CS Discussion 9

TA: Albert Ji

Friday 1st December, 2023

Information

- HW4 is out.
- Due Dec 10, 11:59 pm.

Prime Implicates/Implicants

- Prime implicates:
  - Clauses
  - Subsumption
    - Clause α subsumes clause β iff
    - $\alpha \models \beta$
    - $\alpha \subseteq \beta$
  - From CNF to prime implicates: resolution

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  - From CNF to prime implicates: resolution
- Prime implicants:
  - Terms
  - Subsumption condition:
    - Term α subsumes term β iff
    - $\beta \models \alpha$
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  - From DNF to prime implicants: consensus

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  - Subsumption condition:
    - Term α subsumes term β iff
    - $\beta \models \alpha$
    - $\alpha \subseteq \beta$
  - From DNF to prime implicants: consensus
- PI/IP representation:
  - Given Δ, we can write it as a conjunction of its prime implicates
  - Or as a disjunction of its prime implicants

Universal Literal Quantification

- $\forall x \cdot \Delta = \Delta|x \wedge (x$
- Example: $\Delta = (\neg$ $x \wedge \neg y) \vee (y \wedge z))$
- $\forall x \cdot \Delta = \Delta|x \wedge (x$

$$= (y \wedge (\neg y \vee (y \wedge z))) \wedge (x \vee (y \wedge z))$$
$$= (y \wedge z) \wedge (x \vee (y \wedge z))$$
$$= y \wedge z$$

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- Assume $I = \{x, y, z\}$, then $\forall I \cdot \Delta = \forall x, y, z, \cdot \Delta$
- Note that $\forall x, y \cdot \Delta = \forall x \cdot (\forall y \cdot \Delta) = \forall y \cdot (\forall x \cdot \Delta)$
- $\forall x, y, z \cdot \Delta = \forall y, z \cdot (y \wedge z)$

$$= \forall z \cdot (y \wedge z)$$
$$= y \wedge z$$

Sufficient/Necessary Reasons

- Let I be the instance and Δ be its class formula
- The complete reason: $\forall I . \Delta$
- Sufficient reasons:
  - Prime implicants of the complete reason
- Necessary reasons:
  - Prime implicates of the complete reason

Universal Literal Quantification

- Definition for disc
- $\forall x_i \cdot \Delta = \Delta|x_i \wedge \Delta|\neg x_i$ (or $\Delta|x_j$)
- Example: $\Delta = (x_1$ [illegible] $)) \wedge (\neg x_3 \vee y_3)$

Universal Literal Quantification

- Definition for disc
- $\forall x_i \cdot \Delta = \Delta|x_i \wedge \Delta|\neg x_i \wedge \bigwedge_{j \neq i} (x_j \vee \Delta|x_j)$
- Example: $\Delta = (x_1 \vee (\neg x_2 \wedge \neg x_3)) \wedge (\neg x_3 \vee y_3)$
- $\begin{aligned} \forall x_2 \cdot \Delta &= \Delta|x_2 \wedge (x_2 \vee \Delta|x_1) \wedge (x_2 \vee \Delta|x_3) \\ &= (y_1 \wedge z_1) \wedge (x_2 \vee \top) \wedge (x_2 \vee ((y_1 \vee z_1) \wedge y_3)) \\ &= (y_1 \wedge z_1) \wedge \top \wedge (x_2 \vee (y_3 \wedge z_1)) \\ &= (y_1 \wedge z_1) \wedge (x_2 \vee y_3) \\ &= x_2 \wedge y_1 \wedge z_1 \end{aligned}$

Universal Literal Quantification

- Definition for disc
- $\forall x_i \cdot \Delta = \Delta|x_i \wedge \Delta|\neg x_i$
- Example: $\Delta = (x_1 \vee (y_1 \wedge z_1)) \wedge (\neg x_3 \vee y_3)$
- $\forall x_2 \cdot \Delta = \Delta|x_2 \wedge (x_2 \vee \Delta|x_1) \wedge (x_2 \vee \Delta|x_3)$
  $= (y_1 \wedge z_1) \wedge (x_2 \vee \top) \wedge (x_2 \vee ((y_1 \vee z_1) \wedge y_3))$
  $= (y_1 \wedge z_1) \wedge \top \wedge (x_2 \vee (y_3 \wedge z_1))$
  $= (y_1 \wedge z_1) \wedge (x_2 \vee y_3)$
  $= x_2 \wedge y_1 \wedge z_1$
- Since Δ is $\vee$-decomposable
- $\forall x_2 \cdot \Delta = (\forall x_2 \cdot x_1 \vee (y_1 \wedge z_1)) \wedge (\forall x_2 \cdot \neg x_3 \vee y_3)$
  $= (\bot \vee (y_1 \wedge z_1)) \wedge (x_2 \vee y_3)$
  $= x_2 \wedge y_1 \wedge z_1$

General Reasons

- $\overline{\forall} x_i \cdot \Delta = \Delta \wedge \Delta|_{\overline{x_i}}$
- Alternatively, $\overline{\forall} x_i \Delta = \Delta|_{x_i} \wedge \Delta|_{\overline{x_i}} \wedge \bigwedge_{j \neq i} (\neg x_j \vee \Delta|_{x_j})$.
- Example: $\Delta = (x_1 \vee (y_1 \wedge z_1)) \wedge (\neg x_3 \vee y_3)$

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- Example: $\Delta = (x_1 \vee (y_1 \wedge z_1)) \wedge (\neg x_3 \vee y_3)$
- $\overline{\forall} x_2 \cdot \Delta = \Delta|x_2 \wedge \Delta$
  $= (y_1 \wedge z_1) \wedge (x_1 \vee (y_1 \wedge z_1)) \wedge (\neg x_3 \vee y_3)$
  $= y_1 \wedge z_1 \wedge \neg x_3$

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- Example: $\Delta = (x_1 \vee (y_1 \wedge z_1)) \wedge (\neg x_3 \vee y_3)$
- $\overline{\forall} x_2 \cdot \Delta = \Delta|_{x_2} \wedge \Delta|_{\overline{x_2}}$
  $= (y_1 \wedge z_1) \wedge (x_1 \vee (y_1 \wedge z_1)) \wedge (\neg x_3 \vee y_3)$
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- Similarly, $\overline{\forall} x, y, z \cdot \Delta = \overline{\forall} z \cdot (\overline{\forall} y \cdot (\overline{\forall} x \cdot \Delta))$.

General Reasons

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- Alternatively, $\overline{\forall} x_i \cdot \Delta = \Delta|_{x_i} \wedge \bigwedge_{j \neq i} (\neg x_j \vee \Delta|_{x_j})$.
- Example: $\Delta = (x_1 \vee (y_1 \wedge z_1)) \wedge (\neg x_3 \vee y_3)$
- $\overline{\forall} x_2 \cdot \Delta = \Delta|_{x_2} \wedge \Delta|_{\overline{x}_2}$
  $= (y_1 \wedge z_1) \wedge (x_1 \vee (y_1 \wedge z_1)) \wedge (\neg x_3 \vee y_3)$
  $= y_1 \wedge z_1 \wedge (\neg x_3 \vee y_3)$
- Similarly, $\overline{\forall} x, y, z \cdot \Delta = \overline{\forall} z \cdot (\overline{\forall} y \cdot (\overline{\forall} x \cdot \Delta))$.
- A general sufficient reason (GSR) is a variable-minimal prime implicant of the general reason $\overline{\forall} I \cdot \Delta$,
- A general necessary reason (GNR) is a variable-minimal prime implicate of the general reason $\overline{\forall} I \cdot \Delta$.

Consensus

Let $\alpha = l_1 \wedge \alpha'$, $\beta = l_2 \wedge \beta'$, where l_1, l_2 are literals of variable L. Then the consensus of α and β on L is $(l_1 \vee l_2) \wedge \alpha' \wedge \beta'$.

- Example: $\alpha = x_{12} \wedge y_{12}$, $\beta = x_{13} \wedge y_{13} \wedge z_1$
- The consensus of α and β on X is $x_{123} \wedge y_1 \wedge z_1$.
- If X has only three states $\{x_1, x_2, x_3\}$, then the consensus is $y_1 \wedge z_1$.

Consensus

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Resolution

Let $\alpha = l_1 \vee \alpha'$, $\beta = l_2 \vee \beta'$, where l_1, l_2 are literals of variable L. Then the resolvent of α and β on L is $(l_1 \wedge l_2) \vee \alpha' \vee \beta'$.

- Example: $\alpha = x_{12} \vee y_{12}$, $\beta = x_{13} \vee y_{13}$.
- The resolvent of α and β on X is $x_1 \vee y_{123}$.
- If Y has only three states $\{y_1, y_2, y_3\}$, then the resolvent is $\top$, which means the resolution is meaningless.

Decision and Classifier Bias

- A decision on instance X is biased iff it can be different on an instance Y that disagrees with X on protected features only
- A decision is biased iff all of its sufficient reasons mention some protected features
- A classifier is biased iff one of its decisions is biased.

Decision and Classifier Bias

- Consider classifier $y) \vee (y \wedge z) \vee (x \wedge \neg z) \vee (\neg x \wedge \neg y \wedge z)$
- The prime implica re:
  - $x \wedge y$
  - $y \wedge z$
  - $x \wedge \neg z$
  - $\neg x \wedge z$
- Let x be a protected feature. Is the decision $\{x, y, z\}$ biased?

Decision and Classifier Bias

- Consider classifier $(x \wedge y) \vee (y \wedge z) \vee (x \wedge \neg z) \vee (\neg x \wedge \neg y \wedge z)$
- The prime implicants are:
  - $x \wedge y$
  - $y \wedge z$
  - $x \wedge \neg z$
  - $\neg x \wedge z$
- Let x be a protected feature. Is the decision $\{x, y, z\}$ biased?
  - The sufficient reasons are $(x \wedge y)$ and $(y \wedge z)$
  - Not biased

Decision and Classifier Bias

- Consider classifier $(x \wedge y) \vee (y \wedge z) \vee (x \wedge \neg z) \vee (\neg x \wedge \neg y \wedge z)$
- The prime implicants are:
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  - $y \wedge z$
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  - $\neg x \wedge z$
- Let x be a protected feature. Is the decision $\{x, y, z\}$ biased?
  - The sufficient reasons are $(x \wedge y)$ and $(y \wedge z)$
  - Not biased
- Is the classifier biased?

Decision and Classifier Bias

- Consider classifier $(x \wedge y) \vee (y \wedge z) \vee (x \wedge \neg z) \vee (\neg x \wedge \neg y \wedge z)$
- The prime implicants are:
  - $x \wedge y$
  - $y \wedge z$
  - $x \wedge \neg z$
  - $\neg x \wedge z$
- Let x be a protected feature. Is the decision $\{x, y, z\}$ biased?
  - The sufficient reasons are $(x \wedge y)$ and $(y \wedge z)$
  - Not biased
- Is the classifier biased?
- Yes, since the decision $\{x, y, \neg z\}$ is biased.

HW3: Q1

Q1

- $f = (\neg A \vee \neg B \vee C) \wedge (B \vee \neg C)$
- What is the compressed (X, Y)-partition of function f, where $X = \{A, B\}$ and $Y = \{C\}$?

HW3: Q1

Q1

- $f = (\neg A \vee \neg B \vee (\quad \neg C)$
- What is the compressed (X, Y)-partition of function f, where $X = \{A, B\}$ and $Y = \{C\}$?

-

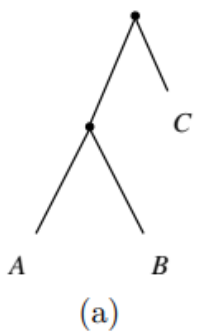
Prime	Sub
$A \wedge B$	C
$A \wedge \neg B$	$\neg C$
$\neg A \wedge B$	$\top$
$\neg A \wedge \neg B$	$\neg C$

- The compressed (X, Y)-partition is:
- $\Delta = (\neg B, \neg C),\ (A \wedge B, C),\ (\neg A \wedge B, \text{true})$

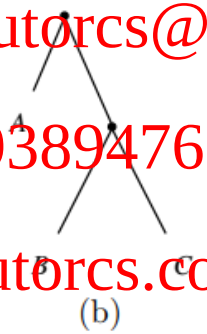
HW3: Q1

Q1

- $f = (\neg A \vee \neg B \vee ($ [illegible] $\neg C)$
- What is the compressed (X, Y)-partition of function f, where $X = \{A, B\}$ and $Y = \{C\}$?
- To construct an SDD using the (X, Y)-partition above, which of the vtrees should be used?



(a)



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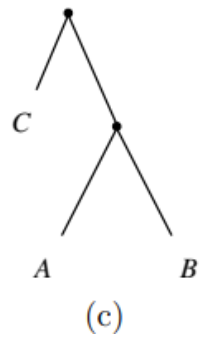

(c)

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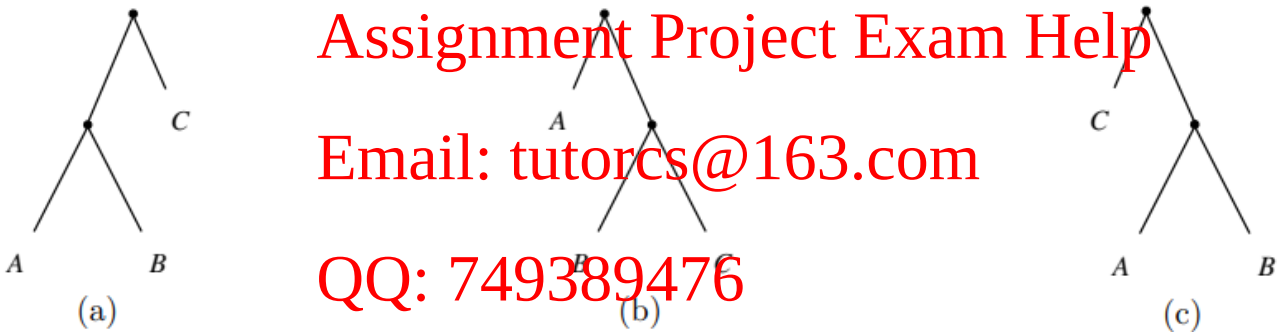
- To construct an SDD using the (X, Y)-partition above, which of the vtrees should be used?

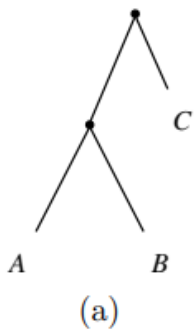


- Vtree (a)

HW3: Q1

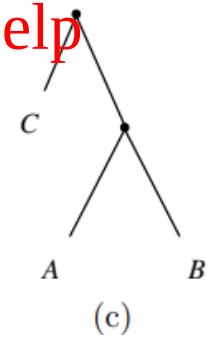
Q1

- $f = (\neg A \vee \neg B \vee C) \wedge (D \vee \neg C)$
- Which vtrees will lead to an SDD that corresponds to an OBDD?



(a)

(b)

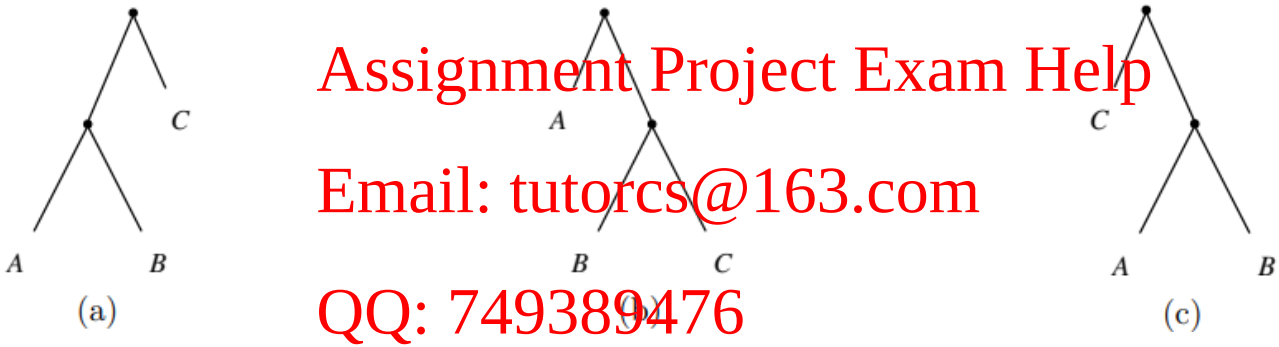


(c)

HW3: Q1

Q1

- $f = (\neg A \vee \neg B \vee (\quad \neg C)$
- Which vtrees will lead to an SDD that corresponds to an OBDD?



- Vtrees (b) and (c) since they are right-linear.

HW3: Q2

Q2

- $f = (A \wedge B) \vee (B \wedge C) \vee (C \wedge D)$
- Construct the compressed (X, Y)-partitions for f and $\neg f$, where $X = \{A, C\}$ and $Y = \{B, D\}$.

HW3: Q2

Q2

- $f = (A \wedge B) \vee (B \wedge D)$
- Construct the compressed $(X, Y)-$partitions for f and $\neg f$, where $X = \{A, C\}$ and $Y = \{B, D\}$.

Prime	Sub
$A \wedge C$	$B \vee D$
$A \wedge \neg C$	B
$\neg A \wedge C$	$B \vee D$
$\neg A \wedge \neg C$	$\bot$

- $\Delta = (C, B \vee D), (A \wedge \neg C, B), (\neg A \wedge \neg C, \text{false})$

HW3: Q2

Q2

- $f = (A \wedge B) \vee (B \wedge C \wedge D)$
- Construct the compressed $(X, Y)-$partitions for f and $\neg f$, where $X = \{A, C\}$ and $Y = \{B, D\}$.

Prime	Sub
$A \wedge C$	$\neg(B \vee D)$
$A \wedge \neg C$	$\neg B$
$\neg A \wedge C$	$\neg(B \vee D)$
$\neg A \wedge \neg C$	$\top$

- $\Delta = (C, \neg B \wedge \neg D), (A \wedge \neg C, \neg B), (\neg A \wedge \neg C, \text{true})$

HW3: Q2

Q2

- $f = (A \wedge B) \vee (B \wedge C) \vee (C \wedge D)$
- Derive a general rule for finding an (X, Y)-partition for any function $\neg f$ from an (X, Y)-partition of function f.

HW3: Q2

Q2

- $f = (A \wedge B) \vee (B \wedge C) \vee (C \wedge D)$
- Derive a general rule for finding an (X, Y)–partition for any function $\neg f$ from an (X, Y)–partition of function f.
- Negate the subs of an (X, Y)-partition of f to get the (X, Y)-partition of $\neg f$.

HW3: Q3

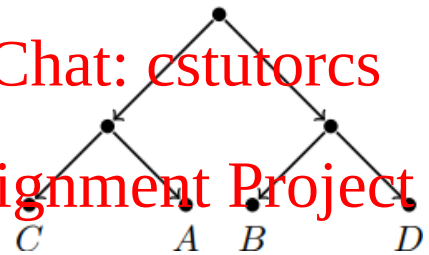
Q3

- Construct an SDD for the function f
- $f = (A \wedge \neg B) \vee (\neg B \wedge C) \vee (C \wedge D)$

HW3: Q3

Q3

- Construct an SDD for the function f
- $f = (A \wedge \neg B) \vee (\neg A \wedge \neg B) \vee (C \wedge D)$



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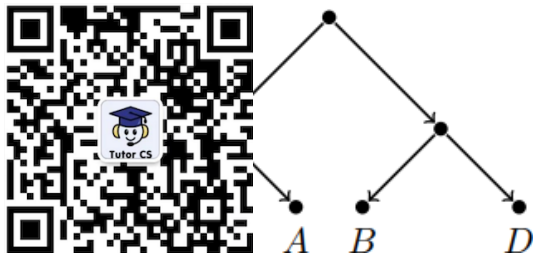
Prime	Sub
$A \wedge C$	$\neg B \vee D$
$A \wedge \neg C$	$\neg B$
$\neg A \wedge C$	$\neg B \vee D$
$\neg A \wedge \neg C$	$\bot$

- $f = (C, \neg B \vee D),\ (A \wedge \neg C, \neg B),\ (\neg A \wedge \neg C, \text{false})$

HW3: Q3



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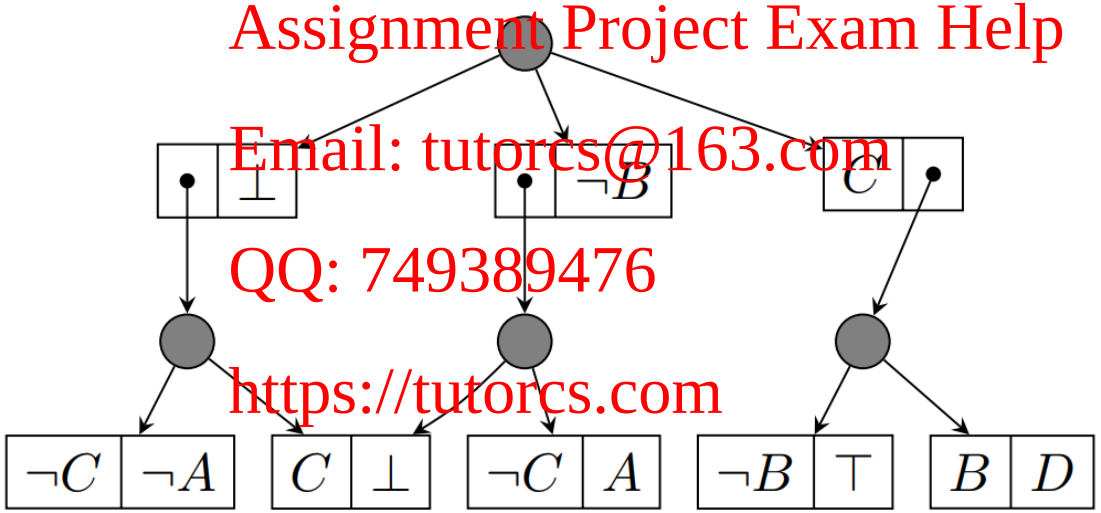


HW3: Q4

Q4

- Consider a structured space which corresponds to selecting k or more items from a set of n items
- Suppose we use the Boolean variable x_i to indicate whether item i is selected
- Describe a CNF that captures this structured space

Q4

- Consider a structu[illegible] which corresponds to selecting k or more items from a[illegible]tems
- Suppose we use the Boolean variable A_i to indicate whether item i is selected
- Describe a CNF that captures this structured space

- Consider a DNF for selecting fewer than k items
- Define S_{n-k+1} as all possible sets of combinations of integers $1, 2, \ldots, n$ of size $(n-k+1)$
- DNF: $\bigvee_{S \in S_{n-k+1}} (\bigwedge_{i \in S} \neg A_i)$
- CNF: $\neg \bigvee_{S \in S_{n-k+1}} (\bigwedge_{i \in S} \neg A_i) = \bigwedge_{S \in S_{n-k+1}} (\bigvee_{i \in S} A_i)$
- Number of clauses: $\binom{n}{n-k+1}$

HW3: Q4

Q4

- Consider a structured space which corresponds to selecting k or more items from a set of n items
- Describe a DNF that captures this structured space

HW3: Q4

Q4

- Consider a structured space which corresponds to selecting k or more items from a set of n items
- Describe a DNF that captures this structured space

- Define S_k as all possible sets of combinations of integers $1, 2, \ldots, n$ of size k
- DNF: $\bigvee_{s \in S_k} (\bigwedge_{i \in s} A_i)$
- Number of terms: $\binom{n}{k}$

HW3: Q4

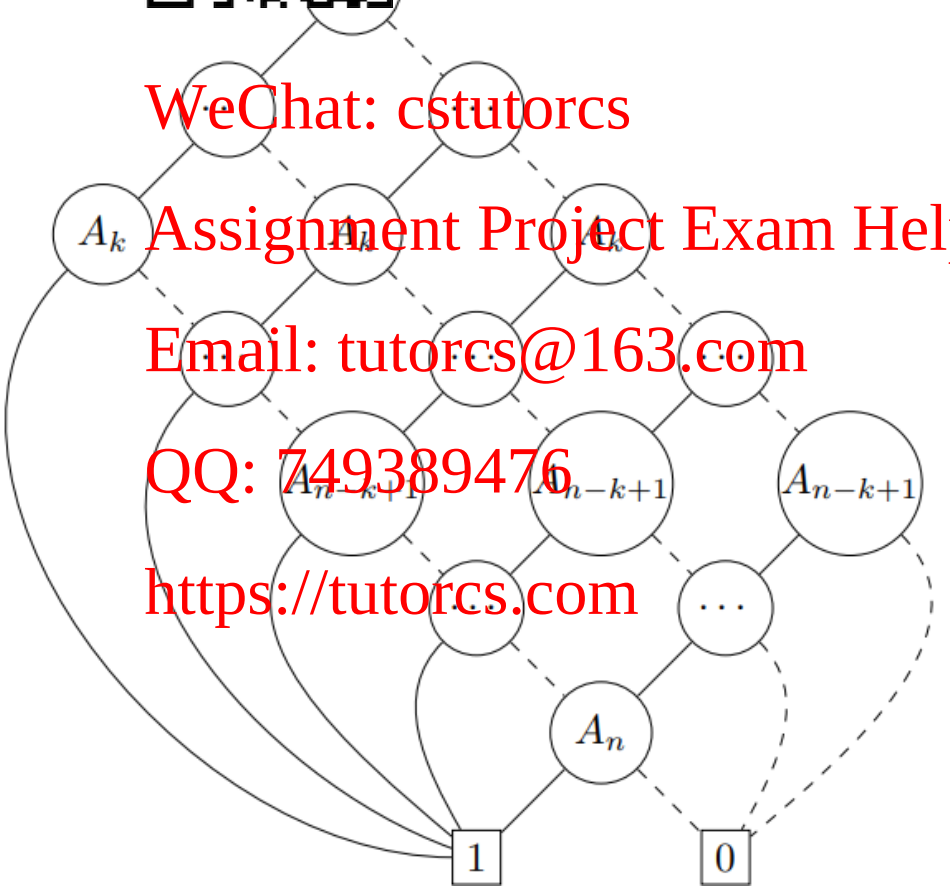
Q4

- Can you capture this structured space more efficiently using an OBDD?

HW3: Q4

Q4

- Can you capture this required space more efficiently using an OBDD?



HW3: Q4

- OBDD is more efficient. There are $k \cdot (n - k + 1) + 2$ nodes.

HW3: Q5

Q5

- $\Delta = (\neg A \vee B \vee \neg C) \wedge (\neg A \vee B \vee C) \wedge (A \vee \neg B \vee \neg C) \wedge (A \vee \neg B \vee C) \wedge (A \vee B \vee C)$
- List the prime implicates of Δ

HW3: Q5

Q5

- $\Delta = (\neg A \vee B \vee \neg C) \wedge (\neg A \vee B \vee C) \wedge (A \vee \neg B \vee \neg C) \wedge (A \vee \neg B \vee C) \wedge (A \vee B \vee C)$
- List the prime implicates of Δ

- To obtain the prime implicates, we do resolutions and remove subsumptions

- Resolve $(\neg A \vee B \vee \neg C)$ and $(\neg A \vee B \vee C)$
  - $\neg A \vee B$

- Resolve $(A \vee \neg B \vee \neg C)$ and $(A \vee \neg B \vee C)$
  - $A \vee \neg B$

HW3: Q5

Q5

- $\Delta = (\neg A \vee B \vee \neg C) \wedge (\neg A \vee \neg B \vee \neg C) \wedge (A \vee \neg B \vee \neg C) \wedge (A \vee \neg B \vee C) \wedge (A \vee B \vee C)$

- Resolve $(\neg A \vee B \vee \neg C)$ and $(\neg A \vee B \vee C)$
  - $\neg A \vee B$
- Resolve $(A \vee \neg B \vee \neg C)$ and $(A \vee \neg B \vee C)$
  - $A \vee \neg B$
- Resolve $(\neg A \vee B)$ and $(A \vee B \vee C)$
  - $B \vee C$
- Resolve $(A \vee \neg B)$ and $(B \vee C)$
  - $A \vee C$

HW3: Q5

Q5

- $\Delta = (\neg A \vee B \vee \neg C) \wedge (\neg A \vee \neg B \vee \neg C) \wedge (A \vee \neg B \vee \neg C) \wedge (A \vee \neg B \vee C) \wedge (A \vee B \vee C)$

- Resolve $(\neg A \vee B \vee \neg C)$ and $(\neg A \vee B \vee C)$
  - $\neg A \vee B$
- Resolve $(A \vee \neg B \vee \neg C)$ and $(A \vee \neg B \vee C)$
  - $A \vee \neg B$
- Resolve $(\neg A \vee B)$ and $(A \vee B \vee C)$
  - $B \vee C$
- Resolve $(A \vee \neg B)$ and $(B \vee C)$
  - $A \vee C$
- The prime implicates are $(\neg A \vee B)$, $(A \vee \neg B)$, $(B \vee C)$, $(A \vee C)$

HW3: Q5

Q5

- List the prime implicants of $\neg\Delta$

HW3: Q5

Q5

- List the prime implicants of $\neg\Delta$

- The prime implicates of Δ are $(\neg A \vee B)$, $(A \vee \neg B)$, $(B \vee C)$, $(A \vee C)$
- $\Delta = (\neg A \vee B) \wedge (A \vee \neg B) \wedge (B \vee C) \wedge (A \vee C)$

HW3: Q5

Q5

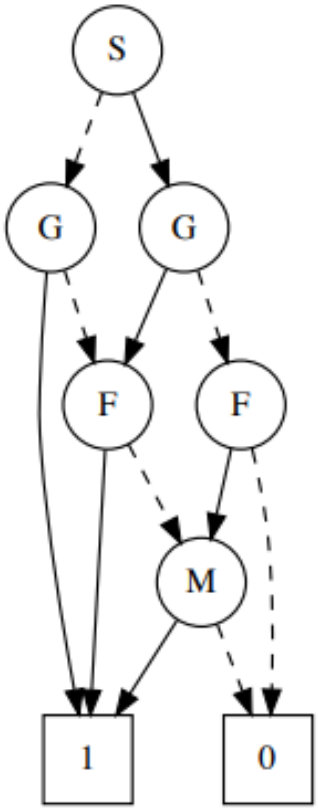
- List the prime implicants of $\neg\Delta$

- The prime implicates of Δ are $(\neg A \vee B)$, $(A \vee \neg B)$, $(B \vee C)$, $(A \vee C)$

- $\Delta = (\neg A \vee B) \wedge (A \vee \neg B) \wedge (B \vee C) \wedge (A \vee C)$
- $\neg\Delta = (A \wedge \neg B) \vee (\neg A \wedge B) \vee (\neg B \wedge \neg C) \vee (\neg A \wedge \neg C)$

- The prime implicants of $\neg\Delta$ are $(A \wedge \neg B)$, $(\neg A \wedge B)$, $(\neg B \wedge \neg C)$, $(\neg A \wedge \neg C)$

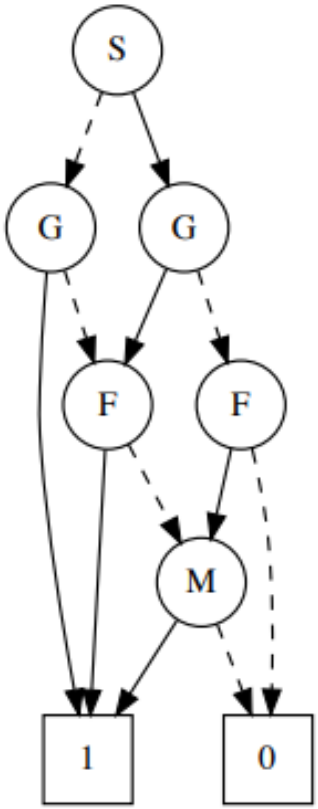
HW3: Q6



Q6

- $\{S = 1, G = 0, F = 1, M = 1\}$
- Identify a smallest set of features that renders the remaining features irrelevant to the decision on this instance

HW3: Q6

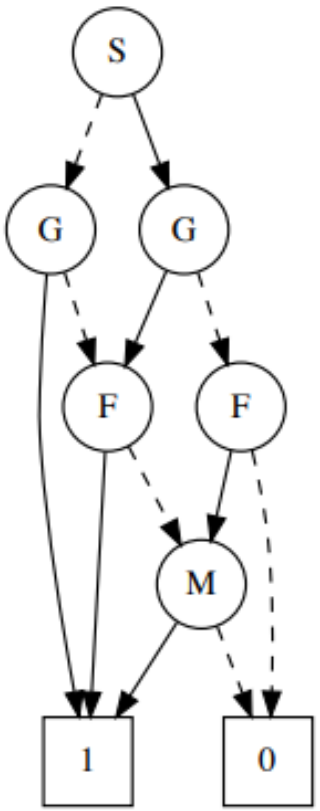


$\{S = 1, G = 0, F = 1, M = 1\}$

- Identify a smallest set of features that renders the remaining features irrelevant to the decision on this instance

- First method:
- The circuit has 5 prime implicants:
- $(\neg S \wedge G)$, $(\neg S \wedge F)$, $(F \wedge M)$, $(G \wedge M)$, $(G \wedge F)$
- The prime implicant explanation is $(F \wedge M)$

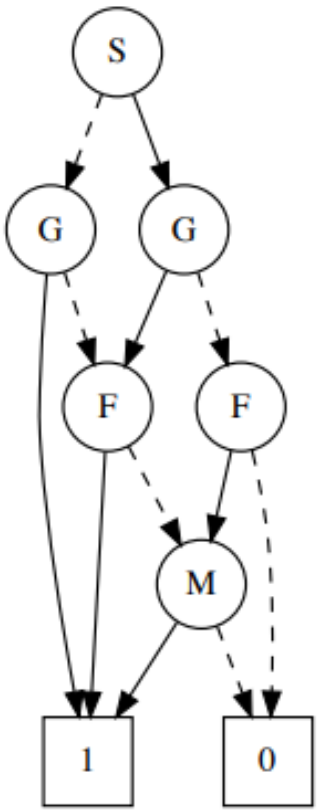
HW3: Q6



- $\{S = 1, G = 0, F = 1, M = 1\}$
- Identify a smallest set of features that renders the remaining features irrelevant to the decision on this instance.

- Second method:
- Try to remove $F = 1$ and check whether the decision is still guaranteed.
- No.

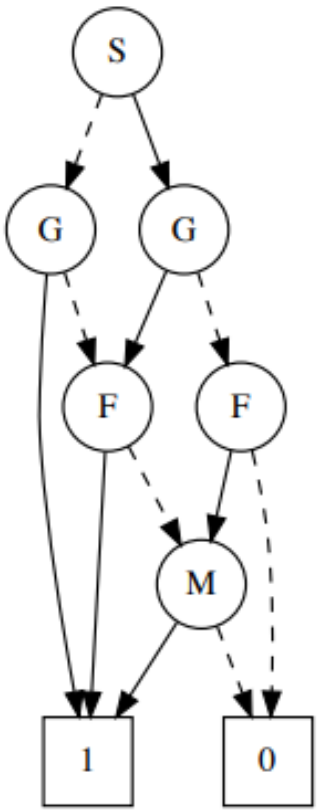
HW3: Q6



- $\{S = 1, G = 0, F = 1, M = 1\}$
- Identify a smallest set of features that renders the remaining features irrelevant to the decision on this instance.

- Second method:
- Try to remove $M = 1$ and check whether the decision is still guaranteed.
- No.

HW3: Q6

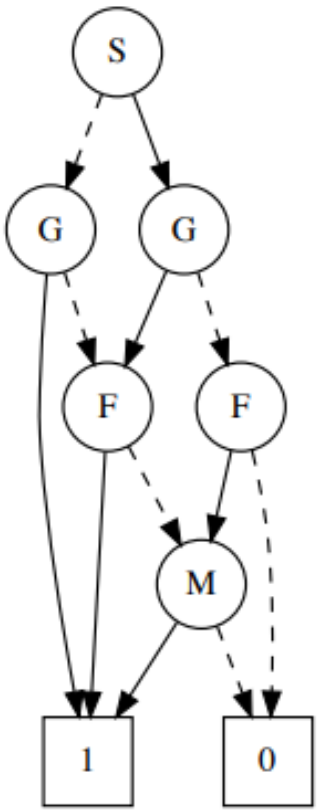


$\{S = 1, G = 0, F = 1, M = 1\}$

- Identify a smallest set of features that renders the remaining features irrelevant to the decision on this instance.

- Second method:
- Try to remove $S = 1$, $G = 0$.
- Both can be removed.
- The prime implicant is $(F \wedge M)$

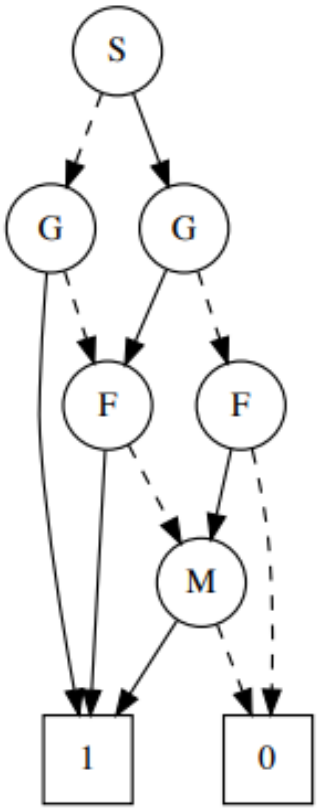
HW3: Q6



Q6

- Compute the complete reason for $\Delta = [G \wedge (\neg S \vee F)] \vee (F \wedge M) \vee [(\neg S \vee G) \wedge (F \vee M)]$ and $I = \{S = 0, G = 1, F = 0, M = 1\}$.

HW3: Q6

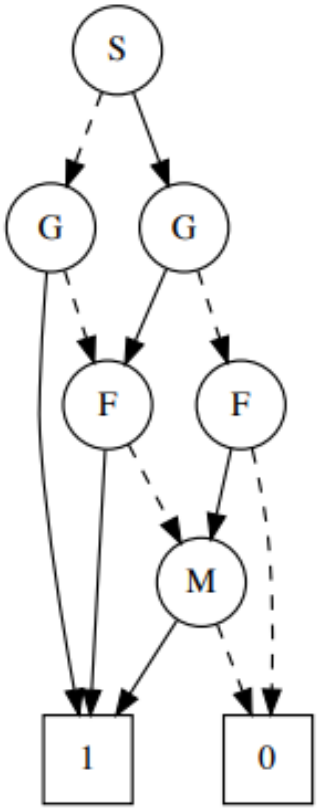


Q6

te the complete reason for $\Delta = \neg S \vee F)] \vee (F \wedge M) \vee [(\neg S \vee G) \wedge (F \vee M)]$ $= \{S = 0, G = 1, F = 0, M = 1\}$.

- $\forall \neg S \cdot \Delta = (G \vee ((F \wedge M) \vee (F \vee M)) \wedge (\neg S \vee [(G \wedge F) \vee (F \wedge M) \vee (G \wedge (F \vee M))]) = (G \wedge \neg S) \vee (F \wedge G) \vee (M \wedge \neg S) \vee (G \wedge F) \vee (F \wedge M) \vee (G \wedge M)$.
- $\forall G, \neg S \cdot \Delta = (\neg S \vee F \vee M) \wedge (G \vee (F \wedge \neg S) \vee (M \wedge \neg S) \vee (F \wedge M)) = (G \wedge \neg S) \vee (F \wedge \neg S) \vee (M \wedge \neg S) \vee (G \wedge F) \vee (F \wedge M) \vee (G \wedge M)$
- $\forall \neg F, G, \neg S \cdot \Delta = [(G \wedge \neg S) \vee (M \wedge \neg S) \vee (G \wedge M)] \wedge [\neg F \vee G \vee M \vee \neg S] = (G \wedge \neg S) \vee (M \wedge \neg S) \vee (G \wedge M)$
- $\forall M, \neg F, G, \neg S \cdot \Delta = (G \vee \neg S) \wedge (M \vee (G \wedge \neg S)) = (G \wedge \neg S) \vee (M \wedge \neg S) \vee (G \wedge M)$

HW3: Q6



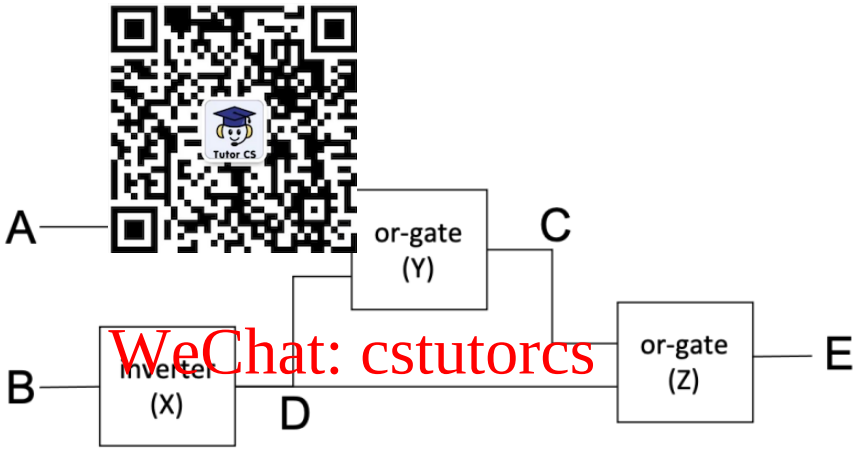
Q6

- Compute the sufficient/necessary reasons.

- Sufficient reasons: $(G \wedge \neg S)$, $(G \wedge M)$, $(M \wedge \neg S)$.

- Necessary reasons: $(G \vee \neg S)$, $(G \vee M)$, $(M \vee \neg S)$.

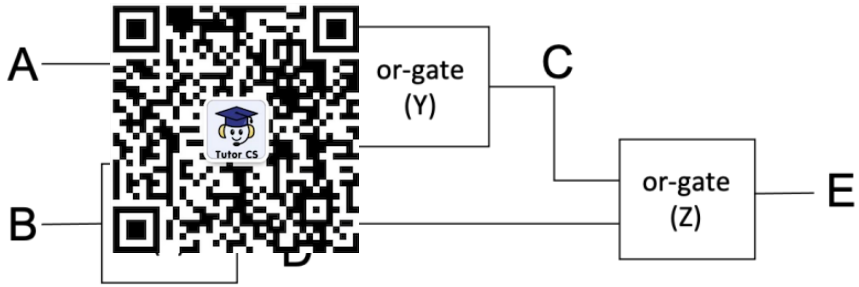
HW3: Q7



Q7

- Write the system description as a CNF

HW3: Q7

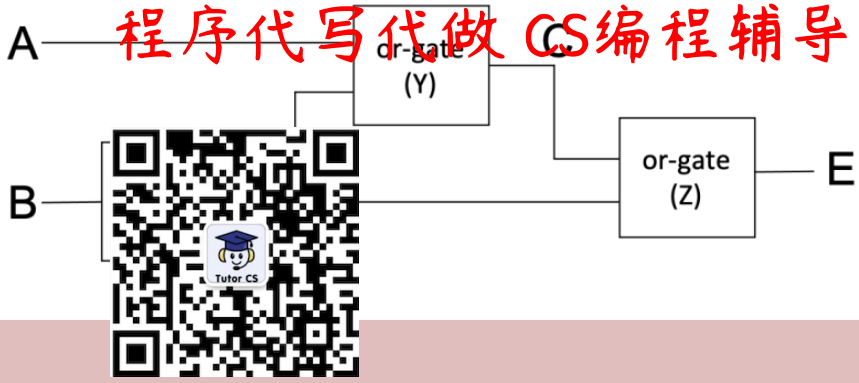


Q7

- Write the system description as a CNF

$$\Delta = (OK_X \Rightarrow (B \Leftrightarrow \neg D)) \wedge (OK_Y \Rightarrow ((A \vee D) \Leftrightarrow C)) \wedge (OK_Z \Rightarrow ((C \vee D) \Leftrightarrow E))$$

HW3: Q7



Q7

- Write the system description as a CNF

$$
\begin{aligned}
&A \leftrightarrow (\neg OK_X \vee \neg B \vee \neg D) \\
&\quad \wedge (\neg OK_X \vee B \vee D) \\
&\quad \wedge (\neg OK_Y \vee \neg A \vee C) \\
&\quad \wedge (\neg OK_Y \vee C \vee \neg D) \\
&\quad \wedge (\neg OK_Y \vee A \vee \neg C \vee D) \\
&\quad \wedge (\neg OK_Z \vee \neg C \vee E) \\
&\quad \wedge (\neg OK_Z \vee \neg D \vee E) \\
&\quad \wedge (\neg OK_Z \vee C \vee D \vee \neg E)
\end{aligned}
$$

HW3: Q7

Q7

- Suppose the system input is $A = 1, B = 0$ and the system output is $E = 0$
- Construct the health condition for the system using directed resolution

HW3: Q7

Q7

- Suppose the syste $A = 1, B = 0$ and the system output is $E = 0$
- Construct the health condition for the system using directed resolution

A	$\{\neg OK_Y, \neg A, C\}, \{\neg OK_Y, \neg C, A, D\}, A$	
B	$\{\neg OK_X, \neg B, \neg D\}, \{\neg OK_X, D, B\}, \neg B$	
C	$\{\neg OK_Y, \neg D, C\}, \{\neg OK_Z, \neg C, E\}, \{\neg OK_Z, \neg E, C, D\}$	$(\neg OK_Y, C)$
D	$\{\neg OK_Z, \neg D, E\}$	$\{\neg OK_Y, \neg OK_Z, \neg D, E\}, \{\neg OK_X, D\}$
E	$\neg E$	$\{\neg OK_X, \neg OK_Y, \neg OK_Z, E\}, \{\neg OK_Y, \neg OK_Z, E\}, \{\neg OK_X, \neg OK_Z, E\}$
		$\{\neg OK_X, \neg OK_Y, \neg OK_Z\}, \{\neg OK_X, \neg OK_Z\}, \{\neg OK_Y, \neg OK_Z\}$

- $\exists A, B, C, D, E \cdot (\Delta \wedge A \wedge \neg B \wedge \neg E) = \neg OK_Z \vee (\neg OK_X \wedge \neg OK_Y)$

HW3: Q7

Q7

- health$(\Delta, \alpha) = \neg OK_Z \vee (\neg OK_X \wedge \neg OK_Y)$
- List the kernel diagnoses

- The kernel diagnoses are $\neg OK_Z$ and $(\neg OK_X \wedge \neg OK_Y)$

HW3: Q7

Q7

- health$(\Delta, \alpha) = \neg OK_Z \vee (\neg OK_X \wedge \neg OK_Y)$
- List the minimal-cardinality diagnoses

- The minimal-cardinality diagnosis is $\neg OK_Z$, i.e., $OK_Z = \text{false}$