INF

July 1, 2021

## 1 Importing p

[78]:
```
#importing packages
import statsmodels.api as sm
from statsmodels.tsa.stattools import adfuller
import pandas as pd
import numpy as np
import statsmodels.formula.api as smf
from sklearn import linear_model
import matplotlib.pyplot as plt
from statsmodels.tsa.arima.model import ARIMA
```



## 2 Reading Excel file saved in hard drive

[79]:
```
#reading the file
df = pd.read_excel("C:\\Users\\Aluck\\OneDrive\\fisher_update.xlsx")
df
```

[79]:

| | DATE | P | R |
|---|---|---|---|
| 0 | 1969-12-01 | 17.1 | 6.65 |
| 1 | 1970-03-01 | 17.3 | 7.15 |
| 2 | 1970-06-01 | 17.5 | 8.70 |
| 3 | 1970-09-01 | 17.6 | 6.35 |
| 4 | 1970-12-01 | 17.9 | 6.50 |
| .. | ... | ... | ... |
| 166 | 2011-06-01 | 178.3 | 4.99 |
| 167 | 2011-09-01 | 179.4 | 4.81 |
| 168 | 2011-12-01 | 179.4 | 4.51 |
| 169 | 2012-03-01 | 179.5 | 4.44 |
| 170 | 2012-06-01 | 180.4 | 3.49 |

[171 rows x 3 columns]

## 3 Calculating annual inflation from quarterly CPI



[80]:
```
#computing the inflation rate
df['INF'] = 400*np.log(df['P']/df['P'].shift(1))
df.head()
```

[80]:

| | DATE | P |
|---|---|---|
| 0 | 1969-12-01 | 17.1 |
| 1 | 1970-03-01 | 17.3 |
| 2 | 1970-06-01 | 17.5 |
| 3 | 1970-09-01 | 17.6 |
| 4 | 1970-12-01 | 17.9 |


[81]:
```
df.tail()
```

[81]:

| | DATE | P | R | INF |
|---|---|---|---|---|
| 166 | 2011-06-01 | 178.3 | 4.99 | 3.605658 |
| 167 | 2011-09-01 | 179.4 | 4.81 | 2.460170 |
| 168 | 2011-12-01 | 179.4 | 4.51 | 0.000000 |
| 169 | 2012-03-01 | 179.5 | 4.44 | 0.222903 |
| 170 | 2012-06-01 | 180.4 | 3.49 | 2.000560 |



## 4 Selecting sample data from row 57:Qtr 1 1984 to row 170: Qtr 2012



[82]:
```
#Selecting the sample from
dta =df.iloc[57:170]
dta
```



[82]:

| | DATE | P | R | INF |
|---|---|---|---|---|
| 57 | 1984-03-01 | 65.2 | 13.57 | -1.836269 |
| 58 | 1984-06-01 | 65.4 | 12.81 | 1.225116 |
| 59 | 1984-09-01 | 66.2 | 10.53 | 4.863282 |
| 60 | 1984-12-01 | 67.2 | 12.34 | 5.997114 |
| 61 | 1985-03-01 | 68.1 | 15.29 | 5.321586 |
| .. | ... | ... | ... | ... |
| 165 | 2011-03-01 | 176.7 | 4.92 | 6.159232 |
| 166 | 2011-06-01 | 178.3 | 4.99 | 3.605658 |
| 167 | 2011-09-01 | 179.4 | 4.81 | 2.460170 |
| 168 | 2011-12-01 | 179.4 | 4.51 | 0.000000 |
| 169 | 2012-03-01 | 179.5 | 4.44 | 0.222903 |

[113 rows x 4 columns]

## 5 Plotting the time series: Inflation



[83]:
```
#plotting the series
plt.plot(dta['INF'],label='Inflation')
plt.legend(loc='b                rge')
plt.show()
```




[84]:
```
dt = dta['INF'].dropna()
dt
```



[84]:
```
57    -1.836269
58     1.225116
59     4.863282
60     5.997114
61     5.321586
          ...
165    6.159232
166    3.605658
167    2.460170
168    0.000000
169    0.222903
Name: INF, Length: 113, dtype: float64
```

## 6 ADF test of stationarity and unit root



[85]:
```
#ADF Test under (i) Constant (no linear trend)
X = dt.values
result = adfuller(X, maxlag=None, regression='c', autolag='BIC', store=False,␣
 ↪regresults=Fals
print(f'ADF Statis
print(f'n_lags: {
print(f'p-value:
for key, value in
        print('\t
if result[0] < re
        print("Reject Ho_ Time Series is then stationary")
else:
        print("Failed to Reject Ho_ Time Series is then non-stationary")
```



```
ADF Statistic: -3.820525408886768
n_lags: 0.0027041579302507688
p-value: 0.0027041579302507688
        1%:-3.491
        5%:-2.888
        10%:-2.581
Reject Ho_ Time Series is then stationary
```

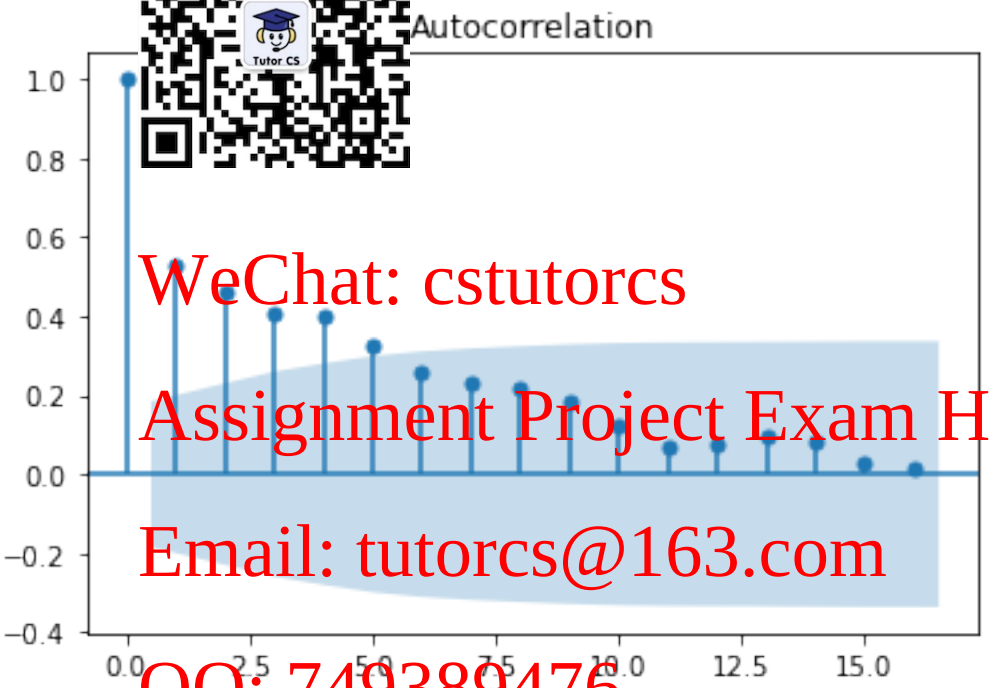
[86]:
```
# ADF test under (ii) Constant , Linear trend
X = dt.values
result = adfuller(X, maxlag=None, regression='ct', autolag='BIC', store=False,␣
 ↪regresults=False)
print(f'ADF Statistic: {result[0]}')
print(f'n_lags: {result[1]}')
print(f'p-value: {result[1]}')
for key, value in result[4].items():
        print('\t%s:%.3f'%(key,value))
if result[0] < result [4] ["1%"]:
        print("Reject Ho_ Time Series is then stationary")
else:
        print("Failed to Reject Ho_ Time Series is then non-stationary")
```

```
ADF Statistic: -4.707778614704212
n_lags: 0.0006768517326375754
p-value: 0.0006768517326375754
        1%:-4.043
        5%:-3.451
        10%:-3.151
Reject Ho_ Time Series is then stationary
```

## 7 Correlogram: ACF and PACF



[87]:
```
#running ACF and PACF
sm.graphics.tsa.plot_acf(dt.values.squeeze(),lags=16)
sm.graphics.tsa.pl                    .squeeze(),lags=16)
plt.show()
```






[88]:
```
# Generating the Q tables
import numpy as np
r,q,p = sm.tsa.acf(dt.values.squeeze(), qstat=True)
data = np.c_[range(1,41), r[1:], q, p]
table = pd.DataFrame(data, columns=['lag', "AC", "Q", "Prob(>Q)"])
print (table.set_index('lag'))
```

```
            AC          Q      Prob(>Q)
lag
1.0   0.526177   32.123408  1.446835e-08
2.0   0.461362   57.042746  4.105113e-13
3.0   0.405887   76.505017  1.723856e-16
4.0   0.398961   95.481302  9.003058e-20
5.0   0.328190  108.441254  8.746813e-22
6.0   0.260200  116.663817  8.174178e-23
7.0   0.228314  123.054314  1.769702e-23
8.0   0.215547  128.804379  5.005157e-24
9.0   0.182584  132.969880  2.903758e-24
10.0  0.124870  134.937112  4.582456e-24
11.0  0.070530  135.570871  1.293531e-23
12.0  0.073664  136.269055  3.386720e-23
13.0  0.095310  137.449513  6.819758e-23
14.0  0.080457  138.299210  1.544904e-22
15.0  0.031527  138.431012  4.692586e-22
16.0  0.015339  138.462534  1.440337e-21
```

```
17.0  0.073788  139.199540  3.120838e-21
18.0 -0.005333  139.203436  9.118893e-21
19.0  0.073269  139.945566  1.869803e-20
20.0  0.097139  141.264071  2.913081e-20
21.0  0.030206  141.[illegible]  7.[illegible]e-20
22.0  0.035012  14[illegible]  e-19
23.0 -0.040731  14[illegible]  e-19
24.0 -0.035400  14[illegible]  e-18
25.0 -0.066250  14[illegible]  e-18
26.0 -0.123986  14[illegible]  e-18
27.0 -0.119284  14[illegible]  e-18
28.0 -0.061785  14[illegible]  e-18
29.0 -0.104202  149.348204  3.792051e-18
30.0 -0.047342  149.699109  7.583105e-18
31.0 -0.087683  150.917517  1.051183e-17
32.0 -0.072328  151.786788  1.681406e-17
33.0 -0.163640  156.106553  6.505437e-18
34.0 -0.122372  158.569818  5.366036e-18
35.0 -0.106180  160.448125  5.569460e-18
36.0 -0.119128  162.843188  4.683319e-18
37.0 -0.122013  165.388711  3.698133e-18
38.0 -0.106321  167.347329  3.670295e-18
39.0  0.006632  167.[illegible]  7.787[illegible]e-18
40.0 -0.083014  168.581796  1.015628e-17
```

```
C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:657:
FutureWarning: The default number of lags is changing from 40 tomin(int(10 *
np.log10(nobs)), nobs - 1) after 0.12is released. Set the number of lags to an
integer to  silence this warning.
  warnings.warn(
C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:667:
FutureWarning: fft=True will become the default after the release of the 0.12
release of statsmodels. To suppress this warning, explicitly set fft=False.
  warnings.warn(
```

#ARMA(1,1)

[89]:
```
arima=ARIMA(dt.values,exog=None, order=(1, 0, 1), seasonal_order=(0, 0, 0, 0),␣
 ↪trend=None, enforce_stationarity=True, enforce_invertibility=True,␣
 ↪concentrate_scale=True)
results = arima.fit()
print(results.summary())
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  113
Model:                 ARIMA(1, 0, 1)   Log Likelihood                -261.473
Date:                Thu, 01 Jul 2021   AIC                            530.945
Time:                        23:24:09   BIC                            541.855
```

```
Sample:                                 0   HQIC                           535.372
                                      - 113 Scale                          5.959
Covariance Type:                        opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.3
ar.L1          0.8                   12.806      0.000       0.741       1.009
ma.L1         -0.5                   -5.172      0.000      -0.708      -0.319
==============================================================================
===
Ljung-Box (L1) (Q):                   0.04   Jarque-Bera (JB):
80.62
Prob(Q):                              0.85   Prob(JB):
0.00
Heteroskedasticity (H):               0.32   Skew:
0.63
Prob(H) (two-sided):                  0.05   Kurtosis:
6.94
==============================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
```

#ARMA(2,0)

[90]:
```
arima_1=ARIMA(dt.values,exog=None, order=(2, 0, 0), seasonal_order=(0, 0, 0, 0),␣
 ↪trend=None, enforce_stationarity=True, enforce_invertibility=True,␣
 ↪concentrate_scale=True)
results_1 = arima_1.fit()
print(results_1.summary())
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  113
Model:                 ARIMA(2, 0, 0)   Log Likelihood                -262.721
Date:                Thu, 01 Jul 2021   AIC                            533.442
Time:                        23:24:09   BIC                            544.352
Sample:                             0   HQIC                           537.869
                                  - 113   Scale                            6.096
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.3784      0.701      4.820      0.000       2.005       4.752
ar.L1          0.4007      0.061      6.555      0.000       0.281       0.520
```

```
ar.L2          0.2691      0.093      2.893      0.004       0.087       0.451
==============================================================================
===
Ljung-Box (L1) (Q):                   0.22   Jarque-Bera (JB):
65.00
Prob(Q):                              0.64   Prob(JB):
0.00
Heteroskedasticity                    0.52   Skew:
0.60
Prob(H) (two-sided                    0.05   Kurtosis:
6.52
==============================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
```



ARMA (1,2)

[91]:
```
arima_2=ARIMA(dt.values,exog=None, order=(1, 0, 2), seasonal_order=(0, 0, 0, 0),␣
 ↪concentrate_scale=True,trend=None, enforce_stationarity=True,␣
 ↪enforce_invertibility=True,)
results_2 = arima_2.fit()
print(results_2.summary())
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  113
Model:                 ARIMA(1, 0, 2)   Log Likelihood                -261.428
Date:                Thu, 01 Jul 2021   AIC                            532.855
Time:                        23:24:02   BIC                            546.492
Sample:                             0   HQIC                           538.389
                                  - 113   Scale                            5.954
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.3129      0.824      4.021      0.000       1.698       4.928
ar.L1          0.8670      0.076     11.333      0.000       0.717       1.017
ma.L1         -0.5203      0.103     -5.047      0.000      -0.722      -0.318
ma.L2          0.0311      0.104      0.300      0.765      -0.172       0.234
==============================================================================
===
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):
76.28
Prob(Q):                              0.96   Prob(JB):
0.00
```

```
Heteroskedasticity (H):                 0.53   Skew:
0.64
Prob(H) (two-sided):                    0.05   Kurtosis:
6.82
===================================================================================
===

Warnings:
[1] Covariance mat                    ng the outer product of gradients (complex-
step).
```

#ARMA(2,2)

[92]:
```
arima_3=ARIMA(dt.values,exog=None, order=(2, 0, 2), seasonal_order=(0, 0, 0, 0),␣
 ↪concentrate_scale=True,trend=None, enforce_stationarity=True,␣
 ↪enforce_invertibility=True)
results_3 = arima_3.fit()
print(results_3.summary())
```

```
                               SARIMAX Results
===================================================================================
Dep. Variable:                          y   No. Observations:                  113
Model:                     ARIMA(2, 0, 2)   Log Likelihood                -261.005
Date:                    Thu, 21 Jul 2021   AIC                           534.010
Time:                            23:24:09   BIC                           550.375
Sample:                                 0   HQIC                          540.651
                                      - 113   Scale                           5.904
Covariance Type:                      opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.2844      0.738      4.449      0.000       1.838       4.731
ar.L1          1.7513      0.331      5.287      0.000       1.102       2.401
ar.L2         -0.7879      0.274     -2.872      0.004      -1.326      -0.250
ma.L1         -1.4184      0.337     -4.212      0.000      -2.078      -0.758
ma.L2          0.5338      0.158      3.374      0.001       0.224       0.844
===================================================================================
===
Ljung-Box (L1) (Q):                     0.00   Jarque-Bera (JB):
64.01
Prob(Q):                                0.99   Prob(JB):
0.00
Heteroskedasticity (H):                 0.56   Skew:
0.60
Prob(H) (two-sided):                    0.08   Kurtosis:
6.49
===================================================================================
===
```

```
Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
```

#ARMA (0,2)

[93]:
```
arima_4=ARIMA(dt.y, order=(0, 0, 2), seasonal_order=(0, 0, 0, 0),␣
 ↪concentrate_sca, enforce_stationarity=True,␣
 ↪enforce_inverti
results_4 = arima.
print(results_4.su
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  113
Model:                 ARIMA(0, 0, 2)   Log Likelihood                -268.884
Date:                Thu, 01 Jul 2021   AIC                            545.769
Time:                        23:24:09   BIC                            556.678
Sample:                             0   HQIC                           550.196
                                - 113   Scale                            6.812
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.5175      0.429      8.193      0.000       2.676       4.359
ma.L1          0.4118      0.070      5.911      0.000       0.275       0.548
ma.L2          0.2178      0.128      2.171      0.030       0.027       0.529
===================================================================================
===
Ljung-Box (L1) (Q):                   0.17   Jarque-Bera (JB):
40.10
Prob(Q):                              0.68   Prob(JB):
0.00
Heteroskedasticity (H):               0.37   Skew:
0.68
Prob(H) (two-sided):                  0.00   Kurtosis:
5.58
===================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
```

[94]:
```
name= ['ARMA_1_1','ARMA_2_0','ARMA_1_2','ARMA_2_2','ARMA_0_2']
aic =[results.aic,results_1.aic, results_2.aic,results_3.aic,results_4.aic]
ret= (name,aic)
```

```
ret
```

[94]:
```
(['ARMA_1_1', 'ARMA_2_0', 'ARMA_1_2', 'ARMA_2_2', 'ARMA_0_2'],
 [530.9451650125998,
  533.4422760944168,
  532.8552441569485,
  534.0102545166145,
  545.7687072920195])
```

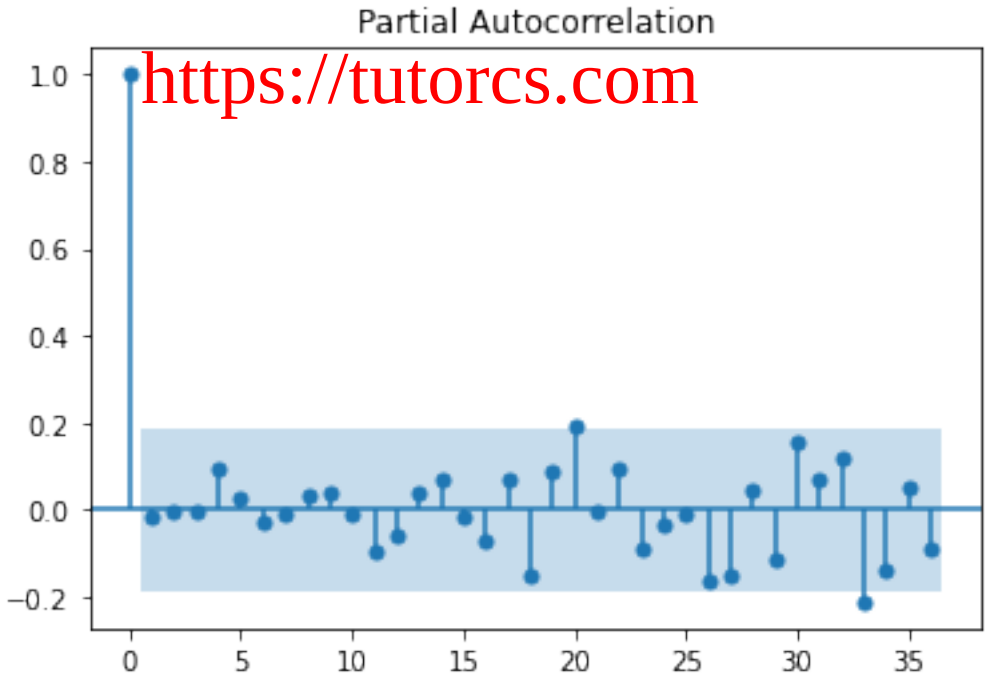
[95]:
```
name= ['ARMA_1_1','ARMA_2_0','ARMA_1_2','ARMA_2_2','ARMA_0_2']
bic =[results.bic,results_1.bic,results_2.bic,results_3.bic,results_4.bic]
ret_1= (name,bic)
ret_1
```

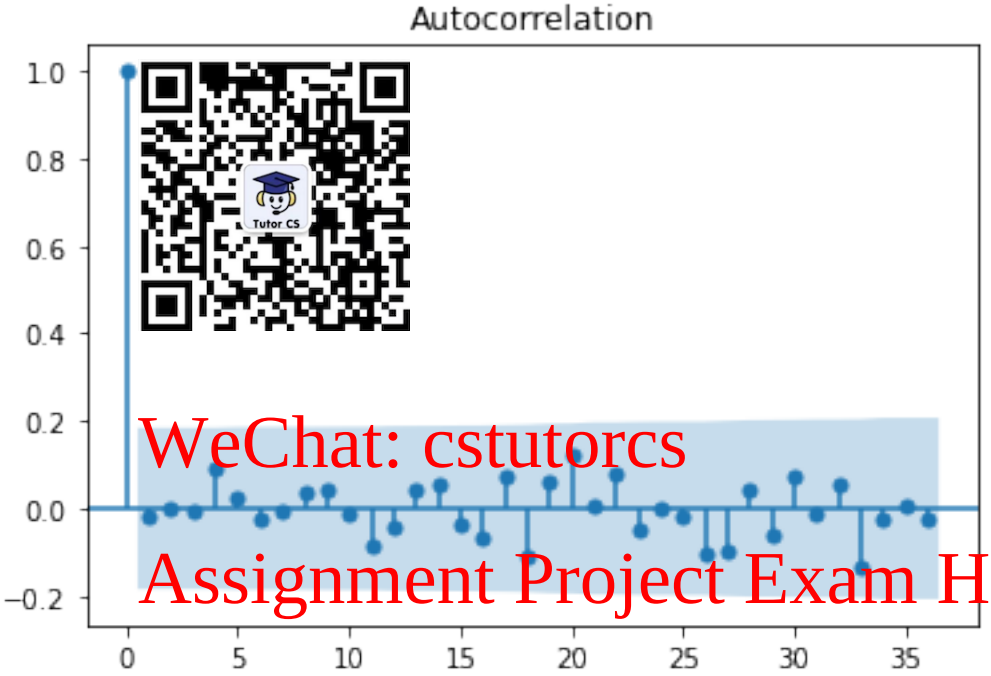
[95]:
```
(['ARMA_1_1', 'ARMA_2_0', 'ARMA_1_2', 'ARMA_2_2', 'ARMA_0_2'],
 [541.8547162874492,
  544.3518273692656,
  546.4921832505099,
  550.3745814288889,
  556.6782585668686])
```

## 8 Diagnostic tests of ARMA (1,1)

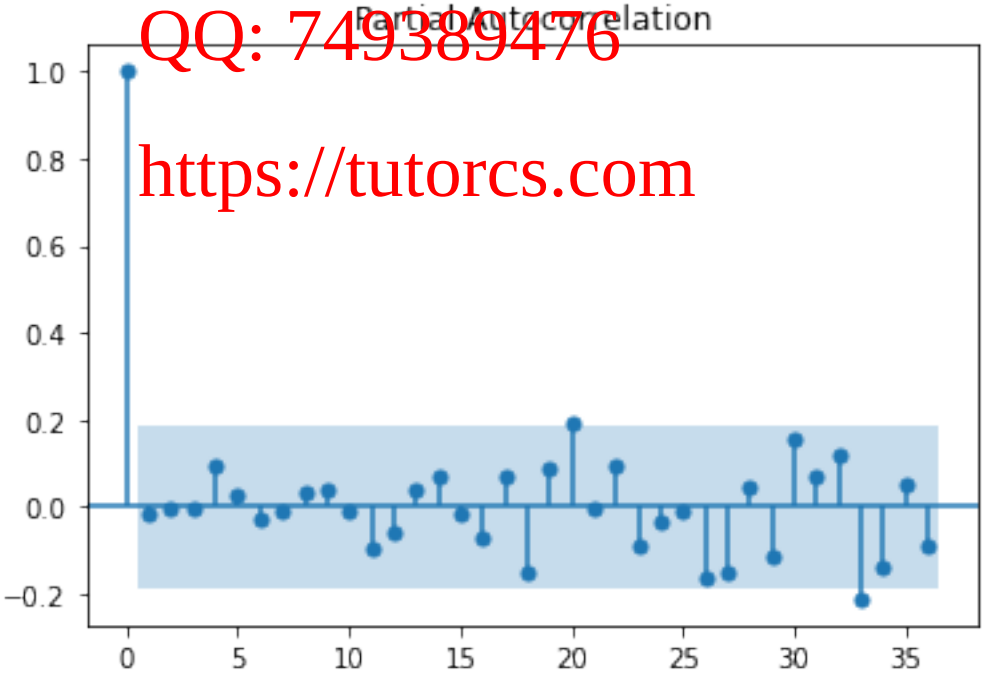
[96]:
```
dtr = results.resid
sm.graphics.tsa.plot_acf(dtr.squeeze(),lags=36)
sm.graphics.tsa.plot_pacf(dtr.squeeze(),lags=36)
```

[96]:







[97]:
```
from scipy import stats
stats.describe(dtr)
```



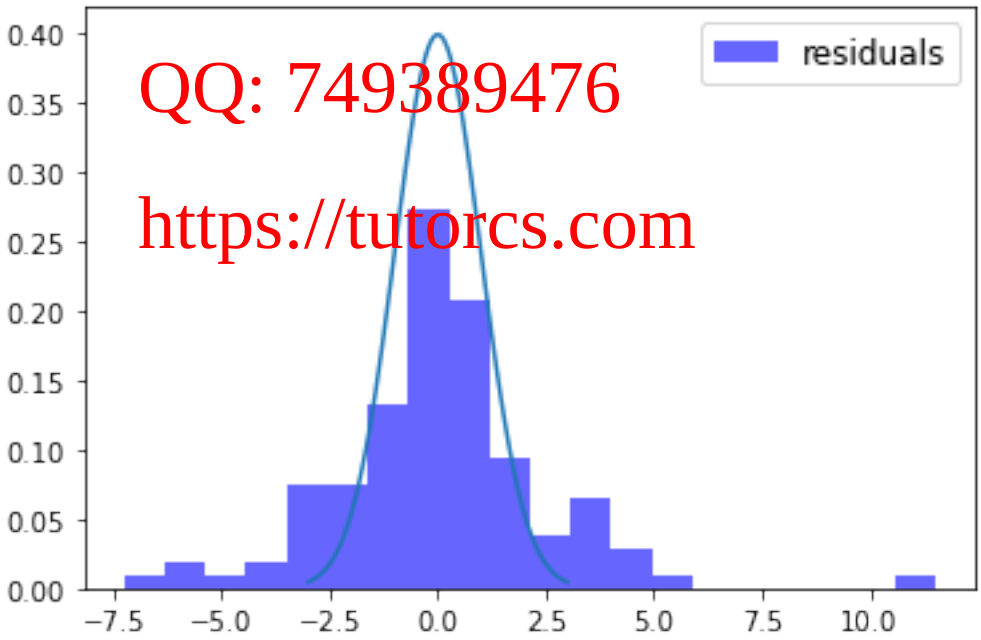
[97]:
```
DescribeResult(nobs=113, minmax=(-7.254783393877607, 11.5183532221989),
mean=0.047921103437431[illegible]6.098915686575292,
skewness=0.587362[illegible]s=3.853285721150856)
```

[98]:
```
JB_resid= stats.ja[illegible]
JB_resid
```

[98]:
```
Jarque_beraResult([illegible]146239956, pvalue=0.0)
```

[99]:
```
#Plot histogram for residuals
import math
plt.hist(dtr,bins=20,label='residuals', density=True, alpha=0.6, color='b')
plt.legend(loc='best', fontsize='large')
#plotting the normal distribution curve
mu = 0
variance = 1
sigma = math.sqrt(variance)
x = np.linspace(mu - 3*sigma, mu + 3*sigma, 100)
plt.plot(x, stats.norm.pdf(x, mu, sigma))
plt.show()
```





## 9 BDS



[100]:
```
#computing the standardised residuals as residuals from ARMA(1,1) divided by std
 ↪error of the model
import statistics
var= statistics.va                sid)
se= var**0.5
std_res=results.re
```

[101]:
```
#Computing the BD
import statsmodels                stat
bds = stat.bds(st                 psilon=None, distance = 1.5)
print('bds_stat, pvalue:{}'.format(bds))
```

```
bds_stat, pvalue:(array(1.74252253), array(0.08141705))
```



## 10 Forecasting



[102]:
```
from statsmodels.tsa.arima_model import ARIMA
```

In and Out-of-sample forecast (data sets start from 1 to 113 + h=1)



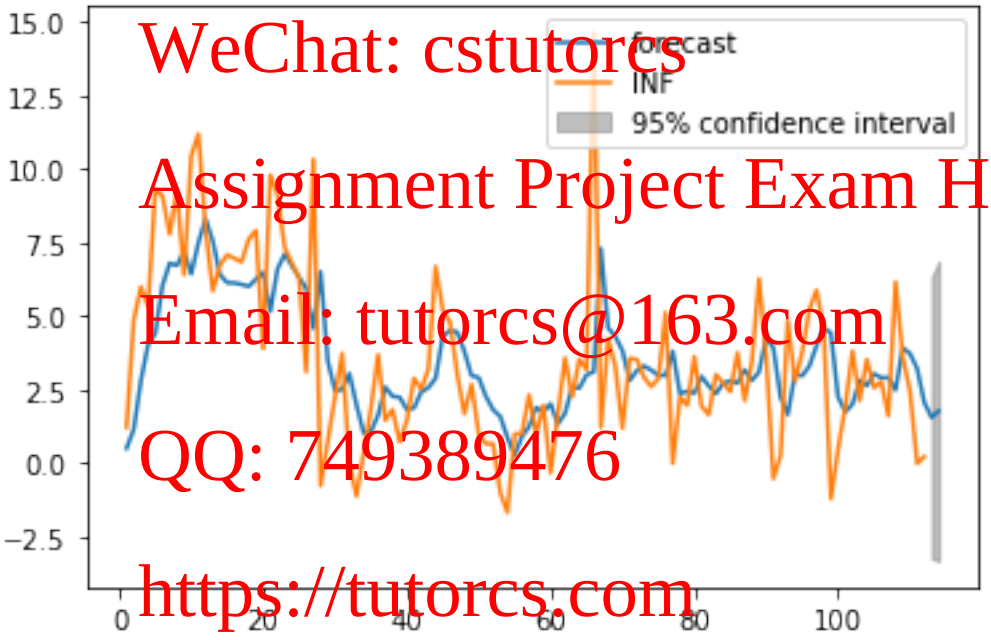
[103]:
```
#Static forecast
model = ARIMA(endog=dt.dropna(), order=(1, 0, 1))
results = model.fit()
results.plot_predict(1,114, dynamic=False)
```

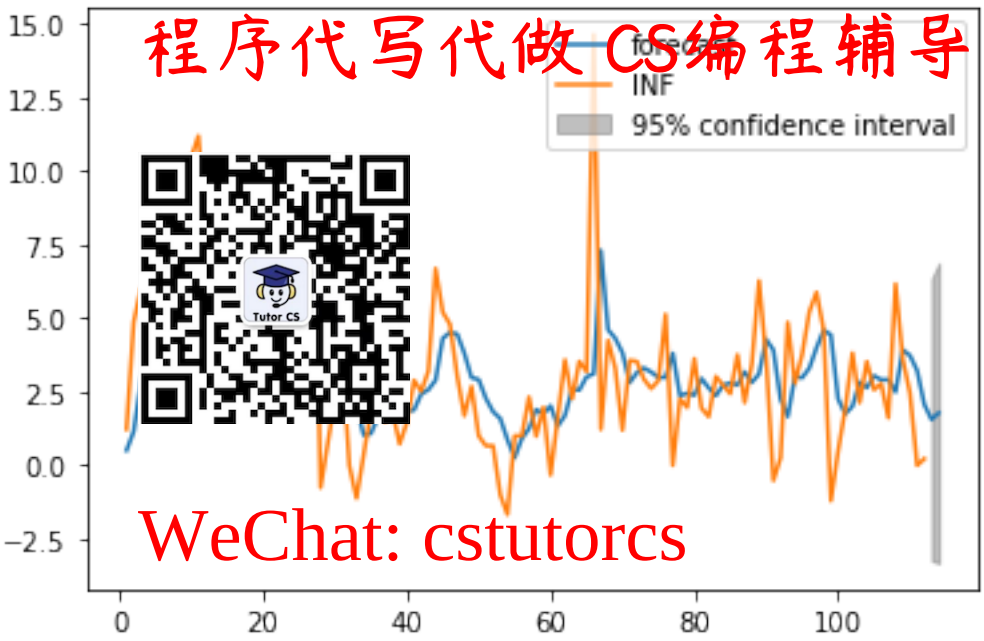


```
C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\arima_model.py:472:
FutureWarning:
statsmodels.tsa.arima_model.ARMA and statsmodels.tsa.arima_model.ARIMA have
been deprecated in favor of statsmodels.tsa.arima.model.ARIMA (note the .
between arima and model) and
statsmodels.tsa.SARIMAX. These will be removed after the 0.12 release.

statsmodels.tsa.arima.model.ARIMA makes use of the statespace framework and
is both well tested and maintained.

To silence this warning and continue using ARMA and ARIMA until they are
removed, use:

import warnings
warnings.filterwarnings('ignore', 'statsmodels.tsa.arima_model.ARMA',
                        FutureWarning)
warnings.filterwarnings('ignore', 'statsmodels.tsa.arima_model.ARIMA',
                        FutureWarning)

  warnings.warn(ARIMA_DEPRECATION_WARN, FutureWarning)
C:\Users\rluck\anaconda3\lib\site-
```

```
packages\statsmodels\tsa\base\tsa_model.py:578: ValueWarning: An unsupported
index was provided and will be ignored when e.g. forecasting.
  warnings.warn('An unsupported index was provided and will be'
C:\Users\rluck\anaconda3\lib\site-
packages\statsmodels\tsa\base\tsa_model.py:376: ValueWarning: No supported index
is available. Prediction will be given with an integer index beginning
at `start`.
  warnings.warn('No supported index is available.'
C:\Users\rluck\anaconda3\lib\site-
packages\statsmodels\tsa\base\tsa_model.py:376: ValueWarning: No supported index
is available. Prediction will be given with an integer index beginning
at `start`.
  warnings.warn('No supported index is available.'
```

[103]:





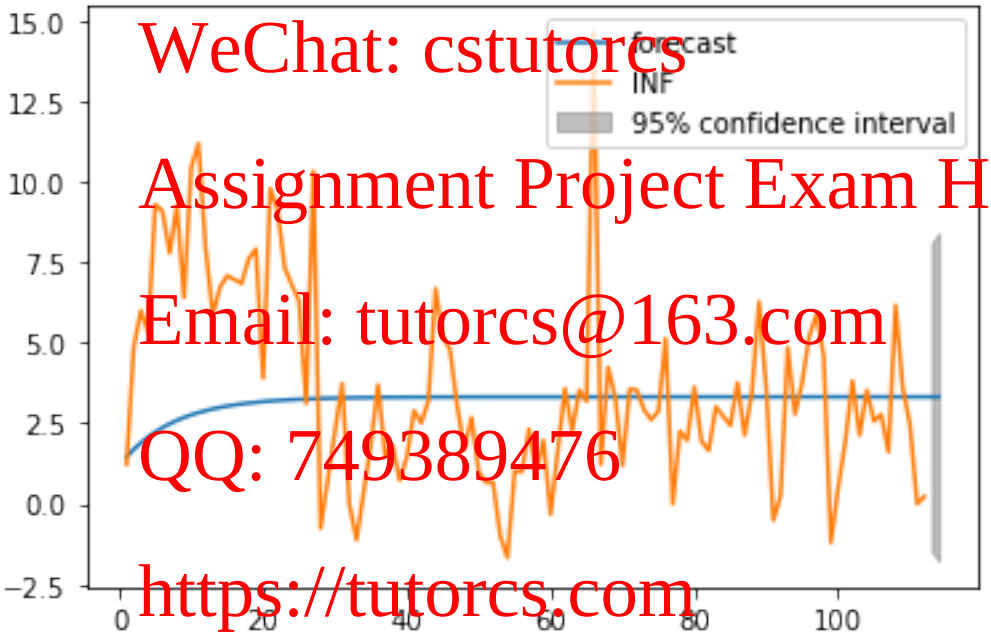


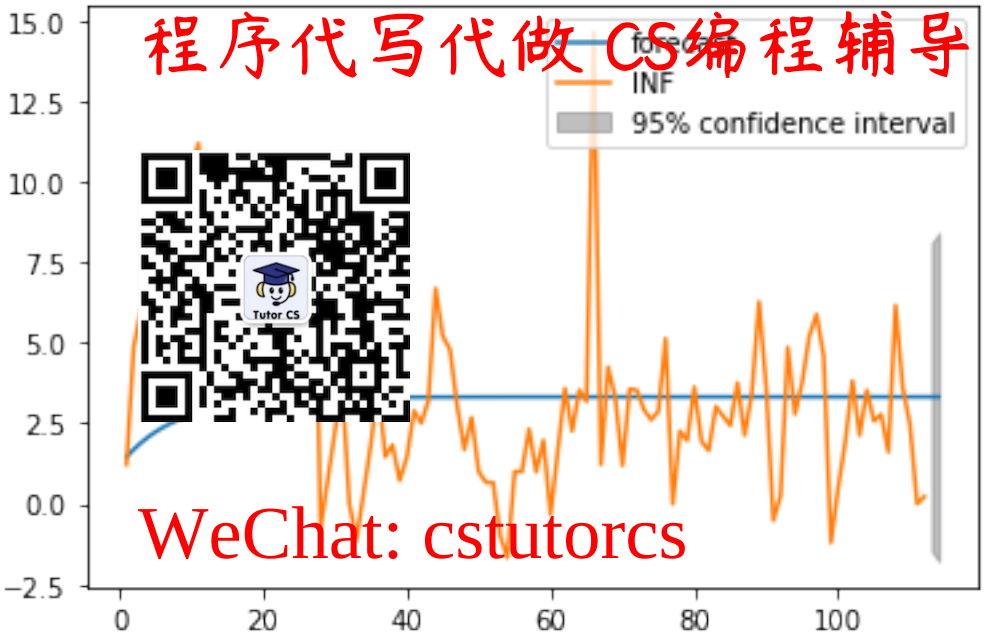

[104]:
```
#Dynamic forecast
model = ARIMA(endog=dt, order=(1, 0, 1))
results = model.fit()
results.plot_predict(1, 114, dynamic=True)
```

```
C:\Users\rluck\anaconda3\lib\site-packages\statsmodels\tsa\arima_model.py:472:
FutureWarning:
statsmodels.tsa.arima_model.ARMA and statsmodels.tsa.arima_model.ARIMA have
been deprecated in favor of statsmodels.tsa.arima.model.ARIMA (note the .
between arima and model) and
statsmodels.tsa.SARIMAX. These will be removed after the 0.12 release.

statsmodels.tsa.arima.model.ARIMA makes use of the statespace framework and
is both well tested and maintained.

To silence this warning and continue using ARMA and ARIMA until they are
removed, use:

import warnings
warnings.filterwarnings('ignore', 'statsmodels.tsa.arima_model.ARMA',
                        FutureWarning)
warnings.filterwarnings('ignore', 'statsmodels.tsa.arima_model.ARIMA',
                        FutureWarning)

  warnings.warn(ARIMA_DEPRECATION_WARN, FutureWarning)
C:\Users\rluck\anaconda3\lib\site-
```

```
packages\statsmodels\tsa\base\tsa_model.py:578: ValueWarning: An unsupported
index was provided and will be ignored when e.g. forecasting.
  warnings.warn('An unsupported index was provided and will be'
C:\Users\rluck\anaconda3\lib\site-
packages\statsmodels\tsa\base\tsa_model.py:376: ValueWarning: No supported index
is available. Prediction will be given with an integer index beginning
at `start`.
  warnings.warn('No supported index is available.'
C:\Users\rluck\anaconda3\lib\site-
packages\statsmodels\tsa\base\tsa_model.py:376: ValueWarning: No supported index
is available. Prediction will be given with an integer index beginning
at `start`.
  warnings.warn('No supported index is available.'
```

[104]:








[105]:
```
from sklearn.metrics import mean_squared_error
from sklearn.metrics import mean_squared_error
pred_s= results.predict(1,113,dynamic =False)
stats_s= mean_squared_error(dt,pred_s)
print('mean squared error for static forecast:{}'.format(stats_s))
pred_d= results.predict(1,113,dynamic =True)
stats_d= mean_squared_error(dt,pred_d)
print('mean squared error for dynamic forecast:{}'.format(stats_d))
```

```
mean squared error for static forecast:2.4633509757276286
mean squared error for dynamic forecast:9.95112632140072

C:\Users\rluck\anaconda3\lib\site-
packages\statsmodels\tsa\base\tsa_model.py:376: ValueWarning: No supported index
is available. Prediction results will be given with an integer index beginning
at `start`.
  warnings.warn('No supported index is available.'
C:\Users\rluck\anaconda3\lib\site-
packages\statsmodels\tsa\base\tsa_model.py:376: ValueWarning: No supported index
is available. Prediction results will be given with an integer index beginning
at `start`.
  warnings.warn('No supported index is available.'
```

[ ]: