

程序代写代做 CS编程辅导

FIT2014 Theory of Computation



Lecture 20

Decidability
WeChat: cstutorcs

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Overview

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- ▶ Decision problems
- ▶ Decidable problems and languages

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- ▶ Deciders

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- ▶ Closure

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Deciders

Reminder:

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A decider is a Turing Machine that halts for every input.



A language is decidable if it is Accept(L) for some decider.

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... in which case, its complement is Reject(L) for the same decider.

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Examples:

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▶ Regular Languages

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▶ Context Free Languages

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▶ $\{a^n b^n a^n : n \geq 0\}$

Decidable: synonyms

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ble

recursive

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... sometimes, though “computable” has
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Decision Problems

INPUT: an integer

QUESTION: Is it even?

INPUT: a string.

QUESTION: Is it a palindrome?

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INPUT: an expression in propositional logic

QUESTION: Is it ever True?

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INPUT: a graph G , and two vertices s and t

QUESTION: is there a path from s to t in G ?

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INPUT: a Python program

QUESTION: is it syntactically correct?

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INPUT: a Finite Automaton

QUESTION: Does it define the empty language?

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INPUT: two Regular Expressions

QUESTION: Do they define the same language?



INPUT: a Finite Automaton

QUESTION: Does it define an infinite language?

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INPUT: a Context Free Grammar

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QUESTION: Does it define the empty language?

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INPUT: a Context Free Grammar

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QUESTION: Does it generate an infinite language?

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INPUT: a Context Free Grammar and a string w

QUESTION: Can w be generated by the grammar?

Decision Problems

A **decision problem** is a problem where, for each input, the answer is Yes or No.

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A decider **solves** a decision problem if

- ▶ Accepts an input for which the answer is Yes, and
- ▶ Rejects any input for which the answer is No.



Decision problem \longrightarrow language

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- ▶ { YES-inputs }

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Language \longrightarrow decision problem

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- ▶ INPUT: a string
(over some alphabet, usually representing some object)

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QUESTION: Is the string in the language?

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Thus, a decider solves a decision problem if and only if it is a decider for its corresponding language.

Encoding of Input

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The input and output for a Turing Machine is always a string.

For any object, O , $\langle O \rangle$ will denote the encoding of the object as a string.

If we have several objects, O_1, \dots, O_n , we denote their encoding into a single string by $\langle O_1, \dots, O_n \rangle$.

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Testing Emptiness of Regular Languages

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Decision Problem:

INPUT: a Finite Automaton

QUESTION: Does it define the empty language?

Language:

FA-Empty := $\{ \langle A \rangle : A \text{ is a FA and } L(A) = \emptyset \}$

Theorem.

FA-Empty is decidable.

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Testing Emptiness of Regular Languages

Theorem.

FA-Empty is decidable.

Proof. (outline)

Algorithm:

Input: $\langle A \rangle$ where A is a Finite Automaton.

1. Mark the Start State of A .

2. Repeat until no new states get marked:

- ▶ Mark any state that has a transition coming into it from any state that is already marked.
- ▶ If no final state is marked, Accept, otherwise Reject.

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Testing Equivalence of Regular Expressions

Decision Problem:

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REGULAR EXPRESSION EQUIVALENCE

INPUT: two Regular Expressions

QUESTION: Do they define the same language?



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For a Regular expression R , let $L(R)$ be the language defined by R .

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Language:

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$\text{RegExpEquiv} := \{ \langle A, B \rangle \mid A, B \text{ are regular expressions and } L(A) = L(B) \}$

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Theorem.

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RegExpEquiv is decidable.

Testing Equivalence of Regular Expressions

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Proof.

Algorithm:



Input: $\langle A, B \rangle$ where A and B are regular expressions

1. Construct a FA, C , that defines the language

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$$(L(A) \cap \overline{L(B)}) \cup (\overline{L(A)} \cap L(B))$$

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2. Run the previous Turing Machine, T , on C .

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3. If T accepts C , then Accept, else Reject.

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Testing Emptiness of Context Free Language

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Decision Problem:

INPUT: a Context Free Grammar

QUESTION: Does it define the empty language?



Language:

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CFG-Empty $:= \{ G : G \text{ is a CFG and } G \text{ defines the empty language} \}$

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Theorem.

CFG-Empty is decidable.

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Testing Emptiness of Context Free Language

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Algorithm:

Input: $\langle A \rangle$ where A is a Context Free Grammar.

1. Mark all the terminal symbols in A .
2. Repeat until no new symbols get marked:
 - ▶ Mark any non-terminal X that has a production which has all the right-hand symbols marked.
 - ▶ If Start Symbol is not marked, Accept, else Reject.

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Some Decidable Problems

INPUT: a Finite Automaton

QUESTION: Does it define the empty language?

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INPUT: two Regular Expressions

QUESTION: Do they define the same language?



INPUT: a Finite Automaton

QUESTION: Does it define an infinite language?

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INPUT: a Context Free Grammar

QUESTION: Does it define the empty language?

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INPUT: a Context Free Grammar

QUESTION: Does it generate an infinite language?

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INPUT: a Context Free Grammar and a string w

QUESTION: Can w be generated by the grammar?

Language classes

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Decidable

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CFLs

regular
languages

Closure properties

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If L is decidable, then \bar{L} is decidable

If L_1 and L_2 are decidable, then

- ▶ $L_1 \cup L_2$
- ▶ $L_1 \cap L_2$
- ▶ $L_1 L_2$
- ▶ ...



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Exercise:

Formulate and prove more closure results.

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Revision

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- ▶ Decidable Problems, decidability, and the link between them.
- ▶ Decision problems, relation to Turing machines
- ▶ Examples of Decidable Problems.
- ▶ Closure properties

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Reading: Sipser, Section 4.1, pp. 190–201.

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Preparation: Sipser, Section 4.2, pp. 201–213, especially pp. 207–209.

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