COMP4121 Advanced Algorithms

Aleks Ignjatović

School of Computer Science and Engineering
University of New South Wales Sydney

Recommender Systems

Recommender Systems

- Main purpose: the noble goal of selling you as much stuff as possible, regardless of whether you need it or not.

- Examples of recommender systems:
  - Netflix's, to recommend to you which movie to see next.
  - Amazon's, to recommend to you which book to buy next.
  - Kogan's to recommend which gizmo to buy next.
  - IEEE's Xplore: to recommend which articles might be of interest to you, given what article you have just look at.
- Two major kinds of recommender systems:
  - **content based**: items are recommended by their intrinsic similarity (i.e., similar properties, qualities, kind etc.)
    For example a book might be recommended because you bought a book on a similar topic)
  - **collaborative filtering**: items are recommended based on some similarity measure between users and between items based on ratings of items by the community of users.

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- Collaborative filtering techniques are superior in performance and does not rely on human advice.
- **A Representative Example:** Assume users are rating movies that they have seen. On the basis of such information we would like to recommend to a user a movie he has not already seen.

- Two main approaches: the **Neighbourhood Method** and the **Latent Factor Method**

- The Neighbourhood Method comes in two flavours:

  (I) based on similarity of users:
  - assume it happens that two users A and B gave “similar” evaluations to movies that they have both seen;
  - if there is a movie which user A liked a lot but user B has not seen,
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- (II) based on similarity of items:
  - assume it happens that two movies M_1 and M_2 received similar ratings by most [illegible]
  - a user has seen [illegible] and liked it;
  - then it is reasonable to recommend movie M_2 to such a user.

- Note that in both approaches movies are not categorised and compared by their "intrinsic" features but we rely only on the "wisdom of the crowd".
- This is an example of *collaborative filtering*.

- We now want to explore how such similarities of users and of items are measured in a most informative way.

- We can construct a sparsely populated table of ratings R; the rows will correspond to movies, the columns to users. The entry $r(j, i)$ of the table, if non empty, represents the rating user U_i gave to movie M_j (in general, *item* M_j).

- Usually, such a rating is the "number of stars", in range $0 - 5$ (or a similar, relatively small rating range, usually with at most 10 or so levels).

- (II) based on similarity of items:
  - assume it happens that two movies M_1 and M_2 received similar ratings by most users;
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- We replace these integers with more informative numbers.
- A more informative number can be obtained by computing the mean $\overline{r}$ of all ratings of all users for [illegible] (thus, the mean of all numbers in our partial table of ratings R)
- We now obtain from t[illegible]w table $\overline{R}$ by replacing all ratings $r(j,i)$ in R by the values $r^*(j,i) =$ [illegible]
- Now numbers $r^*(j,i)$ are already more informative: if $r^*(j,i) > 0$ this means, in a sense, that user U_j has liked movie M_i above the "global average".
- The fact that numbers $r^*(j,i)$ can be both positive and negative with about equal likelihood is important for the subsequent steps to be taken.
- Some users are more generous and tend to give higher scores than the average user; some are more critical and tend to give lower scores.
- We are not interested in evaluating generosity of users, we want to assess only the "taste" of users: w[illegible] and what they like less.
- Similarly, some movies get higher scores because they are popular at the moment for whatever [illegible]s have less "hype" about them because they might be older and less trendy.
- Again, we are not interested in "absolute popularity" or "trendiness" of a movie, rather, we would like to assess how "intrinsically likeable" a movie is.

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- Now numbers $r^*(j,i)$ are already more informative: if $r^*(j,i) > 0$ this means, in a sense, that user U_j has liked movie M_i above the "global average".
- The fact that numbers $r^*(j,i)$ can be both positive and negative with about equal likelihood is important for the subsequent steps to be taken.
- Some users are more generous and tend to give higher scores than the average user; some are more critical and tend to give lower scores.
- We are not interested in evaluating generosity of users, we want to assess only the "taste" of users: what they like more and what they like less.
- Similarly, some movies get higher scores because they are popular at the moment for whatever reason and other movies have less "hype" about them because they might be older and less trendy.
- Again, we are not interested in "absolute popularity" or "trendiness" of a movie, rather, we would like to assess how "intrinsically likeable" a movie is.

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- For that reason we want to remove the "systematic biases" of both the users and the movies, thus taking out the individual "generosity" of each user and the "hype" of each movie.
- For that purpose we introduce for every user U_i a variable υ_i standing for the "individual bias" of user U_i, reflecting his tendency to give overall higher or lower scores.
- We also introduce for each movie M_j a variable μ_j standing for the "hype bias" of movie M_j which is due to how "fashionable" the movie is (which anyhow usually quickly fades with time).
- We now remove both such systematic biases by seeking the values of variables υ_i and variables μ_j which minimises the expression

$$S(\vec{\upsilon},\vec{\mu}) = \sum_{(j,i)\in R} \left(r^*(j,i) - \upsilon_i - \mu_j\right)^2$$

- Note that μ 's are constant shifts of rows (each row corresponding to a movie) and υ's are constant shifts of columns (each corresponding to a user)
- We chose such constant shifts of each row and of each column which minimise the residuals.
- Each such residual $\widetilde{r}(j,i) = r^*(j,i) - \upsilon_i - \mu_j$ better represents the "intrinsic" sentiment of a user U_i for a movie M_j than $\widetilde{r}(j,i)$.

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- This is a *Least Squares* problem and is easily reducible to a system of linear equations:
- $S(\vec{v}, \vec{\mu})$ achieves a min [illegible] $\vec{\mu}$ for which all the partial derivatives $\frac{\partial}{\partial \mu_j} S(\vec{v}, \vec{\mu})$ for all j an [illegible]) for all i are equal to 0:

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$$= -2 \sum_{i:(j,i)\in R} (r^*(j,i) - v_i - \mu_j) = 0$$

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- Unfortunately, Least Squares fits usually suffer from overfitting: they often minimise the objective function by choosing excessively large values for the variables, relying on m[illegible]elations of positive and negative terms.
- The solution to this pr[illegible]lled *regularisation*: we introduce a term which penalises for lar[illegible] the variables.
- Thus, instead, we mini[illegible]m

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where λ is a suitably chosen small positive constant, usually $10^{-10} \leq \lambda \leq 10^{-2}$.

- Optimal value of λ can be "learned" in a way to be described later.
- We now obtain from table $\overline{R}$ a new table $\widetilde{R}$ by replacing all $r^*(j,i)$ in $\overline{R}$ with values $\widetilde{r}(j,i) = r^*(j,i) - v_i - \mu_j$ where $v's$ and $\mu's$ were obtained by our regularised least squares fit.
- Having removed the systematic biases of users and trendiness of movies, we are now ready to estimate similarities of users and similarities of movies.

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- Thus, instead, we minimise the following sum

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Neighbourhood Method - similarity of users

- One of the most frequently used measure of similarity of users is the *cosine similarity measure.*
- Let us first compare two users U_i and U_k. We find all movies that both users have ranked and delete all other entries $\widetilde{r}(j,i)$ and $\widetilde{r}(j',k)$ in the corresponding columns of these two users in the partial table $\widetilde{R}$ (thus, we remove ratings of movies which only one of the two users have seen and all the blank spaces).
- In this way we obtain two column vectors $\vec{u}_i$ and $\vec{u}_k$ such that the coordinates of vector $\vec{u}_i$ are the rankings of user U_i and the coordinates of vector $\vec{u}_k$ are rankings of user U_k of all the movies seen by both users.
- The similarity of the two users is measured by the cosine of the angle between these two vectors.
- Intuitively, these two users have similar tastes if the two vectors point in "similar directions".
- Recall that

$$\cos(u_i, u_k) = \frac{\langle \vec{u}_i, \vec{u}_k \rangle}{\|\vec{u}_i\| \cdot \|\vec{u}_k\|}$$

where $\langle \vec{u}_i, \vec{u}_k \rangle = \sum_p (\vec{u}_i)_p (\vec{u}_k)_p$ is the scalar product of vectors $\vec{u}_i$ and $\vec{u_k}$ and $\|\vec{u}_k\| = \sqrt{\sum_p (u_k)_p^2}$ is the norm (the "length") of vector $\vec{u}_k$.

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- Intuitively, these two users have similar tastes if the two vectors point in "similar directions".
- Recall that

$$\cos(u_i, u_k) = \frac{\langle \vec{u}_i, \vec{u}_k \rangle}{\|\vec{u}_i\| \cdot \|\vec{u}_k\|}$$

where $\langle \vec{u}_i, \vec{u}_k \rangle = \sum_p (\vec{u}_i)_p (\vec{u}_k)_p$ is the scalar product of vectors $\vec{u}_i$ and $\vec{u}_k$ and $\|\vec{u}_k\| = \sqrt{\sum_p (u_k)_p^2}$ is the norm (the "length") of vector $\vec{u}_k$.

- One of the most frequently used measure of similarity of users is the *cosine similarity measure.*
- Let us first compare tw[illegible] and U_k. We find all movies that both users have ranked and delete [illegible] entries $\widetilde{r}(j,i)$ and $\widetilde{r}(j',k)$ in the corresponding columns of these two u[illegible] partial table $\widetilde{R}$ (thus, we remove ratings of movies which only one [illegible] users have seen and all the blank spaces).
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- This is not a good feature; $\text{sim}(U_i, U_k)$ should depend only on the "intrinsic similarity" of tastes of the two users and thus it should not depend on irrelevant things such as the number of movies they have both seen.

- Dividing the scalar product of the two vectors by the product of their norms results in a quantity depending only the angle between the two vectors, which more properly reflects similarity of the tastes of the two users.

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$$\mathrm{pred}(j, i) = \overline{r} + v_i + \mu_j + \frac{\sum_{1 \le l \le L} \mathrm{sim}(U_i, U_{k_l})\, \widetilde{r}(j, k_l)}{\sum_{1 \le l \le L} |\mathrm{sim}(U_i, U_{k_l})|}$$

- We then recommend to user U_i movie M_j for which the predicted rating $\mathrm{pred}(j, i)$ is the highest.
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- We can in a similar way estimate similarity of movies, working on columns of table $\widetilde{R}$ (instead of rows).

- For any two movies M_j [illegible] M_n consider all users which have rated both movies and form two v[illegible] and $\vec{m}_n$ with coordinates which are the ratings of the form $\widetilde{r}(j$ [illegible] $l)$ where l ranges over all users who rated both movies.
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- A very different commonly used method is the **Latent Factor Method**.
- Heuristics behind the method:
  - One can argue that there is only a relatively small number (up to a few hundreds) of features a movie might posses to various extents which appeal to users' tastes and which determine how much a particular user would like such a movie.
  - Examples of such features are "action movie", "romantic movie", "famous actors", "special effects", "violence", "humour", etc.
  - Let us enumerate all of these features as $f_1, f_2, \ldots, f_N$ where N is of the order of a few tens to a few hundreds.
  - A movie can have each of these features say f_i to an extent e_i, where e_i is, say, between 0 and 10.
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Latent Factor Method

- A very different commonly used method is the **Latent Factor Method**.
- Heuristics behind the method:
  - One can argue t[illegible] is only a relatively small number (up to a few hundreds) of [illegible] a movie might posses to various extents which appeal to [illegible] tastes and which determine how much a particular user would like such a movie.
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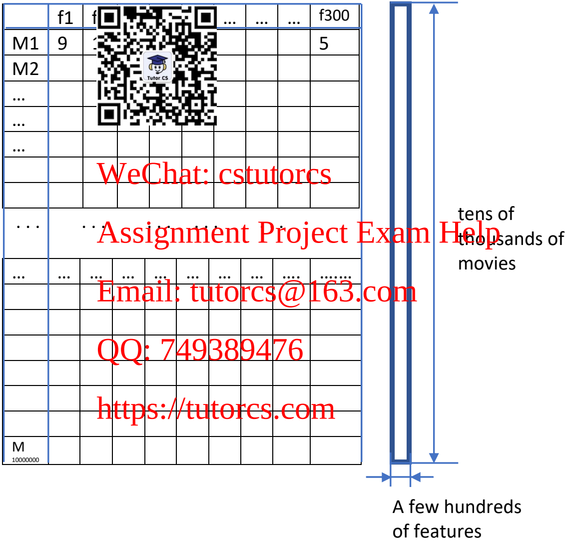
Latent Factor Method

- Thus, if feature f_1 is “action movie” and if $F(1, 1) = 9$ this would mean that the first movie on our list has a very significant action component.



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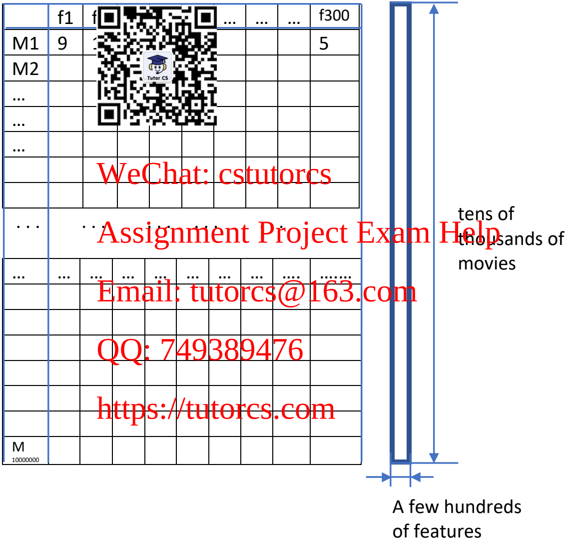

Latent Factor Method

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- On the other hand, if feature f_2 is "romantic" and the value of $(\vec{l^1})_2$ is 1, this would mean that user U_1 does not like very much movies with lots of romance.
- We can now form a matrix L whose rows correspond to features and columns correspond to users.
- If feature f_m is "special effects" and entry $L(m,i)$ in m^{th} row and i^{th} column is, say, 5, this would mean that user U_i is ambivalent towards feature f_m: he neither likes nor dislikes movies which have lots of special effects.

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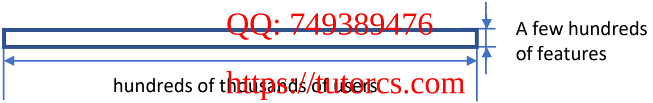
- If feature f_1 is "action movie" and feature f_2 is "romantic movie" and if $L(1, 1) = 9$ and $L(2, 1) = 1$, this would mean that the first user on our list likes movies with lots of action but does not like movies with lots of romance.

.	U1	U2	…	…	…					…	…	…	…	…	…	…	…	…	….	U 10000000
f1	9									…										
f2	1									…										
f3	7									…										
…	…									…										
fm	5									…										
…	…									…										
f300										…										



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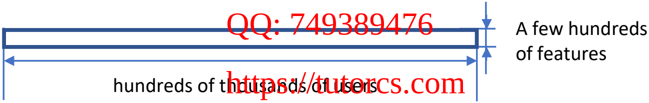

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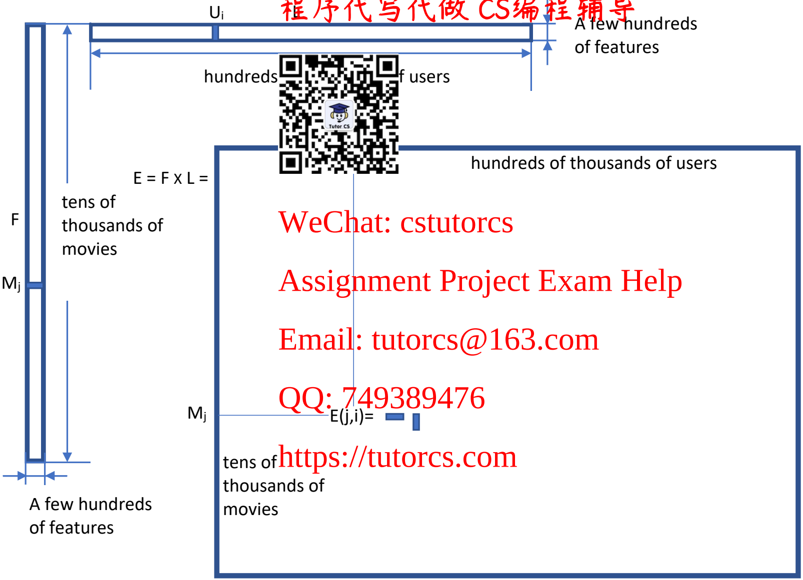

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- **Solution**: all of these should be "learned" from the partial table of the existing ratings of movies!
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- However, there is a ve[illegible]roblem with such an approach to prediction how much would a use[illegible] movie M_j.
- How can we determine [illegible]he relevant few dozens to few hundreds of features needed to describe a movie exhaustively?
- Who would assess each movie objectively according to how much of each feature such amovie has?
- Even worse, how would we determine objectively how much each feature is important to each user?
- **Solution**: all of these should be "learned" from the partial table of the existing ratings of movies!
- We even do not need to know what the features are or what they mean.
- These features should also "emerge" from the partial table of user ratings R!

- Let N be the number of "features" we want to let emerge (with no meaning assigned whatsoever). In applications N ranges between 20 and up to 200.
- Let $\#M$ be the number of movies in the database and $\#U$ be the number of users.
- **Idea:** Fill matrices F of size $\#M \times N$ and L of size $N \times \#U$ with variables $F(j,m)$ and $L(m,i)$ which yet have to be determined.
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minimize

$$S(\vec{F}, \vec{L}) = \sum_{(j,i):R(j,i)\text{exists}} \left(\sum_{1 \leq m \leq N} F(j,m) \cdot L(m,i) - R(j,i) \right)^2$$

- Note that the total number of variables is $(\#M + \#U) \times N$.
- So N should be chosen so that $(\#M + \#U) \times N$ is a fraction of the total number of existing entries in the partially filled table R of user's ratings.
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- However, there is a serious problem with this approach: setting the partial derivatives of the objective $S(\vec{F},\vec{L})$ with respect to all variables to zero results in the following system of equations:

$$\begin{aligned}
\frac{\partial}{\partial F(j,m)} S(\vec{F},\vec{L}) &= \frac{\partial}{\partial F(j,m)} \sum_{(j,i):R(j,i)\ \text{exists}} \left(\sum_{1\le m\le N} F(j,m)\cdot L(m,i) - R(j,i) \right)^2 \\
&= 2 \sum_{i:R(j,i)\ \text{exists}} \left(\sum_{1\le m\le N} F(j,m)\cdot L(m,i) - R(j,i) \right) L(m,i) = 0;
\end{aligned}$$

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- This is a huge system of cubic equations and cannot be solved feasibly.
- Worse, such an optimis[illegible] problem is even not convex, so search for the optimal solution can end up in a local minimum.
- We apply an iterative [illegible] find an approximate solution.
- Note that we apply su[illegible] to "raw data" - no de-biasing like the one we performed in the N[illegible]od Method.
- Steps:
  - We initially set all variables $F(j,m)$ to the same value $F^{(0)}(j,m)$, say a median va[illegible]
  - we now solve the following Least Squares problem in variables $\{L(m,i) : 1 \leq m$ [illegible] only

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  - Note that, since $F^{(0)}(j,m)$ are concrete numbers rather than variables, such a Least Squares problem does reduce to a system of linear equations after we find the partials and set them to zero.
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  - Let $F^{(1)}(j,m)$ be the solutions to such a Least Squares problem; we now use these values to solve the following Least Squares problem in variables $\{L(m,i) : 1 \leq m \leq N; 1 \leq i \leq \#U\}$ only:

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- Steps (continued):
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  - We now solve the following Least Squares problem in variables $\{F(j,m) : 1 \leq$ [illegible] $i \leq \#U\}$ only:

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- Steps (continued):

  - *minimize*

$$\sum_{(j,i):R(j,i)} \left(\sum_{m\le N} F^{(1)}(j,m)\cdot L(m,i) - R(j,i)\right)^2$$

  - We keep alternating between taking either $\{F(j,m) : 1 \leq m \leq N; 1 \leq j \leq \#U\}$ or $\{L(m,i) : 1 \leq m \leq N; 1 \leq i \leq \#U\}$ as free variables, fixing the values of the other ones from the previously obtained solution to the corresponding Least Squares problem.
  - This method is sometimes called "*Method of Alternating Projections*".
  - We stop such iterations when

$$\sum_{(j,m)} (F^{(k)}(j,m) - F^{(k-1)}(j,m))^2 + \sum_{(i,m)} (L^{(k)}(m,i) - L^{(k-1)}(m,i))^2$$

becomes smaller than an accuracy threshold $\varepsilon > 0$.

- Steps (continued):

  - *minimize*

$$\sum_{(j,i):R(j,i)} \left(\sum_{1\le m\le N} F^{(1)}(j,m) \cdot L(m,i) - R(j,i) \right)^2$$

  - We keep alternating between taking either $\{F(j,m) : 1 \le m \le N; 1 \le i \le \#U\}$ or $\{L(m,i) : 1 \le m \le N; 1 \le i \le \#U\}$ as free variables, fixing the values of the other set from the previously obtained solution to the corresponding Least Squares problem.

  - This method is sometimes called "*Method of Alternating Projections*".
  - We stop such iterations when

$$\sum_{(j,m)} (F^{(k)}(j,m) - F^{(k-1)}(j,m))^2 + \sum_{(i,m)} (L^{(k)}(m,i) - L^{(k-1)}(m,i))^2$$

becomes smaller than an accuracy threshold $\varepsilon > 0$.

Latent Factor Method

- Steps (continued):
  - *minimize*

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- Steps (continued):

  - After we obtain the values $F^{(k)}(j,m)$ and $L^{(k)}(m,i)$ from the last iteration k, we f[illegible]rresponding matrices F of size $\#M \times N$ and L of size N[illegible]

$$\widetilde{F} = \Big([illegible] \Big) : 1 \leq j \leq \#M; \;\; 1 \leq m \leq N \Big);$$

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  - We finally set $E = F \times L$ as the final matrix of predicted ratings of all movies by al[illegible] where $E(j,i)$ is the prediction of the rating of movie M_j by user U_i.

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- Most likely this is because they are able to leverage the "global information", based on the relationship of ALL ratings, more effectively than the Neighbourhood Methods which use ratings in a more "localised way".

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  - the Latent Factor Method, which can be deployed with different number N of "latent factors" (in applications usually between 20 and 200)
- So how do we decide which one we should use in a particular application?
- How can we evaluate how effective a particular choice of a recommender system is?
- **Idea:** We use real existing data. As an example we use the *Netflix Challenge* competition.
- Netflix provided approximately 100 million actual ratings of 480, 000 users, rating 17, 770 movies.
- The competition was to stay open till a submission was able to beat the Netflix's own recommender system *Cinematch* by more than 10% and then all the competitors had 30 days to submit an algorithm which was the final entry.
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- But how was the performance of the proposed algorithms measured?
- 100 million ratings that were made available was to serve as the training data set R for the algorithms.
- The test consisted of about 1.4 million ratings which were NOT included in the 100 million ratings training data set R and not available to the teams.
- However, all the users and movies of these 1.4 million ratings were included in the 100 million ratings made available (but with these users rating different movies in the training data set than the movies they rated in these 1.4 million test ratings).
- The algorithms had to predict these 1.4 million ratings on the basis of such 100 million ratings training data set.
- The accuracy was measured by the RMS (Root Mean Square) error

$$rms\_error = \sqrt{\frac{\sum_{(j,i)\in T}(T(j,i) - P_a(j,i))^2}{1.4 \times 10^6}}$$

- Here $T(j,i)$ are the actual ratings included in the test set T (but not included in the training set R made available to the competitors)
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- But instead of picking one recommender system over another one, we can also combine several recommender systems as follows.
- Let $P_k(j,i)$ be the predicted ratings of a recommender system S_k, $1 \leq k \leq B$, where we have B many recommender systems.
- We can now form a combined prediction as a weighted average

$$= \sum_{1 \leq k \leq B} w_k P_k(j,i)$$

where $\sum_{1 \leq k \leq B} w_k = 1$ are positive weight factors.

- But how do we determine optimal weights w_k, and also optimal values of other parameters such as the regularisation factors λ and the number N of Latent Factors?
- The answer is pretty mundane: by an arduous trial and error procedure:
- If we have a massive training data set as in the case of the Netflix prize, we can remove quite a few smaller testing subsets T_q of ratings and then use the algorithm with different values of the parameters to predict these removed test ratings.
- We can then measure the RMS error of the predictions on these test data sets T_q with different values of the parameters, trying to tweak the parameters till we get as smaller error as possible, but making sure that we do not overfit, by using reasonably diverse and numerous tests sets T_q.

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- But how do we determine optimal weights w_k, and also optimal values of other parameters such as the regularisation factors λ and the number N of Latent Factors?
- The answer is pretty mundane: by an arduous trial and error procedure:
- If we have a massive training data set as in the case of the Netflix prize, we can remove quite a few smaller testing subsets T_q of ratings and then use the algorithm with different values of the parameters to predict these removed test ratings.
- We can then measure the RMS error of the predictions on these test data sets T_q with different values of the parameters, trying to tweak the parameters till we get as smaller error as possible, but making sure that we do not overfit, by using reasonably diverse and numerous tests sets T_q.

Recommender Systems - conclusions

- But instead of picking one recommender system over another one, we can also combine several recommender systems as follows.
- Let $P_k(j,i)$ be the predicted ratings of a recommender system S_k, $1 \leq k \leq B$, where we have B many recommender systems.
- We can now form a combined prediction as a weighted average

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Recommender Systems - conclusions

- In fact, the best performing algorithms at the Netflix competition were combinations of dozens of components with empirically tuned parameters.
- Further improvements in performance can be achieved by giving lower weights to older ratings of movies, thus also introducing the temporal dimension.
- **Conclusion:**
- The Recommender Systems, just as the Google PageRank algorithm, exemplify a design paradigm:
- The ingredient "baseline" algorithms have a sound basis employing increasingly sophisticated mathematical concepts and theorems.
- However, the final product is an empirically obtained "tweak" of such component algorithms.
- Unlike Physics, Computer Science cannot seek "definitive", exact methods and theories, especially for applications which involve subjective human factors such as taste or human opinion.
- We look for good approximations of complex and "noisy" reality, obtained from mathematically based components through empirical testing and tweaking.
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