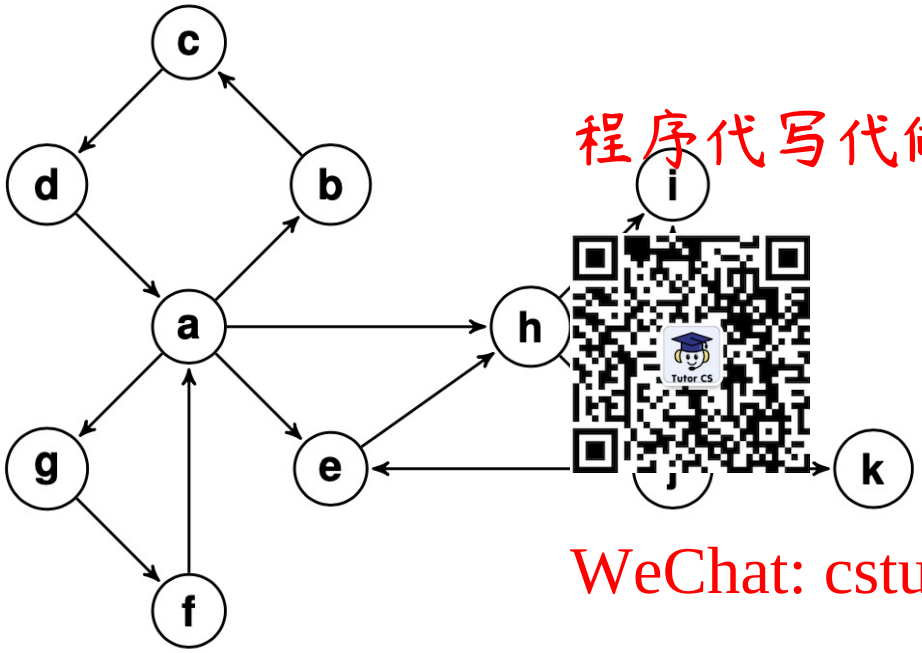

Q1: Find the SCCs

$C_1 = \{a,b,c,d,f,g\}$

$C_2 = \{e,h,j\}$

$C_3 = \{i\}$

$C_4 = \{k\}$

Q2: Find the component graph

$V_{SCC} = \{C_1, C_2, C_3, C_4\}$

$E_{SCC} = \{(C_1,C_2),(C_2,C_3),(C_2,C_4)\}$

Q3: Algorithm to compute a component graph

1. Compute SCCs and label all vertices u with their SCC C_u
2. Create a vertex for each SCC
3. For each vertex u:

   For each outgoing edge (u,v):

   If v belongs to different SCC then add an edge (C_u,C_v)

Q4: Complexity

O(V+E)

Theorem: In DFS of a connected undirected graph G, we get only tree and back edges. No forward or cross edges.

Proof (no forward ec

- Let (u, v) be a forw .
- Thus, v is a descendant of u but (u,v) is not a tree edge.
- By parenthesis theorem, d(u) < d(v) < f(v) < f(u).
- When (u,v) explored, v is?
  - White? No because it would be a tree edge
  - Black? No because (v,u)=(u,v) would already be labelled as a back edge
  - Gray? No. DFS would explore all paths out of u before reaching and continue processing v ⇒ f(u) < f(v), which contradict the hypothesis v is a descendant of u

There are other ways to build your proof.

Theorem: In DFS of a connected undirected graph G, we get only tree and back edges. No forward or cross edges.

Proof (no cross edge

- Let (u, v) be a cros
- Thus, neither u or v is a descendant of the other.
- By parenthesis theorem, d(u) < f(u) < d(v) < f(v) or d(v) < f(v) < d(u) < f(u).
- Assume u is discovered first. At d(u), v is still white.
- Since v can be reached from u. v is a descendant of u by the White path theorem. Not possible.
- Thus v is discovered first, and at d(u), v is still white.
- But u can be reached from v because G is undirected and there is an edge (u,v)=(v,u). By white path theorem, u is a descendant of v, which also contradict the hypothesis.