CIT 596 **Spring 2023**

Module 5–6

Name: NAME HERE Collaborator: COLLABORATOR NAME HERE

Answers that describe an algorithm should always include a proof of correctness and running time analysis. Algorithms may be described in English or in clear pseudocode with explanatory comments.

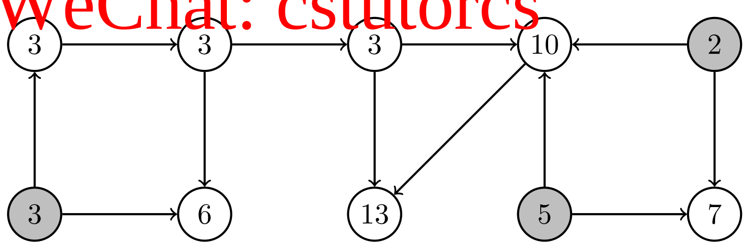
1. You are given an integer $r \in [1..n]$ and a sequence $\sigma = s_1, s_2, \ldots, s_n$ of n distinct elements in which elements are presented one at a time. When element s_i is presented, you can no longer access any of $s_1, \ldots, s_{i-1}$ unless your algorithm has stored them. You are asked to output the r^{th} smallest element in σ. Design an algorithm that can accomplish this using $O(r)$ space and $O(n)$ *expected time.* **(25 points)**

2. You are given the adjacency-list representation of a directed acyclic graph $G = (V, E)$ with n vertices and m edges. Furthermore, for every vertex $v \in V$ with in-degree zero, you are given a number $value(v)$. For every vertex $v \in V$ with positive in-degree, we define

$$value(v) = \sum_{(u,v) \in E} value(u),$$

the sum of the values of all of v's predecessors in the graph.

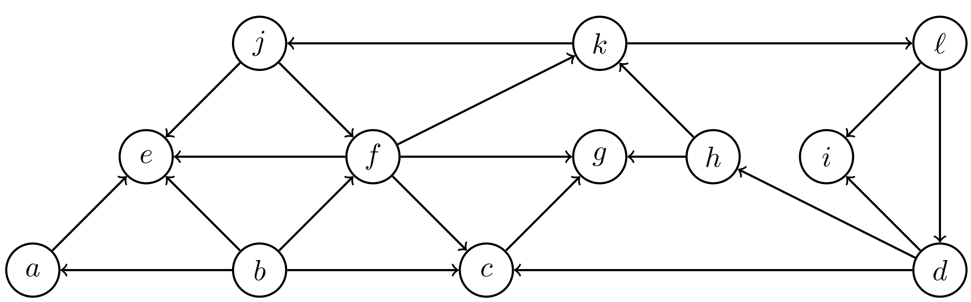
For example, in the graph below, the gray vertices have in-degree zero, so their values determine the values of all other vertices in the graph.



Design a $O(n + m)$-time algorithm to compute $value(v)$ for all vertices $v \in V$. **(25 points)**

3. You are given the adjacency list representation of a directed graph $G = (V, E)$ with n vertices and m edges. We define the set $S = \{v \in V : \text{ some cycle in } G \text{ is reachable from } v\}$.

For example, in the graph below, $S = \{b, d, f, h, j, k, \ell\}$.



Design an $O(n+m)$ time algorithm to find the set S. (**25 points**)

4. Let $G(V, E)$ be an undirected graph with n vertices and m edges such that G has two vertices s and t where the shortest path from s to t has length strictly more than $n/2$. Then, prove that there must be a vertex $w \in V \setminus \{s, t\}$ such that every path from s to t must pass through w. Furthermore, assuming that G is given to you in the adjacency-list representation along with vertices s and t, design an $O(n+m)$ time algorithm to output such a vertex w. (**25 points**)