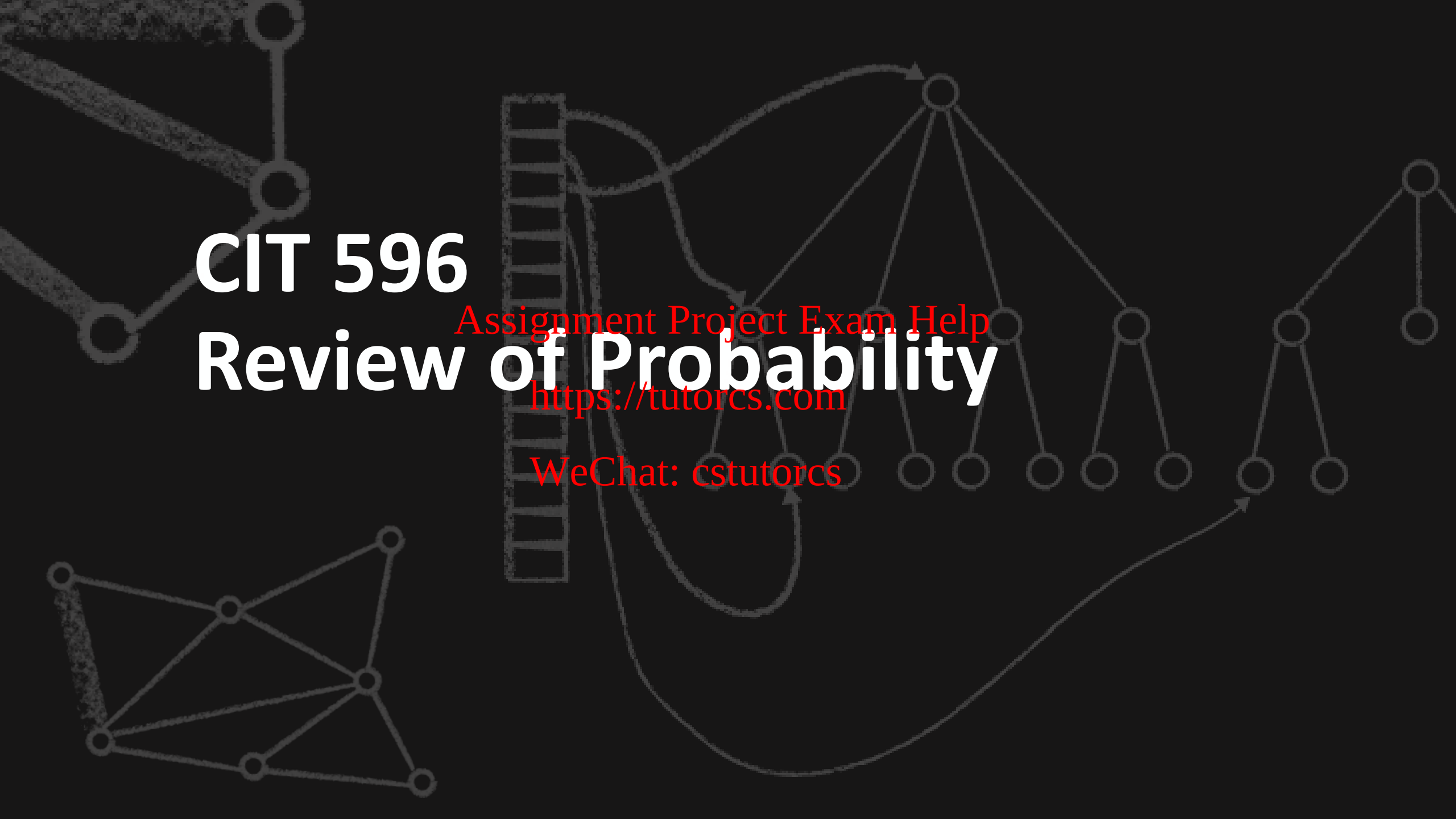

CIT 596
Review of Probability

REVIEW OF PROBABILITY

Experiment: something where the outcome is uncertain.

- e.g. tossing a coin, rolling one die, predicting the weather tomorrow.

Sample space Ω: set of possible outcomes of an experiment.

- For a coin-tossing experiment, $\Omega = \{H, T\}$
- For a normal die-rolling experiment, $\Omega = \{1, 2, 3, 4, 5, 6\}$
- For weather prediction, $\Omega = \{sunny, cloudy, rainy, snowy\}$

Event: a set of outcomes.

- For a die-rolling experiment, an event could be “an odd roll”.
  This would correspond to the set $E = \{1, 3, 5\}$

REVIEW OF PROBABILITY

Probability space: sample space with probability for each outcome.

- For a fair coin-tossing experiment, the probability space would be $(\{\mathrm{H}, \mathrm{T}\}, P)$ where P maps each outcome to the number $\frac{1}{2}$.
- In general, the probability for any outcome is between 0 and 1.
- The sum of all outcome probabilities is 1.
- We will stick to finite sample spaces (and avoid subtle math concerns).

REVIEW OF PROBABILITY

Random Variable: A function X from the outcomes of an experiment to numbers, i.e. a function $X : \Omega \to \mathbb{R}$

Example: Toss a coin 10 times. Sample space has 2^{10} elements, one for each sequence of Hs and Ts, and each with probability $\frac{1}{2^{10}} = 2^{-10}$.

Suppose we want to count the number of Hs.

Define a random variable X that maps each point to its number of Hs.

- e.g. $X(HHTTTHTHTT) = 4$

What is the probability of observing 4 Hs?

It is the probability that $X = 4$, written as $\Pr[X = 4]$.

REVIEW OF PROBABILITY

$\Pr[X = 4]$: How many sequences in the sample space have 4 Hs?

$$\binom{10}{4} = 210$$

What is the probability of each sequence?

$$2^{-10}$$

So…

$$\Pr[X = 4] = \frac{210}{2^{10}} \approx \frac{1}{5}$$

X can take values from 0 to 10 with differing probabilities:

Heads	0	1	2	3	4	5	6	7	8	9	10
Probability $(\times 2^{10})$	1	10	45	120	210	252	210	120	45	10	1

REVIEW OF PROBABILITY

Another example: $X =$ "the sum of two die rolls"

X can take on values in $[2..12]$

- e.g. $X = 5$ can result from one of $\{(1,4),(2,3),(3,2),(4,1)\}$

Probability of each outcome is $1/36$ so $\Pr[X = 5] = 4 \cdot \frac{1}{36} = \frac{1}{9}$

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability $(\times\ 36)$	1	2	3	4	5	6	5	4	3	2	1

Expectation of a random variable: weighted average of the values it can take

$$\mathbf{E}[X] = \sum_{x=2}^{12} (x \cdot \Pr[X = x]) = 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + \cdots + 12 \cdot \frac{1}{36} = 7$$