Monash University
Faculty of Information Technology

FIT2014 Theory of Computation

Lecture 19

Universal Turing Machines

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based in part on previous slides by David Albrecht

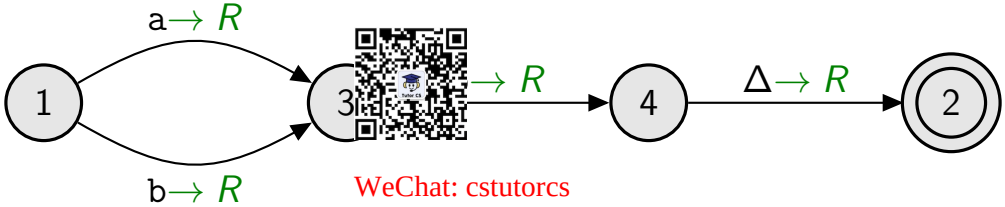
Overview

- Tables for Turing Machines
- Encoding
- Decoding
- Definition of a Universal Turing Machine
- Algorithm for a Universal Turing Machine
- Existence of Universal Turing Machines

Assumptions

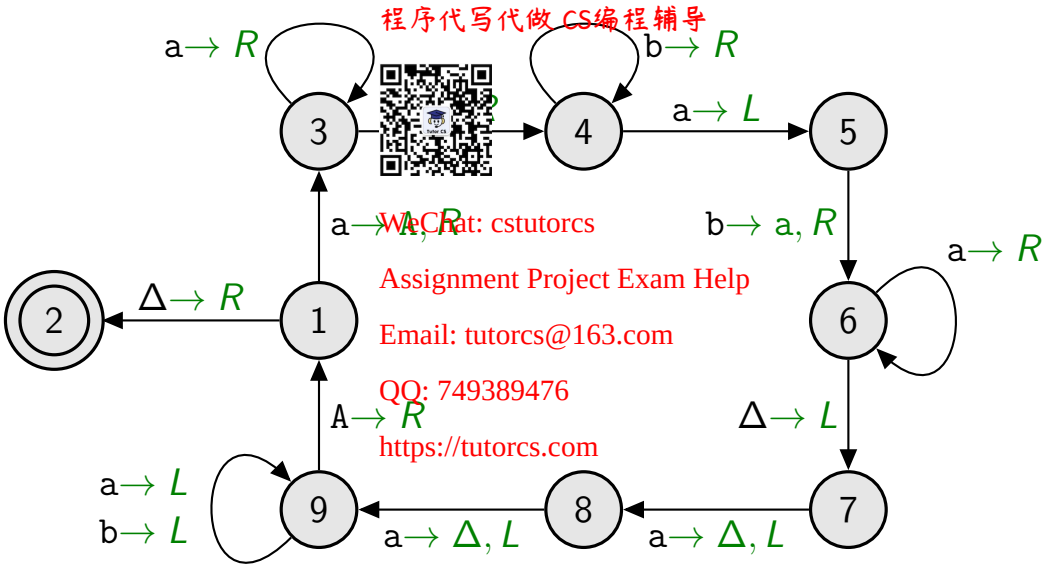
- Input Alphabet: {a,b}
- Tape Alphabet: {a,b,#}
- Start State: numbered 1
- Accept State: numbered 2

Tables for TMs



From	To	Read	Write	Move
1	3	a	a	R
1	3	b	b	R
3	4	b	b	R
4	2	Δ	Δ	R

TM for $\{a^n b^n a^n : n \geq 0\}$



Table

From	To	Read	Write	Move
1	3	a	#	R
3	3	a	a	R
3	4	b	b	R
4	4	b	b	R
4	5	a	a	L
5	6	b	a	R
6	6	a	a	R
6	7	Δ	Δ	L
7	8	a	Δ	L
8	9	a	Δ	L
9	9	a	a	L
9	9	b	b	L
9	1	#	#	R
1	2	Δ	Δ	R

Conditions to Check

Check whether there is a row with a 1 in the From column.

Check that there is no row with a 2 in the From column.

Check there are no two rows with the same numbers in the From and the same letter in the Read column.

Coding

State number

Code
$a^n b$

Letter	Code
a	aa
b	ab
Δ	ba
#	bb

Direction	Code
L	a
R	b

Coding the Table

From	To	Read	Write	Move	Code
1	3	a	#	R	abaaabaabbb
3	3	a	a	R	abaaabaaaab
3	4	b	b	R	abaaaababab
4	4	b	b	L	abaaaababab
4	5	a	a	L	aaaabaaaaabaaaaa
5	6	b	a	R	aaaaabaaaaaababaab
6	6	a	a	R	aaaaaabaaaaaabaaaab
6	7	Δ	Δ	L	aaaaaabaaaaaaabbabaa
7	8	a	Δ	L	aaaaaaabaaaaaaaabaabaa
8	9	a	Δ	L	aaaaaaaabaaaaaaaaabaabaa
9	9	a	a	L	aaaaaaaaabaaaaaaaaabaaaaa
9	9	b	b	L	aaaaaaaaabaaaaaaaaabababa
9	1	#	#	R	aaaaaaaaababbbbbb
1	2	Δ	Δ	R	abaabbabab

Encoding of the TM

abaaabaabbbaaabaaabaaaaba[illegible]babbbaaabaaaababbbaaaabaaaaabaaaaa
aaaaabaaaaaababaabaaaaabaaaaaabaaaaabaaaaaabaaaaaaabbabaa
aaaaaaabaaaaaaaabaabaaaaaaaaaabaaaaaaaabaabaaaaaaaaaaabaaaaaaaaabaaaaa
aaaaaaaaabaaaaaaaaababa[illegible]babbbbbbbaaabbbabab

. . . as one long string, without breaks or spaces.

Code Word Language (CWL)

The **Code-Word Language (CWL)** is the regular language defined by the regular expression

$$(a^*b(a \cup b)^5)^*$$

Words which encode a TM belong to CWL.

BUT: Not all words in CWL encode a TM.

Quantifier practice:

$\neg\forall w \in \text{CWL} \quad \exists M \quad : \quad w$ encodes M

$\exists w \in \text{CWL} \quad \neg\exists M \quad : \quad w$ encodes M

$\exists w \in \text{CWL} \quad \forall M \quad : \quad \neg(w$ encodes $M)$

$\exists w \in \text{CWL} \quad \forall M \quad : \quad w$ does not encode M

Decode

abaaabaaaababaaababababbaaa  babbaaaabaabbabab

From	To	Read	Write	Move
1	3	a	a	R
1	3	b	b	R
3	4	b	b	R
4	2	Δ	Δ	R

Algorithm

While there are unread letters:

1. Read and count the next clump of a's, then read the b after it.
   - Interpret clump of a's as the state number, in unary, that this transition goes from.
2. Read and count the next clump of a's, then read the b after it.
   - Interpret clump of a's as the state number, in unary, that this transition goes to.
3. Read the next two letters.
   - Interpret it as the letter to be read for this transition.
4. Read the next two letters.
   - Interpret it as the letter to be written for this transition.
5. Read the next letter.
   - Interpret it as the direction for this transition.

Universal Turing Machine (UTM)

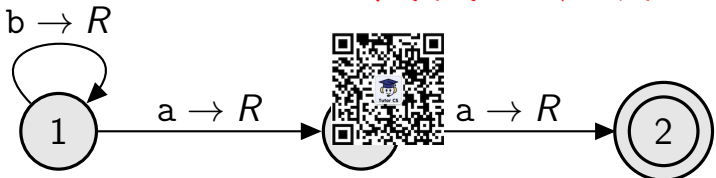
Definition

A **Universal Turing Machine (UTM)** is a Turing Machine that takes, as input,

- an encoding of some Turing Machine M, together with
- a string x, to be used as input to M

and simulates the execution of M on x.

Example

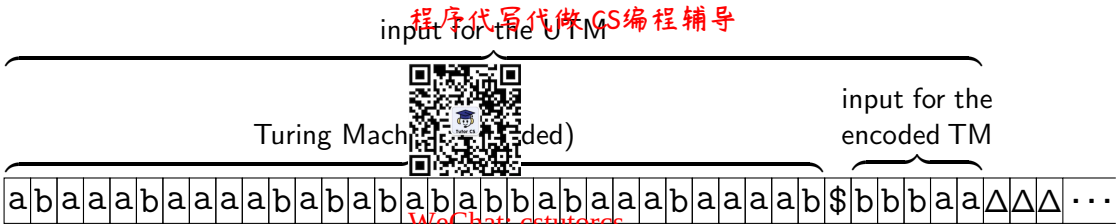


Suppose we want a UTM to simulate the execution of this TM on the string bbbaa.

Input to the UTM:

Turing Machine:	abaaabaaaabababababbaaabaabaaaab
Data:	bbbaa

Input for UTM



Algorithm for a UTM

1. Move rightwards to first letter of the encoded TM's input.
   Read it. Mark it, so we can come back to it (e.g., $\mathtt{a} \mapsto \mathtt{A}$, $\mathtt{b} \mapsto \mathtt{B}$).
   - Remember it (in choice of state).
2. Move leftwards to first instruction.
3. If the next state in current instruction in encoded TM is the Accept state
   Find (from current instruction) what to write and direction of next move.
   Remember it (in choice of state).
   Move rightwards back to current position in encoded TM's input.
   Write the required letter, move in the required direction, and Accept.

   else
   Find (from current instruction) what to write and direction of next move.
   Remember it (in choice of state).
   Move rightwards back to current position in encoded TM's input.
   Write the required letter, move in the required direction.
   Read current letter in encoded TM's input. Mark it, so we can come back to it.
   Remember it (in choice of state).
   Move leftwards to find the next instruction (using remembered letter).

Exercise

Suppose:

- U is a UTM,
- T is a TM
- x is an input string for T, with $|x| = n$.
- When T is run on input x, it takes time t and visits at most s tape cells.

Using the algorithm outline of the previous slide, and the encoding scheme for TMs given in this lecture:

- Determine an upper bound for the time taken by U to simulate the running of T on input x.
- Give the bound in terms of t, s and n.

Importance of UTMs

- theoretical model of one computer simulating another
- Stored-program computer
- von Neumann architecture

Revision

- Know how to encode a Turing Machine.
- Know how to decode Turing Machine representation.
- Know what a Universal Turing Machine is, and what it does.
- Understand why UTMs exist.