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University of New South Wales
School of Computer Science and Engineering
Foundations of Computer Science (COMP9020)
FINAL EXAM — Session 1, 2017

This paper must be submitted and cannot be retained by the student

Instructions:

- **Ensure you enter your correct name and student number above!**
- This exam paper contains 10 multiple-choice questions (pages 1-3) plus 5 open questions (pages 4-8).
  Each multiple-choice question is worth 4 marks ($10 \times 4 = 40$).
  Each open question is worth 12 marks ($5 \times 12 = 60$).
  Total exam marks = 100.

- Only use a blue or black pen. **All answers must be recorded in this paper.**
- For the multiple-choice questions, tick **one** box for your answer directly (each multiple-choice question has only one correct answer).
  To make a correction, tick *all* boxes, then *circle* one box for your answer.
- For the open questions, write your answer in the space provided (if you need more space, you can write on the back of the sheet).
- A separate white booklet is provided for scratch work only. **Do not write your answers in the Examination Answer Book, it will not be marked.**
- Time allowed – 120 minutes + 10 minutes reading time.
- The exam is *closed book*. Reference materials are not allowed, apart from one A4-sized sheet (double-sided is ok) of your own notes.
- Number of pages in this exam paper: 8 (in addition to this cover sheet).

1. How m[illegible]nterval $[-100, 100]$ are divisible by 5 **or** 7 (or both)?

   - ☐ 6[illegible]
   - ☒ 6[illegible]
     $N = \lfloor 100/5 \rfloor + \lfloor 100/7 \rfloor - \lfloor 100/35 \rfloor) + 1 = 2 \cdot (20 + 14 - 2) + 1 = 65$
   - ☐ 67
   - ☐ 68

2. Consider the alphabets $\Sigma = \{s, e, a\}$ and $\Psi = \{a, r, t\}$. How many words are in the set $\{\omega \in (\Sigma \setminus \Psi)^* : \text{length}(\omega) \leq 2\}$?

   - ☐ 2
   - ☐ 4
   - ☐ 6

   - ☒ 7

3. Which of the following is **not** a correct equivalence?

   - ☒ $\neg A \vee B \equiv \neg(B \wedge \neg A)$
   - ☐ $A \wedge \neg B \equiv \neg(B \vee \neg A)$

   - ☐ $A \Rightarrow \neg B \equiv B \Rightarrow \neg A$
   - ☐ $\neg(A \Rightarrow B) \equiv \neg B \wedge A$

4. Consider the functions $f : \mathbb{N} \longrightarrow \{0, 1, 2\}$ and $g : \{0, 1, 2\} \longrightarrow \{0, 1, 2\}$ defined by

$$f(x) = x \bmod 3$$
$$g(x) = |x - 2|$$

   Which of the following statements is true?

   - ☐ $f \circ f \neq f$
   - ☒ $g \circ g = \text{Id}_{\{0,1,2\}}$
   - ☐ $f \circ g$ is **not** onto
   - ☐ $g \circ f$ is **not** onto

5. Consid  on $S = \{1, 2, 3, 4, 6, 12\}$ defined by

   y if $x \mid y$ (i.e., x is a divisor of y)

   Which t true?

   - ☐ lu
   - ☒ glb($\{4, 6, 12\}$) = 1
     correct is glb($\{4, 6, 12\}$) = 2
   - ☐ $(S, \leq)$ is a lattice
   - ☐ $1 < 3 < 2 < 6 < 4 < 12$ is a topological sort of $(S, \leq)$

6. All connected graphs with n vertices and k edges satisfy

   - ☐ $n \geq k + 1$
   - ☐ $n \geq k$
   - ☐ $n \leq k$
   - ☒ $n \leq k + 1$
     a tree has $k + 1$ vertices

7. We would like to prove that $P(n)$ for all $n \geq 0$.
   Which of the following conditions imply this conclusion?

   - ☐ $P(0)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(n + 1))$
   - ☐ $P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \wedge P(n + 1) \Rightarrow P(n + 2))$
   - ☒ $P(0)$ and $P(1)$ and $\forall n \geq 0\,(P(n) \wedge P(n + 1) \Rightarrow P(n + 2))$
     True
   - ☐ $P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(n + 2))$

8. Consid  n by $T(1) = 1$ and $T(n) = 4 \cdot T(\frac{n}{2}) + n$.
This ha

- ☐ O
- ☐ O
- ☒ O
  master theorem
- ☐ $O(2^n)$

9. Let $S = \{1, 2, 3\}$ and $\mathbb{B} = \{0, 1\}$.
How many different *onto* functions $f : S \longrightarrow \mathbb{B}$ are there?

- ☐ 0
- ☒ 6
  $2^3 - 2 = 6$ since there are $|\mathbb{B}|^{|S|} = 2^3$ functions in total, and two of them are not onto: $f_1 : s \mapsto 0$ and $f_2 : s \mapsto 1$
- ☐ 8
- ☐ 9

10. Which of the following is true for all A, B?

- ☒ $P(A \cap B|B) \geq P(A|B)$
- ☐ $P(A \cap B) = P(B) \cdot P(B|A)$
- ☐ $P(A \cup B) \geq P(A) + P(B)$
- ☐ $P(A|B) + P(A|\bar{B}) = 1$

11. Consid formulae:

$$\phi = \neg(A \Rightarrow (B \wedge C))$$
$$\psi = \neg A \vee C$$

(a) T *nctive* normal form (DNF).
(b) P .e., $\neg B$ is a logical consequence of ϕ and ψ).
(c) Is $\phi \vee \psi$ a tautology (i.e., always true)? **Explain your answer.**

(a) $\overline{\overline{A} + BC} = \overline{\overline{A}} \cdot \overline{BC} = A \cdot (\overline{B} + \overline{C}) = AB + A\overline{C}$

(b) From ψ it follows that $\neg(A \wedge \neg C)$.
From (a) it then follows that $A \wedge \neg B$, which implies $\neg B$.

Alternative solution using a truth table:

A	B	C	ϕ	ψ	$\neg B$
F	F	F	F	T	T
F	F	T	F	T	T
F	T	F	F	T	F
F	T	T	F	T	F
T	F	F	T	F	T
T	F	T	T	T	T
T	T	F	F	F	F
T	T	T	F	T	F

(c) $\phi \vee \psi$ is always true:
Case 1: A is false or C is true. Then ψ is true.
Case 2: Case 1 is false, then $A \wedge \neg C$, hence ϕ is true according to (a).

Alternative solution extends the truth table from above by $\phi \vee \psi$.

12. Prove t[illegible]ions $\mathcal{R}_1 \subseteq S \times S$ and $\mathcal{R}_2 \subseteq S \times S$ the following holds:

[illegible]ymmetric, then $\mathcal{R}_1 \setminus \mathcal{R}_2$ is symmetric.

If $(x,$ [illegible]$) \in \mathcal{R}_1$ and $(x, y) \notin \mathcal{R}_2$.

By symmetry of $\mathcal{R}_1$ and $\mathcal{R}_2$ it follows that $(y, x) \in \mathcal{R}_1$ and $(y, x) \notin \mathcal{R}_2$.

Hence $(y, x) \in \mathcal{R}_1 \setminus \mathcal{R}_2$.

Alternative proof by contradiction:

If $\mathcal{R}_1 \setminus \mathcal{R}_2$ is not symmetric, then there exist $x, y \in S$ such that $(x, y) \in \mathcal{R}_1$ and $(x, y) \notin \mathcal{R}_2$ but $(y, x) \notin \mathcal{R}_1 \setminus \mathcal{R}_2$.

From $(y, x) \notin \mathcal{R}_1 \setminus \mathcal{R}_2$ it follows that $(y, x) \notin \mathcal{R}_1$ or $(y, x) \in \mathcal{R}_2$.

But $(y, x) \notin \mathcal{R}_1$ contradicts $(x, y) \in \mathcal{R}_1$ given that $\mathcal{R}_1$ is symmetric,
and $(y, x) \in \mathcal{R}_2$ contradicts $(x, y) \notin \mathcal{R}_2$ given that $\mathcal{R}_2$ is symmetric.

13. The *Fi* defined as follows:

$= 1; \quad F_i = F_{i-1} + F_{i-2}$ for $i \geq 3$

Write a r the statement that every *third* Fibonacci number (th is even (i.e., divisible by 2).

Base case $n = 3$:

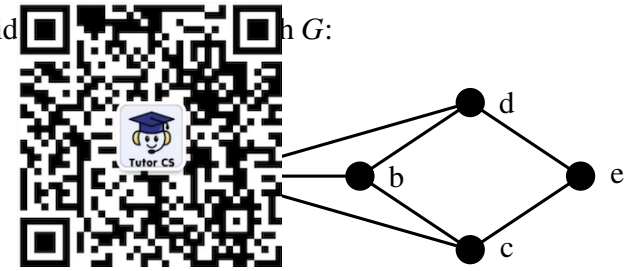
$F_1 = 1;\ F_2 = 1;\ F_3 = 2$. Hence, $2 \mid F_3$.

Inductive step $n \longrightarrow n + 3$: By definition,

$$\begin{aligned} F_{n+3} &= F_{n+2} + F_{n+1} \\ &= (F_{n+1} + F_n) + F_{n+1} \\ &= 2 \cdot F_{n+1} + F_n \end{aligned}$$

From the induction hypothesis $2 \mid F_n$ it follows that $2 \mid (2F_{n+1} + F_n)$.

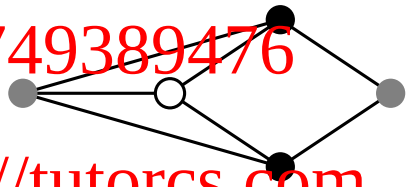
14. Consid[illegible] G:



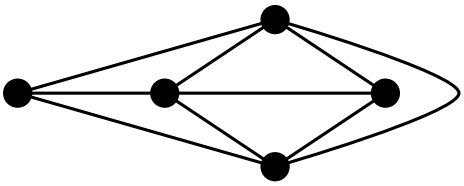
(a) Give all 3-cliques of G.
(b) What is the chromatic number $\chi(G)$ of G? **Explain your answer.**
(c) What is the maximal number of edges that can be added to G such that G remains planar? **Explain your answer.**

(a) {a,b,e}, {a,b,d}

(b) $\chi(G) = 3$.
3 colours are necessary because G contains a 3-clique.
3 colours are also sufficient:



(c) A maximum of 2 edges can be added, for example:



3 edges cannot be added since this would result in K_5, which is not planar.

15. Consid s containing 2 jacks and 4 aces. One card is randon ck at a time. Calculate the expected number of drawin e is drawn:
    (a) if k into the deck after each drawing;
    (b) if back into the deck after each drawing.

    Briefly rs.

(a) Each drawing event has the probability $p = \frac{4}{6} = \frac{2}{3}$.
Hence, the expected number of drawing attempts is $\frac{1}{p} = 1.5$

(b) $1 \cdot \frac{4}{6} + 2 \cdot \frac{2}{6} \cdot \frac{4}{5} + 3 \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot 1 = \frac{2}{3} + \frac{8}{15} + \frac{1}{5} = \frac{21}{15} = \frac{7}{5} = 1.4$

— END OF EXAM —