Lecture - 1

Digital Signal Processing

ENGN 4537 / 6537

Semester 2, 2023

Textbook

A.V. Oppenheim & R.W. Schafer, "Discrete-Time Signal Processing", New International Edition (3e), Pearson, 2013.

* Chapter 2 (2.0-2.9) : Discrete-Time Signals and Systems
* Chapter 3 (3.0-3.7) : The z-Transform
* Chapter 4 (4.0-4.6, 4.8) : Sampling of Continuous-Time Signals

* Chapter 5 (5.0-5.7.1) : Transform Analysis of Linear Time-Invariant Systems
* Chapter 6 (6.0-6.5, 6.7-6.8) : Structures for Discrete-Time Systems

* Chapter 7 (7.0-7.5) : Filter Design Techniques
* Chapter 8 (8.0-8.7) : The Discrete Fourier Transform
* Chapter 9 (9.0-9.5) : Computation of the Discrete Fourier Transform
* Chapter 10 (10.0-10.3) : Fourier Analysis of Signals Using the Discrete Fourier Transform

Another useful resource is the MIT open Courseware: `https://ocw.mit.edu/courses/6-341-discrete-time-signal-processing-fall-2005/`

Teaching and Learning Activities

Lectures (Weeks 1 to 12):

* Lecture A: Tuesd[illegible]05:00 PM, PHYS T (DNF Dunbar)
* Lecture B: Wedne[illegible]0 - 03:00 PM, PHYS T (DNF Dunbar)

Tutorial (Weeks 2 to 12):

* Session 1: Thursday, 02:00 - 3:30 PM, Ian Ross Seminar Rm 214
* Session 2: Friday, 09:00 - 10:30 AM, Ian Ross Seminar Rm 214

You are required to attend one tutorial session every week. Tutorial sign-up is set up on the MyTimetable system.

Project Drop-in Sessions (In-person & online via Zoom):

* Week 6: Thursday, 11:00 AM-12:30 PM, Brian Anderson A203
* Weeks 8 to 10: Thursday, 11:00 AM-12:30 PM, Birch 1.33 Teaching Lab

Students can attend (not compulsory) these sessions to discuss your doubts regarding the MATLAB project with tutors.

Assessment

There are FOUR components to the assessment for this course:

	Components	Due
1	Weekly Quiz	Weeks 3 to
2	Mid-term Test	Week 7
3	MATLAB Project	Week 11
4	Final Exam	Exam perio

e: Students have to score a minimum of 50% to pass this course)

* Starting in Week 3, tutorials will include an in-class quiz based on topics discussed in the previous week. *Each quiz is worth 1% and there is a total of 10 quizzes*. Submission of the quiz will be allowed only during the tutorial.
* MATLAB Project details will be released in Week 4. It is an individual project. ENGN6537 students will have an extra component. Project is due in Week 11 (4:59 PM on Friday, 20 October 2023). No late submission is allowed.

  Any plagiarism and cheating will be dealt with seriously and according to ANU rules. You are responsible to familiarize yourself with these rules.

FAQ

Please carefully read the course outline available on Wattle. Most of your questions are already answered there.

Student Reps Are Vital!

We need 1 class  ntative each for ENGN4537 and ENGN6537.

Please see the slides available on Wattle. If you are interested, apply by 3rd August.

A little bit about me and my research interests

I'm from the Acousti dio Research Group

Surround Sound · Noise Cancellation · Blind Sound Signal analysis · Smart Homes · Virtual Reality




Dolby Labs · Sony Corporation · Australian Signals Directorate · Facebook Reality Labs

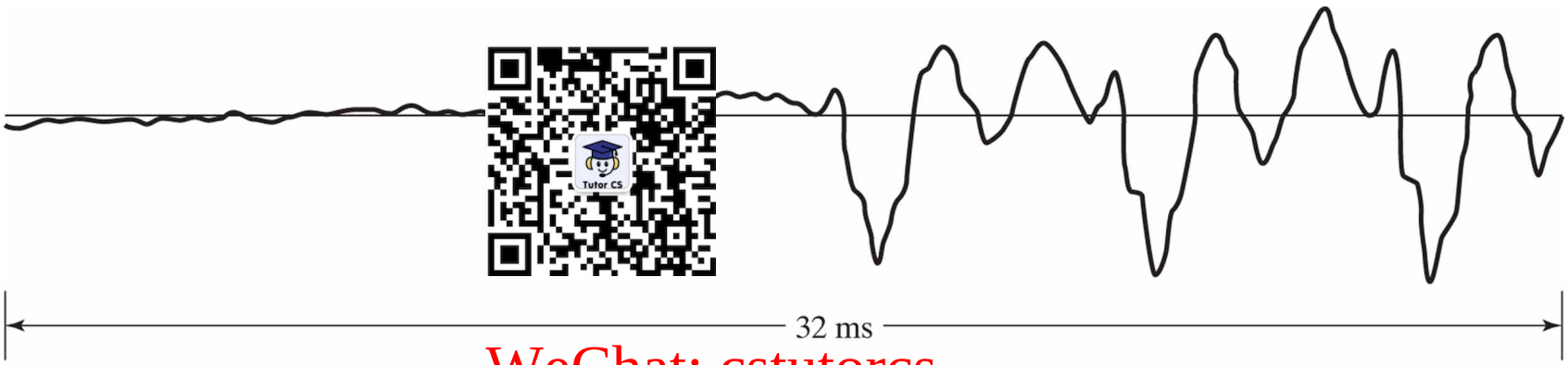
About this Course

Digital Signal Proces

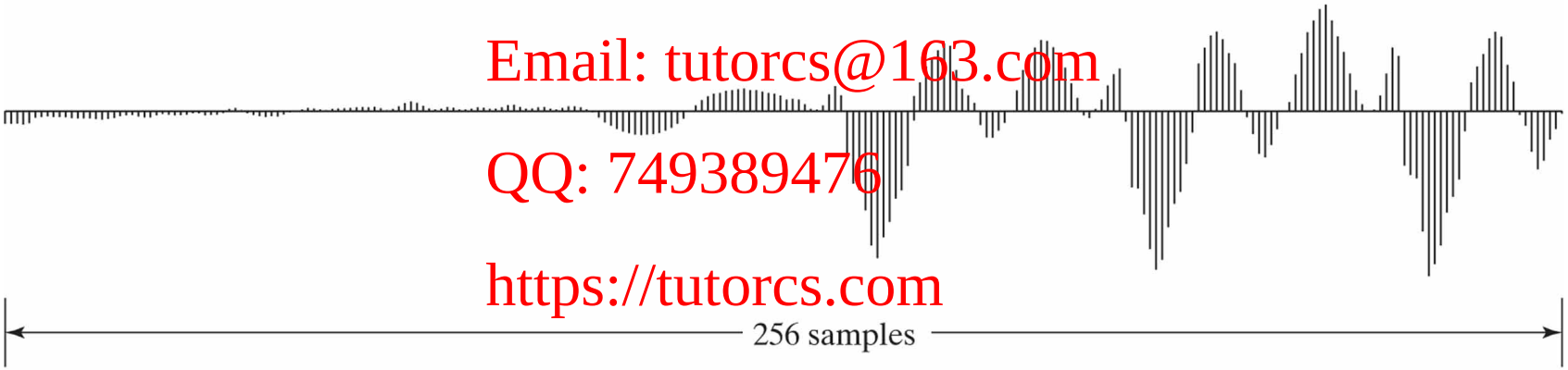
- Continuous-time signals (or analog signals, subject of ENGN 2228): defined along a continuum of time.

- Discrete-time signals (subject of this course ENGN 4537/6537) including digital signals as a special case: defined at discrete-times.

Example



(a)



(b)

About this Course

Digital Signal Processing

* Signals convey information about the state or behavior of a physical system.

About this Course

Digital Signal Processing

* Signals convey information about the state or behavior of a physical system.

* Signal Processing deals with the representation, transformation and manipulation of signals and the information the signals contain.

About this Course

Digital Signal Processing

* Signals convey information about the state or behavior of a physical system.

* Signal Processing deals with the representation, transformation and manipulation of signals and the information the signals contain.

* Signal Processing applications includes communications, medical imaging, audio & video systems, consumer electronics, robotics, remote sensing, finance etc.

About this Course

What goes on in a Zoom

Course Outline

* Discrete-time Signals and Systems
* The Z-Transform
* Transform Analysis of Linear Time-Invariant (LTI) Systems
* Sampling of Continuous-Time Signals
* The Discrete Fourier Transform (DFT)
* Computation of DFT using FFT algorithms
* Structure of Discrete-time Systems
* Filter Design Techniques
* Fourier Analysis of Signals using DFT
* Short Time Fourier Transform (STFT)

Course Outline

* Discrete-time Signals and Systems
* The Z-Transform
* Transform Analysis of Linear Time-Invariant (LTI) Systems
* Sampling of Continuous-Time Signals
* The Discrete Fourier Transform (DFT)
* Computation of DFT using FFT algorithms
* Structure of Discrete-time Systems
* Filter Design Techniques
* Fourier Analysis of Signals using DFT

Discrete-Time Signals

Definition (Signals)

Signals describe the evolution of a physical phenomenon and carry information in their variations. Mathematically signals are functions of one or more independent variables.

Signals are generally connected with physical quantities that vary with "independent variables", such as time, space or both.

Discrete-Time Signals

Examples of signals are:

* temperature (we

* pressure (sound)

* voltages, currents (electrical signals, radio)
* gray level on paper (black-and-white photography)
* ...

Discrete-Time Signals (dimension)

The independent variables can be 1D (one dimensional), 2D, 3D, etc.

* A sound signal varies with time (1D)

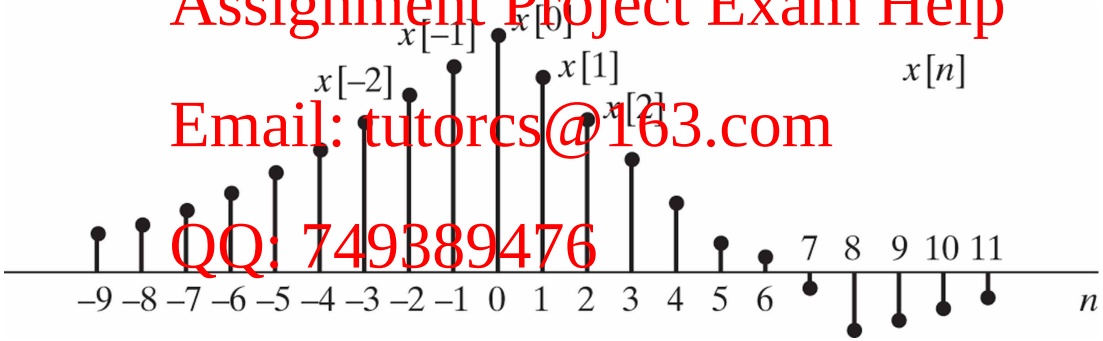
* An image varies with cartesian coordinates x and y in space (2D)

* The temperature can vary with position in a room horizontal x and y and vertical z (3D)
* A movie is a 2D image that varies with time (3D)

Digital Signals vs Discrete-Time Signals

What is the differenc

* **Digital signals** for which both time and amplitude are discrete. They are a special case of discrete-time signals.



Digital Systems vs Discrete-Time Systems

Signal Processing Sy also be classified as continuous-time syst ete-time systems and digital systems based on the input a signals.

* continuous-time systems: both inputs and outputs are continuous-time signals

* Discrete-time systems: both inputs and outputs are discrete-time signals

* Digital systems: both inputs and outputs are digital signals

Discrete-Time Signals: Notation

* $x[n]$, n is a disc  or an integer.

Discrete-Time Signals: Notation

* $x[n]$, n is a discr  or an integer.

* Can arise in periodic sampling of a continuous-time signal

$$x[n] = x_a(nT), \quad -\infty < n < \infty$$

where $x_a(t)$ is the corresponding continuous-time signal and T is the sampling period ($1/T$ is the *sampling frequency*).

Discrete-Time Signals: Notation

* $x[n]$, n is a discr  or an integer.

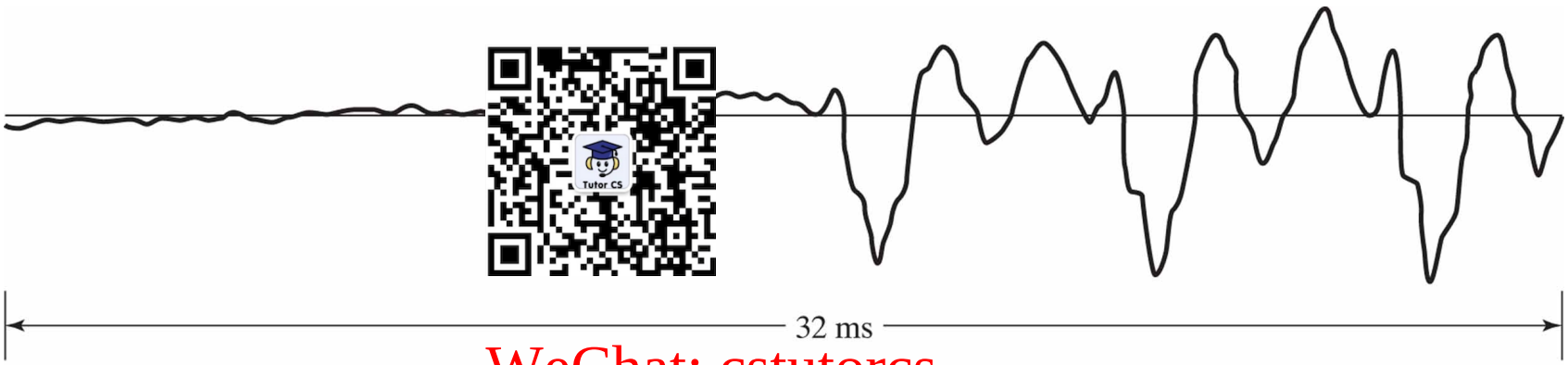
* Can arise in periodic sampling of a continuous-time signal

$$x[n] = x_a(nT), \quad -\infty < n < \infty$$

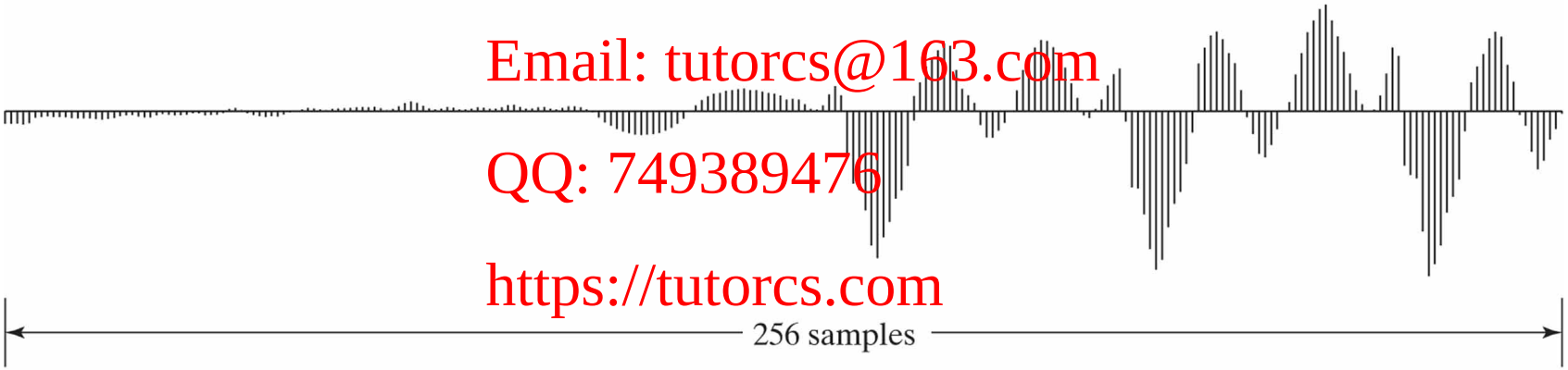
where $x_a(t)$ is the corresponding continuous-time signal and T is the sampling period ($1/T$ is the *sampling frequency*).

* $x[n]$ is **only** defined for integer values of n.

Example of Sampling



(a)



(b)

Basic Discrete-Time Signals

* Unit sample seq  impulse) $\delta[n]$

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0. \end{cases}$$

Basic Discrete-Time Signals

* Unit step sequen

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0. \end{cases}$$

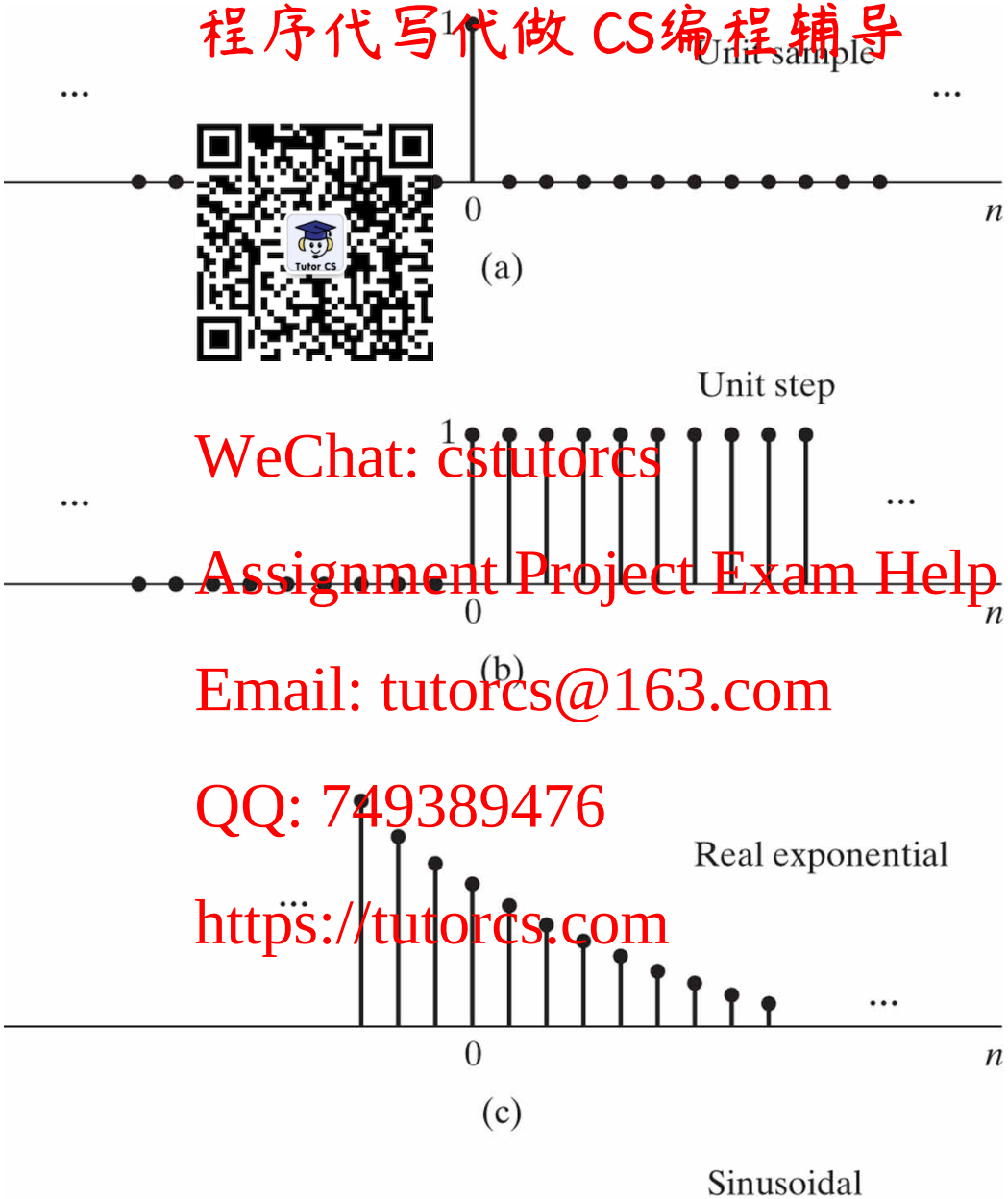
Basic Discrete-Time Signals

* Real exponential (A and α are real)

$$x[n] = A\alpha^n$$

* A and α are complex too, which results in a complex exponential sequence.

Pictures of Basic Discrete-Time Signals



Exercise

* Can you express discrete-time signal $x[n]$ in terms of $\delta[n]$?

Exercise

* Can you express discrete-time signal $x[n]$ in terms of $\delta[n]$?

* Express $u[n]$ in terms of $\delta[n]$.

Exercise

* Can you express discrete-time signal $x[n]$ in terms of $\delta[n]$?

* Express $u[n]$ in terms of $\delta[n]$.

* Express $\delta[n]$ in terms of $u[n]$.

Important Result

* Any input $x[n]$ can be written as a weighted sum of delayed impulses i.e.,

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Discrete Complex Exponential Signals

* $x[n] = A\, e^{j\omega_0 n}$.

Discrete Complex Exponential Signals

* $x[n] = A\,e^{j\omega_0 n}$.

* Continuous-time counterpart is $x(t) = Ae^{j2\pi ft} = Ae^{j\omega_0 t}$.

Discrete Complex Exponential Signals - Two important properties

* Frequency characteristics

Discrete Complex Exponential Signals - Two important properties

* Frequency characteristics

* Periodicity

Complex Exponential Signals - Frequency Spectrum

* Consider $x[n] = $  $r)n$ where $r \in \mathbb{Z}$ is an integer:

$$x[n] = A\,e^{j(\omega_0+2\pi r)n}$$

$$= A\,e^{j\omega_0 n}e^{j2\pi rn}$$

$$= A\,e^{j\omega_0 n}\underbrace{e^{j2\pi rn}}_{1}$$

$$= A\,e^{j\omega_0 n}$$

Complex Exponential Signals - Frequency Spectrum

* $x[n] = A\,e^{j\omega_0 n} = e^{j(\omega_0 + 2\pi r)n}$, $r \in \mathbb{Z}$.

Complex Exponential Signals - Frequency Spectrum

* $x[n] = A\,e^{j\omega_0 n} = e^{j(\omega_0 + 2\pi r)n}$, $r \in \mathbb{Z}$.

* **Important:** Complex exponential sequences with frequencies $(\omega_0 + 2\pi r)$, where r is an integer, are indistinguishable from one another.

Complex Exponential Signals - Frequency Spectrum

* $x[n] = A\,e^{j\omega_0 n} = e^{j(\omega_0 + 2\pi r)n}$, $r \in \mathbb{Z}$.

* **Important:** Complex exponential sequences with frequencies $(\omega_0 + 2\pi r)$, where r is an integer, are indistinguishable from one another.

* Thus, for a discrete-time signal $x[n] = Ae^{j\omega_0 n}$, we only need to consider frequencies in an interval of length 2π. Typically, one can choose, $-\pi < \omega_0 \leq \pi$ or $0 \leq \omega_0 < 2\pi$.

Complex Exponential Signals - Periodicity

* Recall: in contir e, complex exponentials (and sinusoids) are pe time with period equal to 2π divided by the frequency $= e^{j2\pi ft} = e^{j\omega_0 t}$, then the period is

$$T_0 = \frac{1}{f} = \frac{2\pi}{\omega_0}$$

Complex Exponential Signals - Periodicity

* Recall: in contin[illegible]e, complex exponentials (and sinusoids) are pe[illegible]time with period equal to 2π divided by the frequency[illegible] $= e^{j2\pi ft} = e^{j\omega_0 t}$, then the period is

$$T_0 = \frac{1}{f} = \frac{2\pi}{\omega_0}$$

* New: in discrete-time case, a periodic signal has the property $x[n] = x[n+N]$ for all n, where N is the period which has to be an integer.

Complex Exponential Signals - Periodicity

* For a complex exponential sequence

$$x[n] = e^{j\omega_0 n} = e^{j\omega_0(n+N)} = e^{j\omega_0 n} e^{j\omega_0 N}$$

Thus, we need $e^{j\omega_0 N} = 1 = e^{j2\pi k}$.

Complex Exponential Signals - Periodicity

* For a complex exponential sequence

$$x[n] = e^{j\omega_0 n} = e^{j\omega_0(n+N)} = e^{j\omega_0 n} e^{j\omega_0 N}$$

Thus, we need $e^{j\omega_0 N} = 1 = e^{j2\pi k}$.

Complex Exponential Signals - Periodicity

$\omega_0 N = 2\pi k$, where k is an integer: $k \in \mathbb{Z}$.

Complex Exponential Signals - Periodicity

* Hence, Complex exponential (and sinusoidal) are periodic only when

$$N = \frac{2\pi k}{\omega_0}$$

is an integer.

* **Thus, depending on the values of ω_0, some complex exponentials may not be periodic at all.**

Complex Exponentials: Example

* Example of a co[illegible]ponential discrete-time signal, which is periodic;

Complex Exponentials: Example

* Example of a complex exponential discrete-time signal, which is periodic;

Periodic signal: $x_1[n] = e^{j(\frac{\pi}{4}n)}$, $\omega_0 = \frac{\pi}{4}$, and $N = \frac{2\pi}{\omega_0} = 8$.

$$x_1[n+N] = e^{j(\frac{\pi}{4}(n+8))} = e^{j(\frac{\pi}{4}+2\pi)} = e^{j(\frac{\pi}{4}n)} = x_1[n]$$

Complex Exponentials: Example

* Example of a complex exponential discrete-time signal, which is not periodic;

Complex Exponentials: Example

* Example of a complex exponential discrete-time signal, which is not periodic;

Aperiodic signal: $x_2[n] = e^{j(n)}$; $\omega_0 = 1$, and $2\pi/\omega_0 = 2\pi$ (not an integer)

Complex Exponentials: $e^{j\omega_k n}$

* Remember: ω_0 and $\omega_0 + 2\pi r$, $r \in \mathbb{Z}$ are **indistinguishable** frequencies.

Complex Exponentials: $e^{j\omega_k n}$

* Remember: ω_0 and $\omega_0 + 2\pi r$, $r \in \mathbb{Z}$ are **indistinguishable** frequencies.

* If the period is N, then we have $\omega_0 N = 2\pi k$, hence

Complex Exponentials: $e^{j\omega_k n}$

* Remember: ω_0 and $\omega_0 + 2\pi r$, $r \in \mathbb{Z}$ are **indistinguishable** frequencies.

* If the period is N, then we have $\omega_0 N = 2\pi k$, hence

There are N **distinguishable** frequencies for which $e^{j\omega_k n}$ is periodic with period N, for example frequencies $\omega_k = 2\pi k/N$, where $k = 0, 1, \ldots, N-1$

* This is the basis for discrete-time Fourier analysis.

Low and High Frequencies

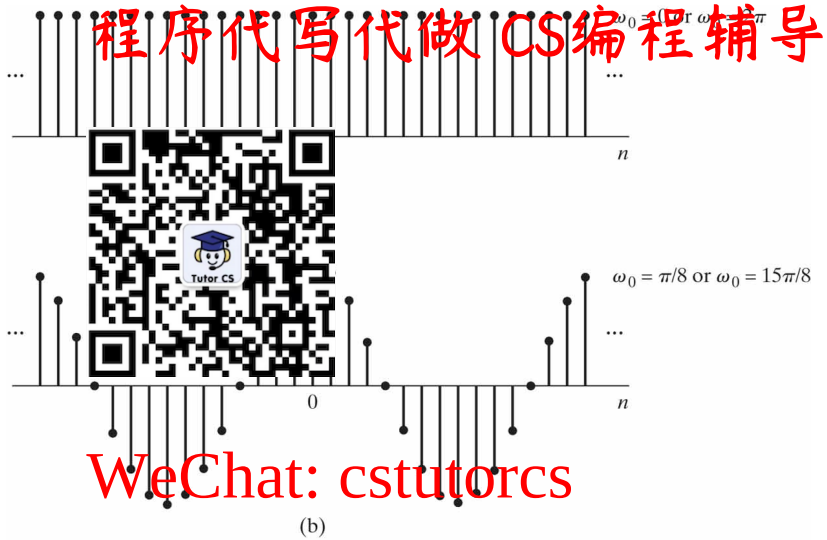
* Recall: for a continuous-time signal $x(t) = \cos \Omega_0 t$, as frequency Ω_0 increases, $x(t)$ oscillates progressively more rapidly.

Low and High Frequencies

* Recall: for a continuous-time signal $x(t) = \cos \Omega_0 t$, as frequency Ω_0 increases, $x(t)$ oscillates progressively more rapidly.

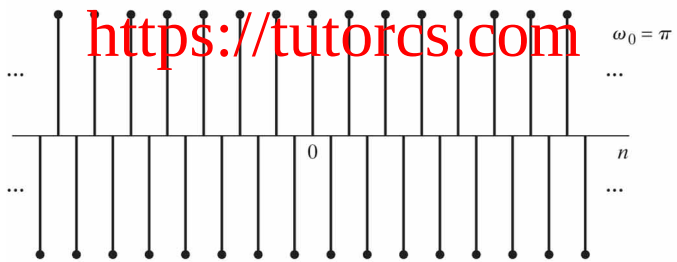
* New: for the discrete-time signal $x[n] = \cos \omega_0 n$, as ω_0 increases from 0 to π, $x[n]$ oscillate more rapidly. However, as ω_0 increases from π to 2π, the oscillation become slower.

$x[n] = \cos \omega_k n$



(b)

(c)



(d)

Low and High Frequencies

* Frequencies in the vicinity of $\omega_0 = 2\pi k$ are referred as *low frequencies*, whereas frequencies in the vicinity of $\omega_0 = \pi + 2\pi k$ are referred as *high frequencies* (for integer k).

Problem Set for this week

* Read and understand Chapter 2 of the textbook
* Problems Related to Lecture 01: 2.7, 2.21, 2.22, and 2.40.