1. State some ⬛⬛⬛⬛⬛ autoencoders and PCA.

2. What is the ⬛⬛⬛⬛⬛ back-propagation algorithm for an autoencoder with L layer ⬛⬛⬛⬛ layer?

3. Assume that you initialize all weights in a neural net to the same value and you do the same for the bias terms. Is this a good idea? Justify your answer.

4. An autoencoder is a neural network designed to learn feature representations in an unsupervised manner. Unlike a standard multi-layer network, an autoencoder has the same number of nodes in its output layer as its input layer. An autoencoder is trained to reconstruct its own input x, i.e. to minimize the reconstruction error. An autoencoder is shown below.

| Input layer (L_1) | Hidden layer (L_2) | Output layer (L_3) |
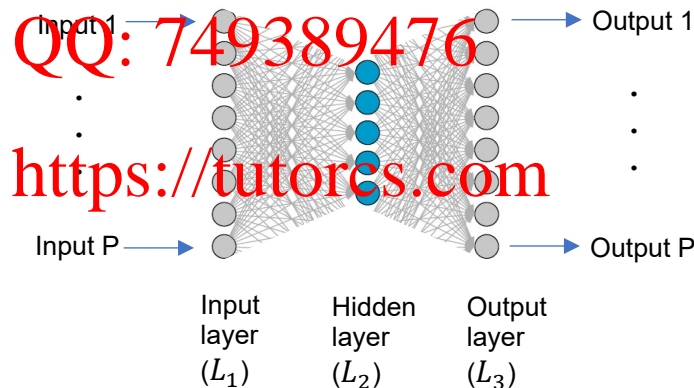
Suppose the input is a set of P-dimensional unlabelled data $\{x^{(i)}\}_{i=1}^{N}$. Consider an autoencoder with H hidden units in the second layer L_2. We will use the following notation for this autoencoder:

- W^e denotes the $P \times H$ weight matrix between L_1 and L_2
- W^d denotes the $H \times P$ weight matrix between L_2 and L_3
- σ denotes the activation function for L_2 and L_3
- $s_j^{(i)} = \sum_{k=1}^{P} W_{kj}^e x_k^{(i)}$
- $h_j^{(i)} = \sigma(\sum_{k=1}^{P} W_{kj}^e x_k^{(i)})$
- $t_j^{(i)} = \sum_{k=1}^{H} W_{kj}^d h_k^{(i)}$
- $\hat{x}_j^{(i)} = \sigma\left(\sum_{k=1}^{H} W_{kj}^d h_k^{(i)}\right)$

- $J(W^e, W^d)^{(i)} = \frac{1}{2} \|x^{(i)} - \hat{x}^{(i)}\|^2 = \sum_{j=1}^{P} (x_j^{(i)} - \hat{x}_j^{(i)})^2$ is the reconstruction error for example $x^{(i)}$
- $J(W^e, W^d) = \sum_{j=1}^{N} J(W^e, W^d)^{(i)}$ is the total reconstruction error
- (We add a bias to the input layer and hidden layer so that no bias term has to be explicitly included)

Fill in the following three equations for W^e and W^d. Use the notation defined above. There is no new notation needed.

$$\frac{\partial J^{(i)}}{\partial W_{kl}^d} = \sum_{j=1}^{P} \left(\boxed{} \cdot \frac{\partial \hat{x}_j^{(i)}}{\partial W_{kl}^d} \right)$$

$$\frac{\partial \hat{x}_l^{(i)}}{\partial W_{kl}^d} = \sigma' \left(\sum_{k=1}^{H} W_{kj}^e x_k^{(i)} \right) \cdot \boxed{}$$
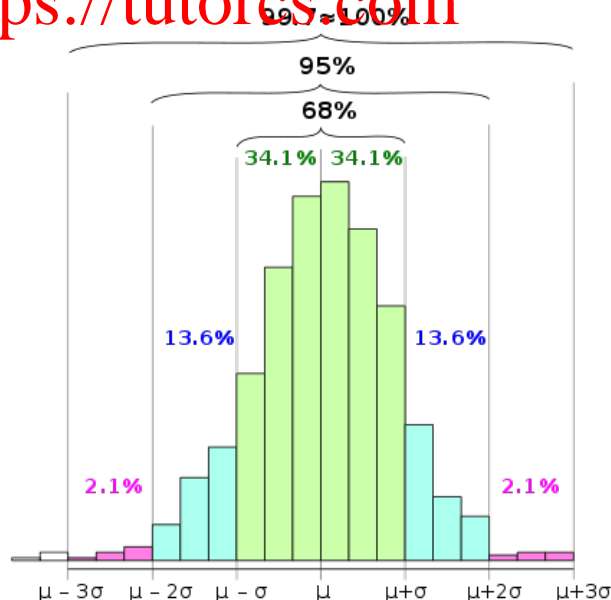
$$\frac{\partial J^{(i)}}{\partial W_{kl}^e} = \frac{\partial J^{(i)}}{\partial s_j^{(i)}} \cdot \boxed{}$$

$$\frac{\partial J^{(i)}}{\partial s_j^{(i)}} = \sum_{k=1}^{H} \left(\frac{\partial J^{(i)}}{\partial u_k^{(i)}} \cdot \boxed{} \cdot \sigma'(s_j^{(i)}) \right)$$

5. 3σ rule is a common technique used for anomaly detection. Describe what is the intuition of this rule for anomaly detection? How our result will be effected if we use other values of σ (e.g., 2σ, or 4σ)?

6. In the VAE, how sampling of the latent code is different during training and generation (generating a new sample)?