1. Estimating MA

Consider an invertible MA(1) model $y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1}$, $\varepsilon_t \sim iid\ \text{WN}(0, \sigma^2)$. Suppose that we know ε_0 (shock at $t=0$) and 3 observations on y_t, ie, $\{y_1, y_2, y_3\}$. Express ε_t in terms of the parameters (μ, θ_1) and $(\varepsilon_0, y_1, y_2, y_3)$ for $t = 1,2,3$. As we can use $\mu + \theta_1 \varepsilon_{t-1}$, which is in the information set Ω_{t-1}, to forecast y_t, what is the interpretation for the shock ε_t? Further, how do you apply the "least squares" principle to estimate the parameters (μ, θ_1)? Just specify the objective function. The minimisation operation (first order derivatives) is not required. Now assume that the shocks ε_t are normally distributed. Write down the log-likelihood function in terms of (μ, θ_1) and y_1, y_2, y_3 and simplify removing all terms not influencing the optimization problem. Compare the objective function for the least squares and the MLE.

2. Find the unconditional variance of ARMA(1,1) model

$$y_t = \mu + \theta_1 \varepsilon_{t-1} + \varphi y_{t-1} + \varepsilon_t, \ \varepsilon_t \sim iid\ \text{WN}(0, \sigma^2)$$

Note that ε_{t-1} and y_{t-1} are now linearly dependent. Therefore you also need to consider covariance between these terms when you compute the variance.

3 Computing Exercise. Box-Jenkins methodology

This question is based on the data in the Excel file **fisher_update.XLS**. The file contains 171 quarterly observations, from 1969Q4 to 2012Q2, on the Australian Consumer price Index (P) and on the yield to maturity of 90-day bank accepted bills (R).

(a)   Generate the inflation rate as: INF=400*(log(P(1))-log(P)). When we construct the inflation rate this way, we lose the last observation, namely, 2012Q2. We change the sample to 1984Q1 to 2012Q1, which is the post-float period of the exchange rate.

Perform an ADF test for a unit root for inflation over the period 1984Q1-2012Q1. Comment on the results. Would you conclude that INF is stationary?

(b)   Generate the correlogram of INF (16 lags). Comment on which ARMA models would fit the data.

(c)   Estimate the models you considered in (b). Then select one model by using AIC/BIC.

(d)   Perform diagnostic checks on the model of your choice. Comment on whether or not the model fits the data well.

(e)   Now you are _____ ng exercise, using the model you are happy with (the output of (d)). Re-esti_____ F over the sample period 1984Q1-2009Q4, thereby keeping the last nine _____-of-sample forecasting exercise.

* First, change the s_____ Q1 2009Q4. Then estimate your model. I
* Then perform (pse_____ forecast for **2010Q1 2012Q1**; generate both
  * **Static forecast** (meaning 1-step ahead forecast based on most recent available observations for each *t*)
  * **Dynamic Forecast**.
* Compare these forecasts with actuals Inflation series.