Solution to Homework Assignment 4

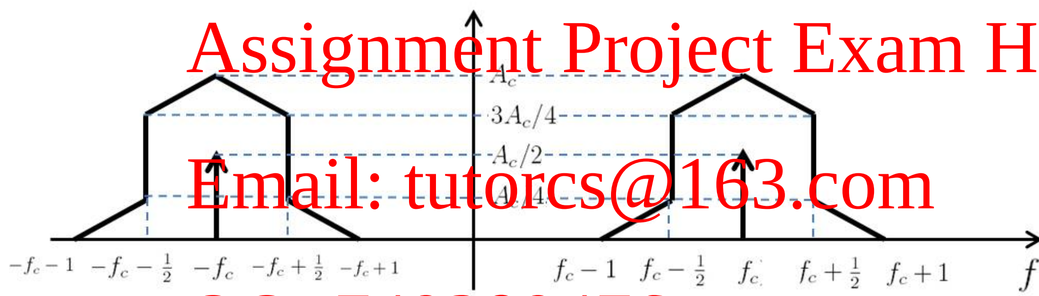
Solution to Problem 1: when $k_a = 1$, the modulated signal is expressed as

$$s(t) = [1 + k_a m(t)]c(t) = [1 + sinc(t) + sinc^2(t)]A_c cos(2\pi f_c t).$$

a) Via Fourier trans[illegible]es,

$$M([illegible] \frac{f}{2}\Big).$$

$$\begin{aligned} S(f[illegible] \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)] \\ = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c}{2}[\mathrm{rect}(f - f_c) + \mathrm{rect}(f + f_c)] \\ + \frac{A_c}{2}\left[\Lambda\left(\frac{f - f_c}{2}\right) + \Lambda\left(\frac{f - f_c}{2}\right)\right]. \end{aligned}$$



b) The bandwidth of $m(t)$ is $W = 1$Hz. The bandwidth of $s(t)$ is $B_T = 2W = 2$Hz.

Solution to Problem 2:

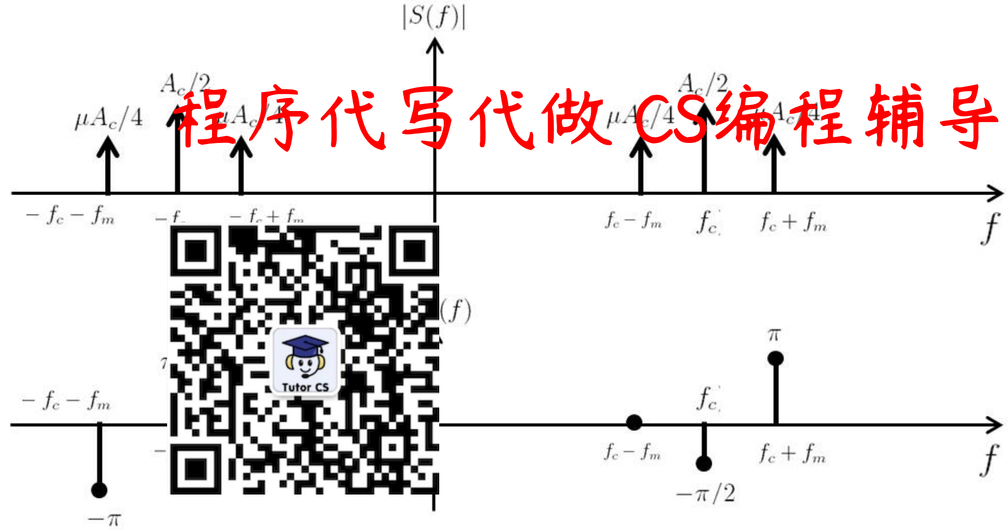
$$\begin{aligned} s(t) &= A_c[1 + k_a A_m \sin(2\pi f_m t)] \sin(2\pi f_c t) \\ &= A_c \sin(2\pi f_c t) + \frac{\mu A_c}{2}[\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c + f_m)t)] \quad \text{where} \quad \mu = k_a A_m \end{aligned}$$

$$\begin{aligned} S(f) = &\frac{A_c}{2j}[\delta(f - f_c) - \delta(f + f_c)] \\ &+ \frac{\mu A_c}{4}[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)] \\ &- \frac{\mu A_c}{4}[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]. \end{aligned}$$

The magnitude spectrum and phase spectrum can be seen in the following figure.

(b) Comparing the spectra with that in lectures, it can be seen that

- The frequency locations of the spectral components of these two AM waves are identical.
- The two AM waves have the same magnitude spectrum.

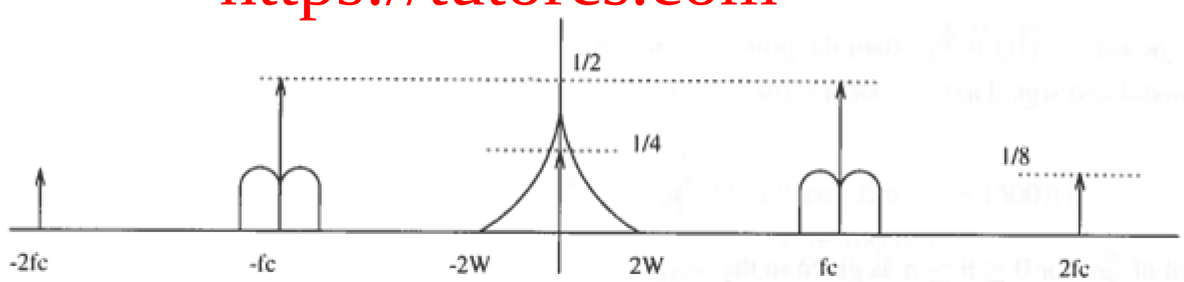


- The difference is in the phase of the upper sideband at frequency $\pm(f_c+f_m)$ (phase change is π) and the carrier frequency $\pm f_c$ (phase change is $\mp\frac{\pi}{2}$).

Solution to Problem 3:

$$
\begin{aligned}
y(t) &= m(t) + \cos(2\pi f_c t) + \frac{1}{2}m(t)^2 + \frac{1}{2}\cos^2(2\pi f_c t) + m(t)\cos(2\pi f_c t) \\
&= m(t) + \cos(2\pi f_c t) + \frac{1}{2}m(t)^2 + \frac{1}{2}\left[\frac{1+\cos(4\pi f_c t)}{2}\right] + m(t)\cos(2\pi f_c t) \\
&= \frac{1}{4} + m(t) + \frac{1}{2}m(t)^2 + \cos(2\pi f_c t) + m(t)\cos(2\pi f_c t) + \frac{1}{4}\cos(4\pi f_c t).
\end{aligned}
$$

$$
\begin{aligned}
Y(f) &= \frac{1}{4}\delta(f) + M(f) + \frac{1}{2}M(f) * M(f) + \frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c) \\
&\quad + \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c) + \frac{1}{8}\delta(f-2f_c) + \frac{1}{8}\delta(f+2f_c).
\end{aligned}
$$



Solution to Problem 4: i). $M(f) = \text{rect}\left(\frac{f}{4}\right)$. For the signal at point (b), $s_b(t)$, we have

$$
S_b(f) = M(f)\text{rect}\left(\frac{f}{2}\right) = \text{rect}\left(\frac{f}{2}\right). \quad s_b(t) = 2\text{sinc}(2t).
$$

For the signal at point (c), $s_c(t)$, we have (Notice that $f_c = 2$)

$$
S_c(f) = \frac{1}{2}S_b(f-f_c) + \frac{1}{2}S_b(f+f_c) = \frac{1}{2}\text{rect}\left(\frac{f-f_c}{2}\right) + \frac{1}{2}\text{rect}\left(\frac{f+f_c}{2}\right).
$$

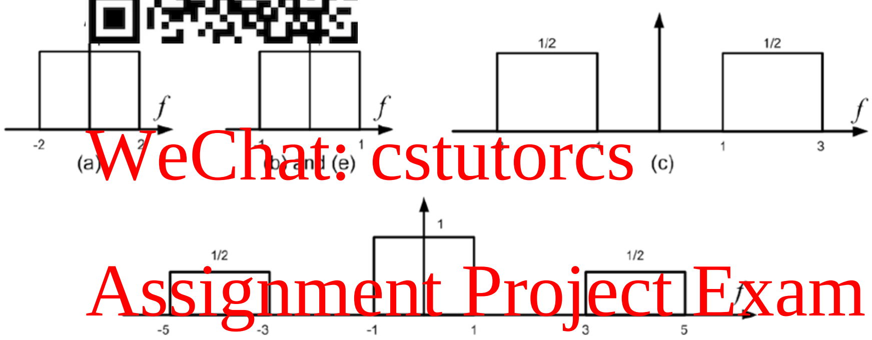
For the signal at point (d), $s_d(t)$, we have

$$s_d(t) = 2s_c(t)\cos(2\pi f_c t) = 2s_b(t)\cos^2(2\pi f_c t) = s_b(t) + s_b(t)\cos(4\pi f_c t)$$
$$S_d(f) = S_b(f) + \frac{1}{2}S_b(f-2f_c) + \frac{1}{2}S_b(f+2f_c) = \text{rect}\left(\frac{f}{2}\right) + \frac{1}{2}\text{rect}\left(\frac{f-2f_c}{2}\right) + \frac{1}{2}\text{rect}\left(\frac{f+2f_c}{2}\right).$$

After the LPF, at po

$$\tilde{M}(f) = \text{rect}\left(\frac{f}{2}\right),$$

which is the same a $= s_b(t)$.



(a) (b) and (e) (c)

(d)

ii). To find the minimum value of f_c, we should consider the spectrum of the signal at point (c). For the signal at point (e) to be equal to the signal at point (b), the two rectangular pulses should not overlap. Thus, $-f_c + 1 \leq f_c - 1 \Rightarrow f_c \geq 1$. Therefore, the minimum value of f_c is 1.

Solution to Problem 5:

$$m(t) = e^{j2\pi f_0 t} \rightleftharpoons M(f) = \delta(f - f_0)$$
$$\hat{M}(f) = -j\text{sgn}(f)M(f) = -j\text{sgn}(f)\delta(f-f_0) = -j\text{sgn}(f_0)\delta(f-f_0).$$

Thus $\hat{m}(t) = -j\text{sgn}(f_0)e^{j2\pi f_0 t}$.