ECON3206/5206: Financial Econometrics

Week-1: A Review of Random Variables and Distributions

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Random Variables: Definitions

- RV and probability distribution
  - Two types of RV
    - Discrete RV: it ta countable number of values.
    - Continuous RV: i y value in a interval.
  - Probability distribution (how likely the values occur)
    - Discrete RV

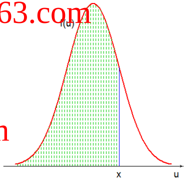
X	x_1	x_2	x_3	...
$P(X = x_i)$	p_1	p_2	p_3	...

      - Probability mass (pmf):
      - Cumulative distribution (cdf): $F(x) = P(X \le x) = \sum_{x_i \le x} p_i$

    - Continuous RV
      - Probability density (pdf): $f(u)$
      - Cumulative distribution (cdf):

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)du$$

Area under $f(u)$ curve up to x



Random Variables: Unconditional (marginal) Expectations (moments)

- Characterise RVs
  - Mean or expecte
    - The weighted ave ssible values, a measure of location
    - Discrete RV: $= \sum_i x_i P(X = x_i)$
    - Continuous RV: $E(X) = \int_{-\infty}^{\infty} u f(u) du$
  - Mean of $g(X)$ (eg. $X^2, e^X, \ldots$)
    - Discrete RV: $E[g(X)] = \sum_i g(x_i) P(X = x_i)$
    - Continuous RV: $E[g(X)] = \int_{-\infty}^{\infty} g(u) f(u) du$
  - Variance of X:
    - A measure of the amount of variation in all possible values

$$\mathrm{Var}(X) = E\{[X - E(X)]^2\} = E(X^2) - [E(X)]^2$$

  - Covariance between X and Y
    - A measure of association

$$\mathrm{Cov}(X, Y) = E\{[X - E(X)][Y - E(Y)]\}$$

Random Variables: Conditioning

- Conditional dis… of Y given X
  - The distributio… X is "known".
  - It depends on … X.
  - It is denoted as $Y|X$.

  eg. the distribution of *wage* for *age* = 20,

  the distribution of *wage* for *age* = 30, …

  wage|*age* depends on *age*.

bhp$_t$|**bhp**$_{t-1}$:

dist. of **bhp**$_t$ when **bhp**$_{t-1}$ is "fixed"

- Conditional expectation $E[g(Y)|X]$
  - It is calculated with the conditional distribution, treating X as "known" or "fixed".
  - It depends on X and, hence, is also a RV.

Random Variables

– Conditional mea[illegible]en X

- $E(Y|X)$ genera[illegible] on X.

eg. Linear regression $Y = a + bX + \varepsilon$.

$$E(Y|X) = a + bX + E(\varepsilon|X) = a + bX.$$

– Conditional variance of Y give X

- $\mathrm{Var}(Y|X)$ generally depends on X.

eg. Linear regression $Y = a + bX + \varepsilon$.

$$\mathrm{Var}(Y|X) = \mathrm{Var}(\varepsilon|X).$$

Properties of Expectation operator

Let X and Y be RV[illegible] a constant.

– Expectation is [illegible]erator

$E(a) = a$, Var[illegible]

$E(aX) = aE(X)$;

$E(X + Y) = E(X) + E(Y)$;

$E[g(X)|X] = g(X)$ for any function $g(\cdot)$;

$E(Y) = E[E(Y|X)]$ (iterated expectations);

$E(Y|X) = E(Y)$ if Y is independent of X;

$$\boxed{E\left(\frac{X}{Y}\right) \neq \frac{E(X)}{E(Y)}}$$

– Variance in a nonlinear operator

$\text{Var}(aX) = a^2\text{Var}(X)$;

$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y)$;

$\text{Var}(Y) = E[\text{Var}(Y|X)] + \text{Var}[E(Y|X)]$.

Sample moments

- Suppose we have ies) observations $\{X_t\}_1^T = \{X_1, X_2, \ldots,$ $\{Y_t\}_1^T = \{Y_1, Y_2, \ldots, Y_T\}$.
- Sample mean
  - $\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_t$
  - a measure of location (central tendency)
  - an estimator of population mean $E(X)$
- Sample variance and standard deviation
  - $\hat{\sigma}_X^2 = \frac{1}{T-1}\sum_{t=1}^{T}(X_t - \bar{X})^2, \qquad \hat{\sigma}_X = \sqrt{\hat{\sigma}_X^2}$
  - a measure of variation
  - an estimator of the population variance $\mathrm{Var}(X)$

Behold the summation operator

Linear operator:

$$\sum_{t=1}^{T}(a+bx_t+cy_t) = \ldots x_t + c\sum_{t=1}^{T} y_t$$

Summation of a constant:

$$\sum_{t=1}^{T} a = Ta$$

Nonlinear relations:

$$\sum_{t=1}^{T} x_t y_t \neq \sum_{t=1}^{T} x_t \sum_{t=1}^{T} y_t$$

$$\sum_{t=1}^{T} \frac{x_t}{y_t} \neq \frac{\sum_{t=1}^{T} x_t}{\sum_{t=1}^{T} y_t}$$