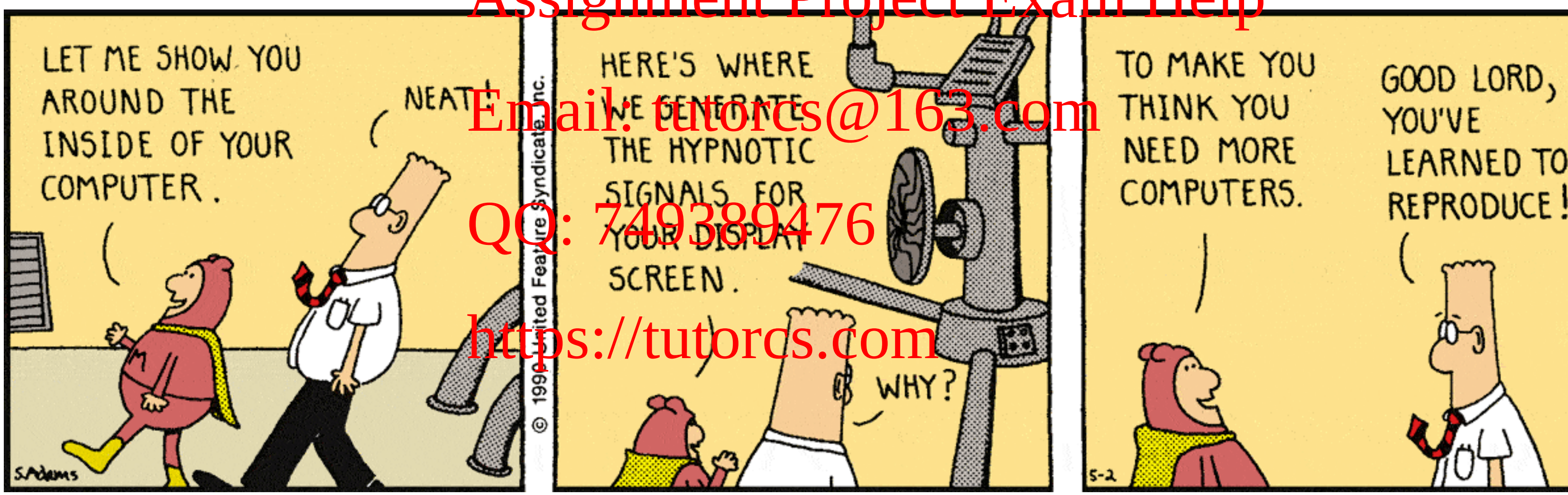
Lecture 2

Digital Devices and : Binary and Hexadecimal Numbers



Last Time: Computers

We have distinguished between

- General-purpose com… g., desktops, laptops
- Embedded computers … crocontrollers

Two things both classes have in common is: They are **digital electronic computers**

Digital computers process information in discrete form, in particular **binary** two values: **0** and **1**

Electronic computers as opposed to mechanical systems

Vacuum tubes ➡️ Transistors ➡️ MOSFET ➡️ **Integrated Chips (IC)**

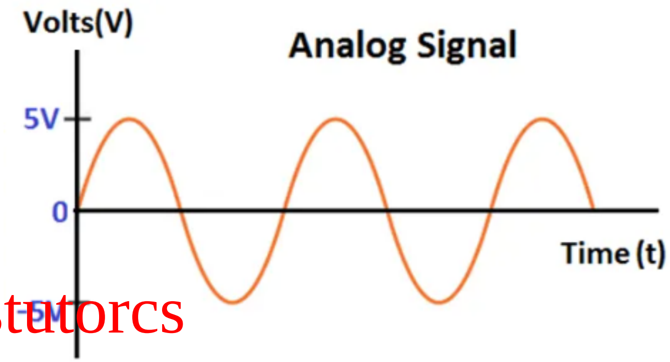
➡️ Quantum computers

➡️ ???

Analog vs Digital

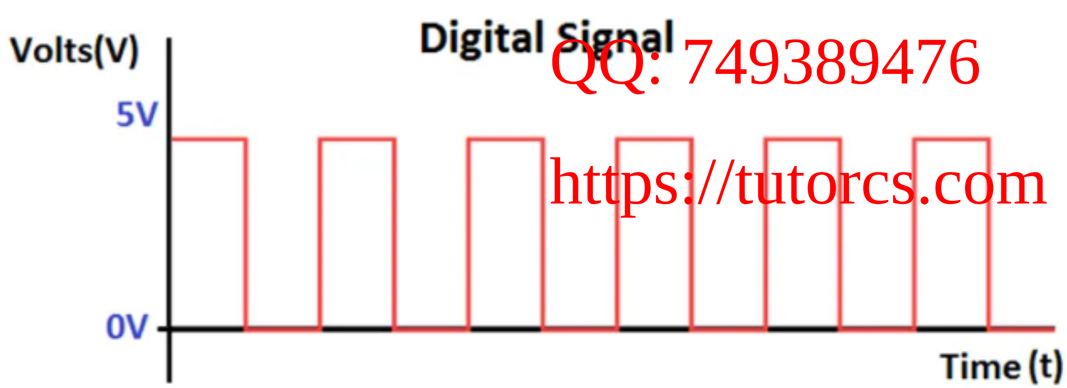
An **analog signal** takes a continuous range of values

⇒ infinite set of possible



A **digital signal** takes values from **a finite set**

Often this finite set has only **two possible values** **0** and **1**



aka **logic signal**
or **binary signal**

The Digital Revolution

The **digital revolution** changed human civilization

Agricultural Rev.
~10,000 BCE

[illegible]trial Rev.
[illegible]50-1850

Digital Rev.
~1950

Agricultural Age

Industrial Age

Information Age

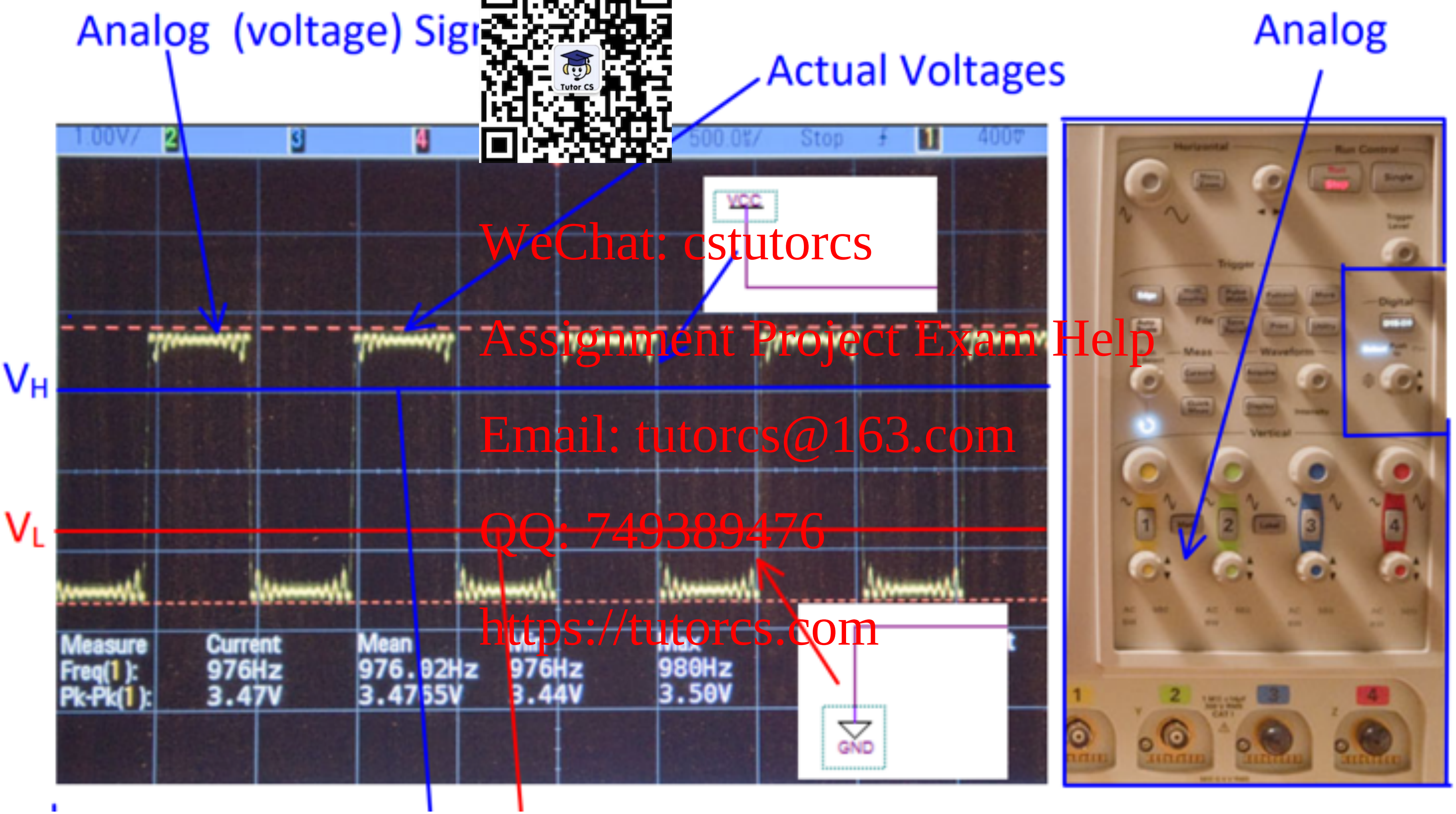
What is the digital revolution?

- Transition from analog to digital technology
- Transistor (1947) ⇒ Integrated Circuits (IC) ⇒ Computers
- Digital signals and digital logic
- Digital communication ⇒ The Internet

YET digital signals are ~~a big fat lie~~ only an abstraction

Digital Signals

Physical digital signals do not exist – a digital signal is an abstraction



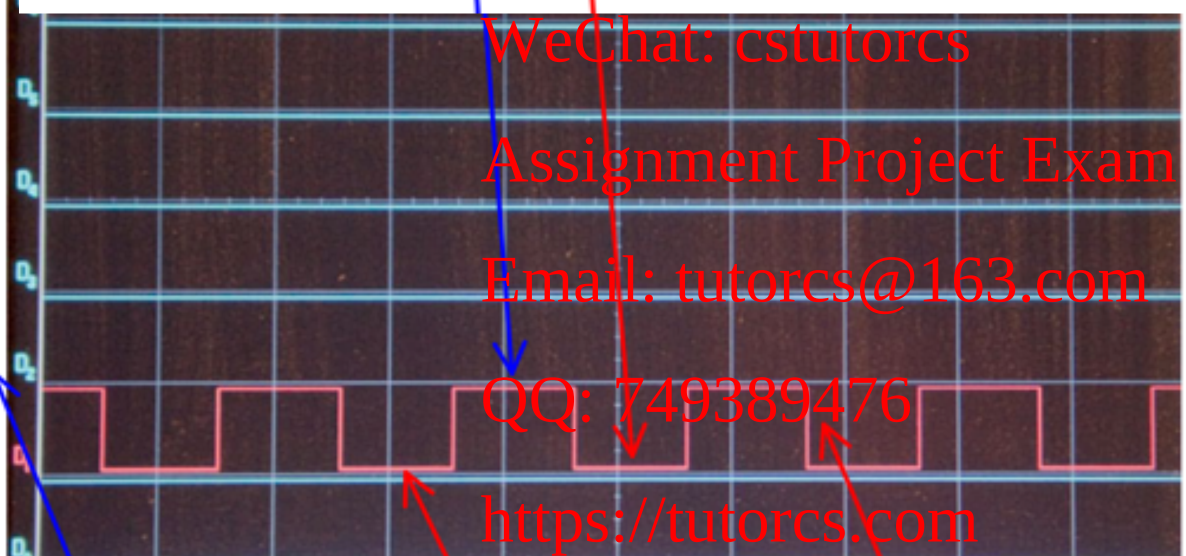
Digital Signals

When a mixed signal oscilloscope displays a digital signal it is an abstraction

Interpretation

Digital signal is Hi or 1 if analog signal voltage > blue voltage

Digital signal is Lo or 0 if analog signal voltage < red voltage



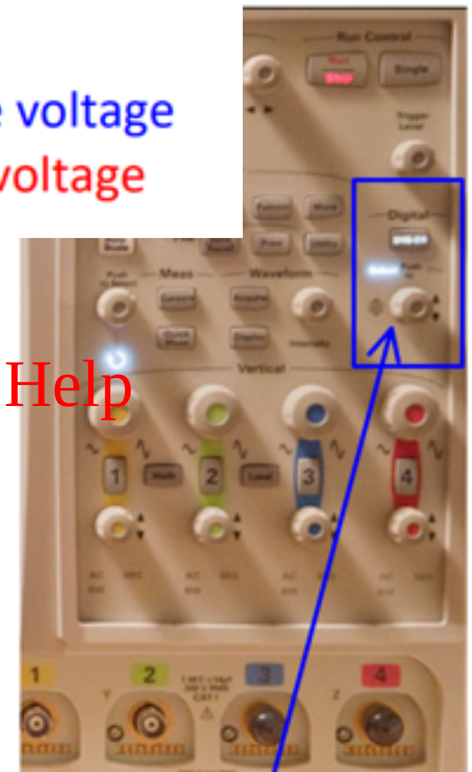
Hi, 1

Lo, 0

Logical Values not actual Voltages

1 Bit (Binary Digit)

Digital Signal



Digital

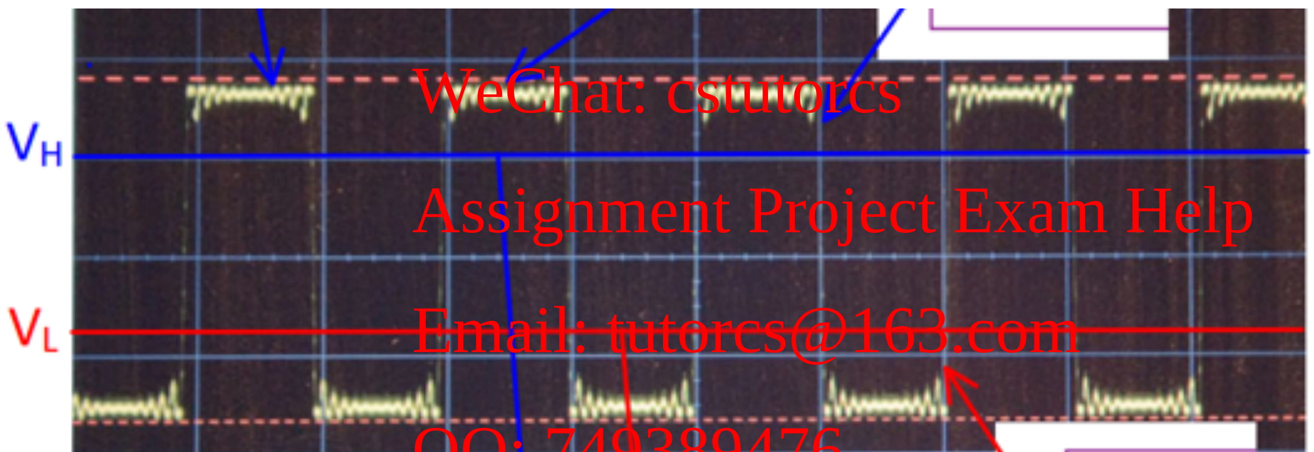
Digital Signals

This makes **digital signals very resilient to noise**

Digital signal is interprete

1 if (analog signa > blue voltage

0 if (analog signa < red voltage



Noise will have **no effect** as long as it is not too big

Listen to AM radio (analog) and HD radio and compare the sound quality

(Slightly) scratch a vinyl recording and a CD and see what happens

Binary Numerals

Digital signals take two values: **0** and **1**

Hardware can distinguish … **two stable states**: **0** and **1**

- Transistor and capacit… ic RAM)
- Flip-flops (static RAM)
- Magnetic material (HDD)
- Cells in solid-state drives (SSD)

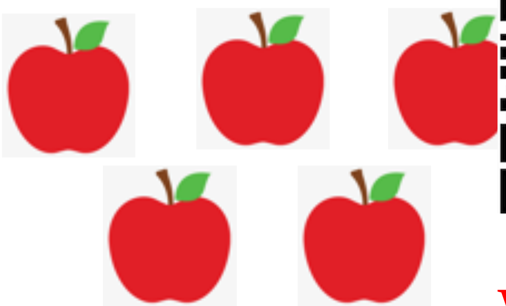
⇒ We have to learn to do everything with **0** and **1** only

What do computers do? Duh, they **compute**!

- **Math – Numbers and operations** ⇒ **We have to re-learn to do math using 0 and 1 only**
- Logic – True/False ✓

Numbers and Numeral Systems

A **number** is a mathematical object (abstraction) used to count and measure



"five"

Physical objects

Mathematical abstraction

There are multiple symbolic ways to express the number **"five"**

Tally marks

V

Roman numerals

5

Numerals we use in everyday life

101

Numerals used by computers

Numeral Systems

A **numeral system** is a mathematical notation for representing numbers

We focus on **positional** n[illegible]stems where the **value** of the expressed number depends on num[illegible]ols (**digits**) and their **position**

decimal base-10	tens ones 11	= 1x10 + 1x1	"eleven"
binary base-2	twos ones 11	= 1x2 + 1x1	"three"
hexa-decimal base-16	16s ones 11	= 1x16 + 1x1	"seventeen"

Decimal Numerals

Base 10 ("ten")

- Each position is a pow
- **Digits** 0, 1, 2, …, 9

Why base 10?

Example:

100 10 1 0.1 0.01

721.53

But we all know this!

Binary Numerals

Base 2 ("two")

Why base 2?

- Each position is a pow
- **Digits 0 and 1** **git = bit**

Example:

8 4 2 1 ½ ¼

1101.01

represents value

1x8 + 1x4 +0x2 + 1x1 + 0x ½ + 1x ¼ = 13.25 in decimal

Notation: $(1011)_2$ 0b1011 1011b

Binary to Decimal Conversion

32 16 8 4 2 1

101011

$= 32 + 8 + 2 + 1 = 43$ in decimal

Decimal to Binary Conversion

e.g. $41 = 32 + 8 + 1$

32	16	8	4	2	1
1	0	1	0	0	1

More systematically:

$41 = 2 \times 20 + 1$

$20 = 2 \times 10 + 0$

$10 = 2 \times 5 + 0$

$5 = 2 \times 2 + 1$

$2 = 2 \times 1 + 0$

$1 = 2 \times 0 + 1$

What about 52579? Bookmark your favorite decimal-to-binary converter!!

Hexadecimal Numerals

Base 16 ("sixteen")

Why base 16?

- Each position is a pow
- **Digits** 0, 1, 2, …, 9, A, , F

Example:

256 16 1

0x 3AD

represents value

$$3 \times 256 + 10 \times 16 + 13 \times 1 = (941)_{10}$$

in binary

0011 1010 1101

0x3 0xA 0xD

Hexadecimal to Binary

One hexadecimal digit encodes four **bits** (i.e., four binary digits)

Decimal	Hexadecimal	Binary
0	[illegible]	0000
1	[illegible]	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Hexadecimal ↔ Binary

$\overbrace{0110}^{6}\,\overbrace{1111}\,\overbrace{1011}$

$\underbrace{0010}_{2}\,\underbrace{1100}_{C}\,\underbrace{0111}_{7}$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Bookmark your favorite number converter!!

Operations with Binary Numerals

We can easily add two binary numbers

carry 1 1

```
   01101011        107
 + 11010010      + 210
 ----------      -----
  100111101        317
```


Note that the result is 9 bits

This would be an overflow in a 8-bit register

Operations with Binary Numerals

We can easily multiply two binary numbers

```
          1011
   x      1101
   ------------
          1011
         0000
        1011
   +   1011
   ------------
      10001111
```

```
     11
   x 13
   ----
    143
```

Operations with Binary Numerals

We can subtract two binary numbers – as long as the result is not negative

At this point we do not kn represent negative numbers with 0 and 1

Next time: 1’s complement, 2’s complement, signed numbers …