

# COMP 251

程序代写代做 CS编程辅导

Algorithms



Structures (Winter 2022)

Algorithm Paradigms – Dynamic Programming 2

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# Announcements

程序代写代做 CS编程辅导

- A2 is out (just after this lecture).

- Please start working

- Good for you.
- Good for us.



- A2 is a good preparation for the midterm.

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- You have 3 weeks; however this assignment is harder than A1.

- Plus the reading week is in the middle.

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- Midterm will be held two days after the due date.

- No extensions.

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# Outline

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- Complete Search
- Divide and Conquer
- Dynamic Programming
  - Introduction.
  - Examples.
- Greedy.



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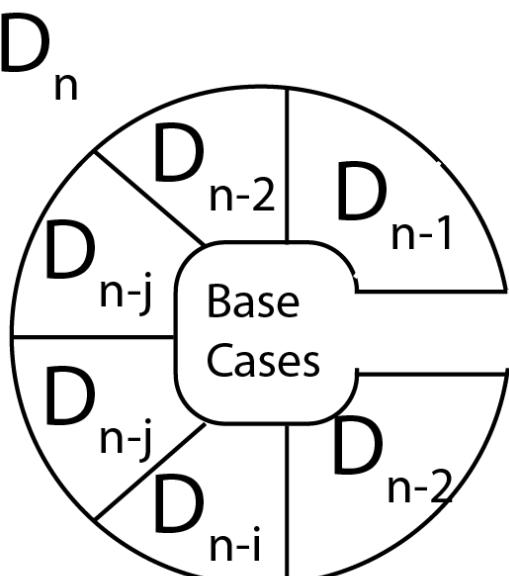
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# Dynamic Programming— Take home picture

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## Paradigm



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Overlapping subproblems

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AND

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Optimal substructure

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## Solution



Memoization  
Top-Down Approach

OR

Tabulation  
Bottom-up Approach

# Dynamic Programming— 2D - Knapsack

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- Given  $n$  objects and a "knapsack."
- Item  $i$  weighs  $w_i > 0$  and has value  $v_i > 0$ .
- Knapsack has capacity  $W$ .
- Goal: fill knapsack so as to maximize total value.



Ex.  $\{1, 2, 5\}$  has value 35. WeChat: cstutorcs

Ex.  $\{3, 4\}$  has value 40.

Ex.  $\{3, 5\}$  has value 46 (but exceeds weight limit).

$i$	$v_i$	$w_i$
1	1	1
2	6	2
3	18	5
4	22	6
5	28	7

knapsack instance  
(weight limit  $W = 11$ )



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weight: 3  
value: 20

weight: 2  
value: 40

weight: 1  
value: 10

Taken from baeldung.com

# Dynamic Programming – 2D - Knapsack

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**Step 1:** Identify the problem (in words).

**Step 1.1:** Identify the problem.



Let  $\text{OPT}(i)$  be the maximum total value of items 1 to  $i$  (i.e., value of the optimal solution to the problem including activities 1 to  $i$ ).

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I just copy the same definition used for the weighted interval scheduling

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- Let  $\text{OPT}(i)$  be the maximum total weight of compatible activities 1 to  $i$  (i.e., value of the optimal solution to the problem including activities 1 to  $i$ ).

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# Dynamic Programming – 2D - Knapsack

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**Step 2:** Find the recurrence.



**Step 2.1:** What decision do we make at every step?.

**Case 1:** OPT does not select (activity) item  $i$

- Must include optimal solution on other (activities) items  $\{1, 2, \dots, i-1\}$ .

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**Case 2:** OPT selects (activity) item  $i$

- (activity) Add weight  $w_i$  Email: [tutors@163.com](mailto:tutors@163.com) and value  $v_i$
- (activity) Cannot use incompatible activities – (item) ??
- (activity) Must include optimal solution on remaining compatible activities  $\{1, 2, \dots, p(j)\}$ . -- (item) ??
  - Selecting item  $i$  does not necessarily imply that we will have to reject other items
  - Without knowing what other items were selected before  $i$ , we do not even know if we have enough room for  $i$ .

# Dynamic Programming – 2D - Knapsack

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Step 2: Find the recurrence.

Case 2: OPT selects item i

- Selecting item i does not immediately imply that we will have to reject other items
- Without knowing what other items were selected before i, we do not even know if we have enough room for i.



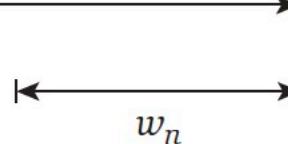
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Conclusion: We need more subproblems!!!!

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After item  $n$  is included in the solution, a weight of  $w_n$  is used up and there is  $W - w_n$  available weight left

# Dynamic Programming – 2D - Knapsack

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**Step 1:** Identify the subproblems (in words).

**Step 1.1:** Identify the possible subproblems.



Let  $\text{OPT}(i, w)$  be the maximum profit subset of items 1 to  $i$  with weight limit  $w$ .

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# Dynamic Programming – 2D - Knapsack

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Step 2: Find the recurrence.



Case 1: OPT does not select item  $i$

- OPT selects best of  $\{1, 2, \dots, i-1\}$  using weight limit  $w$ .

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Optimal substructure property

Case 2: OPT selects item  $i$

- New weight limit =  $w - w_i$
- OPT selects best of  $\{1, 2, \dots, i-1\}$  using this new weight limit.

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$$OPT(i, w) = \begin{cases} 0 & \text{if } i = 0 \\ OPT(i-1, w) & \text{if } w_i > w \\ \max\{ OPT(i-1, w), v_i + OPT(i-1, w-w_i) \} & \text{otherwise} \end{cases}$$

# Dynamic Programming – 2D - Knapsack

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KNAPSACK ( $n, W, w_1, \dots, w_n, v_1, \dots, v_n$ )

FOR  $w = 0$  TO  $W$

$M[0, w] \leftarrow 0.$



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FOR  $i = 1$  TO  $n$

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FOR  $w = 1$  TO  $W$

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IF ( $w_i > w$ )  $M[i, w] \leftarrow M[i-1, w].$

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ELSE  $M[i, w] \leftarrow \max \{M[i-1, w], v_i + M[i-1, w - w_i]\}.$

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RETURN  $M[n, W].$

# Dynamic Programming – 2D - Knapsack

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i	v <sub>i</sub>	w <sub>i</sub>
1	1	1
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Max weight W = 11

# Dynamic Programming – 2D - Knapsack

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M	0	1	2	3	4	5	6	7	8	9	10	11
{}	0	0	0	0	0	0	0	0	0	0	0	0
{1}	0											
{1,2}	0											
{1,2,3}	0											
{1,2,3,4}	0											
{1,2,3,4,5}	0											



W

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i ↓

# Dynamic Programming – 2D - Knapsack

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i	v <sub>i</sub>	w <sub>i</sub>
1	1	1
2	6	2
3	18	5
4	22	6
5	28	7

W = 11

FOR i



FOR j

IF

ELSE

$M[i, w] \leftarrow M[i-1, w]$ .

$M[i, w] \leftarrow \max \{ M[i-1, w], v_i + M[i-1, w-w_i] \}$

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M	0	1	2	3	4	5	6	7	8	9	10	11
{}	0	0	0	0	0	0	0	0	0	0	0	0
{1}	0	1	1	1	1	1	1	1	1	1	1	1
{1,2}	0											
{1,2,3}	0											
{1,2,3,4}	0											
{1,2,3,4,5}	0											

# Dynamic Programming – 2D - Knapsack

程序代写代做 CS 编程辅导

i	v <sub>i</sub>	w <sub>i</sub>
1	1	1
2	6	2
3	18	5
4	22	6
5	28	7

W = 11

```
FOR i
    FOR w
        IF M[i, w] <= M[i-1, w].
        ELSE
            M[i, w] ← max { M[i-1, w], vi + M[i-1, w - wi] }
```

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M	0	1	2	3	4	5	6	7	8	9	10	11
{}	0	0	0	0	0	0	0	0	0	0	0	0
{1}	0	1	1	1	1	1	1	1	1	1	1	1
{1,2}	0	1										
{1,2,3}	0											
{1,2,3,4}	0											
{1,2,3,4,5}	0											

# Dynamic Programming – 2D - Knapsack

程序代写代做 CS 编程辅导

i	v <sub>i</sub>	w <sub>i</sub>
1	1	1
2	6	2
3	18	5
4	22	6
5	28	7

W = 11

```
FOR i
    FOR w
        IF M[i, w] <= M[i-1, w].
        ELSE
            M[i, w] ← max { M[i-1, w], vi + M[i-1, w - wi] }
```

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M	0	1	2	3	4	5	6	7	8	9	10	11
{}	0	0	0	0	0	0	0	0	0	0	0	0
{1}	0	1	1	1	1	1	1	1	1	1	1	1
V <sub>2</sub> +M(i-1, W <sub>2</sub> )	1	6	11	16	21	26	31	36	41	46	51	56
{1,2,3}	0											
{1,2,3,4}	0											
{1,2,3,4,5}	0											

M(i-1, w)

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# Dynamic Programming – 2D - Knapsack

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i	v <sub>i</sub>	w <sub>i</sub>
1	1	1
2	6	2
3	18	5
4	22	6
5	28	7

W = 11

```
FOR i
    FOR w
        IF M[i, w] < M[i-1, w].
        ELSE
            M[i, w] ← max { M[i-1, w], vi + M[i-1, w - wi] }
```

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M	0	1	2	3	4	5	6	7	8	9	10	11
{}	0	0	0	0	0	0	0	0	0	0	0	0
{1}	0	1	1	1	1	1	1	1	1	1	1	1
{1, 2}	0	0	6	7	M(i-1,w)							
{1, 2, 3}	0											
{1, 2, 3, 4}	0											
{1, 2, 3, 4, 5}	0											

V<sub>2</sub>+M(0,1)w=7M<sub>2</sub>9389476

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# Dynamic Programming – 2D - Knapsack

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i	v <sub>i</sub>	w <sub>i</sub>
1	1	1
2	6	2
3	18	5
4	22	6
5	28	7

W = 11

```
FOR i
    FOR w
        IF M[i, w] <= M[i-1, w].
        ELSE
            M[i, w] ← max { M[i-1, w], vi + M[i-1, w - wi] }
```

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M	0	1	2	3	4	5	6	7	8	9	10	11
{}	0	0	0	0	0	0	0	0	0	0	0	0
{1}	0	1	1	1	1	1	1	1	1	1	1	1
{1,2}	0	1	6	7	7	7	7	7	7	7	7	7
{1,2,3}	0											
{1,2,3,4}	0											
{1,2,3,4,5}	0											

# Dynamic Programming – 2D - Knapsack

程序代写代做 CS 编程辅导

i	v <sub>i</sub>	w <sub>i</sub>
1	1	1
2	6	2
3	18	5
4	22	6
5	28	7

W = 11

```
FOR i
    FOR w
        IF M[i, w] <= M[i-1, w].
        ELSE
            M[i, w] ← max { M[i-1, w], vi + M[i-1, w - wi] }
```

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M	0	1	2	3	4	5	6	7	8	9	10	11
{}	0	0	0	0	0	0	0	0	0	0	0	0
{1}	0	1	1	1	1	1	1	1	1	1	1	1
{1,2}	0	1	6	7	7	7	7	7	7	7	7	7
{1,2,3}	0	1	6	7	7	18	19	24	25	25	25	25
{1,2,3,4}	0											
{1,2,3,4,5}	0											

# Dynamic Programming – 2D - Knapsack

程序代写代做 CS 编程辅导

i	v <sub>i</sub>	w <sub>i</sub>
1	1	1
2	6	2
3	18	5
4	22	6
5	28	7

W = 11

```
FOR i
    FOR w
        IF M[i, w] <= M[i-1, w].
        ELSE
            M[i, w] ← max { M[i-1, w], vi + M[i-1, w - wi] }
```

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M	0	1	2	3	4	5	6	7	8	9	10	11
{}	0	0	0	0	0	0	0	0	0	0	0	0
{1}	0	1	1	1	1	1	1	1	1	1	1	1
{1,2}	0	1	6	7	7	7	7	7	7	7	7	7
{1,2,3}	0	1	6	7	7	18	19	24	25	25	25	25
{1,2,3,4}	0	1	6	7	7	18	22	24	28	29	29	40
{1,2,3,4,5}	0											

# Dynamic Programming – 2D - Knapsack

程序代写代做 CS 编程辅导

i	v <sub>i</sub>	w <sub>i</sub>
1	1	1
2	6	2
3	18	5
4	22	6
5	28	7

W = 11

FOR  $i$   
FOR  $w$   
IF  $M[i, w] < M[i-1, w]$   
ELSE  $M[i, w] \leftarrow \max \{ M[i-1, w], v_i + M[i-1, w - w_i] \}$

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Item 3 in

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solution

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Item 4 in  
solution

M	0	1	2	3	4	5	6	7	8	9	10	11
{}	0	0	0	0	0	0	0	0	0	0	0	0
{1}	0	1	6	7	7	7	7	0	0	0	0	0
{1,2}	0	1	6	7	7	7	7	0	0	0	0	0
{1,2,3}	0	1	6	7	7	18	19	24	25	25	25	25
{1,2,3,4}	0	1	6	7	7	18	22	24	28	29	29	40
{1,2,3,4,5}	0	1	6	7	7	18	22	28	29	34	35	40

# Dynamic Programming – 2D - Knapsack

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**Theorem.** There exists an algorithm to solve the knapsack problem with  $n$  items and maximum weight  $W$  in  $\Theta(n W)$  time and  $\Theta(n W)$  space.

Pf.

- Takes  $O(1)$  time per table entry.
- There are  $\Theta(n W)$  table entries. ← "pseudo-polynomial"
- After computing optimal values, can trace back to find solution:  
take item  $i$  in  $OPT(i, w) \neq M[i, w] < M[i-1, w]$ .

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weights are integers  
between 1 and  $W$

# Dynamic Programming— 2D

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**Problem:** given two strings  $x$  and  $y$ , find the longest common subsequence (LCS) and its length.

**Example:**

- $x$  : ABCBDAB
- $y$  : BDCABC
- “BCAB” is the longest subsequence found in both sequences, so the answer is 4.

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# Dynamic Programming— 2D

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**Step 1:** Identify the sub-problems (in words).

**Step 1.1:** Identify the main problem.

Let  $C_{nm}$  be the length of the LCS of  $x_{1..n}$  and  $y_{1..m}$



**Step 1.2:** Identify the possible sub-problems.

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Let  $C_{ij}$  be the length of the LCS of  $x_{1..i}$  and  $y_{1..j}$

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- A B C B D A B

B							
D							
C							
A							
B							
C							
B							
C							

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# Dynamic Programming— 2D

程序代写代做 CS编程辅导

## Step 2: Find the recurrence

### Step 2.1: What decisions make at every step?.

Two options. To contribute to the LCS length or not.

- If  $x_i = y_j$ , they both contribute to the LCS => match
- If  $x_i \neq y_j$ , either  $x_i$  or  $y_j$  does not contribute to the LCS, so one can be dropped

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# Dynamic Programming— 2D

程序代写代做 CS编程辅导

## Step 2: Find the recurrence

### Step 2.1: What decisions make at every step?.

Two options. To contribute to the LCS length or not.

- If  $x_i = y_j$ , they both contribute to the LCS => match
- If  $x_i \neq y_j$ , either  $x_i$  or  $y_j$  does not contribute to the LCS, so one can be dropped

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Let  $X = \langle x_1, x_2, \dots, x_m \rangle$  and  $Y = \langle y_1, y_2, \dots, y_n \rangle$  be sequences, and let  $Z = \langle z_1, z_2, \dots, z_k \rangle$  be any LCS of  $X$  and  $Y$ .

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1. If  $x_m = y_n$ , then  $z_k = x_m = y_n$  and  $Z_{k-1}$  is an LCS of  $X_{m-1}$  and  $Y_{n-1}$ .  
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2. If  $x_m \neq y_n$ , then  $z_k \neq x_m$  implies that  $Z$  is an LCS of  $X_{m-1}$  and  $Y$ .
3. If  $x_m \neq y_n$ , then  $z_k \neq y_n$  implies that  $Z$  is an LCS of  $X$  and  $Y_{n-1}$ .

Optimal substructures

# Dynamic Programming– 2D

程序代写代做 CS编程辅导

Let  $X = \langle x_1, x_2, \dots, x_m \rangle$  and  $Y = \langle y_1, y_2, \dots, y_n \rangle$  be sequences, and let  $Z = \langle z_1, z_2, \dots, z_k \rangle$  be any LCS of  $X$  and  $Y$ .



1. If  $x_m = y_n$ , then  $z_k = x_m = y_n$  and  $Z_{k-1}$  is an LCS of  $X_{m-1}$  and  $Y_{n-1}$ .
- If  $z_k \neq x_m$ , then we could append  $x_m = y_n$  to  $Z$  to obtain a common subsequence of  $X$  and  $Y$  of length  $k+1$ , contradicting the supposition that  $Z$  is a LCS of  $X$  and  $Y$ .
- The prefix  $Z_{k-1}$  is a common subsequence of  $X_{m-1}$  and  $Y_{n-1}$  with length  $k-1$ . We wish to show that it is an LCS.
  - Suppose for the purpose of contradiction that there exists a common subsequence  $W$  of  $X_{m-1}$  and  $Y_{n-1}$  with length greater than  $k-1$ . Then, appending  $x_m = y_n$  to produce  $W$  produces a common subsequence of  $X$  and  $Y$  whose length is greater than  $k$ , which is a contradiction.

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# Dynamic Programming— 2D

程序代写代做 CS 编程辅导

Let  $X = \langle x_1, x_2, \dots, x_m \rangle$  and  $Y = \langle y_1, y_2, \dots, y_n \rangle$  be sequences, and let  $Z = \langle z_1, z_2, \dots, z_k \rangle$  be any LCS of  $X$  and  $Y$ .

2. If  $x_m \neq y_n$ , then  $z_k \neq y_n$ . This implies that  $Z$  is an LCS of  $X_{m-1}$  and  $Y$ .



- If  $z_k \neq x_m$ , then  $Z$  is a common subsequence of  $X_{m-1}$  and  $Y$ . If there were a common subsequence  $W$  of  $X_{m-1}$  and  $Y$  with length greater than  $k$ , then  $W$  would also be a common subsequence of  $X_m$  and  $Y$ , contradicting the assumption that  $Z$  is an LCS of  $X$  and  $Y$ .

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# Dynamic Programming— 2D

程序代写代做 CS编程辅导

Let  $X = \langle x_1, x_2, \dots, x_m \rangle$  and  $Y = \langle y_1, y_2, \dots, y_n \rangle$  be sequences, and let  $Z = \langle z_1, z_2, \dots, z_k \rangle$  be any LCS.



1. If  $x_m = y_n$ , then  $z_k = x_m = y_n$ .  
**WeChat: cstutorcs**  
 $Z_{k-1}$  is an LCS of  $X_{m-1}$  and  $Y_{n-1}$ .
2. If  $x_m \neq y_n$ , then  $z_k \neq x_m$ .  
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 $Z$  is an LCS of  $X_{m-1}$  and  $Y$ .
3. If  $x_m \neq y_n$ , then  $z_k \neq y_n$ .  
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implies that  $Z$  is an LCS of  $X$  and  $Y_{n-1}$ .

- Overlapping

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- To find an LCS of  $X$  and  $Y$ , we may need to find the LCSs of  $X$  and  $Y_{n-1}$  and of  $X_{m-1}$  and  $Y$ . But each of those subproblems has the subsubproblem of finding an LCS of  $X_{m-1}$  and  $Y_{n-1}$ .

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# Dynamic Programming— 2D

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## Step 2: Find the recurrence

### Step 2.1: What decisions make at every step?.

Two options. To contribute to the LCS length or not.

- If  $x_i = y_j$ , they both contribute to the LCS => match
- If  $x_i \neq y_j$ , either  $x_i$  or  $y_j$  does not contribute to the LCS, so one can be dropped

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-	A	B	C	B	D	A	B
-	Assignment Project Exam Help						
B							
D							
C							
A							
B							
C							

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# Dynamic Programming– 2D

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## Step 2: Find the recurrence

- If  $x_i = y_j$ , they both contribute to the LCS => match
  - $C_{ij} = C_{i-1,j-1} + 1$
- Otherwise, either  $x_i$  or  $y_j$  does not contribute to the LCS, so one can be dropped
  - $C_{ij} = \max\{C_{i-1,j}, C_{i,j-1}\}$



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## Step 3: Recognize and solve the base cases.

- $C_{i0} = C_{0j} = 0$ .

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$$c[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0, \\ c[i - 1, j - 1] & \text{if } i, j > 0 \text{ and } x_i = y_j, \\ \max(c[i, j - 1], c[i - 1, j]) & \text{if } i, j > 0 \text{ and } x_i \neq y_j. \end{cases}$$

# Dynamic Programming— 2D

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## Step 4: Implement a solving methodology.



```
for(i=0;i<=n;i++) c[i][0]=0;
for(j=0;j<=m;j++) c[0][j]=0;
for(i=1;i<=n;i++){
    for(j=1;j<=m;j++){
        if(x[i]==y[j])
            c[i][j]=c[i-1][j]+1;
        else
            c[i][j]=max(c[i-1][j],c[i][j-1]);
    }
}
```

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# Dynamic Programming— 2D

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Step 4: Implement a solving methodology.

		A	C	B	D	A	B
		-	0	0	0	0	0
		B	0	0	WeChat:1cstutorcs	1	1
		D	0	0	Assignment Project Exam Help	2	2
		C	0	0	Email: tutorcs@163.com	2	2
		A	0	0	QQ:1749389476	2	3
		B	0	0	https://tutorcs.com	2	3
		C	0	0	1	2	4



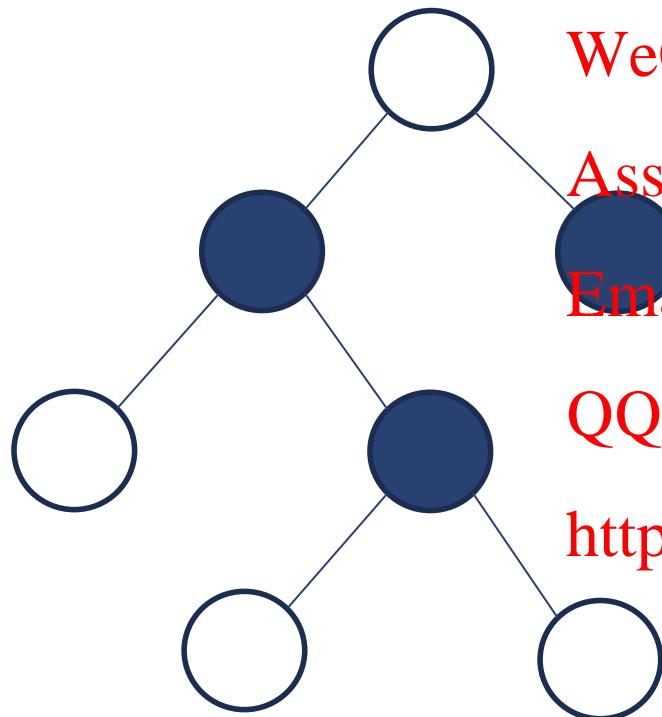
# Dynamic Programming— trees

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**Problem:** given a tree, find the size of the **Largest Independent Set** (LIS). A set of nodes is an independent set if there are no edges between the nodes.



**Example:**



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The largest independent set  
(LIS) is in white. The size of  
the LIS is 5.

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# Dynamic Programming— trees

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**Step 1:** Identify the sub-problems (in words).

**Step 1.1:** Identify the root problem.

MIS(r) denote the size of the largest independent set in the tree with root at r.



**Step 1.2:** Identify the possible subproblems.

MIS(v) denote the size of the largest independent set in the subtree rooted at v.

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# Dynamic Programming— trees

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## Step 2: Find the recurrence

## Step 2.1: What decisions make at every step?.

# Two options.

- Include the node.  
• A set that includes  $v$  necessarily excludes all of  $v$ 's children.
  - Do not include the current node (root).  
• Any independent set is the union of independent sets in the subtrees rooted at the children of  $v$ .



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**- or -**

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# Dynamic Programming— trees

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## Step 2: Find the recurrence

### Step 2.1: What decisions make at every step?.

Two options.

- Include the node.
  - A set that includes  $v$  necessarily excludes all of  $v$ 's children.
- Do not include the current node (root).
  - Any independent set is the union of independent sets in the subtrees rooted at the children of  $v$ .



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$$MIS(v) = \max \left\{ \sum_{w \downarrow v} MIS(w), 1 + \sum_{w \downarrow v} \sum_{x \downarrow w} MIS(x) \right\}$$

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notation  $w \downarrow v$  means “ $w$  is a child of  $v$ ”

children  $w$  of  $v$

grandchildren  $x$  of  $v$

# Dynamic Programming— trees

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$$MIS(v) = \min \left\{ MIS(w), 1 + \sum_{w \downarrow v} \sum_{x \downarrow w} MIS(x) \right\}$$



## Step 4: Implement a solving methodology.

- What data structure should we use to memoize this recurrence?
  - Array, 2D array, tree?
- What's a good order to consider the subproblems?
  - The subproblems associated with any node  $v$  depends on the subproblems associated with the children and grandchildren of  $v$ .
    - We can visit the nodes in any order we like, provided that every vertex is visited before its parent.
      - Pre-order? In-Order? Post-Order?

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# Dynamic Programming— trees

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## Step 4: Implement a solving methodology.

- What data structure should we use to memoize this recurrence?
    - The most natural choice is a tree itself! Specifically, for each vertex  $v$ , we store the result of  $MIS(v)$  in  $v.MIS$
  - What's a good order to consider the subproblems?
    - The subproblems associated with any node  $v$  depends on the subproblems associated with the children and grandchildren of  $v$ .
    - We can visit the nodes in any order we like, provided that every vertex is visited before its parent.
    - Post-Order traversal.
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Dynamic programming is *not* about filling in tables.

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It's about smart recursion!

# Dynamic Programming— trees

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## Step 4: Implement a solving methodology.

- We can derive an efficient algorithm by defining two separate functions over the nodes of the tree.
  - Let  $MISyes(v)$  denote the size of the largest independent set of the subtree rooted at  $v$  that includes  $v$ .
  - Let  $MISno(v)$  denote the size of the largest independent set of the subtree rooted at  $v$  that excludes  $v$ .

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$$MISyes(v) = 1 + \sum_{w \downarrow v} MISno(w)$$

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$$MISno(v) = \max \{ MISyes(w), MISno(w) \}$$



# Dynamic Programming— trees

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$$MISyes(v) = \sum_{w \in v} MISno(w)$$



$$MISno(v) = \max_{w \in v} \{MISyes(w), MISno(w)\}$$

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# Dynamic Programming— trees

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$$MISyes(v) = \sum_{w \in v} MISno(w)$$



$$MISno(v) = \max_{w \in v} \{MISyes(w), MISno(w)\}$$

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TREEMIS(s) Assignment Project Exam Help

$$v.MISno \leftarrow 0$$

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$$v.MISyes \leftarrow 1$$

for each child  $w$  of  $v$

$$v.MISno \leftarrow v.MISno + \text{TREEMIS2}(w)$$

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$$v.MISyes \leftarrow v.MISyes + w.MISno$$

return  $\max\{v.MISyes, v.MISno\}$

# Dynamic Programming— trees

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TREEMIS2( $v$ ):

$v.MISno \leftarrow 0$

$v.MISyes \leftarrow 1$

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return  $\max\{v.MISyes, v.MISno\}$



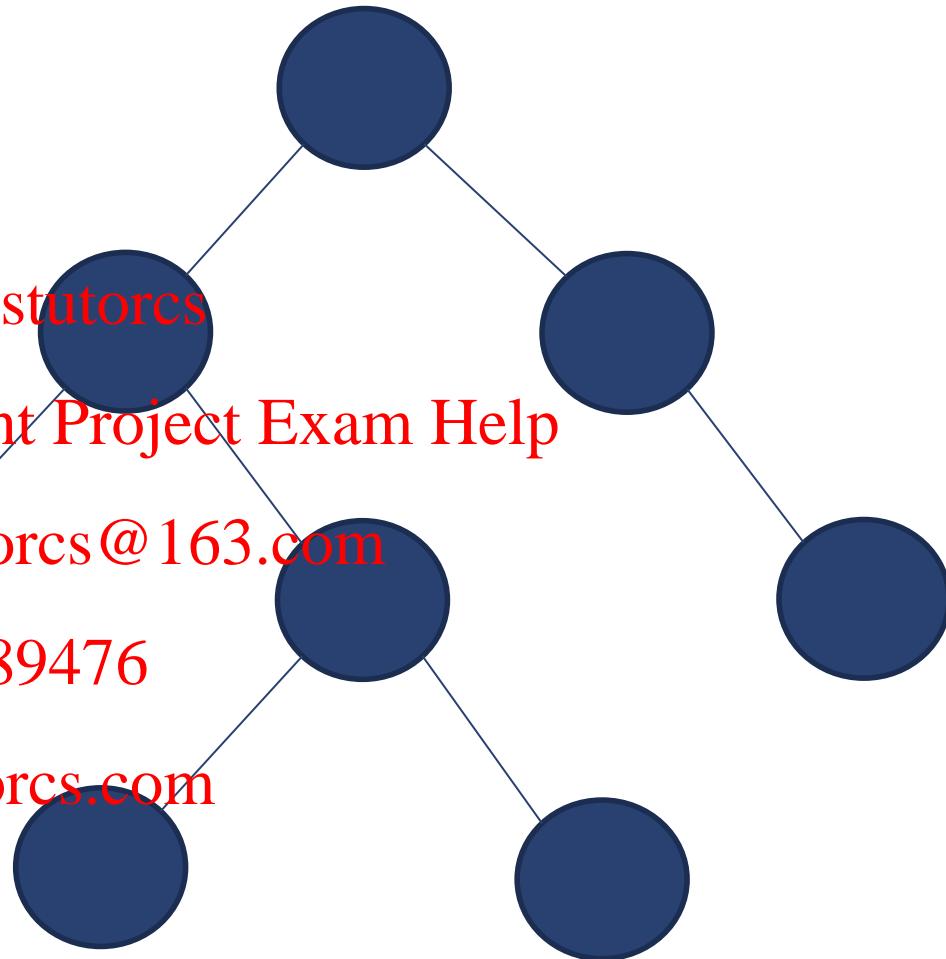
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# Dynamic Programming— trees

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N = 0  
Y = 1

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# Dynamic Programming— trees

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N = 0  
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# Dynamic Programming— trees

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# Dynamic Programming— trees

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# Dynamic Programming— trees

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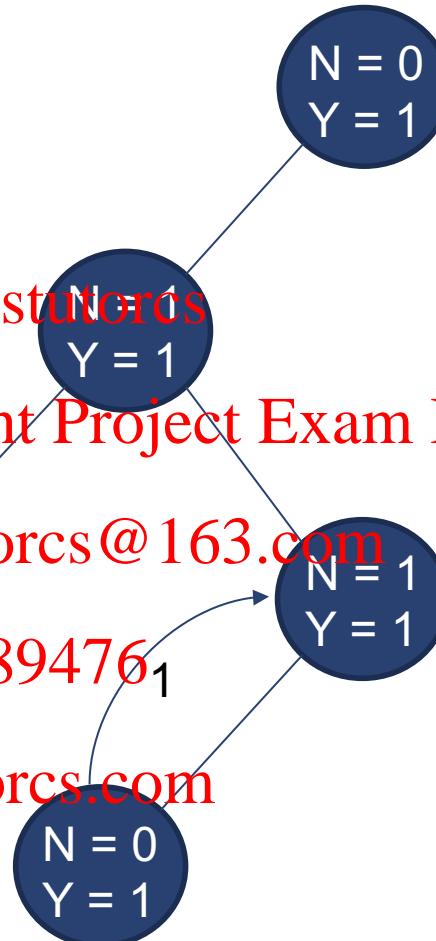
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# Dynamic Programming— trees

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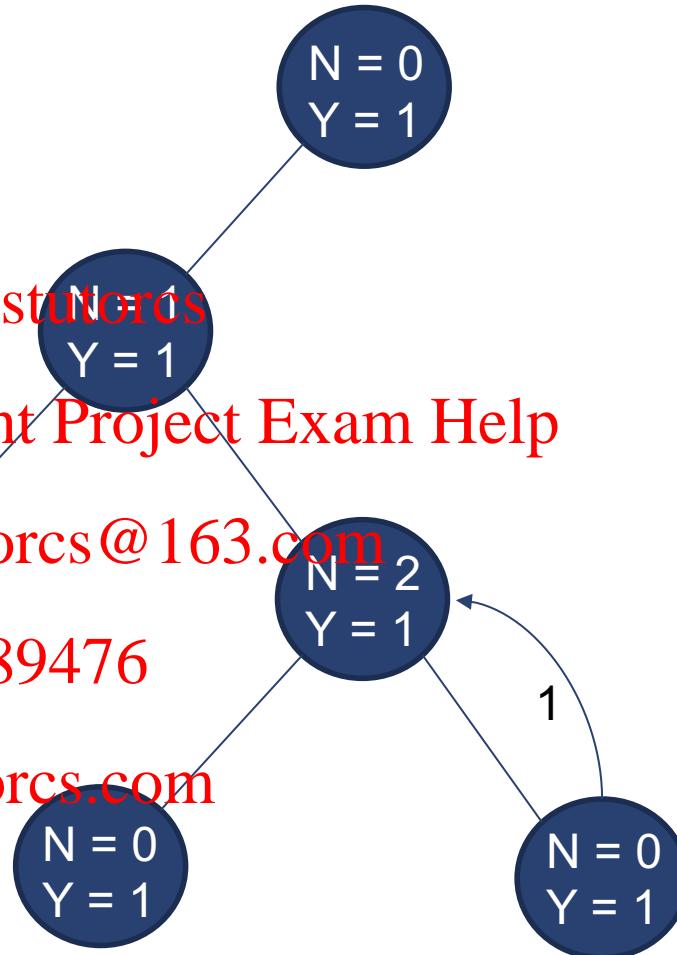
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# Dynamic Programming— trees

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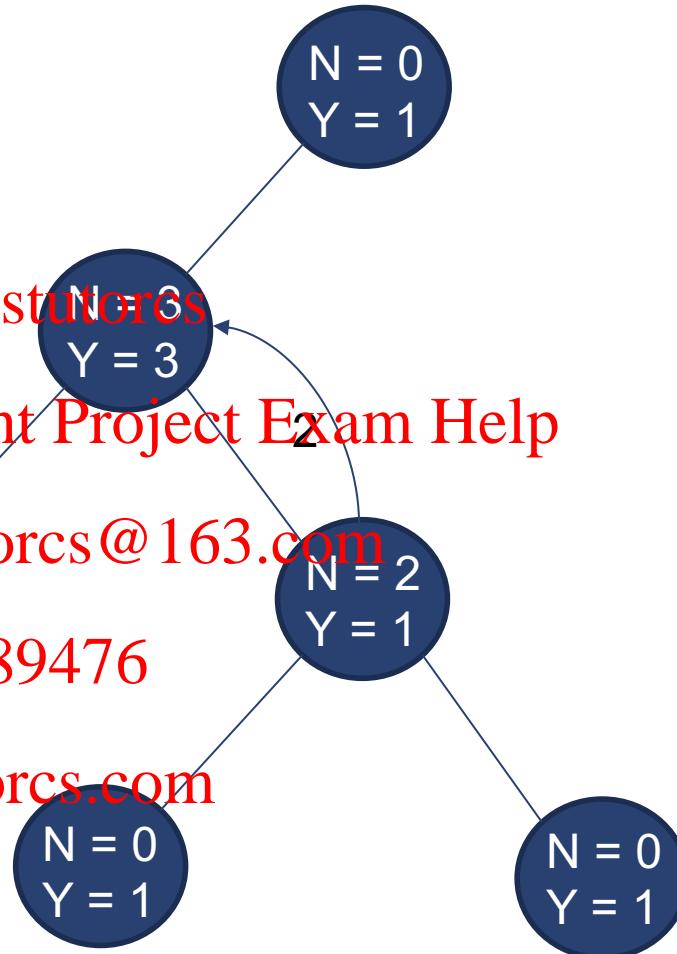
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# Dynamic Programming— trees

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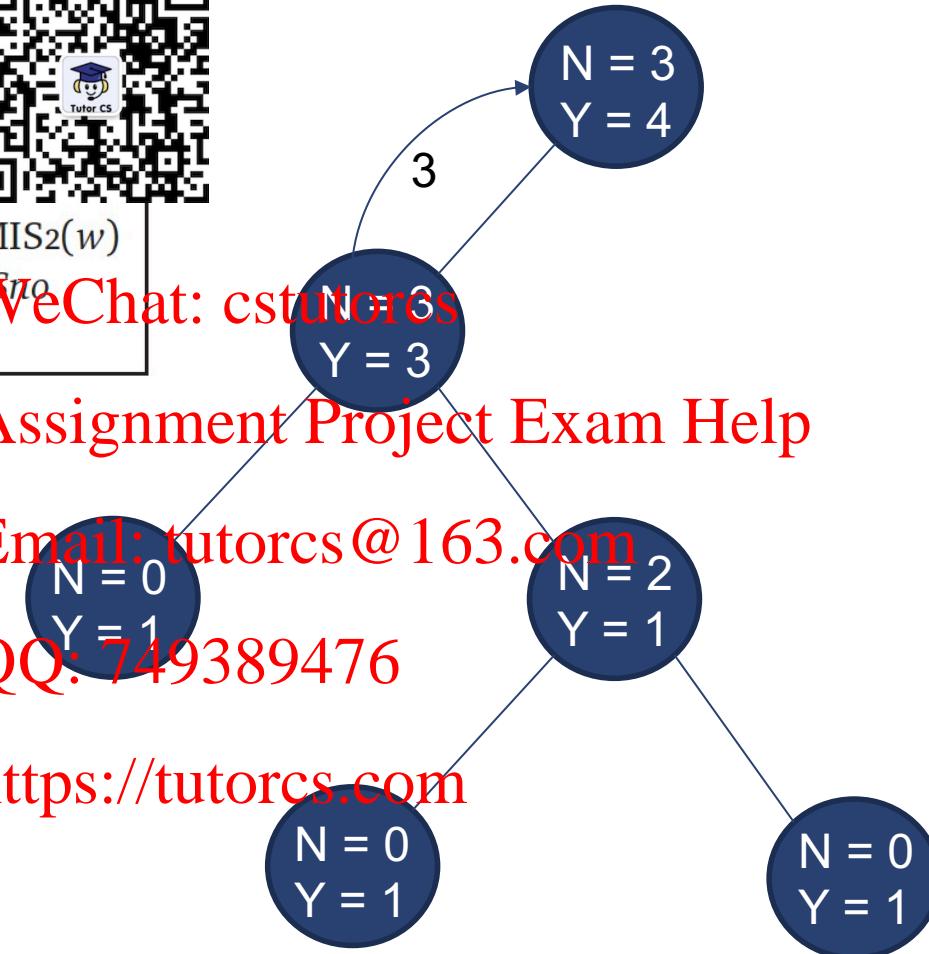
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# Dynamic Programming— trees

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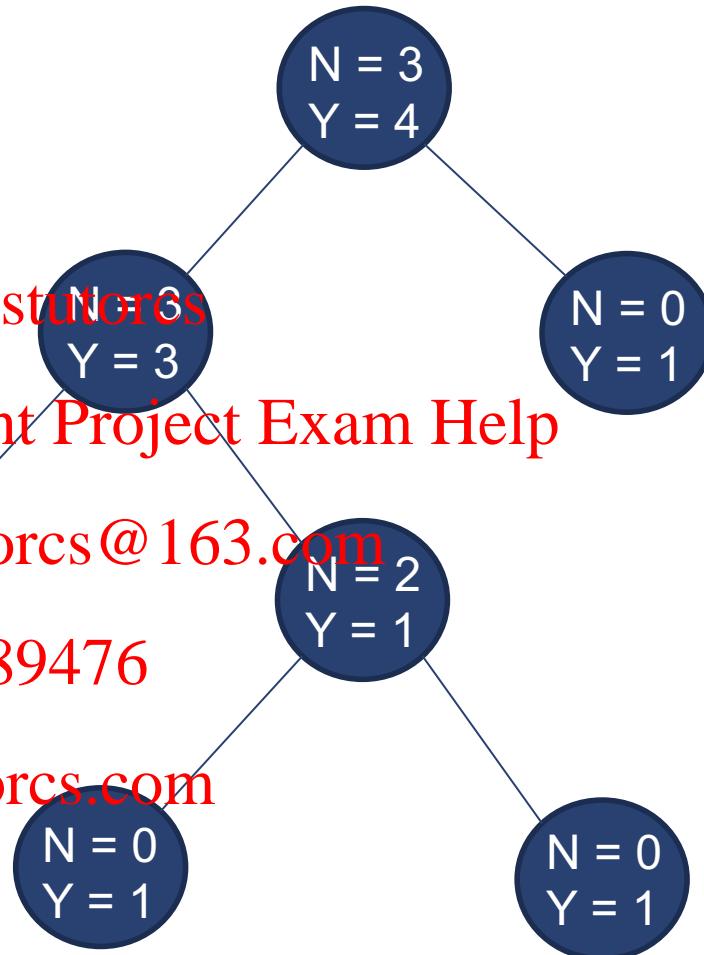
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# Dynamic Programming— trees

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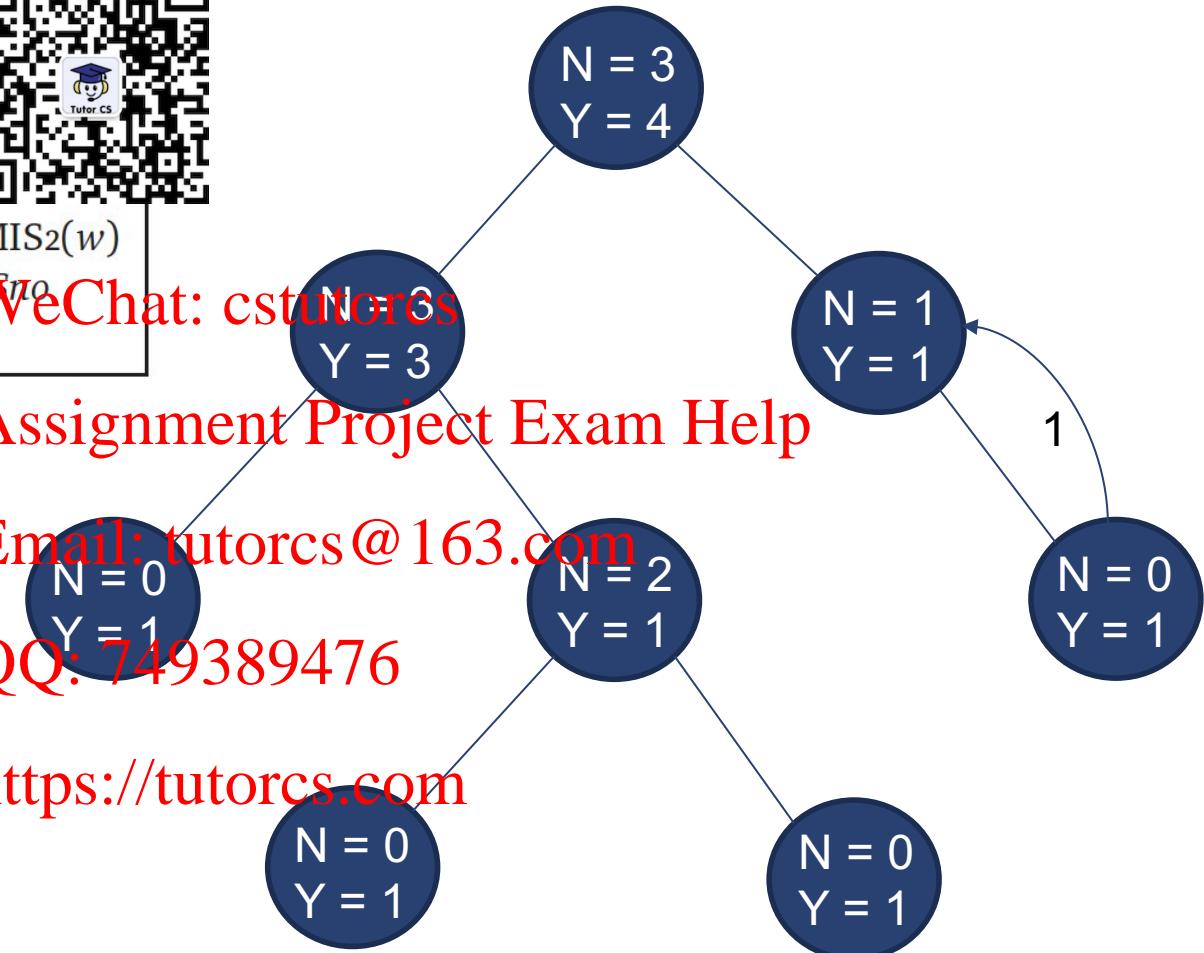
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# Dynamic Programming— trees

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    return max{ $v.MISyes, v.MISno$ }
```



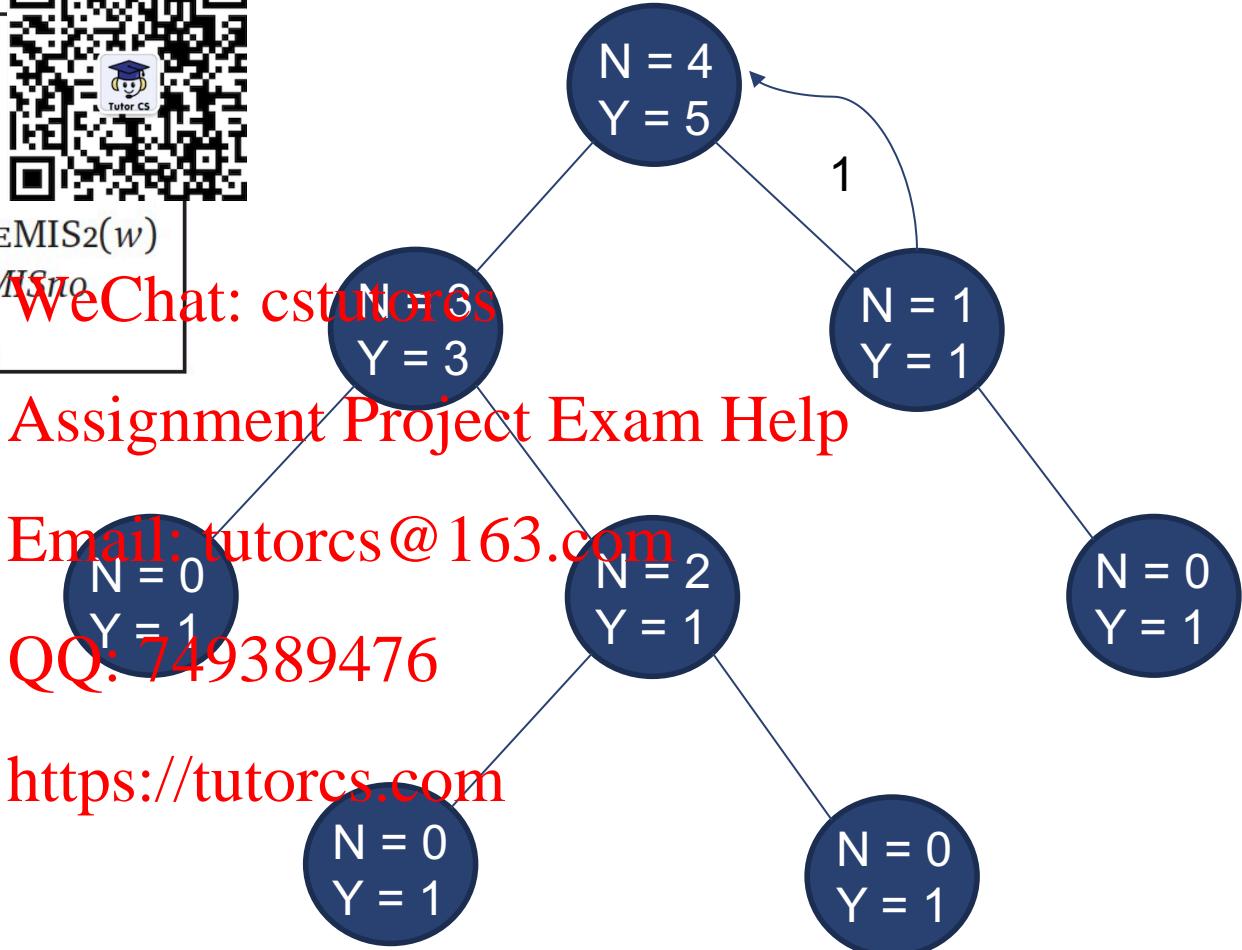
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# Outline

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- Complete Search
- Divide and Conquer
- Dynamic Programming
  - Introduction.
  - Examples.
- Greedy.



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