COM[illegible]0

Foundations of Computer Science

Lecture 15: Probability

Outline

Elementary Discrete Probability

Independence

Infinite Sample Spaces

Recursive Probability Computations

Conditional Probability

Indpendence, revisited

Outline

Elementary Probability

Definition

Sample space:

$$\Omega = \{\omega_1, \dots, \omega_n\}$$

Each point represents an outcome.

Event: a collection of outcomes = subset of Ω

Probability distribution: A function $P : \mathrm{Pow}(\Omega) \to \mathbb{R}$ such that:

- $P(\Omega) = 1$
- E and F disjoint events then $P(E \cup F) = P(E) + P(F)$.

Fact

$$P(\emptyset) = 0, \quad P(E^c) = 1 - P(E)$$

Examples

Tossing a coin: $\Omega = \{H, T\}$

$$P(H) = P(T) = 0.5$$

Rolling a die: $\Omega = \{1, 2, 3, 4, 5, 6\}$

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

Uniform distribution

Each outcome ω_i equally likely:

$$P(\omega_1) = P(\omega_2) = \ldots = P(\omega_n) = \frac{1}{n}$$

This a called a **uniform probability distribution** over Ω

Examples

Tossing a coin: $\Omega = \{H, T\}$

$$P(H) = P(T) = 0.5$$

Rolling a die: $\Omega = \{1, 2, 3, 4, 5, 6\}$

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

Computing Probabilities by Counting

Computing probabilities with respect to a *uniform* distribution comes down to coun[illegible]ze of the event.
If $E = \{e_1, \ldots, e_k\}$ t[illegible]

$$P(E) = \sum_{i=1}^{k} P(e_i) = \sum_{i=1}^{k} \frac{1}{|\Omega|} = \frac{|E|}{|\Omega|}$$

Most of the counting rules carry over to probabilities wrt. a uniform distribution.

Important!

The expression “selected at random”, when not further qualified, means:

“subject to / according to / . . . a *uniform* distribution.”

Combining events

We can create complex events by combining simpler ones. Common constructions:

- A and B: $A \cap B$
- A or B: $A \cup B$
- Not A: $\Omega \setminus A$
- A followed by B

The first three involve events from the same set of outcomes. The last may involve events from different sets of outcomes (e.g. roll die and flip coin).

Inclusion-exclusion rule

Fact

$$
\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\
&\quad - P(A \cap B) - P(B \cap C) - P(C \cap A) \\
&\quad + P(A \cap B \cap C)
\end{aligned}
$$

Exercises

$\boxed{\text{RW: 5.2.7}}$ Suppose an experiment leads to events A, B with probabilities $P(A) =$ [illegible] $= 0.8$, $P(A \cap B) = 0.4$.
Find

- $P(B^c)$
- $P(A \cup B)$
- $P(A^c \cup B^c)$

$\boxed{\text{RW: 5.2.8}}$ Given $P(A) = 0.6$, $P(B) = 0.7$, show $P(A \cap B) \geq 0.3$

Examples

Example

A four-digit number [illegible]ed at random (i.e. randomly from $[1000 \ldots 9999]$). Fin[illegible]ability p that n has each of 0, 1, 2 among its digits.

Let $q = 1 - p$ be the complementary probability and define

$$A_i = \{n : \text{no digit } i\}, A_{ij} = \{n : \text{no digits } i, j\}, A_{ijk} = \{n : \text{no } i, j, k\}$$

Then define

$T = A_0 \cup A_1 \cup A_2 = \{n : \text{ missing at least one of } 0, 1, 2\}$

$S = (A_0 \cup A_1 \cup A_2)^c = \{n : \text{ containing each of } 0, 1, 2\}$

Example (cont'd)

Once we find the car[...] T, the solution is

$$q = \frac{|T|}{9000}, \; p = 1 - q$$

To find $|A_i|, |A_{ij}|, |A_{ijk}|$ we reflect on how many choices are available for the first digit, for the second etc. A special case is the leading digit, which must be $1, \ldots, 9$

Examples

Example (cont'd)

$$
\begin{aligned}
|A_0| &= 9^4, \quad |A_1| = |A_2| = 8 \cdot 9^3 \\
|A_{01}| &= |A_{02}| = 8^4, \quad |A_{12}| = 7 \cdot 8^3 \\
|A_{012}| &= 7^4
\end{aligned}
$$

$$
\begin{aligned}
|T| &= |A_0 \cup A_1 \cup A_2| \\
&= |A_0| + |A_1| + |A_2| - |A_0 \cap A_1| - |A_0 \cap A_2| - |A_1 \cap A_2| \\
&\qquad + |A_0 \cap A_1 \cap A_2| \\
&= 9^4 + 2 \cdot 8 \cdot 9^3 - 2 \cdot 8^4 - 7 \cdot 8^3 + 7^4 \\
&= 25 \cdot 9^3 - 23 \cdot 8^3 + 7^4 = 8850
\end{aligned}
$$

$$
q = \frac{8850}{9000}, \quad p = 1 - q \quad \approx 0.01667
$$

Example

Previous example ge[illegible] Probability of an r-digit number having all of 0,1,2,3 [illegible] digits.
We use the previous notation: A_i — set of numbers n *missing* digit i, and similarly for all $A_{ij\ldots}$.
We aim to find the size of $T = A_0 \cup A_1 \cup A_2 \cup A_3$, and then to compute $|S| = 9 \cdot 10^{r-1} - |T|$.

$$\begin{aligned}|A_0 \cup A_1 \cup A_2 \cup A_3| &= \text{sum of } |A_i| \\ &- \text{sum of } |A_i \cap A_j| \\ &+ \text{sum of } |A_i \cap A_j \cap A_k| \\ &- \text{sum of } |A_i \cap A_j \cap A_k \cap A_l|\end{aligned}$$

Exercises

Exercises

$\boxed{\text{RW: 5.6.38}}$ (Supp) C[illegible]blems, 75 are ‘easy’ and 40 ‘important’.

(b) n problems chosen randomly. What is the probability that all n are important?

Exercises

Exercises

$\boxed{\text{RW: 5.2.3}}$ A 4-letter word is selected at random from Σ^4, where $\Sigma = \{a, b, c, d, e\}$. Wh[illegible]robability that

(a) the letters in the w[illegible]distinct?

(b) there are no vowels (“a”, “e”) in the word?

(c) the word begins with a vowel?

Outline

- Elementary Discrete [illegible]
- Independence
- Infinite Sample Spaces
- Recursive Probability Computations
- Conditional Probability
- Indpendence, revisited

Unifying sets of outcomes

To combine events from different sets of outcomes we unify the sample space using the **product space**: $\Omega_1 \times \Omega_2 \times \ldots \times \Omega_n$.

Example

Flipping a coin and rolling a die:

$$\Omega_1 = \{\text{heads}, \text{tails}\} \qquad \Omega_2 = \{1, 2, 3, 4, 5, 6\}$$

$$\Omega = \Omega_1 \times \Omega_2 = \{(\text{heads}, 1), (\text{heads}, 2), \ldots\}$$

NB

This approach can also be used to model sequences of outcomes.

Events in the product space

Events are lifted into the product space by restricting the appropriate co-ordinate. E.g. $A \subseteq \Omega_1$ translates to $A' = A \times \Omega_2 \times \ldots \times$

Example

Coin shows heads and die shows an even number:

$$\Omega_1 = \{\text{heads}, \text{tails}\} \qquad A = \{\text{heads}\}$$
$$\Omega_2 = \{1, 2, 3, 4, 5, 6\} \qquad B = \{2, 4, 6\}$$

$$\Omega = \Omega_1 \times \Omega_2 = \{(\text{heads}, 1), (\text{heads}, 2), \ldots\}$$
$$A' = A \times \Omega_2 \qquad B' = \Omega_1 \times B$$

"A and B" or "A followed by B" corresponds to:

$$A' \cap B' = (A \times \Omega_2) \cap (\Omega_1 \times B) = A \times B$$

Probability in the product space

NB

Cannot assume that $P(A \times B) = P(A)P(B)$

Example

Toss two coins.

- A: First coin shows heads
- B: Both coins show tails

$$
\begin{array}{ll}
\Omega_1 = \{H, T\} & \Omega_2 = \{HH, HT, TH, TT\} \\
A = \{H\} & A' = \{(H, HH), (H, HT), (H, TH), (H, TT)\} \\
B = \{TT\} & B' = \{(H, TT), (T, TT)\} \\
A' \cap B' = A \times B = \{(H, TT)\} &
\end{array}
$$

$$P(A) = \frac{1}{2} \quad P(B) = \frac{1}{4} \quad P(A' \cap B') = 0$$

Product distribution

Given probability distributions on the component spaces, there is a natural probability distribution on the product space:

$$P(E_1 \times E_2 \times \cdots \times E_n) = P_1(E_1) \cdot P_2(E_2) \cdots P_n(E_n)$$

Intuitively, the probability of an event in one dimension is not affected by the outcomes in the other dimensions.

Fact

If the P_i are uniform distributions then so is the product distribution.

Independence

Informally, events are *independent* if the outcomes in one do not affect the outcomes in the other.
More generally, we define independence on events of the **same** sample space.

Definition

A and B are **(stochastically) independent** (notation: $A \perp B$) if
$P(A \cap B) = P(A) \cdot P(B)$

NB

Informal notion of independence corresponds to the stochastic independence of the "lifted" events A' *and* B'

Important!

Unless specified otherwise, we assume independence when unifying events (where appropriate).

Independence of multiple events

Independence of $A_1, \ldots, A_n$ ($A_1 \perp A_2 \perp \cdots \perp A_n$)

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdots P(A_{i_k})$$

This is often called (for emphasis) a *full* independence

Pairwise independence is a *weaker* concept.

Example

Toss of two coins

$$\left.\begin{array}{l} A = \langle \text{first coin } H \rangle \\ B = \langle \text{second coin } H \rangle \\ C = \langle \text{exactly one } H \rangle \end{array}\right\} \begin{array}{l} P(A) = P(B) = P(C) = \frac{1}{2} \\ P(A \cap B) = P(A \cap C) = P(B \cap C) = \frac{1}{4} \\ \text{However: } P(A \cap B \cap C) = 0 \end{array}$$

One can similarly construct a set of n events where any k of them are independent, while any $k+1$ are dependent (for $k < n$).

Example: Dependent events

Example

Basic non-independent events (assuming non-trivial probabilities)

- $A \subseteq B$
- $A \cap B = \emptyset$
- Any pair of one-point events $A = \{x\}$, $B = \{y\}$:
  either $x = y$ and $A \subseteq B$
  or $x \neq y$ and $A \cap B = \emptyset$

Exercise

Exercise

$\boxed{\text{RW: 9.1.25}}$ Does $A \perp B \perp C$ imply $(A \cap B) \perp (A \cap C)$?

Example: Sequences of independent events

Example

Team A has probability $p = 0.5$ of winning a game against B. What is the probability of A winning a best-of-seven match if

- (a) A already won the first game?
- (b) A already won the first two games?
- (c) A already won two out of the first three games?

(a) Sample space S — 6-sequences formed from wins (W) and losses (L)

$$|S| = 2^6 = 64$$

Favourable sequences F — those with three to six W

$$|F| = \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6} = 20 + 15 + 6 + 1 = 42$$

Therefore $P_{0.5} = \frac{42}{64} \approx 66\%$(b) Sample space S — 5-sequences of W and L

$$|S| = 2^5 = 32$$

Binomial distribution

A useful corollary:

Fact

In a sequence of n independent trials, each with a probability of p of success:

$$P(\textit{exactly } k \textit{ successes}) = \binom{n}{k} p^k q^{n-k}$$

where $q = (1 - p)$.

NB

This leads to a probability distribution on sequences of outcomes, known as the **binomial distribution**.

Exercises

Exercise

RW: 5.2.11 Two dice, a red die and a black die, are rolled. What is the probability that

(a) the sum of the values is even?

(b) the number on the red die is bigger than on the black die?

(c) the number on the black die is twice the one on the red die?

Exercises

Exercise

$\boxed{\text{RW: 5.2.12}}$ Two dice, a red die and a black die, are rolled. What is the probabili

(a) the maximum o bers is 4?

(b) their minimum is 4?

Exercise

Exercises

RW: 5.2.5 An urn co[illegible]ed and 4 black balls. 3 balls are removed without rep[illegible] What are the probabilities that

(a) all 3 are red

(b) all 3 are black

(c) one is red, two are black

Outline

Elementary Discrete [illegible]y

Independence

Infinite Sample Spaces

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Infinite sample spaces

Probability distributions generalize to infinite sample spaces **with some provisos**.

- In continuous spaces (e.g. $\mathbb{R}$):
  - Probability distributions are *measures*;
  - Sums are *integrals*;
  - Non-zero probabilities apply to *ranges*;
  - Probability of a single event is 0. Note: Probability 0 is not the same as impossible.
- In discrete spaces (e.g. $\mathbb{N}$):
  - Probability 0 is the same as impossible.
  - No uniform distribution!
  - Non-uniform distributions exist, e.g. $P(0) = 1$, $P(n) = 0$ for $n > 0$; or $P(0) = 0$, $P(n) = \frac{1}{2^n}$ for $n > 0$.
  - May consider limiting probabilities if that makes sense.

Asymptotic Estimate of Relative Probabilities

Example

Event $A \overset{\text{def}}{=}$ one die rolled n times and you obtain two 6's
Event $B \overset{\text{def}}{=}$ n dice rolled simultaneously and you obtain one 6

$$P(A) = \frac{\binom{n}{2} \cdot 5^{n-2}}{6^n} \qquad P(B) = \frac{\binom{n}{1} \cdot 5^{n-1}}{6^n}$$

Therefore $\frac{P(A)}{P(B)} = \frac{\binom{n}{2}}{\binom{n}{1}} \cdot \frac{1}{5} = \frac{n(n-1)}{2} \cdot \frac{1}{5n} = \frac{n-1}{10} \in \Theta(n)$

n	1	2	3	4	...	11	...	20	...
$P(A)$	0	$\frac{1}{36}$	$\frac{5}{72}$	$\frac{25}{216}$	...	0.296	...	0.198	...
$P(B)$	$\frac{1}{6}$	$\frac{10}{36}$	$\frac{25}{72}$	$\frac{125}{324}$	...	0.296	...	0.104	...

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Use of Recursion in Probability Computations

Question

Given n tosses of a c[illegible] *is the probability of two* HEADS *in a row?*

Answer

Recall $N(n)$: the number of sequences without HH.

$$N(n) = N(n-1) + N(n-2): \; N(n) = \textsc{fib}(n+1)$$

$$N(n) \approx \frac{1}{\sqrt{5}} \left(\frac{\sqrt{5}+1}{2}\right)^{n+1} \approx 0.72 \cdot (1.6)^n$$

$$p_n = 1 - \frac{\textsc{fib}(n+1)}{2^n} \approx 1 - 0.72 \cdot (0.8)^n$$

Example

Question

Given n tosses, what is the probability q_n of at least one HHH?

$q_0 = q_1 = q_2 = 0$; q_3

Then recursive computation:

$$
\begin{aligned}
q_n = {} & \frac{1}{2} q_{n-1} && \text{(initial: T)} \\
& + \frac{1}{4} q_{n-2} && \text{(initial: HT)} \\
& + \frac{1}{8} q_{n-3} && \text{(initial: HHT)} \\
& + \frac{1}{8} && \text{(start with: HHH)}
\end{aligned}
$$

Example

Question

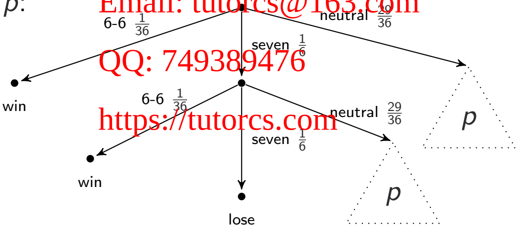
Two dice are rolled repeatedly. What is the probability that '6–6' will occur before two … *ive (back-to-back) 'totals seven'?*

NB

The probability of either occurring at a given roll is the same: $\frac{1}{36}$.

Let $p = P(\text{6–6 first})$

p:



Example

Question

A coin is tossed 'indefinitely'. Which pattern is more likely (and by how much) to appea[illegible]H or HHT?

Difficult probability calculations

NB

The majority of problems in probability and statistics do not have such elegant solutions. Hence the use of computers for either precise calculations or approximate simulations is mandatory. However, it is the use of recursion that simplifies such computing or, quite often, makes it possible in the first place.

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Conditional probability

Definition

Conditional probabi[illegible]**iven** S:

$$P(E|S) = \frac{P(E \cap S)}{P(S)}, \quad E, S \subseteq \Omega$$

It is defined only when $P(S) \neq 0$

NB

$P(A|B)$ and $P(B|A)$ are, in general, not related — one of these values predicts, by itself, essentially nothing about the other. The only exception, applicable when $P(A), P(B) \neq 0$, is that $P(A|B) = 0$ iff $P(B|A) = 0$ iff $P(A \cap B) = 0$.

Conditional probability in uniform distributions

If P is the uniform distribution over a finite set Ω, then

$$P(E \mid S) = \frac{\frac{|E\cap S|}{|\Omega|}}{\frac{|S|}{|\Omega|}} = \frac{|E \cap S|}{|S|}$$

This observation can [illegible] calculations...

Example

RW: 9.1.6 A coin is tossed four times. What is the probability of

(a) two consecutive HEADS

(b) two consecutive HEADS *given* that ≥ 2 tosses are HEADS

Conditional probability in uniform distributions

If P is the uniform distribution over a finite set Ω, then

$$P \ldots \frac{\frac{|E\cap S|}{|\Omega|}}{\frac{|S|}{|\Omega|}} = \frac{|E\cap S|}{|S|}$$

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T	T	T	T	H	T	T	T
T	T	T	H	H	T	T	H
T	T	H	T	H	T	H	T
T	T	H	H	H	T	H	H
T	H	T	T	H	H	T	T
T	H	T	H	H	H	T	H
T	H	H	T	H	H	H	T
T	H	H	H	H	H	H	H

Conditional probability in uniform distributions

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T	T	T	H	H	T	T	H
T	T	H	T	H	T	H	T
T	T	H	H	H	T	H	H
T	H	T	T	H	H	T	T
T	H	T	H	H	H	T	H
T	H	H	T	H	H	H	T
T	H	H	H	H	H	H	H

(a) $\frac{8}{16}$

Conditional probability in uniform distributions

If P is the uniform distribution over a finite set Ω, then

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This observation can … alculations...

Example

$\boxed{\text{RW: 9.1.6}}$ A coin is tossed four times. What is the probability of

(a) two consecutive HEADS

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T	T	T	T	H	T	T	T
T	T	T	H	H	T	T	H
T	T	H	T	H	T	H	T
T	T	H	H	H	T	H	H
T	H	T	T	H	H	T	T
T	H	T	H	H	H	T	H
T	H	H	T	H	H	H	T
T	H	H	H	H	H	H	H

(a) $\frac{8}{16}$ (b) $\frac{8}{11}$

Some General Rules

Fact

- $A \subseteq B \to P(A|$ [illegible] $)$
- $A \subseteq B \to P(B|$ [illegible]
- $P(A \cap B|B) = P(A|B)$
- $P(\emptyset|A) = 0$ *for* $A \neq \emptyset$
- $P(A|\Omega) = P(A)$
- $P(A^c|B) = 1 - P(A|B)$

NB

- $P(A|B)$ *and* $P(A|B^c)$ *are not related*
- $P(A|B), P(B|A), P(A^c|B^c), P(B^c|A^c)$ *are not related*

Example

Example

Two dice are rolled and the outcomes recorded as b for the black die, r for the red die and $s = b + r$ for their total.
Define the events $B = \{b \geq 3\}$, $R = \{r \geq 3\}$, $S = \{s \geq 6\}$.

$$P(S|B) = \frac{4+5+6+6}{24} = \frac{21}{24} = \frac{7}{8} = 87.5\%$$

$$P(B|S) = \frac{4+5+6+6}{26} = \frac{21}{26} = 80.8\%$$

The (common) numerator $4 + 5 + 6 + 6 = 21$ represents the size of the $B \cap S$ — the common part of B and S, that is, the number of rolls where $b \geq 3$ and $s \geq 6$. It is obtained by considering the different cases: $b = 3$ and $s \geq 6$, then $b = 4$ and $s \geq 6$ etc.

The denominators are $|B| = 24$ and $|S| = 26$

Example

Example (cont'd)

Recall: $B = \{b \geq 3\}$, $R = \{r > 3\}$, $S = \{s \geq 6\}$

$$P([illegible]R) = 2/3 = 66.7\%$$

$$P(S) = \frac{5+6+5+4+3+2+1}{36} = \frac{26}{36} = 72.22\%$$

$$P(S|B \cup R) = \frac{2+3+4+5+6+6}{32} = \frac{26}{32} = 81.25\%$$

The set $B \cup R$ represents the event 'b or r'.
It comprises all the rolls except for those with *both* the red and the black die coming up either 1 or 2.

$$P(S|B \cap R) = 1 = 100\% \text{ — because } S \supseteq B \cap R$$

Exercise

Exercise

RW: 9.1.9 Consider t[illegible] and eight black marbles; draw two without replacement. [illegible] b_1 — Black on the first draw, b_2 — Black on the s[illegible]w, r_1 — Red on first draw, r_2 — Red on second draw

Find the probabilities

(a) both Red:

Exercise

(b) both Black:

(c) one Red, one Bla

Exercise

Exercise

RW: 9.1.12 What is the probability of a flush given that all five cards in a Poker hand are red?

Exercise

Exercise

$\boxed{\text{RW: 9.1.22}}$ Prove the following:

If $P(A|B) > P(A)$ ("[illegible]orrelation") then $P(B|A) > P(B)$

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Stochastic Independence, again

Definition

A and B are **(stochastically) independent** (notation: $A \perp B$) if $P(A \cap B) = P(A) \cdot P(B)$

If $P(A) \neq 0$ and $P(B) \neq 0$, all of the following are *equivalent* definitions:

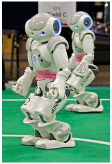
- $P(A \cap B) = P(A)P(B)$
- $P(A|B) = P(A)$
- $P(B|A) = P(B)$
- $P(A^c|B) = P(A^c)$ or $P(A|B^c) = P(A)$ or $P(A^c|B^c) = P(A^c)$

The last one claims that

$$A \perp B \leftrightarrow A^c \perp B \leftrightarrow A \perp B^c \leftrightarrow A^c \perp B^c$$

Using independence to simplify calculations

Independence of events, not just pairwise independence, can greatly simplify computations and reasoning in AI applications. It is common for many expert systems to make an approximating assumption of independence, even if it is not completely satisfied.



$$P(\mathsf{sense}_t \mid \mathsf{loc}_t, \mathsf{sense}_{t-1}, \mathsf{loc}_{t-1}, \ldots) = P(\mathsf{sense}_t \mid \mathsf{loc}_t)$$

Exercise

Exercise

$\boxed{\text{RW: 9.1.7}}$ Suppose t[illegible]periment leads to events A, B and C with $P(A) = 0.3$, $P($[illegible] and $P(A \cap B) = 0.1$

(a) $P(A|B) = \frac{P(A \cap B)}{P(B)}$ [illegible]

(b) $P(A^c) = 1 - P(A) = 0.7$

(c) Is $A \perp B$? No. $P(A) \cdot P(B) = 0.12 \neq P(A \cap B)$

(d) Is $A^c \perp B$? No, as can be seen from (c).

Note: $P(A^c \cap B) = P(B) - P(A \cap B) = 0.4 - 0.1 = 0.3$
$P(A^c) \cdot P(B) = 0.7 \cdot 0.4 = 0.28$

Exercise

Exercise

$\boxed{\text{RW: 9.1.8}}$ Given $A \perp B$, $P(A) = 0.4$, $P(B) = 0.6$

$P(A|B) =$

$P(A \cup B) =$

$P(A^c \cap B) =$

Supplementary Exercise

Exercise

$\boxed{\text{RW: 9.5.5}}$ (Supp) W... n two events with

$P(A) = \frac{1}{4}$, $P(B) = $ ...

True, false or could b...

a. $P(A \cap B) = \frac{1}{12}$
b. $P(A \cup B) = \frac{7}{12}$
c. $P(B|A) = \frac{P(B)}{P(A)}$
d. $P(A|B) \geq P(A)$
e. $P(A^c) = \frac{3}{4}$
f. $P(A) = P(B)P(A|B) + P(B^c)P(A|B^c)$