

COMP4121hAdvanced Algorithms

Assignment Project Exam Help

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• Assume that we have n sensors such that the errors of readings X_1, \ldots, X_n of these same are independent, unbiased and normally distribute that the different and **known** standard deviations $\sigma_1, \ldots, \sigma_n$ ample, we have tested them all in a lab doing many mental than a comparing their readings with the true value obtained from a precise instrument). We chat: cstutores

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- Assume that, using these sensors, we have obtained measurements X_1, \ldots, X_n , which Assignments Project Exactified the true value of the quantity being measured.

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- Assume that, using these sensors, we have obtained measurements X_1, \ldots, X_n , which Assignibelik Project Exactified the true value of the quantity being measured. Email: tutorcs@163.com
 • Let the true value of the measured quantity be equal to μ .

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- Since the sensors are inblased the expected value of the readings of all sensors is equal to μ i.e. $E[X_i] = \mu$.

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- Since the sensors are inblased the expected value of the readings of all sensors is equal to μ i.e. $E[X_i] = \mu$.
- We can now compute the **likelihood** that a particular μ produced measurements X_1, X_2, \ldots, X_n and then pick the value of μ for which such a likelihood is the largest.

• Since the errors are independent was since $E[X_i] = \mu$, the probability to obtain readings $\mathbf{X} = X_1, \dots, X_n$ is

$$\mathcal{L}_{n}(\mu|\mathbf{X}) = \prod_{i=1}^{n} \frac{1}{\sigma_{i} \sqrt{2\pi}} \sum_{i=1}^{n} \frac{1}{\sigma_{i}^{2}} = \left(\prod_{i=1}^{n} \frac{1}{\sigma_{i} \sqrt{2\pi}}\right) e^{-\frac{1}{2} \sum_{i=1}^{n} \frac{(X_{i} - \mu)^{2}}{\sigma_{i}^{2}}}$$

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• Differentiating with respect to μ and setting the derivative equal to zero we get

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$$\frac{d}{d\mu}\mathcal{L}_n(\mu|\mathbf{X}) = \prod_{\substack{n=1 \ \text{odd}}} \frac{1}{\sqrt{2\pi}} \sum_{\substack{n=1 \ \text{odd}}} \frac{1}{\sqrt{2\pi}} \sum_{i=1}^n \frac{(X_i - \mu)^2}{\sigma_i^2} \sum_{i=1}^n \frac{(X_i - \mu)}{\sigma_i^2}$$

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• Differentiating with respect to μ and setting the derivative equal to zero we get

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$$\frac{d}{d\mu}\mathcal{L}_n(\mu|\mathbf{X}) = \prod_{\substack{n=1 \ \text{odd}}} \frac{1}{\sqrt{2\pi}} \frac{\text{tutores @ $\frac{1}{2}6\Sigma_i^n \text{com} \frac{(X_i - \mu)^2}{\sigma_i^2}}}{\sqrt{2\pi}} \sum_{i=1}^n \frac{(X_i - \mu)}{\sigma_i^2}$$

• Thus,

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$$\frac{d}{d\mu}\mathcal{L}_n(\mu|\mathbf{X}) = 0 \quad \Leftrightarrow \quad \sum_{i=1}^n \frac{X_i}{\sigma_i^2} - \mu \sum_{i=1}^n \frac{1}{\sigma_i^2} = 0 \quad \Leftrightarrow \quad \mu = \frac{\sum_{i=1}^n \frac{X_i}{\sigma_i^2}}{\sum_{j=1}^n \frac{1}{\sigma_j^2}}$$

• Since the Maximun程底的局位像的網絡結構是 true value is

$$\mu = \frac{1}{n} \frac{\frac{1}{\sigma_i^2}}{\frac{1}{\sigma_i^2}} = \sum_{i=1}^n \frac{\frac{1}{\sigma_i^2}}{\sum_{j=1}^n \frac{1}{\sigma_j^2}} X_i$$

we see that the maximum likelihood estimate is a weighted mean of the readings of all schedis weighted inversely project by arriance.

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• Since the Maximun 程序的局位像的系统转换 true value is

$$\mu = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} X_{i}}{\sum_{i=1}^{n} \frac{1}{\sigma_{i}^{2}}} = \sum_{i=1}^{n} \frac{\frac{1}{\sigma_{i}^{2}}}{\sum_{j=1}^{n} \frac{1}{\sigma_{j}^{2}}} X_{i}$$

we see that the maximum likelihood estimate is a weighted mean of the readings of all schedis weighted inversely proportional Project Exam Help

• Let us find the variance of such an estimator:

$$\begin{split} V(M(\mathbf{X})) &= E \left(\sum_{i=1}^{n} \frac{\frac{1}{\sigma_{i}^{2}}}{\sum_{j=1}^{n} \frac{1}{\sigma_{j}^{2}}} \underbrace{\sum_{j=1}^{n} \frac{1}{\sigma_{j}^{2}}}_{j=1}^{n} \underbrace{\sum_{j=1}^{n} \frac{1}{\sigma_{j}^{2}}}_{j=1}^{n} \underbrace{\sum_{j=1}^{n} \frac{1}{\sigma_{j}^{2}}}_{j=1}^{n} \underbrace{\sum_{j=1}^{n} \frac{1}{\sigma_{j}^{2}}}_{j=1}^{n} \underbrace{\sum_{k=1}^{n} \frac{1}{\sigma_{k}^{2}}}_{j=1}^{n} \underbrace{\sum_{k=1}^{n} \frac{1}{\sigma_{k}^{2}}}_{j=1}^{n}}_{j=1}^{n} \underbrace{\sum_{k=1}^{n} \frac{1}{\sigma_{k}^{2}}}_{j=1}^{n}}_{j=1}^{n} \underbrace{\sum_{k=1}^{n} \frac{1}{\sigma_{k}^{2}}}_{j=1}^{n}}_{j=1}^{n} \underbrace{\sum_{k=1}^{n} \frac{1}{\sigma_{k}^{2}}}_{j=1}^{n}}_{j=1}^{n}}_{j=1}^{n} \underbrace{\sum_{k=1}^{n} \frac{1}{\sigma_{k}^{2}}}_{j=1}^{n}}_{j=1}^{n}}_{j=1}^{n} \underbrace{\sum_{k=1}^{n} \frac{1}{\sigma_{k}^{2}}}_{j=1}^{n}}_{j=1}^{n}}_{j=1}^{n} \underbrace{\sum_{k=1}^{n} \frac{1}{\sigma_{k}^{2}}}_{j=1}^{n}}_{j=1}^{n}}_{j=1}^{n}}_{j=1}^{n}$$

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• Since the errors of I airwise independent, we have $E((X_i - \mu)(X_j - \mu)) = i \neq j$; thus, from

$$V(M(\mathbf{X})) = \left(\frac{1}{\sigma_i^2} (X_i - \mu) \frac{1}{\sigma_j^2} (X_j - \mu) \right)$$

$$V(M(\mathbf{X})) = \left(\frac{1}{\sigma_i^2} \frac{1}{\sigma_i^2} \frac{1}{\sigma_k^2} \frac{1}{\sigma_k^2} \frac{1}{\sigma_k^2} \frac{1}{\sigma_k^2} \right)$$

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• Since the errors of i = i airwise independent, we have $E((X_i - \mu)(X_j - \mu)) = i \neq j$; thus, from

$$V(M(\mathbf{X})) \underbrace{\text{We Chat:}}_{i,j=1}^{n} \underbrace{\frac{1}{\sigma_i^2}(X_i - \mu)}_{k=1} \underbrace{\frac{1}{\sigma_j^2}(X_j - \mu)}_{\sum_{k=1}^{n} \frac{1}{\sigma_k^2}} \underbrace{\sum_{k=1}^{n} \frac{1}{\sigma_k^2}}_{\sum_{k=1}^{n} \frac{1}{\sigma_k^2}}$$

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we obtain

$$V(M(\mathbf{X})) = E \left(\sum_{i=1}^{n} \frac{\frac{1}{\sqrt[n]{2}} (X_i 749389476}{\left(\underbrace{\sum_{i=1}^{n} \frac{1}{\sqrt[n]{2}} (X_i 749389476}_{\text{originators}} + \underbrace{\sum_{i=1}^{n} \frac{1}{\sqrt[n]{2}$$

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• Since for all i

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$$\frac{1}{\sum_{j=1}^{n} \frac{1}{\sigma_j^2}} < \frac{1}{\frac{1}{\sigma_i^2}} = \sigma_i^2$$

we get that also V^{\bullet}

 $\min_{1 \le i \le n} \sigma_i^2$

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• Since for all i

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we get that also $V(m(\mathbf{X}))$

• Thus, we see that sweethat of the best of our sensors!

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• Since for all i

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we get that also $V(M, \mathbf{x})$

 $\min_{1 \le i \le n} \sigma_i^2$

• Thus, we see that see that see that to as the best of our sensors!

the best of our sensors!

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Can we find a more accurate estimator which is also unbiased?

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we get that also $V(m(\mathbf{x})) < 1$

• Thus, we see that see Charto GS IMD restimate is more accurate than

- the best of our sensors!

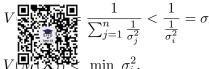
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 Can we find a more accurate estimator which is also unbiased?
- It turns out that in Ethis cast one ML63stimator achieves the smallest possible variance among all unbiased estimators.

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• Since for all i

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we get that also $V(\pi(\mathbf{X})) < \min_{1 \le i \le n} \sigma$

- Thus, we see that see Chartogst MDrestimate is more accurate than the best of our sensors!
- the best of our sensors!

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 Can we find a more accurate estimator which is also unbiased?
- It turns out that in Ethis cast one M163stianator achieves the smallest possible variance and many all unbiased estimators.
- Such a lower bound for the variances of all unbiased estimators is given by the Cramentep Rature Research which you can look up in the lecture notes on Iterative Filtering which you can find at the class website, look at page 6 of https:

//www.cse.unsw.edu.au/~cs4121/lectures_2019/IF.pdf

Application of Markov's Inequality: Chebyshev's Inequality (Markov was a student of Chebyshev)

expectation E[X]

• Theorem: Let $X \square X \square X \square X$ Indom variable with a finite \mathbf{Z} e variance V[X]. Then

 $E[X] \ge t \le \frac{V[X]}{t^2}$

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Application of Markov's Inequality: Chebyshev's Inequality (Markov was a student of Chebyshev).

• Theorem: Let X in Indom variable with a finite expectation E[X] and E[X] e variance V[X]. Then $F[X] = \frac{V[X]}{t^2}$

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• **Proof:** Note that

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$$\mathcal{P}(|X - E[X]| \ge t) = \mathcal{P}((X - E[X])^2 \ge t^2)$$
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Application of Markov's Inequality: Chebyshev's Inequality (Markov was a student of Chebyshev).

• Theorem: Let X on the index point E[X] and E[X] in the expectation E[X] are variance V[X]. Then $E[X] = E[X] | \ge t$

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- Proof: Note that Assignment Project Exam Help $\mathcal{P}(|X E[X]| > t) = \mathcal{P}((X E[X])^2 \ge t^2)$ Email: Tutores @ 163.com
- Also, $Y = (X E[X])^{\frac{1}{2}}$ is a non-negative random variable with a finite expectation F[Y] = V[X] = V[X].

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Application of Markov's Inequality: Chebyshev's Inequality (Markov was a student of Chebyshev).

• Theorem: Let X ndom variable with a finite expectation E[X] and the expectation E[X] are variance V[X]. Then F[X] = E[X] = t

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• Proof: Note that Assignment Project Exam Help

$$\mathcal{P}(|X - E[X]| \ge t) = \mathcal{P}((X - E[X])^2 \ge t^2)$$

- Also, $Y = (X E[X])^{\frac{1}{2}}$ is a non-negative random variable with a finite expectation F[Y] is $Y = (X E[X])^{\frac{1}{2}} = V[X]$.
- \bullet Thus, we can apply the Markov inequality to Y to obtain

$$\mathcal{P}(|X - E[X]| \ge t) = \mathcal{P}((X - E[X])^2 \ge t^2) = \mathcal{P}(Y > t^2) \le \frac{E[Y]}{t^2} = \frac{V[X]}{t^2}$$

• If we get a large number of integration E[X] we would expect that the mean of these samples should be $\operatorname{clos}_{\bullet\bullet}$ pected value E[X] of X. The Law of large numbers bounds the parameters has such a mean is further away from E[X] than a prescribed value $\operatorname{clos}_{\bullet\bullet}$

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• If we get a large number of interpretate (X, X_n) of a random variable X with an expectation E[X] we would expect that the mean of these samples should be $\operatorname{clos}_{\bullet,\bullet}$ pected value E[X] of X. The Law of large numbers bounds the parameter $\operatorname{clos}_{\bullet,\bullet}$ that such a mean is further away from E[X] than a prescribed value $\operatorname{clos}_{\bullet,\bullet}$

$$\mathcal{P}\left(\left|\frac{X}{n}\right| + \frac{1}{n} + \frac{1}{n} - E[X] \right| > \varepsilon\right) \le \frac{V[X]}{n\varepsilon^2}$$

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• If we get a large number of integerited which is X, X_n of a random variable X with an expectation E[X] we would expect that the mean of these samples should be $\operatorname{clos}_{\blacksquare}$ pected value E[X] of X. The Law of large numbers bounds the parameters is that such a mean is further away from E[X] than a prescribed value X.

$$\mathcal{P}\left(\left|\frac{X}{\mathbb{D}}\right| + \frac{1}{n} + \frac{1}{n} + \frac{1}{n} - E[X]\right| > \varepsilon\right) \le \frac{V[X]}{n\varepsilon^2}$$

• Proof: By the Chebyshev inequality,

$$\mathcal{P}\left(\left|\frac{X_1+\ldots+X_n}{n}\right| - \underbrace{As}\left[\underset{n}{\overset{X_1+X_2+\ldots+X_n}{\text{lightment}}} \underbrace{Project^n}_{n}\right] \times \underbrace{As}\left[\underset{\varepsilon^2}{\overset{X_1+X_2+\ldots+X_n}{n}}\right] \times \underbrace{As}\left[\underset{n}{\overset{X_1+\ldots+X_n}{\text{lightment}}} \underbrace{As}\left[\underset{n}{\overset{X_1+\ldots+X_n}{\text{lightment}}}\right] \times \underbrace{As}\left[\underset{n}{\overset{X_1+\ldots+X_n}{\text{lightment}}} \underbrace{As}\left[\underset{n}{\overset{X_1+\ldots+X_n}{\text{lightment}}}$$

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• If we get a large number of interpretate (X, X_n) of a random variable X with an expectation E[X] we would expect that the mean of these samples should be $\operatorname{clos}_{\bullet,\bullet}$ pected value E[X] of X. The Law of large numbers bounds the parameter $\operatorname{clos}_{\bullet,\bullet}$ that such a mean is further away from E[X] than a prescribed value $\operatorname{clos}_{\bullet,\bullet}$

$$\mathcal{P}\left(\left|\frac{X \bigcap_{i=1}^{n} \sum_{i=1}^{n} \dots + X_n}{n} - E[X]\right| > \varepsilon\right) \le \frac{V[X]}{n\varepsilon^2}$$

• Proof: By the Chebyshev inequality,

$$\mathcal{P}\left(\left|\frac{X_1+\ldots+X_n}{n}\right| - \mathbf{As}\left[\mathbf{X_1} + X_2 + \mathbf{Y_1} - \mathbf{X_1} + \mathbf{X_2} + \mathbf{X_1} - \mathbf{X_1$$

• We now use the fact that the expectation is a linear operator and all X_i are equally distributed to conclude that

$$E\left[\frac{X_1 + X_2 + \dots + X_n}{n}\right] = \frac{E[X_1] + E[X_2] + \dots + E[X_n]}{\text{https://tutorcs.com}} = \frac{nE[X]}{n} = E[X]$$

• If we get a large number of integrates an expectation E[X] we would expect that the mean of these samples should be $\operatorname{clos}_{\blacksquare}$ pected value E[X] of X. The Law of large numbers bounds the parameters are that such a mean is further away from E[X] than a prescribed value E[X] of X.

$$\mathcal{P}\left(\left|\frac{X}{n}\right| + \frac{X}{n} - E[X] \right| > \varepsilon\right) \le \frac{V[X]}{n\varepsilon^2}$$

• Proof: By the Chebyshev inequality,

$$\mathcal{P}\left(\left|\frac{X_1+\ldots+X_n}{n}\right| - \mathbf{As}\left[\mathbf{X_1} + X_2 + \mathbf{Y_1} - \mathbf{X_1} + \mathbf{X_2} + \mathbf{X_1} - \mathbf{X_1$$

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• On the other hand, variance of any random variable X satisfies $V[a X] = a^2 V[X]$.

• If we get a large numb軽痒讷他蛋似做 級编程貓旱., X_n of a random variable X with an expectation E[X] we would expect that the mean of these samples should be closest pected value E[X] of X. The Law of large numbers bounds the p \mathbb{Z} hat such a mean is further away from E[X]than a prescribed valu.

$$\mathcal{P}\left(\left|\frac{X}{n}\right| + \frac{X}{n} - E[X] \right| > \varepsilon\right) \le \frac{V[X]}{n\varepsilon^2}$$

• Proof: By the Chebyshev inequality.

$$\mathcal{P}\left(\left|\frac{X_1+\ldots+X_n}{n}\right| - \mathbf{As}\left[\mathbf{X_1} + X_2 + \mathbf{Y_1} - \mathbf{X_1} + \mathbf{X_2} + \mathbf{X_1} - \mathbf{X_1$$

• We now use the fact that the expectation is a linear operator and all X_i are equally distributed to conclude that

$$E\left[\frac{X_1 + X_2 + \dots + X_n}{n}\right] = \frac{E[X_1] + E[X_2] + \dots + E[X_n]}{\text{https://tutorcs.com}} = \frac{nE[X]}{n} = E[X]$$

- On the other hand, variance of any random variable X satisfies $V[a X] = a^2 V[X].$
- Also, for INDEPENDENT variables X_1, \ldots, X_n we have $V[X_1 + \ldots + X_2] = V[X_1] + \ldots + V[X_n].$ (日) (日) (日) (日) (日)

• Thus

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$$\mathcal{P}\left(\left|\frac{X_1+\ldots+\sum_{n=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{n}+\ldots+X_n]}{n}\right]\right|>\varepsilon\right)\leq \frac{V\left[\frac{X_1+\ldots+X_n]}{n}\right]}{\varepsilon^2}$$

implies

$$\mathcal{P}\left(\left|\frac{X_1 + \dots + X_n}{\mathbf{W}_n^2} - E[X]\right| > \varepsilon\right) \le \frac{\frac{nV[X]}{n^2}}{\varepsilon^2} = \frac{V[X]}{n\varepsilon^2}$$

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Thus

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$$\mathcal{P}\left(\left|\frac{X_1+\ldots+\sum_{n=1}^{n} \frac{1}{n} + \ldots + X_n]}{n}\right|\right| > \varepsilon\right) \le \frac{V\left[\frac{X_1+\ldots+X_n]}{n}\right]}{\varepsilon^2}$$
 implies
$$\mathcal{P}\left(\left|\frac{X_1+\ldots+X_n}{N} - E[X]\right| > \varepsilon\right) \le \frac{\frac{nV[X]}{n^2}}{\varepsilon^2} = \frac{V[X]}{n\varepsilon^2}$$

• To summarise, if X_1, \ldots, X_n are independent, equally distributed random variables with a finite $x_i \in X_i \cap X_i \cap$