

**Question 1.** Any algorithm that solves  $\mathcal{P}$  can be used to sort  $L$  by first using it to place the items of  $L$  in  $T$ , and then traversing  $T$  in linear time and printing an inorder listing of the items in it (the inorder listing produced is a sorted version of  $L$ ). In other words if  $\mathcal{P}$  could be solved faster than the claimed lower bound, then we could sort faster than the known  $\Omega(n \log n)$  lower bound for sorting, a contradiction.

**Question 2.** Without loss of generality assume  $n_A < n_B$  (otherwise simply interchange the roles of  $A$  and  $B$  in what follows).

1. Initialize the answer set  $S$  to be empty.
2. Sort  $A$  in  $O(n_A \log n_A)$  time.
3. For every element  $x$  of  $B$ , binary search for  $x$  in the sorted version of  $A$ : If that binary search finds  $x$  then include  $x$  in  $S$ . This step takes  $O(n_B \log n_A)$  time, because there are  $n_B$  binary searches each of which takes  $O(\log n_A)$  time.
4.  $S$  now contains the elements that are in both  $A$  and  $B$ .

The total time is  $O(n_A \log n_A + n_B \log n_A)$ , which is  $O(n_B \log n_A)$  because  $n_A \leq n_B$ .

**Question 3.** For every vertex  $v$  do the following.

1. Do a breadth-first search starting at  $v$  and stop that search as soon as you encounter a non-tree edge (this is why this search from  $v$  takes  $O(n)$  time). Suppose the non-tree edge encountered is between vertices  $x$  and  $y$ , and let  $T$  be the breadth-first tree created before edge  $(x, y)$  was encountered.
2. Let  $f(v)$  be the length of the cycle consisting of edge  $(x, y)$  together with the  $x$ -to- $y$  path along the edges of  $T$ .

The length of a shortest cycle is simply  $\min_v f(v)$ . There can be no shorter cycle because if there were, then the breadth-first search started at a vertex  $w$  on that cycle would have given an  $f(w)$  equal to the length of that cycle. Note, however, that if  $G$  contained odd cycles, then we would no longer be justified in stopping the breadth-first as soon as a non-tree edge is encountered (because a better odd-length cycle may lie ahead and we would miss it).

**Question 4.** For part 1 see the figure on the next page. Forward edges are  $(d, g)$ ,  $(b, e)$ . Backward edges are  $(b, a)$ ,  $(f, d)$ ,  $(j, h)$ ,  $(k, c)$ . Cross edges are  $(g, f)$ ,  $(h, d)$ ,  $(j, d)$ ,  $(l, j)$ ,  $(k, g)$ ,  $(m, k)$ . The strongly connected components are  $\{a, b\}$ ,  $\{c, k, i, m\}$ ,  $\{h, j, l\}$ ,  $\{d, f, e, g\}$ .

**Question 5.** See the figure on the next page.

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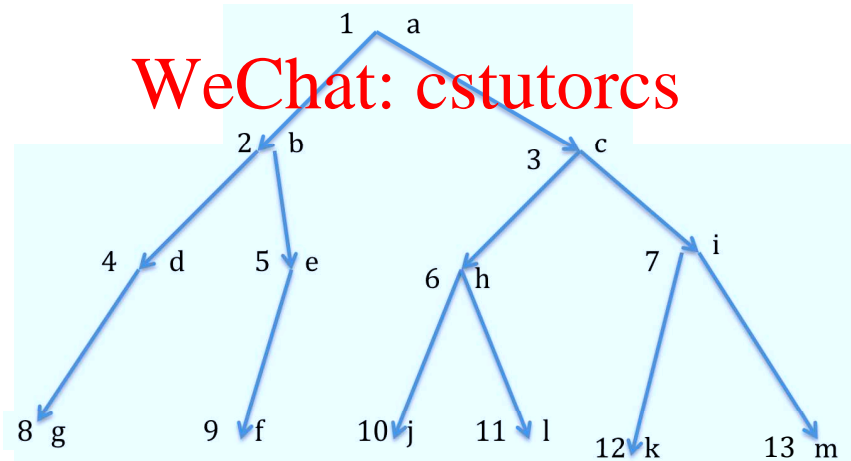


Figure for Question 5

Figure 1: The figures for questions 4.1 and 5