

# ECON7350: Applied Econometrics for Macroeconomics and Finance

## Tutorial 8: Modelling Volatility - I

At the end of this tutorial you should be able to:

- use R to infer the presence of heteroscedasticity using the Breusch-Pagan test;
- construct an adequate set of models with possible GARCH errors;
- use R to estimate volatilities based on models with GARCH errors.

## Assignment Project Exam Help Problems with Solutions

Consider the daily share prices of Commonwealth Bank (CWB) for the period 5 September 1996—30 August 2006 ( $T = 2605$ ) in the data file `cwb.csv`. Let  $\{y_t\}$  denote the process of share prices.

1. Plot the *share prices* ( $y_t$ ) and comment on the possible features of the DGP.

---

**Solution** For this tutorial, we load the following useful packages.

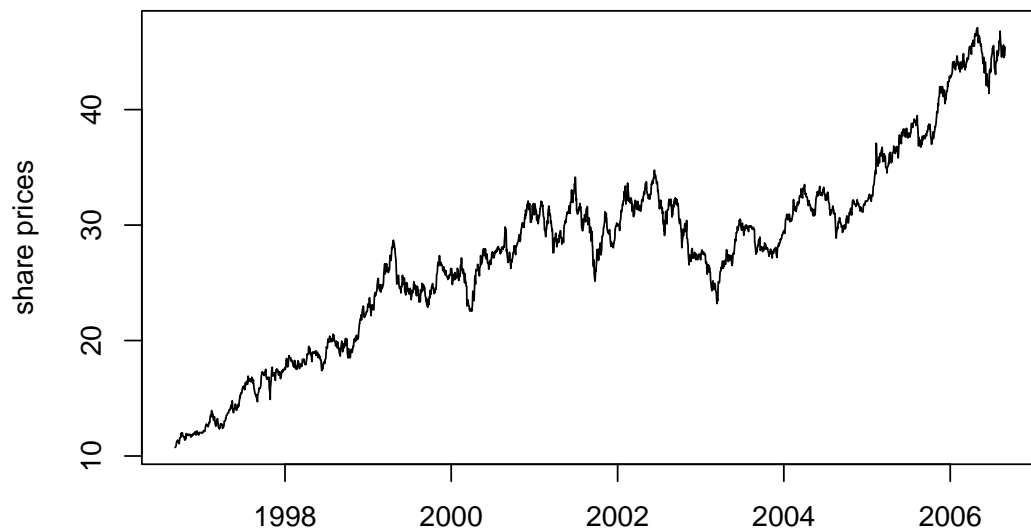
```
library(forecast)
library(dplyr)
library(rugarch)
```

The package `rugarch` provides functions for working with conditional heteroscedasticity specifications. In this tutorial we will use the functions `ugarchspec` and `ugarchfit` to estimate volatilities based on standard GARCH models.

Next, load the data and extract the variables.

```
mydata <- read.delim("cwb.csv", header = TRUE, sep = ",")

date <- as.Date(mydata$date, format = "%m/%d/%Y")
y <- mydata$y
r <- diff(log(y))
plot(date, y, type = "l", xlab = "", ylab = "share prices")
```



## Assignment Project Exam Help

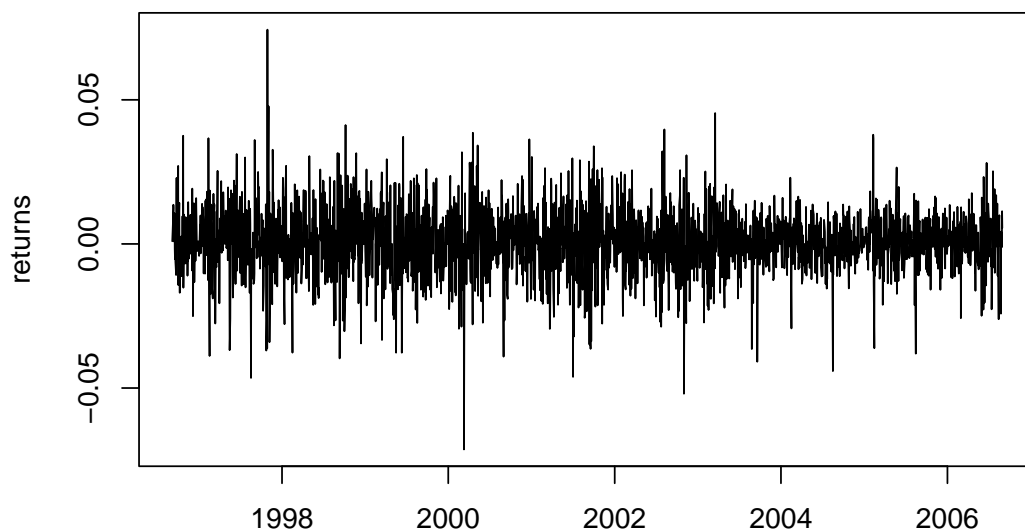
Share prices are clearly trending upwards, being around \$10 in the beginning of the sample and reaching about \$45 towards the end.

<https://tutores.com>

WeChat: cstutorcs

2. Plot the *returns* ( $r_t = \ln y_t - \ln y_{t-1}$ ) and comment on the possible features of the DGP.

```
plot(date[-1], r, type = "l", xlab = "", ylab = "returns")
```



## Assignment Project Exam Help

**Solution**

Returns look stable with possible heteroscedasticity. Observe that if returns are heteroscedastic then so must be the share prices. However, evidence of heteroscedasticity may be less obvious in plots of share prices than in plots of returns.

<https://tutores.com>

**WeChat: cstutorcs**

3. Assuming homoscedasticity, identify an adequate set of ARMA models for  $r_t$ .

**Solution** We use the familiar approach to building an adequate set of homoscedastic ARMA models.

```
ARMA_est <- list()
ic_arma <- matrix( nrow = 4 * 4, ncol = 4 )
colnames(ic_arma) <- c("p", "q", "aic", "bic")
for (p in 0:3)
{
  for (q in 0:3)
  {
    i <- p * 4 + q + 1
    ARMA_est[[i]] <- Arima(r, order = c(p, 0, q))
    ic_arma[i,] <- c(p, q, ARMA_est[[i]]$aic, ARMA_est[[i]]$bic)
  }
}
```

```

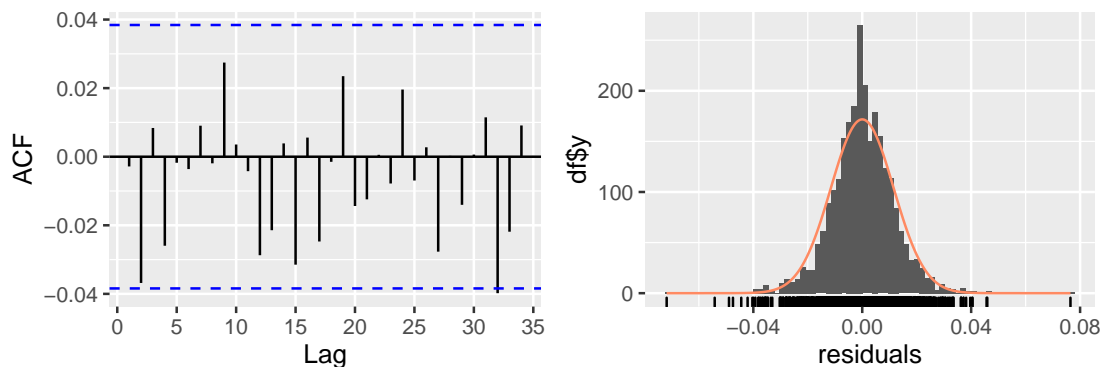
ic_aic_arma <- ic_arma[order(ic_arma[,3]),][1:10,]
ic_bic_arma <- ic_arma[order(ic_arma[,4]),][1:10,]

ic_int_arma <- intersect(as.data.frame(ic_aic_arma),
                        as.data.frame(ic_bic_arma))

adq_set_arma <- as.matrix(arrange(as.data.frame(
                                rbind(ic_int_arma[c(1:3, 6),],
                                ic_bic_arma[2,])), p, q))
adq_idx_arma <- match(data.frame(t(adq_set_arma[, 1:2])),
                    data.frame(t(ic_arma[, 1:2])))

nmods <- length(adq_idx_arma)
for (i in 1:nmods)
{
  checkresiduals(ARMA_est[[adq_idx_arma[i]])]
}

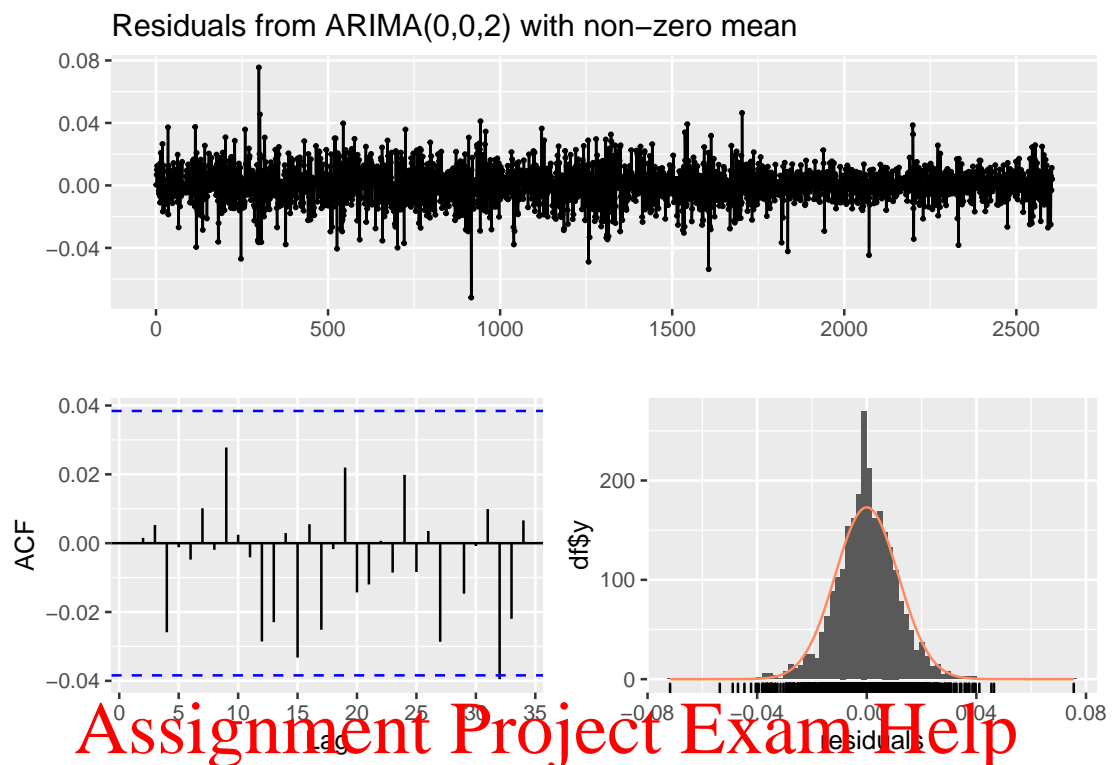
```



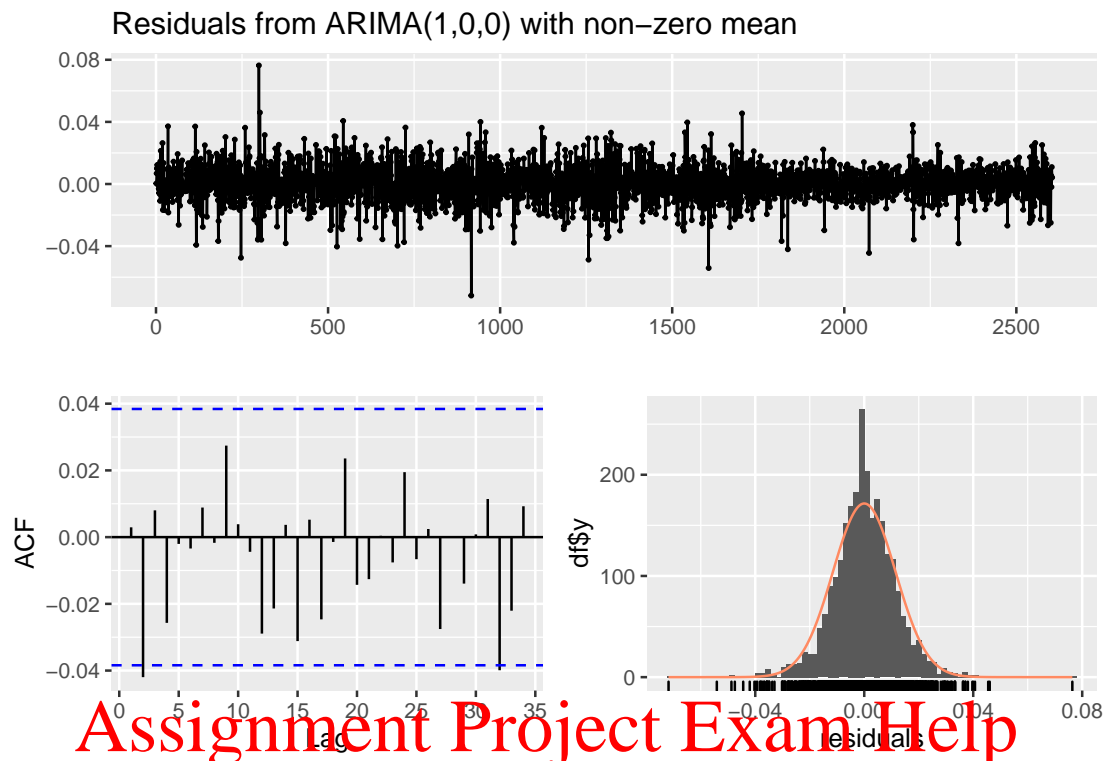
```

##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(0,0,1) with non-zero mean
## Q* = 7.7683, df = 8, p-value = 0.4564
##
## Model df: 2.   Total lags used: 10

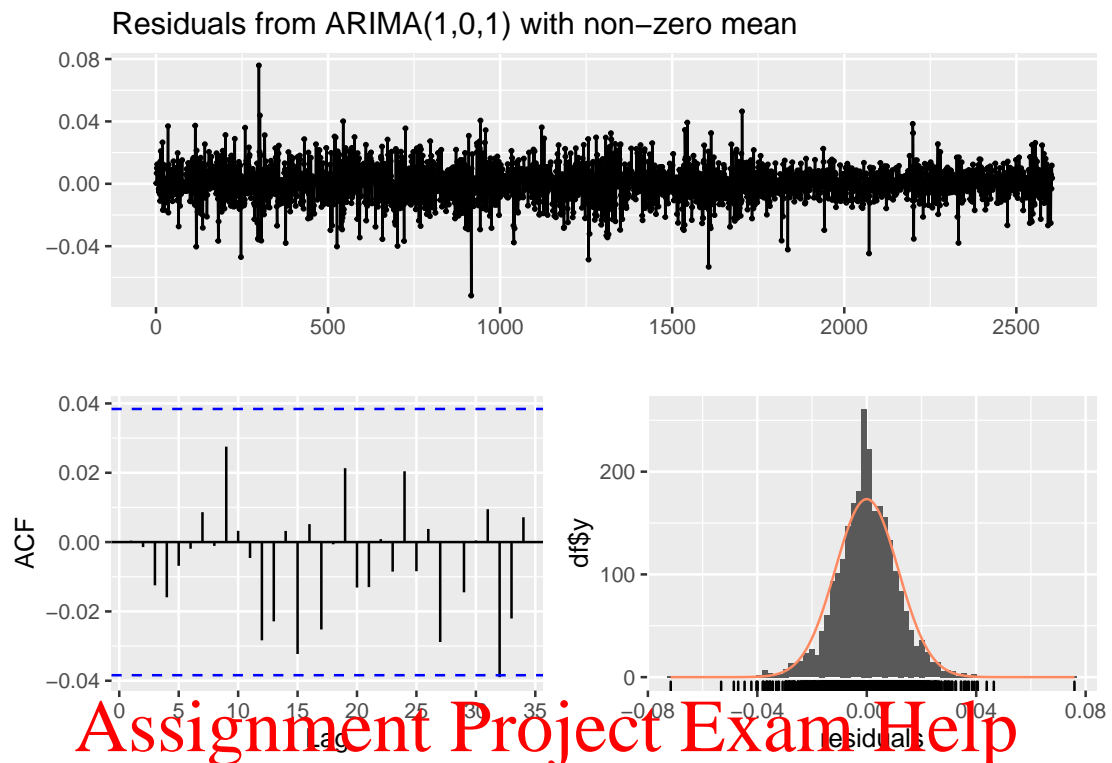
```



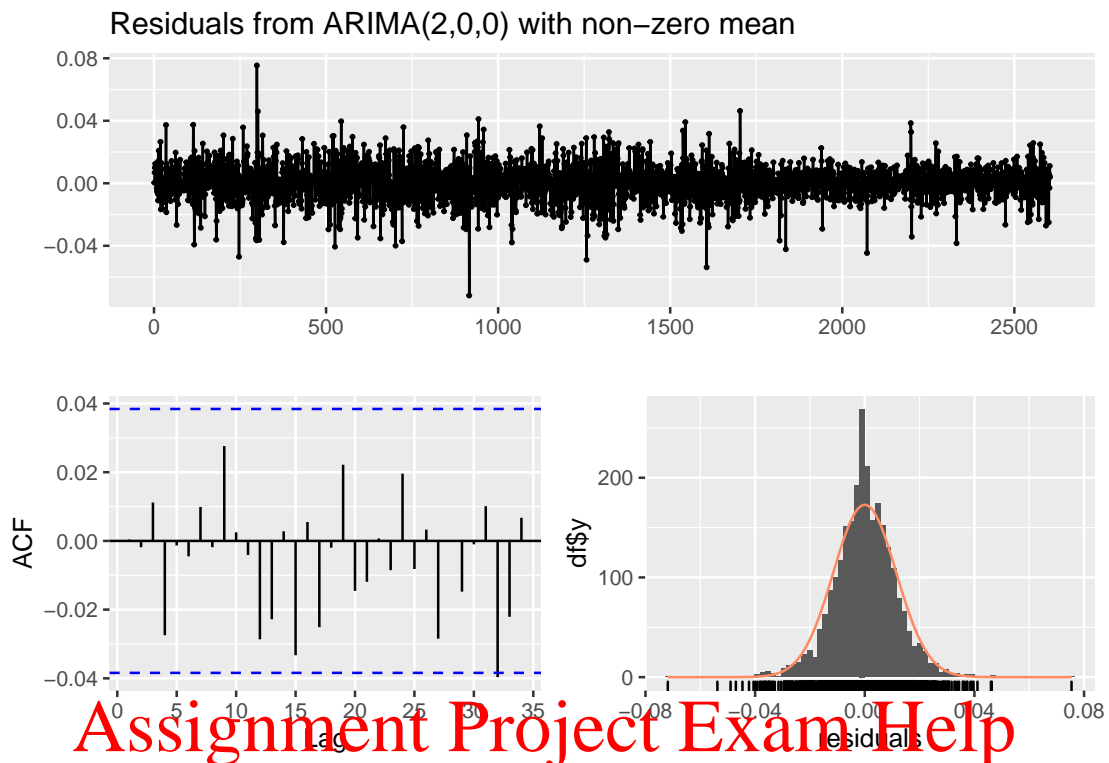
##  
 ## Ljung-Box test  
 ##  
 ## data: Residuals from ARIMA(0,0,2) with non-zero mean  
 ## Q\* = 4.2044, df = 7, p-value = 0.756  
 ##  
 ## Model df: 3. Total lags used: 10



##  
 ## Ljung-Box test  
 ##  
 ## data: Residuals from ARIMA(1,0,0) with non-zero mean  
 ## Q\* = 8.7674, df = 8, p-value = 0.3623  
 ##  
 ## Model df: 2. Total lags used: 10



##  
 ## Ljung-Box test  
 ##  
 ## data: Residuals from ARIMA(1,0,1) with non-zero mean  
 ## Q\* = 3.4143, df = 7, p-value = 0.8442  
 ##  
 ## Model df: 3. Total lags used: 10



```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(2,0,0) with non-zero mean
## Q* = 4.632, df = 7, p-value = 0.7048
##
## Model df: 3.    Total lags used: 10
```

The adequate set consists of five models: MA(1), MA(2), AR(1), AR(2) and ARMA(1, 1).

- 
4. Generate estimated squared residuals for the set of models chosen in Question 3. Plot the squared residuals along with the sample ACFs. Interpret your findings.
- 

**Solution** For each model in the adequate set, extract the estimated residuals using the `resid` function. Then, use `plot` and `acf` to obtain the desired graphs.

```
e2_arma <- list()
for (i in 1:nmods)
{
  e2_arma[[i]] <- resid(ARMA_est[[adq_idx_arma[i]]]) ^ 2
}
```

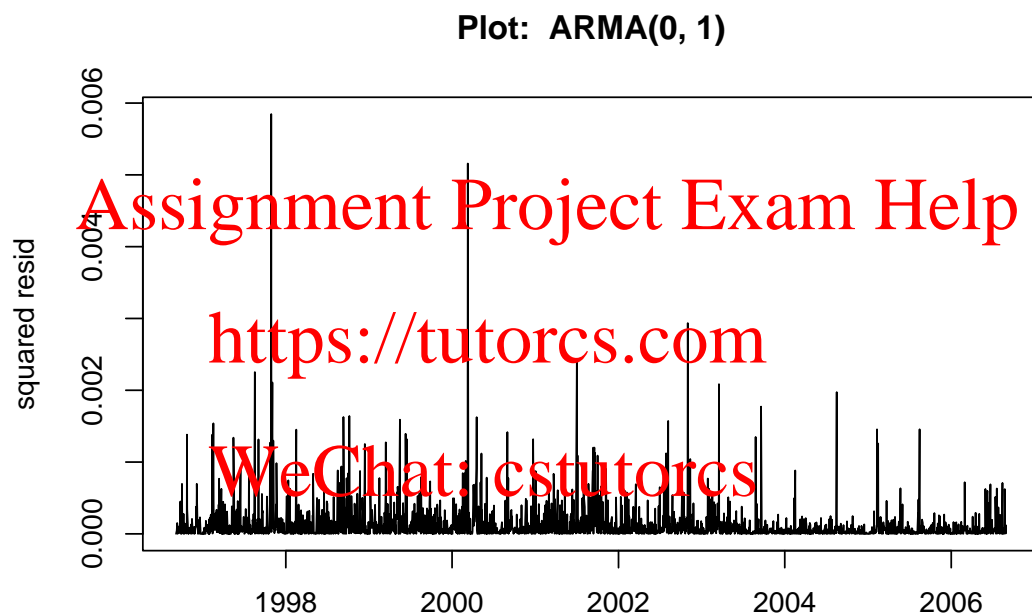


```

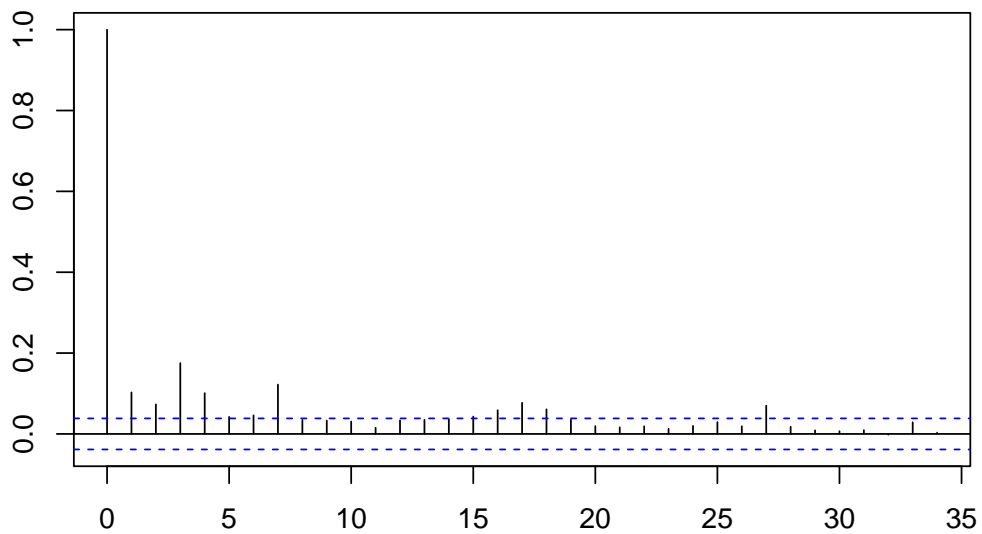
title_p_q <- paste("ARMA(",
                    as.character(adq_set_arma[i, 1]), ", ",
                    as.character(adq_set_arma[i, 2]), ")",
                    sep = "")
plot(date[-1], e2_arma[[i]], type = "l",
      xlab = "", ylab = "squared resid",
      main = paste("Plot: ", title_p_q))

acf(e2_arma[[i]], xlab = "", ylab = "",
     main = paste("SACF: ", title_p_q))
}

```



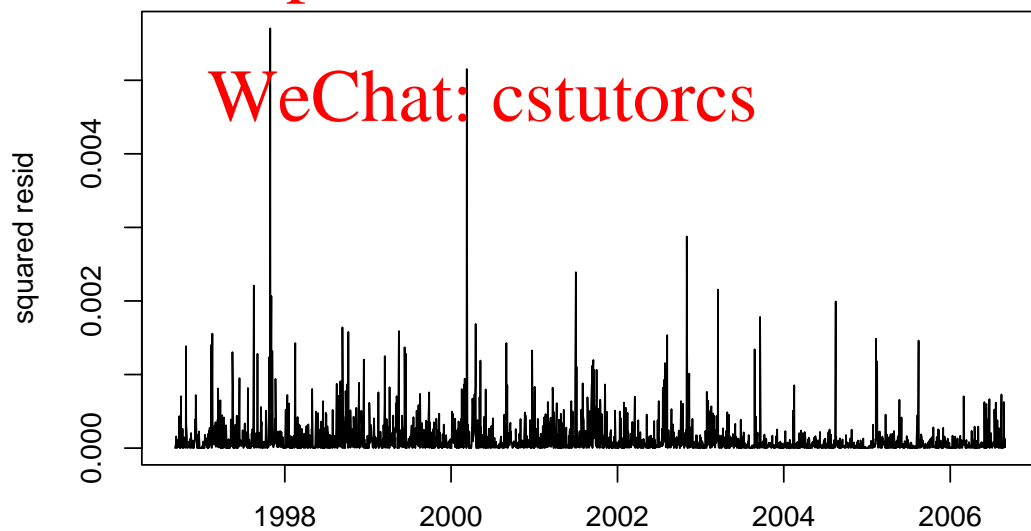
**SACF: ARMA(0, 1)**



Assignment Project Exam Help

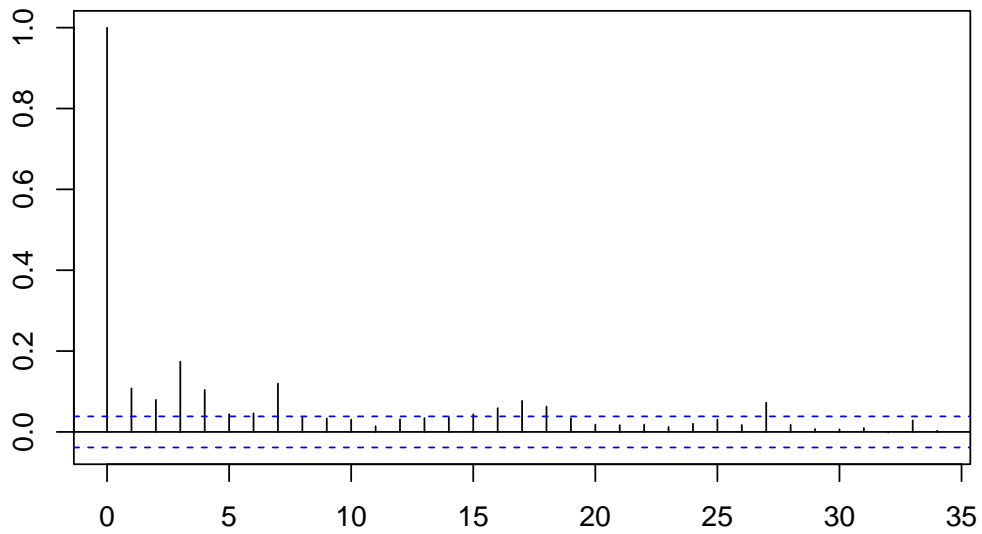
<https://tutorcs.com>

**Plot: ARMA(0, 2)**



WeChat: cstutorcs

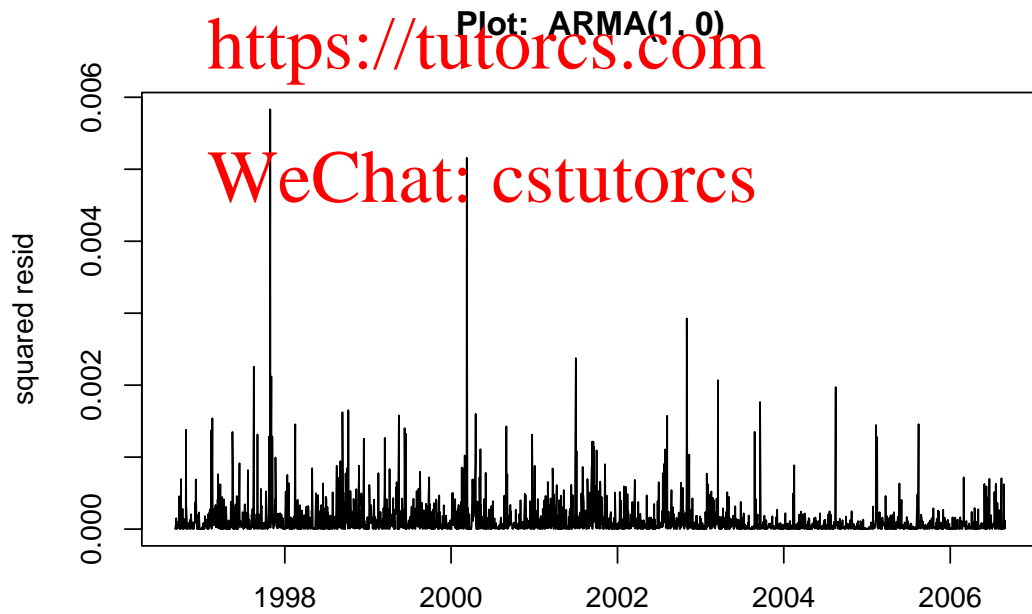
**SACF: ARMA(0, 2)**



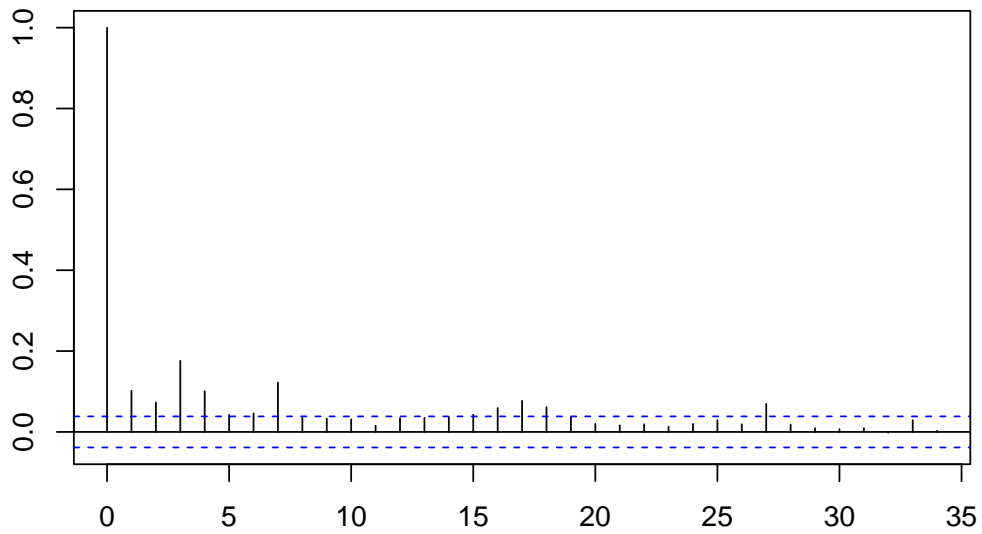
Assignment Project Exam Help

<https://tutorcs.com>

WeChat: cstutorcs



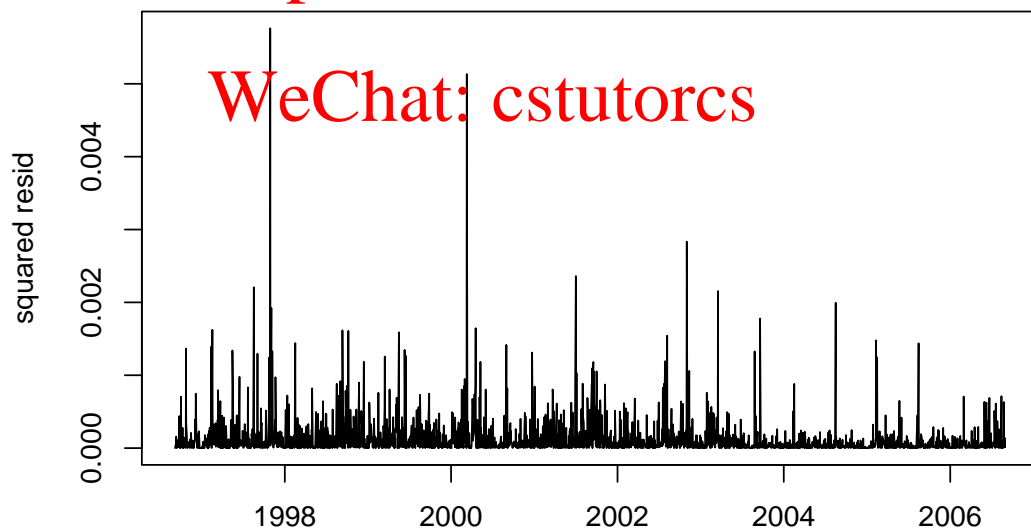
**SACF: ARMA(1, 0)**



Assignment Project Exam Help

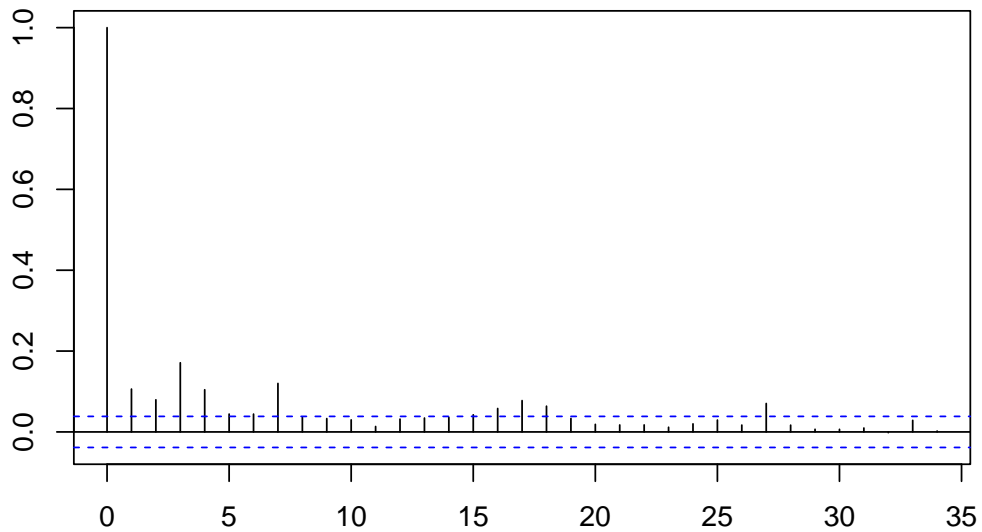
<https://tutorcs.com>

**Plot: ARMA(1, 1)**



WeChat: cstutorcs

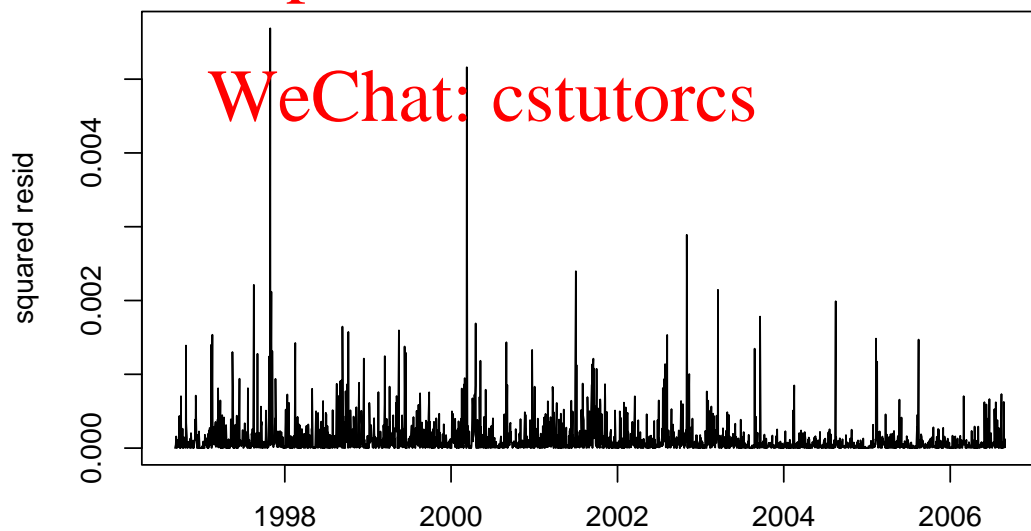
**SACF: ARMA(1, 1)**



Assignment Project Exam Help

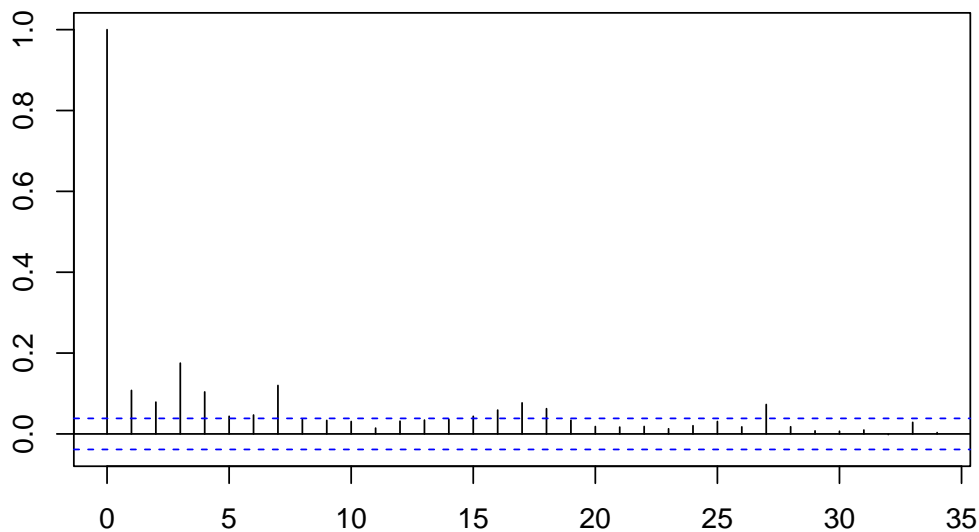
<https://tutorcs.com>

**Plot: ARMA(2, 0)**



WeChat: cstutorcs

### SACF: ARMA(2, 0)



## Assignment Project Exam Help

Squared residuals appear to be strongly autocorrelated even though autocorrelation in the residual levels appears to be quite low. We interpret this as evidence of conditional heteroscedasticity possibly being present in the DGP.

<https://tutorcs.com>

---

## WeChat: cstutorcs

5. Test if the errors in your set of models contain ARCH or GARCH effects.

---

**Solution** We use the Breush-Pagan test, which is a type of Lagrange-Multiplier (LM) test. The test is implemented for every model in the adequate set constructed in Question 3. It also requires a specification of  $j$  lags in the regression of squared residuals on lags. We will try lags ranging from 1 to 10. If there are discrepancies, we will need to delve deeper and construct an adequate set of B-P regressions for the test using the same approach we have learned based on information criteria and residuals analysis.

Note that the residual in the B-P regression involving squared residuals from the ARMA specification is assumed to be mean-independent. This is necessary to justify the  $\chi^2$  distribution of the LM test statistic. You should notice the connection to the corresponding role of mean-independence in the ADF test.

Also, the B-P regression is in fact an  $AR(j)$ . However, because we need the  $R^2$  from the B-P regression, it is easier to use the `lm` function in R rather than `Arima`.

```

bptest <- matrix(nrow = 10 * nmods, ncol = 5)
colnames(bptest) <- c("p", "q", "j", "LM-stat", "p-value")
for (i in 1:nmods)
{
  e2_i <- as.vector(e2_arma[[i]])
  f <- formula(e2_i ~ 1)
  for (j in 1:10)
  {
    # lag lengths in the auto-regression of squared residuals
    k <- 10 * (i - 1) + j
    f <- update.formula(f, paste("~ . + lag(e2_i, n = ", j, ")"));
    bp_reg_j <- lm(f)
    LM_j <- length(e2_i) * summary(bp_reg_j)$r.squared
    p_val_j <- 1 - pchisq(LM_j, df = j)
    bptest[k,] <- c(adq_set_arma[i, 1:2], j, LM_j, p_val_j)
  }
}

```

In specifications, the null hypothesis of homoscedastic errors is rejected in favor of ARCH/GARCH errors at very low significance levels. This suggests we should consider expanding our adequate set of models to include conditional heteroscedasticity.

<https://tutorcs.com>

- Expand the adequate set of models to specifications with heteroscedasticity. To this end, only consider conditional variances modelled with ARCH/GARCH residuals. Hint: Use the `rugarch` package with `ugarchspec` and `ugarchfit` functions.

**Solution** Based on the adequate set constructed with homodcedastic ARMAs, we begin by considering ARMA variants with  $p_m = 0, \dots, 2$  and  $q_m = 0, \dots, 2$  in the conditional mean along with GARCH lags  $p_h = 0, \dots, 2$  and  $q_h = 0, \dots, 2$  in the conditional variance.

There is a total of  $3^4 = 81$  models altogether—it will take a few minutes to estimate them all! Also, some specifications will be so bad that numerical optimisation will fail, so it is helpful to use the `try` function to avoid interruptions.

```

ARMA_GARCH_est <- list()
ic_arma_garch <- matrix( nrow = 3 ^ 4, ncol = 6 )
colnames(ic_arma_garch) <- c("pm", "qm", "ph", "qh", "aic", "bic")
i <- 0
for (pm in 0:2)
{

```

```

for (qm in 0:2)
{
  for (ph in 0:2)
  {
    for (qh in 0:2)
    {
      i <- i + 1
      ic_arma_garch[i, 1:4] <- c(pm, qm, ph, qh)

      if (ph == 0 && qh == 0)
      {
        # for models with constant variance, the ugarchspec and
        # ugarchfit functions do not work well; instead, the
        # documentation advises to use arfimaspec and arfimafit
        ARMA_GARCH_mod <- arfimaspec(
          mean.model = list(armaOrder = c(pm, qm)))

        ARMA_GARCH_est[[i]] <- arfimafit(ARMA_GARCH_mod, r)
        ic_arma_garch[i,5:6] <- infocriteria(
          ARMA_GARCH_est[[i]])[1:2]
      }
      else
      {
        try(silent = T, expr =
        {
          ARMA_GARCH_mod <- ugarchspec(
            mean.model = list(armaOrder = c(pm, qm)),
            variance.model = list(garchOrder = c(ph, qh)))

          ARMA_GARCH_est[[i]] <- ugarchfit(ARMA_GARCH_mod, r,
            solver = 'hybrid')

          ic_arma_garch[i,5:6] <- infocriteria(
            ARMA_GARCH_est[[i]])[1:2]
        })
      }
    }
  }
}

ic_aic_arma_garch <- ic_arma_garch[
  order(ic_arma_garch[,5]),][1:40,]
ic_bic_arma_garch <- ic_arma_garch[

```

Assignment Project Exam Help

<https://tutorcs.com>

WeChat: cstutorcs



```

order(ic_arma_garch[,6]),][1:40,]

ic_int_arma_garch <- intersect(as.data.frame(ic_aic_arma_garch),
                              as.data.frame(ic_bic_arma_garch))

```

There are *a lot* of models that have comparable AIC and BIC measures! We select the first 36 in the intersecting set.

```

adq_set_arma_garch <- as.matrix(arrange(as.data.frame(
  ic_int_arma_garch[1:36,]), pm, qm, ph, qh))
adq_idx_arma_garch <- match(
  data.frame(t(adq_set_arma_garch[, 1:4])),
  data.frame(t(ic_arma_garch[, 1:4])))

```

Next, we check the residuals for specifications in the adequate set. Note that the standard Ljung-Box test is not well-defined for heteroscedastic models because the derivation of the sampling distribution assumes homoscedasticity. We will therefore avoid relying on the hypothesis test, but still examine the SACF for obvious evidence of substantial autocorrelation.

```

nmods <- length(adq_idx_arma_garch)
sacf_garch <- matrix(nrow = nmods, ncol = 14)
colnames(sacf_garch) <- c("pm", "qm", "ph", "qh", 1:10)
for (i in 1:nmods) {
  sacf_garch[i,1:4] <- adq_set_arma_garch[i,1:4]
  sacf_garch[i,5:14] <-
    acf(ARMA_GARCH_est[[adq_idx_arma_garch[i]]]@fit$z,
        lag = 10, plot = F)$acf[2:11]
}

```

Residual autocorrelations appear to be relatively small for all models in the adequate set.

- 
7. Estimate each model in the set identified in Question 6 and plot the estimated volatilities ( $\hat{h}_t$ ). Interpret the results.
- 

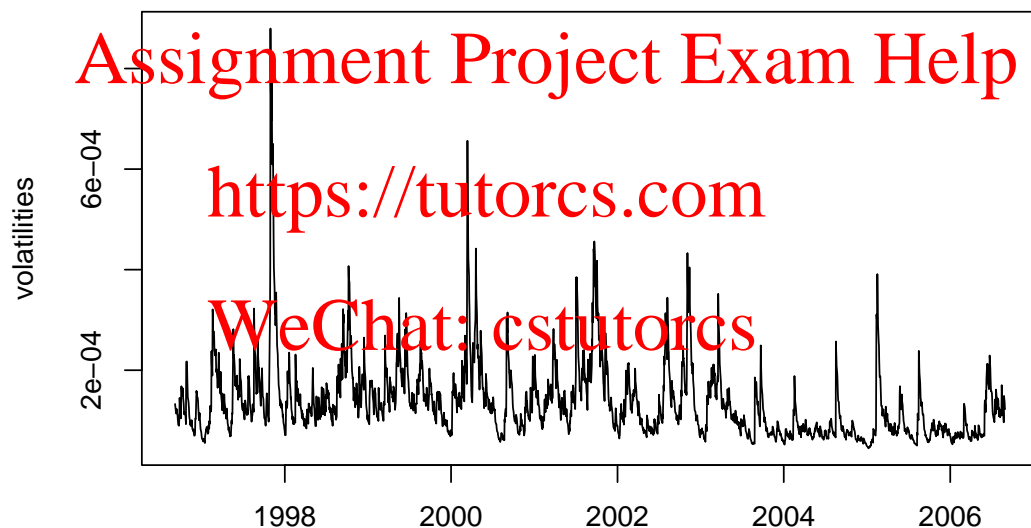
**Solution** We have already estimated all the models in the adequate set, so the only thing left to do is plot the estimated volatilities. Unfortunately, the **rugarch** package *does not* provide standard errors or confidence intervals for estimated volatilities. We would need to write a customised function to compute these (left as practice... or for an advanced econometrics course)!

```

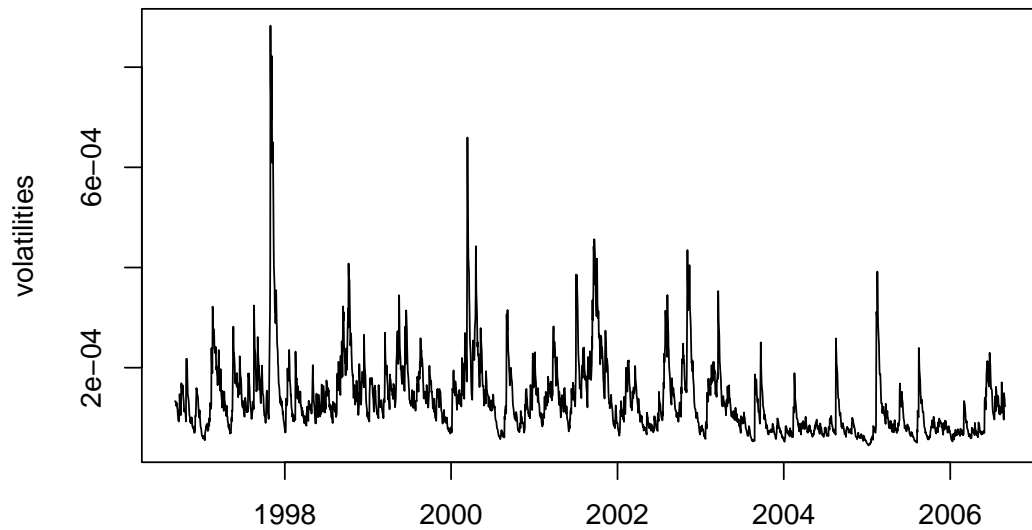
for (i in 1:nmods)
{
  title_p_q <- paste("ARMA(",
                    as.character(adq_set_arma_garch[i, 1]), ", ",
                    as.character(adq_set_arma_garch[i, 2]),
                    ")-GARCH(",
                    as.character(adq_set_arma_garch[i, 3]), ", ",
                    as.character(adq_set_arma_garch[i, 4]), ")",
                    sep = "")
  plot(date[-1], ARMA_GARCH_est[[adq_idx_arma_garch[i]]]@fit$var,
       type = "l", xlab = "", ylab = "volatilities",
       main = title_p_q)
}

```

### ARMA(0, 0)-GARCH(1, 1)



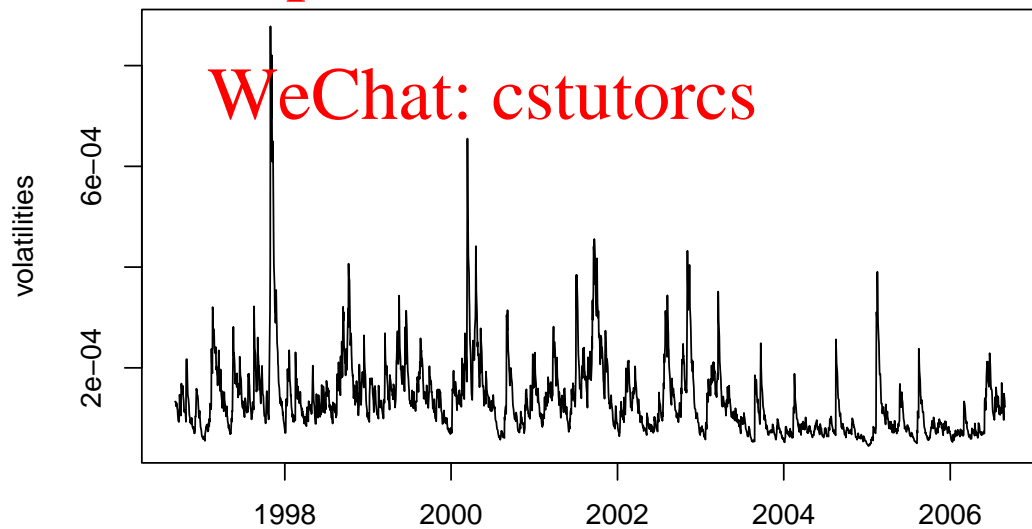
**ARMA(0, 0)–GARCH(1, 2)**



Assignment Project Exam Help

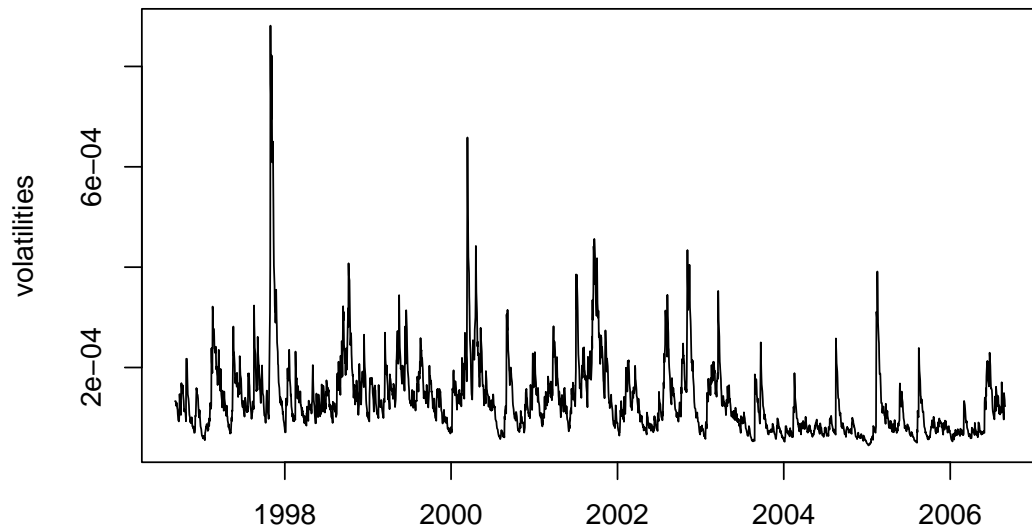
<https://tutorcs.com>

**ARMA(0, 0)–GARCH(2, 1)**



WeChat: cstutorcs

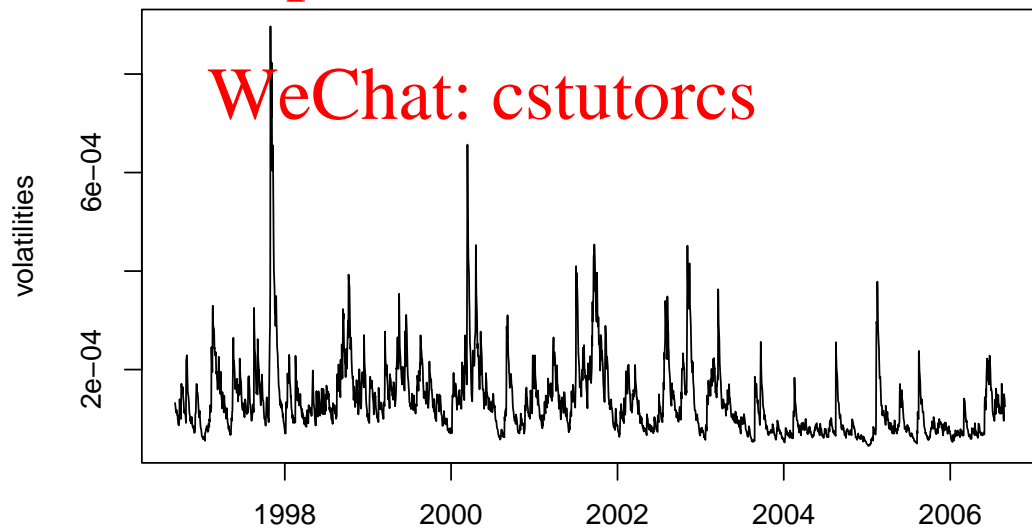
**ARMA(0, 0)–GARCH(2, 2)**



Assignment Project Exam Help

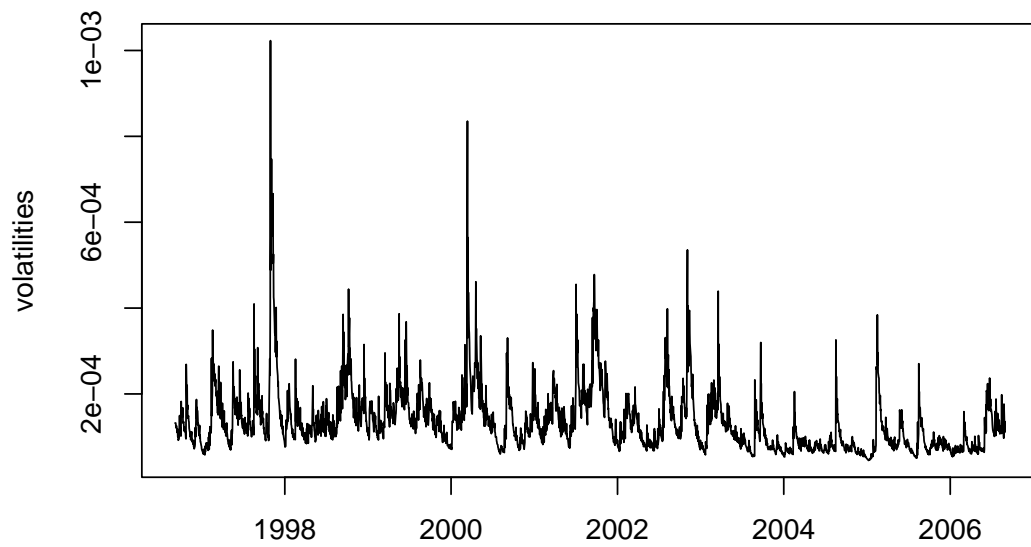
<https://tutorcs.com>

**ARMA(0, 1)–GARCH(1, 1)**



WeChat: cstutorcs

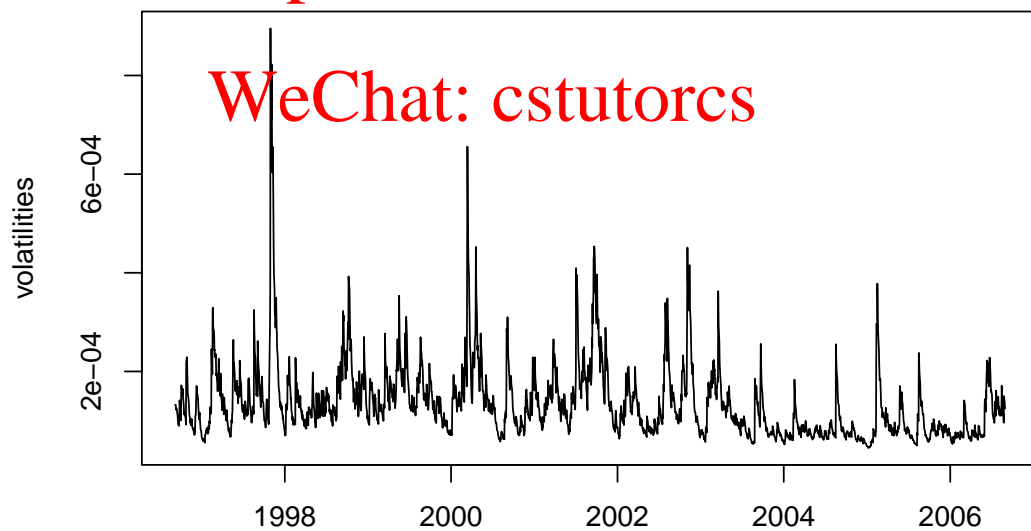
**ARMA(0, 1)–GARCH(1, 2)**



Assignment Project Exam Help

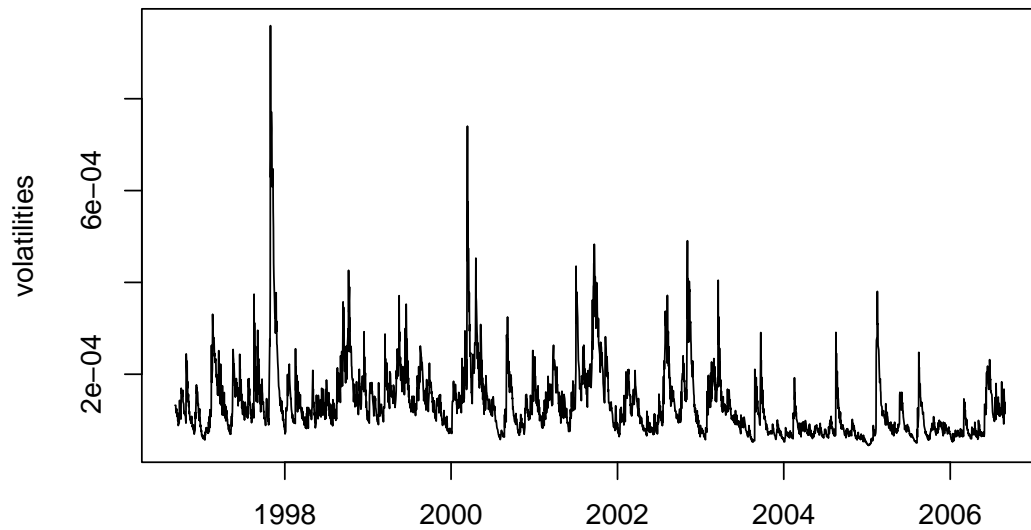
<https://tutorcs.com>

**ARMA(0, 1)–GARCH(2, 1)**



WeChat: cstutorcs

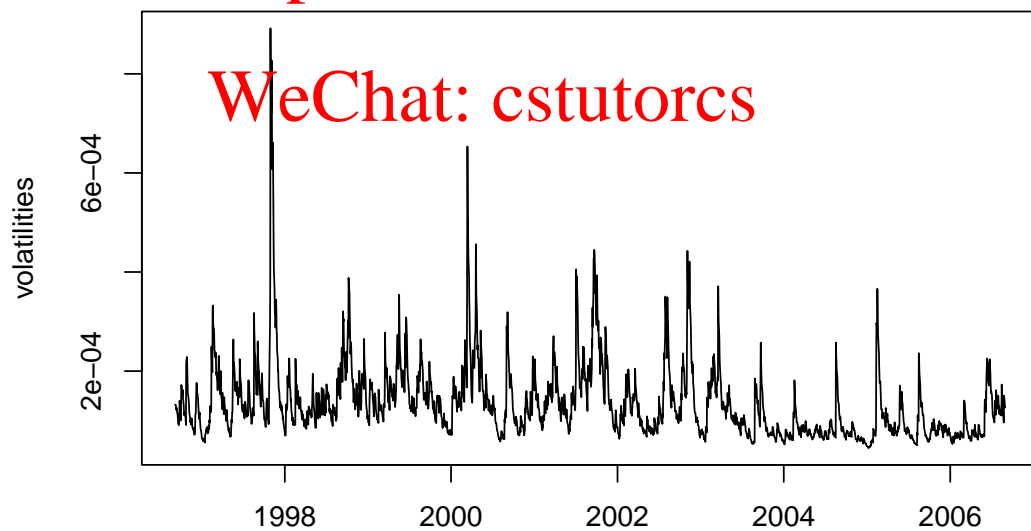
**ARMA(0, 1)–GARCH(2, 2)**



Assignment Project Exam Help

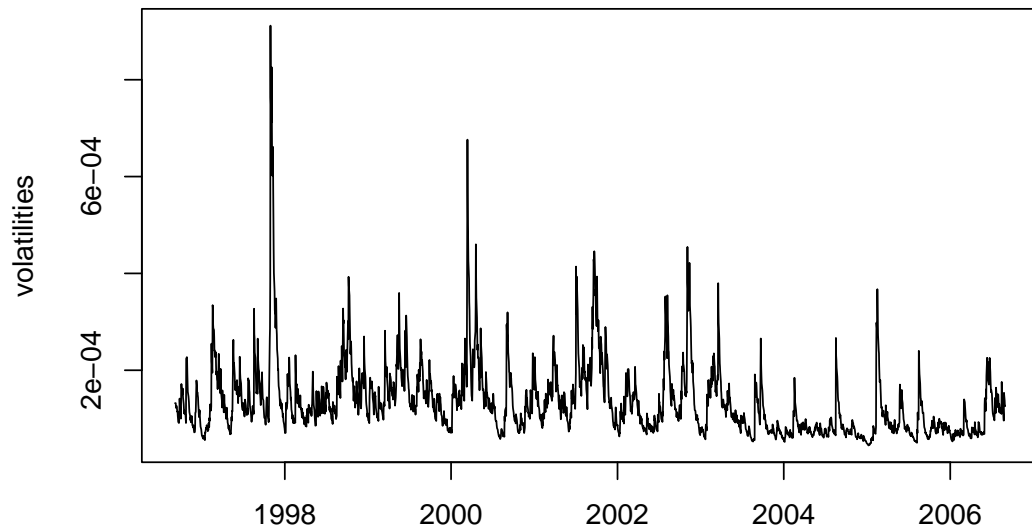
<https://tutorcs.com>

**ARMA(0, 2)–GARCH(1, 1)**



WeChat: cstutorcs

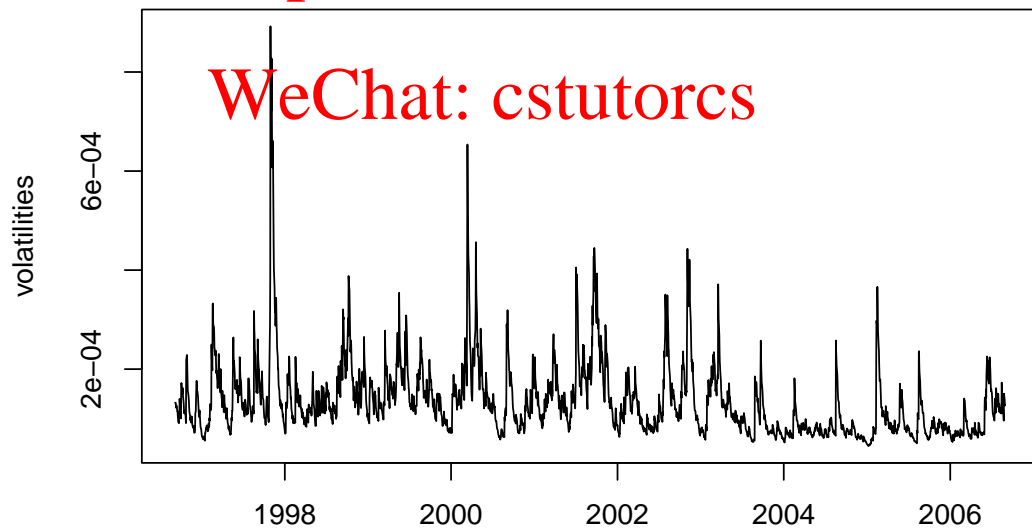
**ARMA(0, 2)–GARCH(1, 2)**



**Assignment Project Exam Help**

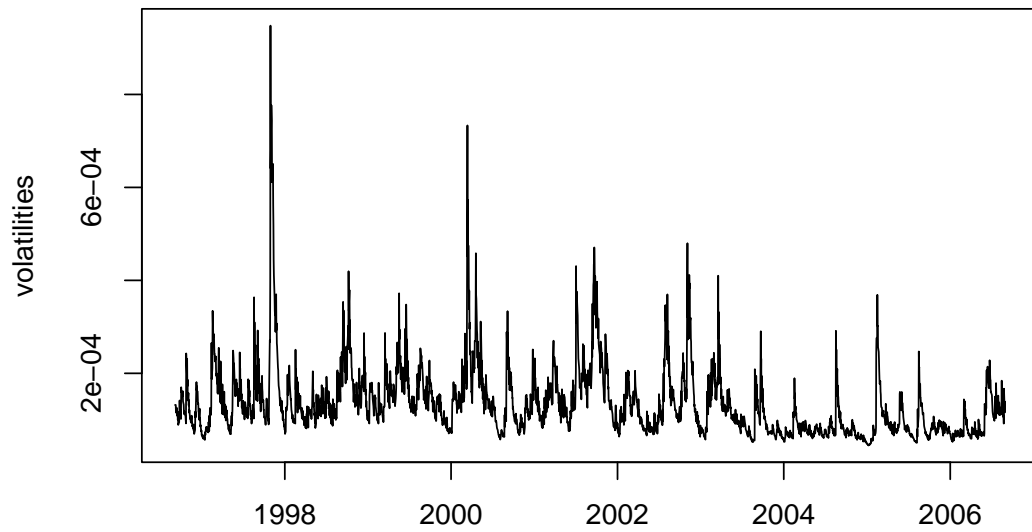
**<https://tutorcs.com>**

**ARMA(0, 2)–GARCH(2, 1)**



**WeChat: cstutorcs**

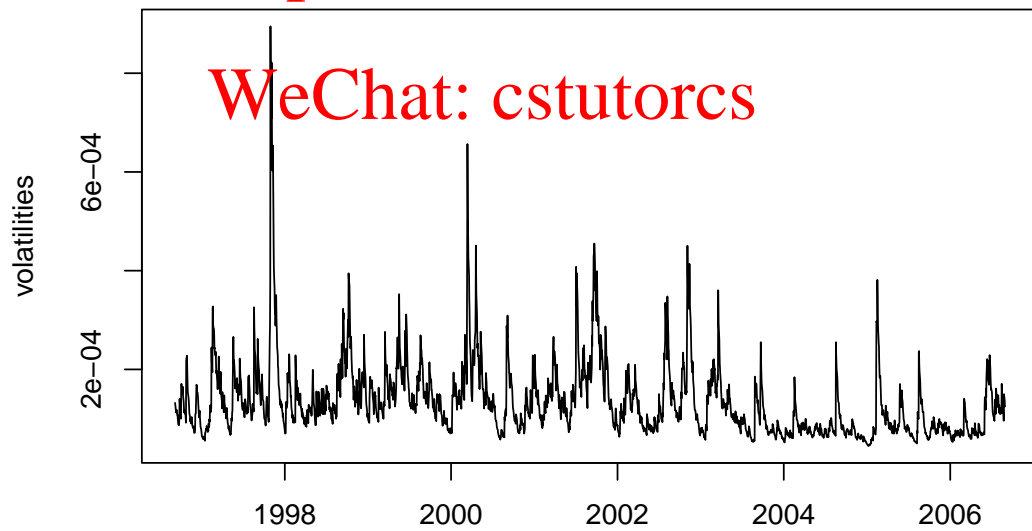
**ARMA(0, 2)–GARCH(2, 2)**



Assignment Project Exam Help

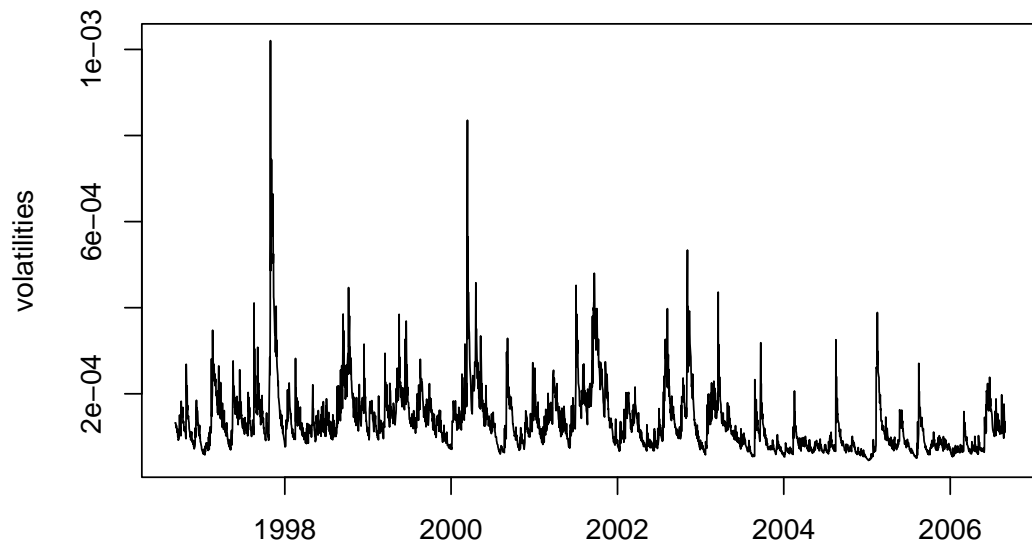
<https://tutorcs.com>

**ARMA(1, 0)–GARCH(1, 1)**





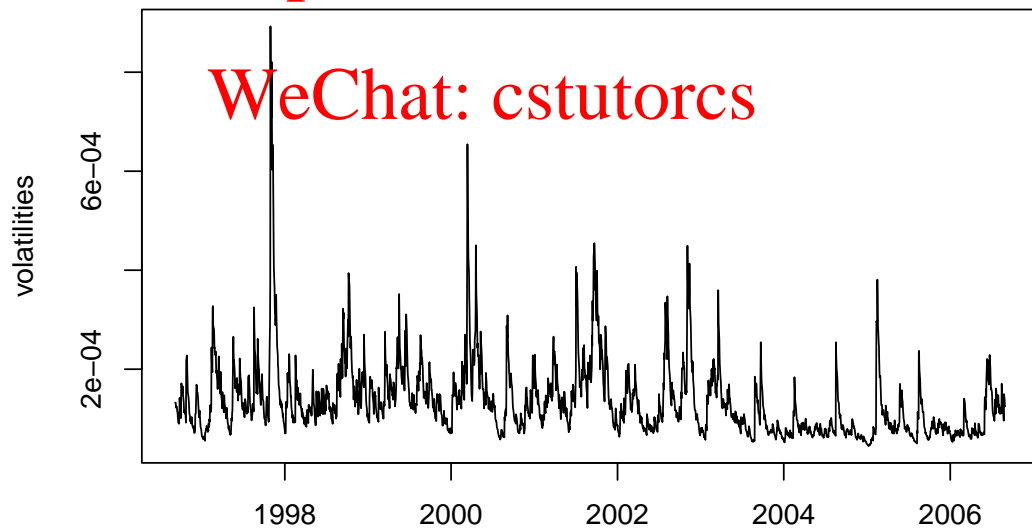
**ARMA(1, 0)–GARCH(1, 2)**



Assignment Project Exam Help

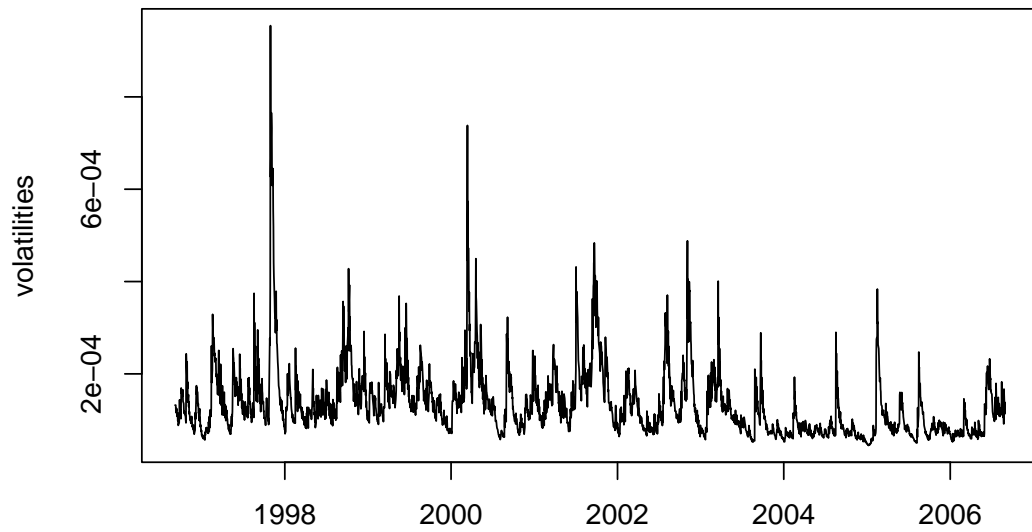
<https://tutorcs.com>

**ARMA(1, 0)–GARCH(2, 1)**



WeChat: cstutorcs

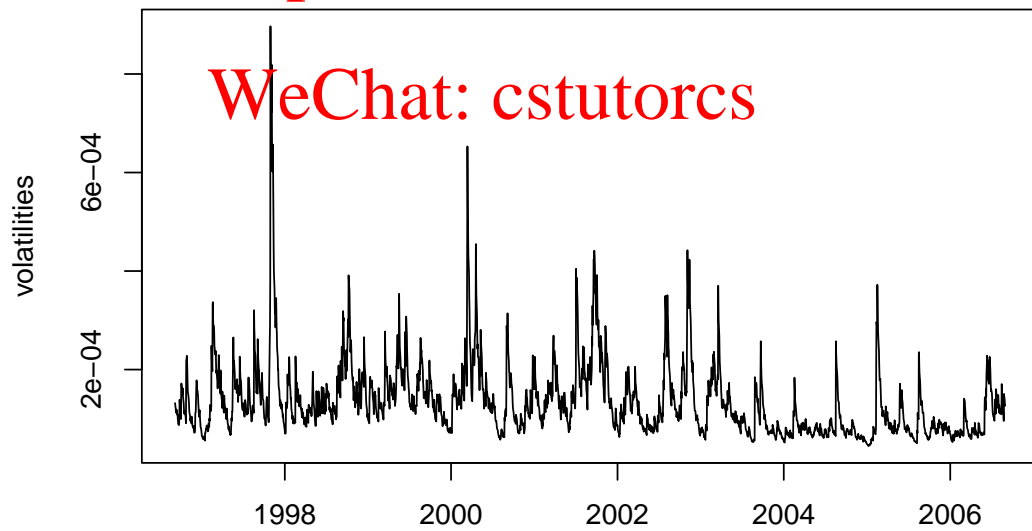
**ARMA(1, 0)–GARCH(2, 2)**



Assignment Project Exam Help

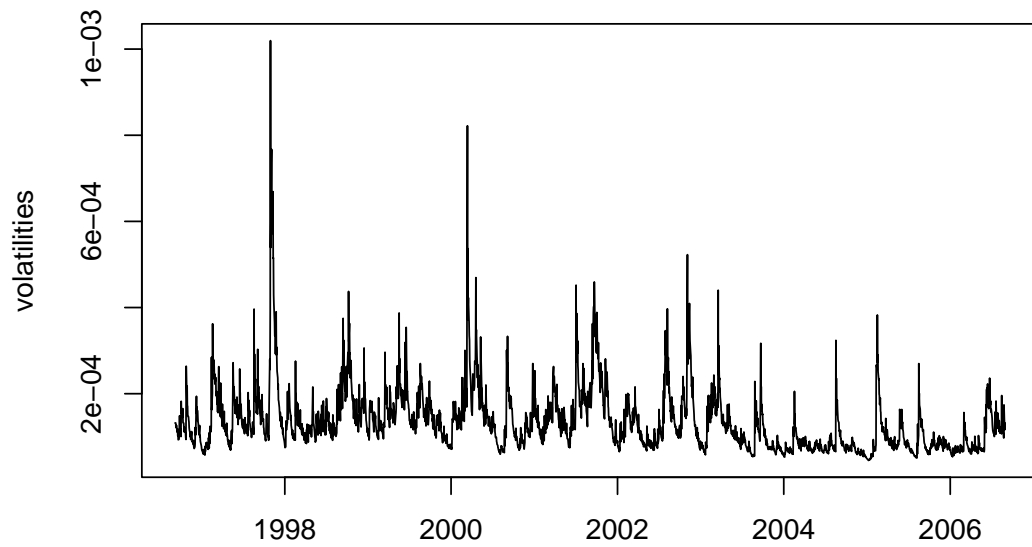
<https://tutorcs.com>

**ARMA(1, 1)–GARCH(1, 1)**



WeChat: cstutorcs

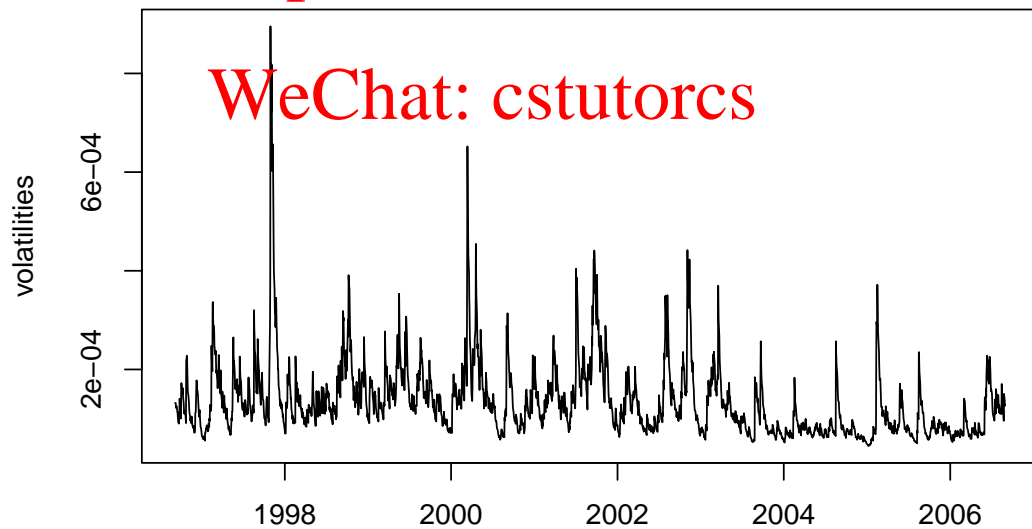
**ARMA(1, 1)–GARCH(1, 2)**



Assignment Project Exam Help

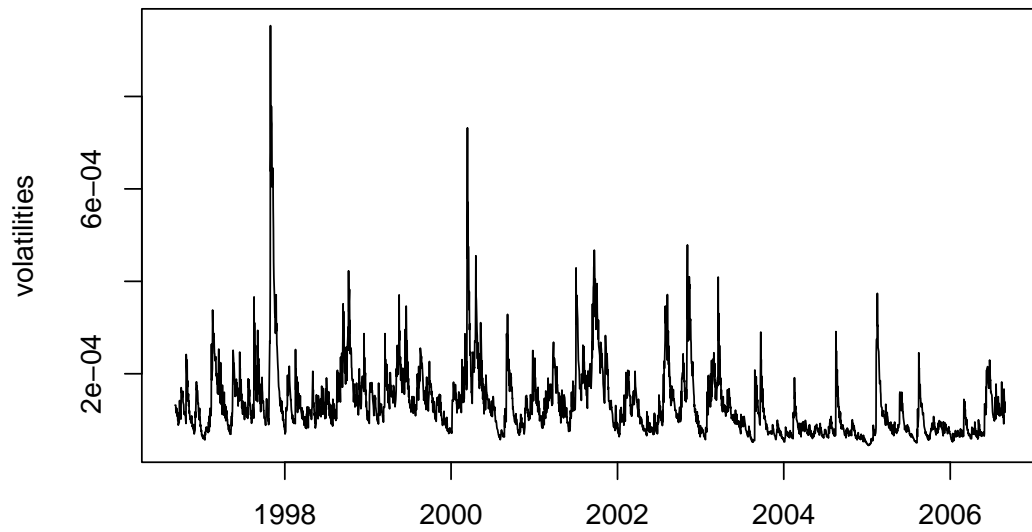
<https://tutorcs.com>

**ARMA(1, 1)–GARCH(2, 1)**



WeChat: cstutorcs

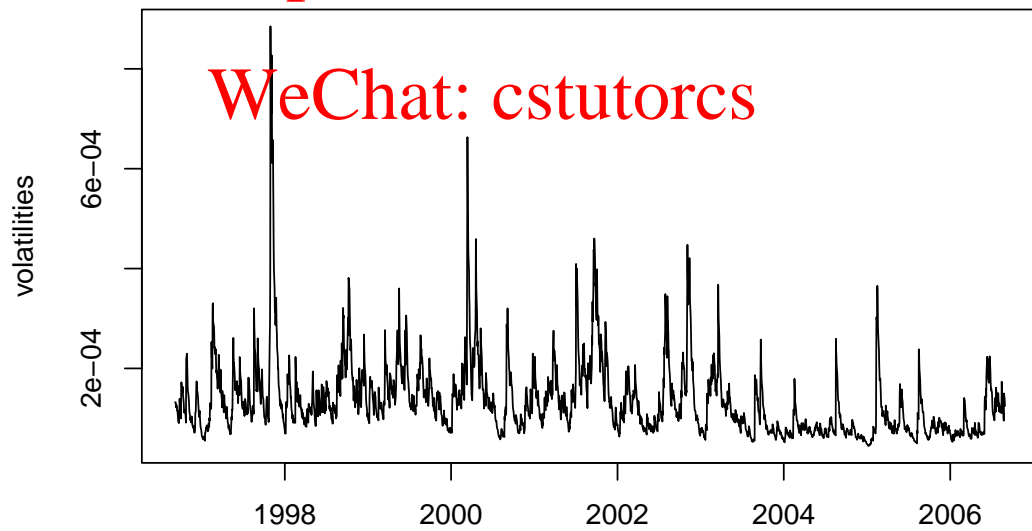
**ARMA(1, 1)–GARCH(2, 2)**



Assignment Project Exam Help

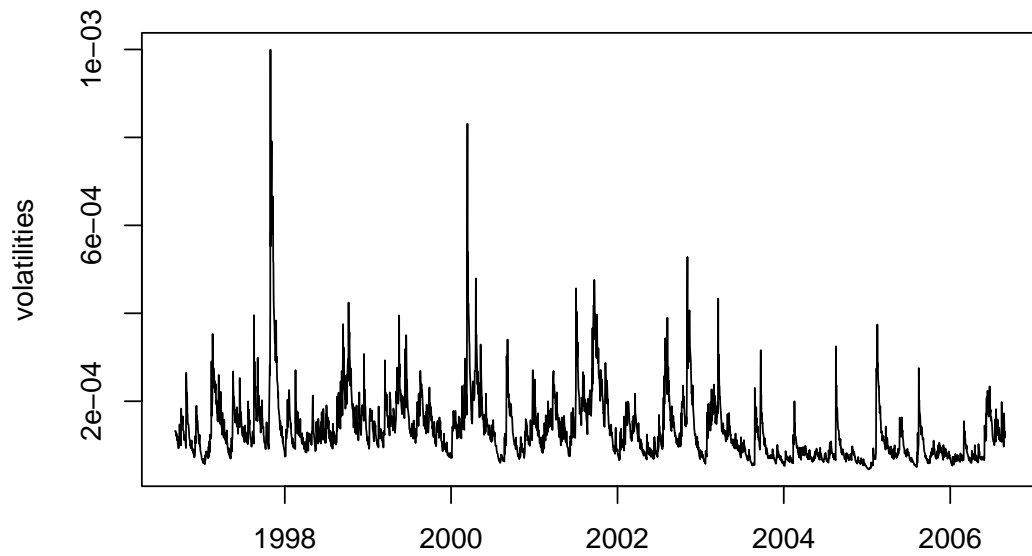
<https://tutorcs.com>

**ARMA(1, 2)–GARCH(1, 1)**



WeChat: cstutorcs

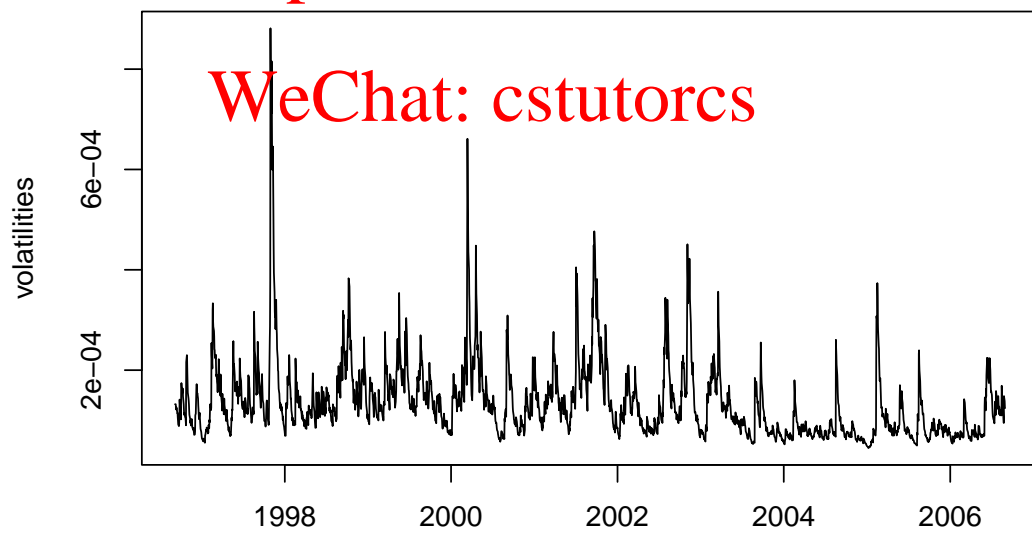
**ARMA(1, 2)–GARCH(1, 2)**



Assignment Project Exam Help

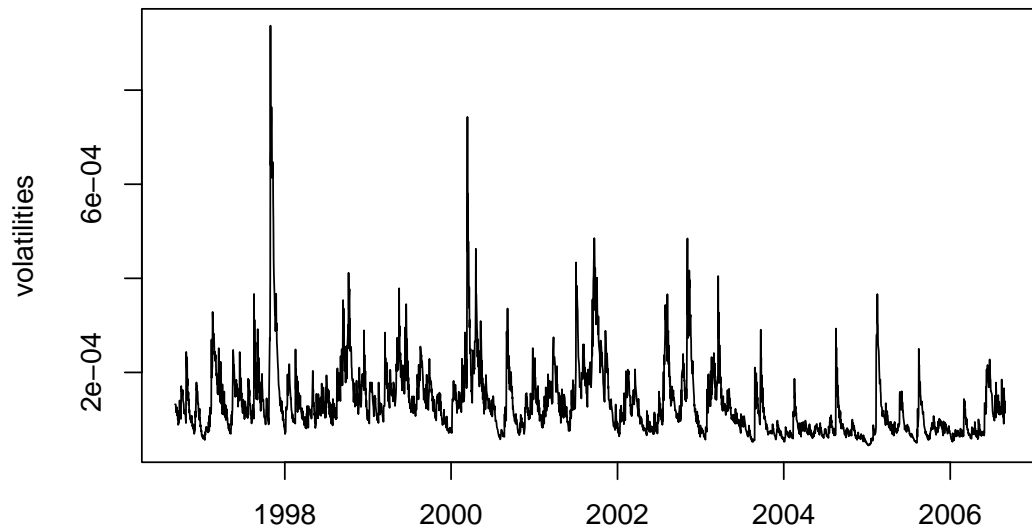
<https://tutorcs.com>

**ARMA(1, 2)–GARCH(2, 1)**



WeChat: cstutorcs

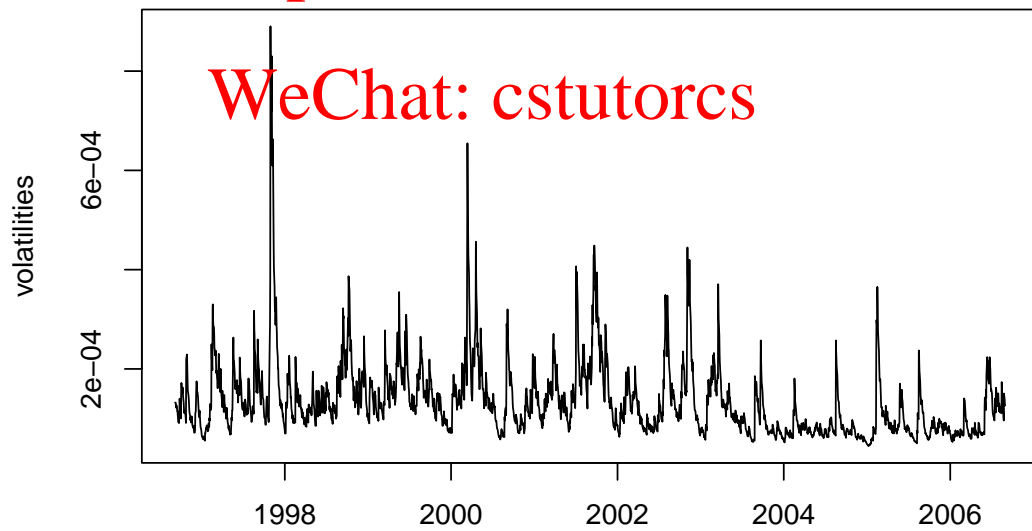
**ARMA(1, 2)–GARCH(2, 2)**



Assignment Project Exam Help

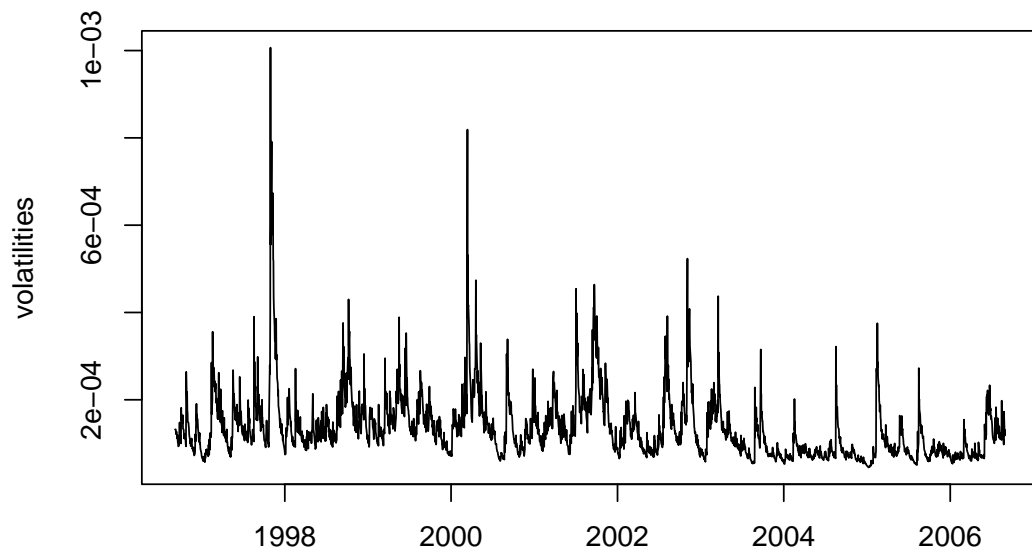
<https://tutorcs.com>

**ARMA(2, 0)–GARCH(1, 1)**



WeChat: cstutorcs

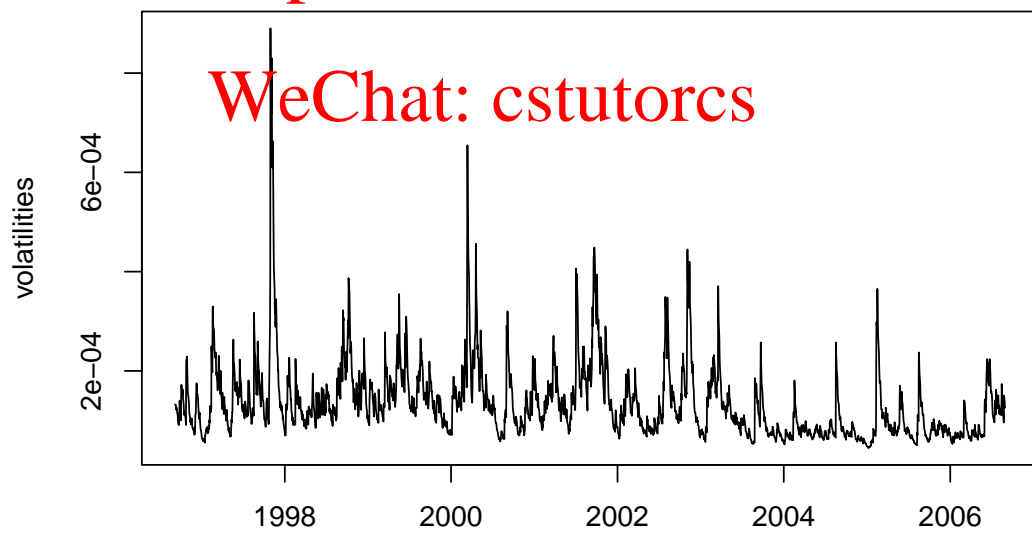
**ARMA(2, 0)–GARCH(1, 2)**



Assignment Project Exam Help

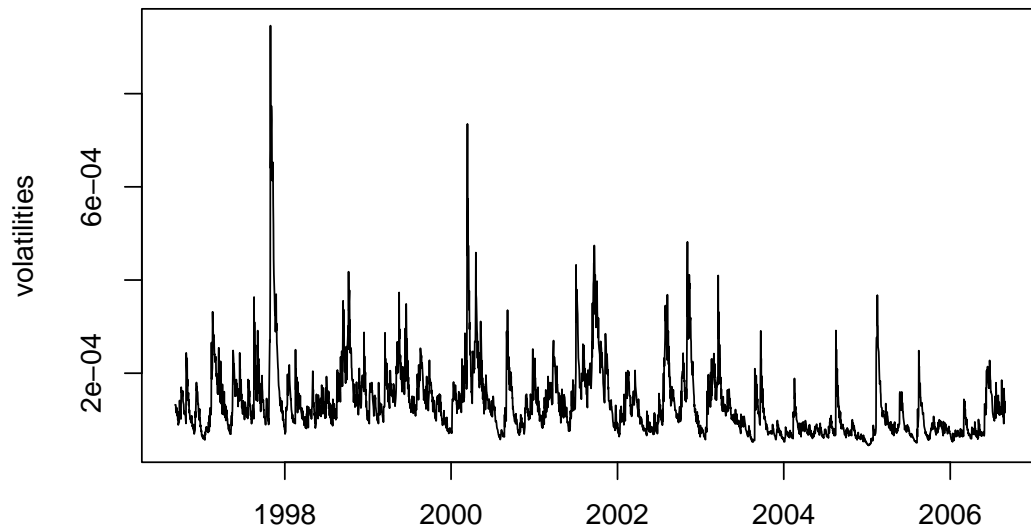
<https://tutorcs.com>

**ARMA(2, 0)–GARCH(2, 1)**



WeChat: cstutorcs

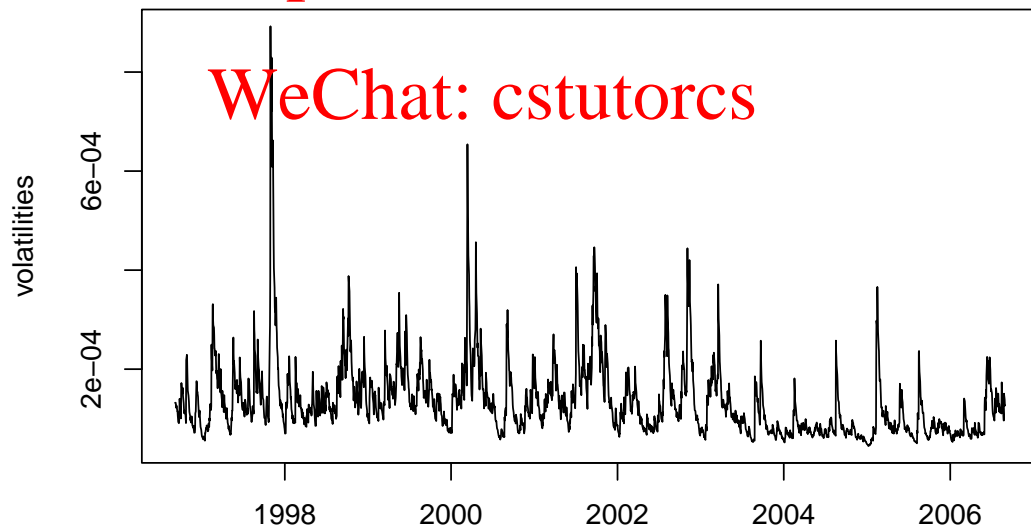
**ARMA(2, 0)–GARCH(2, 2)**



Assignment Project Exam Help

<https://tutorcs.com>

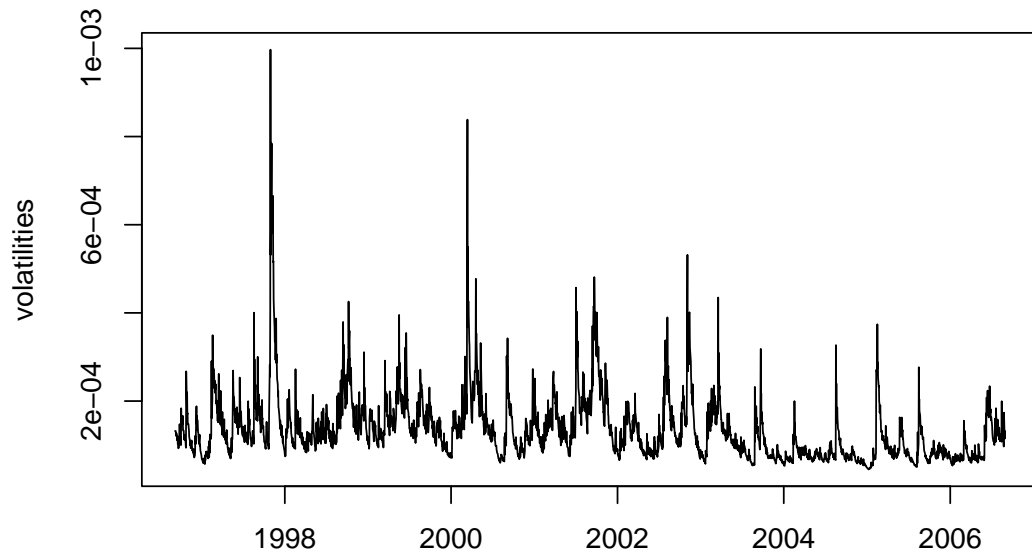
**ARMA(2, 1)–GARCH(1, 1)**



WeChat: cstutorcs



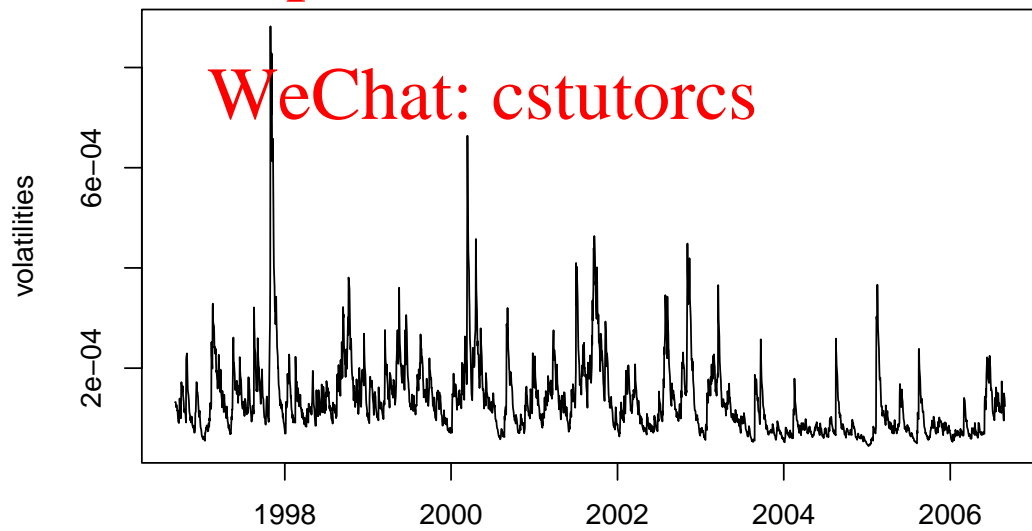
**ARMA(2, 1)–GARCH(1, 2)**



Assignment Project Exam Help

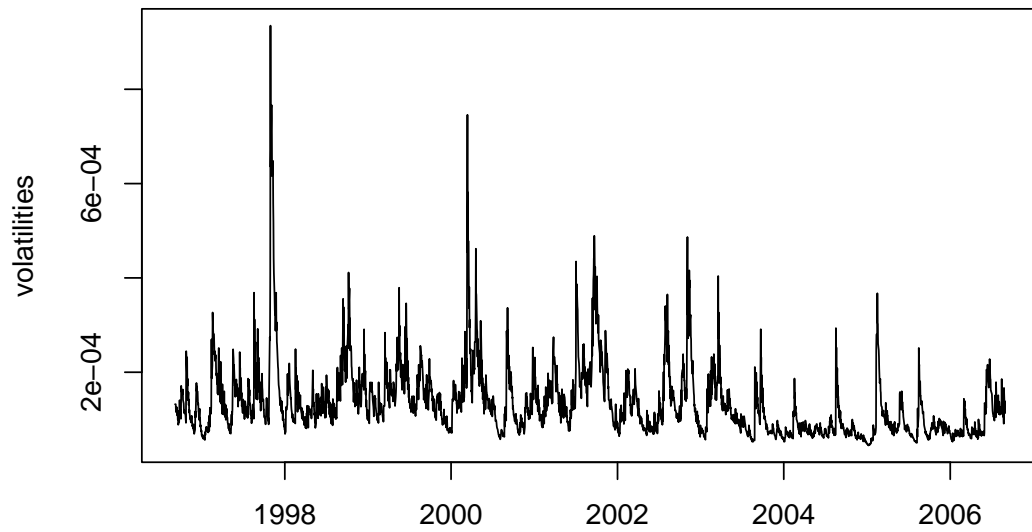
<https://tutorcs.com>

**ARMA(2, 1)–GARCH(2, 1)**



WeChat: cstutorcs

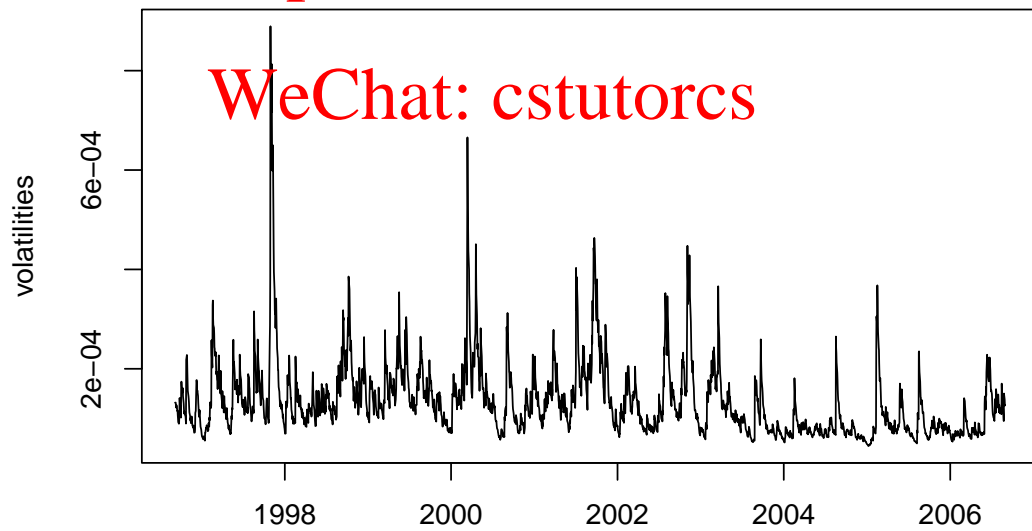
**ARMA(2, 1)–GARCH(2, 2)**



Assignment Project Exam Help

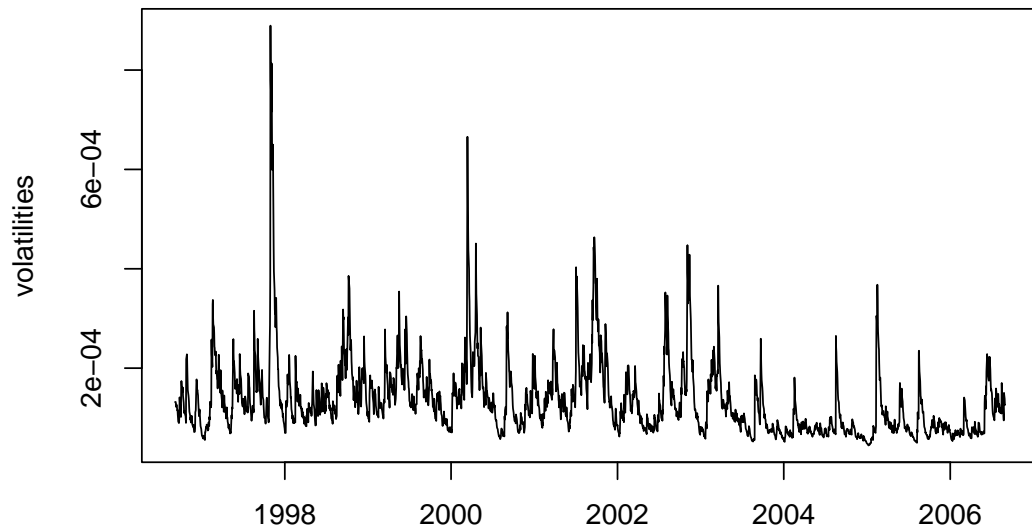
<https://tutorcs.com>

**ARMA(2, 2)–GARCH(1, 1)**



WeChat: cstutorcs

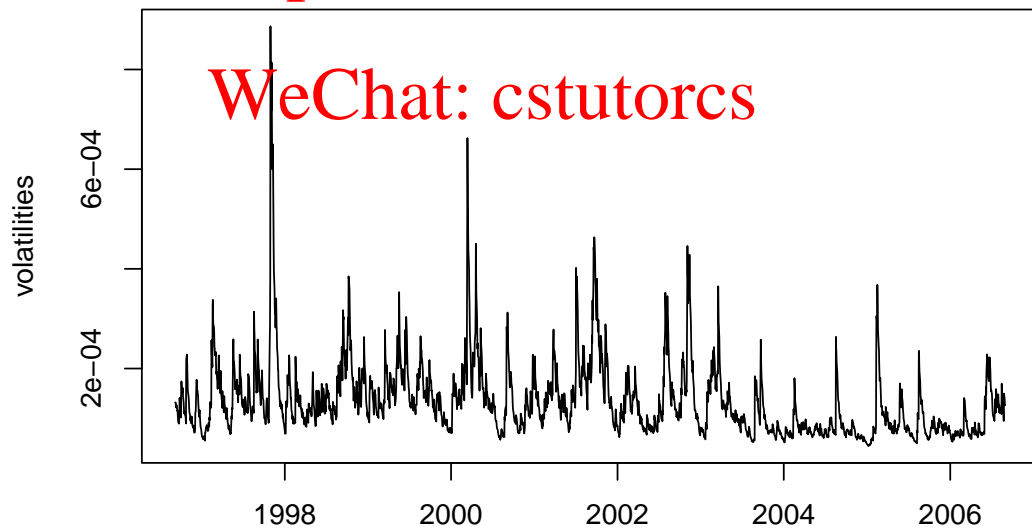
**ARMA(2, 2)–GARCH(1, 2)**



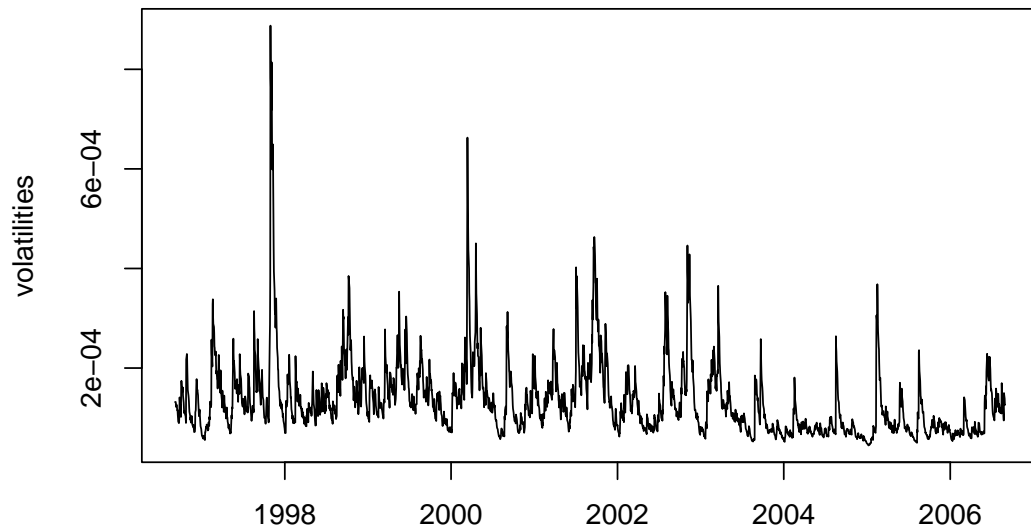
Assignment Project Exam Help

<https://tutorcs.com>

**ARMA(2, 2)–GARCH(2, 1)**



### ARMA(2, 2)–GARCH(2, 2)



## Assignment Project Exam Help

We notice some subtle differences across these specifications, but overall the estimated volatilities over the sample period follow a similar pattern. Hence, even though our adequate set contains many models, the implied high degree of model specification uncertainty is of little practical consequence for inference on volatilities. Put in a positive light: the inference we obtain regarding volatilities is robust to model specification uncertainty.