

程序代写代做 CS编程辅导



UNSW  
SYDNEY



COM

Foundations of Computer Science

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Lecture 8: Graph Theory  
**Assignment Project Exam Help**

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# Outline

程序代写代做 CS编程辅导

Motivation and Appl



Terminology and Notation

Graph Traversals

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Properties of Graphs

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# Graph theory: Historical Motivation

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Bridges of Königsberg



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Can you find a route which crosses each bridge exactly once?

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# Graph theory: Historical Motivation

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Bridges of Königsberg

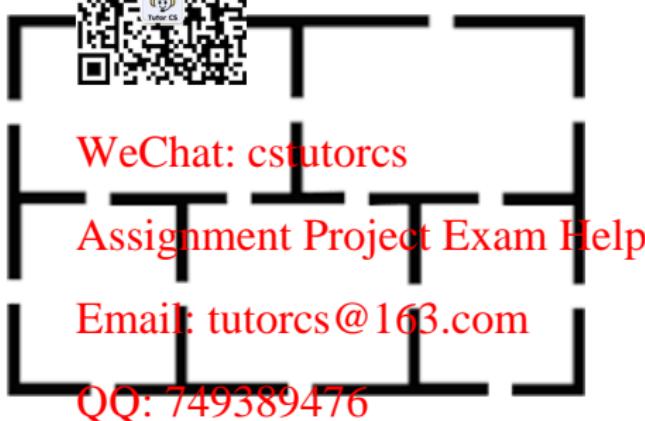


Can you find a route which crosses each bridge exactly once?  
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# Graph theory: Motivation

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Five rooms problem



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Can you find a route which passes through each door exactly once?

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# Graph theory: Motivation

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Crossed house problem



Can you draw this without taking your pen off the paper?

# Graph theory: Motivation

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Three utilities problem



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Can you connect all utilities to all houses without crossing connections?  
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# Graph theory: Motivation

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The following table contains information about several subjects and the students taking them:



Potions	Charms	Herbology	WeChat Tutors	Astronomy	Transfiguration
Harry	Ron	Harry	Hermione	Hermione	
Ron	Luna	George	Neville	Fred	
Malfoy	Ginny	Neville	Seamus	Luna	

How many examination timeslots are needed so that no student has two (or more) exams at the same time?

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# Graphs in Computer Science

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## Examples



- ① The WWW can be considered a massive graph where the nodes are web pages and arcs are hyperlinks.
- ② The possible states of a program form a directed graph.
- ③ Circuit components and their connections form a graph.
- ④ Social networks can be viewed as a graph where the nodes are users and the edges are connections.
- ⑤ The map of the earth can be represented as an undirected graph where edges delineate countries.

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# Graphs in Computer Science

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Applications of graph [QR code] Computer Science are abundant, e.g.

- route planning in navigation systems, robotics
- optimisation, e.g. routes, utilisation of network structures, bandwidth allocation

- compilers using "graph colouring" to assign registers to program variables

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- circuit layout ([Untangle game](#))

- determining the significance of a web page (Google's pagerank algorithm)

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- modelling the spread of a virus in a computer network or news in social network

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# Graphs

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Terminology (the most common; there are many variants):

**Graph** — pair  $(V, E)$



$V$  – set of vertices (or nodes)

$E$  – set of edges

**Undirected graph:** Every edge  $e \in E$  is a two-element set of vertices, i.e.  $e = \{x, y\} \subseteq V$  where  $x \neq y$

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**Directed graph:** Every edge (or arc)  $\in E$  is an ordered pair of vertices, i.e.  $e = (x, y) \in V \times V$ , note  $x$  may equal  $y$ .

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Binary relations on finite sets correspond to directed graphs.

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Symmetric, antireflexive relations correspond to undirected graphs.

# Graph representations

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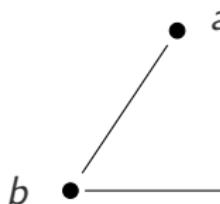
Graph:

$$V = \{a, b, c\}$$

$$E = \{\{a, b\}, \{b, c\}\}$$



Pictorially:



Directed graph:

$$V = \{1, 2, 3\}$$

$$E = \{(1, 2), (2, 3), (3, 2)\}$$

Pictorially:

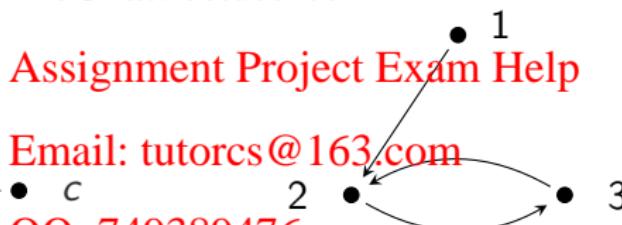
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# Graph representations

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Graph:

$$V = \{a, b, c\}$$

$$E = \{\{a, b\}, \{b, c\}\}$$



Directed graph:

$$V = \{1, 2, 3\}$$

$$E = \{(1, 2), (2, 3), (3, 2)\}$$

Adjacency matrix:

Adjacency matrix:

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$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{array}{l} \text{Assignment Project Exam Help} \\ \text{Email: tutorcs@163.com} \end{array}$$

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# Graph representations

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Graph:

$$V = \{a, b, c\}$$

$$E = \{\{a, b\}, \{b, c\}\}$$



Directed graph:

$$V = \{1, 2, 3\}$$

$$E = \{(1, 2), (2, 3), (3, 2)\}$$

Adjacency list:

$$a : \quad b$$

$$b : \quad a, c$$

$$c : \quad b$$

Adjacency list:

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$$1 : \quad 2$$

$$2 : \quad 3$$

$$3 : \quad 2$$

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# Graph representations

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Graph:

$$V = \{a, b, c\}$$

$$E = \{\{a, b\}, \{b, c\}\}$$



Incidence matrix

(vertices=rows,  
edges=columns):

Directed graph:

$$V = \{1, 2, 3\}$$

$$E = \{(1, 2), (2, 3), (3, 2)\}$$

Incidence matrix

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(vertices=rows,  
edges=columns):  
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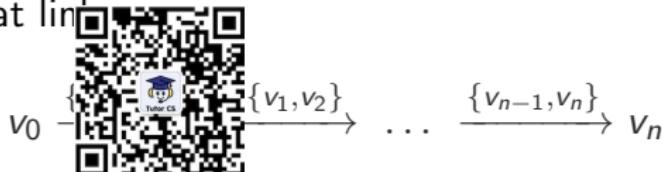
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$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

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# Paths

- A **(directed) path** in a (directed) graph  $(V, E)$  is a sequence of edges that link vertices:



where  $e_i = \{v_{i-1}, v_i\} \in E$  (or  $e_i = (v_{i-1}, v_i) \in E$ )

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- length** of the path is the number of edges:  $n$   
neither the vertices nor the edges have to be all different
- Subpath of length  $r$ :  $(e_m, e_{m+1}, \dots, e_{m+r-1})$
- Path of length 0: single vertex  $v_0$
- Connected set/graph (undirected)** — each pair of vertices joined by a path
- Strongly connected set/graph (directed)** — each pair of vertices joined by a directed path in both directions

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# Vertex Degrees (Undirected graphs)

- Degree of a vertex

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 $\deg(v) = |\{w \in V : \{v, w\} \in E\}|$

i.e., the number of edges attached to the vertex

- Regular graph



degrees are equal

- Degree sequence  $D_0, D_1, D_2, \dots, D_k$  of graph  $G = (V, E)$ ,  
where  $D_i = \text{no. of vertices of degree } i$

Question

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What is  $D_0 + D_1 + \dots + D_k$ ?  
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Fact

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$\sum_{v \in V} \deg(v) = 2 \cdot |E|$ ; so the sum of vertex degrees is always even.  
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Corollary

There is an even number of vertices of odd degree.

# Vertex Degrees (Directed graphs)

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- Out-degree of a vertex



$outdeg(v) = |$

$$\{ w \in V : (v, w) \in E \} |$$

i.e., the number of edges going out of the vertex

- In-degree of a vertex

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 $indeg(v) = | \{ w \in V : (w, v) \in E \} |$

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i.e., the number of edges going in to the vertex

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Fact

$$\sum_{v \in V} outdeg(v) = \sum_{v \in V} indeg(v) = |E|.$$

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# Exercises

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## Exercises

RW: 6.1.13(a) Draw a connected, regular graph on four vertices, each of degree 2



RW: 6.1.13(b) Draw a connected, regular graph on four vertices, each of degree 3  
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RW: 6.1.13(c) Draw a connected, regular graph on five vertices, each of degree 3  
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RW: 6.1.14(a) Graph with 3 vertices and 3 edges  
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RW: 6.1.14(b) Two graphs each with 4 vertices and 4 edges  
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# Exercises

## Exercises

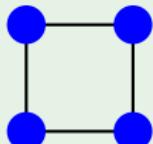
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RW: 6.1.13

Connected



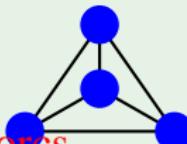
graphs on four vertices



(a)



(b)



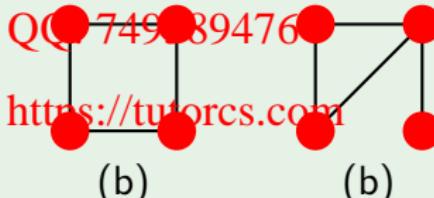
(b)

none  
(c)

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RW: 6.1.14

Graphs with 3 vertices and 3 edges must have a *cycle*



(a) the only one

(b)

(b)

# Exercises

NB

We use the notation

$$n = v(G) = |V| \text{ for vertices of graph } G = (V, E)$$

$$m = e(G) = |E| \text{ for edges of graph } G = (V, E)$$

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## Exercises

RW: 6.1.20(a) Graph With ~~Chat~~ ~~21~~ edges has a degree sequence

$$D_0 = 0, D_1 = 7, D_2 = 3, D_3 = 7, D_4 = ?$$

Find  $v(G)$

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RW: 6.1.20(b) How would your answer change, if at all, when

$$D_0 = 6?$$

# Exercises

NB

We use the notation

$$n = v(G) = |V| \text{ for vertices of graph } G = (V, E)$$

$$m = e(G) = |E| \text{ for edges of graph } G = (V, E)$$

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## Exercises

RW: 6.1.20(a) Graph With ~~21~~ edges has a degree sequence

$$D_0 = 0, D_1 = 7, D_2 = 3, D_3 = 7, D_4 = ?$$

Find  $v(G)$

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$$\sum_v \deg(v) = 2|E|; \text{ here}$$

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$$7 \cdot 1 + 3 \cdot 2 + 7 \cdot 3 + x \cdot 4 = 0.01 \text{ giving } x = 2, \text{ thus}$$

$$v(G) = \sum D_i = 19.$$

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RW: 6.1.20(b) How would your answer change, if at all, when

$$D_0 = 6?$$

No change to  $D_4$ ;  $v(G) = 25$ .

# Cycles

Recall paths  $v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} \dots \xrightarrow{e_n} v_n$

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- *simple path* — all edges of the path ( $i \neq j$ )
- *closed path* —
- **cycle** — closed other  $v_i$  pairwise distinct and  $\neq v_0$
- *acyclic path* —  $v_i \neq v_j$  for all vertices in the path ( $i \neq j$ )

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## NB

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- ①  $C = (e_1, \dots, e_n)$  is a cycle iff removing any single edge leaves an acyclic path. Email: [tutors@163.com](mailto:tutors@163.com)
- ②  $C$  is a cycle if it has the same number of edges and vertices and no proper subpath has this property.  
(Show that the 'subpath' condition is needed, i.e., there are graphs  $G$  that are **not** cycles and  $|E_G| = |V_G|$ ; every such  $G$  must contain a cycle!)

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# Trees

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- **Acyclic graph** — graph that doesn't contain any cycle
- **Tree** — connected [undirected]graph
- A graph is acyclic  if it is a forest (collection of disjoint trees)



## NB

Graph  $G$  is a tree iff WeChat: cstutorcs

- ↔ it is acyclic and  $|V_G| = |E_G| + 1$   
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(Show how this implies that the graph is connected!)
- ↔ there is exactly one simple path between any two vertices.
- ↔  ~~$G$  is connected, but becomes disconnected if any single edge is removed.~~  
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- ↔  ~~$G$  is acyclic, but has a cycle if any single edge on already existing vertices is added.~~  
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# Trees

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A tree with one vertex selected as its *root* is called a *rooted tree*. It imposes an ordering on the edges: ‘away’ from the root — from parent nodes to children. This defines a *level number* (or: *depth*) of a node as its distance from the root.

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Another very common notion in Computer Science is that of a DAG — a *directed, acyclic graph*.

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# Exercise (Supplementary)

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## Exercises

[RW: 6.7.3] (Supp) Tree with  $n$  vertices,  $n \geq 3$ .

Always true, false or weChat: estutorcs

(a)  $e(T) = ?$

(b) at least one vertex of degree exactly 2?

(c) at least two  $v_1, v_2$  s.t.  $\deg(v_1) = \deg(v_2)$

(d) exactly one path from  $v_1$  to  $v_2$

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# Exercise (Supplementary)

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## Exercises

[RW: 6.7.3] (Supp) Tree with  $n$  vertices,  $n \geq 3$ .

Always true, false or wechat: estutorcs

(a)  $e(T) = ?$  — False

(b) at least one vertex of degree exactly 2? — Could be either

(c) at least two  $v_1, v_2$  s.t.  $\deg(v_1) = \deg(v_2)$  — True

(d) exactly one path from  $v_1$  to  $v_2$  — True (characterises a tree)

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# Special Graphs

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- **Complete graph  $K_n$**

$n$  vertices, all pairwise connected,  $\frac{n(n-1)}{2}$  edges.



- **Complete bipartite graph  $K_{m,n}$**

Has  $m + n$  vertices partitioned into two (disjoint) sets, one of  $n$ , the other of  $m$  vertices.

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All vertices from different parts are connected; vertices from the same part are disconnected. No. of edges is  $m \cdot n$ .

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- **Complete  $k$ -partite graph  $K_{m_1, \dots, m_k}$**

Has  $m_1 + \dots + m_k$  vertices, partitioned into  $k$  disjoint sets, respectively of  $m_1, m_2, \dots$  vertices.

No. of edges is  $\sum_{i < j} m_i m_j = \frac{1}{2} \sum_{i < j} m_i m_j$

- These graphs generalise the complete graphs  $K_n = K_{\underbrace{1, \dots, 1}_n}$

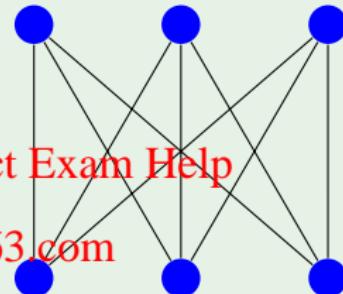
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## Example

$K_5$  :



$K_{3,3}$  :



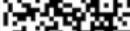
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# Graph Isomorphisms

$\phi : G \longrightarrow H$  is a *graph isomorphism* if 程序代写代做 CS 编程辅导

(i)  $\phi : V_G \longrightarrow V_H$  is a bijection

(ii)  $(x, y) \in E_G$  iff   $(\phi(x), \phi(y)) \in E_H$

Two graphs are called *isomorphic* if there exists (at least one) isomorphism between 

## Example

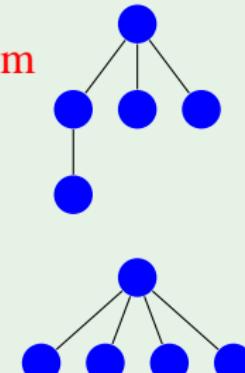
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All nonisomorphic trees on 2, 3, 4 and 5 vertices.

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# Graph exploration

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Often it is useful to **traverse** a graph: visit vertices in some order and examine each one.

- **Search:** Explore the graph until a particular vertex is discovered. [WeChat: cstutorcs](#) [Assignment Project Exam Help](#)
- **Traversal:** Examine all the vertices of the graph  
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# Graph exploration

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Two common graph search/traversal (DFTS/BFS) algorithms are **Depth-first search/traversal** (DFS) and **Breadth-first search/traversal** (BFS).



Both follow the same structure:

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- Examine a vertex  $v$
- Discover new vertices (neighbours of  $v$ )
- Move to the next discovered but not yet examined vertex
- DFS: Examine vertices by most recently discovered
- BFS: Examine vertices by least recently discovered

# DFS vs BFS

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DFS



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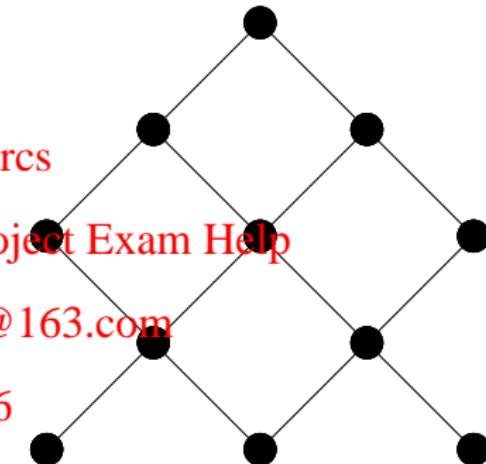
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BFS



# Special types of traversals

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Often we are interested in traversals that have a certain property.  
For example:



- Eulerian traversals: Visit all the edges exactly once
- Hamiltonian traversals: Visit all the vertices exactly once

NB

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In any given graph, these traversals may or may not exist.

Establishing the existence of such a traversal (decision problem) vs finding one if it exists (search problem) are subtly different problems.

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# Edge Traversal

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## Definition

- **Euler path** — [QR code] A path traversing every edge exactly once
- **Euler circuit** — [QR code] A closed Euler path

## Characterisations

- $G$  (connected) has an Euler circuit iff  $\deg(v)$  is even for all  $v \in V$ .
- $G$  (connected) has an Euler path iff either it has an Euler circuit (above) or it has exactly two vertices of odd degree.

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## NB

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- These characterisations apply to graphs with loops as well
- For directed graphs the condition for existence of an Euler circuit is  $\text{indeg}(v) = \text{outdeg}(v)$  for all  $v \in V$

# Exercises

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## Exercises



RW: 6.2.11 Construct a graph with vertex set

$\{0, 1\} \times \{0, 1\} \times \{0, 1\}$  and with an edge between vertices if they differ in exactly two coordinates.

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- (a) How many components does this graph have?
  - (b) How many vertices of each degree?
  - (c) Euler circuit?

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RW: 6.2.12 As Ex. 6.2.11 but with an edge between vertices if they differ in two or three coordinates.

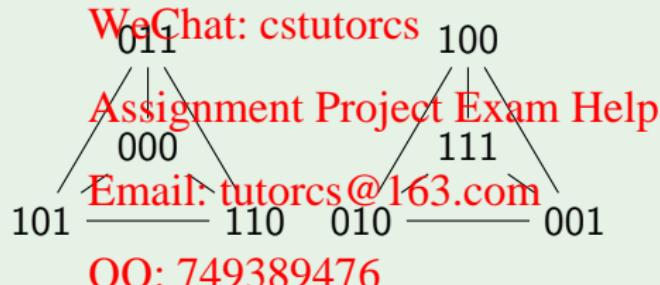
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# Exercises

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## Exercises

RW: 6.2.11 This graph shows the graph of all the *face diagonals* of a cube.  
It has two disjoint cycles.



No Euler circuit

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# Exercises

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## Exercises



RW: 6.2.14 Which complete graphs  $K_n$  have an Euler circuit?  
When do bipartite, 3-partite complete graphs have an Euler circuit?

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# Exercises

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## Exercises

RW: 6.2.14 Which complete graphs  $K_n$  have an Euler circuit?  
When do bipartite, 3-partite complete graphs have an Euler circuit?

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$K_n$  has an Euler circuit for  $n$  odd

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$K_{m,n}$  — when both  $m$  and  $n$  are even

$K_{p,q,r}$  — when  $p + q + r$ ,  $p, q, r$  are all even, ie. when  $p, q, r$  are all even or all odd

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# Bridges of Königsberg

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Bridges of Königsber



Can you find a route which crosses each bridge exactly once? No!  
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# Five rooms problem

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Can you find a route which passes through each door exactly once?  
No!

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# Crossed house problem

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Can you draw this without taking your pen off the paper? Yes!  
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# Vertex Traversal

## Definition

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- **Hamiltonian path** visits every vertex of graph exactly once
- **Hamiltonian cycle** visits every vertex exactly once except the last one, which is the first



## NB

Finding such a cycle, or proving it does not exist, is a difficult problem — the worst case is NP-complete

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## Examples (when the cycle exists)

- All five regular polyhedra (verify!)  
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- $n$ -cube; Hamiltonian circuit = Gray code  
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- $K_m$  for all  $m$ ;  $K_{m,n}$  iff  $m = n$ ;  $K_{a,b,c}$  iff  $a, b, c$  satisfy the triangle inequalities:  $a + b \geq c$ ,  $a + c \geq b$ ,  $b + c \geq a$
- Knight's tour on a chessboard (incl. rectangular boards)

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Examples when a Hamiltonian cycle does not exist are much harder to construct.



Also, given such a graph, it is nontrivial to verify that indeed there is no such a cycle: there is nothing obvious to specify that could assure us about this property.

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In contrast, if a cycle is given, it is immediate to verify that it is a Hamiltonian cycle.

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These situations demonstrate the often enormous discrepancy in difficulty of 'proving' versus (simply) 'checking'.

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# Exercise

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## Exercise

RW: 6.5.5(a)

How many Hamiltonian cycles does  $K_{n,n}$  have?



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# Exercise

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## Exercise

RW: 6.5.5(a)

How many Hamiltonian cycles does  $K_{n,n}$  have?

Let  $V = V_1 \cup V_2$



- start at any vertex in  $V_1$

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- go to any vertex in  $V_2$

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- go to any new vertex in  $V_1$

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- .....

There are  $n!$  ways to order each part and two ways to choose the 'first' part, implying  $c = 2(n!)^2$  circuits.

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# Outline

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Motivation and Appli



Terminology and Notation

Graph Traversals

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Properties of Graphs

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# Colouring

Informally: assigning a “colour” to each vertex (e.g. a node in an electric or transportation network) so that the vertices connected by an edge have different colours.

Formally: A mapping  $c: V \rightarrow [1 \dots n]$  such that for every  $e = (v, w) \in E$



$$c(v) \neq c(w)$$

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The minimum  $n$  sufficient to effect such a mapping is called the **chromatic number** of a graph  $G = (E, V)$  and is denoted  $\chi(G)$ .

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NB

This notion is extremely important in operations research, esp. in scheduling.

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There is a dual notion of ‘edge colouring’ — two edges that share a vertex need to have different colours. Curiously enough, it is much less useful in practice.

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# Properties of the Chromatic Number

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- $\chi(K_n) = n$
- If  $G$  has  $n$  vertices and  $\chi(G) = n$  then  $G = K_n$



## Proof.

Suppose that  $G$  is ‘missing’ the edge  $(v, w)$ , as compared with  $K_n$ .  
Colour all vertices, except  $w$ , using  $n - 1$  colours. Then assign to  $w$  the same colour as that of  $v$ .

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- If  $\chi(G) = 1$  then  $G$  is totally disconnected: it has 0 edges.
- If  $\chi(G) = 2$  then  $G$  is bipartite.
- For any tree  $\chi(T) = 2$ .
- For any cycle  $C_n$ , its chromatic number depends on the parity of  $n$  — for  $n$  even  $\chi(C_n) = 2$ , while for  $n$  odd  $\chi(C_n) = 3$ .

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# Cliques

Graph  $(V', E')$  subgraph of  $(V, E)$  —  $V' \subseteq V$  and  $E' \subseteq E$ .

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## Definition

A **clique** in  $G$  is a complete subgraph of  $G$ . A clique of  $k$  nodes is called  $k$ -clique.



The size of the largest clique is called the *clique number* of the graph and denoted  $\kappa(G)$ .

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## Theorem

$$\chi(G) \geq \kappa(G).$$

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## Proof.

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Every vertex of a clique requires a different colour, hence there must be at least  $\kappa(G)$  colours.



However, this is the only restriction. For any given  $k$  there are graphs with  $\kappa(G) = k$ , while  $\chi(G)$  can be arbitrarily large.

NB

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This fact (and such) is very important in the analysis of parallel computation.



- $\kappa(K_n) = n$ ,  $\kappa(K_{m,n}) = 2$ ,  $\kappa(K_{m_1, \dots, m_r}) = r$ .
- If  $\kappa(G) = 1$  then **WeChat: tutorcs**
- For a tree  $\kappa(T) = 2$ .
- For a cycle  $C_n$ ,  $\kappa(C_3) = 3$ ,  $\kappa(C_4) = 2$ ,  $\kappa(C_5) = 3$ .

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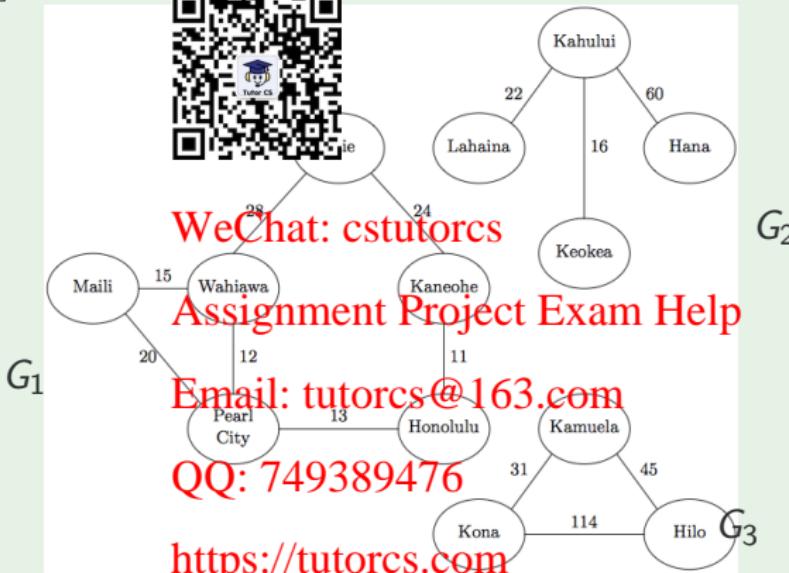
The difference between  $\kappa(G)$  and  $\chi(G)$  is apparent with just **QQ:749389476**.  
 $\kappa(G) = 2$  — this does not imply that  $G$  is bipartite. For example, the cycle  $C_n$  for any odd  $n$  has  $\chi(C_n) = 3$ .

# Exercise

## Exercise

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RW: 9.10.1 (Ullmann)



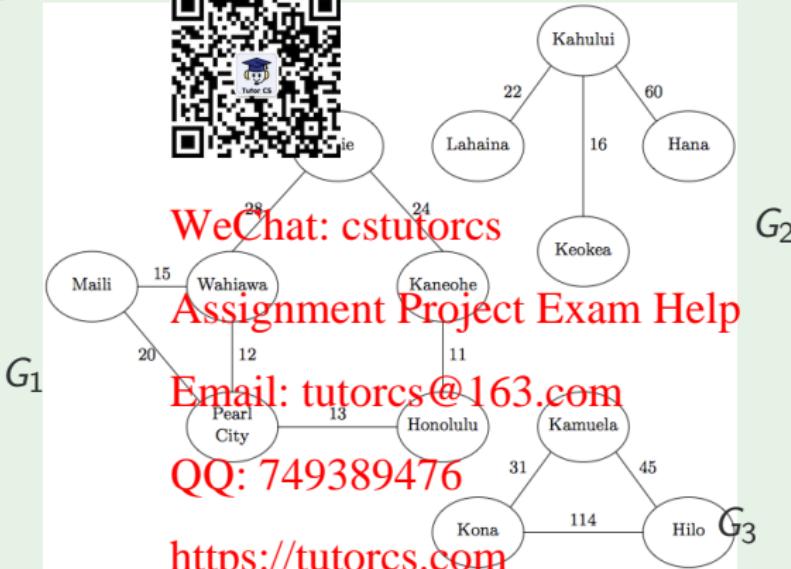
$\chi(G_i)$ ?  $\kappa(G_i)$ ?

# Exercise

## Exercise

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RW: 9.10.1 (Ullmann)



$$\chi(G_i)? \quad \kappa(G_i)? \quad \chi(G_1) = \kappa(G_1) = 3; \quad \chi(G_2) = \kappa(G_2) = 2; \\ \chi(G_3) = \kappa(G_3) = 3$$

# Exercise

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## Exercise

RW: 9.10.3 (Ullmann, 2011) Let  $(V, E)$  be an undirected graph.

What inequalities must hold between

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- the maximal  $\deg(v)$  for  $v \in V$

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- $\chi(G)$

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- $\kappa(G)$

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# Exercise

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## Exercise

RW: 9.10.3 (Ullmann) Let  $(V, E)$  be an undirected graph.

What inequalities must hold between

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- the maximal  $\deg(v)$  for  $v \in V$

- $\chi(G)$

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- $\kappa(G)$

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$$\max_{v \in V} \deg(v) + 1 \geq \chi(G) \geq \kappa(G)$$

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# Timetable scheduling

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The following table contains several subjects and the students taking them:



Potions	Charms	Herbology	WeChat Tutors	Astronomy	Transfiguration
Harry	Ron	Harry	Hermione	Hermione	
Ron	Luna	George	Neville	Fred	
Malfoy	Ginny	Neville	Seamus	Luna	

How many examination timeslots are needed so that no student has two (or more) exams at the same time? 3

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# Planar Graphs

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## Definition

A graph is **planar** if



embedded in a plane without its

## Theorem

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If the graph is planar it can be embedded (without self-intersections) in a plane so that all its edges are straight lines.

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## NB

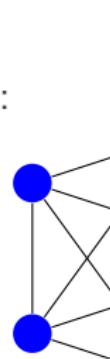
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This notion and its related algorithms are extremely important to VLSI and visualizing <https://tutorcs.com>

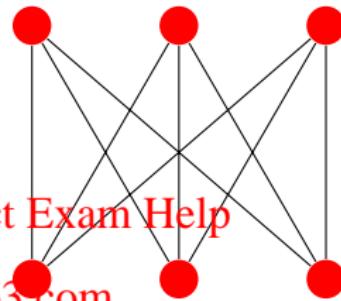
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## Two minimal nonplanar graphs

$K_5$  :



$K_{3,3}$  :



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Try out  $K_5$

Try out  $K_{3,3}$

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# Exercise

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## Exercise

RW: 9.10.2

(Ullmann)



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Is (the undirected version of) this graph planar?

Try it out

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# Exercise

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## Exercise

RW: 9.10.2

(Ullmann)



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Is (the undirected version of) this graph planar? Yes

Try it out

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# Three utilities problem

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Can you connect all utilities to all houses without crossing connections? No!

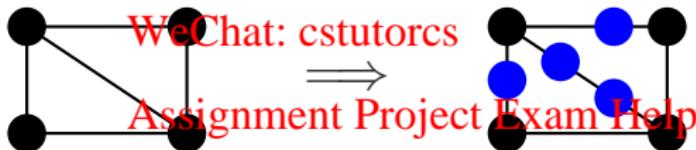
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## Theorem

If graph  $G$  contains, as a subgraph, a nonplanar graph, then  $G$  itself is nonplanar.



For a graph, edge subdivision means to introduce some new vertices, all of degree 2, placing them on existing edges.



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We call such a derived graph a *subdivision* of the original one.

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## Theorem

If a graph is nonplanar then it must contain a subdivision of  $K_5$  or  $K_{3,3}$ .

### Theorem

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$K_n$  for  $n \geq 5$  is nonplanar.



### Proof.

It contains  $K_5$ : choose 5 vertices in  $K_n$  and consider the subgraph they define. □

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### Theorem

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$K_{m,n}$  is nonplanar when  $m \geq 3$  and  $n \geq 3$ .

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### Proof.

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They contain  $K_{3,3}$  — choose any three vertices in each of two vertex parts and consider the subgraph they define. □

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## Question

Are all  $K_{m,1}$  planar?



## Answer

Yes, they are trees of two levels – the root and  $m$  leaves.

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## Question

Are all  $K_{m,2}$  planar? 程序代写代做 CS编程辅导

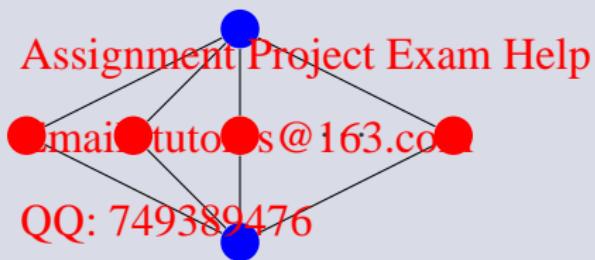
## Answer

Yes; they can be represented by “glueing” together two such trees at the leaves.



Sketching  $K_{m,2}$

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Also, among the  $k$ -partite graphs, planar are  $K_{2,2,2}$  and  $K_{1,1,m}$ . The latter can be depicted by drawing one extra edge in  $K_{2,m}$ , connecting the top and bottom vertices.

Finding a 'basic' nonplanar obstruction is not always simple



It contains a subdivision  $K_{3,3}$ , but not  $K_5$ .

# Strategy for finding a subdivision

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To show  $G$  contains a subdivision of  $H$ :

## Strategy I:



- Start at  $H$
- Perform the following operations as many times as you need:
  - i Subdivide an edge
  - ii Add a vertex
  - iii Add an edge
- Finish with  $G$

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Add a vertex



Add an edge

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## NB

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- Each operation increases  $|V| + |E|$
- Can do all (i) first, then all (ii), then all (iii)

# Strategy for finding a subdivision

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To show  $G$  contains a subdivision of  $H$ :

## Strategy II:

- Start at  $G$
- Perform the following operations as many times as you need:
  - Delete an edge
  - Delete a vertex (and all adjacent edges)
  - Replace a vertex of degree 2 with an edge connecting its neighbours (contracting a vertex)
- Finish with  $H$

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NB

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- Each operation decreases  $|V| + |E|$
- Can do all (i) first, then all (ii), then all (iii)

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# Showing a graph does not contain a subdivision

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## Question

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*What does not change when performing the operations?*

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