Prove that the following languages are undecidable.

See outline of how to solve such problems in the original problem set.

1 $\textsc{AcceptIllini} := \{\langle M\rangle \mid M \text{ accepts the string } \mathit{ILLINI}\}$

Solution:

For the sake of argument, suppose there is an algorithm $\textsc{DecideAcceptIllini}$ that correctly decides the language $\textsc{AcceptIllini}$. Then we can solve the halting problem as follows:

```
DecideHalt(⟨M, w⟩):
   Encode the following Turing machine M′:
      M′(x):
         run M on input w
         return TRUE
   if DECIDEACCEPTILLINI(⟨M′⟩)
      return TRUE
   else
      return FALSE
```

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We prove this reduction correct as follows:

$\Longrightarrow$ Suppose M halts on input w.
Then M' accepts *every* input string x.
In particular, M' accepts the string ILLINI.
So **DecideAcceptIllini** accepts the encoding $\langle M'\rangle$.
So **DecideHalt** correctly accepts the encoding $\langle M, w\rangle$.

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$\Longleftarrow$ Suppose M does not halt on input w.
Then M' diverges on *every* input string x.
In particular, M' does not accept the string ILLINI.
So **DecideAcceptIllini** rejects the encoding $\langle M'\rangle$.
So **DecideHalt** correctly rejects the encoding $\langle M, w\rangle$.

In both cases, **DecideHalt** is correct. But that's impossible, because **Halt** is undecidable. We conclude that the algorithm **DecideAcceptIllini** does not exist.

As usual for undecidablility proofs, this proof invokes *four* distinct Turing machines:

- The hypothetical algorithm **DecideAcceptIllini**.
- The new algorithm **DecideHalt** that we construct in the solution.
- The arbitrary machine M whose encoding is part of the input to **DecideHalt**.
- The special machine M' whose encoding **DecideHalt** constructs (from the encoding of M and w) and then passes to **DecideAcceptIllini**.

2 $\textsc{AcceptThree} := \{\langle M\rangle \mid M \text{ accepts exactly three strings}\}$

Solution:

For the sake of argument, suppose there is an algorithm **DecideAcceptThree** that correctly decides the language AcceptThree. Then we can solve the halting problem as follows:

> DecideHalt($\langle M, w\rangle$):
>   Encode the following Turing machine M':
> $M'(x)$:
>   run M on input w
>   **if** $x = \varepsilon$ or $x = 0$ or $x = 1$
>   return True
>   else
>   return False
>   if DecideAcceptThree($\langle M'\rangle$)
>   return True
>   else
>   return False

We prove this reduction correct as follows:

$\Longrightarrow$ Suppose M halts on input w.
Then M' accepts exactly three strings: ε, 0, and 1.
So **DecideAcceptThree** accepts the encoding $\langle M'\rangle$.
So **DecideHalt** correctly accepts the encoding $\langle M, w\rangle$.

$\Longleftarrow$ Suppose M does not halt on input w.
Then M' diverges on *every* input string x.
In particular, M' does not accept exactly three strings (because $0 \neq 3$).
So **DecideAcceptThree** rejects the encoding $\langle M'\rangle$.
So **DecideHalt** correctly rejects the encoding $\langle M, w\rangle$.

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In both cases, **DecideHalt** is correct. But that's impossible, because Halt is undecidable. We conclude that the algorithm **DecideAcceptThree** does not exist.

3 AcceptPalindrome $:= \{\langle M\rangle \mid M$ accepts at least one palindrome$\}$

Solution:

For the sake of argument, suppose there is an algorithm **DecideAcceptPalindrome** that correctly decides the language **AcceptPalindrome**. Then we can solve the halting problem as follows:

> DecideHalt($\langle M, w\rangle$):
>   Encode the following Turing machine M':
> $M'(x)$:
>   run M on input w
>   return True
>   if DecideAcceptPalindrome($\langle M'\rangle$)
>   return True
>   else
>   return False

We prove this reduction correct as follows:

$\Longrightarrow$ Suppose M halts on input w.
Then M' accepts *every* input string x.
In particular, M' accepts the palindrome $RACECAR$.
So **DecideAcceptPalindrome** accepts the encoding $\langle M' \rangle$.
So **DecideHalt** correctly accepts the encoding $\langle M, w \rangle$.

$\Longleftarrow$ Suppose M does not halt on input w.
Then M' diverges on *every* input string x.
In particular, M' does not accept any palindromes.
So **DecideAcceptPalindrome** rejects the encoding $\langle M' \rangle$.
So **DecideHalt** correctly rejects the encoding $\langle M, w \rangle$.

In both cases, **DecideHalt** is correct. But that's impossible, because HALT is undecidable. We conclude that the algorithm **DecideAcceptPalindrome** does not exist.

Yes, this is ***exactly*** the same proof as for problem 1.

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