ECON3206/5206 Financial Econometrics

Tutorial 3

Question 1. Consider

$$y_t = \alpha + b_1 y_{t-1} + \varepsilon_t \text{ where } \varepsilon_t \sim WN(0,\sigma^2).$$

(a) Calculate unconditional $E(y_t)$, $\text{var}(y_t)$ and $\text{corr}(y_t, y_{t-i})$ for $i = 1,2$.

(b) What is the (optimal) forecast of y_{t+i}, for $i = 1,2$ on the basis of time t information?

(c) Calculate conditional variance $\text{var}(y_{t+1}|\Omega_t)$ and form confidence interval for forecast.

(d) Is y_t a white noise process?

(e) When y_t is a covariance stationary process?

(f) Think about an economic example where AR(1) is relevant?

Question 2. Suppose that a researcher estimated the lag 1 autocorrelation coefficient using a series of T=100 observations, and found it to be equal to 0.15. Is the autocorrelation coefficient significantly different from 0? Specify the null hypothesis, the alternative, test statistics, null distribution and decision criterion.

Question 3. Find the least squares estimator of the coefficient b_1 in the AR(1) model

$$y_t = \alpha + b_1 y_{t-1} + \varepsilon_t, \varepsilon_t \sim WN(0,\sigma^2).$$

*[Show also the under the null hypothesis of the correlation coefficient being zero, the OLS estimator of b_1, $\hat{b}_1$, is asymptotically normally distributed with mean zero and variance 1/T.

You need to use the following elements:

1. Normality of OLS (MLE) estimator
2. Computation of the variance of the OLS estimator for large T.]

Question 4.

Let $f_{t+h|t}$ be the forecast based on Ω_t. Namely, $f_{t+h|t}$ is a function of elements in Ω_t. Which $f_{t+h|t}$ minimises the mean square forecast error (MSFE)?

$$MSFE = E[(y_{t+h} - f_{t+h|t})^2 | \Omega_t].$$

*[Proof your answer formally

Hint: there are several ways to proof this.

Option 1. You may ex e definition of the expectation in terms of the integral (sum for discrete rv, b ntinuous rv in time series). Be careful to specify the correct conditional ex andom variable here. Take non-random terms outside of the expectation and ta

Option 2. Subtract an r in the squared term. Open the squares $(a+b)^2 = a^2 + 2ab + b^2$. Show that the term 2ab is equal to zero using the properties of the conditional expectation. After this, the answer follows automatically as a^2 term is not a function of $f_{t+h|t}$]

5. Estimating the CAPM and making sense of betas

Open the file *CAPM.XLSX* which contains the following daily data for 40 years starting on 12

August 1975 and ending on 12 August 2015 (source Datastream):

Gold Bullion LBM I£/Troy Ounce: the price of Gold

S&P 500 COMPOSITE - PRICE INDEX: a proxy for market portfolio

US T-BILL SEC MARKET 3 MONTH: a proxy for risk-free rate (annualized)

GENERAL ELECTRIC: the price of General Electric (GE) shares

Note: GE is one of the oldest companies in the index. It was founded in late 1800s. One of its co-founders, Thomas Edison, is the inventor of a commercially viable light bulb.

(a) Note that the Tbill interest is quoted on annual basis while the other returns are daily returns. Transform the annual returns to daily returns using compounding formula: $(1+R_d) = (1+R_y)^{1/360}$. Note you may check that the answer is similar to the one where you simply approximate R_d by $R_y/360$.

(b) Calculate the (log) returns for gold **gold_r**, S&P500 **sp500_r** and ge, **ge_r**

(c) Calculate the corresponding excess returns **gold_re, sp500_re, sp500_re**

(d) Plot the excess return of gold "**gold_re**" against the excess market return "**sp500_re**".

(e) Do the same for **ge** excess return

(f) Estimate the CAPM models.

(g) Inspect the estimation output table. Is the CAPM supported by Gold and GE data? Interpret the estimated beta coefficients. Comment on the R-square and the DW statistics.

(h) Find tim esiduals, actual and fitted. Find the histogram for the resid n is normally distributed?

(i) Test y in the residuals.

(j) Test for e residuals.

(k) Is the

These are less routine but more interesting questions

(l) Construct a portfolio based on the market portfolio (S&P 500) and risk-free T-Bill which would yield the same expected return as Gold and GE.

(m) Verify that the expected returns of the original assets and the corresponding constructed portfolios are the same.

(n) Compare the risk (standard deviation) of the original assets and the portfolio replicating the expected returns of these assets. Decompose the risk (st. dev) into the systematic and idiosyncratic risk.

(o) Where would you place Gold on the efficiency frontier, capital allocation line figure?