CS381 **In-class Exam 2 Answers** **Fall 2012**

Question 1. (60 points) Mark by T (= True) or F (= False) each of the following statements:

1. __T__ Let G be a directed graph and assume that a depth-first search in G visits all its vertices. Assume we denote vertices by their depth-first numbers (hence "vertex i" is the vertex whose depth-first number is i). If (i, j) is a non-tree edge, then $i < j$ implies that the edge (i, j) is a forward edge.

2. __F__ Let G be a directed graph and assume that a depth-first search in G visits all its vertices. Assume we denote vertices by their depth-first numbers (hence "vertex i" is the vertex whose depth-first number is i). If (i, j) is a non-tree edge, then $i > j$ implies that the edge (i, j) is a backward edge.

3. __T__ Let G be an n-vertex undirected graph and let v and w be two vertices that are in the same biconnected component of G (assume this biconnected component contains 50 vertices). Then there are two paths between v and w that are vertex-disjoint (i.e., these two paths have no vertex in common other than their endpoints v and w).

4. __F__ Let G be an n-vertex undirected graph and let v and w be two vertices that are in the same biconnected component of G (assume this biconnected component contains 50 vertices). Let P be any given path between v and w. Then there exists another path P' between v and w such that P and P' are vertex-disjoint (i.e., these two paths have no vertex in common other than their endpoints v and w).

5. __F__ Let G be an n-vertex undirected graph (with $n \geq 50$) that is connected and contains no bridges. Let v and w be any two vertices in that graph. Let P be any given path between v and w. Then there exists another path P' between v and w such that P and P' are edge-disjoint (i.e., these two paths have no edge in common).

6. __T__ In a directed graph G, if vertices u and v are in the same strongly connected component, and vertices v and w are in the same strongly connected component, then u and w must be in the same strongly connected component.

7. __F__ In a connected undirected graph G, if vertices u and v are in the same biconnected component, and vertices v and w are in the same biconnected component, then u and w must be in the same biconnected component.

8. __T__ Let T be a depth-first search tree of a directed *acyclic* graph G, and let v be the vertex whose postorder number in T is 1. Then v has an empty adjacency list.

9. __T__ Let T be a depth-first search tree of an n-vertex directed *acyclic* graph G, and let v be the vertex whose postorder number in T is n. Then v does not appear on any of the n adjacency lists.

10. __F__ Suppose that, in a maximum-flow problem, the input directed graph has a capacity of 1 associated with each edge. If k is the value of the maximum flow that can be pushed from vertex s to vertex t, then there are k vertex-disjoint paths from s to t.

11. __F___ Let a problem $\mathcal{P}$ have an $\Omega(n \log n)$ time lower bound, and let A be an algorithm that solves problem $\mathcal{P}$ in $O(n \log n)$ time. There cannot exist any instance of $\mathcal{P}$ that, when given as input to algorithm A, causes A to finish in $O(n)$ time for that particular input.

12. __F___ The $\Omega(n \log n)$ time lower bound proof given in class for the convex hull problem consisted of reducing the convex hull problem to sorting, by showing that any instance of the convex hull can be solved by using sorting.

Question 2. (40 points) Let f be the "failure function" used in the KMP pattern matching algorithm, that is, for a pattern $P = p_1 p_2 \dots p_m$, $f(i)$ is the largest j smaller than i such that the string $p_1 \dots p_j$ matches the string $p_{i-j+1} \dots p_i$ (if no such positive j exists then $f(i)$ is 0). In what follows we assume the pattern is large and that its length is a power of 2, e.g., $m = 2^{20}$.

1. (13 points) Suppose that $f(m) = m - 1$. Write down in the space below an expression for $f(i)$, for every i such that $2 \le i < m$.

   $f(i) = i - 1$.

2. (13 points) Suppose that $f(m) = m - 2$. Write down in the space below an expression for $f(i)$, for every i such that $3 \le i < m$,

   $f(i) = i - 2$

3. (14 points) If $f(2^{11}) = 2^{10}$ then state whether it is possible to have $f(2^{12}) = 2^{11}$, with a (very brief) justification of your answer.

   Impossible, as it would imply that $f(2^{12}) = 3 * 2^{10} = (3/2)2^{11}$, a contradiction to the assumption that $f(2^{12}) = 2^{11}$.