

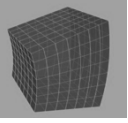
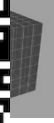

程序代写代做 CS编程辅导

The Jello Cube
Assignment 1, CSCI 520

Jernej Barbic, USC

1

The jello cube



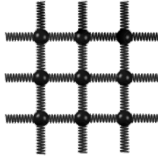
Original cube Deformed cube

The cube is elastic, if compressed, stretched, squeezed, ..., and external forces, it eventually restores to the original shape.

2

Mass-Spring System

- Several mass points
- Connected to each other by springs
- Springs expand and stretch, exerting force on the mass points
- Very often used to simulate cloth
- Examples:
 - [A 2-particle spring system](#)
 - [Another 2-particle example](#)
 - [Cloth animation example](#)



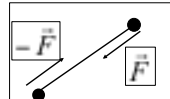
3

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Newton's Laws

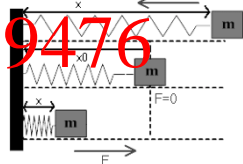
- Newton's 2nd law:
$$\vec{F} = m\vec{a}$$
- Tells you how to compute acceleration, given the force and mass
- Newton's 3rd law: If object A exerts a force \vec{F} on object B, then object B is at the same time exerting force $-\vec{F}$ on A.



4

Single spring

- Obeys the *Hook's law*:
$$F = k(x - x_0)$$
- x_0 = rest length
- k = spring elasticity (aka stiffness)
- For $x < x_0$, spring wants to extend
- For $x > x_0$, spring wants to contract



5

Hook's law in 3D

- Assume A and B two mass points connected with a spring.
- Let \vec{L} be the vector pointing from B to A
- Let R be the spring rest length
- Then, the elastic force exerted on A is:

$$\vec{F} = -k_{\text{Hook}} \left(\left| \vec{L} \right| - R \right) \frac{\vec{L}}{\left| \vec{L} \right|}$$

6

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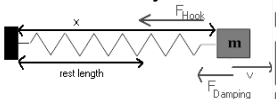
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Damping

- Springs are not completely elastic
- They absorb some of the energy and tend to decrease the velocity of the mass points attached to them
- Damping force depends on the velocity:

$$\vec{F} = -k_d \vec{v}$$


- k_d = damping coefficient
- k_d different than k_{Hook} !!

7



Damping in 3D

Consider two mass points connected with a spring. The vector pointing from B to A is \vec{L} . The damping force exerted on A is:

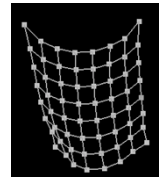
$$\vec{F}_d = -k_d \frac{(\vec{v}_A - \vec{v}_B) \cdot \vec{L}}{|\vec{L}|} \frac{\vec{L}}{|\vec{L}|}$$

\vec{v}_A and \vec{v}_B are velocities of points A and B. Damping force always OPPOSES the motion

8

A network of springs

- Every mass point connected to some other points by springs
- Springs exert forces on mass points
 - Hook's force
 - Damping force
- Other forces
 - External force field
 - Gravity
 - Electrical or magnetic force field
 - Collision force



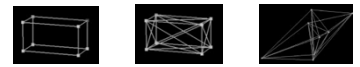
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How to organize the network (for jello cube)

- To obtain stability, must organize the network of springs in some clever way
- Jello cube is a 8x8x8 mass point network
- 512 discrete points
- Must somehow connect them with springs



Basic network Stable network Network out of control

10

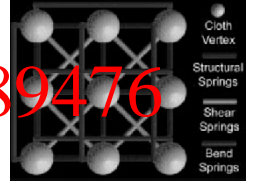
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Solution: Structural, Shear and Band Springs

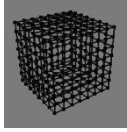
- There will be three types of springs:
 - Structural Springs
 - Shear Springs
 - Band Springs
- Each has its own function



11

Structural springs

- Connect every node to its 6 direct neighbours
- Node (i,j,k) connected to
 - $(i+1,j,k)$, $(i-1,j,k)$, $(i,j+1,k)$, $(i,j-1,k)$, $(i,j,k+1)$, $(i,j,k-1)$ (for surface nodes, some of these neighbors might not exist)
- Structural springs establish the basic structure of the jello cube
- The picture shows structural springs for the jello cube. Only springs connecting two surface vertices are shown.



12

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Shear springs

- Disallow excessive shearing
- Prevent the cube from distorting
- Every node (i,j,k) connected to its diagonal neighbors
- Structural springs = white
- Shear springs = red

A 3D cube (if you can't see it immediately, keep trying)

Shear spring (red) resists stretching and thus prevents shearing

13

Bend springs

from folding over

selected neighbor
per node, node)
springs
angle node
spring (yellow) resists contracting and thus prevents bending

14

External force field

- If there is an external force field, add that force to the sum of all the forces on a mass point

$$\vec{F}_{total} = \vec{F}_{Hook} + \vec{F}_{damping} + \vec{F}_{force\ field}$$

- There is one such equation for every mass point and for every moment in time

15

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Collision detection

- The movement of the jello cube is limited to a bounding box
- Collision detection easy:
 - Check all the vertices if any of them is outside the box
- Inclined plane:
 - Equation: $F(x,y,z) = ax + by + cz + d = 0$
 - Initially, all points on the same side of the plane
 - $F(x,y,z) > 0$ on one side of the plane and $F(x,y,z) < 0$ on the other
 - Can check all the vertices for this condition

16

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Collision response

- When collision happens, must perform some action to prevent the object penetrating even deeper
- Object should bounce away from the colliding object
- Some energy is usually lost during the collision
- Several ways to handle collision response
- We will use the penalty method

17

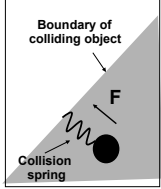
The penalty method

- When collision happens, put an artificial *collision spring* at the point of collision, which will push the object backwards and away from the colliding object
- Collision springs have elasticity and damping, just like ordinary springs

18

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Penalty force




- Direction is normal to the contact surface
- Magnitude is proportional to the amount of penetration
- Collision spring rest length is zero

19

Integrators

Mass points and springs
Hooke's law and Newton's 2nd law
on every mass point
time

Damping provide F
is follows from $F=ma$
acceleration at any given time for
compute the actual motion



20

Integrators (contd.)

- The equations of motion:

$$\frac{d\vec{x}}{dt} = \vec{v}$$

$$\frac{d^2\vec{x}}{dt^2} = \frac{d\vec{v}}{dt} = \vec{a}(t) = \frac{1}{m}(\vec{F}_{\text{Hook}} + \vec{F}_{\text{damping}} + \vec{F}_{\text{force field}})$$

- \vec{x} = point position, \vec{v} = point velocity, \vec{a} = point acceleration
- They describe the movement of any single mass point
- \vec{F}_{Hook} = sum of all Hook forces on a mass point
- \vec{F}_{damping} = sum of all damping forces on a mass point

21

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Integrators (contd.)

- When we put these equations together for all the mass points, we obtain a system of ordinary differential equations.
- In general, impossible to solve analytically
- Must solve numerically
- Methods to solve such systems numerically are called *integrators*
- Most widely used:
 - Euler
 - Runge-Kutta 2nd order (aka the midpoint method) (RK2)
 - Runge-Kutta 4th order (RK4)

22

Integrator design issues

- Numerical stability
 - If time step too big, method “explodes”
 - $\Delta t = 0.001$ is a good starting choice for the assignment
 - Euler much more unstable than RK2 or RK4
 - » Requires smaller time-step, but is simple and hence fast
 - Euler rarely used in practice
- Numerical accuracy
 - Smaller time steps means more stability and accuracy
 - But also means more computation
- Computational cost
 - Tradeoff: accuracy vs computation time

23

Integrators (contd.)

- RK4 is often the method of choice
- RK4 very popular for engineering applications
- The time step should be inversely proportional to the square root of the elasticity k [Courant condition]
- For the assignment, we provide the integrator routines (Euler, RK4)
 - void Euler(struct world * jello);
 - void RK4(struct world * jello);
 - Calls to these routines make the simulation progress one time-step further.
 - State of the simulation stored in ‘jello’ and automatically updated

24

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Tips

- Use double precision for all calculations (double)
- Do not overstretch the z-buffer
 - It has finite precision
 - Ok: `gluPerspective(90.0,1.0,0.01,1000.0);`
 - Bad: `gluPerspective(90.0,1.0,0.0001,100000.0);`
- Choosing the right elasticity and damping parameters is an art
 - Trial and error
 - For a start, can set the ordinary and collision parameters the same
- Read the webpage for updates

25



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