C[illegible] Discussion 10

TA: Albert Ji

Friday 8th December, 2023

Information

- Final
  - Tuesday, December 12
  - 3 pm - 6 pm
  - Franz Hall 1260

Final Review: SDDs

- $\Delta = (C \vee D) \wedge ($
- $X = \{C, D\}, Y =$
- $\Delta = [(C \vee D) \wedge ($ … $(\neg C \wedge \neg D) \wedge (\bot)]$

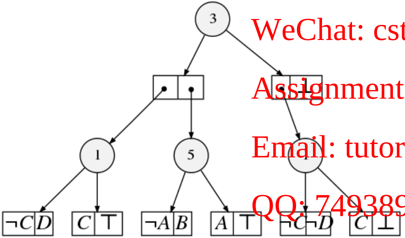


Figure: SDD

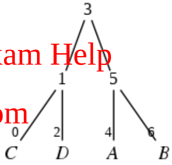
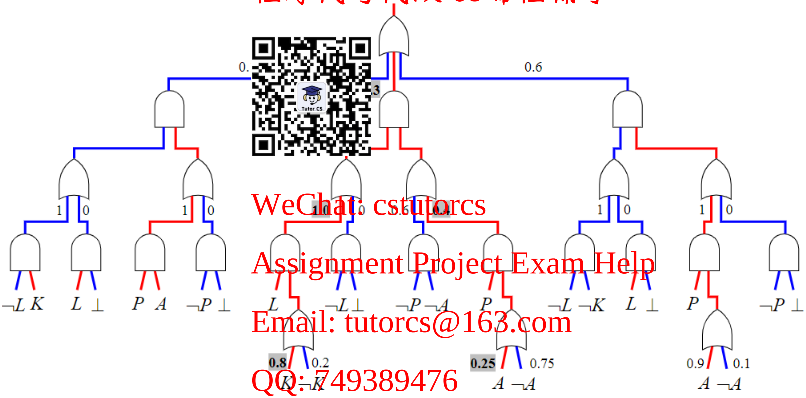


Figure: v-tree

Final Review: PSDDs

Input: L, K, P, A Pr(L,K,P,A) = 0.3 x 1.0 x 0.8 x 0.4 x 0.25 = 0.024

Final Review: PSDDs



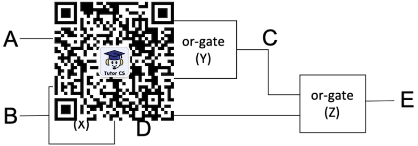
Input: L, K, P, A $Pr(L,K,P,A) = 0.3 \times 1.0 \times 0.8 \times 0.4 \times 0.25 = 0.024$

- Input: L, P, A

Final Review: PSDDs

Input: *L*, *K*, *P*, *A*   Pr(L, K, P, A) = 0.3 x 1.0 x 0.8 x 0.4 x 0.25 = 0.024

- Input: L, P, A
- Pr(L, P, A) $= 0.3 \times 1.0 \times 1.0 \times 0.4 \times 0.25 = 0.03$



- $\Delta = (OK_X \Rightarrow (B \Leftrightarrow \neg D))$
  $\wedge (OK_Y \Rightarrow ((A \vee D) \Leftrightarrow C))$
  $\wedge (OK_Z \Rightarrow ((C \vee D) \Leftrightarrow E))$
- Observation: $A, \neg B, \neg E$
- $\exists A, B, C, D, E \cdot (\Delta \wedge A \wedge \neg B \wedge \neg E) = \neg OK_Z \vee (\neg OK_X \wedge \neg OK_Y)$
- The kernel diagnoses are $\neg OK_Z$ and $(\neg OK_X \wedge \neg OK_Y)$
- The minimal-cardinality diagnosis is $\neg OK_Z$, or $OK_Z = \text{false}$

Final Review: Universal Literal Quantification

- Definition: $\forall x_i \cdot \Delta = \Delta|x_i \wedge \bigwedge_{j \neq i}(x_i \vee \Delta|x_j)$
- Base case: $x_{12} =$ … $|x_1 = x_{12}|x_2 = \top$. $x_{12}|x_3 = \bot$.
- Given a binary cla… nd instance I:
  - Decision: yes …
  - If $I \models \Delta$, the complete reason is $\forall I \cdot \Delta$
  - If $I \not\models \Delta$, the complete reason is $\forall I \cdot \neg\Delta$

Final Review: Universal Literal Quantification

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- Sufficient reasons: prime implicants of the complete reason
- Necessary reasons: prime implicates of the complete reason

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- Given a class formula Δ^c and instance I:
  - The general reason is $\overline{\forall} I \cdot \Delta$

Final Review: Universal Literal Quantification

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- Given a binary cla[illegible]nd instance I:
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- Given a class formula Δ^c and instance I:
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- General sufficient reasons: variable-minimal prime implicants of the general reason

- General necessary reasons: variable-minimal prime implicates of the general reason

Final Review: MAR to Weighted Model Counting

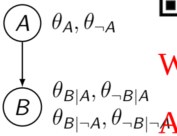

Figure: Bayesian Network N

$$
\begin{aligned}
\Delta = & A \Leftrightarrow P_A, \\
& \neg A \Leftrightarrow P_{\neg A}, \\
& A \wedge B \Leftrightarrow P_{B|A}, \\
& A \wedge \neg B \Leftrightarrow P_{\neg B|A}, \\
& \neg A \wedge B \Leftrightarrow P_{B|\neg A}, \\
& \neg A \wedge \neg B \Leftrightarrow P_{\neg B|\neg A}
\end{aligned}
$$

Final Review: MAR to Weighted Model Counting

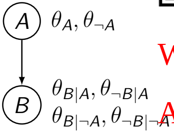

Figure: Bayesian Network $\mathcal{N}$

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\begin{aligned}
\Delta = & I_A \vee I_{\neg A}, \neg I_A \vee \neg I_{\neg A}, \\
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& I_A \wedge I_B \Leftrightarrow P_{B|A}, \\
& I_A \wedge I_{\neg B} \Leftrightarrow P_{\neg B|A}, \\
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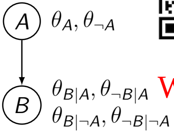

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- N is a Bayesian network about variable A, B with probabilities θ
- Δ is a Boolean formula about variables $I_A, I_{\neg A}, P_A, P_{\neg A}, \ldots$

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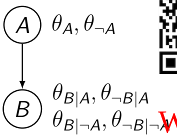

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\end{aligned}
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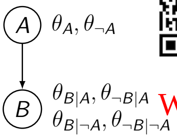
- Set $w(I_X) = w(I_{\neg X}) = 1$ for all variable X
- Set $w(\neg P_{u|v}) = 1$ for all literals u, v (note that u might be a negation like $\neg A$)
- Set $w(P_{u|v}) = \theta_{u|v}$

Final Review: MAR to Weighted Model Counting



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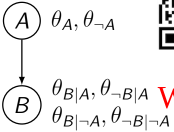
- Computing marginal probability on N is equivalent to weighted model counting on Δ
- For instance, $Pr(\neg B) = wmc(\Delta \wedge I_{\neg B})$



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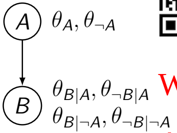
$$

- Assume $\theta_{B|A} = 0$
- Remove $I_A \wedge I_B \Leftrightarrow P_{B|A}$
- Add $\neg I_B \vee \neg I_A$



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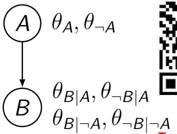
\end{aligned}
$$

- Assume $\theta_{\neg B|A} = 1$.
- Remove $I_A \land \neg I_B \Leftrightarrow P_{\neg B|A}$



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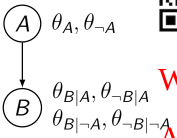
- Remove $I_A \wedge I_B \Leftrightarrow P_{B|A}$
- Add $\neg I_B \vee \neg I_A$
- Assume $\theta_{\neg B|A} = 1$.
- Remove $I_A \wedge \neg I_B \Leftrightarrow P_{\neg B|A}$

Final Review: Local Structures



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\end{aligned}
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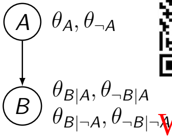
- Assume $\theta_{B|\neg A} = \theta_{\neg B|\neg A}$.

Final Review: Local Structures



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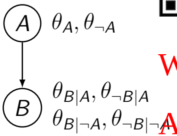
\end{aligned}
$$

- Assume $\theta_{B|\neg A} = \theta_{\neg B|\neg A}$.
- Remove $I_{\neg A} \wedge I_B \Leftrightarrow P_{B|\neg A}$
- Remove $I_{\neg A} \wedge I_{\neg B} \Leftrightarrow P_{\neg B|\neg A}$
- Add $[(I_{\neg A} \wedge I_B) \vee (I_{\neg A} \wedge I_{\neg B})] \Leftrightarrow P'$

Final Review: Local Structures



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Final Review: Arithmetic Circuits

- OR node ($\vee$): $+$
- AND node ($\wedge$): $\times$

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- MAR:
  - Require decomposability and smoothness
  - Substitute λ with $0/1$ and calculate

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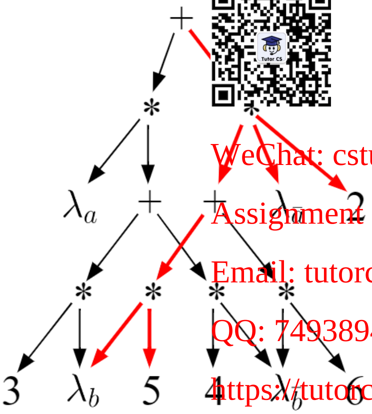
- OR node ($\vee$): $+$
- AND node ($\wedge$): $\times$
- MAR:
  - Require decomposability and smoothness
  - Substitute λ with $0/1$ and calculate
- MPE:
  - Require decomposability, determinism, and smoothness
  - Replace $+$ with max
  - Substitute λ with $0/1$ and calculate

Final Review: Subcircuits



Subcircuits

- One child of $+$ node
- All children of $\times$ node

Final Review: Subcircuits

Subcircuits

- One child of + node
- All children of × node

- Term: $\bar{a}b$
- Coefficient: $5 \times 2 = 10$

My Non-exhaustive Final Review Checklist

- Focus on lectures [illegible] midterm
- OBDDs
- Decomposability, determinism, smoothness
- Bottom-up compilation
- SDDs:
    - Compression
    - Canonicity
    - Polytime apply
- PSDDs:
    - Distribution
    - MAR
- Health gates:
    - System description
    - Health condition
    - Diagnoses:
- XAI:
    - Universal literal quantification
    - Complete reason
    - General reason
    - SR/NR/GSR/GNR
    - Bias
- Encoding Bayesian networks:
    - MAR to WMC
- Arithmetic circuits:
    - MAR
    - MPE
    - Subcircuits