Cardiff University
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CMT10[illegible]ual Computing

Camera Calibration

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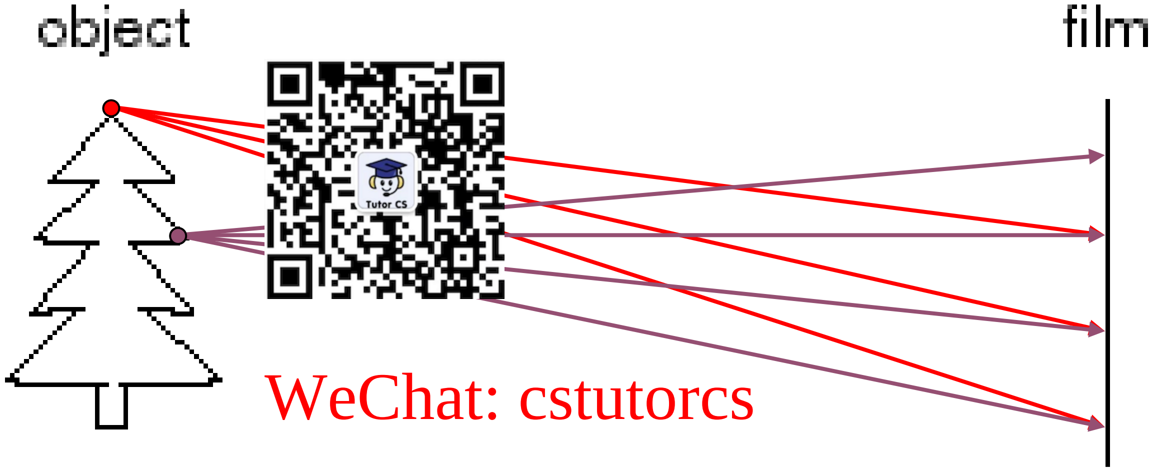
Overview

- Cameras
- Pinhole cameras
  - Vanishing points
- Real camera
  - Aperture adjustment
  - Thin lens formula
  - Lens flaws
- Pinhole camera model
  - Camera parameters: intrinsic parameters, extrinsic parameters
- Camera calibration
  - Linear method

Acknowledgement
The majority of the slides in this section are from Svetlana Lazebnik at University of Illinois at Urbana-Champaign

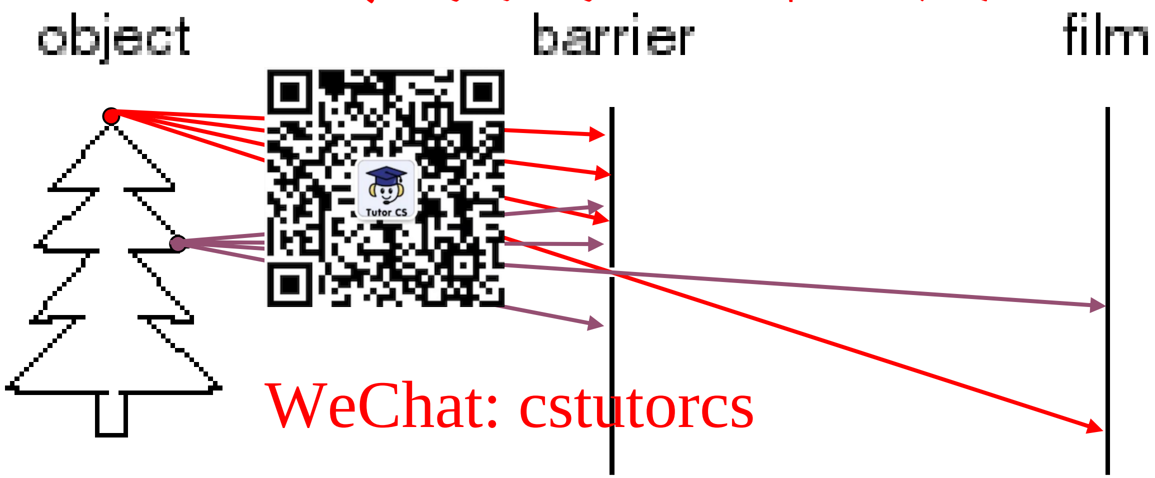
Cameras

Let's Design a Camera



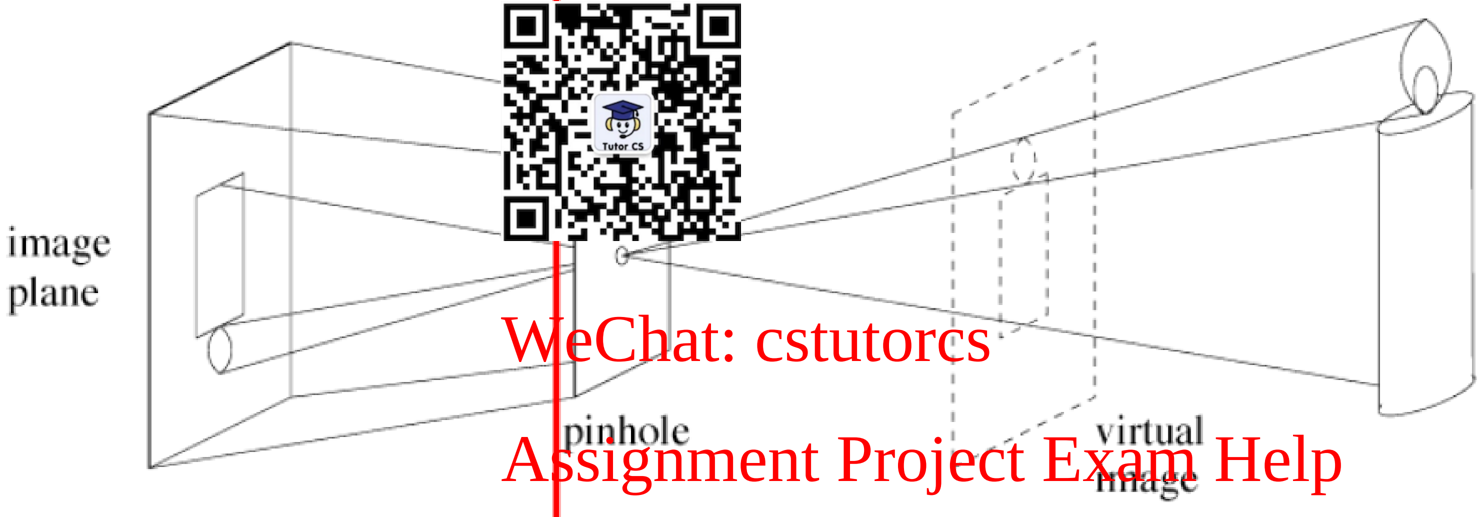
- Idea 1: put a piece of film in front of an object?
- Do we get a reasonable image?

Let's Design a Camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is know as the aperture

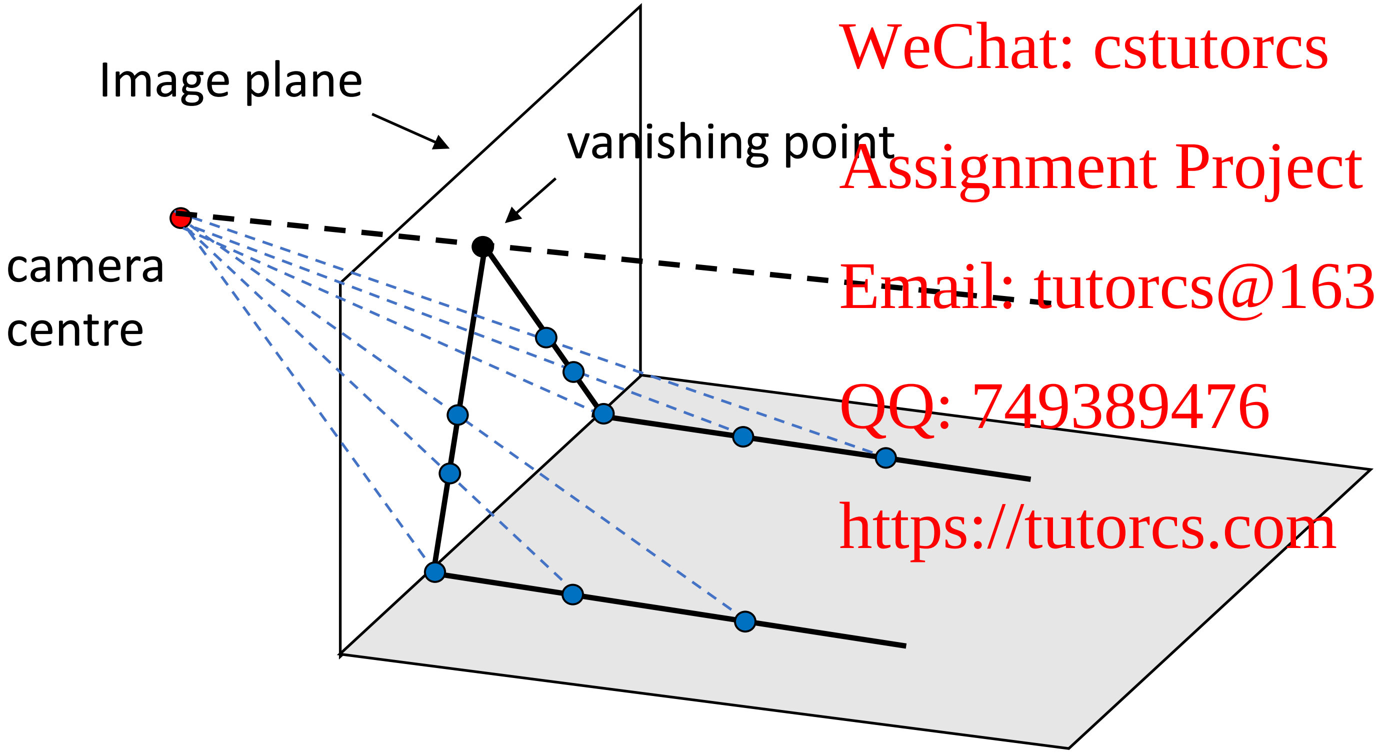
Pinhole Camera Model

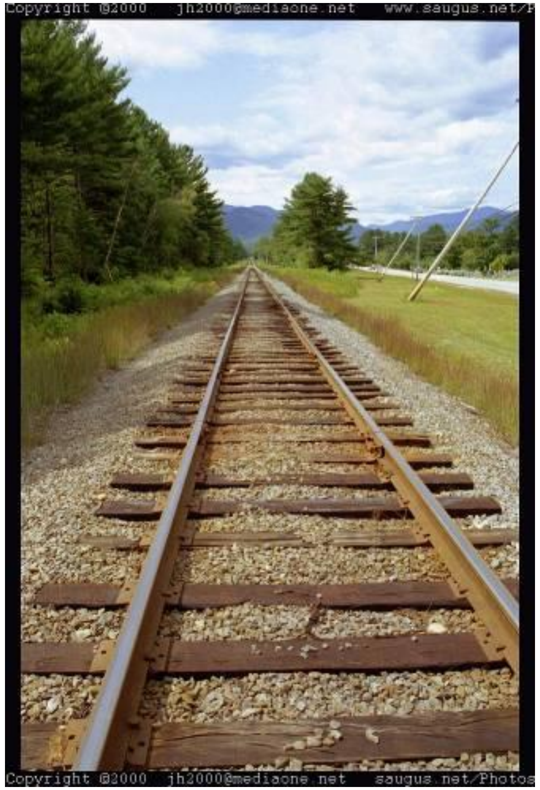


- Pinhole model:
  - Captures pencil of rays – all rays through a single point (pinhole)
  - The point is called centre of projection (focal point)
  - The image is formed on the image plane
  - A virtual image plane is used as mathematical description of the real image plane

Vanishing Points

- Parallel lines are no longer parallel after projection. They converge at a single point on the image plane – vanishing point
- Each direction in space has its own vanishing point
- Exception: directions parallel to the image plane





Building a Real Camera

Home-made Pinhole Camera

- Why so blurry?

  "a larger pinhole to compensate for the smaller amount of light. The result is an image with more blur."

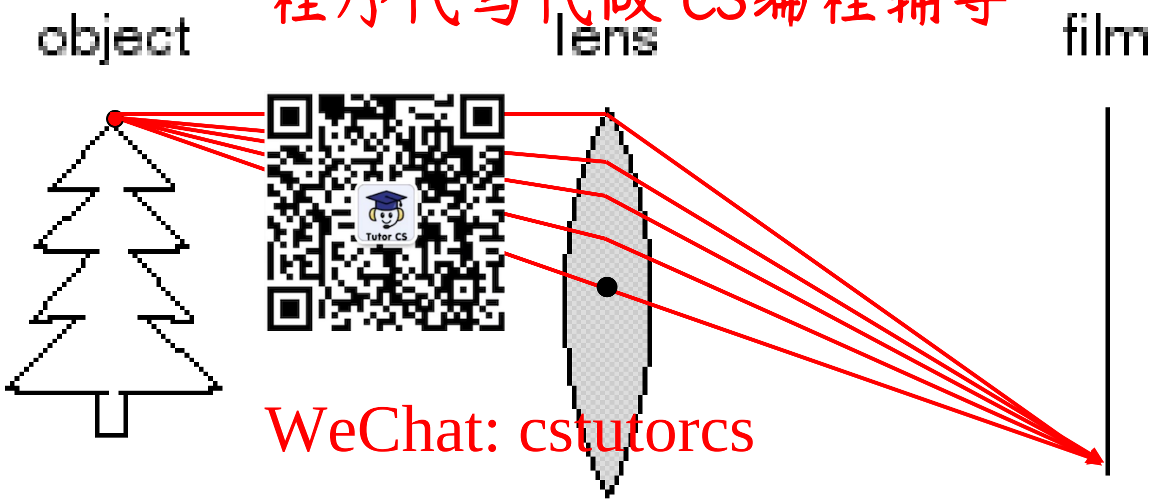
http://www.debevec.org/Pinhole/

Shrinking the Aperture

- Why not make the aperture as small as possible?
  - Less light get through
  - Diffraction effects …

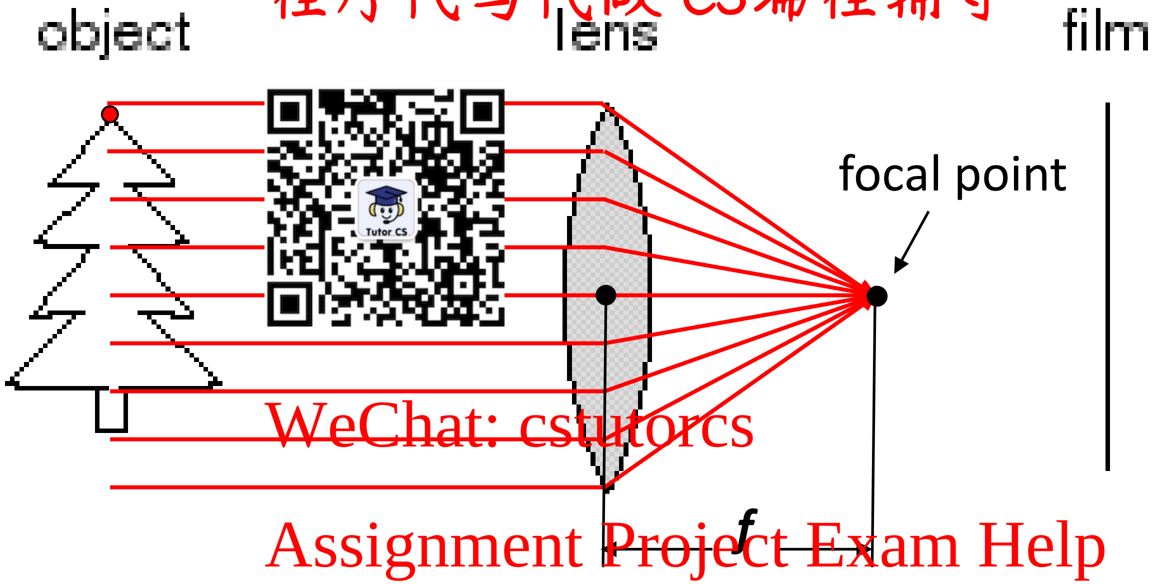
Shrinking the Aperture

Adding a Lens



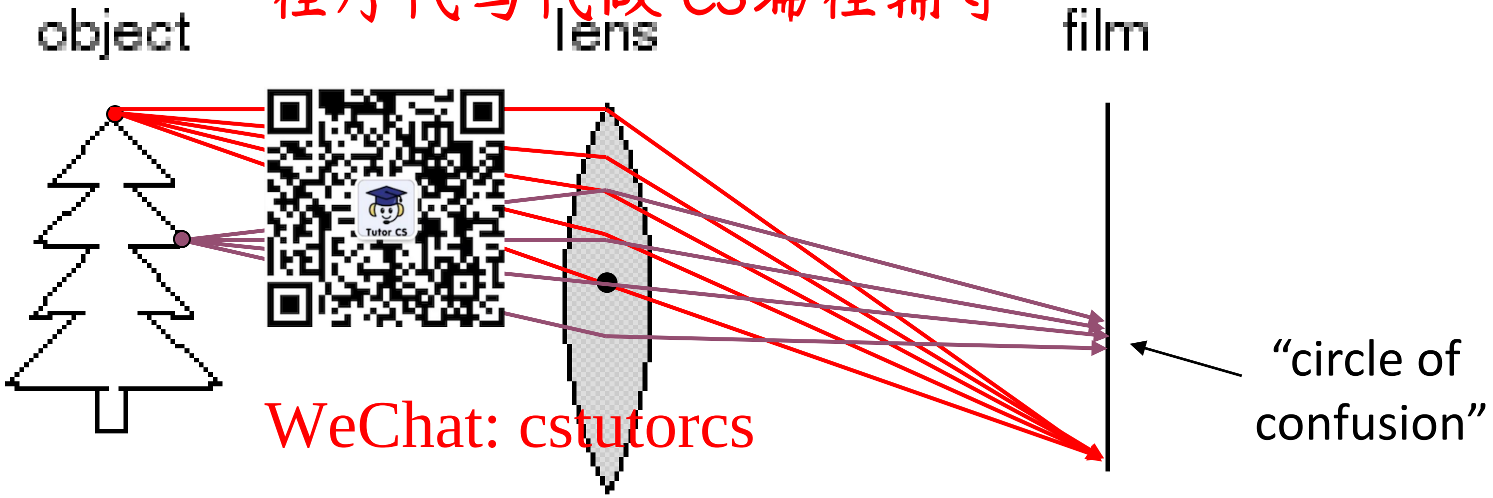
- A lens focuses light onto the film
- Thin lens model:
  - Rays passing through the centre are not deviated (pinhole projection model still holds)

Adding a Lens



- A lens focuses light onto the film
- Thin lens model:
  - Rays passing through the centre are not deviated (pinhole projection model still holds)
  - All parallel rays converge to one point on a plane located at the focal length f

Adding a Lens



- A lens focuses light onto the film
  - There is a specific distance at which an object is "in focus", other points project to a "circle of confusion" in the image

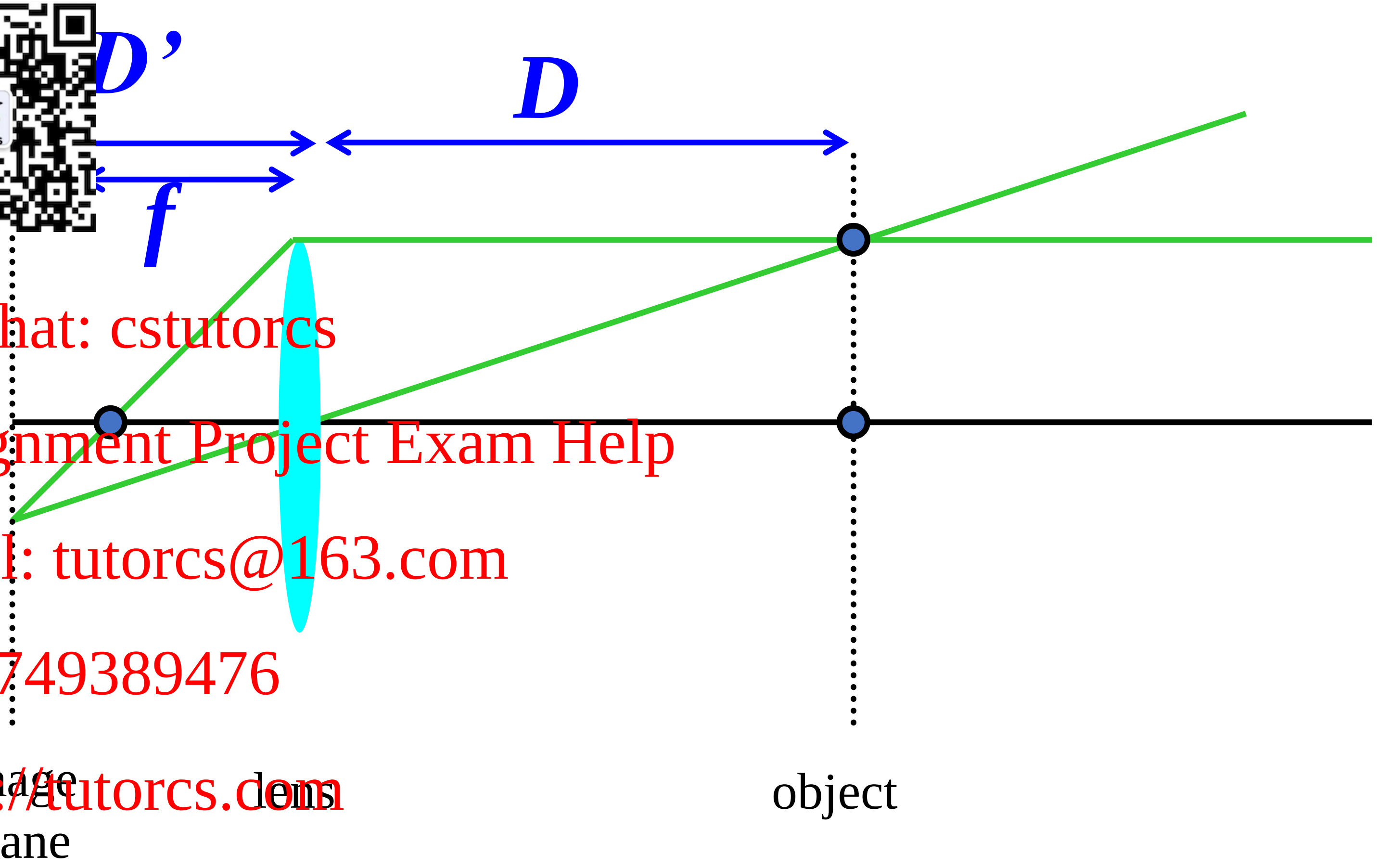
Thin Lens Formula

- What is the relation between:

  The focal length f

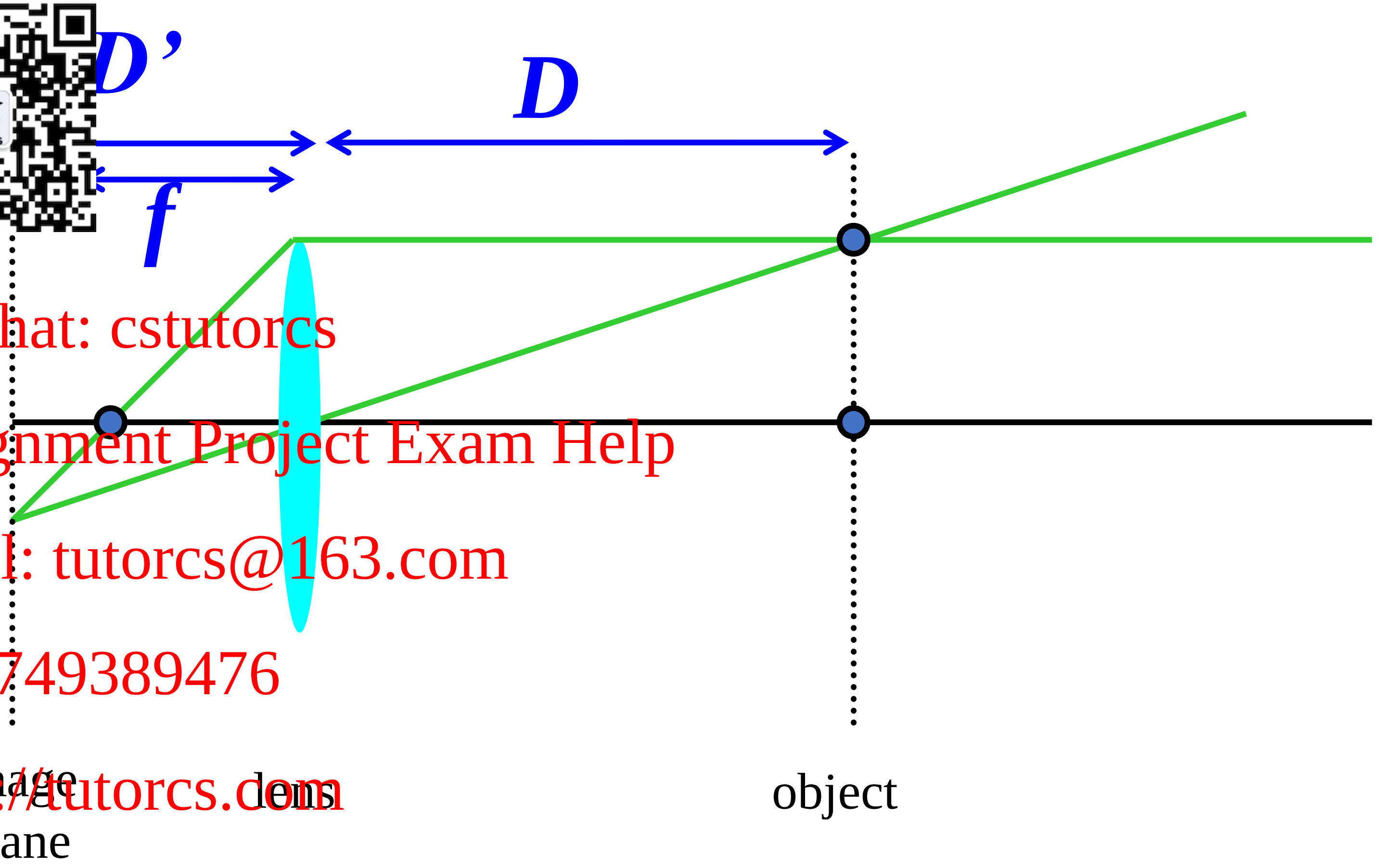
  The distance of the object from the optical centre D

  The distance at which the object will be in focus D'



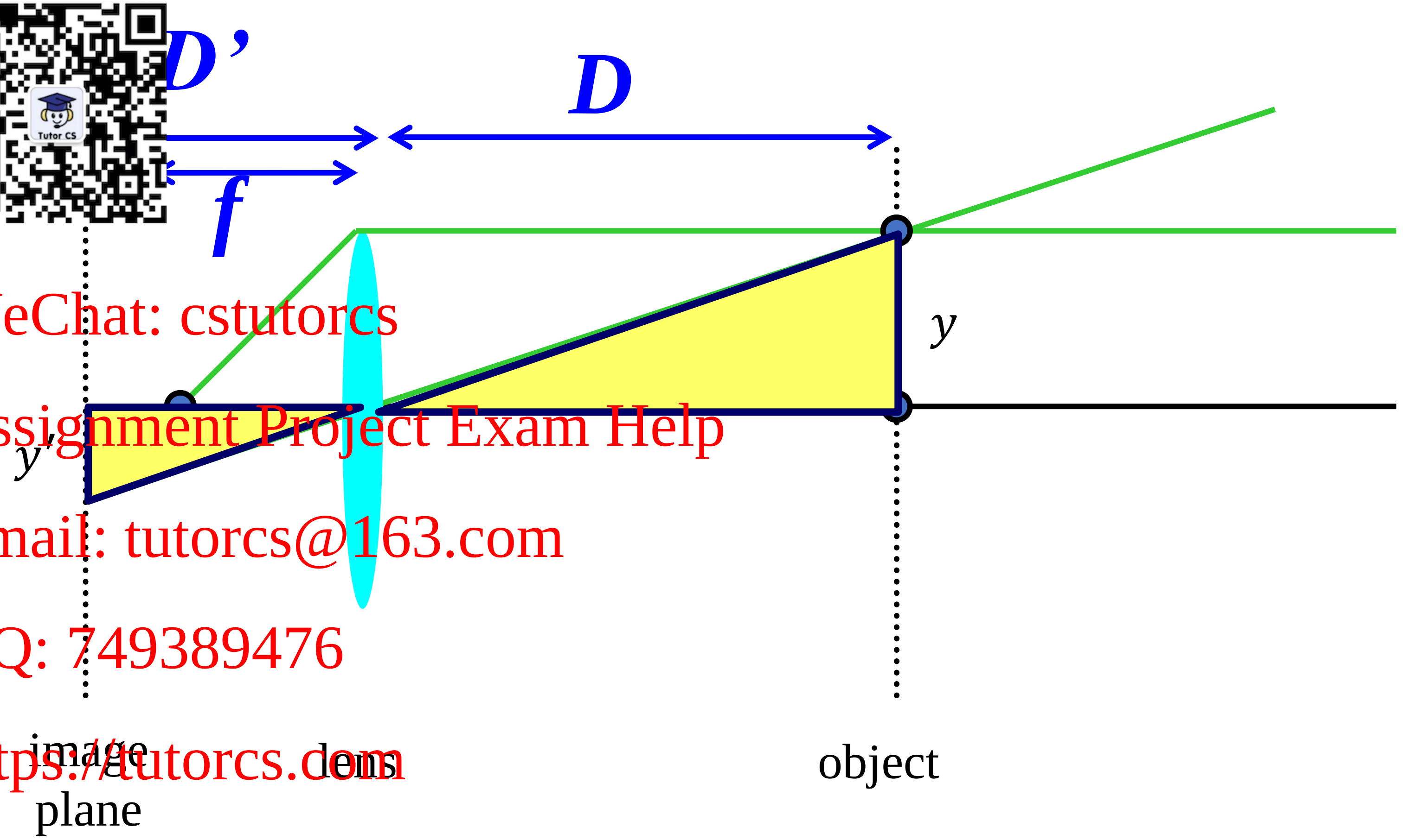
Thin Lens Formula

- Similar triangles everywhere!



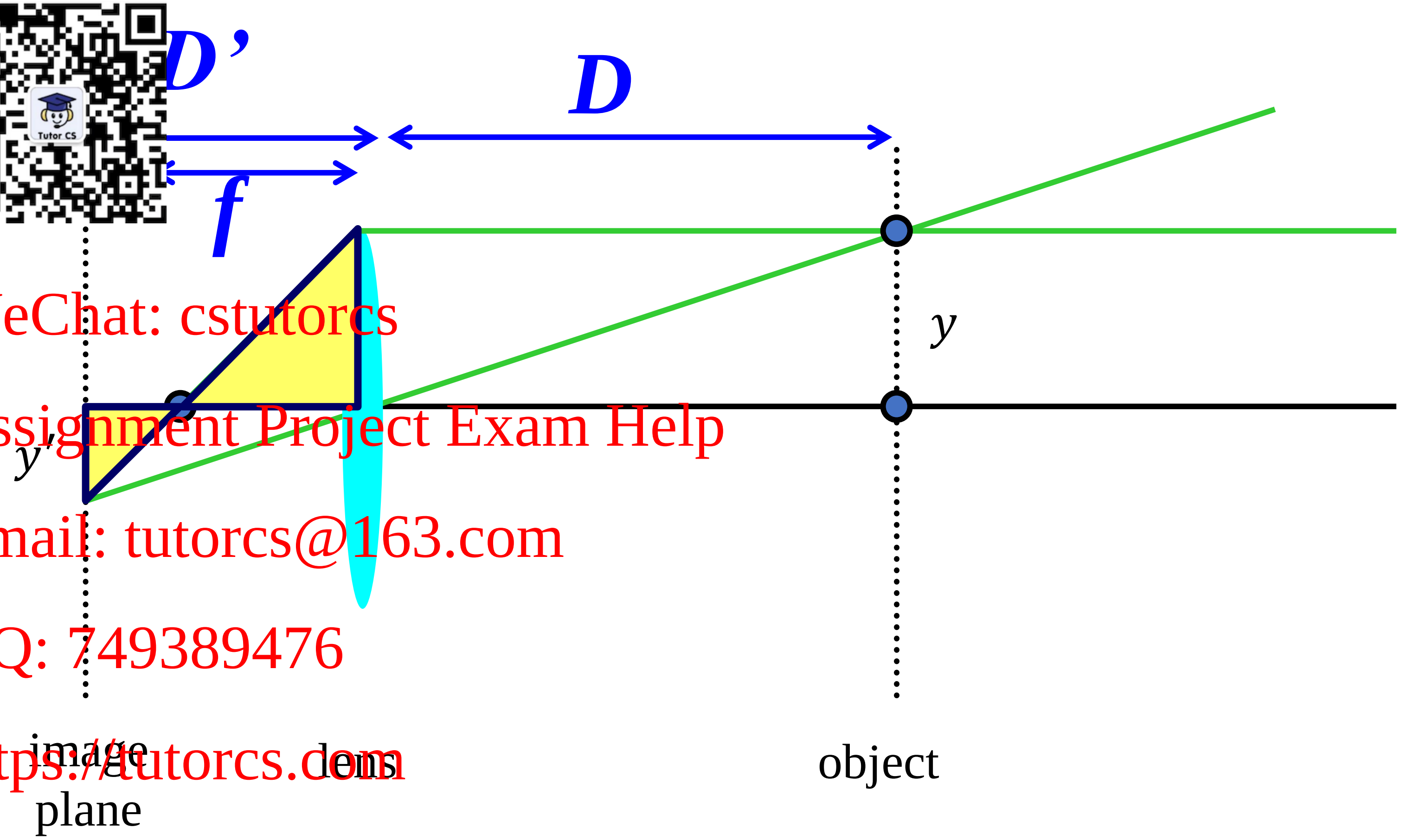
Thin Lens Formula

- Similar triangles everywhere!
- $\frac{y'}{y} = \frac{D'}{D}$



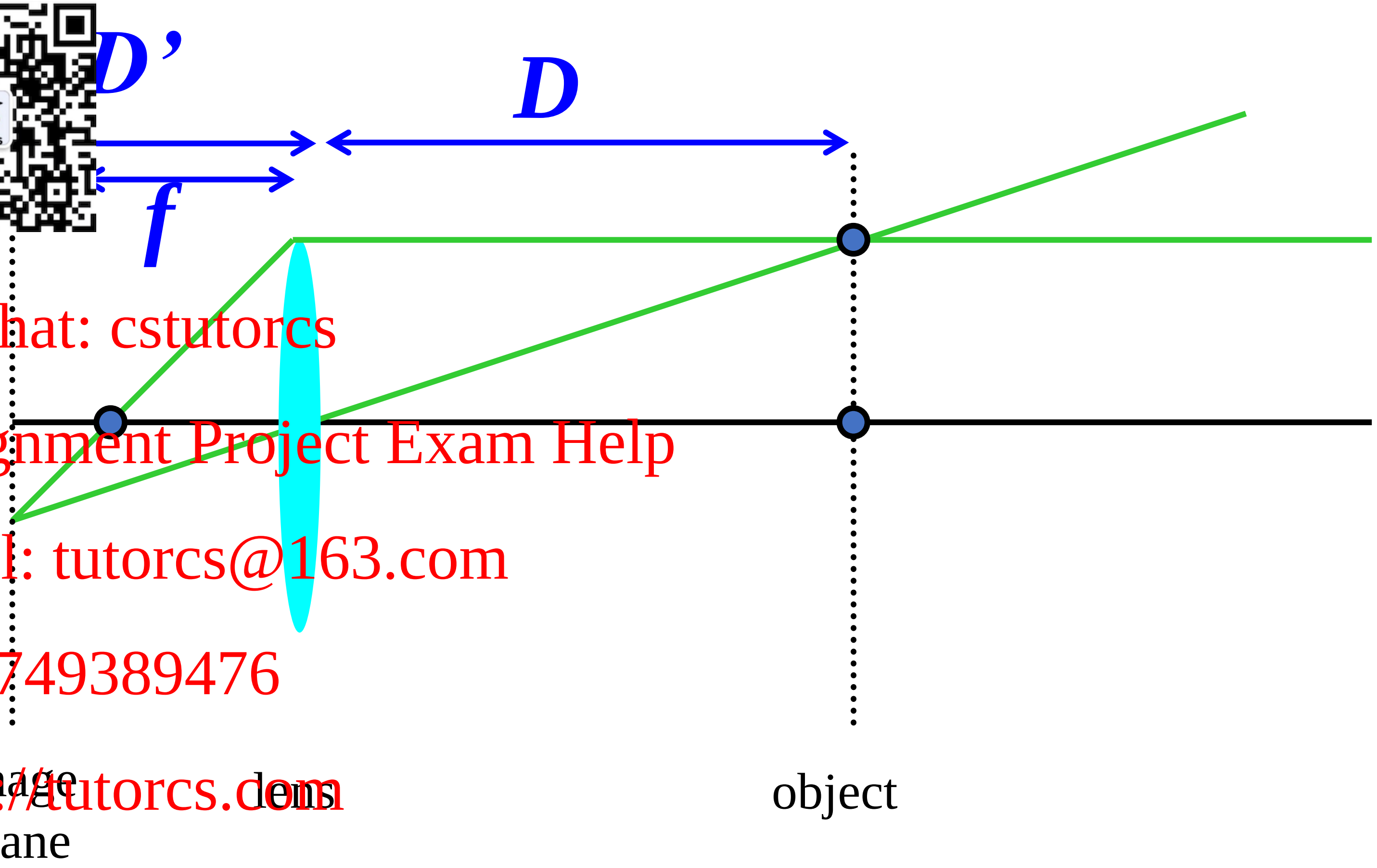
Thin Lens Formula

- Similar triangles everywhere!
- $\frac{y'}{y} = \frac{D'}{D}$
- $\frac{y'}{y} = \frac{D'-f}{f}$

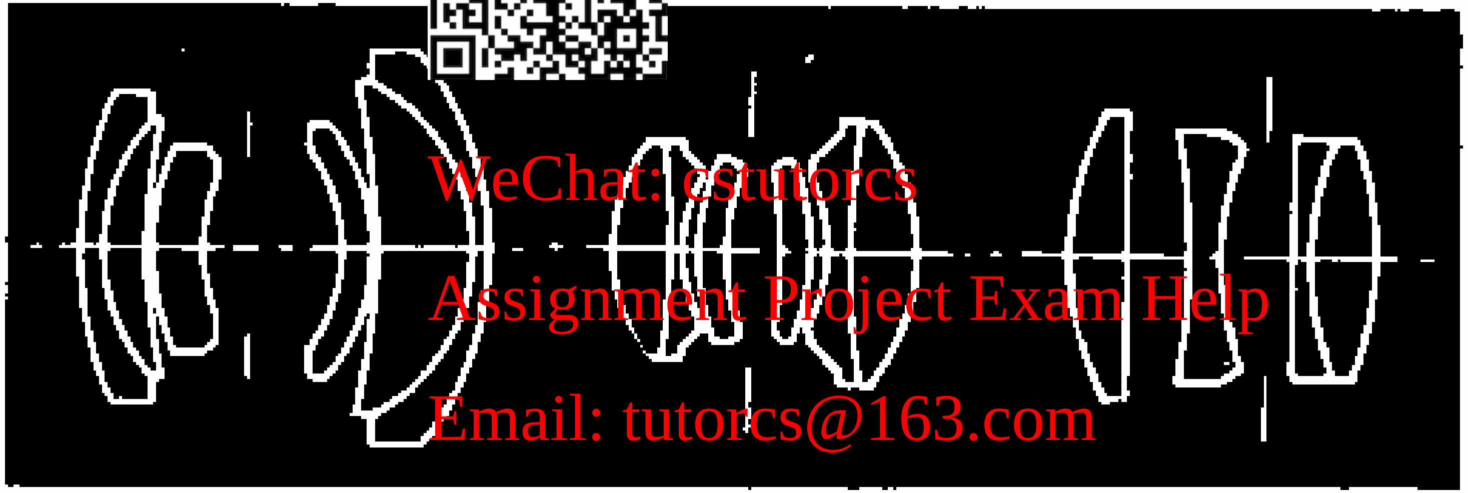


Thin Lens Formula

- $\frac{1}{D} + \frac{1}{D'} = \frac{1}{f}$
- Any point satisfying the thin lens equation is in focus
- As f is fixed, the farther the object, the closer the plane of focus

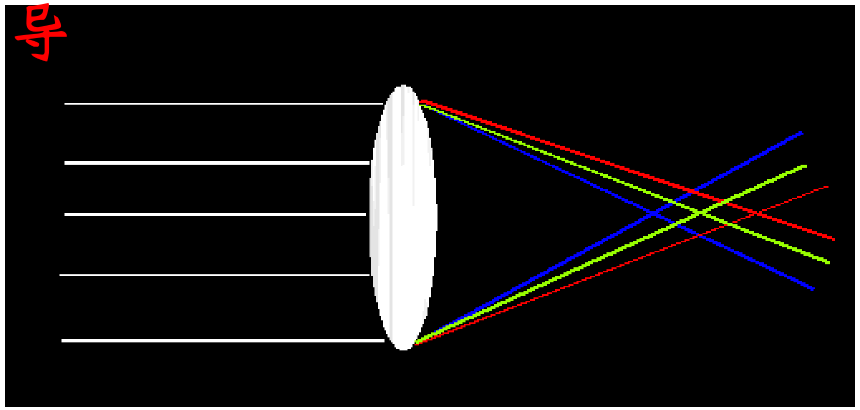


Real Lenses

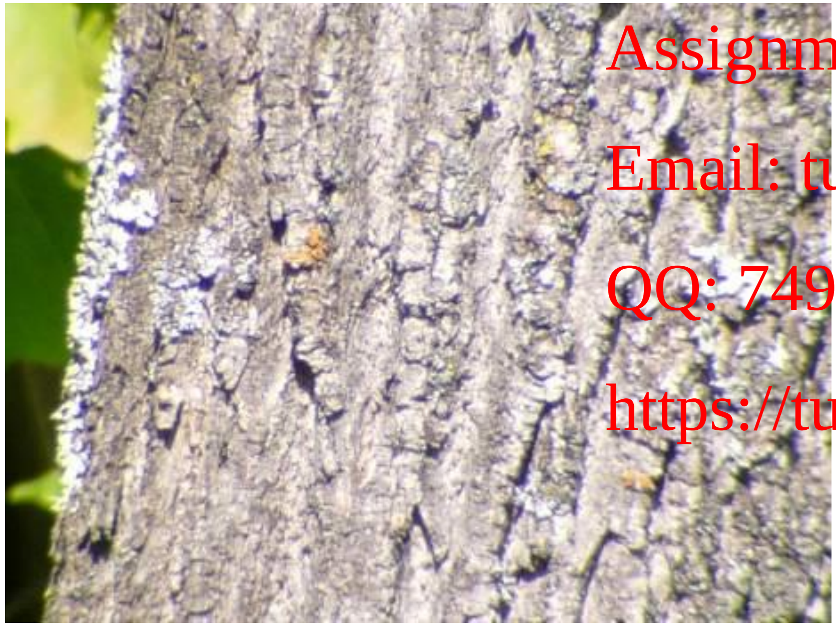


Lens Flaws: Chromatic Aberration

- Lens has different refractive indices for different wavelengths, causes colour fringing



Near Lens centre

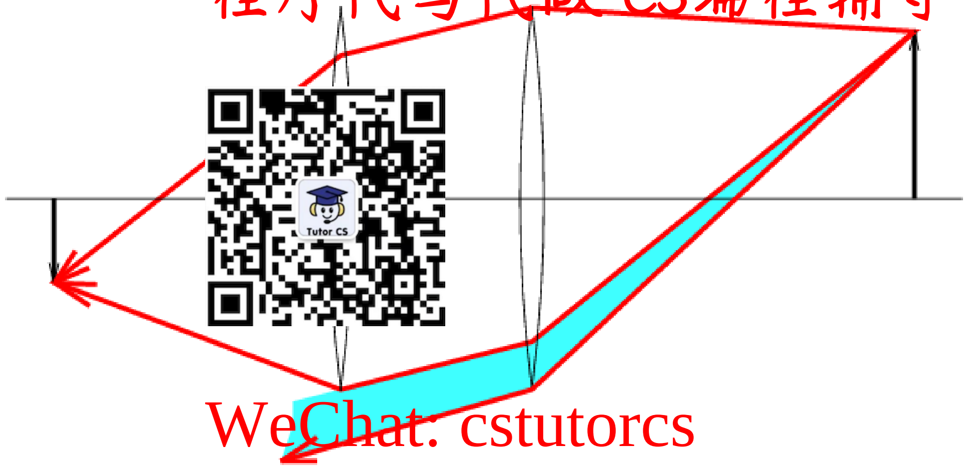


Near Lens Outer Edge

Lens Flaws: Spherical Aberration

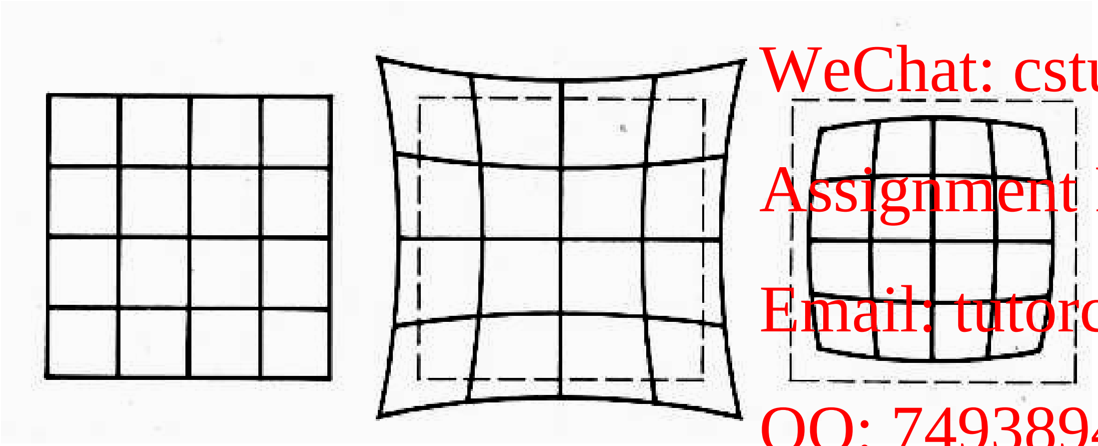
- Spherical lenses do not focus light perfectly
- Rays farther from the optica[illegible]cus closer

Lens Flaws: Vignetting

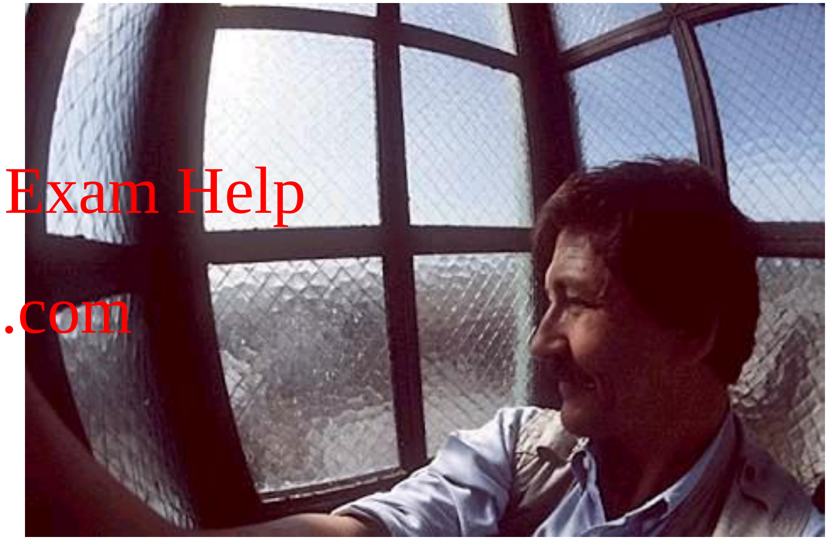


Lens Flaws: Radial Distortion

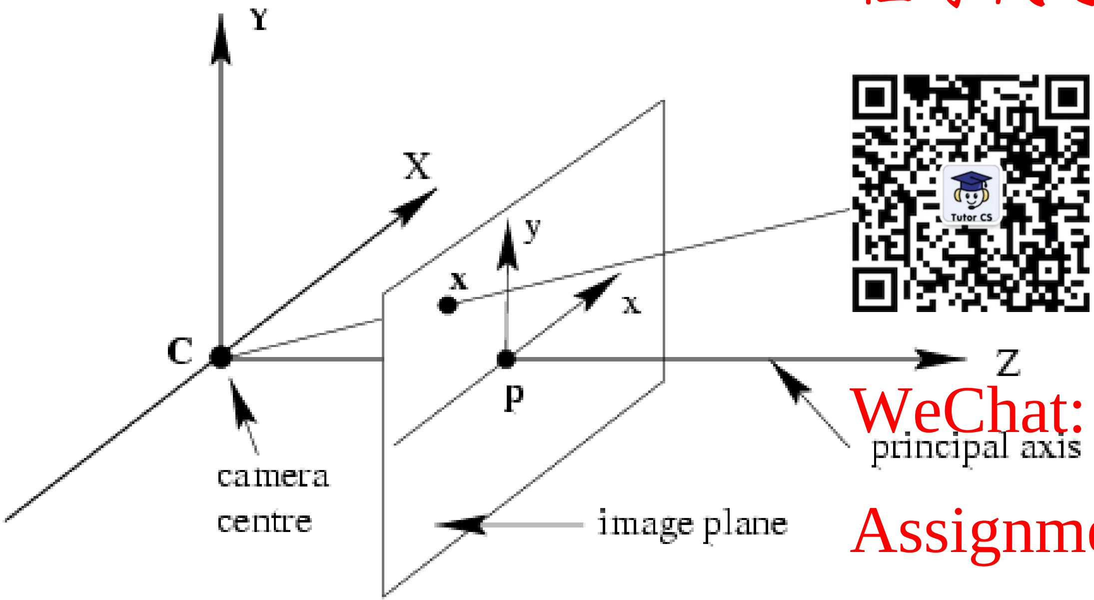
- Caused by imperfect lenses
- Deviations are most noticea[illegible]r the edge of the lens



No distortion   Pin cushion   Barrel

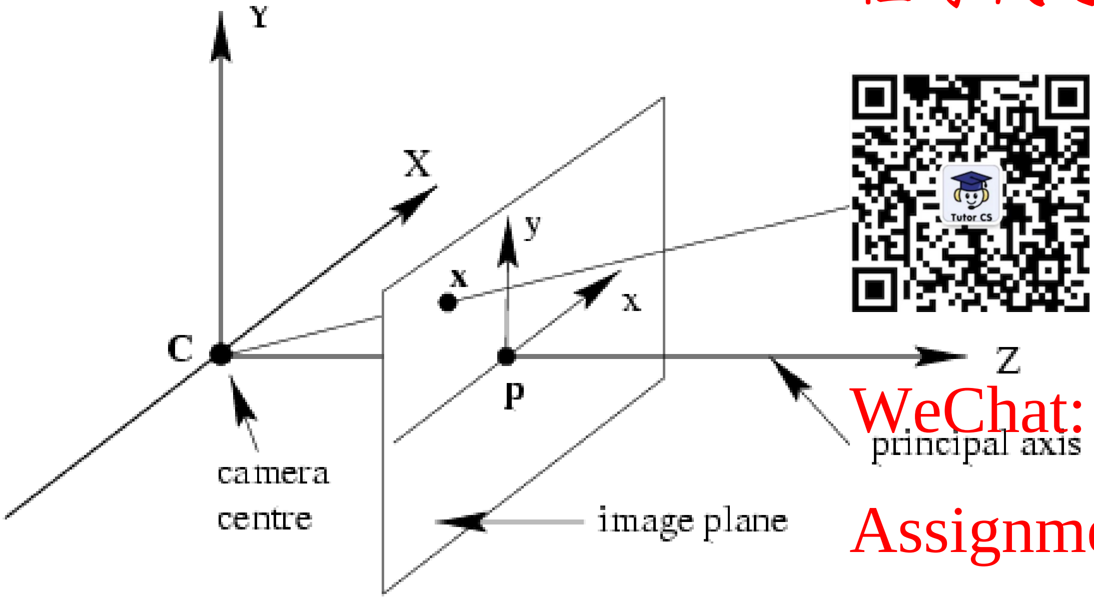


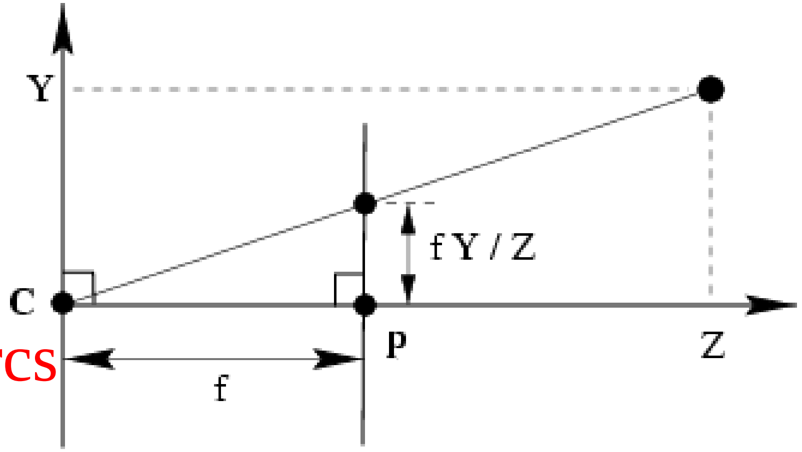
Pinhole Camera Model Revisit



- Principal axis: line from the camera centre perpendicular to the image plane
- Camera coordinate system: camera centre is at the origin and the principal axis is the z-axis

Pinhole Camera Model Revisit



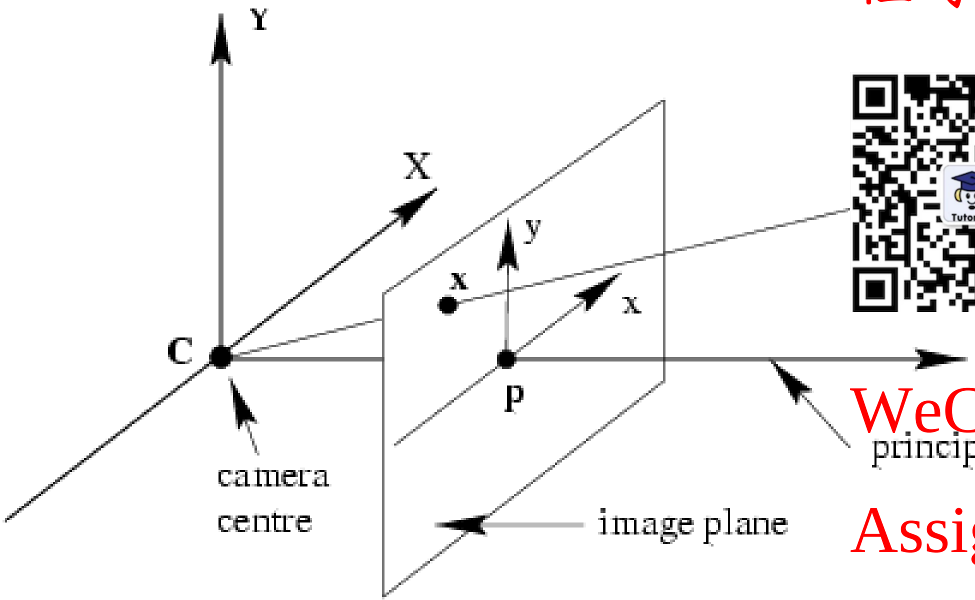


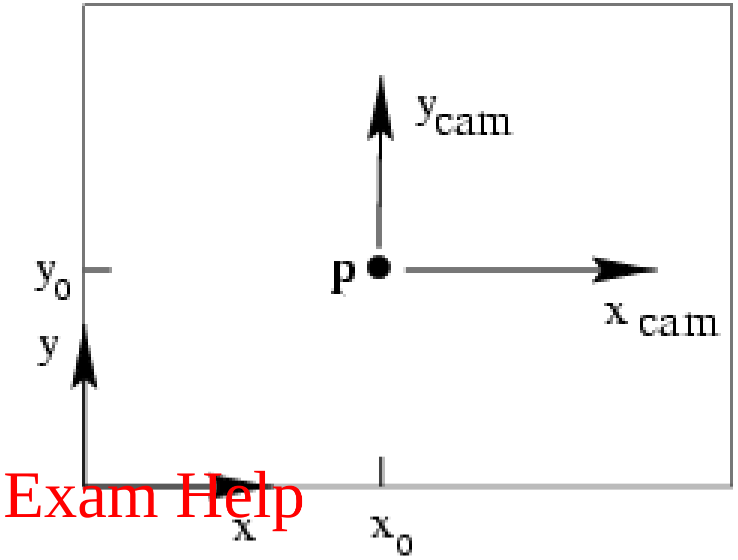
$$(X, Y, Z) \mapsto (f\,X/Z,\ f\,Y/Z)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathrm{x} = \mathrm{P}\mathrm{X}$$

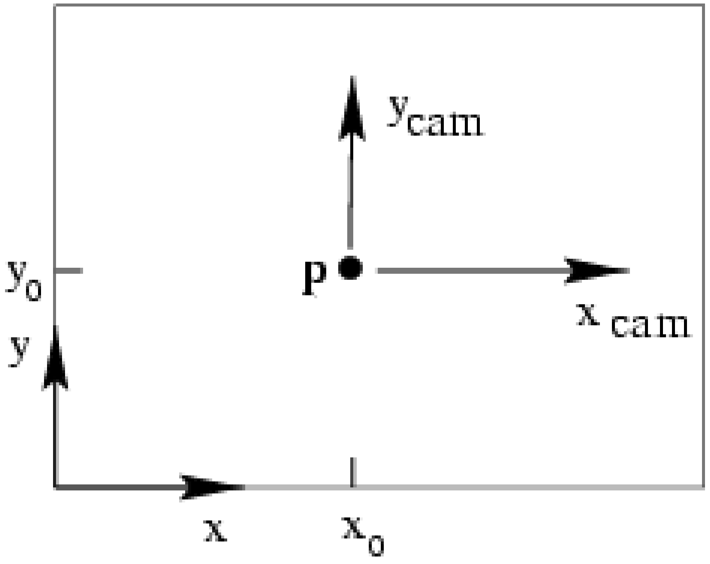
Principal Point





- Principal point (p): point where principal axis intersects the image plane
- Normalised coordinate system: origin is at the principal point
- Image coordinate system: origin is in the corner
- How to go from normalized coordinate system to image coordinate system?

Principal Point Offset



pal point: (p_x, p_y)

$$(X, Y, Z) \mapsto (f\,X/Z + p_x, f\,Y/Z + p_y)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Principal Point Offset

$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

Calibration Matrix $\quad \mathrm{P} = \mathrm{K}[\mathrm{I} \mid 0]$

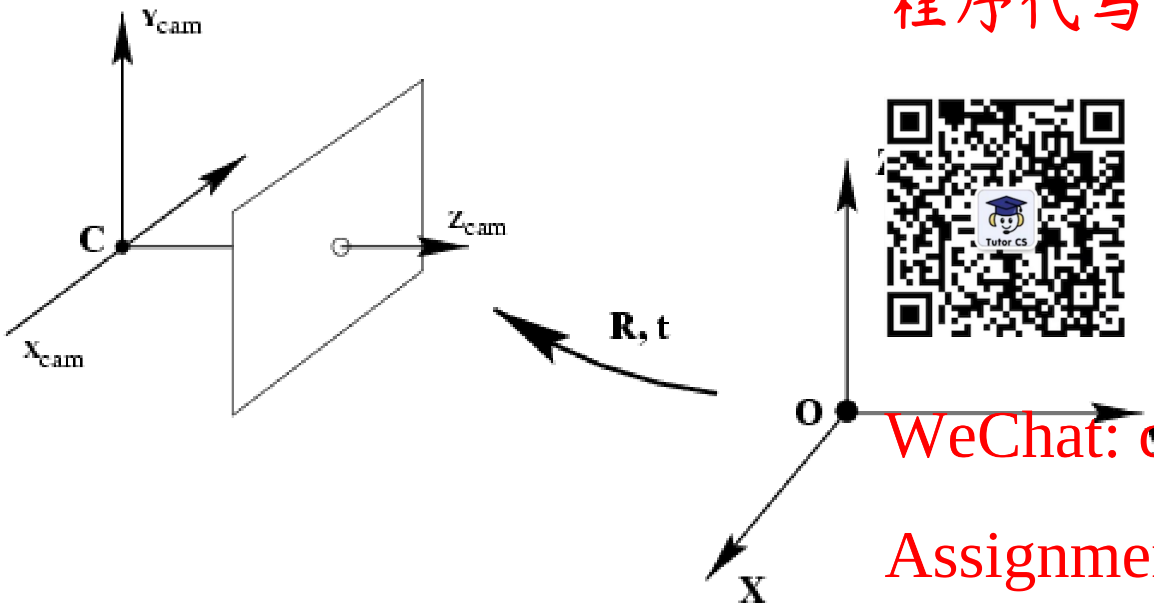
Pixel Coordinates

Pixel size: $\dfrac{1}{m_x} \times \dfrac{1}{m_y}$

- m_x pixels per meter in horizontal direction;
  m_y pixels per meter in vertical direction

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & & \beta_x \\ & \alpha_y & \beta_y \\ & & 1 \end{bmatrix}$$

Camera Rotation and Translation



- In general, the camera coordinate frame will be related to the world coordinate frame by a rotation and a translation

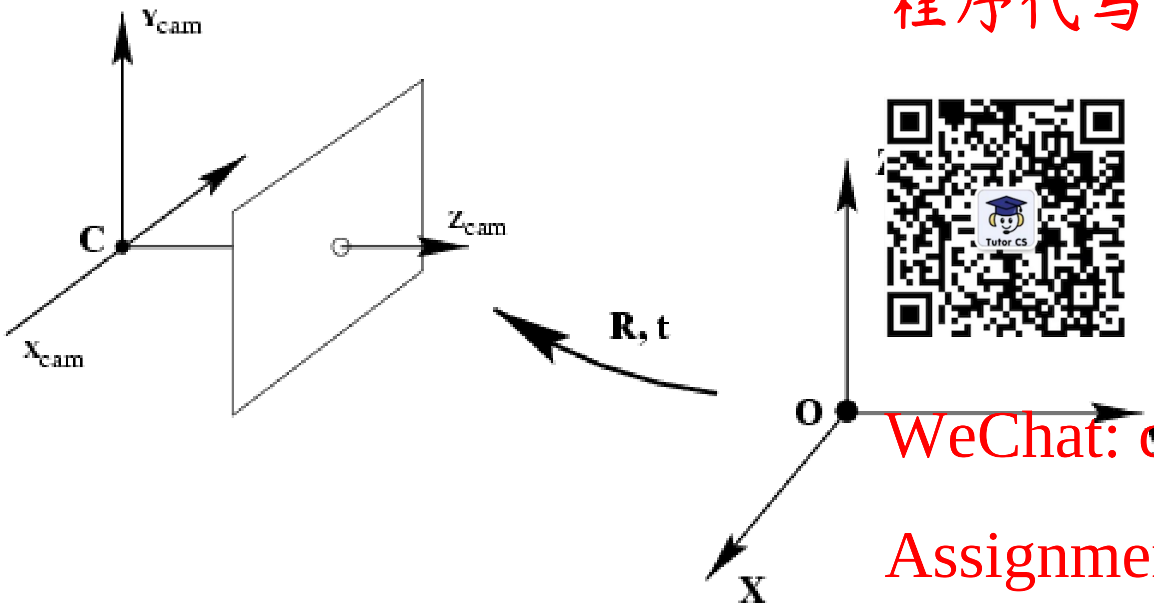
$$X_{cam} = R(\tilde{X} - \tilde{C})$$

coords. of point in camera frame

coords. of a point in world frame (nonhomogeneous)

coords. of camera centre in world frame

Camera Rotation and Translation



- In non-homogenous coordinates

$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

- In homogenous coordinates

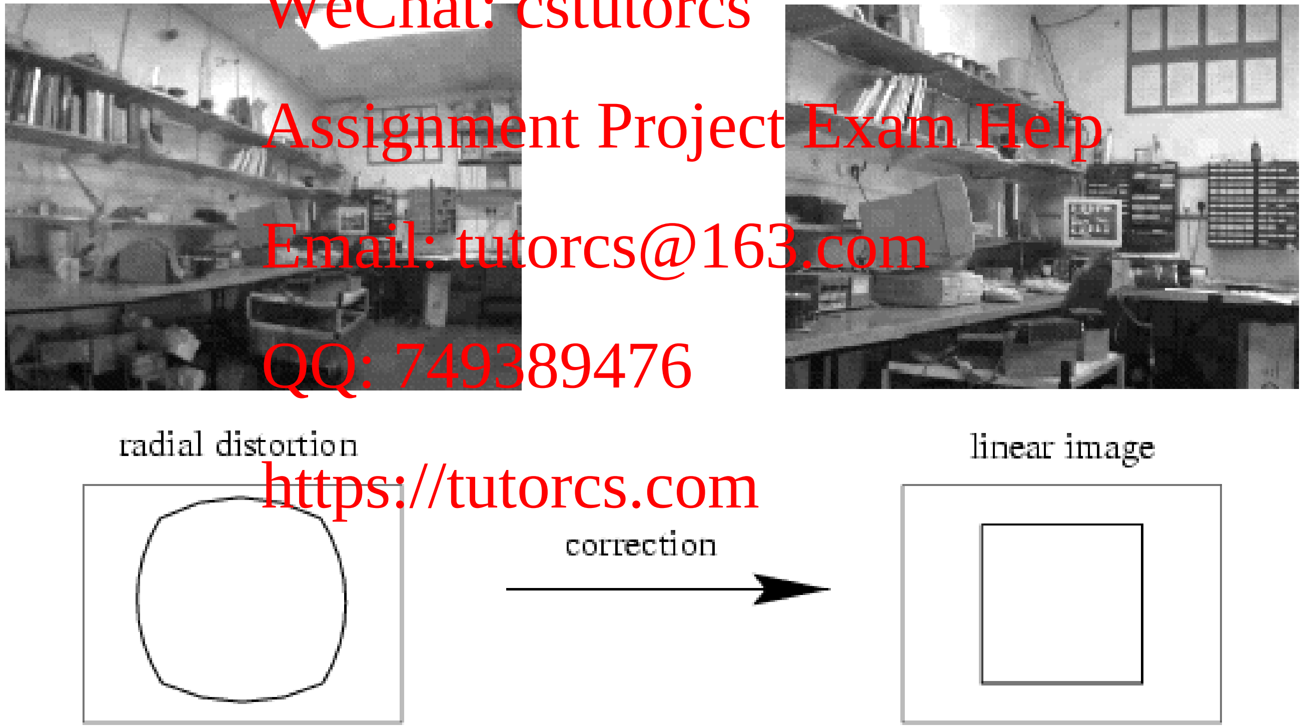
$$X_{cam} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \tilde{X} \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} X$$

$$x = K[I \mid 0]X_{cam} = K\left[R \mid -R\tilde{C}\right]X \qquad P = K[R \mid t], \quad t = -R\tilde{C}$$

Camera Parameters

- Intrinsic parameters
  - Principal point coordinates
  - Focal length
  - Pixel magnification factors
  - Skew (non-rectangular pixels)
  - Radio distortion

$$\begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix}\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & & \beta_x \\ & \alpha_y & \beta_y \\ & & 1 \end{bmatrix}$$



Camera Parameters

- Intrinsic parameters
  - Principal point coordinates
  - Focal length
  - Pixel magnification factors
  - Skew (non-rectangular pixels)
  - Radio distortion

- Extrinsic parameters
  - Rotation and translation relative to world coordinate system

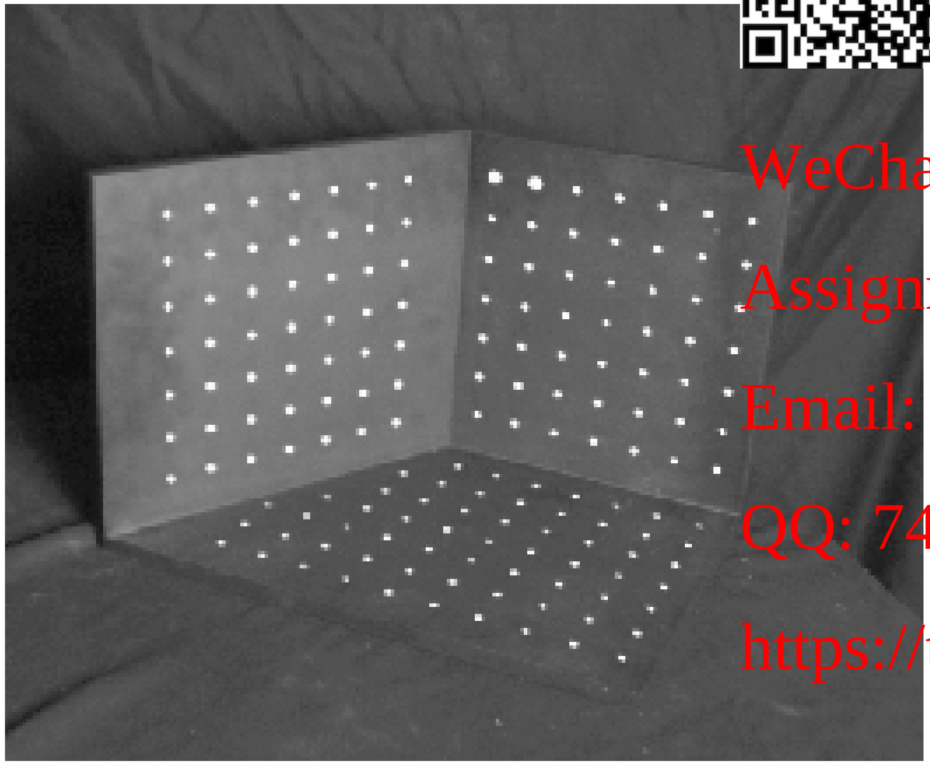
Camera Calibration

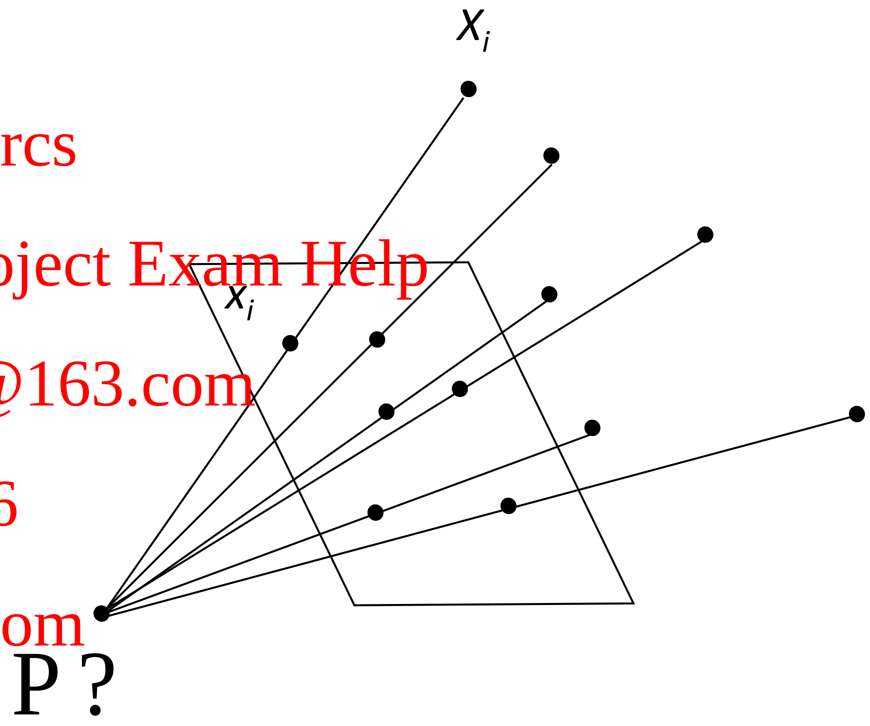
$$\mathbf{x} = \mathbf{P}\mathbf{X} = \mathbf{K}[\mathbf{R}\ \mathbf{T}]\mathbf{X}$$

$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Calibration

- Given n points with known 3D coordinates X_i and known image projections x_i, estimate the camera parameters





Camera Calibration: Linear Method

$$\lambda \mathrm{x_i} = \mathrm{P}\mathrm{X_i} \qquad \mathrm{x_i} \times \mathrm{P}\mathrm{X_i} = 0,$$

$$\mathrm{P}_1^{\mathrm{T}} = [P_{11} \;\; P_{12} \;\; P_{13} \;\; P_{14}]$$
$$\mathrm{P}_2^{\mathrm{T}} = [P_{21} \;\; P_{22} \;\; P_{23} \;\; P_{24}],$$
$$\mathrm{P}_3^{\mathrm{T}} = [P_{31} \;\; P_{32} \;\; P_{33} \;\; P_{34}]$$

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathrm{P}_1^{\mathrm{T}}\mathrm{X_i} \\ \mathrm{P}_2^{\mathrm{T}}\mathrm{X_i} \\ \mathrm{P}_3^{\mathrm{T}}\mathrm{X_i} \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & -\mathrm{X}_i^{T} & y_i\mathrm{X}_i^{T} \\ \mathrm{X}_i^{\mathrm{T}} & 0 & -x_i\mathrm{X}_i^{\mathrm{T}} \\ -y_i\mathrm{X}_i^{\mathrm{T}} & x_i\mathrm{X}_i^{\mathrm{T}} & 0 \end{bmatrix} \begin{bmatrix} \mathrm{P}_1 \\ \mathrm{P}_2 \\ \mathrm{P}_3 \end{bmatrix} = 0$$

Two linearly independent equations

Camera Calibration: Linear Method

$$\begin{bmatrix} 0 & -\mathrm{X}_1^T & y_1\mathrm{X}_1^T \\ \mathrm{X}_1^\mathrm{T} & 0 & -x_1\mathrm{X}_1^\mathrm{T} \\ \cdots & \cdots & \cdots \\ 0 & \mathrm{X}_\mathrm{n}^\mathrm{T} & y_n\mathrm{X}_\mathrm{n}^T \\ \mathrm{X}_\mathrm{n}^\mathrm{T} & 0 & -x_n\mathrm{X}_\mathrm{n}^\mathrm{T} \end{bmatrix} \begin{bmatrix} \mathrm{P}_1 \\ \mathrm{P}_2 \\ \mathrm{P}_3 \end{bmatrix} = 0 \qquad \mathrm{AP} = 0$$

- P has 11 degrees of freedom (12 parameters, but scale is arbitrary
- One 2D/3D correspondence gives two linearly independent equations
- At least 6 correspondences are needed for a solution
- Homogeneous least squares
  - The eigenvector corresponding to the smallest eigenvalue of $\mathrm{A}^\mathrm{T}\mathrm{A}$

Camera Calibration: Linear Method

$$\begin{bmatrix} 0 & -\mathrm{X}_1^T & y_1\mathrm{X}_1^T \\ \mathrm{X}_1^{\mathrm{T}} & 0 & -x_1\mathrm{X}_1^{\mathrm{T}} \\ \cdots & \cdots & \cdots \\ 0 & \mathrm{X}_n^{\mathrm{T}} & y_n\mathrm{X}_n^T \\ \mathrm{X}_n^{\mathrm{T}} & 0 & -x_n\mathrm{X}_n^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathrm{P}_1 \\ \mathrm{P}_2 \\ \mathrm{P}_3 \end{bmatrix} = 0 \qquad \mathrm{AP} = 0$$

- Note: for coplanar points that satisfy $\Pi^T X = 0$, we will get the degenerate solutions: $(\Pi, 0, 0)$, $(0, \Pi, 0)$, or $(0, 0, \Pi)$.

Camera Calibration: Linear Method

- Advantages
  - Easy to formulate and solve
- Disadvantages
  - Doesn’t directly tell you camera parameters
  - Can’t impose constraints, such as known focal length and orthogonality
  - Doesn’t model radial distortion
  - Only an approximate solution
- Non-linear methods are preferred
  - Define error as difference between projected points and measured points
  - Minimise error using Newton’s method or other non-linear optimisation

Summary

- Describe pinhole model.
- What is vanishing point?
- What are intrinsic/extrinsic parameters?
- Describe the linear camera calibration method.
- What are the advantages and disadvantages of linear method for camera calibration?