Design Turing machines $M = (Q, \Sigma, \Gamma, \delta, \text{start}, \text{accept}, \text{reject})$ for each of the following tasks, either by listing the states Q, the tape alphabet Γ, and the transition function δ (in a table), or by drawing the corresponding labeled graph.

Each of these machines uses the input alphabet $\Sigma = \{1, \#\}$; the tape alphabet Γ can be any superset of $\{1, \#, \square, \triangleright\}$ where $\square$ is the blank symbol and $\triangleright$ is a special symbol marking the left end of the tape. Each machine should reject any input not in the form specified below.

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1 On input 1^n, for any non-negative integer n, write $1^n\#1^n$ on the tape and accept.

2 On input $\#^n1^m$, for any non-negative integers m and n, write 1^m on the tape and accept. In other words, delete all the #s and shift the 1s to the start of the tape.

3 On input $\#1^n$, for any non-negative integer n, write $\#1^{2n}$ on the tape and accept. (**Hint:** Modify the Turing machine from problem 1.)

4 On input 1^n, for any non-negative integer n, write 1^{2^n} on the tape and accept. (**Hint:** Use the three previous Turing machines as subroutines.)