Monash University

Information Technology

eory of Computation

FINAL EXAM

2nd semester, 2013

Instructions:

10 minutes reading time.

3 hours writing time.

No books, calculators or devices.

Total marks on the exam = 120.

Working Space

Question 1 **(4 marks)**

You are hunting a mouse in your wardrobe.

Suppose we hav[illegible], C, H and S, with the following meanings:

C: The mous[illegible]
H: The mous[illegible]
S: The mous[illegible]

Use C, H and [illegible]sition, in Conjunctive Normal Form, that is True precisely when the m[illegible]se three locations: your coat, your hat or your shoe.

Question 2 **(4 marks)**

Suppose you have the predicates `computer` and `utm` with the following meanings:

`computer`(X): X is a computer.
`utm`(X): X can simulate any Turing machine.

(a) Write a universal statement in predicate logic with the meaning:

"Everything that can simulate any Turing machine is a computer."

(b) What additional fact would you need to know, to be able to use this statement (and nothing else) to prove that the object `myPhone` is a computer? (Express this fact as an atomic sentence in predicate logic.)

8

Question 3 **(4 marks)**

(a) Write down all strings of at most 8 letters, over alphabet {a,b}, that match the regular expression $(((\mathtt{ab}) \cup (\mathtt{ba}))\mathtt{a})^*$.

(b) Give an NFA with at most 7 states that recognises the language described by this regular expression.

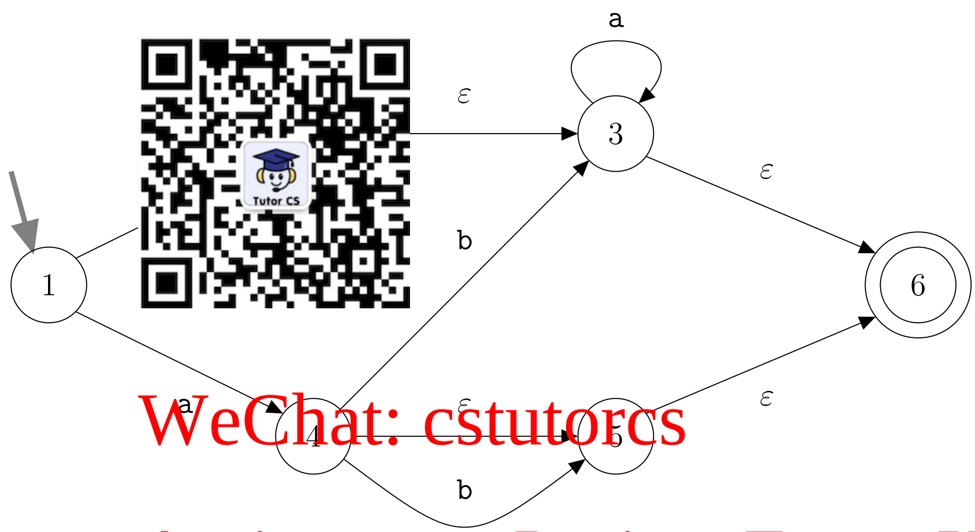
Question 4 **(3 marks)**

A language over alphabet {a,b} is said to be *cofinite* if it contains all strings over that alphabet *except* for some finite number of strings. Prove that every cofinite language is regular.

7

Question 5 **(7 marks)**

Given the following NFA, convert it to a Finite Automaton that recognises the same language.



Represent the FA by filling in the table below.

state	a	b
Start		

7

Question 6 **(3 marks)**

Suppose you have, at your disposal, algorithms to convert NFAs to DFAs, DFAs to Regular Expressions and Regular Expressions to NFAs.

Explain how you would design and implement a lexical analyser to recognise tokens that match a particular r[illegible] In this explanation, do not give code, but do explain where the code com[illegible]

Question 7 **(6 marks)**

Use the Pumping Lemma to prove that the language

$$\{ \mathtt{a}^m \mathtt{b}^n \ : \ m < n \}$$

is not regular.

9

Working Space

Question 8 **(2 marks)**

Give a Context-Free Grammar for the language

$$\{ \mathtt{a}^m \mathtt{b}^n \ : \ m > 2n, \ n \geq 0 \}.$$

Question 9 **(6 marks)**

Consider the following Context-Free Grammar

$$S \rightarrow \mathtt{a}\, S \tag{1}$$

$$S \rightarrow S\, \mathtt{b} \tag{2}$$

$$S \rightarrow \varepsilon \tag{3}$$

Prove by induction that every string of the form $\mathtt{a}^m \mathtt{b}^n$, where $m \geq 0$ and $n \geq 0$, can be generated by this grammar.

8

Working Space

Question 10 **(5 marks)**

Consider the following Context-Free Grammar.

$$
\begin{array}{rcll}
S & \rightarrow & \mathtt{a}B\mathtt{a} & (1) \\
B & \rightarrow & BB & (2) \\
B & \rightarrow & Q & (3) \\
B & \rightarrow & R & (4) \\
Q & \rightarrow & \mathtt{q} & (5) \\
R & \rightarrow & \mathtt{r} & (6)
\end{array}
$$

Give

(a) a derivation, and

(b) a parse tree,

for the string `aqrqa`, labelling each step in the derivation on its right by the number of the rule used. Use the spaces below for your answers.

(a) Derivation:

(b) Parse tree:

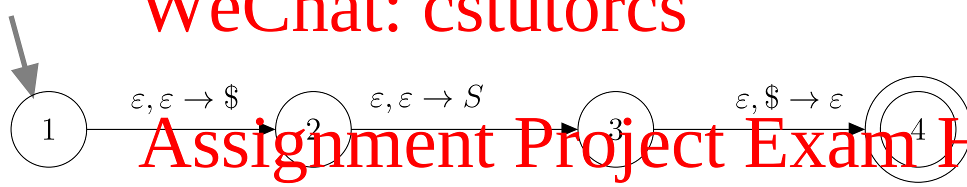
5

Question 11 **(5 marks)**

Given the following Context-Free Grammar:

$$S \rightarrow A \quad (1)$$
$$A \rightarrow AbA \quad (2)$$
$$A \rightarrow a \quad (3)$$

complete the followi a Pushdown Automaton for the language generated by the grammar.



Question 12 **(4 marks)**

State two important results that can be proved using the Chomsky Normal Form for Context-Free Grammars.

9

Question 13 **(6 marks)**

The **2's complement** of a binary string is formed as follows. Flip each bit (i.e., change 0 to 1, and 1 to 0) until you get to the last 1. Keep that last 1, and all 0s after it, unchanged.

For example, if the string is 0110100, then its 2's complement is 1001100.

Draw a Turing [illegible] the 2's complement of any binary string.

6

Question 14 **(4 marks)**

(a) State the Church-Turing thesis.

(b) Give two reasons why the Church-Turing thesis is widely accepted.

Question 15 **(4 marks)**

For each of the following decision problems, indicate whether or not it is decidable.

Decision Problem	your answer (tick **one** box in each row)	
Input: a Turing machine M. Question: Does M eventually halt, if the input is the number 17?	☐ Decidable	☐ Undecidable
Input: a Turing machine M, and a string w. Question: If M is run with input w, does it halt in at most 17 steps?	☐ Decidable	☐ Undecidable
Input: a Turing machine M. Question: Does M have at least 17 states?	☐ Decidable	☐ Undecidable
Input: a Turing machine M. Question: Is the language recognised by M finite?	☐ Decidable	☐ Undecidable

8

Question 16 **(10 marks)**

The Venn diagram on the next page shows several classes of languages. For each language (a)–(j) in the list below, indicate which classes it belongs to, by placing its corresponding letter in the correct region of the diagram.

If a language do[illegible] of these classes, then place its letter above the top of the diagram.

(a) $\{\mathtt{a}^n\mathtt{b}^n : n > 0\}$

(b) $\{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n : n >$ [illegible]

(c) $\{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n\mathtt{d}^n : n > 0\}$

(d) The set of all graphs containing a Hamiltonian circuit.

(e) The set of all graphs containing no circuit.

(f) The set of all correctly formed arithmetic expressions that only use positive integers and the symbols + and −.

(g) The set of all (encodings of) Turing machines that eventually halt, when given themselves as input.

(h) The set of all (encodings of) Turing machines that loop forever, when given themselves as input.

(i) The set of all strings that have ever caused any real computer to eventually halt.

(j) The set of all Context-Free Grammars that define an empty language.


recursively enumerable (r.e.)
decidable
NP
P
Context-Free
Regular
Finite

10

Question 17 **(7 marks)**

Prove that the following problem is undecidable.

Input: a Turing machine M, and a string x.
Question: If M is run on input x, does it eventually **accept** x?

You may use th[illegible]ng Problem is undecidable.

7

Question 18 **(5 marks)**

Let L be a recursively enumerable (r.e.) language. We saw in lectures that there is an enumerator that enumerates L. Here is an attempt at constructing an "enumerator" for L.

Let M be a Turing machine whose set of accepted strings is L. Construct another Turing machine E which do[illegible]

For each string w = [illegible] turn (i.e., sequentially):
{
    Simulate the [illegible] input w.
    If M accepted [illegible]
    Continue to the next iteration.
}

(a) What is wrong with this attempt at an enumerator?

(b) Indicate briefly what would need to be done to fix it. (You don't need to say *in detail* how to fix it, but just indicate in general terms what to do.)

5

Question 19 **(4 marks)**

Suppose M is a Turing machine with time complexity $O(n^3)$ and that U is a Universal Turing Machine that can simulate any t-step Turing machine computation in at most t^4 steps.

Find an upper [illegible]tation and with proof, of the time taken by U to simulate the comput[illegible]nput of size n.

4

Question 20 **(4 marks)**

Prove that the class of regular languages is a subset of the class P.

Question 21 **(2 marks)**

Suppose that a particular decider has polynomial time complexity. When inputs are sufficiently large, what happens to its running time when the length of the input is doubled?

Choose **one** of the answers (a)–(g) below, by circling the appropriate letter.

If more than one answer holds, you must choose the strongest (i.e., most precise) correct answer.

(a) It is raised to at most a constant power.

(b) It is increased by at most a constant factor.

(c) It is increased by at most a constant amount.

(d) It is at most squared.

(e) It is at most doubled.

(f) It increases by at most 2.

(g) It doubles after two years, according to Moore's Law.

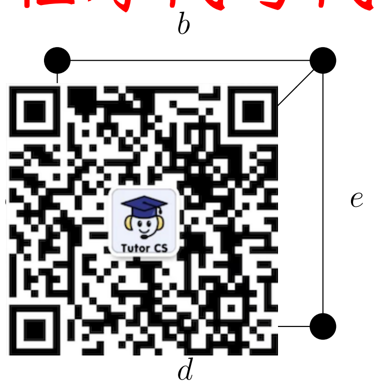
6

Question 22 **(4 + 6 + 7 + 2 + 2 = 21 marks)**

Consider the language NO MONOCHROMATIC TRIANGLE, which consists of all graphs G such that we can assign colours to the edges of the graph so that (i) each edge is either Black or Whit[illegible]o *triangle* (i.e., cycle of length 3) in the graph which is *monochromatic* (i[illegible]e the same colour).

(a) Prove that the l[illegible]CHROMATIC TRIANGLE is in NP.

Now, let W be the following graph.



(b) Construct a Boolean expression E_W in Conjunctive Normal Form such that the satisfying truth assignments for E_W correspond to solutions to the NO MONOCHROMATIC TRIANGLE problem on the above graph W (i.e., they correspond to colourings of the edges of W which have no monochromatic triangle).

(c) Give a polynomial-time reduction from NO MONOCHROMATIC TRIANGLE to SATISFIABILITY.

(d) State the Cook-Levin Theorem.

(e) Given the facts s question:
What else, if anythi to prove, in order to show that NO MONOCHRO-MATIC TRIANGL

END OF EXAMINATION