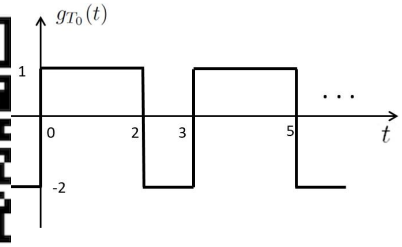
Homework Assignment 3

Due: No need to submit

Problem 1. Consider the periodic signal shown in Figure 1.



Signal for Problem 1.

a) Find the power spectral density of the signal.

b) Find the power and the autocorrelation function of the signal. Draw the autocorrelation function (e.g., using Matlab) and discuss the result.

Problem 2. The unit impulse responses of two linear time-invariant systems are

$$h_1(t) = 400\pi e^{-200\pi t} u(t)$$

$$h_2(t) = 400\pi e^{-200\pi t} \cos(20,000\pi t) u(t).$$

a) Find the magnitude responses of these systems.

b) Determine the filter type and 3-dB cut-off frequency of the first system $h_1(t)$.

c) How about the second system $h_2(t)$?

Problem 3. (Haykin & Moher Problem 2.31 with modifications) A system is a cascade of N identical RC circuits, each has frequency response $H_i(f) = 1/(1 + j2\pi f RC)$ as shown in Figure 2.

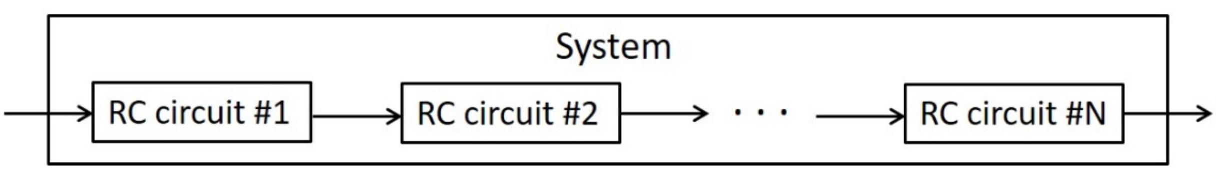


Figure 2: THe systematic diagram in Problem 3.

(a) Determine the overall amplitude response of the system.

(b) Assume that $\tau_0 = RC = T/(2\pi\sqrt{N})$. Show that as N approaches infinity, the amplitude response of the system approaches the Gaussian function $e^{-f^2 T^2/2}$. *Note:* $\lim_{x\to\infty}(1 + \frac{1}{x})^x = e$.

Problem 4. **a)** The signal $g(t) = e^{-|t|}$ is input to a linear time-invariant system whose frequency response is: $H(f) = \sqrt{1 + (2\pi f)^2}$. Calculate the energy of the output.

b) How about the energy of the output when the signal is input to a linear time-invariant system whose frequency response is:

$$H(f) = \begin{cases} -j\sqrt{1+(2\pi f)^2} & 0 < f \le f_0 \\ j\sqrt{1+(2\pi f)^2} & -f_0 \le f < 0 \\ 0 & \text{otherwise} \end{cases}.$$

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Problem 5. Consider as a Fourier transfer depicted in Figure 3. The signals goes through an ideal utoff frequencies at ± 1 Hz. Find the energy spectral density, autocorrelation f of the input and output signals.

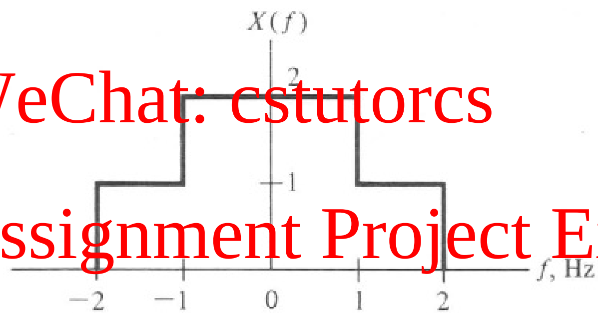

Figure 3: Signal for Problem 5.