Problem Set 2 (Business Cycle Models)
ECON 6002
Due date: Monday, 8 May, 6pm

NOTE: To receiv[illegible] crucial to show all of your workings and not just provide a final al[illegible]al answer. Solve to a higher number of decimals, but report final n[illegible]o two decimal and place a box around the answer. It is also importa[illegible]de answers in your own words. Any quotations from the textbook[illegible] must be in quotation marks and attributed to the original source.

1. Abstracting from long-run growth by setting $n = g = 0$ and from persistent shocks by setting $\rho_A = \rho_G = 0$, with $\tilde{A}_t \equiv lnA_t - ln\bar{A}$ and $\tilde{G}_t \equiv lnG_t - ln\bar{G}$, and normalizing the population to $N = 1$, the following nine equations describe the "baseline" RBC model in Chapter 5:

$$Y_t = C_t + I_t + G_t \tag{1}$$
$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \tag{2}$$
$$K_{t+1} = K_t + I_t - \delta K_t \tag{3}$$
$$\tilde{A}_t = \epsilon_{A,t} \tag{4}$$
$$\tilde{G}_t = \epsilon_{G,t} \tag{5}$$
$$r_t = \alpha(A_t L_t/K_t)^{1-\alpha} - \delta \tag{6}$$
$$w_t = (1-\alpha)(K_t/A_t L_t)^{\alpha} A_t \tag{7}$$
$$\frac{1}{C_t} = e^{-\rho} E_t\left[\frac{1}{C_{t+1}}(1 + r_{t+1})\right] \tag{8}$$
$$\frac{C_t}{1 - L_t} = \frac{w_t}{b} \tag{9}$$

(a) Find the steady state for this economy under the following calibration: $\alpha = \frac{1}{3}$, $\delta = 0.05$, $\bar{r} = 0.03$, $\bar{A} = 1$, and $\bar{L} = 0.5$ and choose $\bar{G}$ such that $\bar{G}/\bar{Y} = 0.2$.In particular, you should find the remaining parameters values b and ρ that are consistent with steady state and determine steady-state values for the endogenous variables, $\bar{Y}$, $\bar{C}$, $\bar{I}$, $\bar{G}$, $\bar{K}$, and $\bar{w}$. (Hint: first solve for ρ using (8), then solve for $\bar{K}$ using (6), then $\bar{I}$ using (3), then $\bar{w}$ using (7), then $\bar{Y}$ using (2), then $\bar{C}$ using (1), then b using (9).)

(b) Now consider the special case of the model where $\delta = 1$ instead of $\delta = 0.05$ and $G_t = 0$ for all t (note: ρ will remain the same and b will be different, but you do not need to solve for it). Solve for Y_t, C_t, I_t, K_{t+1}, r_t, and w_t as analytical expressions of exogenous and predetermined variables A_t and K_t and constants. (Hint: with 100% depreciation, there is a constant saving rate $s = \alpha e^{-\rho}$ and constant labour supply $L_t = \bar{L}$. Given this solution to the household optimization problem, first solve for Y_t, r_t, and w_t from equations (2), (6), and (7) and then the solutions for C_t, I_t, and K_{t+1} are straightforward.)

(c) Again, for the special case of the model, what is the percentage change in output and percentage point change in the interest rate if the economy is at steady state at time $t-1$, but there is a shock $\epsilon_{A,t} = 0.25$ (i.e., 25%) at time t? Explain the economic intuition behind the responses of output and the interest rate in terms of the marginal products of labour and capital. (Hint: note that the $\epsilon_{A,t} = 0.25$ shock is to lnA_t, but the model solution is for the level of A_t. First solve for the steady-state level of output and then solve for output and the real interest rate given the shock.)

2. Consider Calv[illegible] with partial indexation. That is, if a firm is not visited by the Calvo t[illegible]t, its price in t is the previous period's price plus $\gamma\pi_{t-1}$, $0 \leq \gamma \leq 1$. Th[illegible]iod t is $p_t = \alpha x_t + (1-\alpha)(p_{t-1} + \gamma\pi_{t-1})$, where α is the fraction of firm[illegible]o tooth fairy in any given period and x_t is the price they set. The resul[illegible] a hybrid one: $\pi_t = \frac{\gamma}{1+\beta\gamma}\pi_{t-1} + \frac{\beta}{1+\beta\gamma}E_t\pi_{t+1} + \frac{1}{1+\beta\gamma}\kappa\tilde{y}_t$, where $\beta > 0$ i[illegible]r, $\kappa = \frac{\alpha}{1-\alpha}[1 - \beta(1-\alpha)]\phi > 0$ determines the slope of the Phillips curve, y_t is the output gap, and $E_t\pi_{t+1}$ is the expectation (taken at time t) of inflation at $t+1$.

   (a) Show that $x_t - p_t = \frac{1-\alpha}{\alpha}(\pi_t - \gamma\pi_{t-1})$.

   (b) Use the result in (a) and the representative firm's optimal price under Calvo pricing with partial indexation being $x_t = p_t + (1-\beta(1-\alpha))\phi\tilde{y}_t + \beta(1-\alpha)(E_t(x_{t+1} - p_{t+1}) + E_t\pi_{t+1} - \gamma\pi_t)$ to derive the hybrid Phillips curve.

   (c) What value of γ would lead to the highest degree of inflation persistence? Why?

   (d) Now assume that $\beta = 0.9$, $\gamma = 0.5$, and $\kappa = 0.1$. Assume that the central bank has some control of the evolution of $\tilde{y}_t$. Suppose that the central bank announces a *permanent* and *fully credible* reduction in its target or steady-state inflation rate from 7% to 2% at $t = 1$ (prior to this, the economy was at the steady state with 7% inflation and zero output gap). Determine the cost of this disinflation episode. How much is the output gap reduced?