

- 1** Let L be an arbitrary regular language. Prove that the language $reverse(L) := \{w^R \mid w \in L\}$ is regular.
Hint: Consider a DFA M that accepts L and construct a NFA that accepts $reverse(L)$.
- 2** Let L be an arbitrary regular language. Prove that the language $insert1(L) := \{x1y \mid xy \in L\}$ is regular.
 Intuitively, $insert1(L)$ is the set of all strings that can be obtained from strings in L by inserting exactly one 1. For example, if $L = \{\varepsilon, OOK!\}$, then $insert1(L) = \{1, 1OOK!, O1OK!, OO1K!, OOK1!, OOK11\}$.

Work on these later:

- 3** Let L be an arbitrary regular language. Prove that the language $delete1(L) := \{xy \mid x1y \in L\}$ is regular.
 Intuitively, $delete1(L)$ is the set of all strings that can be obtained from strings in L by deleting exactly one 1. For example, if $L = \{101101, 00, \varepsilon\}$, then $delete1(L) = \{01101, 10101, 10110\}$.
- 4** Consider the following recursively defined function on strings:

$$stutter(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ aa \bullet stutter(x) & \text{if } w = ax \text{ for some symbol } a \text{ and some string } x \end{cases}$$

Intuitively, $stutter(w)$ doubles every symbol in w . For example:

- $stutter(PRESTO) = PPRREESSTTOO$
- $stutter(HOCUS \diamond POCUS) = HHCCUUSS \diamond HHCCUUSS$

Let L be an arbitrary regular language.

- Prove that the language $stutter(L) := \{stutter(w) \mid w \in L\}$ is regular.
- Prove that the language $stutter(L) := \{stutter(w) \mid w \in L\}$ is regular.

- 5** Consider the following recursively defined function on strings:

$$evens(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ \varepsilon & \text{if } w = a \text{ for some symbol } a \\ b \cdot evens(x) & \text{if } w = abx \text{ for some symbols } a \text{ and } b \text{ and some string } x \end{cases}$$

Intuitively, $evens(w)$ skips over every other symbol in w . For example:

- $evens(EXPELLIARMUS) = XELAMS$
- $evens(AVADA \diamond KEDAVRA) = VD \diamond EAR.$

Once again, let L be an arbitrary regular language.

- Prove that the language $evens^{-1}(L) := \{w \mid evens(w) \in L\}$ is regular.
- Prove that the language $evens(L) := \{evens(w) \mid w \in L\}$ is regular.