

Assignment Project Exam Help

# Public-Key Crypto

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# Review: Integrity

*Problem:* Sending a message over an **untrusted channel** without being changed

*Provably-secure solution:* **Random function**

*Practical solution:*



e.g. "Attack at dawn", 628369867...

**Pseudorandom function (PRF)**

Input: **arbitrary length  $k$**

Output: **fixed-length value**

Secure if practically indistinguishable from a random function, unless know  **$k$**

*Real-world use:*

**Message authentication codes (MACs)**

built on cryptographic hash functions

Popular example: **HMAC-SHA256<sub>k</sub>(m)**

[Cautions?!]

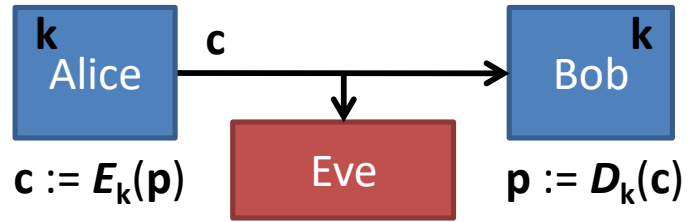
# Review: Confidentiality

*Problem:* Sending message in the presence of an **eavesdropper** without revealing it

*Provably-secure solution:* **One-time pad**

*Practical solution:*

**Pseudorandom generator (PRG)**



Input: fixed-length  $k$

Output: arbitrary-length stream

Secure if practically indistinguishable from a random stream, unless know  $k$

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*Real-world use:*

**Stream ciphers** (can't reuse  $k$ )

Popular example: **AES-128** + **CTR mode**

**Block ciphers** (need **padding/IV**)

Popular example: **AES-128** + **CBC mode**

[Cautions?!]

# Review: Diffie-Hellman Key Exchange

Lets Alice and Bob **agree on a shared secret** value by having a public conversation

**Alice**

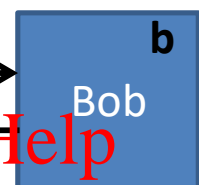
Generates random secret **a** ( $0 < a < p$ )



$g^a \bmod p$

**Bob**

Generates random secret **b** ( $0 < b < p$ )



$g^b \bmod p$



Computes **x**

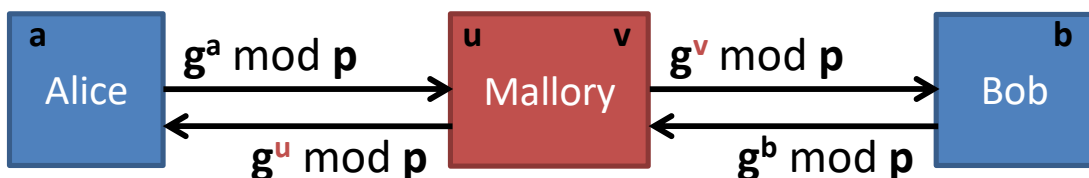
$$\begin{aligned} &= (g^b \bmod p)^a \bmod p \\ &= g^{ba} \bmod p \end{aligned}$$

Computes **x'**

$$\begin{aligned} &= (g^a \bmod p)^b \bmod p \\ &= g^{ab} \bmod p \end{aligned}$$

$$x == x'$$

*Problem:* **Man-in-the-middle attacks**



Caution: D-H gives you a shared secret, but don't know who's on the other end!

Suppose Alice publishes data to lots of people, and they all want to verify integrity...

Can't share an integrity key with *everybody*, or else *anybody* could forge messages

Suppose Bob wants to receive data from lots of people, confidentially...

Schemes we've discussed would require a separate key shared with each person

[What to do?]

Solution:

## Public-key Crypto

So far, encryption key == decryption key  
“**symmetric key crypto**”

**New idea:** Keys are distinct, and  
*you can't find one from the other*

Almost always used by splitting key-pair

Alice keeps one key private (“**private key**”)  
Publishes the other key (“**public key**”)

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Many applications

Invented in 1976 by Diffie and Hellman  
(earlier by Clifford Cocks of British  
intelligence, in secret)

Best known, most common  
public-key algorithm: **RSA**

Rivest, Shamir, and Adleman 1978

# Requirements for a public key crypto system to be secure

1. Computationally easy for B to generate a key pair:  $PU_b, PR_b$
2. Computationally easy for sender A to generate the ciphertext for message M:  $C=E(PU_b, M)$
3. Computationally easy for receiver B to decrypt the ciphertext:  $M=D(PR_b, C)$
4. Computational infeasible to guess  $PR_b$  knowing  $PU_b$ .
5. Computational infeasible to recover M from  $PU_b$  and C.



## A Method for Obtaining Digital Signatures and Public-Key Cryptosystems

R.L. Rivest, A. Shamir, and L. Adleman\*



# How RSA works

## Key generation:

1. Pick large (say, 1024 bits) random primes **p** and **q**
2. Compute **N** := **pq**  
(RSA uses multiplication mod **N**)
3. Pick **e** to be relatively prime to  $(p-1)(q-1)$
4. Find **d** so that  $ed \bmod (p-1)(q-1) = 1$
5. Finally: **Public key** is **(e,N)**  
**Private key** is **(d,N)**

To encrypt:  $E(x) = x^e \bmod N$

To decrypt:  $D(x) = x^d \bmod N$

# Why RSA works

## “It works” theorem:

For all  $0 < x < N$ ,

can show that  $D(E(x)) = x$

Proof:

$$D(E(x)) = (x^e \bmod pq)^d \bmod pq$$

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$$= x^{ed} \bmod pq$$
$$= x^{a(p-1)(q-1)+1} \bmod pq \text{ for some } a$$

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(because  $ed \bmod (p-1)(q-1) = 1$ )

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$$= (x^{(p-1)(q-1)})^a x \bmod pq$$
$$= (x^{(p-1)(q-1)} \bmod pq)^a x \bmod pq$$
$$= 1^a x \bmod pq$$

(because of the fact that if  $p, q$   
are prime, then for all  $0 < x < N$ ,  
 $x^{(p-1)(q-1)} \bmod pq = 1$ )

$$= x$$

# Is RSA secure?

Best known way to compute  $d$  from  $e$   
is factoring  $N$  into  $p$  and  $q$ .

Best known factoring algorithm:

## General number field sieve

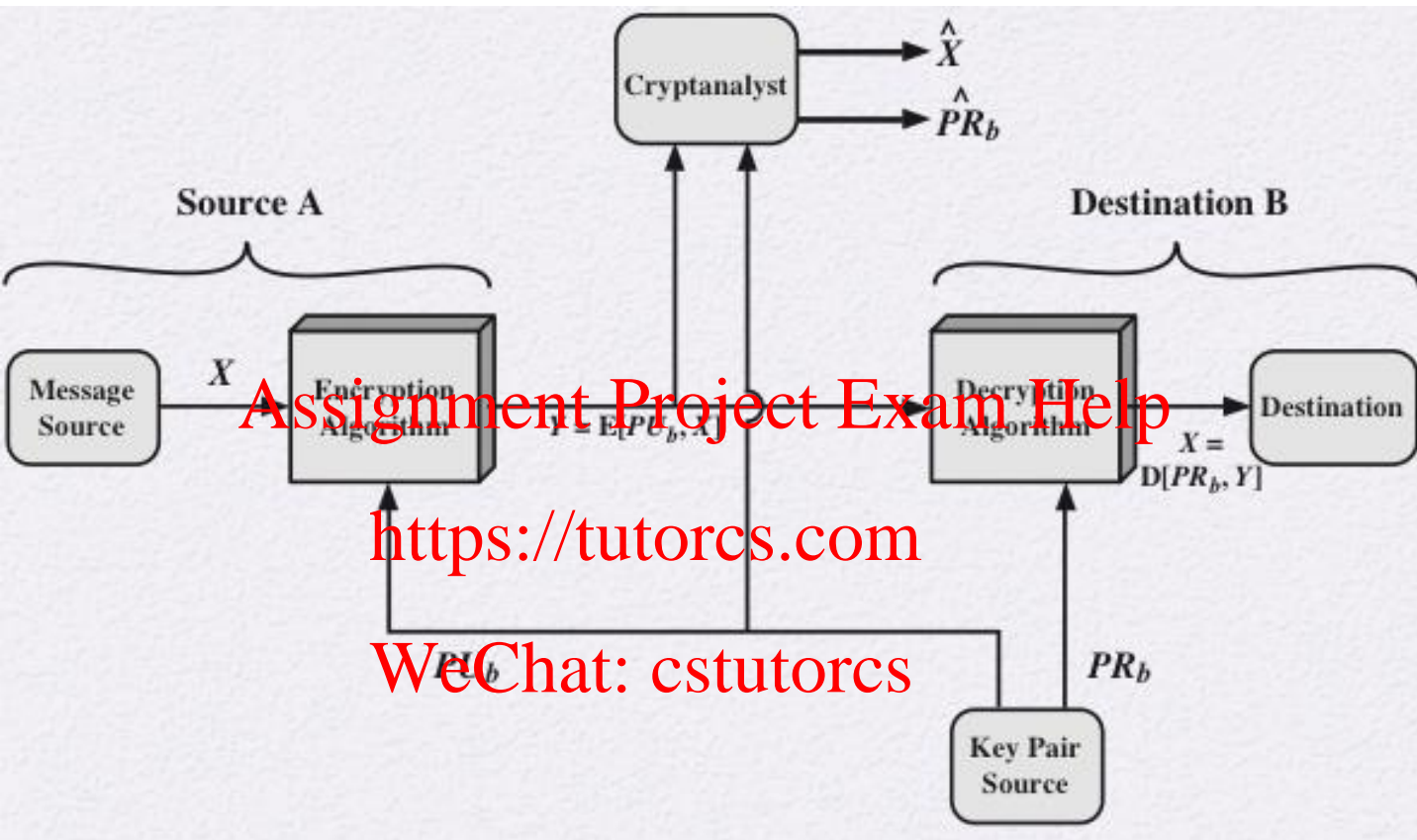
Takes more than polynomial time  
but less than exponential time  
to factor  $n$ -bit number.

(Still takes way too long if  $p, q$   
are large enough and random.)

Fingers crossed...

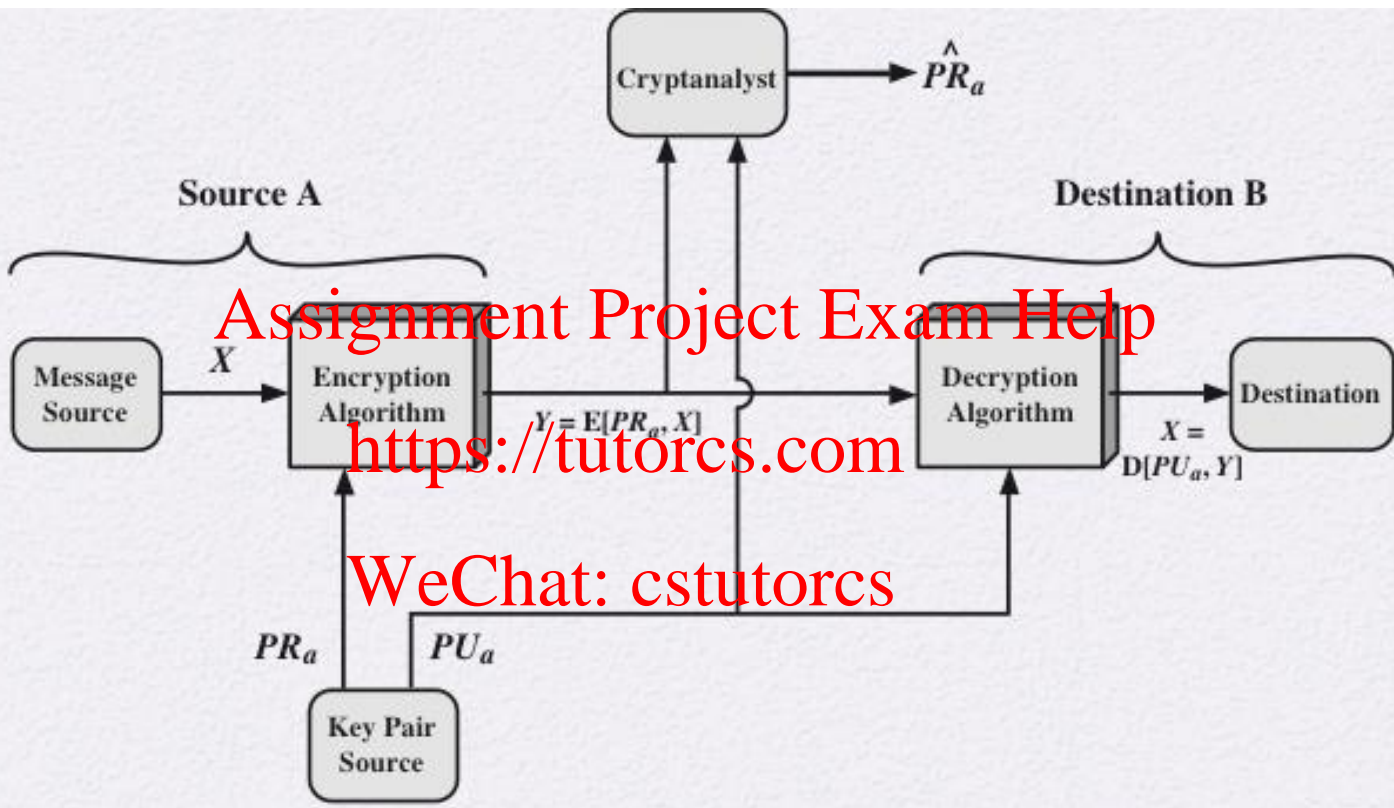
but can't rule out a breakthrough!

Signing with the public key for confidentiality or secrecy:



Does this provide integrity?

Signing with private key for integrity/authentication.



Does this provide confidentiality?

*Subtle fact:* RSA can be used for either confidentiality or integrity

## **RSA for confidentiality:**

Encrypt with public key

Decrypt with private key

“your eyes only” message

## **RSA for integrity**

Encrypt (“sign”) with private key

Decrypt (“verify”) with public key

called a **digital signature**

[What if we want both confidentiality and integrity on the same message?]

# How to have both confidentiality and integrity (using RSA)?

Alice (A) wants to send a secret message  $M$  to Bob (B) so that Bob can verify that it comes from Alice.

Which one(s) is/are secure?

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1.  $E(E(M, PR_A), PU_B)$

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2.  $E(E(M, PU_B), PR_A)$

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3.  $C = E(M, PR_A) \quad t = E(H(C), PU_B)$

- Send  $C || t$

4.  $C = E(M, PU_B) \quad t = E(H(C), PR_A)$

- Send  $C || t$

# Review: Public-key Crypto

So far, encryption key == decryption key  
“**symmetric key crypto**”

New idea: Keys are distinct.

**RSA:  $N := pq$**

**Public key** is  **$(e, N)$**

**Private key** is  **$(d, N)$**

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To encrypt:  $E(x) = x^e \bmod N$

To decrypt:  $D(x) = x^d \bmod N$

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*RSA for confidentiality:*

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Encrypt with public key

Decrypt with private key

*RSA for integrity (**digital signatures**):*

Encrypt (“sign”) with private key

Decrypt (“verify”) with public key

[\[Cautions?!\]](#)



# RSA drawback: Performance

Factor of 1000 or more slower than AES.

Dominated by exponentiation – cost goes up (roughly) as cube of key size.

Message must be shorter than  $N$ .

[How big should the RSA keys be?]

## Use in practice:

*Encryption* **Assignment Project Exam Help**

Use RSA to encrypt a random  $x < N$ ,  
compute  $k := \text{PRF}(x)$ , encrypt message  
using a symmetric cipher and key  $k$

*Signing:*

Compute  $v := \text{PRF}(m)$ , use RSA to sign  
a carefully padded version of  $v$   
(many gotchas!)

Almost always should use crypto libraries  
to get the details right

**True or false:**

Public-key encryption is a general-purpose technique that has made symmetric encryption obsolete

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## True or false:

Key distribution is trivial when using public-key encryption, compared to the cumbersome handshaking involved with key distribution centers for symmetric encryption.

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# Attacks against RSA

1. Brute force: trying all possible private keys
2. Mathematical attacks: factoring
3. Timing attacks: using the running time of decryption
4. Hardware-based fault attack: induce faults in hardware to generate digital signatures
5. Chosen plaintext attack on unpadded RSA

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# Exercise

Suppose Bob uses RSA crypto with a very large modulus  $n$  for which the factorization cannot be found in a reasonable amount of time.

Suppose Alice sends a message to Bob by representing each alphabet letter as an integer between 0 and 25 (A  $\rightarrow$  0, ..., Z  $\rightarrow$  25) and then encrypting each number separately using RSA with large  $e$  and large  $n$ .

Is this method secure?

If yes, why?

If not, how to efficiently attack this encryption method?

## Solution:

For a set of message block values  $SM = \{0, 1, 2, \dots, 25\}$ . The set of corresponding ciphertext block values  $SC = \{0^e \bmod N, 1^e \bmod N, \dots, 25^e \bmod N\}$ , and can be computed by everybody with the knowledge of the public key of Bob.

The most efficient attack is to compute  $M^e \bmod N$  for all possible values of  $M$ , then create a look-up table with a ciphertext as an index, and the corresponding plaintext as a value of the appropriate location in the table.

# Two subtle “textbook” RSA problems:

1. For small  $e$  and  $m$ :

$$m^e \bmod N == m^e$$

*Trivial to decrypt!*

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2. If  $m$  is chosen from a small set, easy to confirm a ciphertext is a given message (anyone can encrypt!)

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*Chosen plaintext attack*

# Solution: RSA Padding

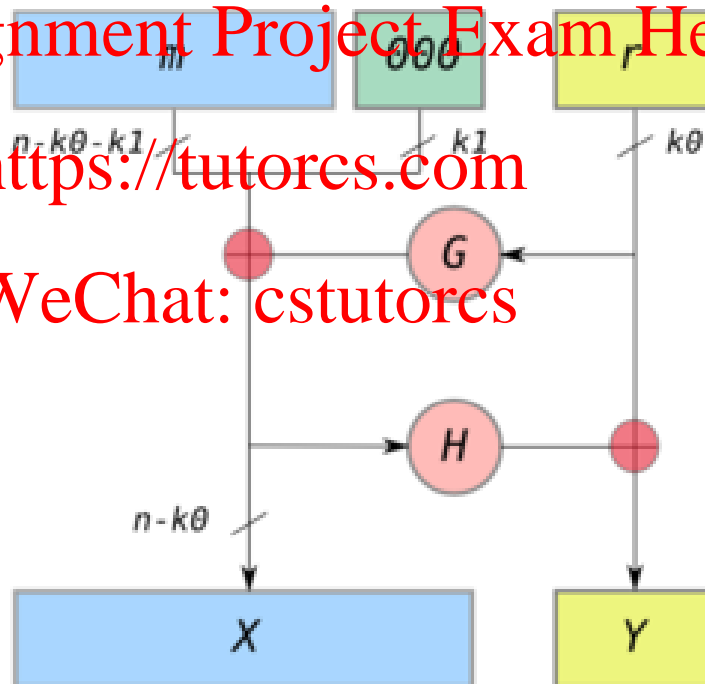
Need to make sure  $m$  is as large enough to wrap around  $N$

Need to *randomize* before encryption

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## So Far:

The Security Mindset

Message Integrity

Confidentiality

Key Exchange

Building a Secure Channel

Public Key Crypto

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## Next Week:

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Begin Web Security Unit

HTTPS: Secure channels for the web