COMP4121 Advanced Algorithms

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The Hidden Markov Models and the Viterbi Algorithm and its applications

The Problem of Speech Recognition

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- Spoken words of a particular language are understood by a listener by parsing them into a sequence of phonemes.
- There is no general consensus what exactly the phonemes are, even for a specific language such as English.
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- For example, if you replace the last phoneme /s/ in “kiss” with phoneme /l/ you obtain a different word, “kill”.
- However, we identify variants of the same sound as a single phoneme, for example the “k” sound in words “cat”, “kit” and “skill” are considered all to be slight variants of a single basic phoneme /k/, and replacing one such variant with another would not cause the listener to mistakenly understand the word being pronounced.

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- From such waveforms they have to "decide" what phonemes were spoken.

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- Note that some phonemes are only slightly different from one another and the phonemes also vary a lot from speaker to speaker;
- It is hard to determine with absolute certainty from a feature vector $\vec{f}[i]$ what phoneme "caused" a waveform that resulted in that particular feature vector.

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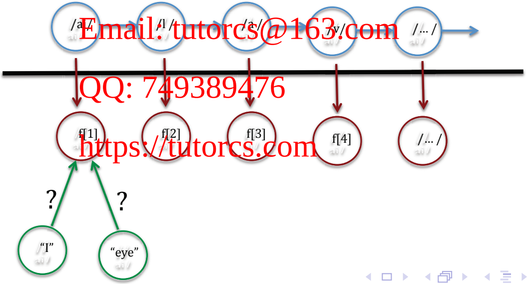

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- The Viterbi algorithm has a huge range of applications, besides speech recognition, for example, for decoding the transmitted convolutional codes in presence of noise in communication channels, which is used in digital telecommunications such as the CDMA mobile phone network and 802.11 wireless LANs; it is also used in many other fields such as bioinformatics and computational linguistics.

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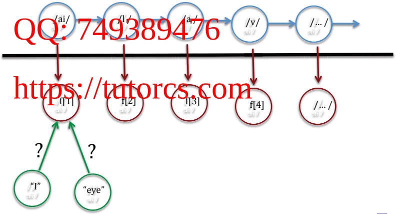

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- Since we can rule out values $p=0$ and $p=1$, this implies $4(1-p)-2p=0$ which yields $p=2/3$ and $1-p=1/3$ which sounds right: the probability of getting a tail is twice as high as the probability of getting a head, and indeed, we have twice as many tails than heads in the observed outcome.

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- You are allowed to draw one single ball and look at its number, and then you have to estimate how many balls there are inside.

Maximum likelihood estimation

- So assume we have an observed outcome, such as $TTHTTH$ in the previous example, generated by [illegible] process with unknown parameters, such as the probability p of ge[illegible] in the previous example. Maximum Likelihood estimation [illegible] the unknown parameters so that the probability of getting [illegible]come is the largest.
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- You are allowed to draw one single ball and look at its number, and then you have to estimate how many balls there are inside.
- Assume that you drew the ball numbered k. Since all balls are equally likely, if there are n balls inside, then the probability of drawing any particular ball is $1/n$.

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- This does not happen for large samples; it can be shown that as the size of the sample increases, ML estimate approaches the best possible estimate.

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- In other words, we are looking for *the most likely cause* of the sequence of observations $(y_1, y_2, \ldots, y_T)$.
- In our speech recognition example we are given a sequence of feature vectors $(\vec{f}[1], \vec{f}[2], \ldots, \vec{f}[10])$ extracted from the sound waveform and we have to find what sequence of phonemes $(\phi_1, \phi_2, \ldots, \phi_{10})$ is the most likely cause of the sequence of the feature vectors $(\vec{f}[1], \vec{f}[2], \ldots, \vec{f}[10])$.

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- This could, theoretically, be done as follows. We first find the probability $P(\vec{x})$ that a sequence $\vec{x}$ occurred at all, as the following product:

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- Next, we compute the probability $P^*(\vec{x}, \vec{y})$ that this particular sequence of states $\vec{x} = (x_1, \ldots, x_T)$ will cause the sequence of observable outcomes $\vec{y} = (y_1, \ldots, y_T)$, as the product of the probabilities $e(x_1, y_1) \cdot \ldots \cdot e(x_T, y_T)$ where each $e(x_i, y_i)$ is the "emission" probability that state x_i produces the observable manifestation y_i.

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- Finally, the probability $\mathcal{P}(\vec{x}, \vec{y})$ that sequence $\vec{x}$ is the cause of the sequence of observable outcomes $\vec{y}$ is the product of the probabilities $P(\vec{x})$ that sequence $\vec{x}$ occurred at all and the probability $P^*(\vec{x}, \vec{y})$ that, if it did occur, it also caused the right observable outcome $\vec{y}$, so in total $\mathcal{P}(\vec{x}, \vec{y}) = P(\vec{x}) \cdot P^*(\vec{x}, \vec{y})$.

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- Instead, the Viterbi algorithm solves this problem in time $O(T \times K^2)$ using dynamic programming.

The Viterbi Algorithm

- **The Viterbi Algorithm:** We solve all subproblems $\mathcal{S}(i,k)$ for every $1 \leq i \leq T$ and every $1 \leq k \leq K$:

  Subproblem $\mathcal{S}(i,k)$: *equence of states* $(x_1, \ldots x_i)$ *such that* $x_i = s_k$ *and such that the prob* *serving the outcome* $(y_1, \ldots, y_i)$ *is maximal.*

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- Thus, we solve subpro[illegible]l truncations $(x_1, \ldots, x_i)$, $1 \leq i \leq T$, but also insisting that the last state x_i of such a truncated subsequence be state s_k.

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- Thus, we solve subproblems not only for all truncations $(x_1, \ldots, x_i)$, $1 \leq i \leq T$, but also insisting that the last state x_i of such a truncated subsequence be state s_k.

- Let us denote by $L(i,k)$ the largest possible probability for which there exists a sequence $(x_1, x_2, \ldots, x_i)$ ending with $x_i = s_k$ causing the subsequence of observable outcomes $(y_1, y_2, \ldots, y_i)$; such probabilities must satisfy the following recursion:

$$L(1,k) = \pi_k \cdot e(s_k, y_1)$$

$$L(i,k) = \max_{m \in S} \left(L(i-1,m)\, p(s_m \to s_k)\, e(s_k, y_i)\right);$$

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- Here $\sigma(i,k)$ stores the value of m for which $L(i-1,m)\, p(s_m \to s_k)\, e(s_k, y_i)$ is the largest which allows us to reconstruct the sequence of states which maximize the probabilities we are tracking.

The Viterbi Algorithm

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- We can now obtain the sequence of states which is most likely cause of the given sequence of observations by backtracking:

$$
\begin{aligned}
x_T &= \arg\max_{m \in S} (L(T,m))\\
x_{i-1} &= \sigma(i, x_i), \quad i = T, T-1, \ldots, 2.
\end{aligned}
$$

The Viterbi Algorithm Example

- In California there are twice as many raccoons as possums. Having gotten a job with Google, you are in California observing a back yard. It is dusk, so the probability that you think you saw a raccoon when you are actually looking at a possum at a distance [illegible] the probability that you think you saw a possum while you are actually looking at a raccoon at a distance is 1/4. Raccoons move in packs; if a raccoon comes to your back yard the probability that the next animal to follow will also be a raccoon is 4/5. Possums are solitary animals, so if a possum comes to your back yard, this does not impact the probabilities what the next animal to come will be (a possum or a raccoon, but recall in California there are twice as many raccoons as possums!) You believe that you saw four animals coming in the following order: a raccoon, a possum, a possum, a raccoon ($rppr$). Given such a sequence of observations, what actual sequence of animals is most likely to cause such a sequence of your observations?

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The Viterbi Algorithm Example

- In California there are twice as many raccoons as possums. Having gotten a job with Google, you are in California observing a back yard. It is dusk, so the probability that you th[illegible]w a raccoon when you are actually looking at a possum at a distanc[illegible]e probability that you think you saw a possum while you are [illegible]oking at a raccoon at a distance is 1/4. Raccoons move in pac[illegible]accoon comes to your back yard the probability that the ne[illegible]to follow will also be a raccoon is 4/5. Possums are solitary animals, so if a possum comes to your back yard, this does not impact the probabilities what the next animal to come will be (a possum or a raccoon, but recall in California there are twice as many raccoons as possums!) You believe that you saw four animals coming in the following order: a raccoon, a possum, a possum, a raccoon ($rppr$). Given such a sequence of observations, what actual sequence of animals is most likely to cause such a sequence of your observations?
- probabilities of initial states: $\pi(R) = 2/3$; $\pi(P) = 1/3$.
- transition probabilities:
  $p(R \to R) = 4/5$; $p(R \to P) = 1/5$; $p(P \to P) = 1/3$; $p(P \to R) = 2/3$.

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- transition probabilities:
  $p(R \to R) = 4/5$; $p(R \to P) = 1/5$; $p(P \to P) = 1/3$; $p(P \to R) = 2/3$.
- emission probabilities:
  $O(r|P) = 1/3$;

The Viterbi Algorithm Example

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- emission probabilities:
  $O(r|P) = 1/3$; $O(p|P) = 2/3$;

The Viterbi Algorithm Example

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- transition probabilities:
  $p(R \to R) = 4/5$; $p(R \to P) = 1/5$; $p(P \to P) = 1/3$; $p(P \to R) = 2/3$.
- emission probabilities:
  $O(r|P) = 1/3$; $O(p|P) = 2/3$; $O(p|R) = 1/4$;

The Viterbi Algorithm Example

- In California there are twice as many raccoons as possums. Having gotten a job with Google, you are in California observing a back yard. It is dusk, so the probability that you th[illegible]w a raccoon when you are actually looking at a possum at a distanc[illegible] probability that you think you saw a possum while you are [illegible]oking at a raccoon at a distance is 1/4. Raccoons move in pac[illegible]accoon comes to your back yard the probability that the ne[illegible]to follow will also be a raccoon is 4/5. Possums are solitary animals, so if a possum comes to your back yard, this does not impact the probabilities what the next animal to come will be (a possum or a raccoon, but recall in California there are twice as many raccoons as possums!) You believe that you saw four animals coming in the following order: a raccoon, a possum, a possum, a raccoon ($rppr$). Given such a sequence of observations, what actual sequence of animals is most likely to cause such a sequence of your observations?
- probabilities of initial states: $\pi(R) = 2/3$; $\pi(P) = 1/3$.
- transition probabilities:
  $p(R \to R) = 4/5$; $p(R \to P) = 1/5$; $p(P \to P) = 1/3$; $p(P \to R) = 2/3$.
- emission probabilities:
  $O(r|P) = 1/3$; $O(p|P) = 2/3$; $O(p|R) = 1/4$; $O(r|R) = 3/4$.

The Viterbi Algorithm Example

- In California there are twice as many raccoons as possums. Having gotten a job with Google, you are in California observing a back yard. It is dusk, so the probability that you th[illegible]w a raccoon when you are actually looking at a possum at a distance[illegible]e probability that you think you saw a possum while you are [illegible]oking at a raccoon at a distance is 1/4. Raccoons move in pac[illegible]accoon comes to your back yard the probability that the ne[illegible]o follow will also be a raccoon is 4/5. Possums are solitary animals, so if a possum comes to your back yard, this does not impact the probabilities what the next animal to come will be (a possum or a raccoon, but recall in California there are twice as many raccoons as possums!) You believe that you saw four animals coming in the following order: a raccoon, a possum, a possum, a raccoon ($rppr$). Given such a sequence of observations, what actual sequence of animals is most likely to cause such a sequence of your observations?
- probabilities of initial states: $\pi(R) = 2/3$; $\pi(P) = 1/3$.
- transition probabilities:
  $p(R \to R) = 4/5$; $p(R \to P) = 1/5$; $p(P \to P) = 1/3$; $p(P \to R) = 2/3$.
- emission probabilities:
  $O(r|P) = 1/3$; $O(p|P) = 2/3$; $O(p|R) = 1/4$; $O(r|R) = 3/4$.
- **Homework:** Execute the Viterbi algorithm by hand (using a calculator) to see what actual sequence of animals is most likely to cause the sequence of observations $rppr$.