Financial Econometrics

Slides-13: Remainaing Issues for GARCH and Alternative Models

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Lecture Plan

- Measure the risk premium effect: GARCH-M model
- Deal with structural break in volatility
- Seasonality and distributional assumptions
- Inclusion of other volatility measures
- SV models

GARCH in mean

- Risk premium effect: investing in a riskier asset should be rewarded by a higher expected return.
- In the context of a market index: investing in a riskier (more volatile) period should be rewarded by a higher expected return.
  - In AR(1)-GARCH, the mean equation $y_t = c + \phi y_{t-1} + \mu_t$: implies the **expected return** $= \hat{y}_t = c + \phi y_{t-1}$, which is unrelated to the volatility or risk measure σ_t.
  - Motivation: investors should be rewarded for taking additional risk by obtaining a higher return
- **GARCH-M** is used to account for the risk premium

$$y_t = c + \delta\sigma_{t-1} + \mu_t \qquad \mu_t|\Omega_{t-1} \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1\mu_{t-1}^2 + \beta_1\sigma_{t-1}^2,$$

where δ **measures the risk premium effect.**
(See Lundblad (2007, JFE, p123-150) among others.)

eg. NYSE composite return No evidence for the "risk premium" effect in any of GARCH(1,1), TGARCH/GJR and EGARCH..

GARCH(1,1)

Dependent Variable: RC
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 3 1931
Included observations: 1929 after adjustments
Convergence achieved after 16 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: OFF
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.076352	0.063905	1.194770	0.2322
C	0.016878	0.050651	0.333226	0.7390
AR(1)	0.103027	0.025309	4.070805	0.0000
Variance Equation				
C	0.013845	0.004594	3.013633	0.0026
RESID(-1)^2	0.120280	0.025125	4.787328	0.0000
GARCH(-1)	0.875387	0.021393	40.91840	0.0000

GJR

Dependent Variable: RC
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 3 1931
Included observations: 1929 after adjustments
Convergence achieved after 17 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: OFF
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*RESID(-1)^2*(RESID(-1)<0)
+ C(7)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.002775	0.065764	0.042196	0.9663
C	0.035056	0.049298	0.711105	0.4770
AR(1)	0.114460	0.024099	4.749571	0.0000
Variance Equation				
C	0.020172	0.004755	4.242354	0.0000
RESID(-1)^2	-0.003509	0.018165	-0.193186	0.8468
RESID(-1)^2*(RESID(-1)<0)	0.210087	0.036850	5.701133	0.0000
GARCH(-1)	0.882810	0.019965	44.21804	0.0000

EGARCH

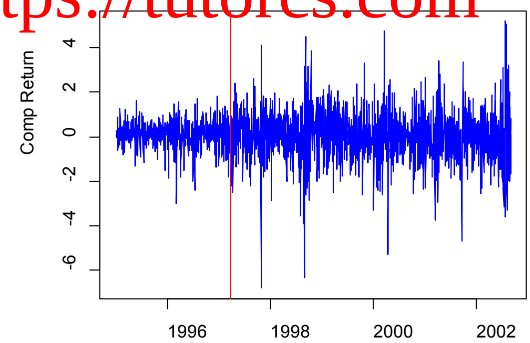
Dependent Variable: RC
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 3 1931
Included observations: 1929 after adjustments
Convergence achieved after 25 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: OFF
LOG(GARCH) = C(4) + C(5)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(6)*RESID(-1)/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.039690	0.063637	0.623694	0.5328
C	-0.005688	0.048548	-0.117162	0.9067
AR(1)	0.118381	0.024124	4.907231	0.0000
Variance Equation				
C(4)	-0.116899	0.022248	-5.254378	0.0000
C(5)	0.139572	0.028321	4.928140	0.0000
C(6)	-0.157712	0.023699	-6.654686	0.0000
C(7)	0.963692	0.007856	122.6648	0.0000

Structural break in Volatility

- The composite return series appears to have a change in its volatility level.
- The change is permanent.
- If ignored, it can result in
  - over-estimating the persistence measure $(\alpha_1 + \beta_1)$;
  - making the unconditional variance estimate inconsistent;
  - reducing the quality of forecasts, and VaR.
- Important to detect and account for the structural break.



Test for structural break

- As the variance is closely related to squared returns, we may check the break in an AR model for the squared returns, using the CUSUM test.

- Model stability: Its structure changes over time?.
  - Recursive parameter estimates. Monitor changes in parameter estimates over time.

$$\begin{array}{ccc} \{y_1, \cdots, y_\tau\}, & \{y_1, \cdots, y_{\tau+1}\}, & \cdots, \{y_1, \cdots, y_T\} \\ \hat{\beta}(\tau), & \hat{\beta}(\tau+1), & \hat{\beta}(T) \end{array}$$

  - Recursive residuals: $e_{\tau+1|\tau} = y_{\tau+1} - X_{\tau+1}\hat{\beta}(\tau)$

$$\begin{array}{ccc} \{y_1, \cdots, y_\tau\}, & \{y_1, \cdots, y_{\tau+1}\}, & \cdots, \{y_1, \cdots, y_\tau\} \\ e_{\tau+1|\tau}, & e_{\tau+2|\tau+1}, & e_{T|T-1} \end{array}$$

  - If the model is stable/correct: $w_{\tau+1|\tau} = \frac{e_{\tau+1|\tau}}{se(e_{\tau+1|\tau})} \sim N(0,1)$

Model stability test: CUSUM

CUSUM test (cumulative sum of standardised recursive residuals)

$$CUSUM_t = \sum_{\tau=K+1}^{t} w_{\tau+1|\tau}, \; t = K+1, K+2, \cdots, T-1$$

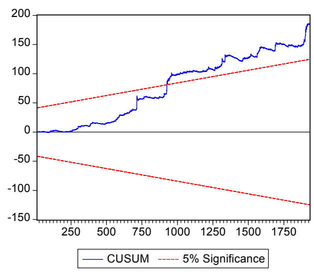
Reject stability if it goes outside the 95% bands.

Eviews: View/Stability Tests/Recursive Estimates after a linear regression is estimated

Test in volatility break: eg. AR(5) for the composite return squared:

$$r_t^2 = a_0 + a_1 r_{t-1}^2 + \cdots + a_5 r_{t-5}^2 + \text{error}_t$$

CUSUM test rejects the null hypothesis of **no break**.



Structural break in volatility

Find the break point

1) Run the restricted regression (no break) and save the log likelihood as ℓ_0.
2) Set $\tau = .15T$ (15% trim). Define the break dummy as $B_{t,\tau}$, which is 0 for $t < \tau$ and 1 for $t \geq \tau$.
3) Run the unrestricted regression

   $$r_t^2 = a_0 + a_1 r_{t-1}^2 + \cdots + a_5 r_{t-5}^2 + \psi B_{t,\tau} + \text{error}_t$$

   and save the log likelihood ℓ_τ and $LR_\tau = 2(\ell_\tau - \ell_0)$.
4) Set $\tau = \tau + 1$. If $\tau \leq .85T$ (15% trim), go to 3). Otherwise go to 5).
5) The break point is estimated as the τ associated with the greatest LR_τ.

It could be used as a test: the null of no break is rejected if max LR > cv.

The cv for **15% trim** is 8.85, see Andrews (1993, Etrca, p821-856).

Structural break in volatility

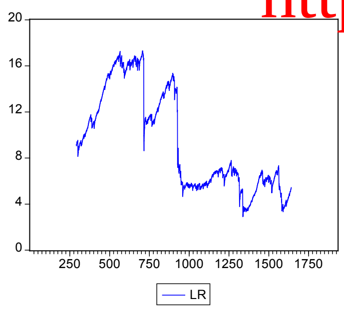
Find the break point

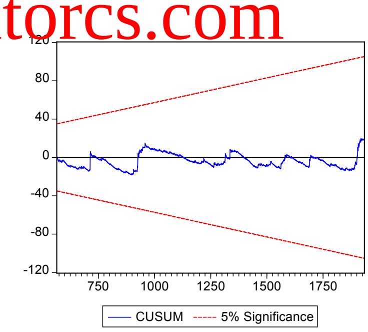
eg. AR(5) for the composite return squared:

$$r_t^2 = a_0 + a_1 r_{t-1}^2 + \cdots + a_5 r_{t-5}^2 + \psi B_{t,\tau} + \text{error}_t$$

The break point = 566.

AR(5) with the break passes the CUSUM test





Incorporating breaks in volatility models

Incorporate a break in GARCH

- Once the break point is known and the break dummy $B_{t,\tau}$ is defined, the break should be included in the conditional variance.
  - GARCH(1,1) :
    $\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \psi B_{t,\tau}$
  - TGARCH/GJR :
    $\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \gamma \mu_{t-1}^2 I_{t-1} + \beta_1 \sigma_{t-1}^2 + \psi B_{t,\tau}$
  - TGARCH/GJR :
    $\ln(\sigma_t^2) = \alpha_0 + \alpha_1 |\nu_{t-1}| + \gamma \nu_{t-1} + \beta_1 \ln(\sigma_{t-1}^2) + \psi B_{t,\tau}$

Seasonality: January Effect

- Including a dummy in the variance equation,
  - GARCH(1,1):
    $\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \lambda J_t$
  - GJR:
    $\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \gamma \mu_{t-1}^2 I_{t-1} + \beta_1 \sigma_{t-1}^2 + \lambda J_t$
  - EGARCH: $\ln(\sigma_t^2) = \alpha_0 + \alpha_1 |\nu_{t-1}| + \gamma \nu_{t-1} + \beta_1 \ln(\sigma_{t-1}^2) + \lambda J_t$

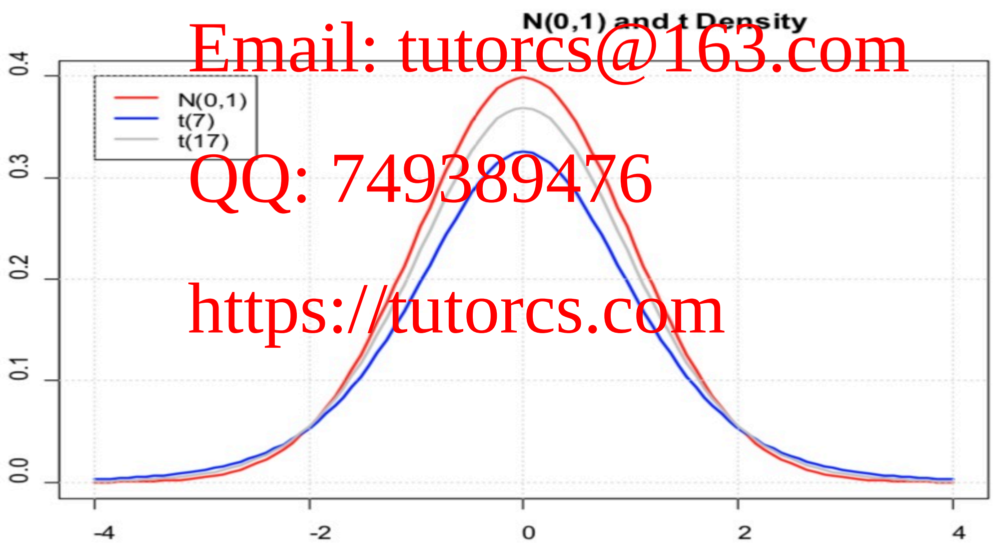
  where J_t is 1 if t is in January and 0 otherwise.

Non-normality

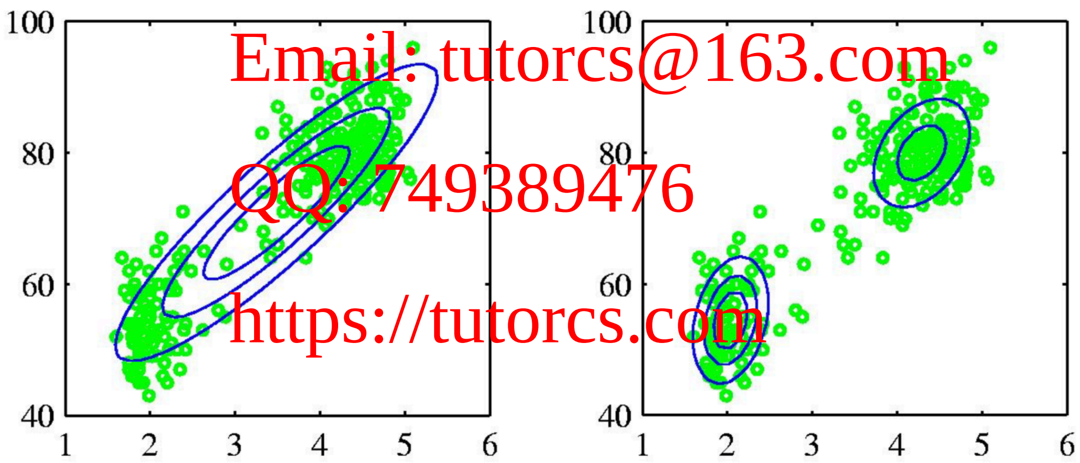
- **Normality alternatives**
  - In our examples, normality is usually rejected owing to
    - heavy tails (Kurtosis > 3) and
    - negative skewness

    in the distribution of the standardised shock ν_t.
  - Alternative distributions may be assumed
    - Student's t: $t(n)$
      with heavy tails but symmetry.
      $t(n) \approx N(0, 1)$ when the df $n \to \infty$
    - Mixture distributions: heavy tails and asymmetry.

Student-t



Mixture of two normals



Single Gaussian | Mixture of two Gaussians

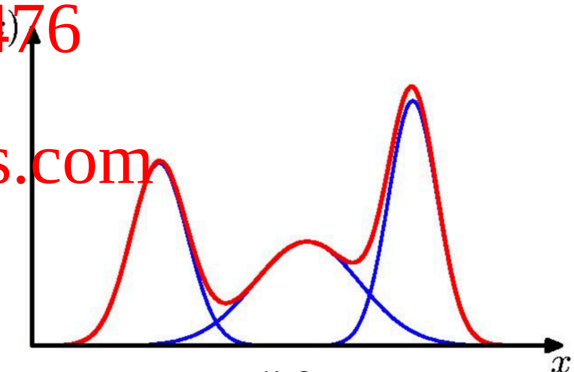
Mixture of gaussians

Combine simple models into a complex model:

$$p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \underbrace{\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)}_{\text{Component}}$$

Mixing coefficient

$$\forall k : \pi_k \geqslant 0 \qquad \sum_{k=1}^{K} \pi_k = 1$$



K=3

Incorporate other volatility measures

- Range and implied volatility

  - In addition to μ_{t-1} or ν_{t-1}, other volatility measures may have predictive power for conditional variance.

    Typically, the range (100ln(high/low)) and implied volatility (IV) are informative measures of volatility.

  - For EGARCH, we may specify

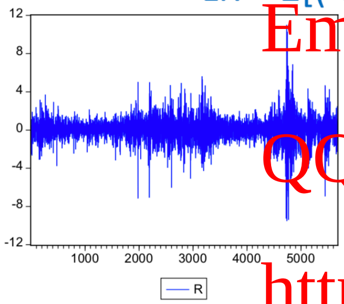
    $\ln(\sigma_t^2) = \alpha_0 + \alpha_1|\nu_{t-1}| + \gamma\nu_{t-1} + \beta_1\ln(\sigma_{t-1}^2) + a_1\text{rng}_{t-1} + a_2\text{iv}_{t-1}$,
    where the range (rng) and IV (iv) are included.

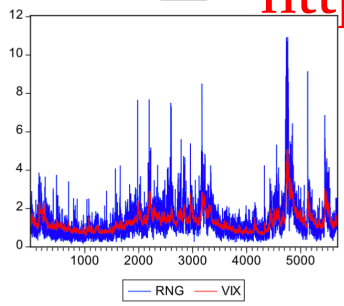
→ It is good for 1-step ahead forecast. However, we need models for the range and IV to do multi-step ahead forecasts.

Example: Range and Implied Volatility in EGARCH

eg. SP500 return, range and VIX: 1990-01-02 to 2012-06-29

LR = 2[(-7587.4)-(-7665.6)] = 156.4, AIC and SIC are all in favour of including RNG and VIX





Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 2 5672
Included observations: 5671 after adjustments
Convergence achieved after 20 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(5)*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.015307	0.011196	1.367157	0.1716
Variance Equation				
C(2)	-0.108049	0.012099	-8.930312	0.0000
C(3)	-0.040400	0.040325	-1.001847	0.3164
C(4)	0.179726	0.040989	4.384704	0.0000
C(5)	-0.111064	0.010764	-10.31786	0.0000
C(6)	0.978467	0.003151	310.5112	0.0000

R-squared	-0.000048	Mean dependent var	0.023481
Adjusted R-squared	-0.000931	S.D. dependent var	1.179539
S.E. of regression	1.180088	Akaike info criterion	2.705549
Sum squared resid	7889.119	Schwarz criterion	2.712577
Log likelihood	-7665.584	Durbin-Watson stat	2.117536

Dependent Variable: R
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 2 5672
Included observations: 5671 after adjustments
Convergence achieved after 21 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(5)*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1)) + C(7)*RNG(-1) + C(8)*VIX(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.013317	0.010704	1.244095	0.2135
Variance Equation				
C(2)	-0.365434	0.057308	-6.376625	0.0000
C(3)	-0.147432	0.039088	-3.771762	0.0002
C(4)	0.214592	0.037860	5.668000	0.0000
C(5)	-0.162571	0.016115	-10.08791	0.0000
C(6)	0.840658	0.024695	34.04156	0.0000
C(7)	0.045700	0.013780	3.316388	0.0009
C(8)	0.177244	0.038160	4.644800	0.0000

R-squared	-0.000074	Mean dependent var	0.023481
Adjusted R-squared	-0.001310	S.D. dependent var	1.179539
S.E. of regression	1.180312	Akaike info criterion	2.678683
Sum squared resid	7889.326	Schwarz criterion	2.688055
Log likelihood	-7587.407	Durbin-Watson stat	2.117481

Stochastic volatility (SV) model: Latent Volatility

- In GARCH type models, the shock μ_{t-1} or ν_{t-1} can be recovered from the mean equation. The conditional variance, as a function of μ_{t-1} is "observable".

- In SV,

$$
\begin{aligned}
y_t &= \mu + \sigma_t \nu_t, \\
\ln(\sigma_t^2) &= \alpha_0 + \beta_1 \ln(\sigma_{t-1}^2) + \eta_t, \ \eta_t \sim \text{iid } N(0, \omega^2)
\end{aligned}
$$

the conditional variance σ_t^2 is latent (unobservable):

  - there are two shocks: ν_t and η_t. Often used in theoretical options pricing literature;
  - it is difficult to estimate (likelihood evaluation is challenging)
  - it is awkward for forecasting, as σ_t^2 is conditional on an unobservable information set.

Summary

- We have seen a variety of models for conditional volatility for niveriate returns models
- Next... Multivariate Volatility models: Portfolio management, hedging strategies...