# Homework 3
## Econ 421 Winter 2019

The due date for this homework is Thursday, March 14 in class.

1. **Screening with two types.** A monopolist is producing a good of quality $q$ at cost $q^2/2$ and offering it at a price $t$ to a consumer with value $\theta q$, where $\theta \in \{L, H\}$ with $0 < L < H < 2L$.

   (a) Assume first that the monopolist can observe $\theta$. What is the profit maximizing mechanism $(q_\theta^{FB}, t_\theta^{FB})$?

   (b) Assume now that the monopolist cannot observe $\theta$, but assumes that $L$ and $H$ are equally likely. He is using a direct mechanism, asking the consumer for his type $\theta$ and assigning him outcome $(q_{\tilde{\theta}}, t_{\tilde{\theta}})$ based on the reported type. Show that if such a mechanism is to induce truth-telling by the consumer and voluntary participation, it must satisfy

$$\begin{aligned} Hq_H - t_H &\geq Hq_L - t_L \qquad (1) \\ Lq_L &\geq t_L \qquad (2) \end{aligned}$$

   (c) Assume that (IC-H) and (IR-L) bind (i.e. are satisfied with equality), and determine the profit-maximizing contract. How do quality levels $q_\theta^*$ compare to the quality levels $q_\theta^{FB}$ from part (a). Explain the difference. How do your findings compare to the model with continuous types discussed in class?



2. **Monopsonistic Screening.** Jeff Bezos is opening a Amazon fulfillment center in South Park, Colorado, and employs Stephen Stotch for $q$ hours a day for a wage of $p$. Jeff's profits are $\pi = V(q) - p$, where $V(q)$ is increasing and concave. Stephen's utility is $u = p - cq$, where $c \sim U[0, 1]$. His outside option is 0 (all the other stores in South Park have shut down).

   Help Jeff maximize profits by proposing mechanism $(p(\tilde{c}), q(\tilde{c}))$, where $\tilde{c}$ is Stephen's self-reported cost type.

   (a) Write down Stephen's utility, $u(c, \tilde{c})$, given his true cost $c$ and his report $\tilde{c}$. Show that in a truthful (i.e. "incentive compatible") mechanism, utility $U(c) = u(c, c)$ is given by

$$U(c) = \int_c^{\bar{c}} q(s)ds + U(\bar{c})$$

   (b) Show that Stephen's ex-ante expected utility is

$$\int_{\underline{c}}^{\bar{c}} U(c)dc = \int_{\underline{c}}^{\bar{c}} [q(c)c]\, dc + U(\bar{c})$$

(c) Show that in Jeff's optimal mechanism, profits equals

$$\pi = \int_{\underline{c}}^{\bar{c}} [V(q(c)) - MC(c)q(c)]dc$$

where $MC(c) = 2c$.

(d) What is the number of hours worked, $q^*(c)$? What is $q^*(c)$ if $V(q) = \log(q)$?

3. **Three types of lemons.** A used cars salesman is selling a car to a competitive market of buyers, who value the car at $v$, while the seller values it at $v - 15$. Only the seller knows $v$, while buyers believe $v$ is equally likely to be $L = 80, M = 100$, or $H = 120$.

   (a) Show that the good car, $H$, cannot be traded in equilibrium.

   (b) Show that there are three equilibria, one in which only bad cars, $L$, are traded, and two in which both bad and medium cars, $L$ and $M$ are traded.[1]

   (c) Which of the equilibria is the most efficient?

4. **Adverse Selection.** A competitive market of health insurers is offering insurance to a customer with privately known cost type $c \sim U[0, 1]$. Serving the customer costs an insurance company $c$, while the value of the insurance to the consumer is $\alpha + \beta c$ where $\alpha \in [0, \frac{1}{2}]$ and $\beta > 1$, capturing risk aversion by the customer, and the fact that insurance companies can bargain discounts with providers.



   (a) Which customer types $c$ will purchase the insurance at a given price $p$.

   (b) Solve for the equilibrium price $p^*$, and show that $\alpha \leq p^* \leq 1/2$, with strict inequalities if $\alpha < 1/2$.



   (c) Under which conditions are all customers insured in equilibrium?

   (d) Assume that the conditions in part (c) are violated. Describe the "unraveling logic" in this application, i.e. assume first that everybody is insured, calculate the break-even price for the insurers, then determine which types will actually purchase insurance, recalculate the break-even price, and so on. Can this market completely unravel, so that no type $c$ purchases insurance?



[1]In one of the latter equilibria, the type $M$ seller is indifferent and mixes over selling and keeping the car.