

# 程序代写代做 CS编程辅导

## ECON 178 WI 2023: Homework 1



, 2023 (by 2:00pm PT)

### Instructions:

- The homework consists of 10 problems. The TAs will randomly pick one problem to grade and this problem will count for 30 points (you will get 30 points if your answers are correct or almost correct). The remaining 10 points will be graded on completion of this assignment.
- There will be two separate submissions: one for your R code and one for your writeup. Please submit both on Gradescope (note details for the submission of the R part are given in “Applied questions”).
- Please follow the policy stated in the syllabus about academic integrity.
- You must read, understand, agree and sign the integrity pledge (<https://academicintegrity.ucsd.edu/take-action/promote-integrity/faculty/excel-with-integrity-pledge.pdf>) before completing any assignment for ECON178. Please include your signed pledge in the submission of your assignment on Gradescope.

Assignment Project Exam Help

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### Conceptual questions

The following questions involve reviewing exercises about expectations, conditional expectations, biases and variances, and basic properties of Normal (also called Gaussian) distributions.

### Question 1

Suppose that we have a model  $y_i = \beta x_i + \epsilon_i$  ( $i = 1, \dots, n$ ) where  $y = \frac{1}{n} \sum_{i=1}^n y_i = 0$ ,  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = 0$ , and  $\epsilon_i$  is distributed normally with mean 0 and variance  $\sigma^2$ ; that is,  $\epsilon_i \sim N(0, \sigma^2)$ . Furthermore,  $\epsilon_1, \epsilon_2, \dots, \epsilon_n$  are independently distributed, and the  $x_i$ s ( $i = 1, \dots, n$ ) are non-random.

- (a) The OLS estimator for  $\beta$  minimizes the Sum of Squared Residuals:

$$\hat{\beta} = \operatorname{argmin}_{\beta} \left[ \sum_{i=1}^n (y_i - \beta x_i)^2 \right]$$

Take the first-order condition to show that

$$\hat{\beta} = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}.$$

- (b) Assume  $\mathbb{E}[\epsilon_i | \beta] = 0$  for all  $i = 1, \dots, n$ . Show that

$$\hat{\beta} = \beta + \frac{\sum_{i=1}^n x_i \epsilon_i}{\sum_{i=1}^n x_i^2}$$

What is  $\mathbb{E}[\hat{\beta} | \beta]$  and  $\text{Var}(\hat{\beta} | \beta)$ ? Use this to show that, conditional on  $\beta$ ,  $\hat{\beta}$  has the following distribution:

$$\hat{\beta} | \beta \sim N\left(\beta, \frac{\sigma^2}{\sum_{i=1}^n x_i^2}\right).$$

- (c) Suppose we believe that  $\beta$  is distributed normally with mean 0 and variance  $\frac{\sigma^2}{\lambda}$ ; that is,  $\beta \sim N(0, \frac{\sigma^2}{\lambda})$ . Assume that  $\beta$  is independent of  $\epsilon_i$  for all  $i = 1, \dots, n$ . Compute the mean and variance of  $\hat{\beta}$ . What is  $\mathbb{E}[\hat{\beta}]$  and  $\text{Var}(\hat{\beta})$ ? (Hint you might find useful:  $\mathbb{E}[w_1 + w_2] = \mathbb{E}[w_1] + \mathbb{E}[w_2]$  and  $\text{Var}(w_1 + w_2) = \text{Var}(w_1) + \text{Var}(w_2)$  if  $w_1$  and  $w_2$  are independent.)
- (d) Since everything is now known, it turns out that

$$\mathbb{E}[\beta | \hat{\beta}] = \mathbb{E}[\beta] + \frac{\text{Cov}(\beta, \hat{\beta})}{\text{Var}(\hat{\beta})} \cdot (\hat{\beta} - \mathbb{E}[\hat{\beta}]).$$

Let  $\hat{\beta}^{RR} = \mathbb{E}[\beta | \hat{\beta}]$ . Compute  $\text{Cov}(\beta, \hat{\beta})$  and use the value of  $\mathbb{E}[\beta]$  along with the values of  $\mathbb{E}[\hat{\beta}]$ ,  $\text{Cov}(\beta, \hat{\beta})$ , and  $\text{Var}(\hat{\beta})$  you have computed to show that

$$\hat{\beta}^{RR} = \mathbb{E}[\beta | \hat{\beta}] = \frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n x_i^2 + \lambda} \cdot \hat{\beta}$$

(Hint:  $\text{Cov}(w_1, w_2) = \mathbb{E}[w_1 w_2] - \mathbb{E}[w_1] \mathbb{E}[w_2]$  and  $\mathbb{E}[w_1 w_2] = \mathbb{E}[w_1 \mathbb{E}[w_2 | w_1]]$  for any random variables  $w_1$  and  $w_2$ )

- (e) Does  $\hat{\beta}^{RR}$  increase or decrease as  $\lambda$  increases? How does this relate to  $\beta$  being distributed  $N(0, \frac{\sigma^2}{\lambda})$ ?

## Question 2

Let us consider the linear regression model  $y_i = \beta_0 + \beta_1 x_i + u_i$  ( $i = 1, \dots, n$ ), which satisfies Assumptions MLR.1 through MLR.5 (see Slide 7 in “Linear\_regression\_review” under “Modules” on Canvas)<sup>1</sup>. The  $x_i$ s ( $i = 1, \dots, n$ ) and  $\beta_0$  and  $\beta_1$  are nonrandom. The randomness comes from  $u_i$ s ( $i = 1, \dots, n$ ) where  $\text{var}(u_i) = \sigma^2$ . Let  $\hat{\beta}_0$  and  $\hat{\beta}_1$  be the usual OLS estimators (which are unbiased for

$\beta_0$  and  $\beta_1$ , respectively) obtained from running a regression of  $\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-1} \\ y_n \end{pmatrix}$  on  $\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix}$  (the intercept

column) and  $\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{pmatrix}$ . Suppose you also run a regression of  $\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-1} \\ y_n \end{pmatrix}$  on  $\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{pmatrix}$  only

(excluding the intercept column) to obtain another estimator  $\tilde{\beta}_1$  of  $\beta_1$ .

<sup>1</sup>The model is a simple special case of the general multiple regression model in “Linear\_regression\_review”. Solving this question does not require knowledge about matrix operations.

- a) Give the expression of  $\tilde{\beta}_1$  as a function of  $y_i$ s and  $x_i$ s ( $i = 1, \dots, n$ ).
- b) Derive  $\mathbb{E}(\tilde{\beta}_1)$  in terms of  $\beta_0$ ,  $\beta_1$ , and  $x_i$ s. Show that  $\tilde{\beta}_1$  is unbiased for  $\beta_1$  when  $\beta_0 = 0$ . If  $\beta_0 \neq 0$ , when will  $\tilde{\beta}_1$  be unbiased for  $\beta_1$ ?
- c) Derive  $\text{Var}(\tilde{\beta}_1)$ , the variance of  $\tilde{\beta}_1$ , in terms of  $\sigma^2$  and  $x_i$ s ( $i = 1, \dots, n$ ).
- d) Show that  $\text{Var}(\tilde{\beta}_1) \leq \text{Var}(\hat{\beta}_1)$ ; that is,  $\text{Var}(\tilde{\beta}_1) \leq \text{Var}(\hat{\beta}_1)$ . When do you have  $\text{Var}(\tilde{\beta}_1) = \text{Var}(\hat{\beta}_1)$ ? You might find useful: use  $\sum_{i=1}^n x_i^2 \geq \sum_{i=1}^n (x_i - \bar{x})^2$  where  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ .
- e) Choosing between  $\tilde{\beta}_1$  and  $\hat{\beta}_1$  is a tradeoff between the bias and variance. Comment on this tradeoff.



### Question 3

Let  $\hat{v}$  be an estimator of the truth  $v$ . Show that  $\mathbb{E}(\hat{v} - v)^2 = \text{Var}(\hat{v}) + [\text{Bias}(\hat{v})]^2$  where  $\text{Bias}(\hat{v}) = \mathbb{E}(\hat{v}) - v$ . (Hint: The randomness comes from  $\hat{v}$  only and  $v$  is nonrandom).

### Applied questions (with the use of R)

For this question you will be asked to use tools from R for coding.

### Installation

- To install R, please see <https://www.r-project.org/>.
- Once you install R, please install also R Studio <https://rstudio.com/products/rstudio/download/>.
- You will need to use R Studio to solve the problem set.

### Download

from Canvas  $\Rightarrow$  Assignments

- data\_ps1.csv;
- template\_ps1.R .

### Submission

- Open the template\_ps1.R file that we provided on Canvas  $\Rightarrow$  Assignments.
- All your solutions and code need to be saved in a single file named template\_ps1\_YOURFIRSTANDLASTNAME.R file. Please use the template\_ps1.R provided in Canvas to structure your answers.
- Any file that is not an .R will not be accepted, and the grade for this exercise will be zero.**
- Please submit your code on **Gradescope**.
- Please follow the policy stated in the syllabus about academic integrity.

## Useful readings

In addition to the lectures provided by the instructor and the TAs, you might find the following readings useful:

- Chapter 2.3 and “An introduction to statistical learning with applications in R”.

## Question 4

This exercise helps you learn basic commands in R by working with the Forest Fires data set (Dua, D. and Graff, C. (2019). UCI Machine Learning Repository;<sup>2</sup> see: <https://archive.ics.uci.edu/ml/machine-learning-repository>). This data set is available on Canvas ⇒ Assignments.

1. Download the dataset from Canvas and open it using the command “read.csv”.
2. Open the data and report how many columns and rows the dataset has;
3. See the names of the variables (see online the command “names”);
4. Run a linear regression with “area” as a function of “temp” using the command “lm”;
5. Report the summary of your results (see online the command “summary”)
6. Plot a scatter plot of the regression (Hint: use `abline()` to draw the regression line)
7. Write down the interpretation of the coefficients as a comment in your .R script (Hint: see template file).

Please write all your answer and code in `template_04.R` file and submit that file on Gradescope as described in the “Submission” section.

<https://tutorcs.com>

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<sup>2</sup>Dua, D. and Graff, C. (2019). UCI Machine Learning Repository [<http://archive.ics.uci.edu/ml>]. Irvine, CA: University of California, School of Information and Computer Science.