F Econometrics

Slides- sed Volatility Models

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Realised volatility (RV)

- Many theoretical finance models treat asset prices as stochastic processes in continuous time, which do not exactly match GARCH type models.
- Intraday observations, as discrete samples on price processes, are informative about volatilities and can be exploited to obtain volatility estimates.
- Realised variance (RV): how to construct daily return variance using intraday returns.

  Models of RV: how to capture the dynamics in RV.
- Andersen et al (2010, JAEtrcs, 25, p233-261) and Corsi (2009, JFEtrcs, 7, p174-196).

RV: Continuous-time model & intraday returns

- A typical continuous-time model (mean equation),
  $$dp(t) = \sigma(t)dW(t) + \kappa(t)dq(t)$$
  $\boxed{r_t = \sigma_t v_t + \text{jump}}$
  - $p(t)$: spot log price;
  - $\sigma(t)$: spot volatility;
  - $W(t)$: standard Brownian motion, independent of $\sigma(t)$;
  - $dq(t)$: 1 if there is a jump and 0 otherwise;
  - $\kappa(t)$: jump size.
- On day t, the return in the time interval $(t_{i-1}, t_i]$:
  $r_{t,i} = p(t_i) - p(t_{i-1})$.
- There are many observable returns on the day:
  $\{r_{t,1}, r_{t,2}, \ldots, r_{t,M}\}$.

RV: Continuous time model & intraday returns

– Daily returns

- For day t, the daily return is given by
  $$r_t = p(t) - p(t-1)\,, \qquad t = 1,2,\ldots$$
  Here t is the day index.
- It is known that, in the absence of jumps,
  $$[\mathrm{IV}_t]^{-1/2} r_t \sim N(0,1)\,,$$
  where $\mathrm{IV}_t = \int_{t-1}^{t} \sigma^2(s)ds$ is the **integrated variance** for day t ("instantaneous" variance of r_t).
- The conditional variance of the return for day t,
  $$\mathrm{Var}(r_t|\Omega_{t-1}) = E(\mathrm{IV}_t|\Omega_{t-1})\,,$$
  is the "σ_t^2" in GARCH type models.

RV: Continuous-time model & intraday returns

– Daily RV

- For day t, the [illegible]V is given by

  $\mathrm{RV}_t = \sum_{i=1}^{M} r_{t,i}^2$.

  It is known that, in the absence of jumps,

  $\mathrm{RV}_t \xrightarrow{p} \mathrm{IV}_t$ as $M \to \infty$. recall $[\mathrm{IV}_t]^{-1/2} r_t \sim N(0,1)$

- In the presence of jumps,

  $\mathrm{RV}_t \xrightarrow{p} \mathrm{IV}_t + \sum_{s=q(t-1)}^{q(t)} \kappa^2(s)$ as $M \to \infty$, but

  $\mathrm{BV}_t = \frac{2}{\pi} \sum_{i=1}^{M} |r_{t,i} r_{t,i-1}| \xrightarrow{p} \mathrm{IV}_t$ as $M \to \infty$.

- RV (or BV) is a good measure of the return volatility.

RV: Continuous time model & intraday returns

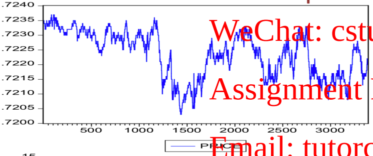
EGARCH:
$$\ln(\sigma_t^2) = \alpha_0 + \beta_1 \ln(\sigma_{t-1}^2) + \alpha_1 |v_{t-1}| + \gamma v_{t-1}$$

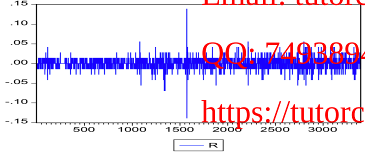
- RV
  - Comparison
    - GARCH models are based on daily returns. The volatility signal is extracted from 1 observation per day.
    - For RV, the volatility signal is extracted from M observations per day. As an estimator of IV, RV makes the volatility *almost* observable.
    - However, intraday returns can be very noisy
      - micro-structure noises: discrete price increments, bid-ask bounce, stale prices, etc, make it impossible to observe the "true" return;
      - when M is really large, the noises will dominate;
      - various strategies: (a) use moderate sampling frequencies; (b) kernel-based RV estimates; (c) two-scales RV that uses both high & moderate frequencies.

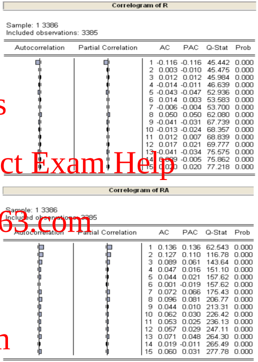
RV: Continuous time model & intraday returns

- Intraday retu

eg. AUD/USD -03,
3386 transaction prices







Correlogram of R

Sample: 1 3386
Included observations: 3385

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.116	-0.116	45.442	0.000
		2	0.003	-0.010	45.475	0.000
		3	0.012	0.012	45.984	0.000
		4	-0.014	-0.011	46.639	0.000
		5	-0.043	-0.047	52.936	0.000
		6	0.014	0.003	53.583	0.000
		7	-0.006	-0.004	53.700	0.000
		8	0.050	0.050	62.080	0.000
		9	-0.041	-0.031	67.739	0.000
		10	-0.013	-0.024	68.357	0.000
		11	0.012	0.007	68.839	0.000
		12	0.017	0.021	69.777	0.000
		13	-0.041	-0.034	75.575	0.000
		14	0.009	-0.005	75.862	0.000
		15	0.020	0.020	77.218	0.000

Correlogram of RA

Sample: 1 3386
Included observations: 3385

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.136	0.136	62.543	0.000
		2	0.127	0.110	116.78	0.000
		3	0.089	0.061	143.64	0.000
		4	0.047	0.016	151.10	0.000
		5	0.044	0.021	157.62	0.000
		6	0.001	-0.019	157.62	0.000
		7	0.072	0.066	175.43	0.000
		8	0.096	0.081	206.77	0.000
		9	0.044	0.010	213.31	0.000
		10	0.062	0.030	226.42	0.000
		11	0.053	0.025	236.13	0.000
		12	0.057	0.029	247.11	0.000
		13	0.071	0.048	264.30	0.000
		14	0.019	-0.011	265.49	0.000
		15	0.060	0.031	277.78	0.000

RV: Continuous time model & intraday returns

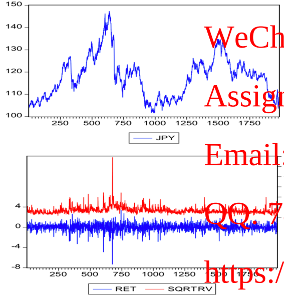
- Intraday re
  eg. AUD/US -11-03,
  3386 transaction prices

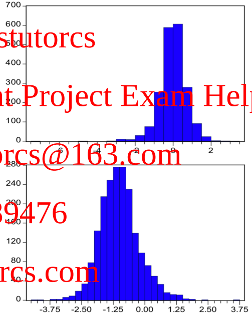
RV: Continuous time model & intraday returns

- Intraday returns
  - Some characteristics of intraday returns
    - Nearly zero mean
    - No or negative autocorrelation in returns
    - Strong positive autocorrelation in absolute returns
    - Non-normal distribution with thick-tails
    - Time-of-day effects
    - News announcements cause bursts of volatility

RV Example: Realised Variance

eg. JPY/USD dai[illegible] & RV
(1996-01-0[illegible]-04-14)





Series: RET	
Sample 1 1980	
Observations 1980	
Mean	-0.005848
Median	0.012435
Maximum	3.354060
Minimum	-7.296780
Std. Dev.	0.715506
Skewness	-1.011394
Kurtosis	10.88356
Jarque-Bera	5464.979
Probability	0.000000

Series: LRV	
Sample 1 1980	
Observations 1980	
Mean	-0.999813
Median	-1.041938
Maximum	3.544347
Minimum	-4.382835
Std. Dev.	0.785164
Skewness	0.422400
Kurtosis	4.377965
Jarque-Bera	215.5293
Probability	0.000000

RV Example: Realised variance

- Realised variance

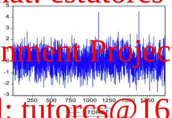
  eg. JPY/USD daily RV (continued),

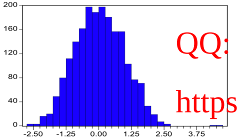
  Standardised return: $\mathrm{RV}_t^{-1/2} r_t$

  Its distribution is close to the normal.

  It and its square have little AC.

Without jumps: $[\mathrm{IV}_t]^{-1/2} r_t \sim N(0,1)$





Series: STDRET	
Sample 1 1980	
Observations 1980	
Mean	0.042329
Median	0.017229
Maximum	4.510362
Minimum	-2.712958
Std. Dev.	0.946469
Skewness	0.133419
Kurtosis	2.939141
Jarque-Bera	6.179789
Probability	0.045507

Correlogram of STDRET

Sample: 1 1980
Included observations: 1980

	AC	PAC	Q-Stat	Prob
1	0.003	0.003	0.0235	0.878
2	-0.014	-0.015	0.4406	0.802
3	0.025	0.025	1.6342	0.652
4	0.003	0.002	1.6507	0.800
5	0.027	0.028	3.0773	0.688
6	0.005	0.004	3.1177	0.794
7	0.016	0.017	3.6331	0.821
8	0.007	0.005	3.7212	0.881
9	0.028	0.029	5.3253	0.805
10	[illegible]	0.008	5.5240	0.854
11	0.006	0.006	5.5954	0.899
12	-0.001	-0.003	5.5961	0.935
13	0.007	0.006	5.6911	0.957
14	-0.003	-0.005	5.7064	0.973
15	-0.031	-0.032	7.6613	0.937
16	0.006	0.004	7.7355	0.956

Correlogram of STDRET2

Sample: 1 1980
Included observations: 1980

	AC	PAC	Q-Stat	Prob
1	-0.044	-0.044	3.8000	0.051
2	-0.033	-0.035	6.0197	0.049
3	0.007	0.004	6.1223	0.106
4	0.017	0.016	6.6706	0.154
5	-0.047	-0.045	11.018	0.051
6	0.021	0.018	11.905	0.064
7	0.001	-0.001	11.907	0.104
8	0.004	0.006	11.937	0.154
9	0.003	0.005	11.961	0.216
10	-0.014	-0.016	12.343	0.263
11	-0.000	0.001	12.343	0.338
12	0.003	0.002	12.365	0.417
13	-0.026	-0.026	13.736	0.393
14	-0.017	-0.018	14.281	0.429
15	-0.020	-0.025	15.115	0.443
16	-0.012	-0.014	15.384	0.497

RV Example: Realised variance (Continued)

eg. JPY/USD daily RV (continued)

logRV has strong, long-lasting AC.

As the decay in AC may not be exponential,

it is regarded as having a "long memory".

Stationary ARMA has exponential decays in AC.

Correlogram of LRV

Sample: 1 1980
Included observations: 1980

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.586	0.586	680.76	0.000
		2	0.496	0.232	1168.6	0.000
		3	0.467	0.174	1601.4	0.000
		4	0.436	0.110	1979.0	0.000
		5	0.420	0.097	2329.6	0.000
		6	0.393	0.056	2637.4	0.000
		7	0.384	0.064	2930.8	0.000
		8	0.368	0.035	3193.0	0.000
		9	0.358	0.051	3448.8	0.000
		10	0.353	0.045	3696.9	0.000
		11	0.321	-0.003	3901.8	0.000
		12	0.324	0.041	4111.6	0.000
		13	0.330	0.049	4328.8	0.000
		14	0.325	0.034	4539.0	0.000
		15	0.327	0.042	4752.1	0.000
		16	0.312	0.014	4947.2	0.000

Correlogram of RET

Sample: 1 1980
Included observations: 1980

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.027	0.027	1.4456	0.229
		2	0.028	0.027	3.0010	0.223
		3	0.010	0.009	3.2178	0.359
		4	-0.001	-0.003	3.2218	0.521
		5	-0.007	-0.007	3.3197	0.651
		6	0.027	0.028	4.7919	0.571
		7	0.013	0.012	5.1035	0.647
		8	0.028	0.026	6.6860	0.571
		9	-0.019	-0.022	7.3914	0.596
		10	-0.003	-0.004	7.4093	0.686
		11	-0.027	-0.026	8.8786	0.633
		12	-0.026	-0.024	10.186	0.600
		13	-0.025	-0.022	11.400	0.577
		14	0.020	0.021	12.194	0.591
		15	-0.034	-0.033	14.446	0.492
		16	0.016	0.017	14.971	0.527

Characteristics of Realised Variance

– Some characteristics of realised variance

- Positive, strong and long-lasting AC
- Approximately bell-shaped distribution for log RV
- Standardised returns $RV_t^{-1/2} r_t$ approximately normal
- True for both equity and foreign exchange markets, see Andersen et al (2001, JFE, 61, p43-76) Andersen et al (2001, JASA, 96, p42-55).

Models for RV: Long Memory Models

- Recall: y_t is said ... if $\Delta y_t = (1 - L)y_t$ is a stationary ARM... self is not.
- I(1) series have "forever" memory.

- RV has long-lasting memory. It should sit between I(1) and stationary ARMA.

- **Fractionally-integrated (FI) process**: y_t is said to be I(d) if $\Delta^d y_t = (1 - L)^d y_t$ is a stationary ARMA but y_t itself is not, where $0 < d < 0.5$.

Models for RV: Long Memory Models

- The fractional c[illegible] is defined by Taylor expansion

$$(1-L)^d = [illegible])L + c_2(d)L^2 + \cdots$$
$$= 1 + \sum_j^{\infty} c_j(d)L^j$$

where the coefficients $c_j(d)$
  - are functions of d;
  - converge to zero non-exponentially.

$\phi(L)\Delta^d$ is an AR(∞) with coefficients converging to zero non-exponentially.

- ARFIMA(p,d,q) model

$$\phi(L)\Delta^d(y_t-\mu) = \theta(L)\varepsilon_t, \quad \varepsilon_t \sim iid \text{ WN}(0,\sigma^2),$$

where $\phi(L)$ and $\theta(L)$ are AR and MA polynomials.

Estimation: maximum likelihood or nonparametrics.

Long Memory Models: Example

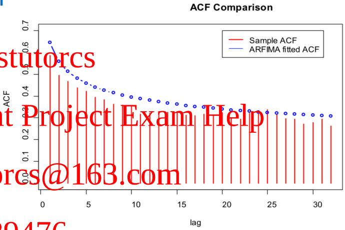
eg. JPY/USD daily l FIMA(0,d,1), $\hat{d} = 0.392$.

Interestingly, A al

(2003, Etrca, 71, p579-625)

find that

$\hat{d} \approx 0.4$ occurs frequently.



– The estimation of ARFIMA is not straightforward. EViews 5 does not have this capacity.

Models for RV: Heterogeneous AR (HAR) model

- Different investors (speculators, pension funds, etc) have different horizons, behave differently, and generate different volatility components.

- In HAR, RV is assumed to consists of short-term (daily), medium-term (weekly) and long-term (monthly) components, reflecting various investment horizons.

- Corsi (2009, JFEtrcs, 7, p174-196)

HAR for ln(RV)

- HAR for ln(RV)

$$h_{d,t} = c + \phi_d h_{d,t-1} + \phi_w h_{w,t-1} + \phi_m h_{m,t-1} + \eta_t,$$

where $h_{d,t}$= ln(daily RV), $h_{w,t}$= ln(weekly RV), $h_{m,t}$= ln(monthly RV), and η_t is a white noise process.

- The HAR model is pragmatic, linear in parameters, and easy to estimate and make forecasts. It can be extended to include quarterly RV, jumps, asymmetric effect, etc.

- **Conditional variance:** As $h_{d,t} = \hat{h}_{d,t} + e_t$ = fitted + resid,

$$\sigma_t^2 = E\{\exp(h_{d,t})|\Omega_{t-1}\} = \exp(\hat{h}_{d,t})\, E\{\exp(e_t)\,|\Omega_{t-1}\}$$
$$\approx \exp(\hat{h}_{d,t})\, E\{\exp(e_t)\} \approx \exp(\hat{h}_{d,t})\, \hat{a}$$

where $\hat{a} = \sum_{t=1}^{T} \exp(e_t)\,/T$.

HAR for ln(RV): Example

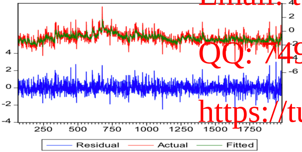
eg. JPY/USD daily ... AR: strong quarterly component

Dependent Variable: LRV
Method: Least Squares

Sample: 67 1980
Included observations: 1914
Newey-West HAC Standard Errors & Covariance (lag truncation=7)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.200512	0.029464	-6.805268	0.0000
LRV1	0.275615	0.033424	8.245911	0.0000
LRV5	0.290716	0.050619	5.743222	0.0000
LRV22	0.083808	0.060197	1.392218	0.1640
LRV66	0.250405	0.053522	4.678537	0.0000

R-squared	0.428369	Mean dependent var	-0.989053
Adjusted R-squared	0.427171	S.D. dependent var	0.786114
S.E. of regression	0.594974	Akaike info criterion	1.802009
Sum squared resid	675.7737	Schwarz criterion	1.816526
Log likelihood	-1719.523	F-statistic	357.6419
Durbin-Watson stat	2.015191	Prob(F-statistic)	0.000000



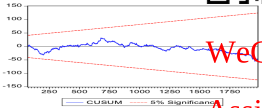
Correlogram of Residuals

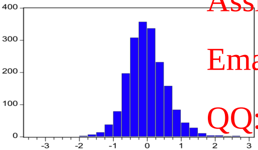
Sample: 67 1980
Included observations: 1914

	AC	PAC	Q-Stat	Prob
1	-0.009	-0.009	0.1435	0.705
2	0.023	0.023	1.1780	0.555
3	0.026	0.026	2.4582	0.483
4	-0.011	-0.011	2.6725	0.614
5	-0.025	-0.026	3.8437	0.572
6	-0.004	-0.004	3.8721	0.694
7	0.018	0.020	4.4825	0.723
8	-0.006	-0.004	4.5567	0.804
9	0.008	0.007	4.6746	0.862
10	0.022	0.020	5.5783	0.849
11	0.027	-0.027	6.9979	0.799
12	0.005	0.004	7.0536	0.854
13	0.014	0.014	7.4275	0.879
14	0.004	0.006	7.4618	0.915
15	0.014	0.014	7.8243	0.931
16	-0.006	-0.008	7.9013	0.952
17	-0.003	-0.004	7.9160	0.968
18	0.019	0.021	8.6079	0.968
19	-0.013	-0.012	8.9171	0.975
20	0.022	0.022	9.8835	0.970
21	-0.015	-0.015	10.348	0.974
22	-0.030	-0.033	12.140	0.954
23	0.012	0.012	12.414	0.964
24	0.007	0.010	12.523	0.973
25	0.050	0.051	17.397	0.867
26	0.013	0.014	17.743	0.885
27	0.014	0.007	18.098	0.901
28	0.009	0.006	18.264	0.919
29	-0.033	-0.030	20.324	0.883
30	-0.013	-0.014	20.668	0.898
31	0.039	0.044	23.630	0.825
32	-0.060	-0.059	30.707	0.532
33	0.004	-0.001	30.745	0.580
34	-0.029	-0.030	32.360	0.548
35	0.012	0.013	32.649	0.582
36	-0.008	-0.002	32.781	0.622

HAR for ln(RV): Example

eg. JPY/USD daily l[illegible]R: passes stability test,
Residuals are n[illegible]and have ARCH effect.





Series: Residuals
Sample 67 1980
Observations 1914

Mean	1.50e-16
Median	-0.032887
Maximum	2.996026
Minimum	-3.292451
Std. Dev.	0.594351
Skewness	0.357813
Kurtosis	4.954691
Jarque-Bera	345.5518
Probability	0.000000

Dependent Variable: LRV
Method: ML - ARCH (Marquardt) - Normal distribution
Sample: 67 1980
Included observations: 1914
Convergence achieved after 11 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.202541	0.022748	-8.903488	0.0000
LRV1	0.269993	0.029769	9.069557	0.0000
LRV5	0.302143	0.045442	6.648917	0.0000
LRV22	0.080293	0.056582	1.419043	0.1559
LRV66	0.247048	0.052834	4.675932	0.0000
Variance Equation				
C	0.087185	0.027577	3.161584	0.0016
RESID(-1)^2	0.116515	0.032085	3.631503	0.0003
GARCH(-1)	0.638255	0.097299	6.559755	0.0000

R-squared	0.428343	Mean dependent var	-0.989053
Adjusted R-squared	0.426244	S.D. dependent var	0.786114
S.E. of regression	0.595455	Akaike info criterion	1.773425
Sum squared resid	675.8041	Schwarz criterion	1.796652
Log likelihood	-1689.168	F-statistic	204.0242
Durbin-Watson stat	2.008304	Prob(F-statistic)	0.000000

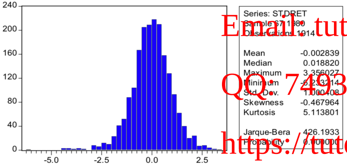
ARCH Test:

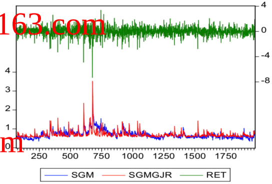
F-statistic	34.71506	Probability	0.000000
Obs*R-squared	34.13136	Probability	0.000000

HAR for ln(RV): Example

eg. JPY/USD daily ... AR:

- Returns standa... the HAR-based conditional variance are non-normal.
- HAR-based σ_t differs markedly from GJR-based σ_t.





Summary

- ARFIMA or HAR[illegible]
  - To account f[illegible]pendence structure in RV, HAR does a great job (ie, residuals are white noise).
  - HAR is easily extended to include asymmetry, jumps, and other components.
  - HAR is trivial to implement. Just OLS!
  - ARFIMA is also good at explaining the dependence structure in RV, but is difficult to implement.
  - Practicality-wise, HAR wins.