CS264A: Automated Reasoning

Fall 2023

Homework 3

Due Date: Monday, Nov 27

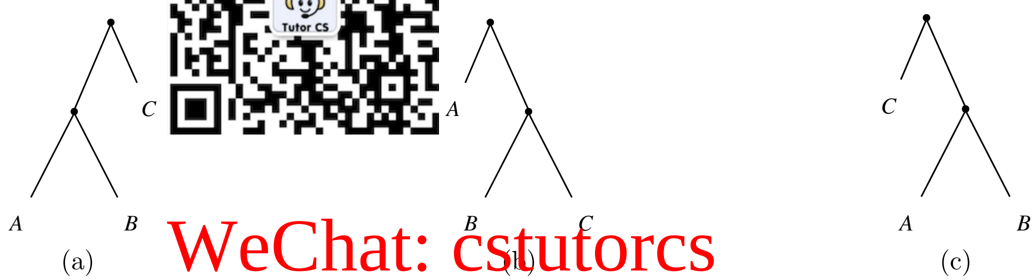


Figure 1: Vtrees over variables A, B, C.

1. [12 pts] Consider the function $f = (\neg A \vee \neg B \vee C) \wedge (B \vee \neg C)$.
   - (6pt) What is the compressed $(\mathbf{X}, \mathbf{Y})$-partition of function f, where $\mathbf{X} = \{A, B\}$ and $\mathbf{Y} = \{C\}$?
   - (3pt) To construct an SDD using the $(\mathbf{X}, \mathbf{Y})$-partition that you derived in the previous question, which of the vtrees in Figure 1 should be used?
   - (3pt) Which vtrees in Figure 1 will lead to an SDD that corresponds to an OBDD?

2. [16 pts] Consider the following function $f = (A \wedge B) \vee (B \wedge C) \vee (C \wedge D)$.
   - (8pt) Construct the compressed $(\mathbf{X}, \mathbf{Y})$-partitions for f and $\neg f$, where $\mathbf{X} = \{A, C\}$ and $\mathbf{Y} = \{B, D\}$.
   - (8pt) Derive a general rule for finding an $(\mathbf{X}, \mathbf{Y})$-partition for any function $\neg f$ from an $(\mathbf{X}, \mathbf{Y})$-partition of function f.

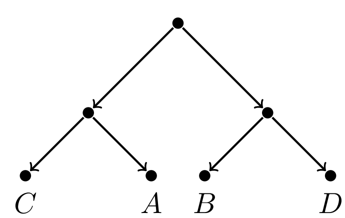


Figure 2: A vtree over variables A, B, C, D.

3. [14 pts] Construct an SDD for the function $f = (A \wedge \neg B) \vee (\neg B \wedge C) \vee (C \wedge D)$ based on the vtree in Figure 2.

4. [12 pts] Consider a structured space which corresponds to selecting k or more items from a set of n items, where $n \geq 1$ and $0 \leq k \leq n$. A Boolean formula Δ captures this space iff there is a one-to-one correspondence between the possible selections and the satisfying assignments of Δ. Suppose we use the Boolean variable A_i to indicate whether item i is selected.

(4pt) Describe a CNF that captures this structured space. How many clauses does the CNF have in terms of n and k.

(4pt) Describe a DNF that captures this structured space. How many terms does the DNF have in terms of n and k.

(4pt) Can you ca[illegible] space more efficiently using an OBDD? If so, describe the OBDD [illegible] of n and k.

5. [12 pts] Conside[illegible]

$$\Delta = (\neg A \vee [illegible] C) \wedge (A \vee \neg B \vee \neg C) \wedge (A \vee \neg B \vee C) \wedge (A \vee B \vee C).$$

(8pt) List the pr[illegible]

(4pt) List the pr[illegible].

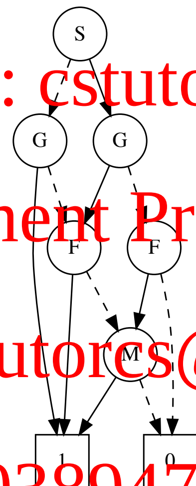


Figure 3: An OBDD representing a classifier.

6. [14 pts] Consider a model that predicts a movie's box success based on four binary features, S (Original Screenplay), G (Great Cinematography), F (Famous Cast), and M (Marketing). The OBDD in Figure 3 describes the classification function, i.e. a feature configuration is evaluated to 1 iff the corresponding movie is predicted to be a success. Please answer the following explanation queries on the OBDD.

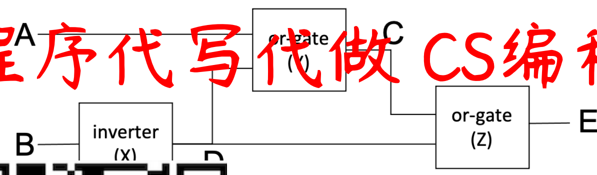
(4pt) **Prime Implicant Explanation:** Consider a movie that is an original screenplay and has poor cinematography, a famous cast, and good marketing $\{S = 1, G = 0, F = 1, M = 1\}$. Identify a smallest set of features α that renders the remaining features β irrelevant to the decision on this instance. That is, if we fix features α to their current values, we can change the values of features β arbitrarily without changing the current decision.

(4pt) **Complete Reason:** The class formula for the positive class is

$$[G \wedge (\neg S \vee F)] \vee (F \wedge M) \vee [(\neg S \vee G) \wedge (F \vee M)].$$

Compute the complete reason for the decision on instance $I = \{S = 0, G = 1, F = 0, M = 1\}$.

(6pt) **Sufficient and Necessary Reasons:** Compute the sufficient reasons and necessary reasons for this decision. That is, compute the prime implicants and prime implicates of the complete reason.



re 4: A system.

7. [20 pts] Consid re 4 and suppose the health of gates X, Y and Z are represented by nd okZ.

(5pt) Write the s a CNF.

Suppose the system input is $A = 1$, $B = 0$ and the system output is $E = 0$. Answer the following questions under this system observation.

(8pt) Construct the health condition for the system using directed resolution.

(4pt) List the kernel diagnoses.

(3pt) List the minimal-cardinality diagnoses.