BASIC PROPERTIES OF TREES

- In a tree, a vertex of degree 1 is called a leaf.
- Theorem: If T is a tree with at least 2 vertices, it has a leaf.
- Proof:

  - Walk starting from some vertex v in T without backtracking on an edge just taken.

  - You cannot revisit any vertex, since that would mean there exists a cycle.
  - So, this walk must terminate because there are only finitely many vertices.

  - Say the walk terminates at x. We came to x on some edge, but we cannot continue the walk, so there must be no other edges incident on x.
  - x is a leaf! QED

BASIC PROPERTIES OF TREES

- Theorem: A tree T with n vertices has $n-1$ edges.
- Proof.

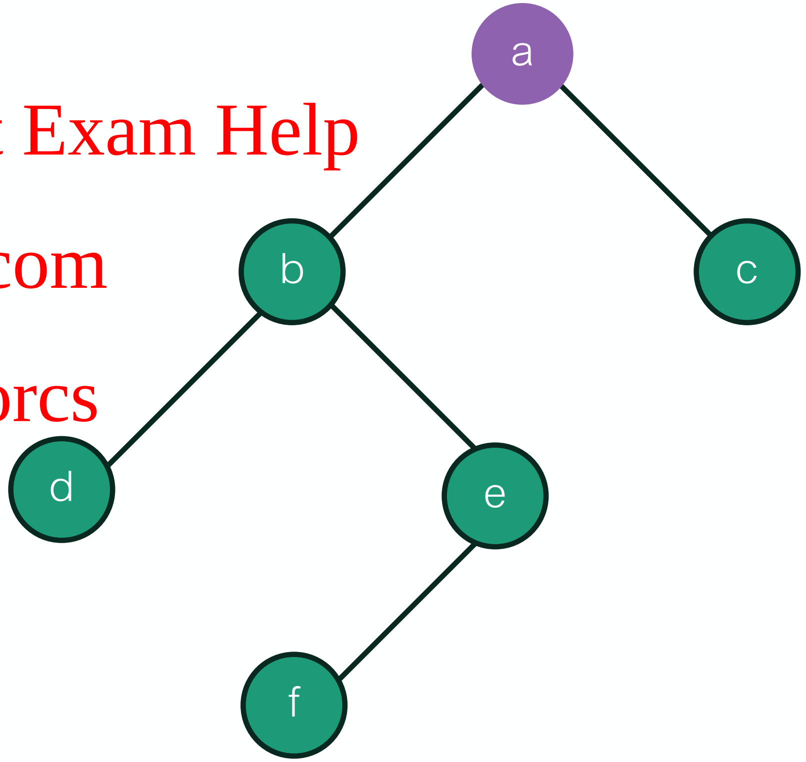
  - Base Case: If the tree has just 1 vertex, it has 0 edges, which aligns with the theorem to prove.
  - Induction Hypothesis: Suppose the theorem is true for all trees with less than n nodes.

  - Induction Step: Consider a tree with n nodes.
  - T has a leaf x.

  - Removing x from T gives us a tree T' with $n-1$ vertices.
  - By the induction hypothesis, T' has $n-2$ edges.
  - Adding back x and the one edge incident on it adds 1 to the number of edges.
  - So T has $n-1$ edges. QED.

TREES – ROOTED AND UNROOTED

- We have seen examples of rooted trees
  - Heaps and binary search trees
- Unrooted trees are just connected, acyclic graphs
  - Can “pick them up” by any vertex, call it the root, and let the tree hang from it
- Trees don’t have to be binary; nodes can have more than two children



TREE PROPERTIES

- Unique path from any vertex to any other in a tree
- If we remove one edge from a tree, the resulting graph has two connected components