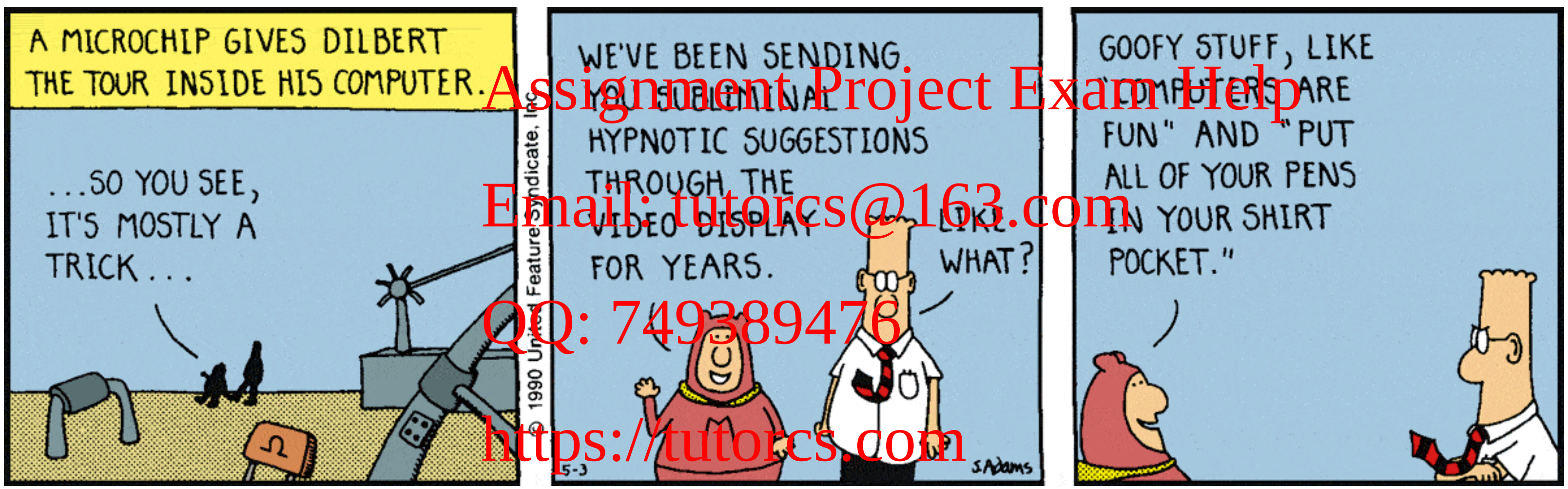
Lecture 3

Signed and Unsigned Numbers



Office Hours

Tentative time and spac

- Tuesdays 1 pm – 2 pm 660
- Tuesdays 2 pm – 3 pm Dreese 331
- Thursdays 1 pm – 2 pm Dreese 331

Will post Quiz #1 on Carmen today (or tomorrow)

n-bit Unsigned Numbers

Unsigned number = positive number

8-bits unsigned numbers n 0 to 255

n-bit unsigned numbers range from 0 to $2^n - 1$

Binary	Decimal
0000 0000	0
0000 0001	1
0000 0010	2
...	
...	
1111 1101	253
1111 1110	254
1111 1111	255

Signed Numbers – First Attempt

How do we represent negative integers in binary?

In decimal we represent n… umbers by prefixing them with a "–" sign

$(-\,0110)_2 \;=\; (-$ … **not** work on computers!!

There is no (–) sign

But we could still use **sign bit** and **magnitude**

0	0	0	0	0	1	1	0	$(6)_{10}$
1	0	0	0	0	1	1	0	$(-\,6)_{10}$
sign bit	magnitude							

0 for positive
1 for negative

Difficult to add two numbers

- Check signs: if both signs are the same, add both numbers …
- … if not compare magnitudes: subtract smaller number from larger one …
- … decide on the sign of the result

Yikes! We need to do better!

Signed Numbers & Complements

The sign and magnitude method does not work well on computers

not for integers or fixed point arithmetic

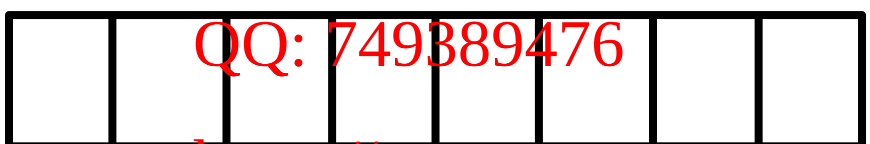
Modern computers use **2'** **ment** for signed numbers

Both 1's and 2's complement work only in context of a **fixed word length**

Two ingredients for complements:

1. **n** = word length in bits

   = size of the register



n = 8 bits

2. **N** = Binary number we want to complement

n-bit Ones' Complement

n-bit 1's complement of a binary number is obtained by flipping its bits

Given binary number **N** a[illegible]r size **n**

- fill the register – i.e., z[illegible]e number as needed to have n bits
- flip all bits – i.e., swap a 0 with a 1 and vice versa

e.g. N = 101101 n = 8 bits ⇒ **8-bit ones' complement**

N = 101101	0	0	1	0	1	1	0	1
8-bit ones' complement	1	1	0	1	0	0	1	0

Same idea for n = 16 bits – only more bits to fill and toggle

Ones' Complement

Why is it called **ones' complement**?

N = 101101			1	0	1	1	0	1
8-bit ones' complement of N	1	1	0	1	0	0	1	0
+								
all ones $2^8 - 1$	1	1	1	1	1	1	1	1

n-bit ones' complement of N $= 2^n - 1 - N$

Ones' Complement

What purpose does the one's complement serve?

$\Rightarrow$ Not much – at least in mputers

However, some earlier co sed 1's complement for signed numbers

i.e., to express – 41 use 1's complement of 41, N = 101001

Does it work? Yes, it does. But there are some issues

e.g., normally 41 + (– 41) = 0

with ones' complement method

00101001 + 11010110 = 11111111

there are two representations of zero with ones' complement

00000000 and 11111111

Two’s Complement

The **2’s complement** of a binary number is obtained by adding 1 to its ones’ complement

Given binary number **N** a[illegible] size **n**

- fill the register – i.e., z[illegible]e number as needed to have n bits
- flip all bits – i.e., 0 ↔1
- **add 1**

N = 101101

0	0	1	0	1	1	0	1

8-bit ones’ complement

1	1	0	1	0	0	1	0

\+ 1

=

1	1	0	1	0	0	1	1

⇒ **8-bit two’s complement**

2’s Complement – The Shortcut

There is a shortcut to write the **2’s complement** of a binary number

Given binary number **N** a[illegible] size **n**

- fill the register – i.e., z[illegible]e number as needed to have n bits
- **leave the least significant zeros and first 1 unchanged**
- **flip all remaining bits**

N = 110100

0	0	1	1	0	1	0	0
1	1	0	0	1	1	0	0

$\Rightarrow$ **8-bit two’s complement**

flip **leave unchanged**

Two's Complement

Why is it called **two's complement**? **Power of two's complement**

N = 101101				1	0	1	1	0	1
8-bit two's complement	+	1	1	0	1	0	0	1	1
Power of 2	1	0	0	0	0	0	0	0	0

$= 2^8$

n-bit two's complement of N $= 2^n - N$ if $N \neq 0$

Two's Complement

A better definition of **two's complement**

$$\text{n-bit two's complement of N} = \begin{cases} 2^n - N & \text{if } N \neq 0 \\ 0 & \text{if } N = 0 \end{cases}$$

Compare to ones' complement

$$\text{n-bit ones' complement of N} = 2^n - 1 - N$$

We see:

2's complement = 1's complement + 1

Works for zero when restricted to n-bits

Signed Numbers w/ 2's Complement

Use **two's complement representation** for signed numbers

modern c – including our MCU – use this method

- If a number N is positi ary representation of N
- If N is negative, use two's complement of absolute value of N

e.g. +/- 43

43		0	0	1	0	1	0	1	1
- 43	+	1	1	0	1	0	1	0	1
	1	0	0	0	0	0	0	0	0

2's complement of |-43|

Does this work?

Is this consistent with the rules of arithmetic?

- N + (– N) = 0 ✓ slide
- – (– N) = N ✓ We get the original N when we complement twice

- Successors and predecessor relationships are consistent with incrementing and decrementing

$(1)_{10}$ = 0001 $(2)_{10}$ = 0010 $(3)_{10}$ = 0011

$(-1)_{10}$ = 1111 $(-2)_{10}$ = 1110 $(-3)_{10}$ = 1101

– 3	– 2	– 1	0	1	2	3
1101	1110	1111	0000	0001	0010	0011

Signed Numbers

8-bits can represent 256 distinct values

8-bit unsigned numbers r… -128 to 127

n-bit signed numbers

-2^{n-1} to $2^{n-1}-1$

not sign-magnitude

	Decimal
1000 0000	-128
1000 0001	-127
…	
1111 1110	-2
1111 1111	-1
0000 0000	0
0000 0001	1
…	
0111 1110	126
0111 1111	127

negative numbers start with a “1”: 2’s complement of the absolute value

positive numbers start with a “0”: binary representation of the number

Signed Numbers

Given **2's complement signed numbers** find the decimal values

- `0110 1001` Positive Number 105
- `1101 0001` Negative Number $(11010001)_2$ = 209

- 47 2's complement is 256 – 209 = 47

2's complement of 11010001 is 00101111

$(00101111)_2$ = 47

- `0010 1010` Positive Number 42

- `1110 1110` Negative Number - 18

Addition of Signed/Unsigned Numbers

Computers add all numbers using the same hardware – they do not
sh between signed or unsigned numbers

Unsigned Number Interpretation		Signed Number Interpretation
43	00101011	43
+ 18	+ 00010010	+ 18
61	00111101	61
overflow possible did not happen		overflow possible did not happen

Addition of Signed/Unsigned Numbers

Computers add all numbers using the same hardware – they do not
sh between signed or unsigned numbers

Unsigned Number Interpretation		Signed Number Interpretation
213	11010101	-43
+ 18	+ 00010010	+ 18
231	11100111	-25

overflow possible did not happen

overflow **not** possible!

Addition of Signed/Unsigned Numbers

Computers add all numbers using the same hardware – they do not
…sh between signed or unsigned numbers

Unsigned Number Interpretation		Signed Number Interpretation
43	00101011	43
+ 238	+ 11101110	+ -18
281	1 $\boxed{00011001}$	25
overflow!		overflow **not** possible!
		carry out of “sign bit”

Addition of Signed/Unsigned Numbers

Computers add all numbers using the same hardware – they do not [illegible]sh between signed or unsigned numbers

<u>Unsigned Number Interpretation</u>		<u>Signed Number Interpretation</u>
213	11010101	-43
+ 238	+ 11101110	+ -18
451	1 $\boxed{11000011}$	-61
overflow!		overflow possible did not happen