Final Review

Instructions on the Final Open-Book Exam

Time: 2:00pm - 4:00pm 24-Apr-2023.
Location: NRE 2-001
Scope: All materials.

1. The exam is open-book. [illegible]tbook, lecture notes, assignments and solutions to the exam. E-versions of the above m[illegible]lease be aware of the University of Alberta's Code for Student Behaviour and complete [illegible] dependently.
2. You may also bring a no[illegible]ator.
3. Show your work by prov[illegible] on calculations and reasoning unless mentioned otherwise.
4. A Fourier transform table, properties of Fourier transform, and commonly used formulas will be provided.
5. One-card (or other photo ID) is required for identification. Please put your ID on the desk before the exam begins.

Course Material Overview
(Please focus on lecture notes posted on eClass. You may refer to the textbook for further information.)

Chapter 1 Introduction

- **Section 1.1** Communication System Structure
- **Section 1.2** Applications (Haykin & Moher 1.1, 1.2)
- **Section 1.3** Primary Resources and Operational Requirements (Haykin & Moher 1.3)
- **Section 1.4** Underpinning Theories and Related Topics (Haykin & Moher 1.4)

Chapter 2 Fourier Representation of Signals and Systems - Review

- **Section 2.1** Fourier Transform (Haykin & Moher 2.1)
- **Section 2.2** Properties of Fourier Transform (Haykin & Moher 2.2)
- **Section 2.3** Fourier Series and Fourier Transform of Periodic Signals (Haykin & Moher 2.4 and 2.5)
- **Section 2.4** Transmission of Signals through Linear Time-Invariant Systems (Haykin & Moher 2.6 partial)
- **Section 2.5** Filters (Haykin & Moher 2.7 partial)
- **Section 2.6** Energy Spectral Density and Autocorrelation Function for Energy Signals (Haykin & Moher 2.8 partial)
- **Section 2.7** Power Spectral Density and Autocorrelation Function for Power Signal (Haykin & Moher 2.9)

Chapter 3 Amplitude Modulation

- **Section 3.1** Fundamentals of AM and Conventional AM (Haykin & Moher 3.1, 3.2)
- **Section 3.2** Double Sideband-Suppressed Carrier Modulation (Haykin & Moher 3.3, 3.4)
- **Section 3.3** Quadrature-Carrier Multiplexing(Haykin & Moher 3.5)
- **Section 3.4** Single Sideband Modulation (Haykin & Moher 3.6)

Section 3.5 Vestige Sideband Modulation (Haykin & Moher 3.7 partial)

Chapter 4 Angle Modulation

Section 4.1 Fundamental Theories of Angle Modulation (Haykin & Moher 4.1)

Section 4.2 Properties of Angle Modulation (Haykin & Moher 4.2, 4.3)

Section 4.3 Spe[illegible]aykin & Moher 4.4, 4.5, 4.6)

Section 4.4 Gen[illegible]ion of FM (Haykin & Moher 4.7,4.8 partial)

Chapter 5 Pulse Modu[illegible]

Section 5.1 Sam[illegible] & Moher 5.1)

Section 5.2 Puls[illegible]on (Haykin & Moher 5.2 partial)

Section 5.3 Pulse-Position Modulation (Haykin & Moher 5.3)

Section 5.4 Time-Division Multiplexing (Haykin & Moher 5.10 partial)

Section 5.5 Quantization: Transition from Analog to Digital Communications (Haykin & Moher 5.5 and 5.6 partial)

Section 5.6 Pulse-Code Modulation (Haykin & Moher 5.4 and 5.6)

Section 5.7 Delta Modulation (Haykin & Moher 5.7)

Section 5.8 Differential Pulse-Code Modulation (Haykin & Moher 5.8)

Section 5.9 Linear Codes (Haykin & Moher 5.9)

Chapter 6 Digital Communications

Section 6.1 Source Coding / Decoding

Section 6.2 Channel Coding / Decoding

Section 6.3 Binary shift keying modulation

Section 6.4 M-ary shift keying modulation

Section 6.5 Constellation Design

Section 6.6 Detection Design

ECE 380

Introduction to Communication Systems

Final Examination

Instructions:

1. **Print your name and ID number** on your answer.

2. Your online submission is due at 4:10pm on Dec. 15, 2021. Please make sure to submit the copy of your answer to eClass successfully. No late submission is accepted.

3. Please follow the definitions of basic functions in the lecture notes and formula sheet posted on eClass.

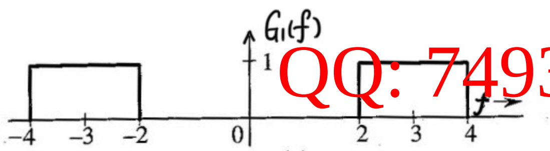
4. **Show your work.**

5. **Cheating is an academic offense.** The University of Alberta is committed to the highest standards of academic integrity and honesty.

Question	Mark Earned	Full Mark
# 1		10
# 2		15
# 3		15
# 4		15
# 5		15
# 6		15
# 7		15
Total		100

Problem 1. (10 points) (a) Use the frequency-shifting property of Fourier Transform to show that if $g(t) \rightleftharpoons G(f)$, then $2g(t)\cos(2\pi f_c t) \rightleftharpoons G(f - f_c) + G(f + f_c)$.

(b) Use (a) to find the time-domain representation and the energy of the signal shown in the following figure.



Problem 2. (15 points) A message $m_0(t)$ is given as follows:

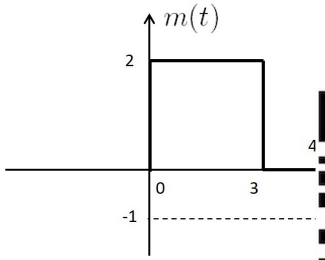
$$m_0(t) = e^{-2|t|} + 100\ \mathrm{sinc}(100t).$$

The signal first passes through an ideal low pass filter whose cutoff frequency is 40Hz. The output of the ideal low pass filter is called $m(t)$.

(a) The upper single-sideband amplitude modulation (USSB AM) is used for $m(t)$ to produce the modulated wave $s(t)$ [illegible]quency is <u>300</u> Hz. Please sketch the frequency spectrum of $s(t)$. Can $m(t)$ be [illegible]y from $s(t)$ using coherent detection? Please explain your answer.

(b)If the double sideband-suppressed carrier (DSB-SC) modulation is used for $m(t)$ to produce the modulated wave $s(t)$ where the carrier frequency is <u>30</u> Hz. Please sketch the frequency spectrum of $s(t)$. Can $m(t)$ be demodulated correctly from $s(t)$ using coherent detection? Please explain your answer.

Problem 3. (15 points)(a) Consider the message $m(t)$ shown in the following figure. If frequency modulation (FM) is used, sketch the modulated FM waveform and discuss your result.



(b) For the modulation in Part (a), if the frequency-sensitivity factor is $k_f = 0.1$, what is the maximum frequency deviation and the maximum phase deviation of the modulated wave?

(c) Consider the FM modulation of a signal whose bandwidth is $W = 10^3$ Hz. The carrier frequency is $f_c = 10^6$ Hz and the maximum frequency deviation is $\Delta f_{\max} = 200$ Hz. Use Carson's rule to approximate the bandwidth of the modulated FM wave.

Problem 4. (15 points) The signal $g(t) = \sin(2\pi t)$ is uniformly sampled where the sampling interval is $T_s = 0.2$ second.

(a) The instantaneous sampled signal is passed through an ideal low-pass filter with cutoff frequency 5 Hz. Find the frequency spectrum of the output signal.

(b) Pulse-coded modulation (PCM) is used for this signal $g(t)$ with the following 4-level quantizer

$$v = Q(m) = \begin{cases} 0.75 & \text{if } 0.5 \le m \le 1 \\ 0.25 & \text{if } 0 \le m < 0.5 \\ -0.25 & \text{if } -0.5 \le m < 0 \\ -0.75 & \text{if } -1 \le m < -0.5 \end{cases}$$

Find the binary PCM coded sequence with natural coding for the first 5 samples starting from t = 0..

(c) If PCM is used for this signal $g(t)$ with a 16-level quantizer, what is the minimum permissible bit rate and its corresponding bit interval?

Problem 5. (15 points) A message sample has the following probability density function (PDF):

$$f_M(m) = \begin{cases} \frac{1}{10} & m \in [-1, 0) \\ \frac{9}{10} & m \in [0, 1) \\ 0 & \text{otherwise} \end{cases} .$$

(a) Design the 2-level r the message sample.

(b) Calculate the mean squared error (MSE) of the quantizer you designed in (a).

(c) Find another 2-level quantizer with a lower MSE and justify your answer.

Problem 6. (15 points) A random source produces X with alphabet $\mathcal{A} = \{a, b, c, d, e, f\}$ with the following probabilities:

$$\mathrm{P}[X = a] = 0.15, \quad \mathrm{P}[X = b] = 0.2, \quad \mathrm{P}[X = c] = 0.08,$$

$$\mathrm{P}[X = d] = 0.15, \quad \mathrm{P}[X = e] = 0.12, \quad \mathrm{P}[X = f] = 0.3.$$

(a) Find the entropy c

(b) Consider the following source coding:

Element	a	b	c	d	e	f
Codeword	000	001	010	011	1	10

Find the average number of bits per symbol. Is the source compressed without distortions using this source coding and why?

(c) Design a Huffman coding scheme for this source. What is the the average number of bits per symbol?

Problem 7. (15 points) Consider a 5-point constellation: $\mathcal{A} = \{0, -2, 2, -j, j\}$.

(a) Find the average transmit energy and the minimum distance of this constellation.

(b) The minimum distance rule is used for detection. If the received signal is $0.5 + 0.4j$, what is the detection result? Justify your answer.

(c) If the minimum distance rule is used for detection, what is the detection region for the constellation element 2 in $\mathcal{A}$? (Simplify your result when possible.)

Properties of Fourier transform

Table 1: Basic Fourier transform p

Time-domain function $g(t)$	Fourier transform $G(f)$
1	$\delta(f)$
$\delta(t)$	1
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\text{rect}\left(\frac{t}{T}\right)$	$T\text{sinc}(fT)$
$2W\text{sinc}(2Wt)$	$\text{rect}\left(\frac{f}{2W}\right)$
$\Delta\left(\frac{t}{T}\right)$	$\frac{T}{2}\text{sinc}^2\left(\frac{fT}{2}\right)$
$e^{-at}u(t)\ \ (a>0)$	$\frac{1}{a+j2\pi f}$
$e^{-a\lvert t\rvert}\ \ (a>0)$	$\frac{2a}{a^2+(2\pi f)^2}$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$

1. **Linearity**: $c_1g_1(t) + c_2g_2(t) \rightleftharpoons c_1G_1(f) + c_2G_2(f)$
2. **Time scaling**: $g(t) \rightleftharpoons G(f)$ then $g(at) \rightleftharpoons \frac{1}{|a|}G(\frac{f}{a})$ (Dilation)
3. **Conjugation**: $g^*(t) \rightleftharpoons G^*(-f)$
4. **Duality**: $g(t) \rightleftharpoons G(f)$ then $G(t) \rightleftharpoons g(-f)$
5. **Time shifting**: $g(t-t_0) \rightleftharpoons e^{-j2\pi f t_0}G(f)$
6. **Frequency shifting:** $g(t)\exp(j2\pi f_c t) \rightleftharpoons G(f-f_c)$ for any constant f_c.
7. **Areas under $g(t)$ and $G(f)$:**

$$\int_{-\infty}^{+\infty} g(t)dt = G(0) \text{ and } \int_{-\infty}^{+\infty} G(f)df = g(0)$$

8. **Differentiation and integration:**

$$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f) \quad \text{and} \quad \int_{-\infty}^{t} g(\tau)d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{1}{2}G(0)\delta(f)$$

9. **Convolution and Modulation:**

$$g_1(t) * g_2(t) \rightleftharpoons G_1(f)\cdot G_2(f) \text{ and } g_1(t)\cdot g_2(t) \rightleftharpoons G_1(f) * G_2(f)$$

10. **Parseval's theorem:**

$$E_g = \int_{-\infty}^{+\infty} |g(t)|^2 dt = \int_{-\infty}^{+\infty} |G(f)|^2 df$$