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Classification

Supervised Learning Technique

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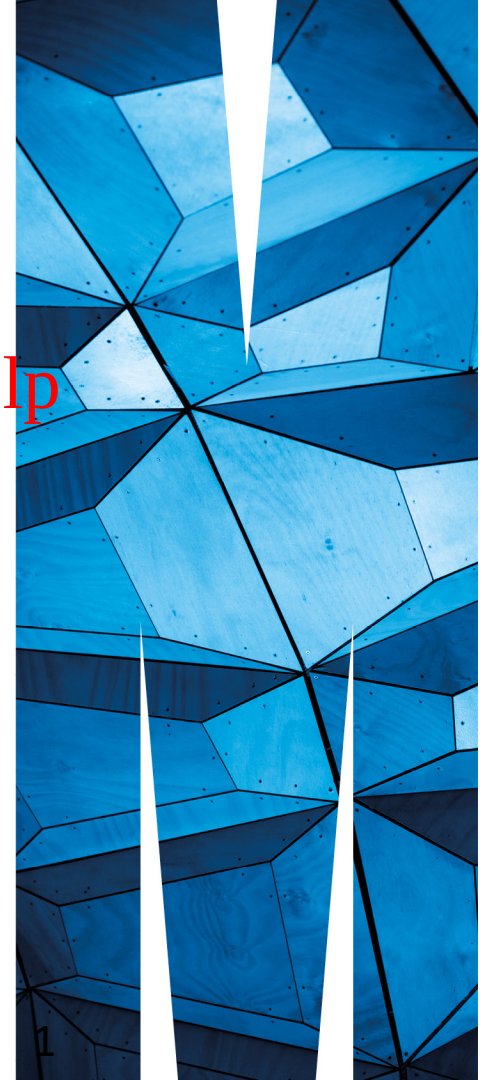

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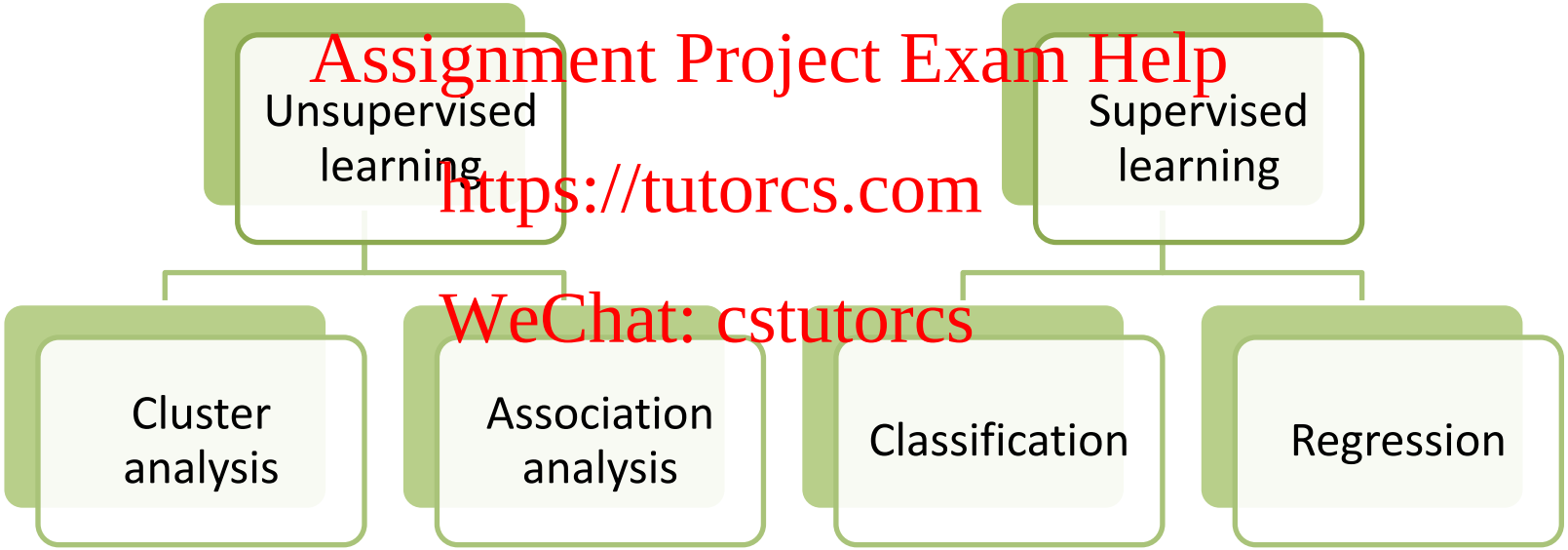
Introduction



Supervised Learning

- Start with some data that provides information about a set of “cases”
- For each case, we have predictor variables and a target variable

- Divide the data up into **training data** and **test data**
  - When dividing, be careful not to “cheat”

  - The test split must have the same form as if you would use in in the real world:
    - Cross-section Prediction: Random Splits
    - Temporal Prediction: Use cases from the future for the test set, not the past

Supervised Learning

- Use the training data to develop a model predicting the value of the target variable for given values of the predictor variables

- The test data must not be viewed at all until the model is built

- The model is judged on how well it "predicts" the test data, in other words we care about the **out of sample performance**. *How well would the model do when new information is collected that it had not seen before?*

Supervised Learning

- Usually try more than one model and compare their performance

- To do this without data snooping, divide the data into three parts: Training data, validating data, and test data

- The test data **must not** be involved in building or choosing the model

Supervised Learning

- The data consists of a (usually large) number of cases
- For each *case*, the data provides inputs $x_1, \ldots, x_n$ and a corresponding value for the target variable

- For example, input variables may be
  Age, Sex, Income, Number of children, Size of loan

- and the target variable may be
  "*Default on loan*" with possible values *Yes* and *No*
- Such data may be divided into parts to be used for training and testing

Advantages of Trees

- There are many classification procedures

- We focus on trees because they
  - Can be built efficiently
  - Easy to interpret
  - Can be building blocks for more accurate

- We consider *classification* and *regression* trees

Advantages of Trees – KTC Data Example

- Looking at those who purchased time share in a holiday house
- Data available
  - Numeric: Age, Income, Number of children
  - Binary: female, married, car loan, mortgage

- Predict price paid

  OR

- Predict bought or not

Classification and Regression Trees CART

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- If the target variable is categorical, e.g. with possible values *Yes* and *No* then for given values of the predictor variables, the decision tree predicts whether the target variable takes the value *Yes*
  - we have a *Classification Tree*

- If the target variable is numerical, then for given values of the predictor variables, the decision tree predicts the value of the target variable
  - we have a *Regression Tree*

- *The predictor variables can be of various types in each case*

Example: Ride-on Mowers

- Predictor variables: Lot size, Income
- Target variable: *Owner*
  Owner of ride-on mower: **Own** or **Not**
- Before introducing a tree classification method, let's investigate how ownership appears to depend on these two variables
- We have a very small data set and we use all the data to set up a model. We'll deal with validation and testing later

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Ride-on Mower Data

HH_ID	Income	Lot_Size	Owner	HH_ID	Income	Lot_Size	Owner
1	60	18.4	Own	13	75	19.6	Not
2	85.5	16.8	Own	14	52.8	20.8	Not
3	64.8	21.6	Own	15	64.8	17.2	Not
4	61.5	20.8	Own	16	43.2	20.4	Not
5	87	23.6	Own	17	84	17.6	Not
6	110.1	19.2	Own	18	49.2	17.6	Not
7	108	17.6	Own	19	59.4	16	Not
8	82.8	22.4	Own	20	66	18.4	Not
9	69	20	Own	21	47.4	16.4	Not
10	93	20.8	Own	22	33	18.8	Not
11	51	22	Own	23	51	14	Not
12	81	20	Own	24	63	14.8	Not

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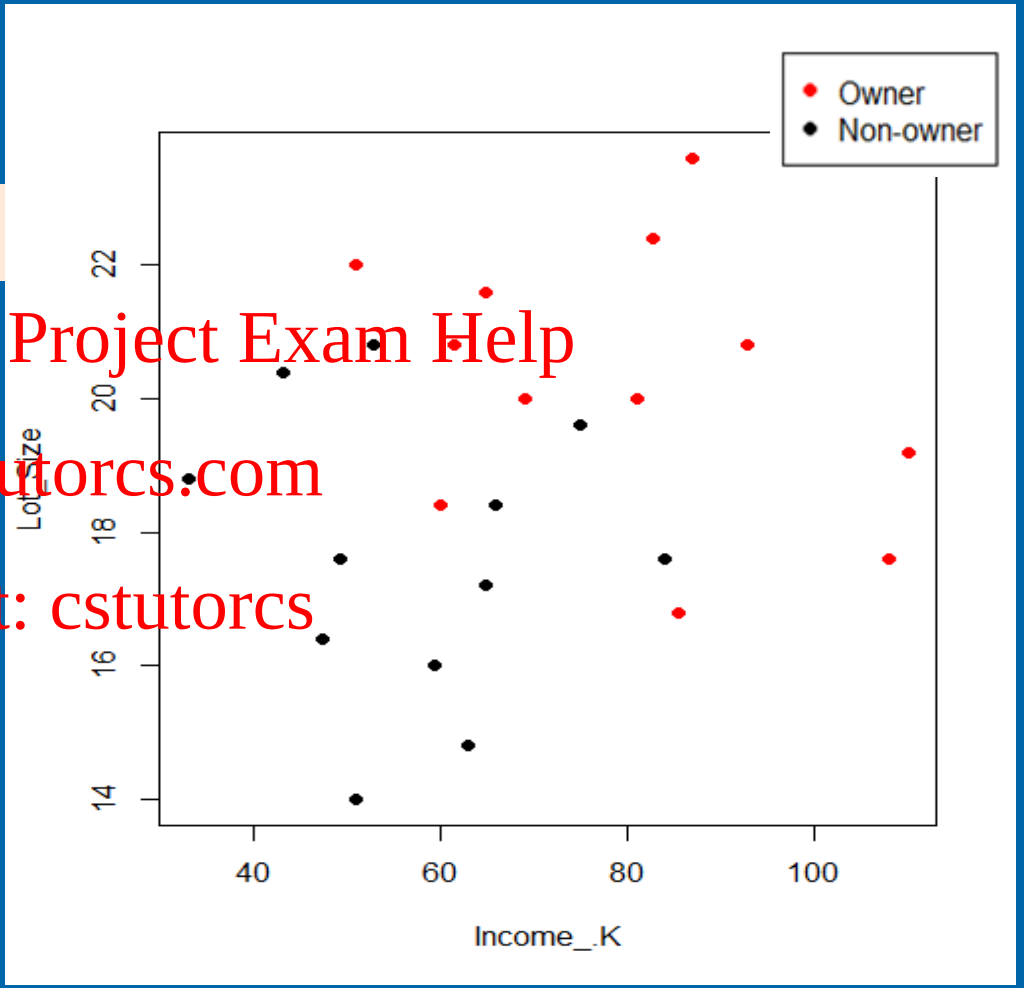
Ride-on Mower Data

We try to divide the data into a small number of rectangles, such that each rectangle has either

- mostly red, or
- mostly black dots.

We only use horizontal or vertical lines.

Here, we try doing this “by hand”



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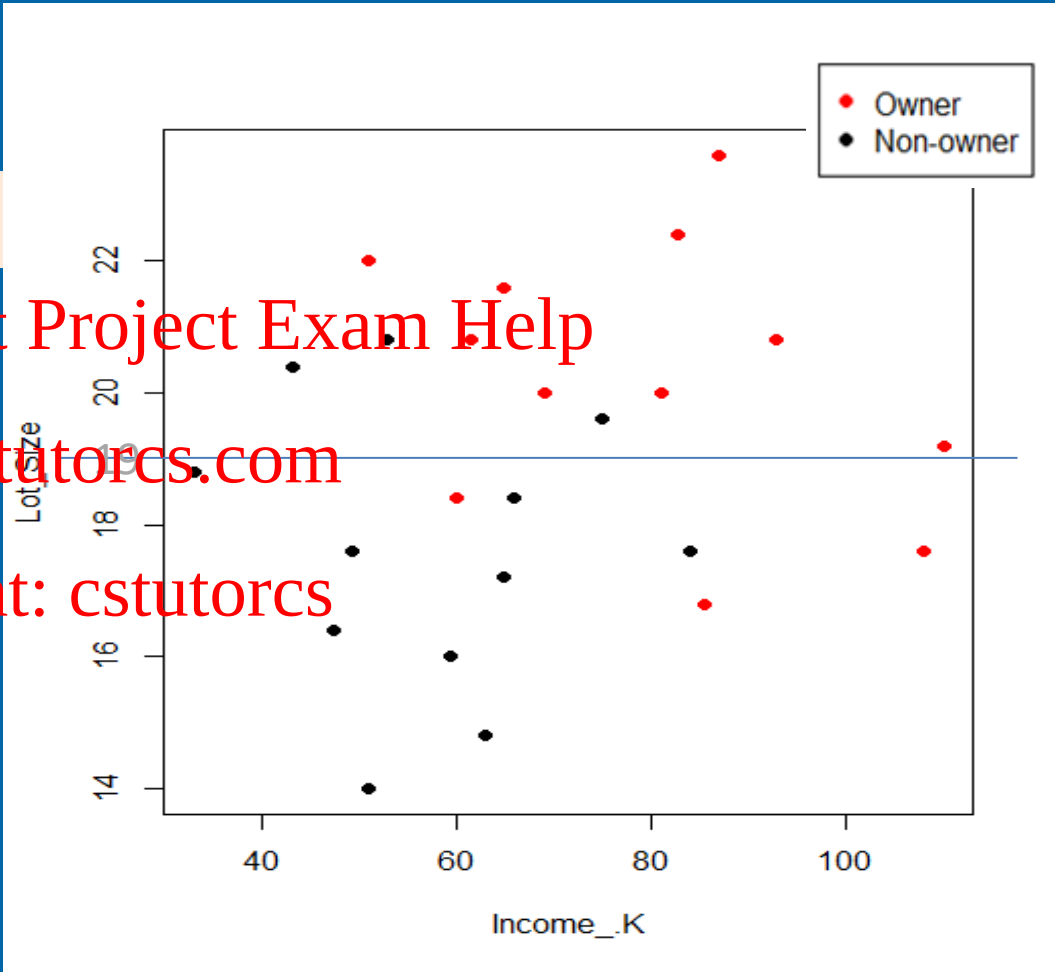
Partition 1

If lot size > 19, likely to be an owner

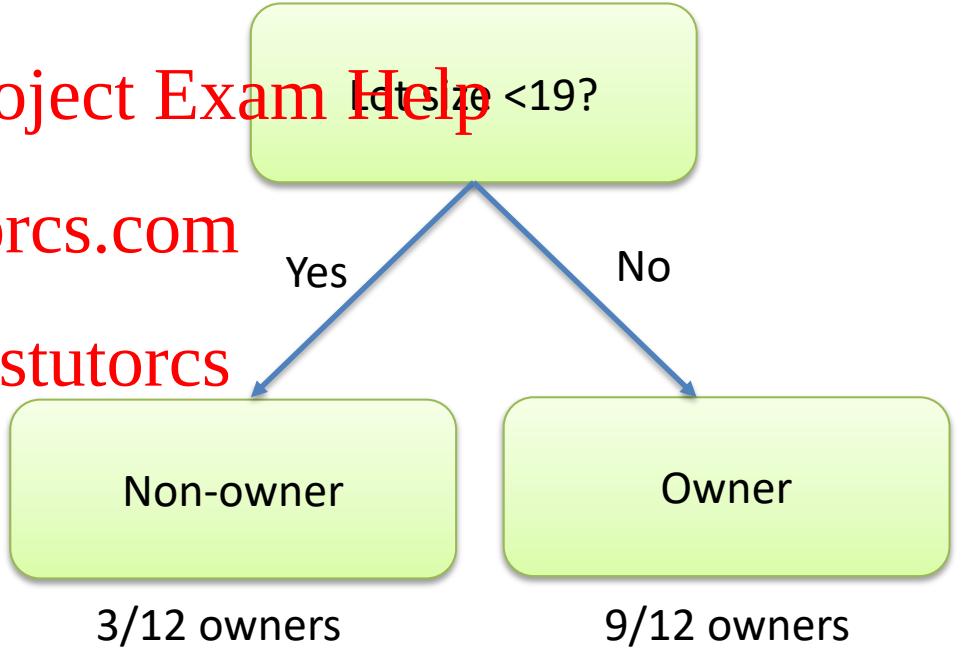
If lot size < 19, likely to be a non-owner

We are trying to find the best separating line



Tree 1

- If I only know about lot size, then based on this tree, I would predict that Lot_size < 19 is a non-owner
- Lot_size > 19 is an owner
- This is based on the largest classification group at the end of the tree (“leaf”)



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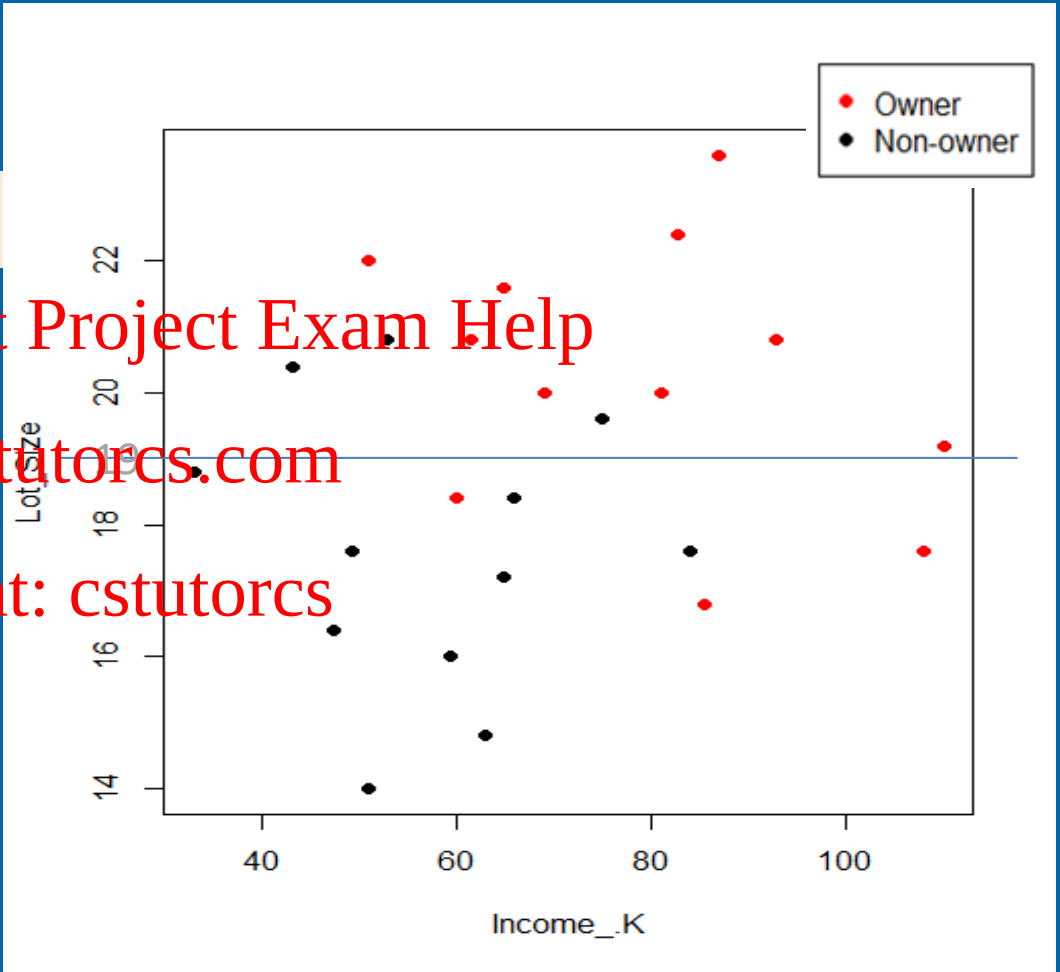
Partition 1

We are trying to choose the division into two parts of the data to minimise the impurity.
We choose which variable to split, and where to split it.
Choices: half way between pairs of consecutive values for the variable

Recursive partitioning



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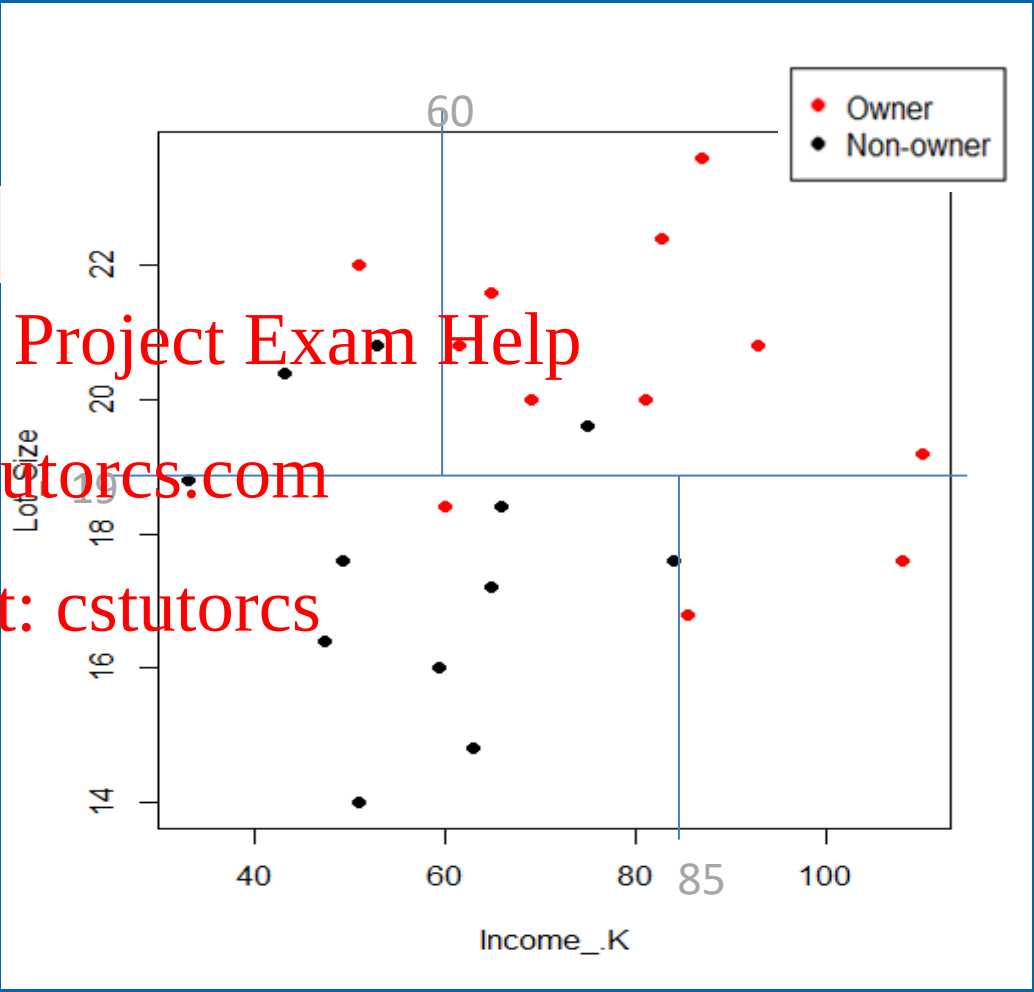
Partition 2

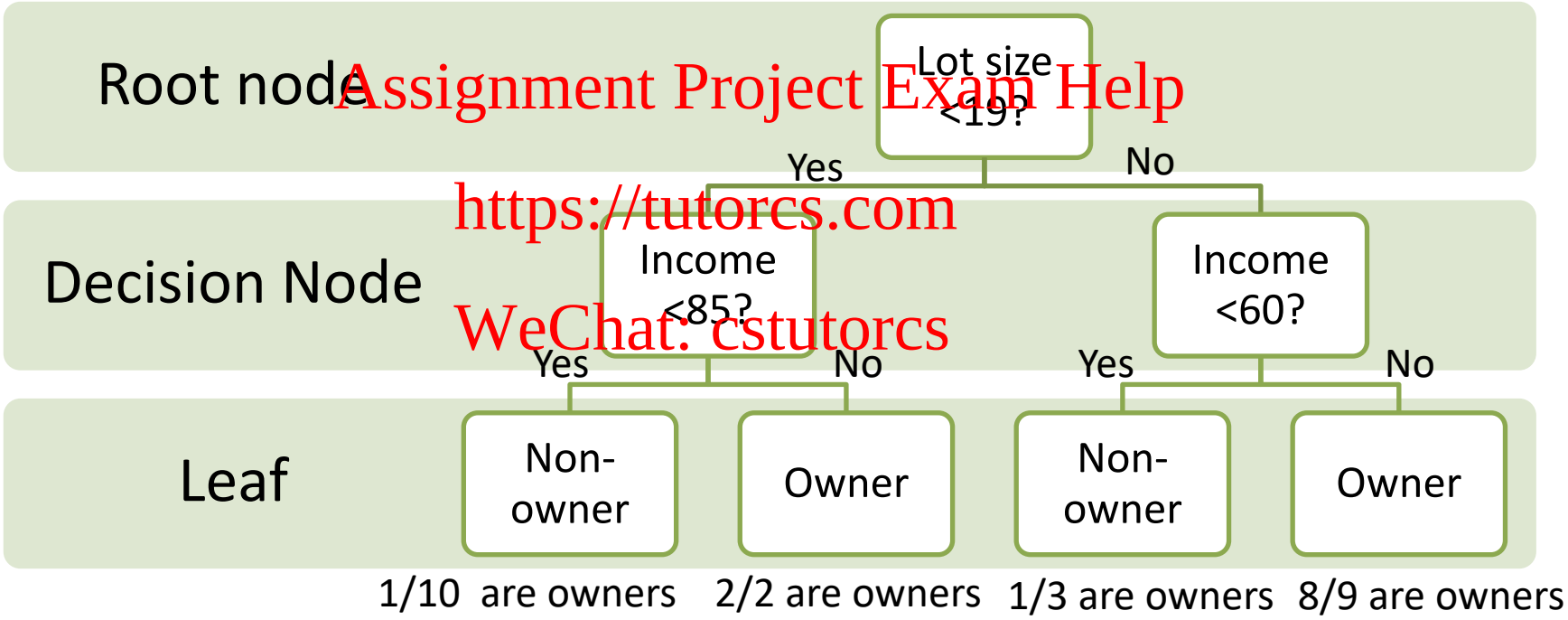
If lot size > 19, and income > 60,
likely to be an owner

If lot size > 19, and income < 60,
likely to be a non-owner

If lot size < 19 and income < 85,
likely to be a non-owner



Tree 2



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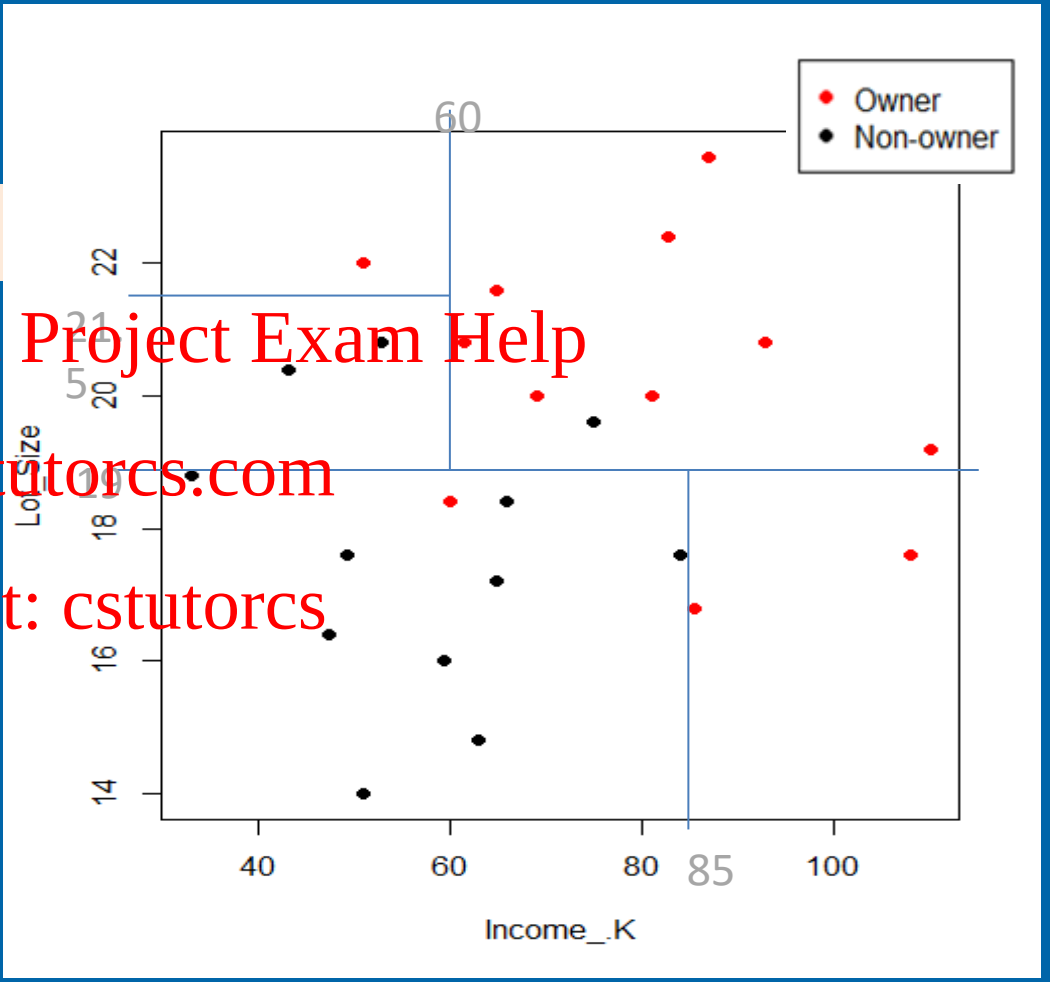
Partition 3

If lot size > 19, and income > 60,
likely to be an owner

If lot size > 19, and income < 60,
likely to be a non-owner

If lot size < 19 and income < 85,
likely to be a non-owner



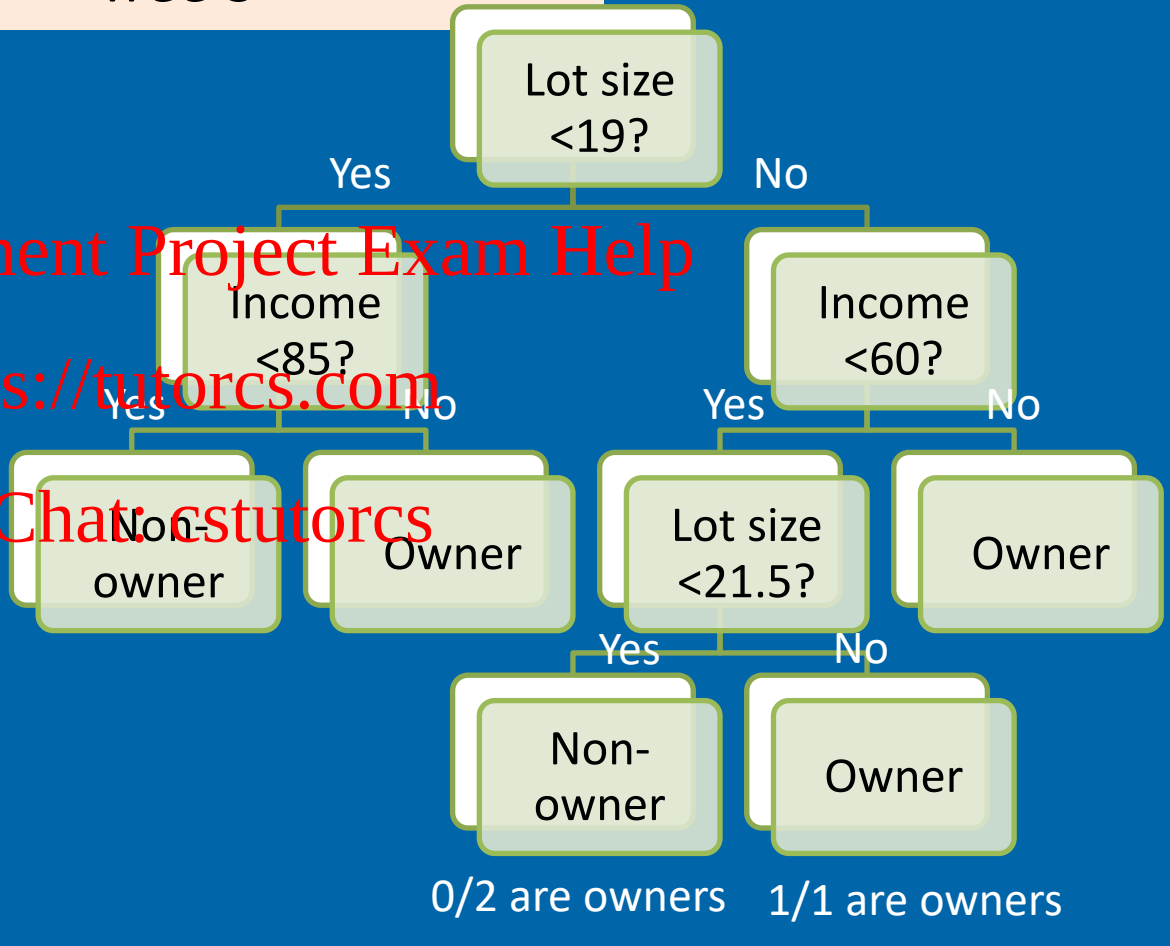
Tree 3

New case: Lot size 18.5, income $61,000
How classified?

Trace a path from the root node to a leaf.

At each node, choose the option corresponding to the new case

At the leaf, if the proportion of non-owners is higher than owners, predict the new case is a non-owner. And vice versa.



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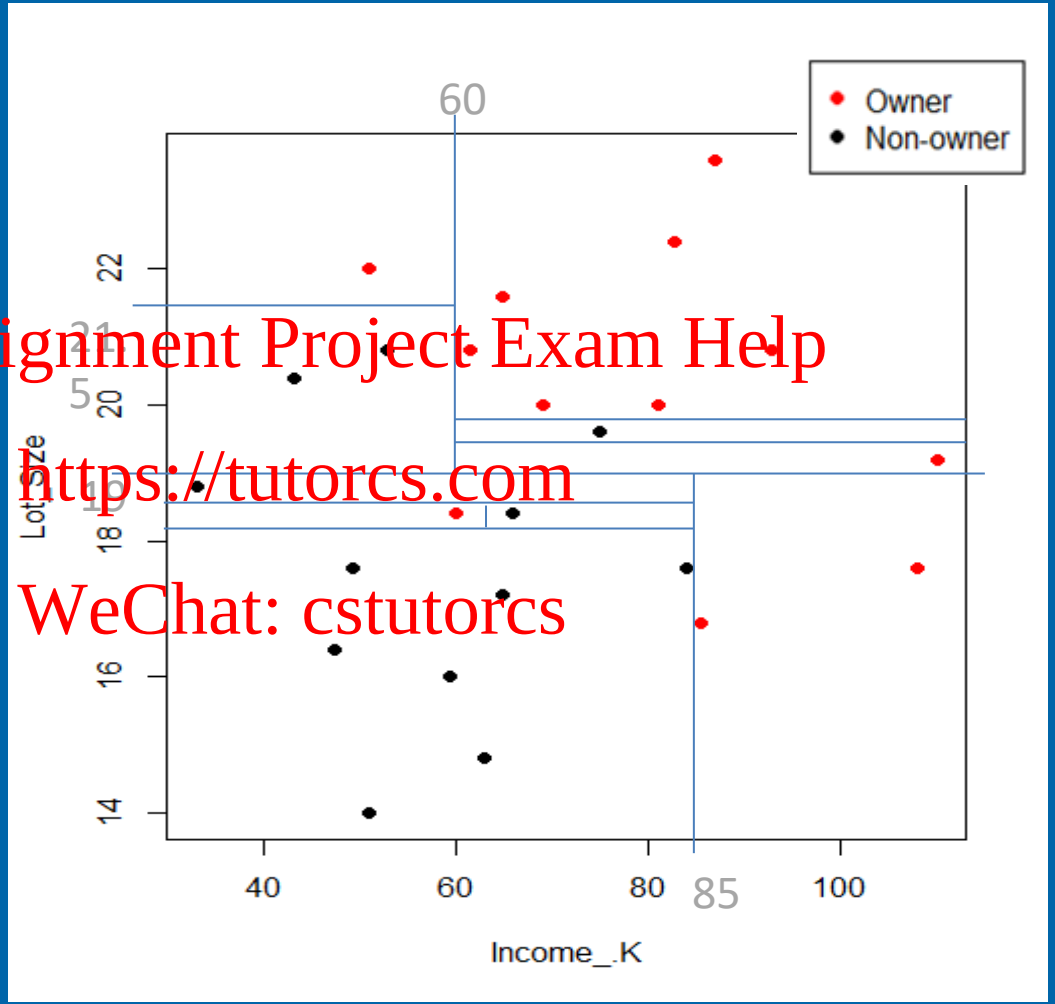
Overfitting



Systematic Construction of Trees

- We have been aiming to minimise disorder – trying to divide the data up so that in each partition there is just one colour, or at least very few of the other colour.
- So far we have done it in an ad hoc way.

If we specify a *measure of disorder*, then we can use computing power to choose at which value of which variable to put the next dividing line.

Choices: half way between pairs of consecutive values for the variable

Measuring Disorder

A measure of disorder is referred to as *deviance*

In this example we have two possible values of the target variable. We look at measures of disorder in this case

- In a particular segment of the space (at a particular leaf):
  - Suppose the proportion of red dots is ***p***, and of black dots is **1 – *p***
  - If *p = 0.5,* we have *maximum disorder*
  - If *p* = 0 or 1, we have *minimum (zero) disorder*

Measuring Disorder

- When there are two values of the target variable, the *Gini index* for disorder is $2p(1-p)$

This is a parabola, maximum at $p = 0.5$ and 0 at $p = 0$ or p = 1

If there are *m* possible outcomes: $Gini = \sum_{k=1}^{m} p_k(1-p_k)$

Measuring Disorder

Other measures of disorder

Entropy = $-2[p\log(p)+(1-p)\log(1-p)]$

- (very similar shape to Gini)

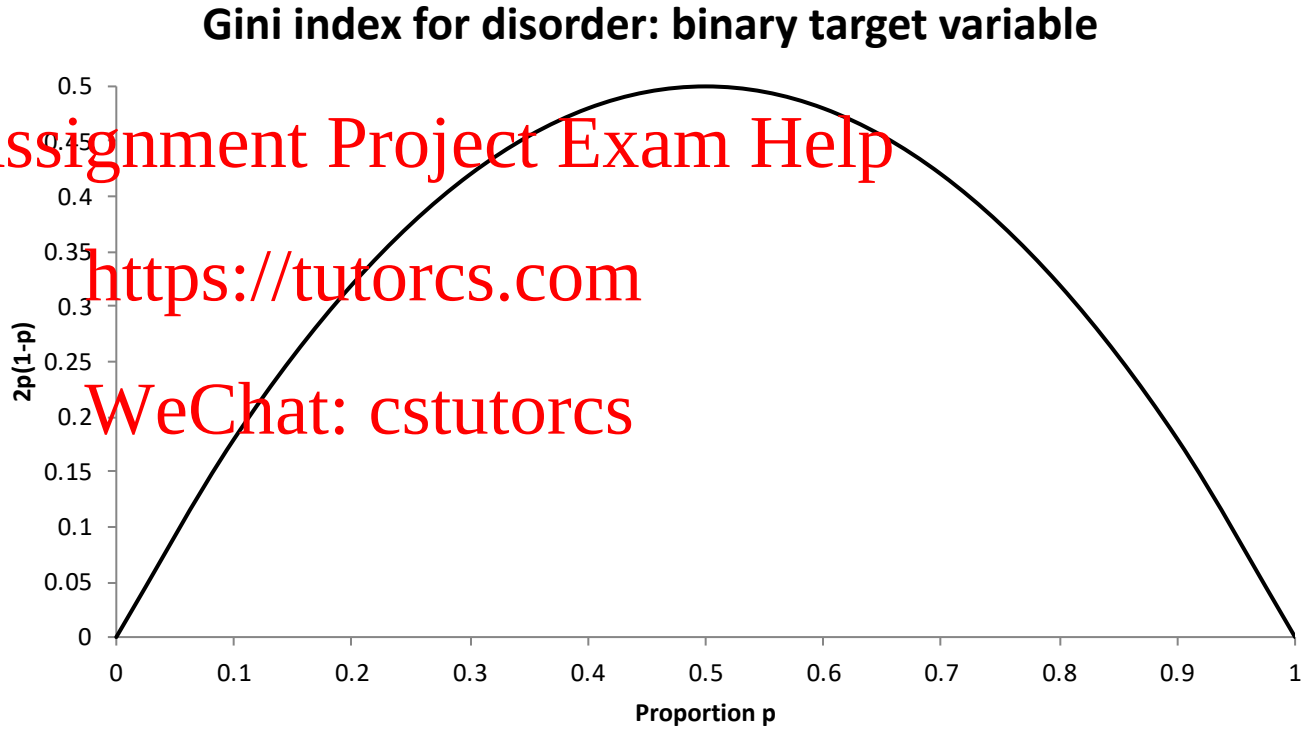
Misclassification index = $\min(p,1-p)$

- are other indexes of disorder (used in r)

Measuring Disorder

The *Gini index for disorder* is a parabola

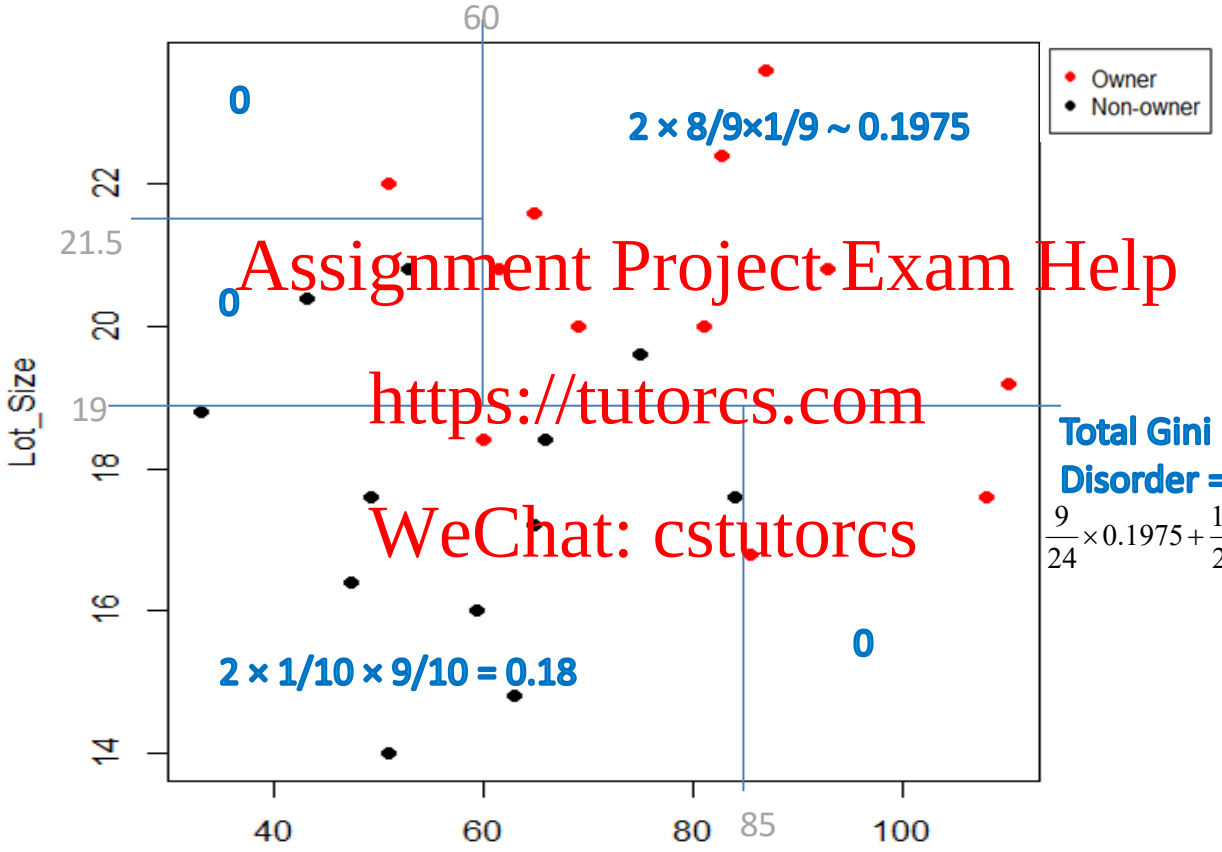
- maximum at $p = 0.5$ and 0 at $p = 0$ or p = 1



Gini Index of Disorder

- To get the Gini index of disorder for the whole tree
  - Calculate the contribution from each leaf as
  - Calculate the weighted sum of Gini indexes for each leaf, weighting by the number of observations in that leaf, as a proportion of all the observations in the sample.
- At each stage, we choose the node and the split at that node to minimise disorder.
- The algorithm is “greedy”: We never undo a partition
- Need to decide when to stop, or go all the way and then prune

Contributions to Disorder Measure



Total Gini measure of Disorder =

$$\frac{9}{24}\times 0.1975+\frac{10}{24}\times 0.18=0.149$$

$$Gini = 2p(1-p)$$

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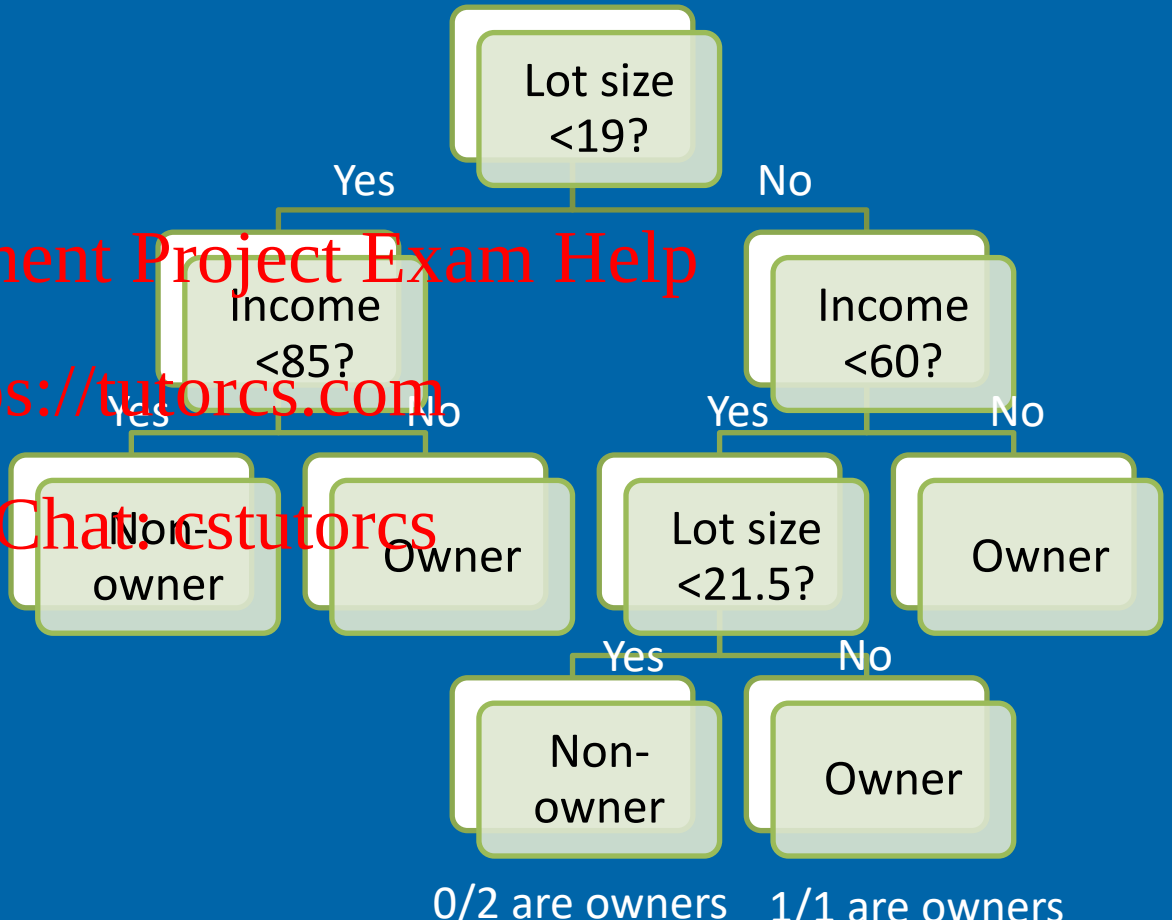
Tree 3



Systematic Classification by Tree

- Picking out by hand classifications that lead to low deviance worked fairly well for this small example
- The computer algorithm can very quickly find at each stage the split that minimises deviance (which variable to split, and where to split it)

Avoiding Overfitting

- We can restrict the size of the tree, or grow a large tree and then prune it back

- These options are investigated in *Workshop 10 Classification trees*

Cross Validation

- Small amount of data
- Wish to decide on optimum tree size
- Divide the data into, say, 4 groups of equal size (more commonly 10.) Call them A, B, C, D
- Leave out one of these four by turn, e.g. leave out A, and prune your tree using the remaining data: B, C, and D.
- Test how well the resulting tree predicts results for A.
- Automated in the command rpart

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Assessing the Model: Confusion Matrix

- We can check how reliable the predictions are
  - For the training data used to create the model
  - For test data where we know the value of the target variable but did not use the data in developing the model
- We consider *confusion matrix* and *error rate*

True Positive
TP = number of correct predictions that target = Own

False Negative
FN = number of incorrect predictions that target = Not

False positive
FP = number of incorrect predictions that target = Own

True negative
TN = number of correct predictions that target = Not

Error rate = $$\frac{FP + FN}{TP + FP + TN + FN}$$

Confusion Matrix:

	Predicted	
Actual	Own	Not
Own	TP	FN
Not	FP	TN

Assessing the Model: Confusion Matrix

- How do you expect the error rate to compare for training and test data?
- How do you expect the error rate to compare for eight-leaf tree and four-leaf tree?

True Positive
TP = number of correct predictions that target = Own

False Negative
FN = number of incorrect predictions that target = Not

False positive
FP = number of incorrect predictions that target = Own

True negative
TN = number of correct predictions that target = Not

Error rate = $$\frac{FP + FN}{TP + FP + TN + FN}$$

Confusion Matrix:

	Predicted	
Actual	Own	Not
Own	TP	FN
Not	FP	TN

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Assessing the Model: Confusion Matrix

- More generally, the *Error Rate* is the proportion of wrong classifications
- So for example in the following confusion matrix

	Predicted		
Actual	Renting	Mortgage	Own outright
Renting	26	**18**	**7**
Mortgage	**15**	56	**20**
Own outright	**2**	**13**	25

Error rate =

$$\frac{\text{Sum of off-diagonals}}{\text{Sum of all cells}} = \frac{18+7+20+15+2+13}{(18+7+20+15+2+13)+(26+56+25)}$$

$$= \frac{75}{182} = 0.412$$

Receiver Operating Characteristic Curve - ROC Curve

- True Positive Rate = $\frac{True\ Positive\ (TP)}{Positive\ (P)}$=TPR
  - Other names: Sensitivity, Recall
- True Negative Rate = $\frac{True\ Negative\ (TN)}{Negative\ (N)}$=TNR
  - Other names: Specificity, Selectivity
- False Positive Rate = $\frac{False\ Positive\ (FP)}{Negative\ (N)}$=FPR = 1-TNR

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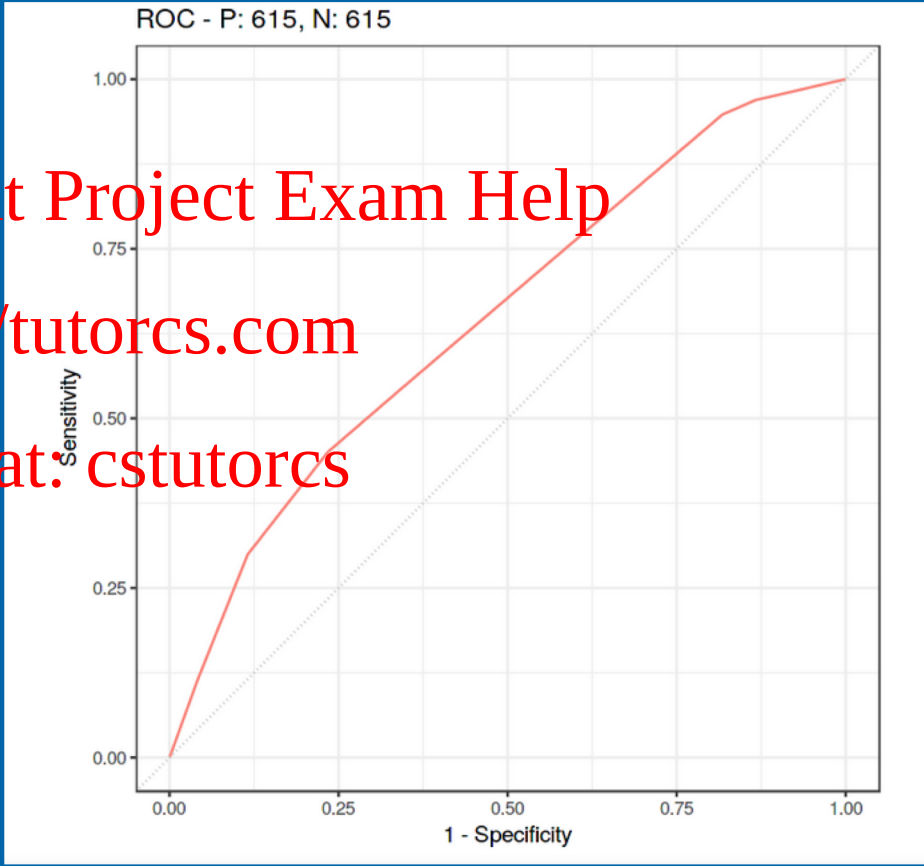
Receiver Operating Characteristic Curve - ROC Curve

AUC: 0.65



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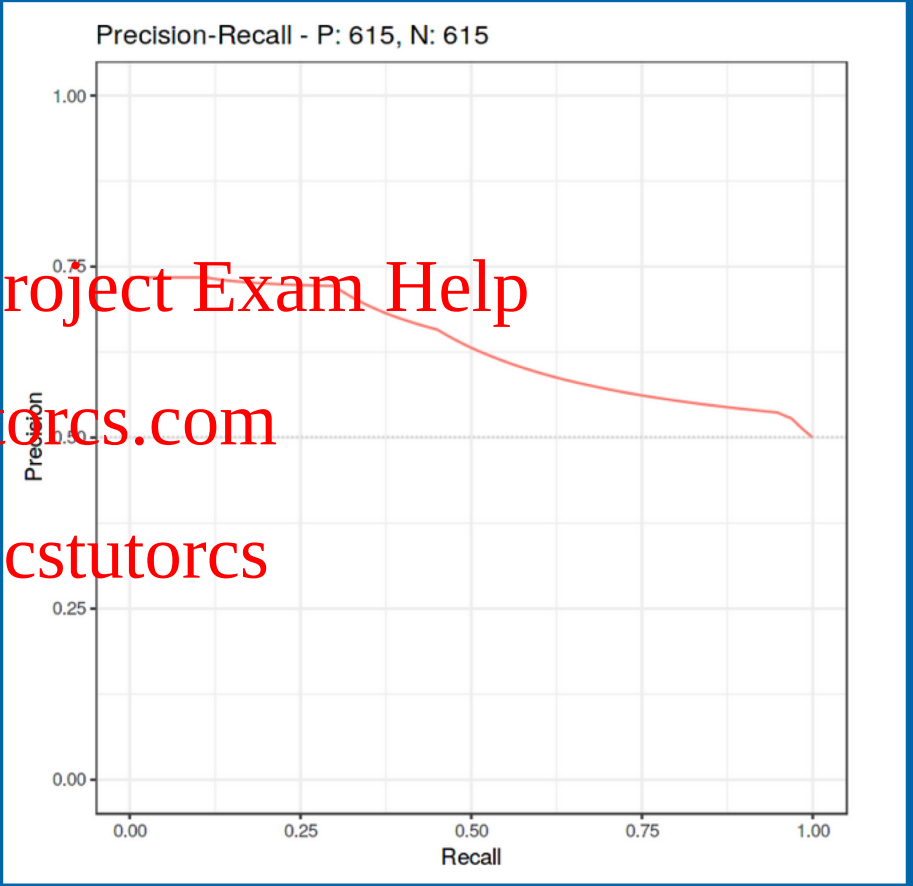
Precision-Recall Curve



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