

Prove that each of the following problems is NP-hard.

- 1 Prove that the following problem is NP-hard: Given an undirected graph  $G$ , find *any* integer  $k > 374$  such that  $G$  has a proper coloring with  $k$  colors but  $G$  does not have a proper coloring with  $k - 374$  colors.
- 2 A **bicoloring** of an undirected graph assigns each vertex a set of *two* colors. There are two types of bicoloring: In a *weak* bicoloring, the endpoints of each edge must use *different* sets of colors; however, these two sets may share one color. In a *strong* bicoloring, the endpoints of each edge must use *distinct* sets of colors; that is, they must use four colors altogether. Every strong bicoloring is also a weak bicoloring.
  - 2.A. Prove that finding the minimum number of colors in a weak bicoloring of a given graph is NP-hard.
  - 2.B. Prove that finding the minimum number of colors in a strong bicoloring of a given graph is NP-hard.

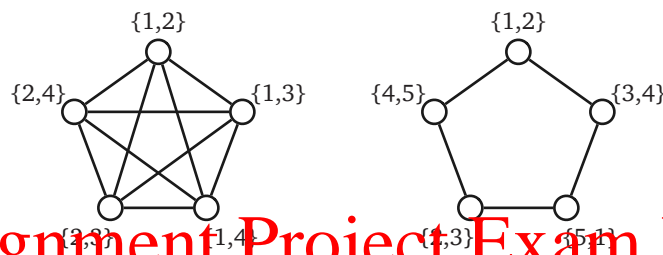


Figure 1: Left: A weak bicoloring of a 5-clique with four colors. Right: A strong bicoloring of a 5-cycle with five colors.

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