

# FIT2014 Theory of Computation



Lecture 3

Propagate Logic

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# Lecture overview

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► Statements with variables

► Predicates

► Definitions and terminology

► Existential quantifier

► Universal quantifier

► Doing logic with quantifiers

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# Statements with variables

Consider these statements:

- ▶  $W$  is negative.
- ▶  $X$  passed this subject.
- ▶  $Y = Z$ .

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These do not yet have truth values.

The variables are **free**, in that no value is (yet) given to them.

You can, if you wish, assign values to them.

Each set of values you give to the variables creates a different specific proposition.

# Statements with variables

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For example, in the statement `W is negative`, the variable `W` is free.  
If we assign values to it, we can evaluate specific propositions:



⋮  
-2 is negative  
-1 is negative  
0 is negative  
1 is negative  
2 is negative  
⋮

⋮  
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True  
True  
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False  
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⋮  
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# Predicates

## Definitions

A **predicate** is a statement with variables such that, for any values of the variables, it is either True or False.

- ▶ i.e., it becomes a proposition



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We treat each variable as ranging over some **domain**.

- ▶ For the predicate  $W$  is negative, we've just been using the domain  $\mathbb{Z}$ .

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The variables of a predicate are also called its **arguments**.

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A predicate is called  **$k$ -ary** if it has  $k$  arguments. Special cases: unary, binary, ternary, ...

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Some alternative terminology:

# arguments	terminology
1	<i>property</i>
$\geq 2$	<i>relation</i>

## Predicates: examples

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# args.	example		domain
1	isNegative( $X$ )		$\mathbb{N}$
1	isNegative( $X$ )		$\mathbb{Z}$
2	$=$		objects
2	$X < Y$		numbers
2	isMotherOf( $X, Y$ ), meaning “ $X$ is the mother of $Y$ ”	WeChat: <a href="#">cstutorcs</a> Assignment Project Exam Help	people
3	gives( $X, Y, Z$ ), meaning “ $X$ gives $Y$ to $Z$ ”		$X, Z$ are people, $Y$ is a gift
$\vdots$	$\vdots$		$\vdots$

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Predicates may be thought of as *truth-valued functions*,  
i.e., functions whose value is always in  $\{\text{True}, \text{False}\}$ .

# Functions

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We'll also use functions whose values aren't necessarily just True or False.



# args.	example	dom	codomain
1	$\sqrt{X}$	non-negative numbers	numbers
1	<code>motherOf(X)</code>	people	people
2	$X + Y$	numbers	numbers
$\vdots$	$\vdots$	$\vdots$	

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Functions with no arguments are called **constants**.

► Examples: 5, Annie, ...

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A function's arguments can be: constants; variables; functions.

# Existential quantifier

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There's a fly in my soup.

$(X \text{ is a fly}) \wedge (X \text{ is in my soup})$ .

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There exists  $W$  :  $W$  is negative.  $\exists W : W < 0$

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If domain of  $W$  is  $\mathbb{N}$ : it's False  $\exists W \in \mathbb{N} : W < 0$

If domain of  $W$  is  $\mathbb{Z}$ : it's True.  $\exists W \in \mathbb{Z} : W < 0$

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## Existential quantifier

Someone did it.  $\exists X : X$  did it.

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It's *sort-of* like a disjunction ..

.....  $\vee$  (Annie did it)  $\vee$  (Edward did it)  $\vee$  (Henrietta did it)  $\vee$  (Radhanath did it)  $\vee$  .....

... *but*:

- ▶ often the domain of a variable is infinite.
- ▶ keep the variables, they're useful.

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The variables are now **bound**. Email: [tutorcs@163.com](mailto:tutorcs@163.com)

You can no longer give specific values to the variables to create specific propositions.

The quantifier has turned the statement into a single proposition about the entire domains of the variables.

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Quantifiers can only be used with *variables*.

Using them with constant objects *makes no sense*:  $\exists 5$ ,  $\exists \text{Annie}$ .

# Existential quantifier

Some computer is human.

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There exists a human computer.

If the domain of  $X$  is  $\{\text{computer}\}$

Predicate:



- ▶  $\text{human}(X)$ :  $X$  is human.

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But what if the domain of  $X$  is  $\{\text{everything on Earth}\}$  ?

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Predicates:

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- ▶  $\text{human}(X)$ :  $X$  is human.
- ▶  $\text{computer}(X)$ :  $X$  is a computer.

# Existential quantifier

Some computer is human.

i.e. 程序代写代做 CS编程辅导

There exists a human computer.

If the domain of  $X$  is  $\{\text{everything on earth}\}$



Predicates:

- ▶  $\text{human}(X)$ :  $X$  is human.
- ▶  $\text{computer}(X)$ :  $X$  is a computer.

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Correct:

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Incorrect:

$\exists X : \text{computer}(X) \wedge \text{human}(X)$

$\exists X : \text{computer}(X) \Rightarrow \text{human}(X)$

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- ▶ “There exists something that is both computer and human.”

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- ▶ “There exists a human computer.”
- ▶ “Some computer is human.”

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- ▶ “There exists something which is not a computer or is human.”

- ▶ “There exists something which is not both a computer and non-human.”
- ▶ “Not everything is a nonhuman computer.”

# Universal quantifier

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Everyone can pass this subject.

$\forall X : \text{canPass}(X).$



for every  $X : X$  can pass this subject.

All numbers are interesting.

$\forall X : X$  is interesting.

$\forall X : \text{isInteresting}(X).$

► True — and we'll prove it!

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For all  $W : W$  is negative

$\forall W : W < 0.$

False.

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Again, the variables are now **bound**.

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# Universal quantifier

Every computer is human.

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If the domain of  $X$  is  $\{\text{computer}\}$   
Predicate:



- ▶  $\text{human}(X)$ :  $X$  is human.

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But what if the domain of  $X$  is  $\{\text{everything on Earth}\}$  ?

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Predicates:

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- ▶  $\text{human}(X)$ :  $X$  is human.
- ▶  $\text{computer}(X)$ :  $X$  is a computer.

# Universal quantifier

Every computer is human.

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If the domain of  $X$  is  $\{\text{everything on earth}\}$



Predicates:

- ▶  $\text{human}(X)$ :  $X$  is human.
- ▶  $\text{computer}(X)$ :  $X$  is a computer.

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Incorrect:

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Correct:

$\forall X : \text{computer}(X) \wedge \text{human}(X)$

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$\forall X : \text{computer}(X) \Rightarrow \text{human}(X)$

- ▶ “Everything is both computer and human.”

- ▶ “Everything is a human computer.”

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- ▶ “For everything, if it’s a computer, then it’s human.”

- ▶ “Everything that’s a computer is also human.”

- ▶ “Every computer is human.”

# Multiple quantifiers

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Thinking of graphs ...

Suppose we have a predicate  $\text{adj}(X, Y)$  meaning that vertices  $X$  and  $Y$  are adjacent.



Some two vertices are not adj.

$$\exists X \exists Y : \neg(X = Y) \wedge \neg \text{adj}(X, Y).$$

$$\exists(X, Y) : \neg(X = Y) \wedge \neg \text{adj}(X, Y).$$

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Every pair of vertices is adjacent.

$$\forall X \forall Y : \neg(X = Y) \Rightarrow \text{adj}(X, Y).$$

$$\forall(X, Y) : \neg(X = Y) \Rightarrow \text{adj}(X, Y).$$

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Some vertex is adjacent to all other vertices.

$$\exists X \forall Y : \neg(X = Y) \Rightarrow \text{adj}(X, Y).$$

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Every vertex has a neighbour.

$$\forall X \exists Y : \text{adj}(X, Y).$$

# Multiple quantifiers

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## Six degrees of separation

Suppose we have a predicate  $\text{knows}(X, Y)$  meaning that person  $X$  knows person  $Y$ .

It has been claimed that, in the human social network,  
the distance between any two people is at most 6.

Exercise: write this claim in predicate logic, using just the predicate  $\text{knows}$ .

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# Doing logic with quantifiers

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If we know that



blah( $X$ )

and **obj** is any specific object (in the domain of  $X$ ),  
then we can deduce that

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We have:

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 $(\forall X \text{ blah}(X)) \Rightarrow \text{blah}(\text{obj})$

Also:

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 $\text{blah}(\text{obj}) \Rightarrow (\exists X \text{ blah}(X))$   
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## Doing logic with quantifiers

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$\forall X (p(X) \wedge q(X))$  is logically equivalent to  $(\forall X p(X)) \wedge (\forall X q(X))$

$\exists X (p(X) \vee q(X))$  is logically equivalent to  $(\exists X p(X)) \vee (\exists X q(X))$

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What about the logical relationship between ...

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$\forall X (p(X) \vee q(X))$  and  $(\forall X p(X)) \vee (\forall X q(X))$  ...?

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... etc

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## Relationship between quantifiers

$\neg \forall Y$  means the same as  $\exists Y \neg$  程序代写代做 CS编程辅导



“Not all dogs are happy.” is the same as ... “There exists an unhappy dog.”

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$\neg \forall X (\text{dog}(X) \Rightarrow \text{happy}(X))$  Not all dogs are happy

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$= \exists X \neg (\text{dog}(X) \Rightarrow \text{happy}(X))$

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$= \exists X \neg (\neg \text{dog}(X) \vee \text{happy}(X))$  (see last lecture)

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$= \exists X (\neg \neg \text{dog}(X) \wedge \neg \text{happy}(X))$  (by De Morgan)

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$= \exists X (\text{dog}(X) \wedge \neg \text{happy}(X))$  There exists an unhappy dog

## Relationship between quantifiers

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Similarly,

$\neg \exists Y$  means the same as  $\forall Y \neg$   
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$\neg \forall Y \neg$  means the same as  $\exists Y$   
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$\neg \exists Y \neg$  means the same as  $\forall Y$   
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