

# Chapter 8. Digital Communications

程序代写代做 CS 编程辅导

Summary: Brief introduction of the components of digital communication system and fundamental rules in digital communication design.



8.1 Source Coding [WeChat: cstutorcs](#)

8.2 Channel coding

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8.3 Binary Shift Keying Modulation

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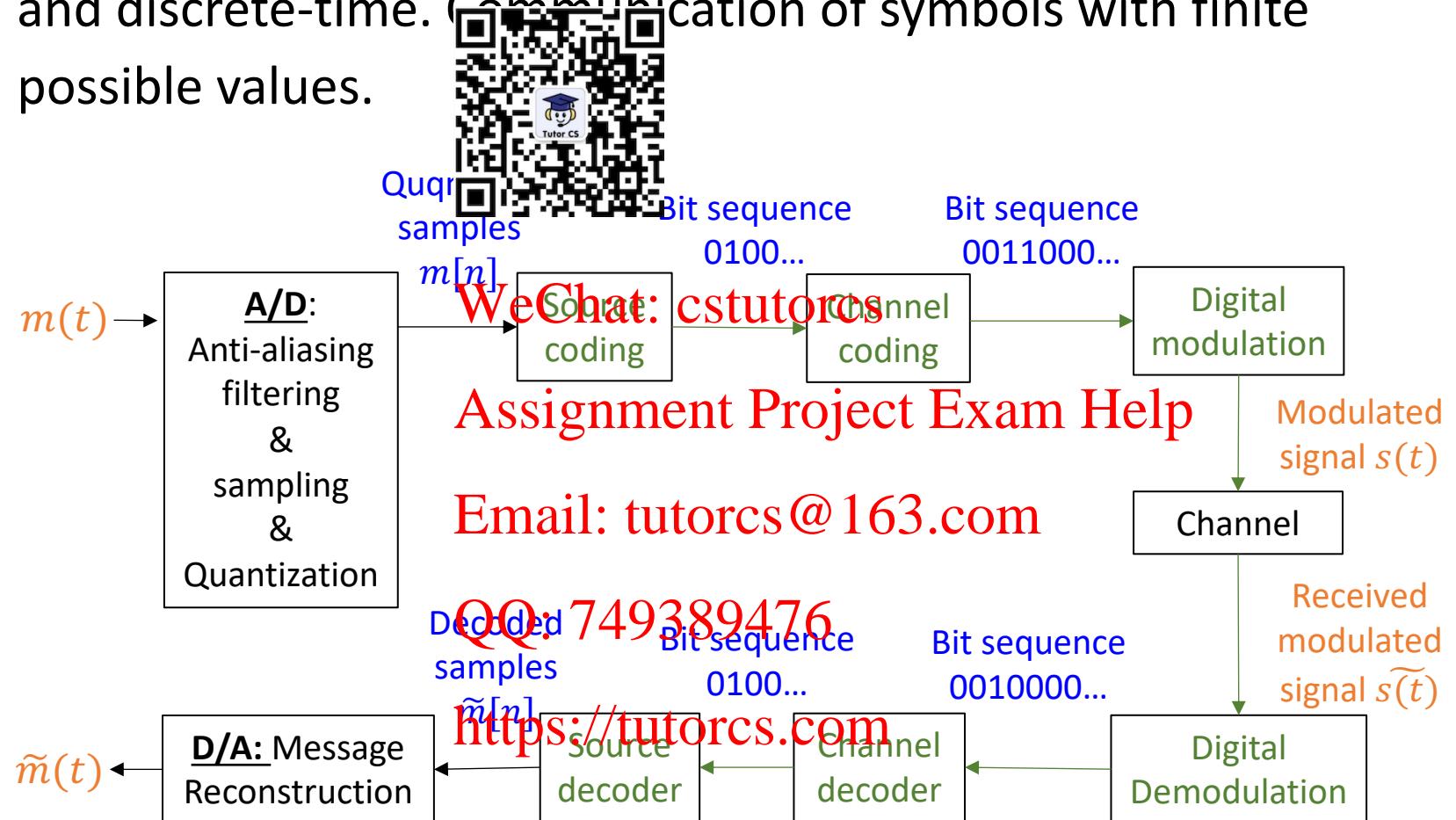
8.4 M-ary Digital Modulation Scheme

8.5 Constellation Design [QQ: 749389476](#)

8.6 Detection Design

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Digital communication 程序代写代做 CS 编程辅导 is digital and discrete-time. Communication of symbols with finite possible values.



# 8.1 Source Coding

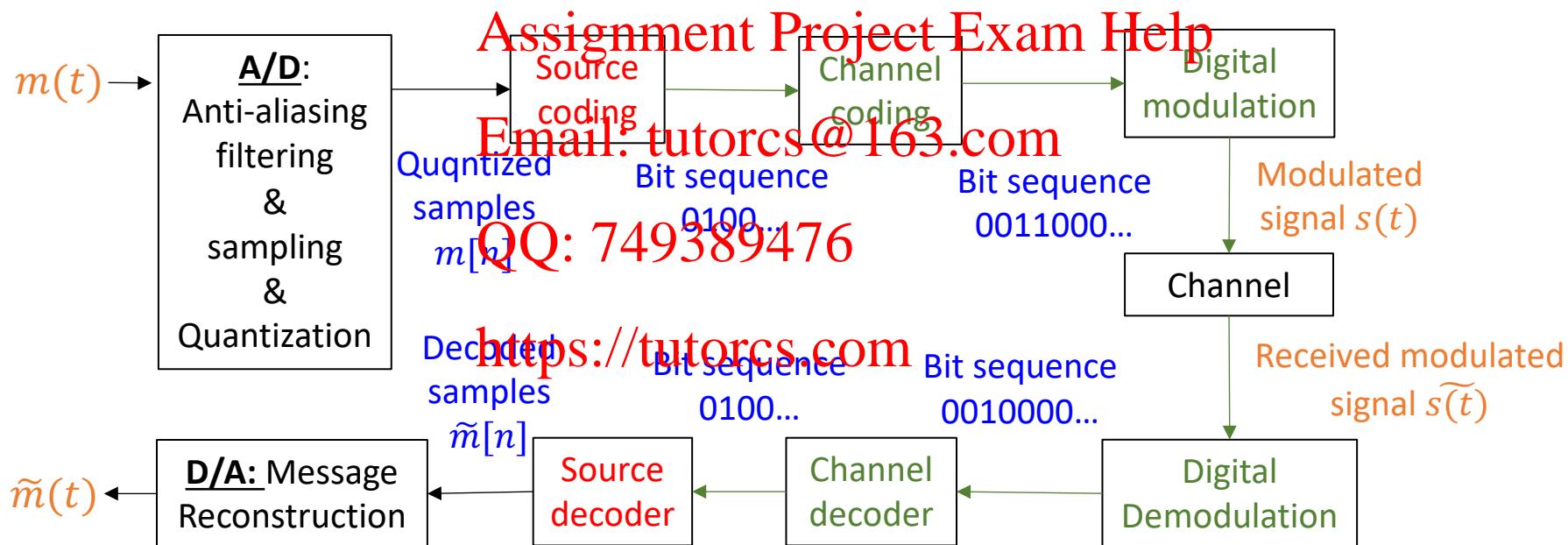
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**Source encoding/data compression:** Efficiently convert message into a binary sequence

- Remove redundancy & achieve low bitrate



**Source decoding:** Interpret the information sequence into message (Reconstruct the original message)



## Coding categories: 程序代写代做 CS编程辅导

- Fixed-length coding
- Variable-length coding



### Fixed-length binary

Example. Extended ASCII (American Standard Code for Information Interchange) code.

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Alphabet	Codeword	Length
A	01000001	8
O	01001111	8
...	..	8
1	00110001	8
...	..	8
\	01011100	8
...	...	8

## Variable-length binary code:

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Statistics of message can be used to design good source encoding alg.

- principles: use long codewords for less frequent symbols; short codewords for more frequent elements.

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$\mathcal{A}$	Prob.	Code 1	Code 2	Average length of codeword: $\bar{l} = \sum_i p(i)l_i$
$a$	$1/2$	00	0	e.g. for codes on the left, $\bar{l}_{code1} = \frac{1}{2} * 2 + \frac{1}{4} * 2 + \frac{1}{8} * 2 + \frac{1}{8} * 2 = 2$
$b$	$1/4$	01	10	Email: tutorcs@163.com QQ: 749389476
$c$	$1/8$	10		$\bar{l}_{code2} = \frac{1}{2} * 1 + \frac{1}{4} * 2 + \frac{1}{8} * 3 + \frac{1}{8} * 3 = 1.75$
$d$	$1/8$	11	111	
Ave. len. $\bar{l}$		2	1.75	

## Variable-length binary code:

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Huffman coding:

- Example of using short codewords for less frequent symbols; frequent elements.
- Greedy algorithm. Build a Huffman tree where two nodes with the smallest probabilities merge to an intermediate.

$\mathcal{A}$	Prob.
$a$	$1/2$
$b$	$1/4$
$c$	$1/8$
$d$	$1/8$

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**Example:** Please design a Huffman code for the following source  
(each letter is shown with its probability in the bracket)  
 $x(3/13)$ ,  $m(3/13)$ ,  $g(3/13)$ , ~~程序代写代做CS编程辅导~~  
 $s(1/13)$ ,  $h(1/13)$ ,  $p(1/13)$ ,  $r(1/13)$



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**Entropy:** For a source  $X$  with certain alphabet and distribution. Its entropy  $H(X)$  is defined as

$$H(X) = -\sum p(x_i) \log_2 p(x_i).$$



**Intuitive example:** Consider weather forecast with 4 possible outcomes: cloudy, snowy, rainy, and sunny.

- For place A, all possible weathers occurs with equal probability;  
→ need 2 bits to deliver a message
- For place B, snowy and raining never happen but cloudy and sunny occurs evenly  
→ Need 1 bit only to deliver a message because less information needs to communicate.

**Entropy:** For a source  $X$  with certain alphabet and distribution. Its entropy  $H(X)$  is defined as

$$H(X) = -\sum p(x_i) \log_2 p(x_i).$$



### The meaning of entropy (Shannon's theorem)

- A source can be compressed without distortion to  $H(X)$  bits per symbol.
- $H(X)$  bits per symbol is the lowest that a source can be compressed without distortion.
- The entropy  $H(X)$  provides a quantitative measure on the amount of information.

In general, for a random source  $X$  which generates alphabet

$$\mathcal{A} = \{x_1, x_2, \dots, x_N\}$$

with the following probability mass function (PMF):

$$p(x_1) = p_1, \quad p(x_2) = p_2, \quad \dots, \quad p(x_N) = p_N$$

the entropy of  $X$  is:

$$\begin{aligned} H(X) &= \text{E}[-\log_2 p(X)] = -\sum_{x \in \mathcal{A}} p(x) \log_2 p(x) = \sum_{x \in \mathcal{A}} p(x) \log_2 [1/p(x)] \\ &= -p_1 \log_2 p_1 - p_2 \log_2 p_2 - \dots - p_N \log_2 p_N. \end{aligned}$$

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- Entropy is non-negative.
- With base-2 log, the unit is bit.
- With the log-function, the joint entropy of 2 independent sources equals the sum of their entropies individually.

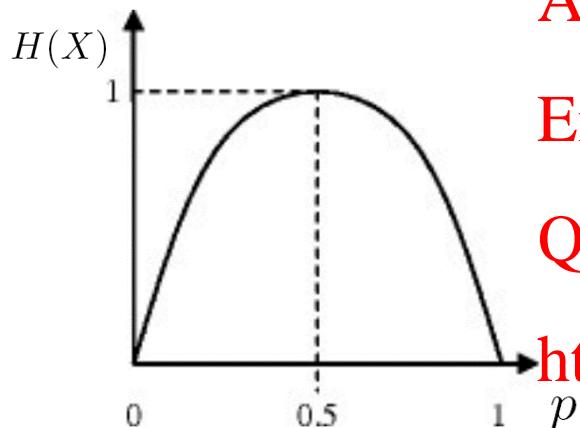
Example: a binary random source ~~程序代与CS编程辅导~~ generates binary bits randomly following some probabilities:

$$p(0) = P[X = 0] = p, \quad p(1) = P[X = 1] = 1 - p.$$

The entropy of the source is

$$H(X) = -p \log_2 p - (1 - p) \log_2(1 - p).$$

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- When  $p \approx 0$  or  $p \approx 1$ , little information is in  $X$ . The entropy achieves its minimum 0.
- When  $p \approx 1/2$ , the maximum amount of information is in  $X$ . The entropy is about its maximum 1.

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**Example:** A source 程序代写代做CS编程辅导 whose alphabet  $\mathcal{A}$  and its probability distribution are as follows. Calculate the entropy.

$$\mathcal{A} = \{a, b, c, d\}.$$

$$p(a) = 1/2, p(b) = 1/4, p(c) = 1/8, p(d) = 1/8.$$



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Example: Consider the following source  $X$  and codes.

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$\mathcal{A}$	Prob.	Code 1	Code 2	Code 3	Code 4
$a$	$1/2$		0	0	0
$b$	$1/4$	01	1	10	01
$c$	$1/8$	10	0	110	011
$d$	$1/8$	11	11	111	111
Ave. len. $\bar{l}$		2	1	1.75	1.75

$H(X) = 1.75$  bits/symbol.

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**Code 1:** Uniquely decodable. Fixed-length codeword. Not the shortest.

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**Code 2:** Not uniquely decodable. Shortest.



**Code 3:** Uniquely decodable.

Prefix-free code: no codeword is a prefix of another.

Instantaneously decodable.

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Optimal in the sense of achieving the entropy bound:  $\bar{l} = H(X)$ .

**Code 4:** Uniquely decodable. Not prefix-free code.

Not instantaneously decodable; need to see following bits to decode.

Optimal:  $\bar{l} = H(X)$ .

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Data compression idea: Use long codewords for less frequent elements; short codewords for more frequent elements.

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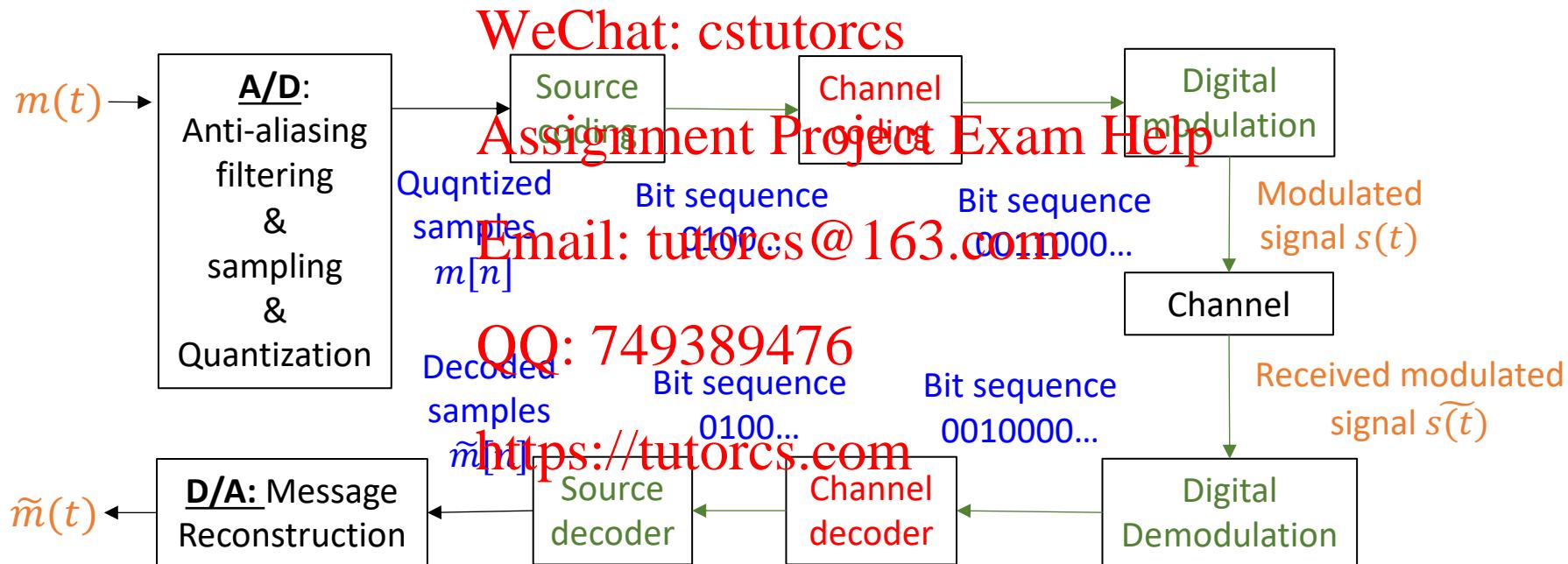
## 8.2 Channel Coding

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**Channel encoding:** Introduce controlled redundancy (extra information) to combat distortions in channels.



**Channel decoding:** Remove controlled redundancy and Reconstruct the compact bit sequence.



## Repetition codes : 程序代碼修改 CS 編程輔導

Example: 3-bit repetition codes

- Encoding: 0 → 000 1 → 111
- Decoding: Majority



Original bit sequence	0	0	1	0	1
After repetition codes	000	000	111	000	111
Received bit sequence	000	001	111	000	101
after decoding	0	0	1	0	1

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Information rate = # message bit / # bit after channel coding

- Introducing more <https://tutorcs.com> may help to handle error, with the penalty of lower information rate and longer bit sequence to be sent.

## (7,4) Hamming code (1950)

- Take 4 information bits:  $d_1, d_2, d_3, d_4$

- Add 3 parity check bits:  $p_1, p_2, p_3$

- Form a 7-bit code word:  $d_2d_3d_4p_1p_2p_3$

- Can correct 1 bit error



XOR operation:  $\oplus$

$$0 \oplus 0 = 1 \oplus 1 = 0$$

$$1 \oplus 0 = 0 \oplus 1 = 1$$

Encoding procedure: WeChat: cstutorcs

$$p_1 = d_1 \oplus d_2 \oplus d_4$$

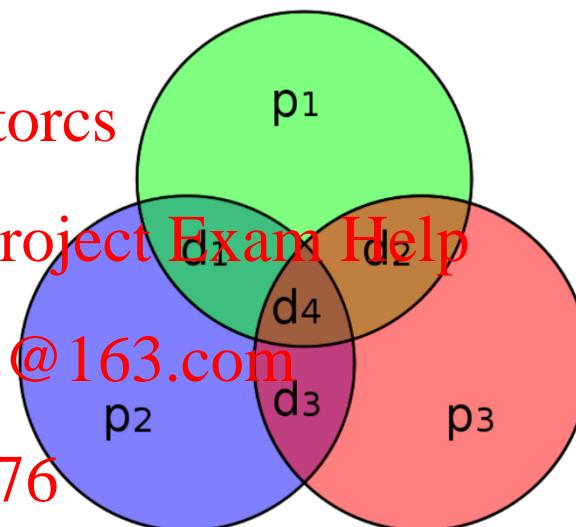
$$p_2 = d_1 \oplus d_3 \oplus d_4$$

$$p_3 = d_2 \oplus d_3 \oplus d_4$$

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Every circle has an even number of 1's.

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[https://en.wikipedia.org/wiki/Hamming\(7,4\)](https://en.wikipedia.org/wiki/Hamming_(7,4)_code)

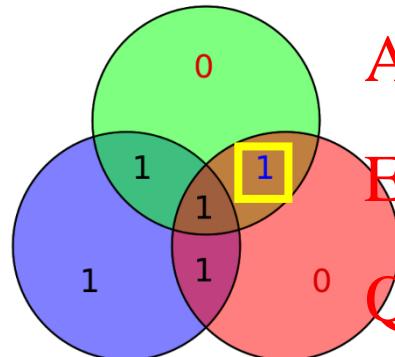
## Decoding procedure:

Put bits in their corresponding positions in the figure, which circles violate parity check. Flip corresponding bit (if any) to ensure the parity checks.

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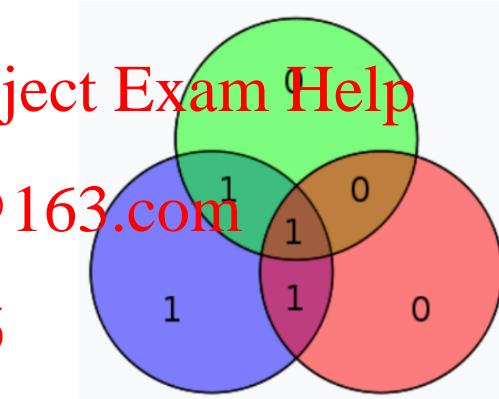
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Data sequence: 1011010  
→ received sequence (1 bit error) 1111010  
→ decoded sequence: 1011010

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Comments:

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- Code rate

= number of info bits / total number of bits of a codeword  
= 4/7.



- Can correct 1 bit error but no more.

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- Repetition code would need 12 bits to protect 4 bits against 1 error.

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- Applications of Hamming codes:
- DRAM memory

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- Satellite communication

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Example: (a) Using (7,4) Hamming code to transmit 0110.

What is the coded segment?

(b) Assume 1001111 is received. Please find the decoding result. Is there any error in the received signal?

(c) If the probability of each bit error is 1% and bit errors are independent. Please calculate the probability that the decoding results is correct in (7,4) Hamming code.



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## How much redundancy can be introduced?

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There is an upper limit of amount of redundancy constrained by channel capacity.



Channel capacity ( $C$ ) is the maximum bit rate of a digital communication systems.

- Characteristic of a channel
- It is a fundamental limit regardless of the coding.

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For example: After the source coding, the bitrate of a message is 0.6Mbps. If we want to transmit the message through a channel with capacity  $C = 1\text{Mbps}$ . Then the maximum redundancy introduced by the channel coding is  $1 - 0.6 = 0.4\text{Mbps}$ .

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## Communication system design in coding ? (Cont.)

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3 factors to be considered:

- The channel bandwidth
  - Measured by 
- The number of levels in digital signals
- The quality of the channel
  - Characterized by channel capacity  $C$
  - Closely related to channel noise level SNR

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Two theorems to relate above three factors:

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- Nyquist's channel theorem
- Shannon's channel coding theorem

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# Communication system design in coding ? (Cont.)

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## Nyquist's Channel

For a noiseless channel, there is an upper limit of bit rate (called the **Nyquist bit rate**), for error-free communication.

$$\text{Nyquist bit rate} = 2 * B * \log_2 M$$



Nyquist bit-rate is the maximum bit rate to represent a message represented by M levels for reliable communication in noiseless channel.

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Ex. Consider a noiseless channel with a bandwidth of 3000Hz. We want to transmit a message with 4 signal levels. What is the Nyquist bit rate?  $\rightarrow 2 * 3000 * \log_2 4 = 12000\text{bps}$

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# Communication system design in coding ? (Cont.)

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Shannon's Channel



theory:

For any given noisy channel, it is possible to communicate digital information error-free when the communicate bit rate is kept below the channel capacity.

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The channel capacity is usually determined by the noise level and available bandwidth.

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$$C = B * \log_2(1 + SNR)$$

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State-of-the-art: Since <https://tutor2000.com>, with advancements in low-density parity-check (LDPC) codes and turbo codes, we are approaching the fundamental limit.

## More about SNR

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$$SNR = \frac{\text{Power of signal (in Watt)}}{\text{power of noise}} = \frac{S}{N} \quad SNR_{dB} = 10 \log_{10}(SNR)$$



Q1: Consider a communication system for which the SNR at the receiver is  $SNR^0$ .

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- (a) If  $SNR^0$  is doubled, how much the  $SNR_{dB}^0$  increases?
  - (b)  $SNR^0$  should be multiplied by which factor to reduce  $SNR_{dB}^0$  by -3 dB?

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Comment: Each 3 dB(-3 dB) increment (decrement) in  $SNR_{dB}$ , increases (decreases) the SNR by a factor of 2 (1/2).

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**Example:** A telephone line normally has a bandwidth of 3000Hz (300Hz-3300Hz) assigned for data communication. The SNR is usually at the level of 35dB. If we want to transmit an signal through this channel,

- (a) Calculate the theoretical channel capacity;
- (b) Calculate the maximum power level.



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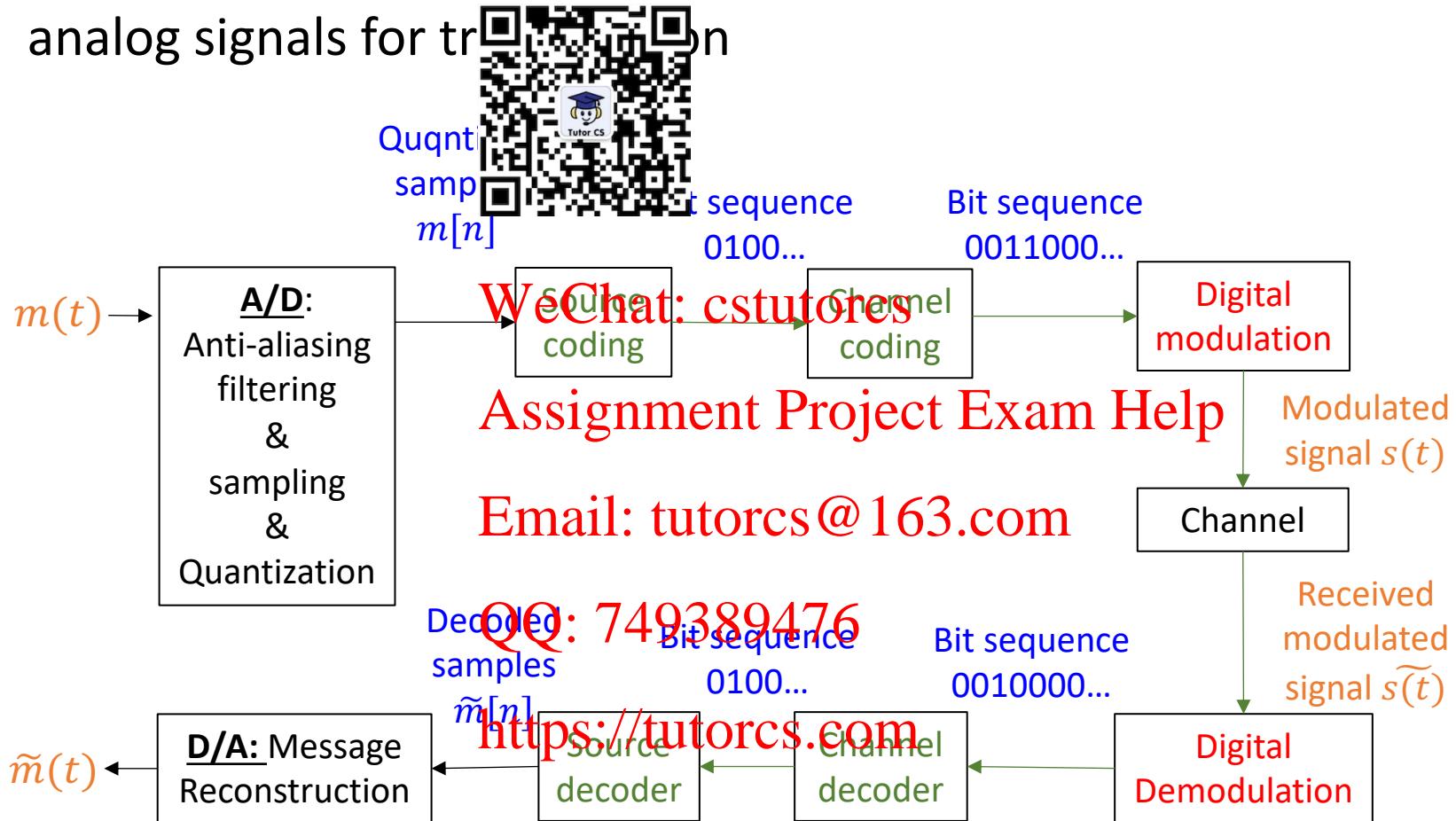
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## 8.3 Binary Shift Keying Modulation

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Shift keying modulation: to convert digital information to analog signals for transmission



# Binary Shift Keying modulation

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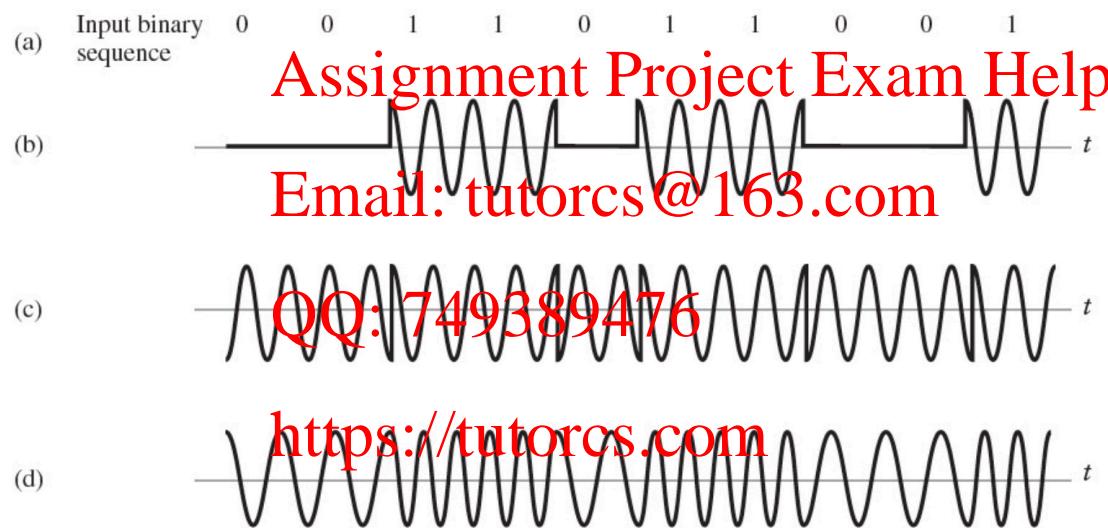
- Input: binary sequence  $b_i = 0/1$

- Carrier:  $c(t) = A \cos(\omega_c t + \varphi_c)$

- Without any loss of generality,  $\varphi_c = 0$

- Output: analog modulated wave whose amplitude, phase, or frequency varying with  $b_i$

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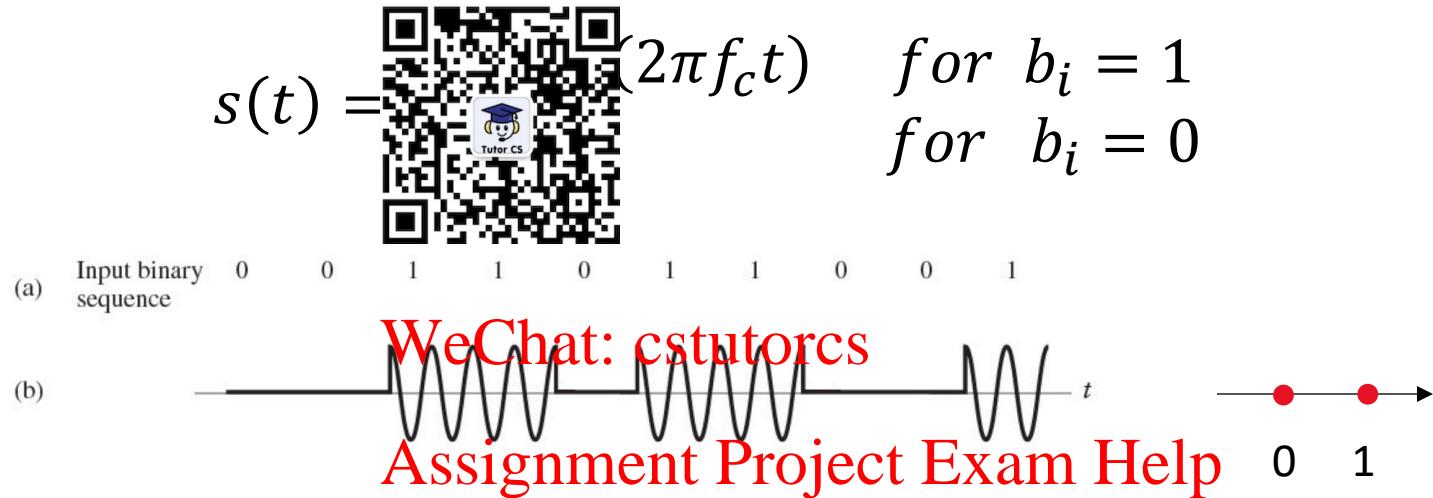


**FIGURE 7.1** The three basic forms of signaling binary information. (a) Binary data stream. (b) Amplitude-shift keying. (c) Phase-shift keying. (d) Frequency-shift keying with continuous phase.

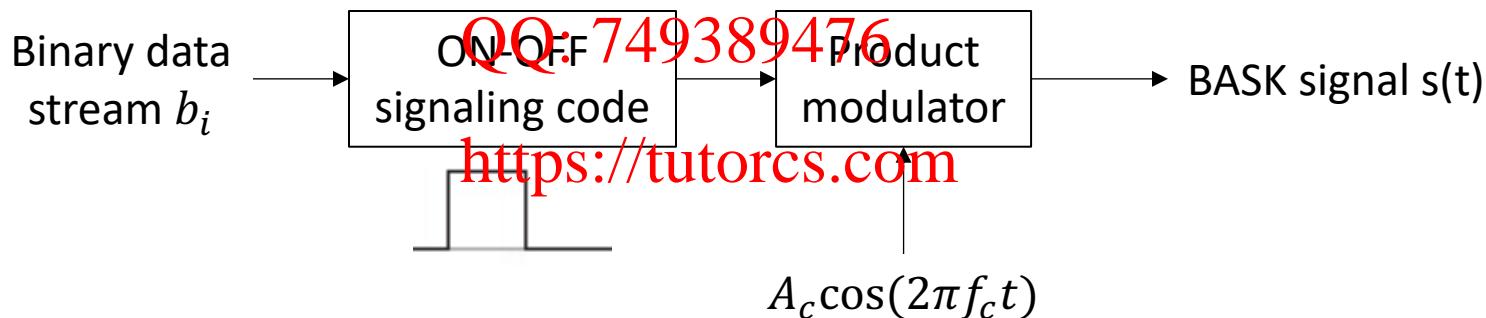
# Binary amplitude Shift Keying (BASK)

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- Amplitude of the carrier varies in accordance with  $b_i = 0/1$



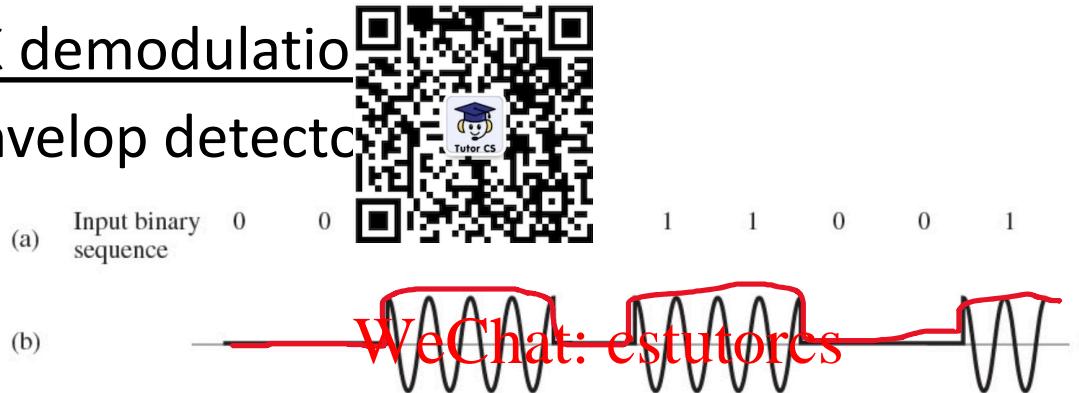
BASK modulation Email: tutorcs@163.com



# Binary amplitude Shift Keying (BASK) 程序代写代做CS编程辅导

## BASK demodulation

- Envelop detector

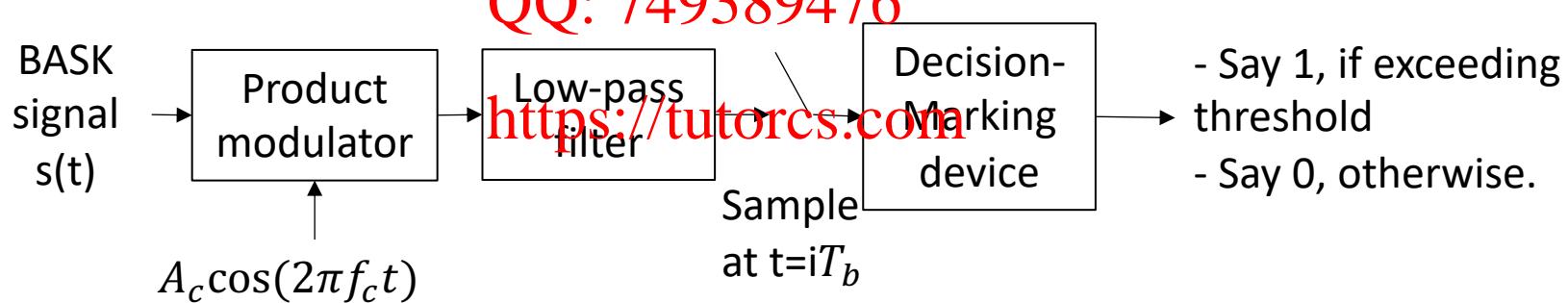


tracing the Assignment Project Exam Help  
nonconstant-envelope of BASK signals

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- Coherence detector – synchronize phase and frequency

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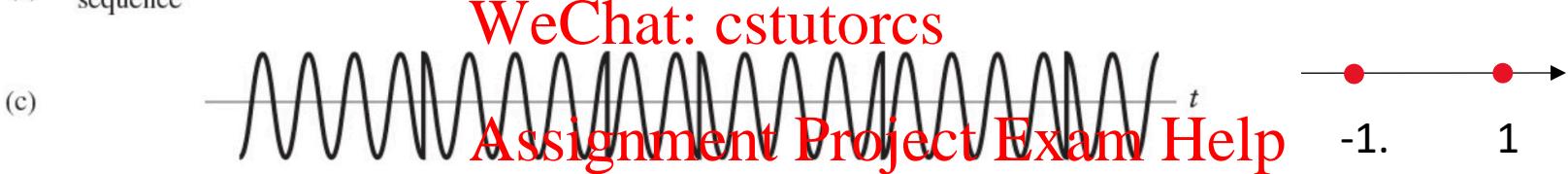
## Binary Phase Shift Keying (BPSK)

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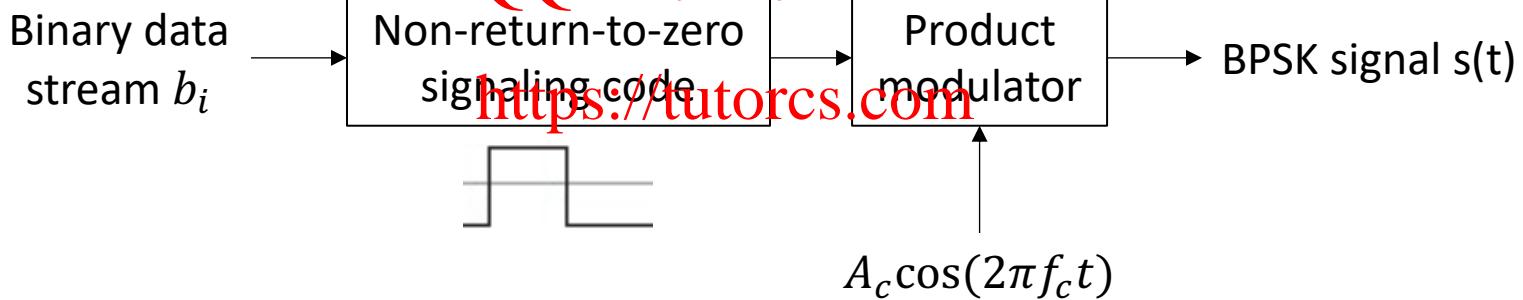
- Phase of the carrier varies in accordance with  $b_i = 0/1$

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t) & \text{for } b_i = 1 \\ -A_c \cos(2\pi f_c t) & \text{for } b_i = 0 \end{cases}$$


(a) Input binary sequence 0 0 1 1 0 1 1 0 0 1



### BPSK modulation



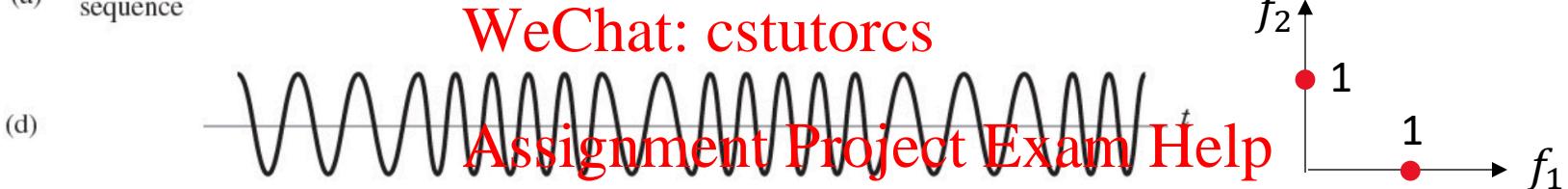
# Binary Frequency Shift Keying (BFSK)

- Frequency of the carrier varies in accordance with  $b_i = 0/1$



$$s(t) = \begin{cases} \pi f_1 t) & \text{for } b_i = 1 \\ \pi f_2 t) & \text{for } b_i = 0 \end{cases}$$

(a) Input binary sequence 0 0 1 1 0 1 1 0 0 1



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Minimum-shift keying (MSK) QQ: 749389476

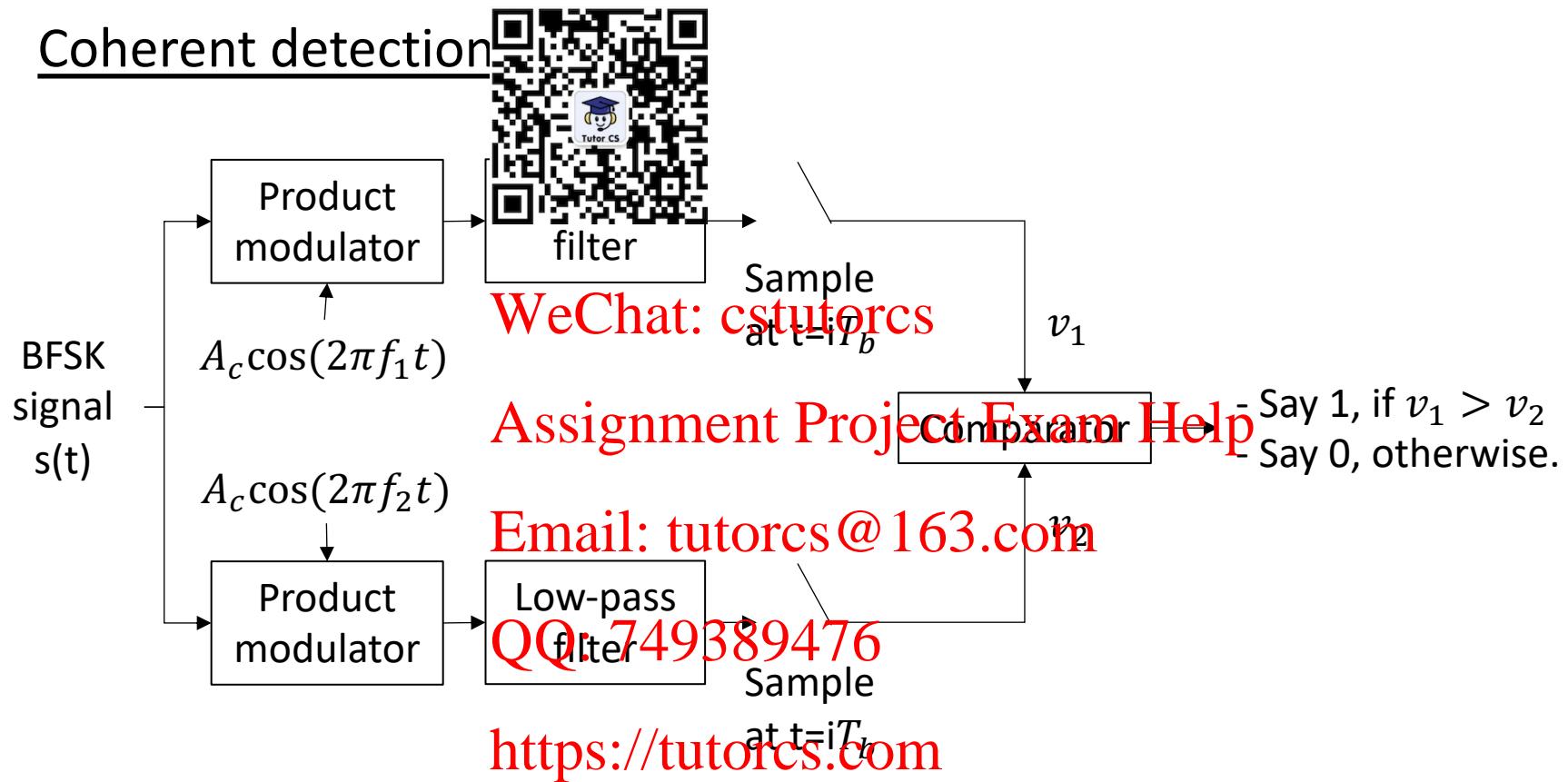
Let  $1/T_b$  and  $f_c$  be the bit rate and carrier frequency. In MSK, <https://tutorcs.com>

$f_1 = f_c + \frac{1}{4T_b}$ ,  $f_2 = f_c - \frac{1}{4T_b}$ . That is,  $f_1 - f_2 = \frac{1}{2T_b}$ .

# Binary Frequency Shift Keying (BFSK)

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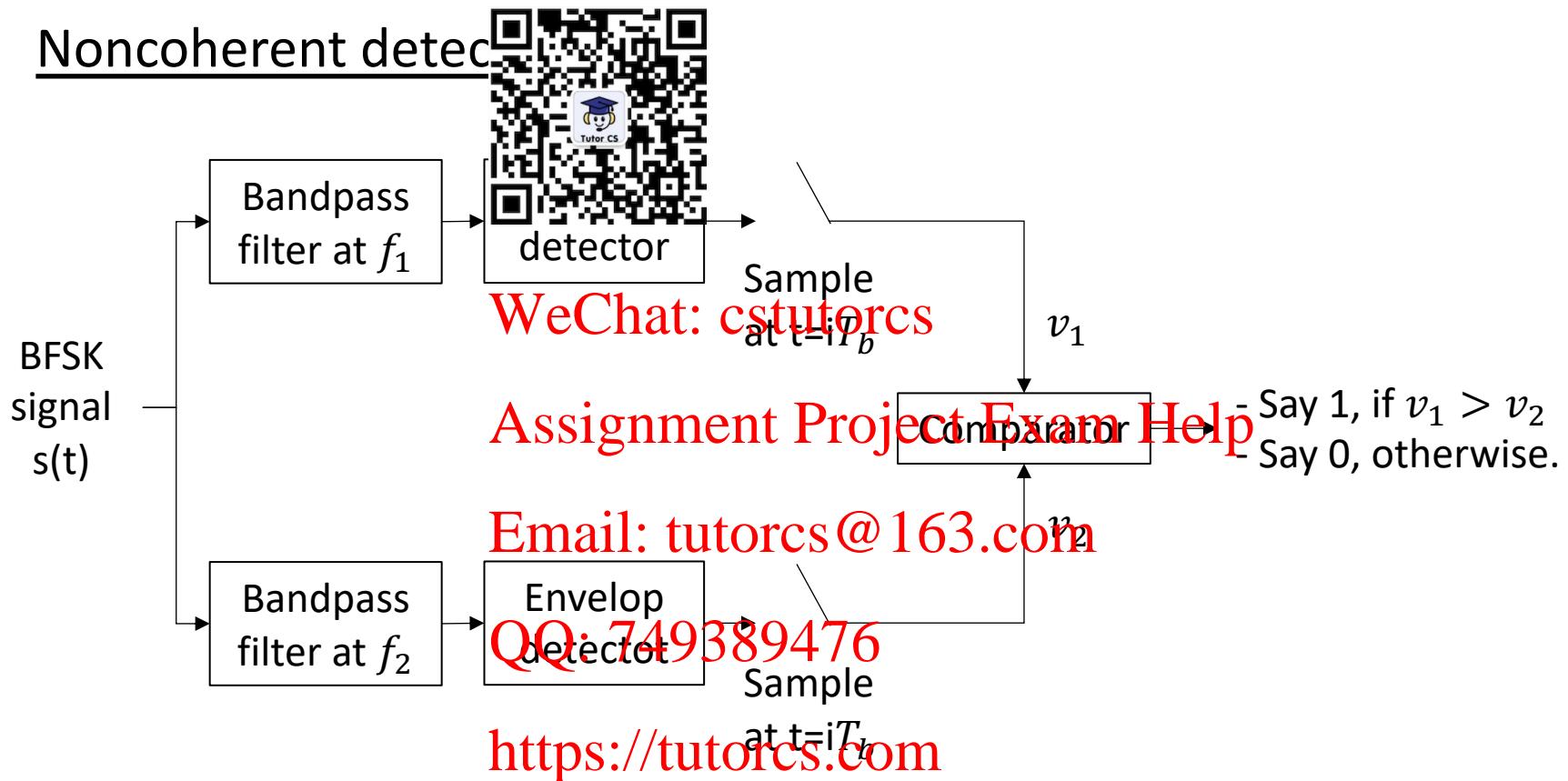
## Coherent detection



# Binary Frequency Shift Keying (BFSK)

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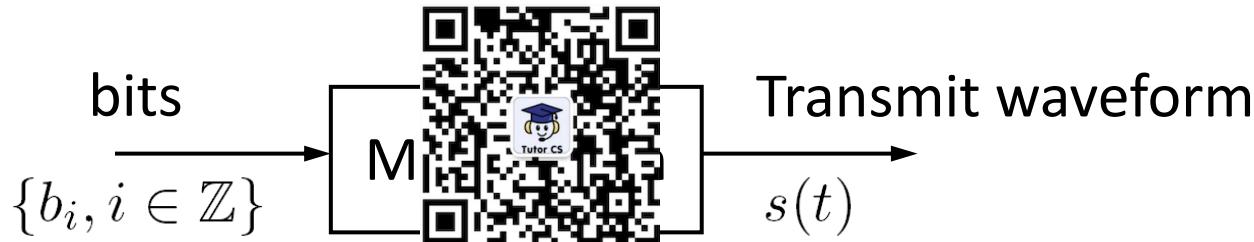
Noncoherent detection



## 8.4 M-ary Shift Keying Modulation

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Binary shift keying is simply, but not bandwidth efficient.



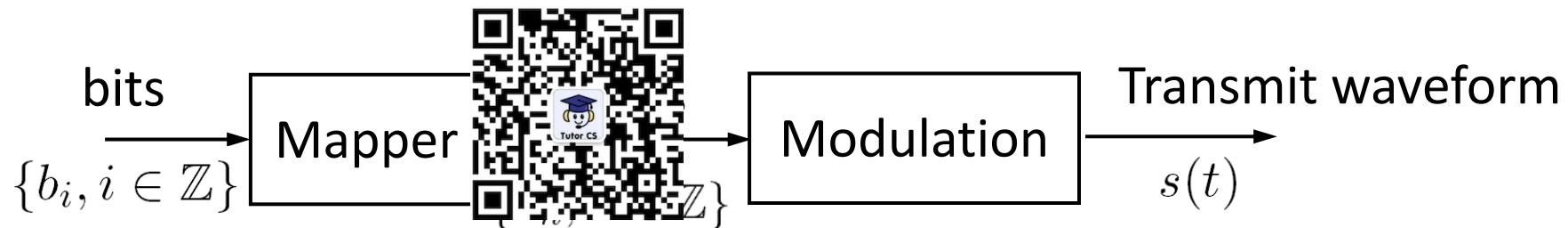
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Question: can we extend binary shift keying in some manner to improve the bandwidth efficiency?  
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Instead, we send one of M possible signals during each signaling (symbol) interval of duration.  
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# M-ary shift keying modulation



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**Mapper:** Takes a group of bits and converts them to a symbol from an alphabet whose elements are real (or complex) numbers.

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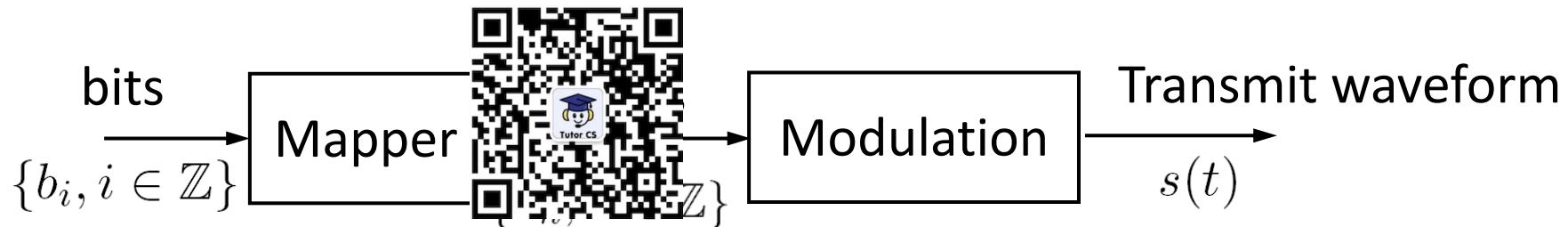
Depending no the mapper, we may have

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- M-ary amplitude shift keying
- M-ary phase shift keying
- M-ary frequency shift keying

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# M-ary Amplitude shift keying modulation



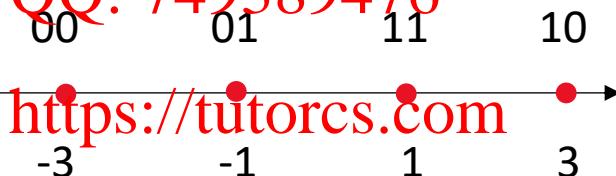
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**Mapper:** Takes a group of bits and converts them to a symbol with different amplitude  $A_i$ .  
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**Example:** Every group of 2 bits can be converted to a symbol

Group of 2 bits

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4-ASK

Transmission wave

$s(t) = A_i \cos(2\pi f_c t)$  for different symbol interval

# M-ary Amplitude shift keying modulation 程序代写 代码代写 CS编程辅导

**Example:** Every group of bits can be converted to a symbol

4-ASK



Gr

00      01      11      10

-3      -1      1      3

Mapping symbols  $A_i$

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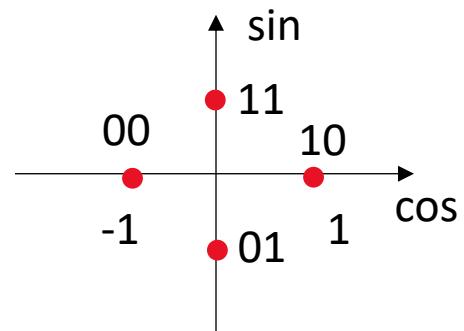
Transmission wave  $s(t) = A_i \cos(2\pi f_c t)$

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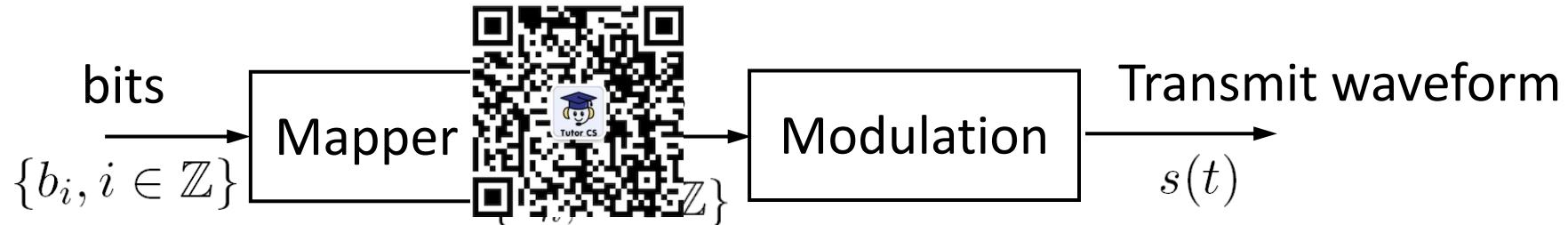
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Quadrature Amplitude modulation(QAM)

- 2 orthogonal carriers  $A_i \cos(2\pi f_c t)$  and  $A_i \sin(2\pi f_c t)$
- Transmission wave is a superposition of the two carriers
- With the same bandwidth efficiency, QAM is more power-efficient than 4-ASK.



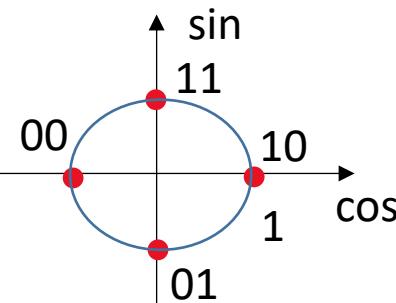
# M-ary Phase shift keying modulation 程序代码 CS 编程辅导



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**Mapper:** Takes a group of bits and converts them to a symbol with different phase on the unit circle in the complex plane.

**Example:** Every group of 2 bits can be converted to a symbol



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**QAM (QPSK):**

<https://tutorcs.com>  $s(t)=\text{Acos}(2\pi f_c t + \frac{2\pi}{4} i)$  for  $i=1,2,3,4$

# M-ary Phase shift keying modulation 程序代码 CS 编程辅导

**Example:** Design 8-PSK

**8-ASK**

Group of



000 001 011 010 110 111 101 100



Mapping symbols  $A_i$

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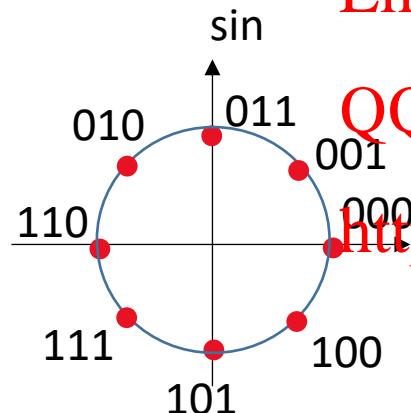
Transmission wave

$$s(t) = A_i \cos(2\pi f_c t)$$

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**8-PSK**

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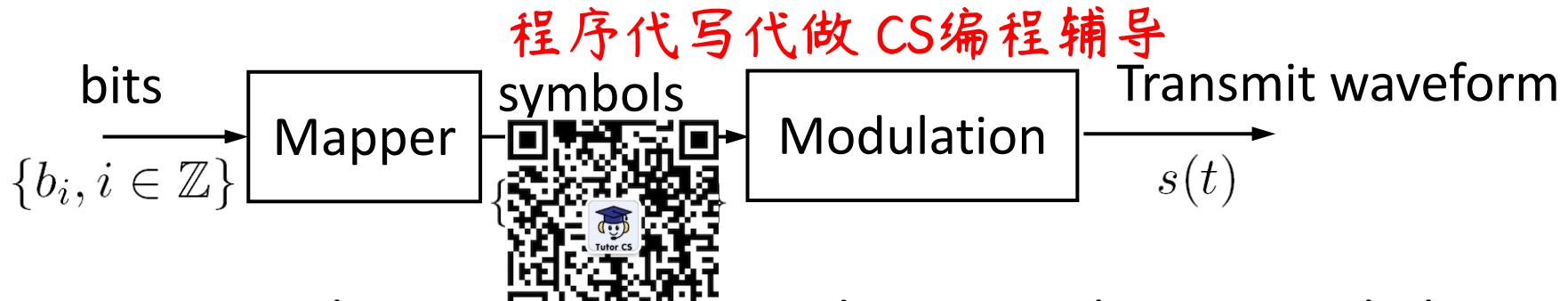


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$$s(t) = A \cos(2\pi f_c t + \frac{2\pi}{4} i) \text{ for } i=1, \dots, 8$$

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## M-ary Frequency shift keying modulation



**Mapper:** Takes a group of bits and converts them to symbols modulated by different carriers

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Note, in M-ary FSK, Any two carriers should be orthogonal:

$$\int_0^T s_i(t)s_j(t)dt = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

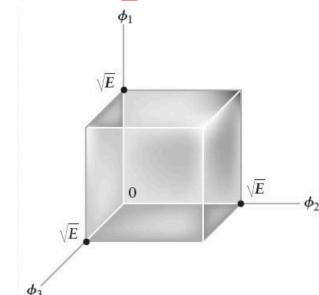
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where  $s_i(t) = \cos(2\pi f_{c_i} t)$ .

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M-ary frequency shift keying is less bandwidth efficiency, in general.



## 8.5 Constellation Design

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Design of the alphabet or symbol set  $\mathcal{A}$ . In other words, design of elements to achieve the best efficiency and/or reliability in communication.



**Design factors:**

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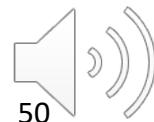
- average energy per symbol;
- minimum distance between elements (determines the reliability).

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# To minimize average energy per symbol

Given a constellation

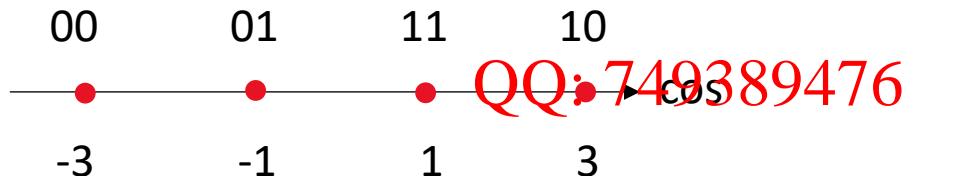
$$\mathcal{A} = \{x_1, x_2, \dots, x_N\},$$

its average transmission power is

$$E_{\mathcal{A}} = \frac{1}{N} \sum_{n=1}^N |x_n|^2 = \frac{|x_1|^2 + \dots + |x_N|^2}{N}$$

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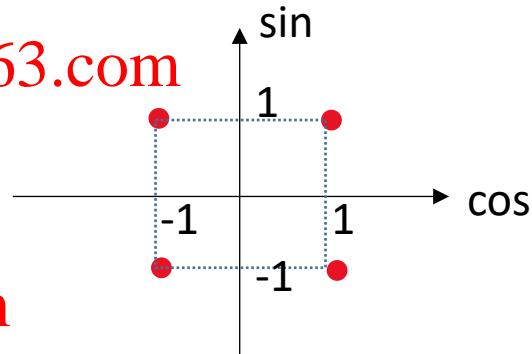
Example. Compare Assignment Project Exam Help of the two constellations.



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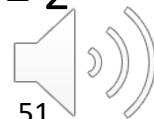
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$$E_{A1} = (9 + 1 + 1 + 9)/4 = 5$$

$$E_{A2} = (2 + 2 + 2 + 2)/4 = 2$$



## To maximize distance between symbols

Given a constellat



the distance between two elements  $x_i, x_j$  is  $|x_i - x_j|$

The minimum distance is

$$d_{\min, \mathcal{A}} = \min_{i \neq j} |x_i - x_j|$$

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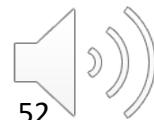
Example. Calculate the minimum distance of two 4-PAMs.

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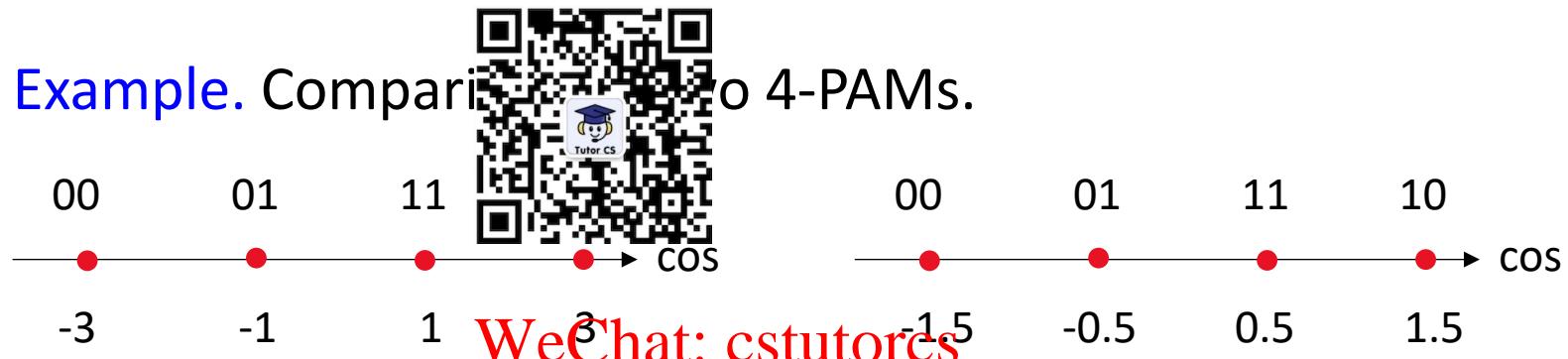
$$d_{\min, A1} = 2$$

$$d_{\min, A2} = 1$$



## To maximize distance between symbols

Example. Comparing to 4-PAMs.



Modulated waveform  $s(t) = A_i \cos(2\pi f_c t)$

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- The left modulation is less energy efficient but can survive with maximal noise level of 2 because  $d_{min,A1} = 2$ .  
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- The right modulation method is more energy efficient but cannot survive when noise level is larger than 1 (i.e.  $d_{min,A2} = 1$ )  
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Large minimum distance suggests reliable the system.



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## Constellation design

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- Given average transmit energy, try to maximize the minimum distance
- Given minimum distance, try to minimize the average transmit energy.



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**Example:** Please depict/design 8-symbol constellations with minimum distance of **1**. Assignment Project Exam Help

(a) 8-ASK;

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(b) 8-PSK;

(c) Is there any constellation that is more power efficient?

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## 8.6 Detector Design

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So far, we discuss demodulation methods without considering noise at the receiver.



In reality, noise is inevitable. With noise,  $y(mT) = a_m + n_m$ .

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where the noise  $n_m$  follows some distribution, usually Gaussian.

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Q: How to recover  $a_m$  from  $y(mT)$ ?  
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This is the **detector design problem**.  
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$y$  : Received value corrupted by noise

$n$  : noise. Usually assume to be Gaussian following  $\mathcal{N}(0, \sigma^2)$ .

$a$  : transmitted symbol. An element in  $\mathcal{A}$ .

$\hat{a}$  : Detection result. An element in  $\mathcal{A}$ .

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Detector design: Design the **decision rule**.

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Objectives:

- Probability of error  $P_e = P[\hat{a} \neq a]$ .
- Complexity. <https://tutorcs.com>



## Detection with minimum distance rule.

$$\hat{a} = \arg \min_{a \in \mathcal{A}} |y - a| = \arg \min_{a \in \mathcal{A}} |y - a|^2.$$



Decision regions.

Probability of error

**Example:** For BPSK, please draw the decision region.

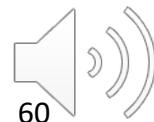
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**Example:** Given a 4-ASK constellation as shown in the figure.

The output of the receiver is  $y = a + n$ , where  $a$  is the transmitted symbol :  
  $\mathcal{N}(0,1)$  is the Gaussian noise.

- (1) If  $y = 0.2$ , estimate the transmitted symbol using the minimum distance rule.
- (2) Find the general decision rule and region for all  $y \in R$ .
- (3) What is the probability of error with this decision rule assuming  $a = 1$ .

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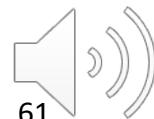
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