程序代写代做 CS编程辅导

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ECON3 Review of CLT & LLN

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- LLN and CLT – pillars of all statistics
  - Let $\{Z_1, Z_2, \dots, Z_T\}$ be a set of *independent* RVs with common mean μ and variance σ^2.
  - **Law of large numbers:** the probability that $\bar{Z} = \frac{1}{T}\sum_{t=1}^{T} Z_t$ differs from μ converges to zero as T goes to infinity.
  - **Central limit theorem:** the distributions of $\frac{\bar{Z}-\mu}{\sqrt{\mathrm{Var}(\bar{Z})}}$ converges to $N(0,1)$ as T goes to infinity.
  - Note that $\mathrm{Var}(\bar{Z}) = \sigma^2/T$ (see Rule 8).

    What happens if $\{Z_1, Z_2, \dots, Z_T\}$ are correlated?

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Distribution of Sample Mean

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Consider N random va[illegible] $\cdots X_N$.

- Let's consider $\bar{X} =$ [illegible] X_k.
- $\bar{X}$ is called the "sample mean" or the "empirical mean".
- $\bar{X}$ is a random variable.

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Suppose we observe values for $X_1, \cdots X_N$ and calculate the empirical mean of the observed values. That gives us one value for $\bar{X}$. But the value of $\bar{X}$ changes depending on the observed values.

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- Suppose we toss a fair coin $N = 5$ times and get H, H, H, T, T. Let $X_k = 1$ when come up heads. Then $\frac{1}{N}\sum_{k=1}^{N} X_k = \frac{3}{5}$
- Suppose we toss the coin another $N = 5$ times and get T, T, H, T, H. Now $\frac{1}{N}\sum_{k=1}^{N} X_k = \frac{2}{5}$

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Distribution of Sample Mean

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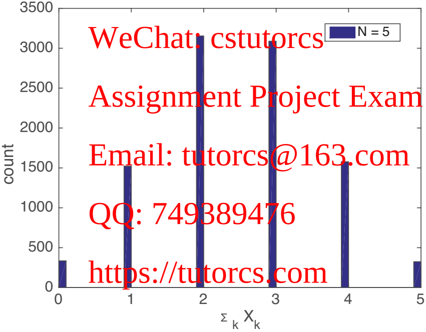
Toss fair coin $N = 5$ ti… alculate $\sum_{k=1}^{N} X_k$. Repeat, and plot histogram of values. It… $Bin(N, \frac{1}{2})$.

- X=[];for i=1:1000… m((rand(1,5)<0.5))]; end; hist(X,50)



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Distribution of Sample Mean

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Random variable $\bar{X} =$ [illegible].

- Suppose the X_k ar[illegible]**dent and identically distributed** (i.i.d)
- Each X_k has mean $E(X_k) = \mu$ and variance $Var(X_k) = \sigma^2$.

Then we can calculate the mean of $\bar{X}$ as:

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$$E(\bar{X}) = E(\frac{1}{N}\sum_{k=1}^{N} X_k) = \frac{1}{N}\sum_{k=1}^{N} E(X_k) = \mu$$

NB: recall linearity of expectation: $E(X + Y) = E(X) + E(Y)$ and $E(aX) = aE[X]$

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- We say $\bar{X}$ is an **unbiased estimator** of μ since $E[\bar{X}] = x$

Distribution of Sample Mean

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We can calculate the variance of $\bar{X}$ as:

$$var(\bar{X}) = var(\frac{1}{N}\sum_{k=1}^{N} X_k) = \frac{1}{N^2} var(\sum_{k=1}^{N} X_k)$$

$$= \frac{1}{N^2}\sum_{k=1}^{N} var(X_k) = \frac{N\sigma^2}{N^2} = \frac{\sigma^2}{N}$$

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NB: recall $Var(aX) = a^2 Var(X)$ and $Var(X+Y) = Var(X) + Var(Y)$ when X, Y independent.

- As N increases, the variance of $\bar{X}$ falls.
- $Var(NX) = N^2 Var(X)$ for random variable X.
- But when add togther **independent** random variables $X_1 + X_2 + \cdots$ the variance is only $NVar(X)$ rather than $N^2 Var(X)$
- This is due to **statistical multiplexing**. Small and large values of X_i tend to cancel out for large N.

Weak Law of Large Numbers[1]

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Consider N independen y distributed (i.i.d) random variables $X_1, \cdots X_N$ each with m variance σ^2. Let $\bar{X} = \frac{1}{N}\sum_{k=1}^{N} X_k$. For any $\epsilon > 0$:

$$P(|\bar{X} - \mu| \geq \epsilon) \to 0 \text{ as } N \to \infty$$

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That is, $\bar{X}$ **concentrates** around the mean μ as N increases.

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Proof:

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- $E(\bar{X}) = E(\frac{1}{N}\sum_{k=1}^{N} X_k) = \frac{1}{N}\sum_{k=1}^{N} E(X_k) = \mu$
- $var(\bar{X}) = var(\frac{1}{N}\sum_{k=1}^{N} X_k) = \frac{N\sigma^2}{N^2} = \frac{\sigma^2}{N}$
- By Chebyshev's inequality: $P(|\bar{X} - \mu| \geq \epsilon) \leq \frac{\sigma^2}{N\epsilon^2}$

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[1] There is also a **strong law of large numbers**, but we won't deal with that here.

Who cares ?

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- Suppose we have
- Define indicator ra able X_i equal to 1 when event E is observed in trial i erwise
- Recall $E[X_i] = P(E)$ is the probability that event E occurs.
- $\bar{X} = \frac{1}{N}\sum_{k=1}^{N} X_k$ is then the relative frequency with which event E is observed over N experiments.
- And ...
  $P(|\bar{X} - \mu| \geq \epsilon) \to 0$ as $N \to \infty$
  tells us that this observed relative frequency $\bar{X}$ converges to the probability $P(E)$ of event E as N grows large.
- So the law of large numbers formalises the intuition of probability as frequency when an experiment can be repeated many times. But probability still makes sense even if cannot repeat an experiment many times – all our analysis still holds.

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Central Limit Theorem (CLT)

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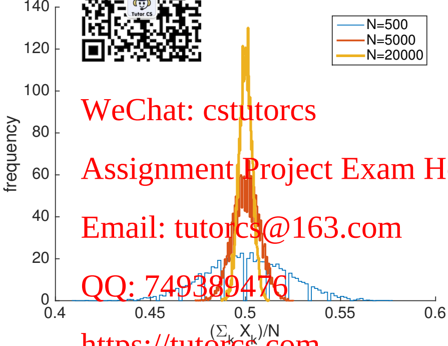
Histogram of $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ as N increases, but now we normalise to keep the area under the curve fixed:



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- See that (i) curve narrows as n increases, it concentrates as we already know from weak law of large numbers.
- Curve becomes more "bell-shaped" as N increases – this is the CLT

The Normal (or Gaussian) Distribution

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Define the following fu

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

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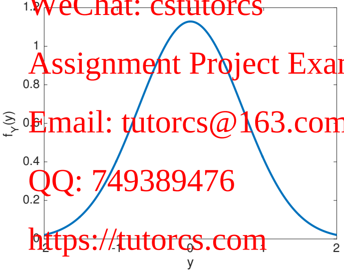

$\mu = 0$, $\sigma = 1$

Central Limit Theorem (CLT)

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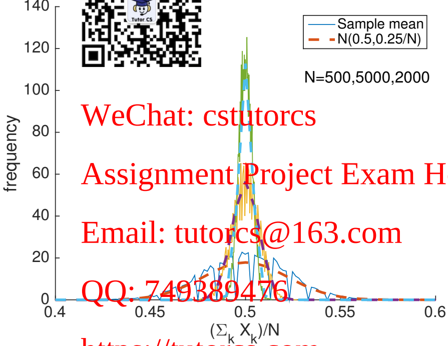
Overlay the Normal distribution, with parameter μ equal to the mean and σ^2 equal to the variance of each of the measured histograms:



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- CLT says that as N increases the distribution of $\bar{X}$ converges to a Normal (or Gaussian) distribution.
- Variance $\to 0$ as $N \to \infty$, i.e. distribution concentrates around its mean as N increases.