Homework 1
Econ 421 Winter 2019

The due date for this homework is Thursday, January 31st in class.

1. **Performance pay for singles and family men:** Your company is employing both singles and family men. They are equally productive, producing output $Z = e + X$, where effort e has cost $C(e) = \frac{1}{2}e^2$, and the random factor X is equally likely to be 0 or 2. Revenue is pZ with $p = 3$. Family men are more risk averse with risk aversion $r_f = 2$ while singles have $r_s = 1/2$.

   (a) What is expectation $E = E[X]$ and variance $V = Var(X)$ of X (and hence of output Z)?

   (b) How much effort will singles and family men exert under a linear contract $W = \alpha + \beta Z$?

   (c) You can offer different contracts to singles and to family men. What value of β_f, β_s maximizes the total certainty equivalent for either kind of employee?

   (d) What wage does the employer need to pay to a single/family-man to make him accept the job (outside option is $\overline{w} = 4$)? Calculate the expected compensation for either type of employee.

   (e) Calculate the expected profit from either kind of employee.

2. **Absolute vs relative performance pay:** You want to incentivize your sales manager responsible for Los Angeles. His sales $Z_A = e_A + X_A + X_C$ depend on his effort e_A (which he controls), the state of the LA economy X_A and the state of the California economy X_C (both of which are random and out of his control). You have a different sales manager for Orange county whose performance $Z_B = e_B + X_B + X_C$ depend on the other manager's effort e_B (which is effectively known in equilibrium), the state of the Orange county economy X_B and X_C as for the LA manager. All random variables X_A, X_B, X_C are independent. On top of the LA manager's base wage α, you want to reward him for his absolute sales Z_A and/or his relative sales compared to Orange county $Z_A - Z_B$.

   (a) Which of these two options (i.e. absolute sales, relative sales) exposes the LA manager to less risk, and therefore requires the lower risk premium?

   (b) Now you consider also combinations of absolute sales, and relative sales, i.e. $\delta Z_A + (1-\delta)(Z_A - Z_B)$. What level of δ minimizes the risk to the LA manager? What happens as X_B is much more uncertain than X_C? Interpret your result.

3. **Multitasking and ownership.** A firm needs the services of a truck driver. The truck driver has two actions. First, she chooses the level of service, $e_1 \in [0, \infty)$, which generates output $y_1(e_1) = e_1$ to the firm. Second, she chooses the maintenance of the

truck, $e_2 \in [0,1]$, which yields benefits pe_2 to whoever owns the truck, where $p > 1$. The two tasks compete for the driver's attention; the cost of effort is

$$C(e_1, e_2) = \begin{cases} 0 & \text{for } e_1 + e_2 \leq 1 \\ \frac{1}{2}(e_1 + e_2 - \overline{e})^2 & \text{for } e_1 + e_2 \geq 1 \end{cases}$$

As in class, we assume that the driver splits her effort, $(e_1, e_2) = (1/2, 1/2)$, when given no explicit incentives. The service level e_1 is observable, but the maintenance level e_2 is not. The firm thus chooses linear compensation schedule $\alpha + \beta e_1$ to maximize it's profits. The driver's outside option is 0.

(a) First assume that the driver is the employee of the firm, so the firm gains the benefit $y_2(e_2)$. What is the driver's optimal choice of (e_1, e_2) when $\beta = 0$, and when $\beta > 0$? What is the firm's optimal choice of β? How does your answer depend on p? Interpret your answer.

(b) Now assume that the firm hires an independent driver who owns her own truck. That is, the driver gains the benefit $y_2(e_2)$ in addition to the $\alpha + \beta e_1$ from the firm. For simplicity, assume $\beta \leq 1$, and recall that $p > 1$ and $e_2 \in [0,1]$. What is the driver's optimal choice of (e_1, e_2)? What is the firm's optimal choice of β?

(c) Now assume that maintenance is important, i.e. $p > 2$. Discuss how your answers in parts (a) and (b) differ.

4. **Testing Incentive Pay (in Theory).** In January 2016 Starbucks introduced incentive pay into its California stores, so managers get 2% of revenue.[1] In Oregon and Washington, stores pay managers a fixed wage.

You initially have cross-sectional data on annual revenue for 2016 for all California, Oregon and Washington stores. You observe that the mean revenue in California stores is significantly larger than Oregon and Washington stores.

(a) Can we conclude that incentive pay raises revenue? Explain how location can cause omitted variable bias. If you know the location of every store, what analysis can you do to reduce this bias? Explain why this analysis may work, and why is may not.

(b) What other store characteristics may interfere with the comparison. What data would you like to help you to make this conclusion?

Suppose we have concluded that incentive pay raises revenue (as best as we can).

(c) Can we conclude that Starbucks should role out the scheme across the US?

(d) Can we conclude that incentive pay makes managers work harder? What extra data would you need to test this? What analysis would you perform?

[1] This is just a thought experiment.