

**1** Suppose you are given a magic black box that somehow answers the following decision problem in *polynomial time*:

- INPUT: A CNF formula  $\varphi$  with  $n$  variables  $x_1, x_2, \dots, x_n$ .
- OUTPUT: TRUE if there is an assignment of TRUE or FALSE to each variable that satisfies  $\varphi$ .

Using this black box as a subroutine, describe an algorithm that solves the following related search problem in *polynomial time*:

- INPUT: A CNF formula  $\varphi$  with  $n$  variables  $x_1, \dots, x_n$ .
- OUTPUT: A truth assignment to the variables that satisfies  $\varphi$ , or NONE if there is no satisfying assignment.

(Hint: You can use the magic box more than once.)

**2** An *independent set* in a graph  $G$  is a subset  $S$  of the vertices of  $G$ , such that no two vertices in  $S$  are connected by an edge in  $G$ . Suppose you are given a magic black box that somehow answers the following decision problem in *polynomial time*:

- INPUT: An undirected graph  $G$  and an integer  $k$ .
- OUTPUT: TRUE if  $G$  has an independent set of size  $k$ , and FALSE otherwise.

**2.A.** Using this black box as a subroutine, describe algorithms that solve the following optimization problem in *polynomial time*:

- INPUT: An undirected graph  $G$ .
- OUTPUT: The size of the largest independent set in  $G$ .

(Hint: You have seen this problem before.)

**2.B.** Using this black box as a subroutine, describe algorithms that solve the following search problem in *polynomial time*:

- INPUT: An undirected graph  $G$ .
- OUTPUT: An independent set in  $G$  of maximum size.

**To think about later:**

**3** Formally, a *proper coloring* of a graph  $G = (V, E)$  is a function  $c: V \rightarrow \{1, 2, \dots, k\}$ , for some integer  $k$ , such that  $c(u) \neq c(v)$  for all  $uv \in E$ . Less formally, a valid coloring assigns each vertex of  $G$  a color, such that every edge in  $G$  has endpoints with different colors. The *chromatic number* of a graph is the minimum number of colors in a proper coloring of  $G$ .

Suppose you are given a magic black box that somehow answers the following decision problem in *polynomial time*:

- INPUT: An undirected graph  $G$  and an integer  $k$ .
- OUTPUT: TRUE if  $G$  has a proper coloring with  $k$  colors, and FALSE otherwise.

Using this black box as a subroutine, describe an algorithm that solves the following *coloring problem* in *polynomial time*:

- INPUT: An undirected graph  $G$ .
- OUTPUT: A valid coloring of  $G$  using the minimum possible number of colors.

(Hint: You can use the magic box more than once. The input to the magic box is a graph and **only** a graph, meaning **only** vertices and edges.)