

程序代写代做 CS编程辅导

FIT2014 Theory of Computation



Lecture 21

Mapping Reductions

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Overview

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- ▶ Mapping reductions: relating one language to another
- ▶ Definition
- ▶ Properties
- ▶ Examples

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Mapping reductions

Definition

A **mapping reduction** from language K to language L is a computable function $f : \Sigma^* \rightarrow \Sigma^*$ that, for every $x \in \Sigma^*$,

$$x \in K \text{ if and only if } f(x) \in L.$$

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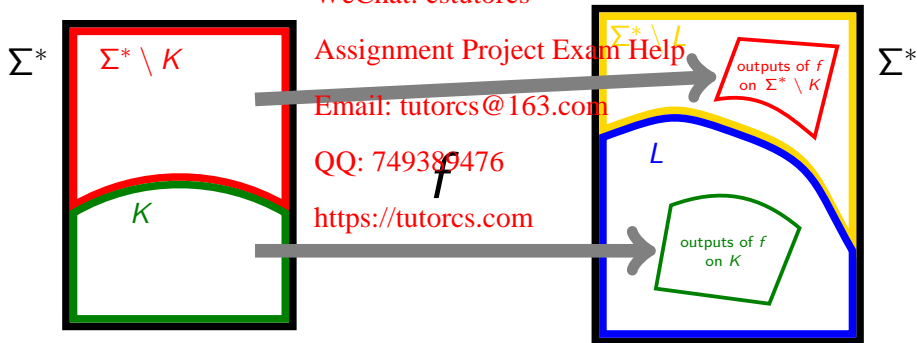
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Mapping reductions

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Notation:

$K \leq_m L$ means mapping reduction from K to L .



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A very simple property:

Every language is mapping-reducible to itself:

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$$\forall L : L \leq_m L$$

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Mapping reductions

Theorem

If there is a mapping reduction f from K to L , then:

If L is decidable, then K is decidable.

Symbolically:

$$(K \leq_m L) \wedge (L \text{ is decidable}) \Rightarrow (K \text{ is decidable})$$

Proof.

Decider for K :

Input: x .

Compute $f(x)$.

Run the Decider for L on $f(x)$.

// This L -Decider accepts $f(x)$ if and only if $x \in K$,
since f is a mapping reduction from K to L . \square

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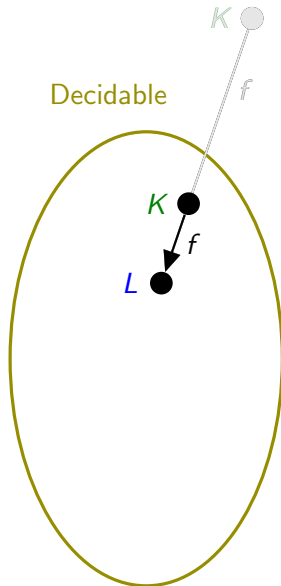
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Mapping reductions

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Corollary

If there is a mapping reduction f to L , then:

If K is undecidable, then L is undecidable.

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Symbolically:

$(K \leq_m L) \wedge (K \text{ is undecidable}) \implies (L \text{ is undecidable})$

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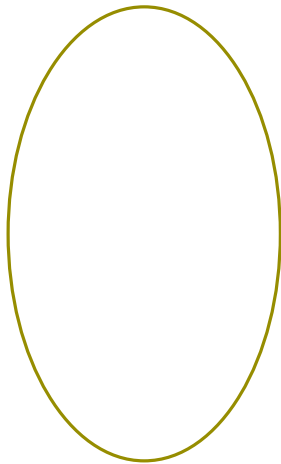
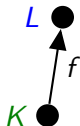
Proof.

Contrapositive of previous Theorem \square

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Decidable



EQUAL to HALF-AND-HALF

Mapping reduction f :

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Input: a word w over $\{a, b\}$

Sort w

Output the sorted word.



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$w \in \text{EQUAL} \iff$ it has the same number of a's as b's

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\iff after sorting, it has the same number of a's as b's
(since sorting does not affect letter frequencies)

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$\iff f(w)$ consists of some number of a's followed by

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the same number of b's
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$\iff f(w) \in \text{HALF-AND-HALF}$

HALF-AND-HALF to PARENTHESES

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Mapping reduction:

Input: a word w

For each letter of w in turn

If previous letter was b and current letter is a

// We have just seen "ba" which is impossible in HALF-AND-HALF.

Output the string $)$.

else

replace current letter as follows:

$a \mapsto ($

$b \mapsto)$

Output: the string obtained from w by doing all these replacements.



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EQUAL to PARENTHESES

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Is there a mapping reduction from EQUAL to PARENTHESES?

Yes! Compose the two previous reductions.



This is a special case of:

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Theorem.

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Mapping reducibility is transitive:

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$$K \leq_m L \leq_m M \Rightarrow K \leq_m M.$$

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Mapping reductions: transitivity

Theorem.

Mapping reducibility is transitive:

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$$K \leq M \implies K \leq_m M.$$


Proof.

Let f be a mapping reduction from K to L and
let g be a mapping reduction from L to M .

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We claim that the composition $g \circ f$, defined for all w by $(g \circ f)(w) = g(f(w))$,
is a mapping reduction from K to M .

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Since f and g are both computable, $g \circ f$ must be too.

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$$\begin{aligned} w \in K &\iff f(w) \in L && \text{(since } f \text{ is a mapping reduction from } K \text{ to } L) \\ &\iff g(f(w)) \in M && \text{(since } g \text{ is a mapping reduction from } L \text{ to } M) \\ &\iff (g \circ f)(w) \in M && \text{(by definition of } g \circ f). \end{aligned}$$

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FA-Empty \longrightarrow No-Digraph-Path

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From previous lecture:



FA-Empty $:= \{ \langle A \rangle : A \text{ is a FA and } L(A) = \emptyset \}$

Digraph-Path $:= \{ \langle G, s, t \rangle : G \text{ is a directed graph, } s, t \text{ are vertices in } G, \text{ and there exists a directed } s-t \text{ path in } G. \}$

No-Digraph-Path $:= \{ \langle G, s, t \rangle : G \text{ is a directed graph, } s, t \text{ are vertices in } G, \text{ and there does not exist a directed } s-t \text{ path in } G. \}$

We give a mapping reduction from FA-Empty to No-Digraph-Path.

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FA-Empty \longrightarrow No-Digraph-Path

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Mapping reduction

Input: $\langle A \rangle$ where A is a DFA.



1. Construct the directed graph G :

- ▶ initially, vertices of $G :=$ states of A
- ▶ every transition $v \xrightarrow{x} w$ in A becomes a directed edge (v, w) from v to w in G .
- ▶ then add a new vertex t
- ▶ for every Final State v of A , add a new directed edge (v, t) from v to t in G .

2. Specify s and t :

- ▶ $s :=$ vertex of Start State of A .
- ▶ t is as created above (the new vertex).

3. Output: $\langle G, s, t \rangle$

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FA-Empty \longrightarrow No-Digraph-Path

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$A \in \text{FA-Empty} \iff$ no sequence of transitions in A leading from Start State to a Final State

\iff there is no path in G leading from s to a vertex representing a Final State

\iff there is no path in G leading from s to t

$\iff \langle G, s, t \rangle \in \text{No-Digraph-Path}$

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RegExpEquiv \longrightarrow FA-Empty

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From previous lecture:

RegExpEquiv $:= \{ \langle A, B \rangle : A, B \text{ are regular expressions and } L(A) = L(B) \}$
FA-Empty $:= \{ \langle A \rangle : A \text{ is a FA and } L(A) = \emptyset \}$

We give a mapping reduction from RegExpEquiv to FA-Empty.

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RegExpEquiv \longrightarrow FA-Empty

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Mapping reduction:

Input: $\langle A, B \rangle$ where A and B are regular expressions

1. Construct a FA, C , that defines the language

$(\overline{L(A)} \cap L(B)) \cup (L(A) \cap \overline{L(B)}).$

2. Output: C

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Reducing *from* a decidable language

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Let's reduce from EnglishPalindromes to YearsOfTransitsOfVenus:



YearsOfTransitsOfVenus $:=$ { n | Transit of Venus occurs in year n }
 $:=$ { ..., 1761, 1769, 1874, 1882, 2004, 2012, 2117, ... }

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Mapping reduction:

Input: a string w over the English alphabet

If w is a palindrome

output 2012

else

output 2021.

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Reducing *from* a decidable language

Theorem.

If L_1 is decidable and L_2 is *any* language except \emptyset and Σ^* then

$$L_1 \leq_m L_2.$$

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Proof. Let D be a decider for L_1 .

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Let $x^{(\text{yes})}$ be any specific word in L_2 .

Let $x^{(\text{no})}$ be any specific word in \bar{L}_2 .

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Mapping reduction from L_1 to L_2 :

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Input: a string w

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1. Run D on w .
2. If D accepts w then output $x^{(\text{yes})}$
else output $x^{(\text{no})}$.

There's not much point
in a mapping reduction
that *decides* L_1 !

Revision

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Reading: Sipser, pp. 234–238.

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