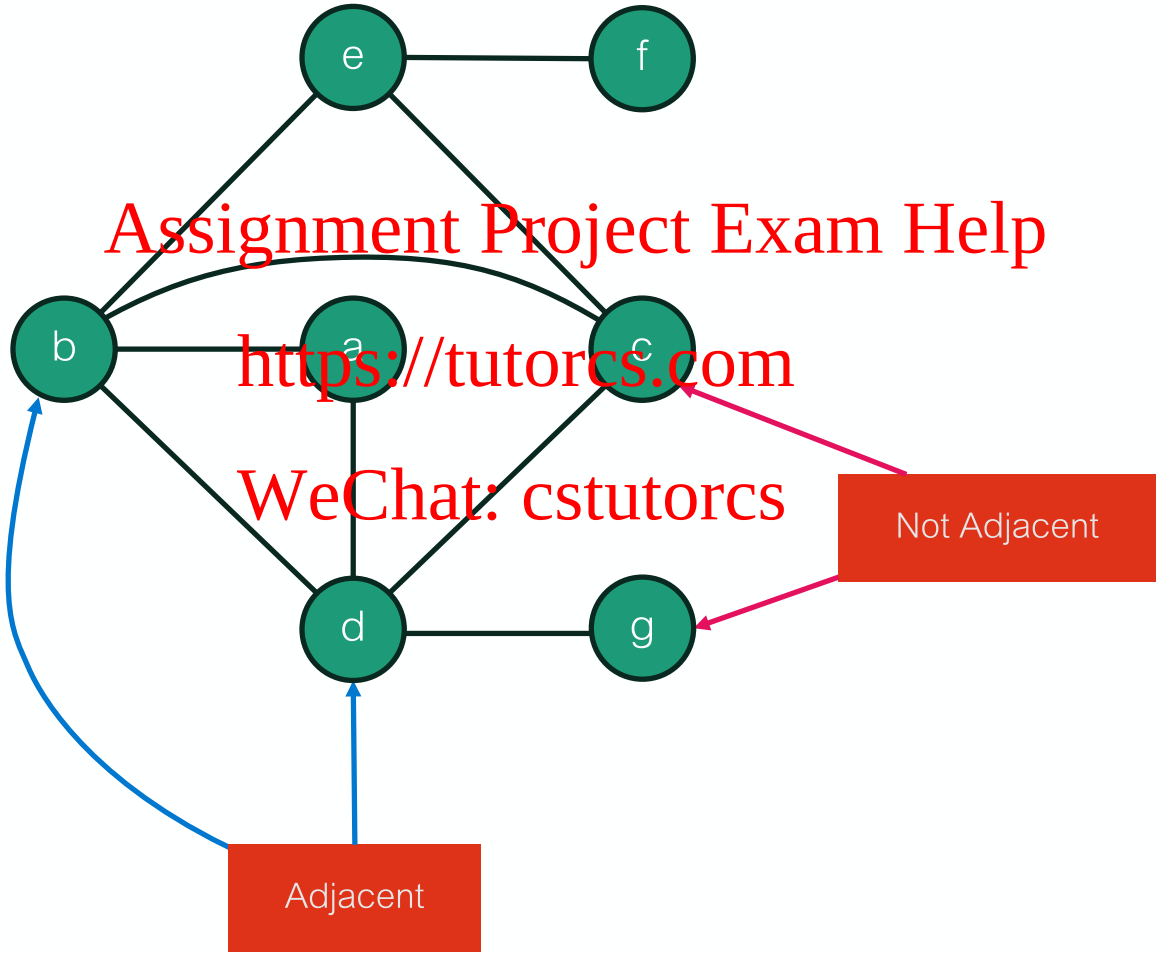
Graph $G = (V, E)$

$V = \{1, 2, 3, 4, 5\}$ (finite set of vertices)

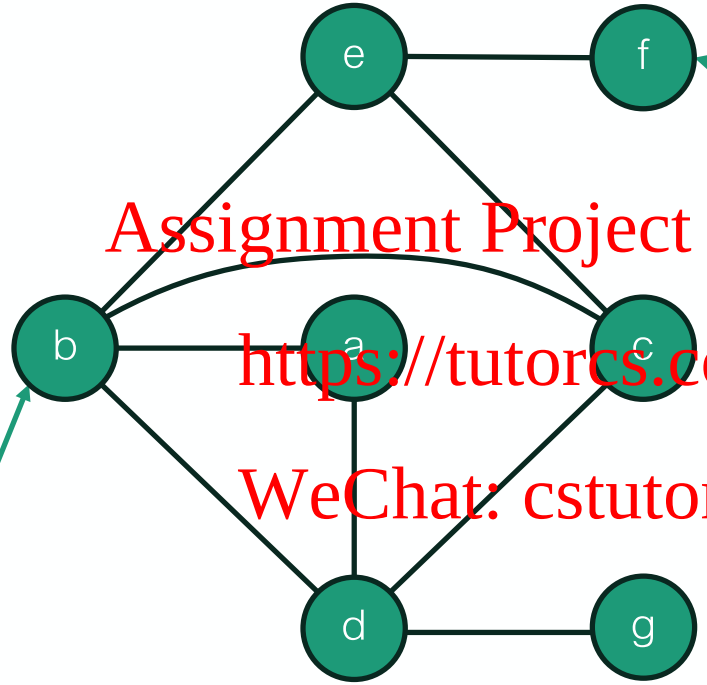
$E = \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 5), (4, 5)\}$ (finite set of edges)

In general, V is a set and E is a set of pairs from V

GRAPH TERMINOLOGY



GRAPH TERMINOLOGY

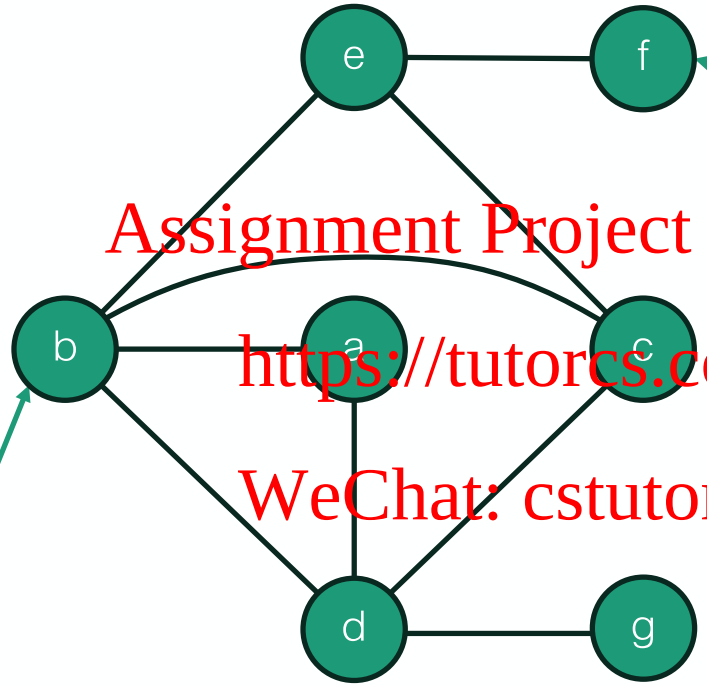


$\text{degree}(f) = 1$

b, a, d, b, e is an example of a path

$\text{degree}(b) = 4$

GRAPH TERMINOLOGY

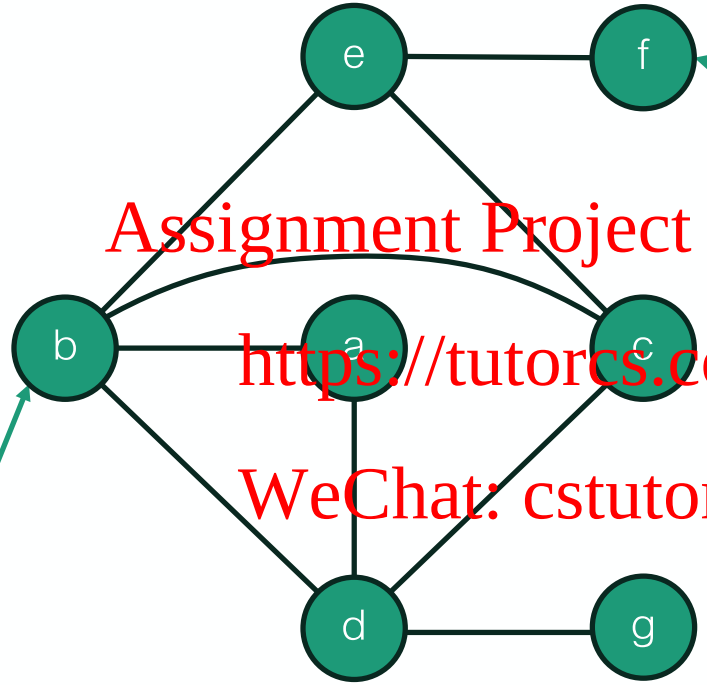


degree$(f) = 1$

degree$(b) = 4$

b, a, d, c is an example of a simple path of length 3

GRAPH TERMINOLOGY



$\text{degree}(f) = 1$

$\text{degree}(b) = 4$

b, a, d, c, b is a cycle but b, a, b and b, a, d, b, e are not cycles

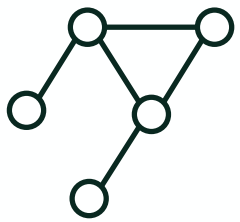
CONNECTED

- Two vertices are connected if there is a (simple) path between them.
- Being connected is an equivalence relation (recall from CIT 592):
  - Reflexive: Every vertex has a path of length 0 to itself.
  - Symmetric: If there is a path from u to v then reverse it to get a path from v to u (only works in undirected graphs)
  - Transitive: If there is a path from u to v and a path from v to w then the paths can be concatenated to make a path from u to w. If this path is not simple, we can remove cycles to make it simple.

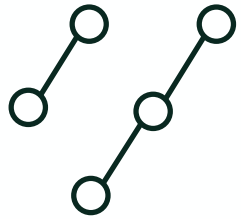
GRAPHS: CONNECTED, ACYCLIC

- Connected components of a graph: equivalence classes of the “connected” relation
- If only one equivalence class, the graph is connected.
- A graph is acyclic if it has no cycles.
- A connected, acyclic graph is called a tree.

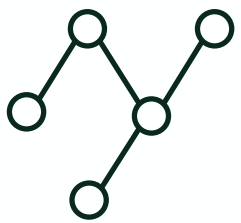
Examples:



a) Connected graph



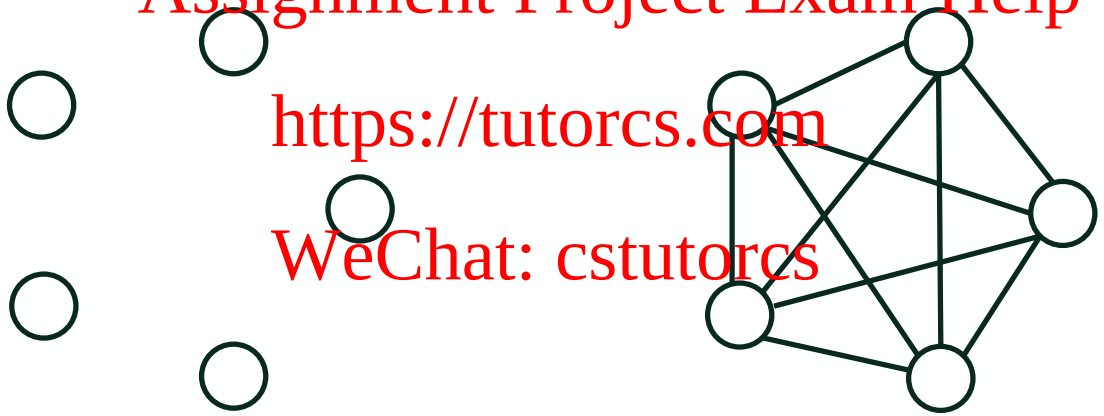
b) Acyclic graph with 2 components



c) Tree

NUMBER OF VERTICES AND EDGES

- Convention: Use n to denote the number of vertices and m to denote the number of edges in a graph.



m can be as low as 0 or as high as $\binom{n}{2}$