UNSW SYDNEY

COMP4161
Advanced Topics in Software Verification

based on slides by J. Blanchette, L. Bulwahn and T. Nipkow

Gerwin Klein, June Andronick, Miki Tanaka, Johannes Åman Pohjola

T3/2022

Content

➜ Foundations & Principles

- Intro, Lambda [illegible] atural deduction [1,2]
- Higher Order [illegible] part 1) [2,3[a]]
- Term rewriting [3,4]

➜ Proof & Specification Techniques

- Inductively defined sets, rule induction [4,5]
- Datatype induction, primitive recursion [5,7]
- General recursive functions, termination proofs [7[b]]
- Proof automation, Isar (part 2) [8]
- Hoare logic, proofs about programs, invariants [8,9]
- C verification [9,10]
- Practice, questions, exam prep [10[c]]

[a] a1 due; [b] a2 due; [c] a3 due

Overview

Au roof and Disproof

➜ Sledgehammer: automatic proofs

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- Quickcheck: counter example by testing

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Based on slides by Jasmin Blanchette, Lukas Bulwahn, and Tobias Nipkow (TUM).

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Dramatic improvemen  automated proofs in the last 2 decades.

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The key:

Efficient reasoning engines, and restricted logics.

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1980s rule[illegible]ns, write ML code

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2000s *embrace external tools, but don't trust them (ATP/SMT/SAT)*

Sledgehammer

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- ➜ *Connects Isab[illegible]TPs and SMT solvers: E, SPASS, Va[illegible]3, Yices, Z3*

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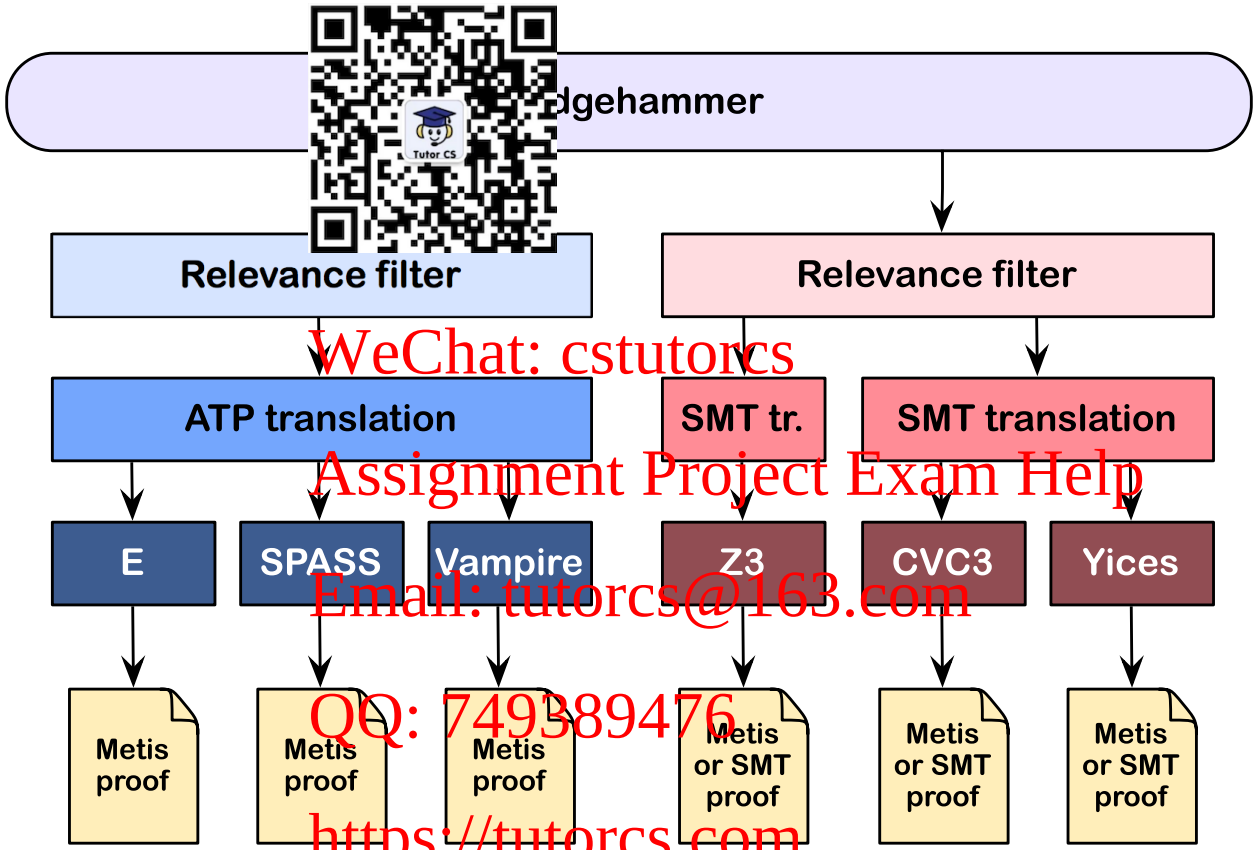
- ➜ *Exploits local parallelism and remote servers*

Demo: Sledgehammer

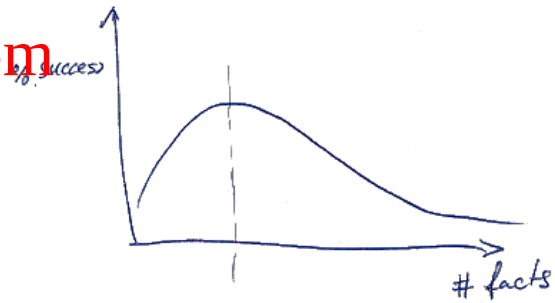
Sledgehammer Architecture



Fact Selection

Provers perform poo[illegible] 1000s of facts.

- ➜ *Best number d[illegible]ends on the prover*
- ➜ *Need to take [illegible]acts we give them*
- ➜ *Idea: order f[illegible]evance, give top n to prover* ($n = 250, 1000, \ldots$)



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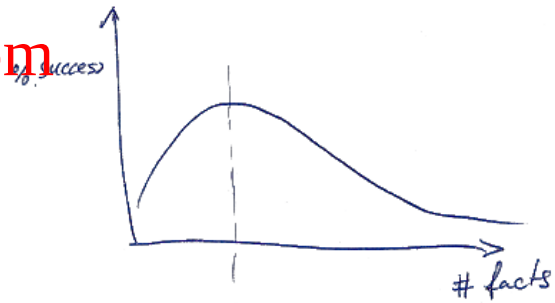

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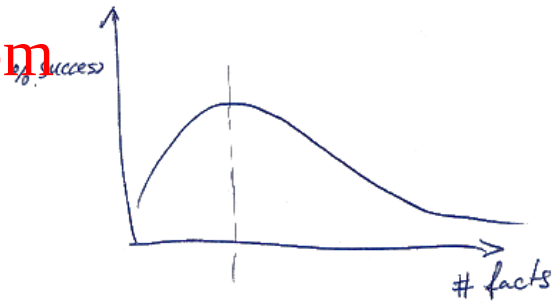
- ➜ *Machine learning method: look at previous proofs to get a probability of relevance*



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- **First-order:**
  - *SK combinators, λ-lifting*
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- **Encode types:**
  - *Monomorphise (generate multiple instances), or*
  - *Encode polymorphism on term level*

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- *Recast into structured Isar proof*
  Fast, not always readable.

Judgement Day (up to 2013)

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- ➜ **2012:** *Better integration with SPASS. 64%*
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- ➜ **2013:** *Machine learning for fact selection. 69%*
  Improves a few percent across provers.

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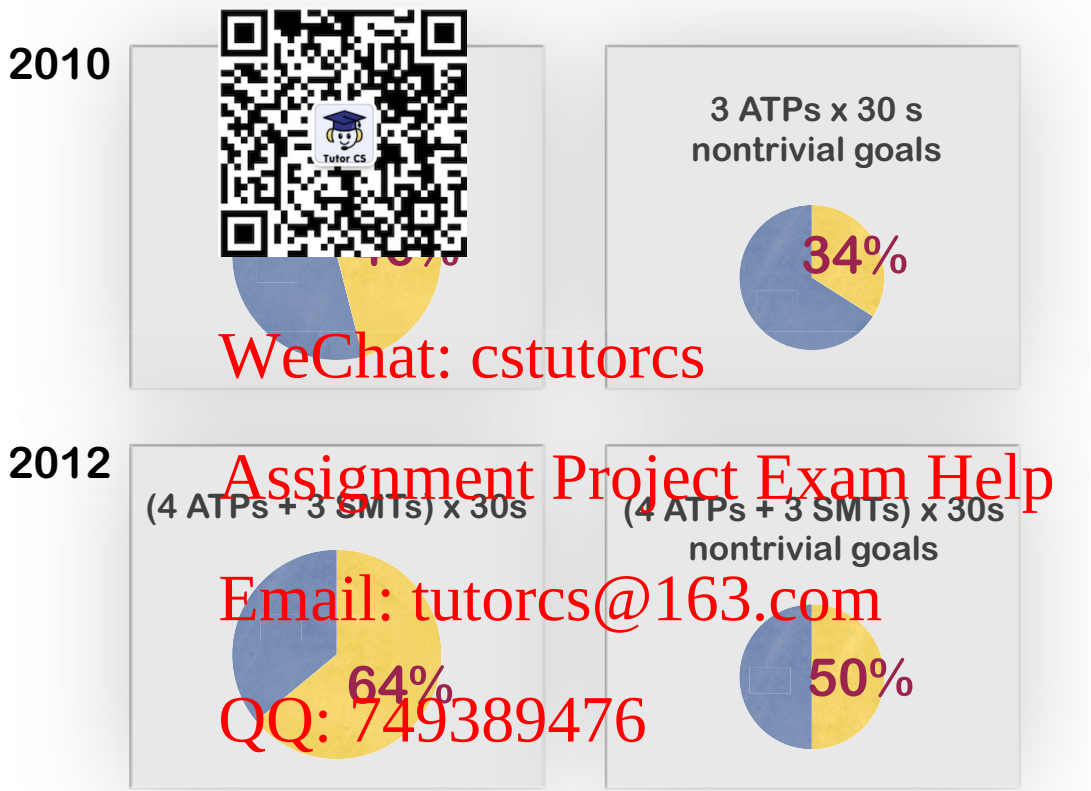

Judgement Day (2016)

Pr	MePo	MaSh	MeSh	Any selector
CV	679	749	**783**	830
E 1.8	622	601	**665**	726
SPASS 3.8ds	678	684	**739**	789
Vampire 3.0	703	698	**740**	789
veriT 2014post	543	556	**590**	655
Z3 4.3.2pre	638	668	**703**	788
Any prover	801	885	**919**	943

Fig. 15 Number of successful Sledgehammer invocations per prover on 1230 Judgment Day goals

$$919/1230 = 74\%$$

Sledgehammer rules!

Example applica

- ➜ *Large Isabelle/ itory of algebras for modelling imperative programs (Kleene Algebra, Hoare logic, . . ., ≈ 1000 lemmas)*
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"The integration of ATP, SMT, and Nitpick is for our purposes very very helpful." – G. Struth

Theorem proving and testing

Testing can show [illegible] presence of errors,
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- ➜ *Most lemma statements are wrong the first time.*
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Find counter examples automatically!

Quickcheck

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Quickcheck

Lightweight validation by testing.

- *Motivated by Haskell's QuickCheck*
- *Uses Isabelle's code generator*
- *Fast*
- *Runs in background, proves you wrong as you type.*

Quickcheck

Covers a numbe g approaches:

- *Random and e esting.*
- *Smart test dat s.*
- *Narrowing-based (symbolic) testing.*

Creates test data generators automatically.

Demo: Quickcheck

Test generators for datatypes

Fast iter  **ontinuation-passing-style**

datatype α list = Nil | Cons α (α list)

Test function:

$\text{test}_{\alpha\ list}$ P = P Nil *andalso* test_{α} (λx. $\text{test}_{\alpha\ list}$ (λxs. P (Cons x xs)))

Test generators for predicates

distinct (remove1 x xs)

Problem:

Exhaustive testing ... many useless test cases.

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Use data flow analysis to figure out which variables must be computed and which generated.

Narrowing

Symbolic execution with demand-driven refinement

- ➜ *Test cases can [illegible] riables*
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Implementation:

Lazy execution with outer refinement loop.

Many re-computations, but fast.

Quickcheck Limitations

Or able specifications!

- ➜ *No equality on functions with infinite domain*
- ➜ *No axiomatic specifications*

Nitpick

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model finder

- ➜ *Based on SAT via Kodkod (backend of Alloy prover)*
- ➜ *Soundly approximates infinite types*

Nitpick Successes

- *Algebraic meth*
- *C++ memory*
- *Found soundn* *TPS and LEO-II*

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Fan mail:

"Last night I got stuck on a goal I was sure was a theorem. After 5–10 minutes I gave Nitpick a try, and within a few secs it had found a splendid counterexample—despite the mess of locales and type classes in the context!"

Demo: Nitpick

We have seen today ...

➜ Proof: Sledgehamm

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Isar

(Part 2)

Datatypes in Isar

Datatype case distinction

proof (cases *term*)
  case Constructor_1
  ⋮
next
⋮
next
  case $(\text{Constructor}_k\ \vec{x})$
  $\ldots\ \vec{x}\ \ldots$
qed

Datatype case distinction

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  case (Constructor$_k$ $\vec{x}$)
  ⋯ $\vec{x}$ ⋯
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case (Constructor$_i$ $\vec{x}$) $\equiv$
fix $\vec{x}$ **assume** Constructor$_i$: "*term* = Constructor$_i$ $\vec{x}$"

Structural induction for nat

show $P\ n$
proof (induct n)
  case 0
  ...
  show ?*case*
next
  case (Suc n)
  ...
  ... n ...
  show ?*case*
qed

$\equiv$ **fix** n **assume** Suc: $P\ n$
  let ?*case* $= P$ (Suc n)

Structural induction: $\Longrightarrow$ and $\bigwedge$

show "$\bigwedge x.\ A\ n \Longrightarrow P\ n$"
proof (induct n)
  case 0
  ...
  show ?*case*
next
  case (Suc n)
  ...
  ... n ...
  ...
  show ?*case*
qed

fix x **assume** 0: "A 0"
let ?*case* = "P 0"

$\equiv$ **fix** n and x
assume Suc: "$\bigwedge x.\ A\ n \Longrightarrow P\ n$"
"A (Suc n)"
let ?*case* = "P (Suc n)"

Demo: Datatypes in Isar

Calculational Reasoning

The Goal

Prove:
$x \cdot x^{-1} = 1$

g:

assoc:	$(x \cdot y) \cdot z = x \cdot (y \cdot z)$
left_inv:	$x^{-1} \cdot x = 1$
left_one:	$1 \cdot x = x$

The Goal

Prove:

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- ➜ Manual: difficult in apply style
- ➜ Isar: with the methods we know, too verbose

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- ➜ **...**: predefined schematic term variable, refers to right hand side of last expression
- ➜ Automatic use of transitivity rules to connect steps

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also
have "$\cdots = t_n$" [proof]

calculation register
"$t_0 = t_1$"
"$t_0 = t_2$"
$\vdots$
"$t_0 = t_{n-1}$"

also/finally

have "$t_0 = t_1$" [proof]
also
have "$\ldots = t_2$" [proo
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$\vdots$
also
have "$\cdots = t_n$" [proof]
finally

calculation register
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$\vdots$
also
have "$\cdots = t_n$" [proof]
finally
show P
— 'finally' pipes fact "$t_0 = t_n$" into the proof

calculation register
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More about also

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Examples:

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- mixed: $[\![a \leq b; b < c]\!] \Longrightarrow a < c$
- substitution: $[\![P\ a; a = b]\!] \Longrightarrow P\ b$

- antisymmetry: $[\![a < b; b < a]\!] \Longrightarrow False$
- monotonicity: $[\![a = f\ b; b < c; \bigwedge x\ y.\ x < y \Longrightarrow f\ x < f\ y]\!] \Longrightarrow a < f\ c$

Demo