Typed λ with Booleans

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Where x is a variable in the λ -Calculus sense.

• Exclude Vumbers to left things implet for ico S

$$\langle T \rangle ::= \langle T \rangle \Rightarrow \langle T \rangle$$
| Bool

Expanding the definition

hiseging Here to the control of the

- A function mapping a Boolean argument to a Boolean result.
- Bool ⇒ Bool ⇒ Bool Boolean argument to a Boolean result.
 - ightharpoonup \Rightarrow is **right associative**, so the above is $Bool \Rightarrow (Bool \Rightarrow Bool)$
- Plus an infinite number of similar variations!

The Typing Relation

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- Explicit Typing (Used in this course).
 - Typing annotations in the syntax functions: https://tutore.s.com
- Implicit Typing (Advanced topic in type theory).
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But consider:

 $\lambda x : Bool.$ if x then s_2 else s_3

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 $x : Bool \vdash t_2 : T_2$

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Typing relation becomes a three-place relation, i.e.

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Context in general

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$$\{w: T_1, x: T_2, y: T_3\} \vdash z: T_4$$
 (1)

where z cantings.../*tutorcs.com

General form

$$\Gamma \vdash t : T$$
 (2)

where Γ is Wet ef war in the relation of the

Called either the typing context or the typing environment.

Well-formed contexts and variables

Formally we have a well-formed context relation:

Assignment Project Exam $\overset{\text{(C-Empty)}}{\text{Help}}$

Well-formed the spin plicitly astrong to the empty context, we instead leave it blank:

$$\frac{x:T\in\Gamma}{\Gamma\vdash x\cdot T}$$

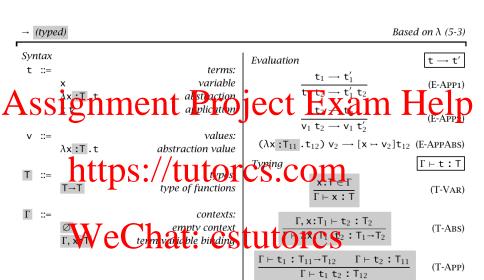
(T-Var)

Q: what happens if we try to "insert" the same x twice?

Function Typing, Correctly

Assignment Project Exam Help, $\Gamma \vdash (\lambda x : T_1.t_2) : T_1 \Rightarrow T_2$

(T-App)



Remark: as is, degenerate.

THEOREM [Uniqueness of Types] In a given typing context Γ , if all the free variables of a term t are in the domain of Γ , t has at most one type. Proof Sketch by Suction on temperature (regianly relies that each typing rule applies to a single term formation rule.

In this case, we say that the typing relation is *syntax directed*. WeChat: CStutorCS

LEMMA [Canonical Forms]

- If v is a value of type Bool then v is either true or false. If v is a value of type $T_1 \Rightarrow T_2$, then v has shape $\lambda x : T_1.t_2$.

Note that type $T_1 \Rightarrow T_2$ may have infinitely many values as inhabitants.

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Progress

Suppose \cdot \vdash t: T. Either t is a value, or else there is some t' such that $t \to t'$.

A later the style of the style

Proof by Induction on Typing Derivations. Each evaluation rule is examined in term. Details use Intelsion and for the Orle tricky case of T-AppAbs, canonical forms are needed.

More about contexts

LEMMA [Permutation invariance]

ALSS 121 M Clare depth as the former.

Proof Sketch; induction on typing derivations.

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We add extra "facts" without changing conclusions: **LEMMA** [Weakening]

If $\Gamma \vdash t : T$ and $x \notin dom(\Gamma)$, then $\Gamma, x : S \vdash t : T$. Moreover, the latter

derivation was the same depth as the former. CSTUTOTCS

Proof Sketch: Induction on typing derivations.

Points of variations show up here, i.e. linear types, union types, dependent types, etc.

LEMMA [Preservation of Types Under Substitution]

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• Proof will proceed by induction over typing derivations, and using a case analysis over typing rules.

As a reminder:

Substitution Lemma II

T-True, T-False, T-If, T-App straightforward.

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- ▶ $[x \mapsto s]z$ would then evaluate to s.
- $x = z \land z = t \implies x = t$ Via the migueness pittite, $x \in S^t : Con = T$ Substituting into lemma statement:

$$\Gamma, x: S \vdash x: S \land \Gamma \vdash s: S \implies \Gamma \vdash s: S$$

- Now to sider # hat cstutores
 Ix size would then evaluate to z (and from there to t).

$$\Gamma, x : S \vdash t : T \land \Gamma \vdash s : S \implies \Gamma \vdash t : T$$

We can now conclude by weakening.

Substitution Lemma III

T-Abs: $t = \lambda y : T_3.t_1 \land T = T_3 \Rightarrow T_4 \land \Gamma, x : S, y : T_3 \vdash t_1 : T_4$ By our meta-rule of substitutions in λ expressions, we derive: Assignment Project Exam Help

Using the the permutation lemma on the rightmost equation:

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Using the weakening lemma on $\Gamma \vdash s : S$:

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By the induction hypothesis:

$$\Gamma, y: T_3 \vdash [x \mapsto s]t_1: T_4.$$

Substitution Lemma IV

Recall T-Abs:

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Applying this to last equation $\Gamma, y : T_3 \vdash [x \mapsto s]t_1 : T_4$, get

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The definition of substitution is:

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- The LHS has type $T_3 \Rightarrow T_4$ from our original case analysis.
- The RHS has the same type from above.