City University of Hong Kong
Department of Economics and Finance
Course EF5213 Assignment #3 **(due March 19, 2023)**

1. Consider the MVO problem that determines the optimal portfolio content **w** and w_0 by minimizing the portfolio risk as

minimize [illegible]

subject to [illegible] $w + w_0 = 1$, and $a_1 \geq w_1 \geq -b_1$, … , $a_n \geq w_n \geq -b_n$

given portfolio mean [illegible] μ_0, asset mean returns $\boldsymbol{\mu}$, and their variance-covariance $\boldsymbol{\sigma}$. There are buy and sell limits in the optimization according to the given positive quantities $\{a_1, \ldots , a_n\}$ and $\{b_1, \ldots , b_n\}$. It should be noted that the optimal portfolio content can be determined through the Kuhn-Tucker conditions as

$$\frac{\partial L}{\partial w_i} = (\boldsymbol{\sigma}\,\mathbf{w} - \lambda_1\,\boldsymbol{\mu} + \mu_0\,\lambda_1\,\mathbf{u})_i = 0 \quad \text{when} \quad a_i \geq w_i \geq -b_i$$

$$< 0 \quad \text{when} \quad w_i = a_i$$

$$> 0 \quad \text{when} \quad w_i = -b_i \quad , \text{for } i = 0, 1, \ldots , n$$

Modify the Markowitz algorithm in the lecture and develop a VBA implementation for the current MVO problem.

(80 points)

Consider the following procedures in your implementation:

(1) Define an *OUT* subset Ω, and separate Ω into two disjoint subsets A and B. You can use the given subroutine *GetSeparation*() for these purposes. Consider the MVO problem with $w_i = -b_i$ for $i \in B$, and $w_i = a_i$ for $i \in A$. The optimal solution of this MVO problem is given by

$$\mathbf{w} = \lambda_1\,(\boldsymbol{\sigma}_m^{-1}\boldsymbol{\mu}_m - \mu_0\,\boldsymbol{\sigma}_m^{-1}\mathbf{u}_m) + \boldsymbol{\sigma}_m^{-1}\boldsymbol{h}\,, \quad \text{where } \boldsymbol{h}_i = \begin{cases} a_i & , i \in A \\ -b_i & , i \in B \\ -\beta_i & , i \notin \Omega \end{cases} \,, \text{ and } \beta_i = \Sigma_{j\in A}\,\sigma_{ij}\,a_j + \Sigma_{j\in B}\,\sigma_{ij}\left(-b_j\right)$$

$$w_0 = 1 - \mathbf{u}^T\mathbf{w}$$

$$\lambda_1 = \frac{\mu_p - \mu_0 + \mu_0\mathbf{u}^T\boldsymbol{\sigma}_m^{-1}\boldsymbol{h} - \mu^T\boldsymbol{\sigma}_m^{-1}\boldsymbol{h}}{C_m\mu_0^2 - 2A_m\mu_0 + B_m}$$

Here, $\{\boldsymbol{\sigma}_m, \boldsymbol{\mu}_m, \mathbf{u}_m\}$ refer to the modified versions of $\{\boldsymbol{\sigma}, \boldsymbol{\mu}, \mathbf{u}\}$ according to the assets in the *OUT* subset Ω.

(2) Check that all the entries of **w** satisfy both the buy and sell limits. If so, proceed to step (3). If this is not the case, return to step (1) and try another separation of Ω or another *OUT* subset.

(3) Check that KKT conditions have been satisfied. If so, w_0 and **w** defined in (1) will be an optimal solution. Otherwise, return to step (1) and try another separation of Ω or another *OUT* subset.