Homework 2
Econ 421 Winter 2019

The due date for this homework is Thursday, February 21 in class.

1. **Tournaments.** There are two agents with output $y_i = e_i + X_i$. Suppose $\{X_i\}_{i=1,2}$ are IID with distribution $X_i \sim N(0, \sigma)$, and the cost of effort is $C(e_i) = e_i^2/2$. As in class, the firm awards a prize of W to the winner, i.e. whoever produces the most output. The loser gets L, where $W > L$.

   (a) In class we showed that in any symmetric pure strategy equilibrium, both agents choose effort e^* determined by

   $$(W - L)h(0) = C'(e^*)$$

   where $h(\cdot)$ is the pdf of $X_2 - X_1$. Using the extra parametric assumptions, what is equilibrium effort?

   (b) Suppose a firm employs both agents, and has profits $\Pi = y_1 + y_2 - W - L$. The agents have outside option zero. What are the optimal prizes? How do these change in σ? What is the intuition?

   (c) So far we've only considered symmetric equilibria in which the agents choose the same effort. Given any prizes (W, L), show there are no asymmetric pure strategy equilibria, where $e_1 \neq e_2$. [Hint: Observe that $h(\cdot)$ is symmetric around zero].

2. **Repeated Partnerships.** Two agents work together, choosing effort e_i at cost $C(e_i) = e_i^2/2$. They produce output $y = e_1 + e_2$ and split the output 50:50.

   (a) What is first-best effort e^{FB}? What is an agent's utility if both exert e^{FB}?

   (b) What is effort in the symmetric pure strategy Nash equilibrium effort, e^*? What is an agent's utility if both exert e^*?

   (c) Suppose agent 2 exerts e^{FB}. What is agent 1's best-response, and what is her resulting utility?

   Suppose the two agents repeatedly play the game over time $t = 1, 2 \ldots$, with a discount factor $\delta \in [0, 1)$.[1] Suppose they can observe each others effort and play the Nash-reversion strategy: "I will play e^{FB} each period, so long as you do likewise; if you deviate I will play e^* forever."

   (d) Show the above strategy is a subgame perfect Nash equilibrium (i.e. no-one wishes to deviate and trigger the punishment) if δ is above a critical threshold, δ^*. Numerically, what is δ^*?

[1] That is, x at time $t + 1$ is worth δx at time t.

3. **Relational Contracts.** A firm employs a worker, and motivates them via a relational contract (i.e. all actions are voluntary). A period has three stages. First, the firm pays the worker a base wage $w \geq 0$. Second, the worker chooses effort $e \geq 0$ at cost $c(e)$, which generates output $y = e$ for the firm. Third, the firm pay the worker a bonus $b \geq 0$. Each period, the worker gets utility $u = w + b - c(e)$, where $c(.)$ is increasing and convex, while the firm gets profit $\pi = e - w - b$.

   (a) Suppose there is only one period. What is the Subgame Perfect Nash Equilibrium of the three stage game?

   Now, suppose the game infinitely repeated, $t = \{0, 1, 2, \ldots\}$, and there is common discount factor δ. We wish to characterize the optimal stationary relational contract, (w, e, b). If either party deviates, the players revert to the SPNE of the one-period game, as in part (a). Let U and Π be the net present value of the agent's utility and the firm's profit.[2]

   (b) Argue that the agent will not deviate if $\delta U \geq c(e) - b$.

   (c) Argue that the firm will not deviate if $\delta \Pi \geq b$.

   (d) Observe that summing the above two constraints implies that

   $$\delta(U + \Pi) \geq c(e) \tag{1}$$

   Suppose we maximize welfare $e - c(e)$ subject to (1).[3] What is the optimal level of effort, e^*? [Hint: Draw a picture].

   (e) Fixing e^*, suppose we want to give all the surplus to the firm, so $U = 0$. What are the optimal wage and bonus (w^*, b^*)? Conversely, suppose we want to give all the surplus to the agent, so $\Pi = 0$. What are the optimal wage and bonus (w^*, b^*)? What is the intuition behind your answers?

4. **Testing Peer Effects (In Theory).** Suppose we have data from Safelite stores across the US. We observe the productivity of each worker each month over 2013-2018. We wish to estimate whether a worker is more productive when they are working with other productive workers.

   (a) We regress the productivity of agent i on the mean productivity of her co-workers and find a positive coefficient. Are there positive peer effects? Why might you worry about self-selection and omitted variable bias?

   (b) Your manager suggests introducing some controls into the regression. What controls might help? How?

   (c) Suppose new recruits are assigned randomly to Safelite stores. How can you overcome the two problems identified in part (a)?

   (d) Suppose new recruits are not assigned randomly, but you know where they live. How can you use an instrument to help solve the two problems identified in part (a)?

[2] Recall, $x + \delta x + \delta^2 x + \ldots = x/(1 - \delta)$.

[3] Analogous to the moral hazard problem we can focus on maximizing total value subject to the joint incentive constraint. As we see in part (e), the surplus can be split arbitrarily between the parties.