# CS/ECE 374 A (Spring 2022)
# Conflict Midterm Exam 1

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**Instructions:**

- Date/Time: Feb 22 Tuesday 7:00pm–9:30pm Central Time.
- Except for your one double-sided handwritten $8.5'' \times 11''$ cheat sheet, exams are **closed-everything**. In particular: No medically unnecessarily electronic devices are allowed in exams, including smart watches and headphones/earbuds.
- You will write your solutions on paper, scan your solutions using a scanning app, convert your scans to PDF, and upload the PDF to Gradescope. (You are **not** allowed to write your solutions on tablets, unlike in some past semesters.) Gradescope will only accept PDF submissions. Please make sure your solution to each numbered problem starts on a new page of your submitted PDF file. Please do not scan your cheat sheets or scratch paper.
- You have **two hours (i.e., 120 minutes)** to write your solutions. We are providing 30 minutes at the end of each exam period for students to scan and upload their exams, and to deal with any unexpected technical difficulties that may arise. Gradescope will stop accepting submissions for each midterm precisely at 9:30pm. We will not accept submissions after the Gradescope deadline. **All work on the exam must stop 30 minutes before the Gradescope deadline, i.e., at 9:00pm.**
- Please make sure that your scans are clear and easy to read. We very strongly recommend that you write with black ink on unlined white paper. Submissions consisting of raw photos or low-quality scans will be penalized.
- If you are ready to scan your solutions before 9:00pm, send a private message to the host of your Zoom call ("Ready to scan") and wait for confirmation before leaving the Zoom call.
- If something goes seriously wrong during the exam, send email to Timothy (tmc@illinois.edu) as soon as possible explaining the situation. If you have already finished the exam but cannot submit to Gradescope for some reason, include a complete scan of your exam as a PDF file in your email. If you are in the middle of the exam, send email to Timothy, continue working until the time limit, and then send a second email with your completed exam as a PDF file. Please do not email raw photos.
- You are reminded about the course's, the department's, and the university's academic integrity policies.
- Avoid the deadly sins. Write complete solutions, not examples. Declare all your variables. Write concisely and precisely.
- Good luck!



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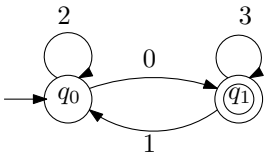
1. [27 PTS] *True or false, with justifications.* For each statement below, determine whether the statement is "True" or "False". You are required provide a short explanation (one or two sentences) for each of your answers.

   (a) The following NFA accepts the language $2^*0(10+3)^*$ (over the alphabet $\{0,1,2,3\}$).




   (b) For every DFA $M$, there exists a regular expression that generates the language accepted by $M$.

   (c) For every language $L$, if $L$ is accepted by a DFA $M$ with $n$ states where exactly one state is an accepting state, then $L^R = \{w^R : w \in L\}$ is accepted by a DFA with $n$ states obtained by reversing the direction of all transitions and reversing the role of the start state and the single accepting state in $M$.

   (d) For every pair of languages $L_1$ and $L_2$, we have $(L_1 \cup L_2)^* L_1^* = (L_1 \cup L_2 L_2^*)^*$.

   (e) The language $\{x \in \{0,1\}^* : x \text{ contains } 0^n 1^n \text{ as a substring for some } n \geq 374\}$ is regular.

   (f) The language $\{x \in \{0,1\}^* : x \text{ contains } 0^n 1^n 0^n \text{ as a substring for some } n \leq 374\}$ is regular.

   (g) For the language $\{x \in \{0,1\}^* : |x| \text{ is divisible by } 2022\}$, there exists a fooling set of size 2022.

   (h) Every context-free language is regular.

   (i) The language generated by the context-free grammar $S \to 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid \varepsilon$ is regular.



2. [24 PTS]

   (a) [8 PTS] Draw an NFA (i.e., a *nondeterministic* finite automaton, possibly with $\varepsilon$-transitions) for the language described by the following regular expression:

$$\big(1(010)^*(11)^* + 010\big)^*.$$

   Justification is not required.

   (b) [8 PTS] Draw an NFA for the following language:

$$\{x \in \{0,1\}^* : (\text{number of 0's in } x \text{ is divisible by 3}) \text{ or } (x \text{ does not end with 0110})\}.$$

   Justification is not required.

   (c) [8 PTS] Draw a DFA (i.e., a *deterministic* finite automaton) for the language described by the following regular expression:

$$(0+1)^*00 \;+\; (0+1)^*11.$$

   If you are using known methods from class, indicate which one(s). If not, briefly explain the meaning of the states in your DFA.

3. [16 PTS]

(a) [8 PTS] Show that the following language is regular:

$$\{0^i 1^j : j \text{ is divisible by } i, \text{ and } 1 \leq i \leq 2022\}.$$

(b) [8 PTS] Describe a DFA for the following language:

$$\{0^n 1^m : n + m \text{ is not divisible by } 2022\}.$$

Do not draw this DFA (there are too many states!). Instead, give a mathematically precise description of the states $Q$, the start state $s$, the accepting states $A$, and the transition function $\delta$. Briefly describe the meaning of the states in your DFA (proof of correctness is not required).

4. [18 PTS] Define the following language:

$$L = \{0^i 1^j 2^k : k \geq \min\{i, j\},\ i, j, k \geq 0\}.$$

(a) [10 PTS] Prove that $L$ is not regular, by using the fooling set method.

(b) [8 PTS] Show that $L$ is context-free, by giving a CFG. Briefly explain the meaning of each non-terminal. (You do not need to provide justification, or proof of correctness.)
[*Hint*: $k \geq \min\{i, j\}$ iff $k \geq i$ or $k \geq j$.]



5. [15 PTS] Given a language $L$ over the alphabet $\Sigma$, define

$$\text{INSERT-FIFTH}(L) = \{xay :\ xy \in L,\ x, y \in \Sigma^*,\ a \in \Sigma,\ |x| = 4\}.$$

(For example, if $1010010011 \in L$, then $1010\mathbf{0}010011$ and $1010\mathbf{1}010011$ are both in $\text{INSERT-FIFTH}(L)$.)

(a) [3 PTS] For $L = (01)^*$, describe $\text{INSERT-FIFTH}(L)$ by a regular expression.

(b) [12 PTS] Prove (in general) that if $L$ is regular, then $\text{INSERT-FIFTH}(L)$ is regular.
[*Hint*: given an NFA (or DFA) $M = (Q, \Sigma, s, \delta, A)$ for $L$, construct an NFA $M' = (Q', \Sigma, s', \delta', A')$ for $\text{INSERT-FIFTH}(L)$. Give a mathematically precise description of $Q'$, $s'$, $\delta'$, and $A'$ in your construction, and a brief explanation of how your NFA works. You don't have to provide meaning of the states or proof of correctness of your NFA.]