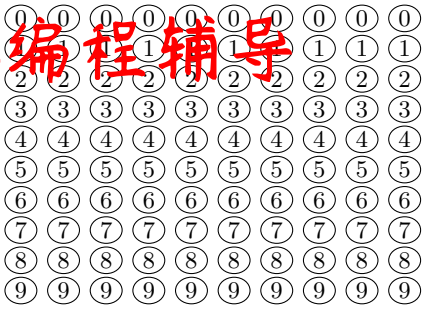
CSC 338H5 S 2019 Midterm Test
Duration — 60
Aids allowed: single-si
non-programmable

tudent Number:



UTORid:

Last Name: _____ **First Name:** _____

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*Do **not** turn this page until you have received the signal to start.*

Fill out the identification section above, write your name and bubble in the Student Number on the top right, and read the instructions below.

Good Luck!

This test consists of 5 questions on 8 pages (including this page). *When you receive the signal to start, please make sure that your copy is complete.*

- Bubble in your student number on the top right corner of this page using either a pen or a pencil.
- If you use any space for rough work, indicate clearly what you want marked.
- There is one blank page at the back of the exam if you need more space.
- **Do not remove any pages from the exam booklet.**

1: _____/ 8
2: _____/ 6
3: _____/ 4
4: _____/10
5: _____/12

TOTAL: _____/40

Question 1. [8 MARKS]

Circle either "True" or "False" for each of the below statements.

1. True False a error is affected by the stability of the algorithm.
2. True False n can be ill-conditioned.
3. True False rizing a non-singular $n \times n$ matrix A into $A = LU$ where and U is upper triangular is well-posed.
4. True False mentary elimination matrix is upper triangular.
5. True False For an $n \times n$ matrix A, if $cond(A) = 1$, then $A = I$ where I is the identity matrix.
6. True False The matrix $A = \begin{bmatrix} 2 & 7 \\ 4 & 15 \end{bmatrix}$ has a large condition number.
7. True False An $n \times n$ matrix A with $cond(A) = 2.13$ is ill-conditioned.
8. True False The product of two upper triangular matrices is upper triangular.

Question 2. [6 MARKS]

Consider the normalized floating-point system $F(\beta = 10, p = 3, L = -10, U = 10)$, where chopping is used for rounding.

Part (a) [1 MARK]

What is the representation of $\frac{1}{3}$ in the floating-point system? (What are the values of the mantissa and exponent?)

Part (b) [1 MARK]

What is the representation of $\frac{1}{3}$ in the floating-point system?

Part (c) [2 MARKS]

Compute $fl(\frac{1}{7}) - fl(\frac{1}{9})$, where the subtraction is floating-point subtraction.

Part (d) [2 MARKS]

What is the relative error of the result of part (c)? Write the relative error as a percentage, rounded to the nearest whole percent.

Question 3. [4 marks]

Consider, again, the normalized floating-point system $F(\beta = 10, p = 3, L = -40, U = 10)$, where chopping is used for rounding.

Suppose that we allow subnormal floating-point numbers in our system. How many subnormal floating-point numbers would we

Question 4. [10 MARKS]

Part (a) [5 MARKS]

Consider the condition number of the function $(f \circ g)(x) = f(g(x))$. Is it true that the condition number of $f \circ g$ is equal to the p[illegible]on numbers of f and the condition number of g? In other words, prove or disprove [illegible]$) = K_f(g(x)) \cdot K_g(x)$.

Part (b) [5 MARKS]

Show that $cond(AB) \leq cond(A)cond(B)$, where A and B are $n \times n$ non-singular matrices.

Question 5. [12 marks]

Part (a) [6 marks]

Consider the following matrix. Find the LU factorization of A using Gauss Elimination. You do not need to use pivoting. Show your [illegible] your final result below.

$$A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \quad U = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

Part (b) [3 MARKS]

Solve the system $Ax = b$ for $b = [7, 4, 2]^T$ using your results from part (a). Show all your work.

Part (c) [3 MARKS]

Compute $||A||_1$, $||x||_1$ and $||b||_1$, where A, b and x are from parts (a) and (b). Why would we expect that $\frac{||b||_1}{||x||_1} \leq ||A||_1$?