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University of New South Wales
School of Economics
al Econometrics
utorial 7

1. (Value at Risk)

The following GARCH(1,1) model has been estimated (such that the parameters are treated as known), using the historical return series $\{y_t\}$ of a portfolio up to date *T*:

$$y_t = c + \varepsilon_t, \qquad \varepsilon_t|\Omega_{t-1} \sim ?\,(0, \sigma_t^2),$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2, \quad t = 1, \ldots, T,$$

where Ω_{t-1} is the information set at $t-1$ and "$?(0, \sigma_t^2)$" is an unknown distribution with mean zero and variance σ_t^2. Suppose that the portfolio's market value at *T* was \$10m. How would you calculate the 99% value-at-risk for the period from *T* to *T*+1? Assume that parameters, ε_T and σ_T^2 are known or already estimated.

2. (GARCH-in-mean model)

Consider the following GARCH-M model

$$y_t = c + \delta\sigma_t^2 + \varepsilon_t, \quad \varepsilon_t|\Omega_{t-1} \sim N(0, \sigma_t^2),$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2, \quad \alpha_0 > 0, \quad \alpha_1 \geq 0, \quad \beta_1 \geq 0, \quad \alpha_1 + \beta_1 < 1,$$

where y_t is the return of a portfolio and Ω_{t-1} is the information set at $t-1$.

(a) Why would the conditional variance σ_t^2 be included in the mean equation? What would be the sign of the parameter δ?

(b) Find $E(y_t|\Omega_{t-1})$ and $E(y_t)$.

3. (EGARCH-model)

Consider the constant conditional mean - EGARCH model

$$y_t = c + \varepsilon_t, \quad \varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2)$$

$$ln(\sigma_t^2) = \alpha_0 + \alpha_1 |v_{t-1}| + \gamma v_{t-1} + \beta_1 ln(\sigma_{t-1}^2), v_{t-1} = \varepsilon_{t-1}/\sigma_{t-1}$$

(a) What are th[illegible]mulation in comparison to GARCH(1,1) model.

(b) Which sign c[illegible]d why?

(c) Compute one-period ahead optimal forecast of y and form 95% confidence bounds.

4. Computing Exercise

(GARCH, extensions and VaR, maybe useful for the project)

This question is based on the data contained in the Excel file *SHARE.XLS*. The file contains daily data on the S&P500 from the 2nd of January, 1998 to the 10th of December, 2001 comprising a total of 994 observations. The S&P500 index is designated *PRICE* in the file. Generate the series for the percentage log return as: *R*=100*(log(*PRICE*)-log(*PRICE*(-1))).

(a) Show a graph of the empirical distribution or CDF for *R*. What is the percentage daily return which cuts off 1% of the left-tail of the empirical distribution? (Or, what is the 1% quantile of the return distribution?).

(b) Assume the mean equation for returns is $R = c + \varepsilon_{t-1}$ and the variance equation for returns is a GARCH(1,1). Estimate the model using the first 900 observations. (We leave the last 94 observations for use in an out-of-sample forecast exercise). Are the sign restrictions for the GARCH specification satisfied? Present a graph of the conditional standard deviation. Comment on the estimation results.

(c) Repeat (b) for GJR model [Use the same clicks as (b) but select **1** for Threshold order], and EGARCH model [Use the same clicks as (b) but select **EGARCH** for Model]. Compare the results and decide which model you prefer, which will be used for the questions below.

(d) For the preferred model, present a histogram and summary statistics of the standardized residuals. Compare and comment the distributions of the residuals and the standardized residuals. What is the 1% quantile of the standardized residuals?

(e) With the preferred model, generate forecasts for returns and for the conditional standard deviation of returns for the out-of-sample observations 901-994.

Do the forecasts of variance show "mean-reverting" behavior?

(f) Suppose a portfolio holds a position of ten million dollars ($10m) in the market portfolio given by *VF*. Calculate the daily empirical 99% Conditional-Value-at-Risk of this portfolio for day 901. (To do this, you will need to look at the entries for series *RF* and *VF* at observation 901. You will also need to use your answers to parts (d) and (e)).