COMP9444 Neural Networks and Deep Learning Term 2, 2020

Solutions to Exercise 7: Reinforcement Learning

This page was last updated: 07/20/2020 10:25:31

Consider an environment with two states $S = \{S_1, S_2\}$ and two actions $A = \{a_1, a_2\}$, where the (deterministic) transitions δ and reward R for each state and action are as follows:

$$\delta(S_1, a_1) = S_1, R(S_1, a_1) = +1$$

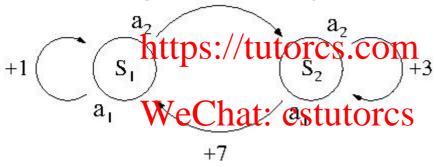
 $\delta(S_1, a_2) = S_2, R(S_1, a_2) = -2$

$$\delta(S_2, a_1) = S_1, R(S_2, a_1) = +7$$

$$\delta(S_2, a_2) = S_2, R(S_2, a_2) = +3$$

1. Draw a picture of this environment, using circles for the states and arrows for the transitions.

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- 2. Assuming a discount factor of γ = 0.7, determine:
 - a. the optimal policy π^* : $S \to A$

$$\pi^*(S_1) = a_2$$

 $\pi^*(S_2) = a_1$

b. the value function $V: S \rightarrow R$

$$V(S_1) = -2 + \gamma V(S_2)$$

$$V(S_2) = +7 + \gamma V(S_1)$$
So $V(S_1) = -2 + 7\gamma + \gamma^2 V(S_1)$
i.e. $V(S_1) = (-2 + 7\gamma)/(1 - \gamma^2) = (-2 + 7 \times 0.7)/(1 - 0.49) = 5.69$
Then $V(S_2) = 7 + 0.7 \times 5.69 = 10.98$

c. the "Q" function Q: $S \times A \rightarrow R$

$$Q(S_1, a_1) = 1 + \gamma V(S_1) = 4.98$$

 $Q(S_1, a_2) = V(S_1) = 5.69$

$$Q(S_2, a_1) = V(S_2) = 10.98$$

 $Q(S_2, a_2) = 3 + yV(S_2) = 10.69$

$$Q(S_2, a_2) = S + \gamma V(S_2) = 10.69$$

Writing the Q values in a matrix, we have:

Q	a ₁	a ₂
S ₁	4.98	5.69
S ₂	10.98	10.69

Trace through the first few steps of the Q-learning algorithm, with a learning rate of 1 and with all Q values initially set to zero. Explain why it is necessary to force exploration through probabilistic choice of actions, in order to ensure convergence to the true Q values.

With a deterministic environment and a learning rate of 1, the Q-Learning update rule is

$$Q(S, a) \leftarrow r(S, a) + \gamma \max_b Q(\delta(S, a), b)$$

Let's assume the agent starts in state S_1 . Since the initial C values are all zero the initial problem be chosen fractionally Xfaction 1 is chosen, the agent will get a reward of +1 and update

$$Q(S_1,a_1) \leftarrow 1 + y \times 0 = 1/tutorcs.com$$

If we do not force exploration, the agent will always prefer action a_1 in state S_1 , and will never explore action a_2 . This means that $Q(S_1, a_2)$ will remain zero forever, instead of the reging to the Salue of 5.69. If we do force exploration, the next steps may look like this:

current state	chosen action	new Q value
S ₁	a ₂	$-2 + \gamma *0 = -2$
S ₂	a ₂	$+3 + \gamma *0 = +3$

At this point, the table looks like this:

Q	a ₁	a ₂
S ₁	1	-2
S ₂	0	3

Again, we need to force exploration, in order to get the agent to choose a_1 from S_2 , and to again choose a_2 from S_1

current state	chosen action	new Q value
S ₂	a ₁	$+7 + \gamma *1 = 7.7$
S ₁	a ₂	$-2 + \gamma*7.7 = 3.39$

Q	a ₁	a ₂
S ₁	1	3.39
S ₂	7.7	3

Further steps will refine the Q value estimates, and, in the limit, they will converge to their true values.

3. Now let's consider how the Value function changes as the discount factor γ varies between 0 and 1.

There are four deterministic policies for this environment, which can be written as π_{11} , π_{12} , π_{21} and π_{22} , where $\pi_{ij}(S_1) = a_i$, $\pi_{ij}(S_2) = a_j$

a. Calculate the value function $V^{\pi}_{(\gamma)}$: $S \to R$ for each of these four policies (keeping γ as a variable)

$$V^{\pi}_{11}(S_1) = +1 + \gamma V^{\pi}_{11}(S_1), \text{ so } V^{\pi}_{11}(S_1) = 1/(1 - \gamma)$$

 $V^{\pi}_{11}(S_2) = +7 + \gamma V^{\pi}_{11}(S_1) = 7 + \gamma/(1 - \gamma)$

$$A_{12}(S_1) = V_{11}(S_1) = 1/(1 - \gamma)$$

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$$V^{\pi}_{21}(S_1) = -2 + 7\gamma + \gamma^2 V^{\pi}_{21}(S_1), \text{ so } V^{\pi}_{21}(S_1) = (-2 + 7\gamma)/(1 - \gamma^2)$$

$$V^{\pi}_{21}(S_2) = (7 - 2\gamma)/(1 - \gamma^2)$$

$$V^{\pi}_{22}(S_1) = -2 + 3\gamma/(1 - \gamma)$$

 $V^{\pi}_{22}(S_2) = -2 + 3\gamma/(1 - \gamma)$ cstutorcs

b. Determine for which range of values of γ each of the policies π_{11} , π_{12} , π_{21} , π_{22} is optimal

 π_{11} is optimal when

$$0 < V^{\pi}_{11}(S_1) - V^{\pi}_{21}(S_1) = ((1 + \gamma) - (-2 + 7\gamma))/(1 - \gamma^2) = (3 - 6\gamma)/(1 - \gamma^2),$$
 i.e. $0 \le \gamma \le 0.5$

 π_{22} is optimal when

$$0 < V^{\pi}_{22}(S_2) - V^{\pi}_{21}(S_2) = (3(1 + \gamma) - (7 - 2\gamma))/(1 - \gamma^2) = (-4 + 5\gamma)/(1 - \gamma^2)$$
, i.e. $0.8 \le \gamma < 1.0$

 π_{21} is optimal for $0.5 \le \gamma \le 0.8$

 π_{12} is never optimal because it is dominated by π_{11} when $\gamma < 2/3$ and by π_{22} when $\gamma > 0.6$