CS264A: Automated Reasoning

Fall 2023

Homework 4

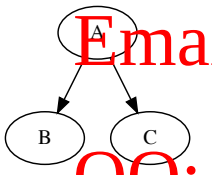
Sunday, December 10

1. (25pt) Consider $f(X, Y, Z) = -6 \cdot X + 5 \cdot Y - 4 \cdot Z + 3$, where X, Y, Z are binary features ($\{0, 1\}$). The classifier labels an instance positively iff $f(X, Y, Z) \geq 0$. For example, since $f(1, 1, 1) = -2 < 0$, the instance $X = 1, Y = 1, Z = 1$ is labeled negatively.

   (a) (7pts) What is the classification function given $X = 1, Y = 1$? In general, what is the form of the classification function after we know the values of features X, Y?

   (b) (15pts) Draw a reduced OBDD representing the decision function of the classifier, using variable order X, Y, Z.

   (c) (3pts) If an instance has $X = 0$ and $Y = 1$, will the value of feature Z affect the instance classification?



A	θ_A
a_0	0.8
a_1	0.2

A	B	$\theta_{B\|A}$
a_0	b_0	0.3
a_0	b_1	0.7
a_1	b_0	0.7
a_1	b_1	0.3

A	C	$\theta_{C\|A}$
a_0	c_0	0.1
a_0	c_1	0.9
a_1	c_0	0.9
a_1	c_1	0.1

Figure 1: Bayesian network.

2. (22pt) Consider the Bayesian network in Figure 1 and suppose we want to compute the most probable explanation (MPE) for this network.

   (a) (12pt) Show the weighted CNF which can be used to compute MPE using weighted MaxSAT.

   (b) (3pt) Modify this weighted CNF so it can be used to compute MPE under evidence $B = b_0$.

   (c) (7pt) What is the MPE instantiation under $B = b_0$, what is the corresponding instantiation of indicator variables, and what is the weight and penalty of this indicator instantiation?

3. (14pt) Consider the following DNF:

$$\Delta = \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + w\bar{x}y\bar{z} + w\bar{x}yz + wxy\bar{z} + wxyz.$$

   (a) (9pt) Compute the prime implicants of Δ using the *consensus* method. That is, close the DNF under consensus and remove subsumed terms.

(b) (5pt) Suppose Δ is a classifier. What are the sufficient reasons for the instance $wxyz$?

4. (20pt) Consider the following classifier and suppose that R is a protected feature.

$$\Delta = [E \wedge [(F \wedge (G \vee W)) \vee (\neg F \wedge R)]] \vee [G \wedge R \wedge W].$$

(a) (5pt) Wha no) on instance $E, \neg F, G, W, R$?

(b) (10pt) Wh e sufficient reasons (PI-explanations) for this decision?

(E, W), $(E, G, R, \neg F)$, (E, G, W).

(c) (5pt) Is th hy?

5. (10pt) Consider the following class formula $\Delta = (x_{12} + y_1) \cdot (x_1 + y_1 + z_1) + (x_3 \cdot y_2 \cdot z_2)$ and the instance $I = \{x_1, y_1, z_1\}$. Suppose X has states $\{x_1, x_2, x_3\}$, Y has states $\{y_1, y_2, y_3\}$, and Z has states $\{z_1, z_2\}$.

(a) (5pt) Compute the complete reason $\forall I \cdot \Delta$.

(b) (5pt) Compute the general reason $\overline{\forall} I \cdot \Delta$.

6. (9pt) True or False (no need to explain)?

(a) (3pt) We can compute marginals on an arithmetic circuit in linear time if it satisfies the decomposability property.

(b) (3pt) We can compute MPE on an arithmetic circuit in linear time if it satisfies the decomposability and smoothness properties.

(c) (3pt) A PSDD is an arithmetic circuit that satisfies the properties of decomposability and determinism.