Leture 1: Introduction

FM321: Risk Management and Modelling

Linyan Zhu

27 September 2022

LSE Finance

Team

- Lecturer: Dr **Linyan Zhu** (L.Zhu3@lse.ac.uk)
  - Assistant Professor of Finance; PhD Economics, UCSD
  - Office hour: Thu 14:00 - 15:00 in MAR 7.22

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- Class teachers:

  Dr **Mesut Tastan** (M.Tastan@lse.ac.uk)

  Office hour: Wed 12:30 - 13:30 in MAR 6.38

  Song Xiao (S.Xiao2@lse.ac.uk)

  Office hour: Mon 15:30 - 16:30 in MAR 6.38

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Main topics

- Introduction to financial returns and risks
- Volatility as a risk measure
  - Univariate volatility models
  - Multivariate volatility models
- Other risk measures beyond volatility
- Implementing risk forecasts
- Backtesting and stress-testing

- There is no textbook that covers the topics in the course exactly the way I plan to.
- Financial Risk Forecasting by Jon Danielsson is the main textbook (required), and closely approximates what we will do for most topics.

- A list of readings is available in the course syllabus for those wishing to study the subject further.

Classes and formative assignments

- A formative assignment will be given every week on Thursday.
- The assignments will ask you to implement the techniques discussed in the lectures.
- They will not be graded, but they will be discussed in the classes the following week (no classes in Week 1).
- Please make every effort to attempt them before class time.
- They will be based on R.

Assessment

5% First summative assignment,

- available on Tue 18 Oct 2022 (Week 4)
- due on Sun 30 Oct 2022, 11:59pm

5% Second summative assignment

- available on Tue 15 Nov 2022 (Week 8)
- due on Sun 27 Nov 2022, 11:59pm

Summative assignments follow the same format as formative ones.

40% Course project

- available in Week 11
- due in Jan 2023

50% In-class assessment (ICA)

- Tue 6 Dec 2022
- closed-book
- All course material is examinable unless stated otherwise.

- Key concepts: price and returns
- Stylized facts about financial returns

- Suppose you have an equity. How to measure its performance?

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- For an equity, total return includes capital gain and dividend yield.
- For a bond, it includes capital gain and coupon payment.

- Time is discrete: $t = 1, \ldots, T$.
- P_t: price of an equity at the end of period t
- D_t: dividend paid out from the equity during period t
- Two types of returns:

  Simple (or arithmetic) returns:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1$$

  Compound (or geometric / logrithmic) returns:

$$r_{t+1} = log\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right)$$

  log: natural log (base $e \approx 2.7182818\ldots$)

Simple vs. log return

- On a day-to-day basis, simple returns are used for a number of purposes (e.g., accounting, or communicating with clients and investors)
- In analytical work, logarithmic returns are preferred for data at high frequencies:
  - Log returns aggregate over time in a simple manner (e.g., the logarithmic return over one-month is the sum of the logarithmic return over the days in that month)
  - Log returns are symmetric
  - Log returns are not bounded below, so one can fit symmetric distributions to the data
  - In many derivatives pricing models, log returns are preferred for theoretical reasons

Simple vs log return

- For small returns (typical with short periods), log returns approximate simple returns well.
- Recall Taylor expansion

$$log(1+x) \approx x \text{ when } x \text{ is small}$$

- Hence,

$$\begin{aligned} r_{t+1} &= log\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right) \\ &= log\left(\frac{P_{t+1}+D_{t+1}}{P_t} - 1 + 1\right) \\ &\approx \frac{P_{t+1}+D_{t+1}}{P_t} - 1 \\ &\equiv R_{t+1} \end{aligned}$$

Prices

- For equities, prices are typically given ex-dividends, so those need to be adjusted
- In general, need to adjust for a wide range of possible activities that can affect the computation of returns, such as:
  - Stock splits
  - Reverse splits
  - Spin-offs
  - Stock Buybacks
  - Price data adjusted for these effects is usually readily available
- For simplicity, from here on, we'll assume that our price data is adjusted for the effects of dividends and corporate actions, so the terms referring to dividends will drop out of the formulas

Portfolio returns

- N assets: $n = 1, 2, \ldots, N$
- $P_{n,t}$: price of asset n at the end of period t
- w_n: weight of asset n in the portfolio
- Price of the portfolio:

$$P_t = \sum_{n=1}^{N} w_n P_{n,t}$$

- Simple return of the portfolio:

$$R_t = \sum_{n=1}^{N} w_n R_{n,t}$$

- However, not true for log returns:

$$r_t = log\left(\frac{P_t}{P_{t-1}}\right) \neq \sum_{n=1}^{N} w_n log\left(\frac{P_{n,t}}{P_{n,t-1}}\right)$$

Equity indices

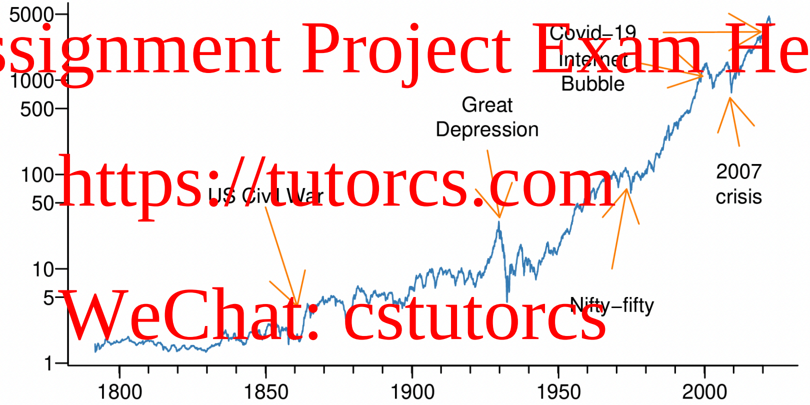
- Show how the price of a representative portfolio of securities in a particular market or asset class has evolved over time

- Multiple weighting schemes are used in practice:

  - Price-weighted (e.g., Dow Jones Industrial Average)

  - Market capitalization or market value weighted (e.g., S&P500)

S&P 500 Price Index



S&P 500 daily returns

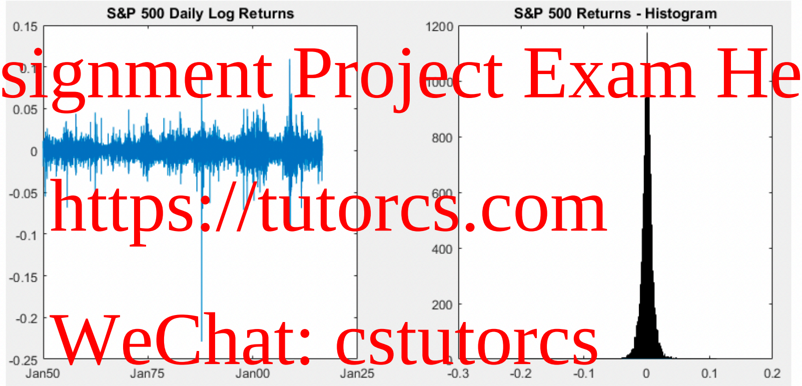


Figure: Sample: 3 Jan 1950 to 8 Aug 2016

S&P 500 daily returns

- Summary statistics:

Average	0.029% (7.33% Ann.)
Std. Dev.	0.97% (15.43% Ann.)
Min.	-22.90% (19 Oct 87)
Max.	10.96% (13 Oct 08)

- Note how small the average is compared to the standard deviation!

Unit conversion

- It is typically a good idea to express descriptive statistics (not the original data) on returns in a standard unit (percent per year).

- With returns that are independent and additive, mean and variance of returns are proportional to the number of periods (T), so standard deviation grows at the rate $\sqrt{T}$.

- We will use this convention in general. The usual factors are:
  - From daily to annual: $T = 252$ (or $T = 250$, or $T = 260$)
  - From monthly to annual: $T = 12$
  - From quarterly to annual: $T = 4$

- Volatility clusters
- Fat tails
- Time-varying correlations

Stylized fact 1

- Volatility, defined as the standard deviation of returns, is the most commonly used measure of risk
- Distinction 1: Volatility can be
  - Unconditional volatility

$$\sigma = \sqrt{E\left[r_t - E(r_t)\right]^2}$$

  - Conditional volatility

$$\sigma_t = \sqrt{E_{t-1}\left[r_t - E_{t-1}(r_t)\right]^2}$$

- Distinction 2:
  - True population volatility: hypothetical quantity that is unknown.
  - Sample unconditional volatility

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left[r_t - \bar{r}\right]^2} \qquad \bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

  - Sample conditional volatility: GARCH etc.

- Daily volatility

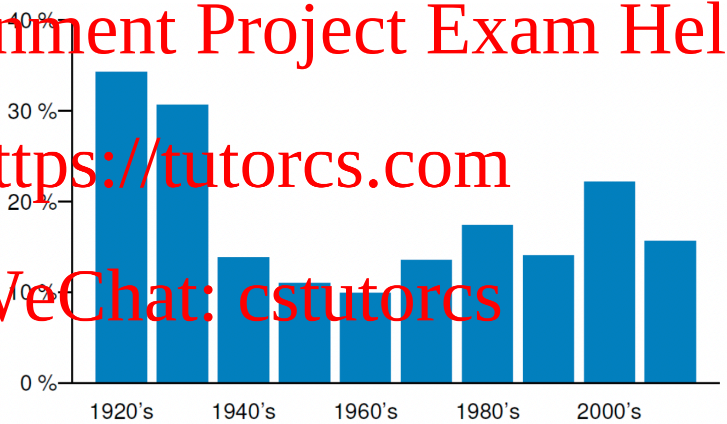
$$\sigma^{daily} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left[r_t - \bar{r}\right]^2}$$

- Annualized volatility

$$\sigma^{ann} = \sqrt{252}\sigma^{daily}$$

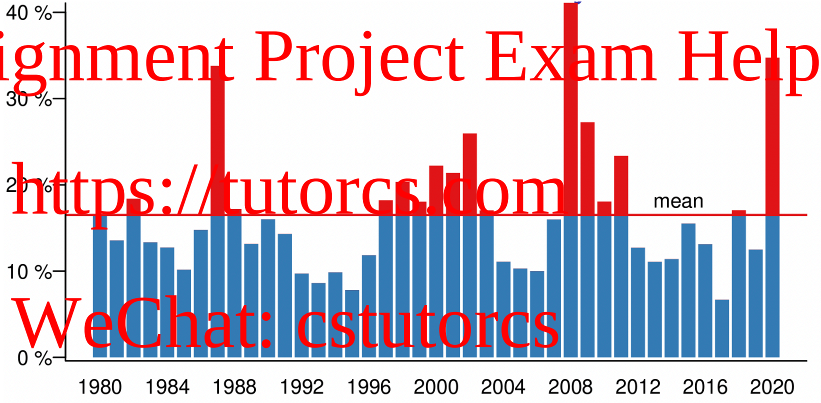
Volatility clusters

- Suppose we use the annualized volatility equation and calculate volatility over a decade



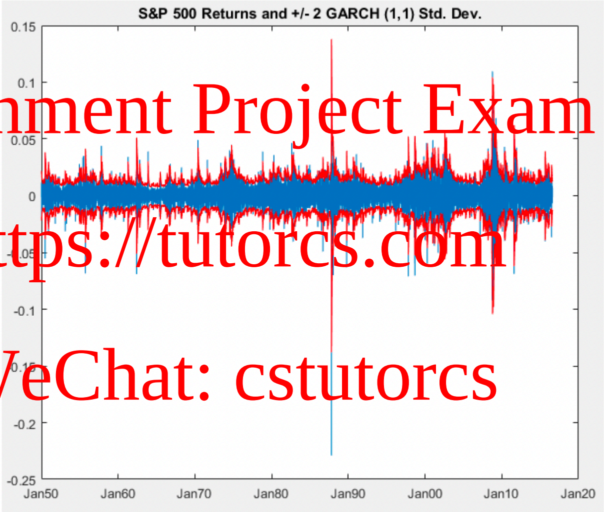
- Then we see that volatility comes in many cycles

- Calculate volatility year by year:



- Both long-run and short run
- We call these volatility clusters

Volatility clusters



Shocks to volatility tends to be persistent, but eventually die out:

- Covariance measures the covariation of two series of returns. Sample covariance:

$$\widehat{Cov}(r_x, r_y) = \frac{\sum_{t=1}^{T}(r_{x,t} - \bar{r}_x)(r_{y,t} - \bar{r}_y)}{T-1}$$

- Correlation is a standardized measure of covariation:

$$\widehat{\rho}_{x,y} \equiv \widehat{Corr}(r_x, r_y) = \frac{\widehat{Cov}(r_x, r_y)}{\hat{\sigma}_x \hat{\sigma}_y}$$

- It is always the case that

$$-1 \leq \hat{\rho}_{x,y} \leq 1$$

- We presented the unconditional sample quantities. Can also study the population or the conditional quantities.

Autocorrelation

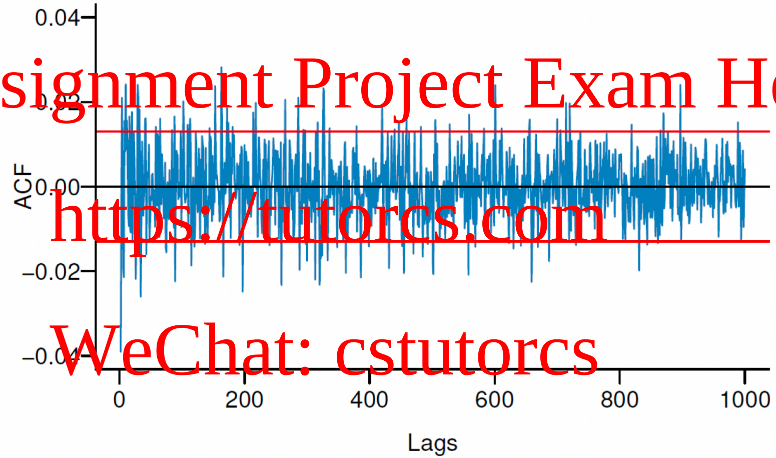
- Measures how a time series is correlated with its own lagged values.

$$\text{1 lag:} \qquad \hat{\rho}_1 = \widehat{Corr}(r_t, r_{t-1})$$

$$\text{k lag:} \qquad \hat{\rho}_k = \widehat{Corr}(r_t, r_{t-k})$$

- If autocorrelation is statistically significant, it shows evidence of predictability in the time series
- The autocorrelation function (ACF) of a time series shows the measured autocorrelation in the series as a function of the number of lags k ($k = 0, 1, 2, \dots$)

ACF of S&P 500 daily returns



Sample: 1928 - mid 2022 Source: Jon Danielsson

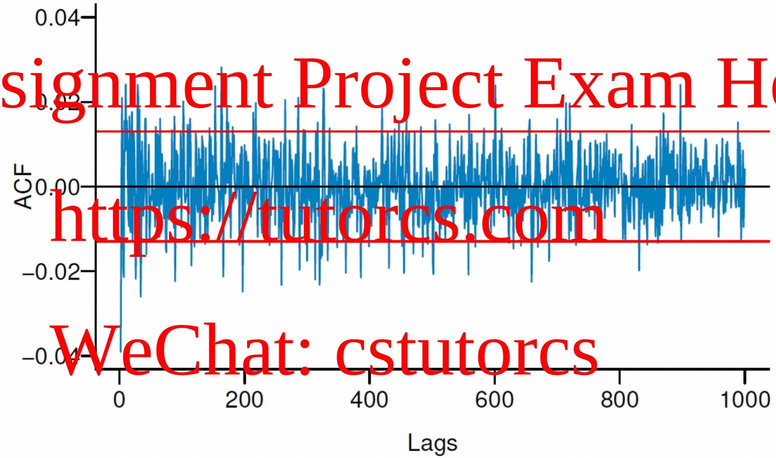
- One can test the significance of individual coefficients ($\hat{\rho}_k$)

- Alternatively, one can test the joint significance of the first K autocorrelation coefficients with the Ljung-Box statistic:

$$J_K = T(T+2)\sum_{k=1}^{K} \frac{\hat{\rho}_k^2}{T-k} \sim \chi^2_K$$

- We will test for both returns (r_t), predictability in mean (price forecasting or alpha)

- And returns squared r_t^2, predictability in volatility.



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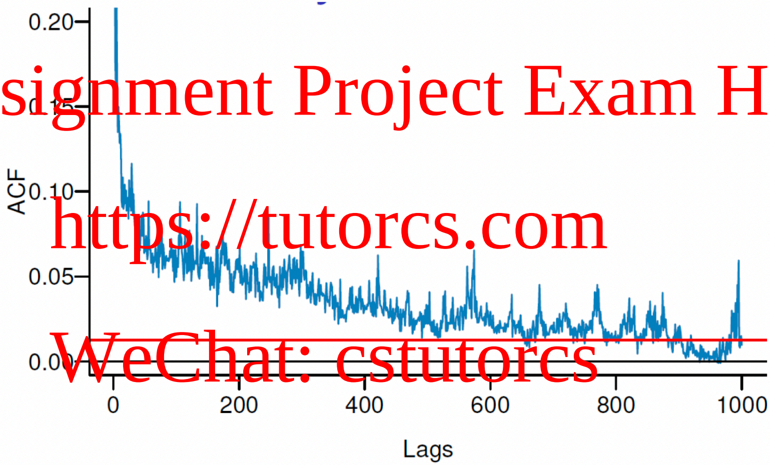
Test for autocorrelation



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Test for autocorrelation

- Testing for the presence of serial correlation yields the following results

	number of lags	test stat	p-value
Returns	100	18.7	0.606
Squared returns	100	46	0.00129

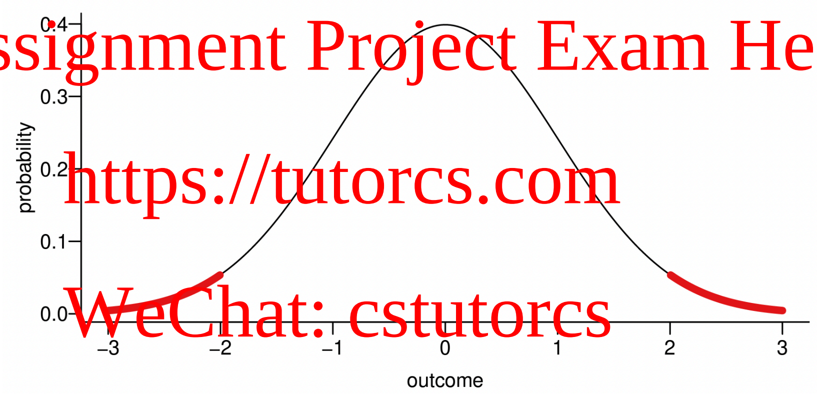
- There is strong evidence of serial correlation for squared returns

Stylized fact 2

- Definition: a series exhibits fat tails if the likelihood of extreme outcomes in that series is higher than that of a normally-distributed random variable with the same mean and standard deviation.

Probabilty distribution of N(0,1)

Tails



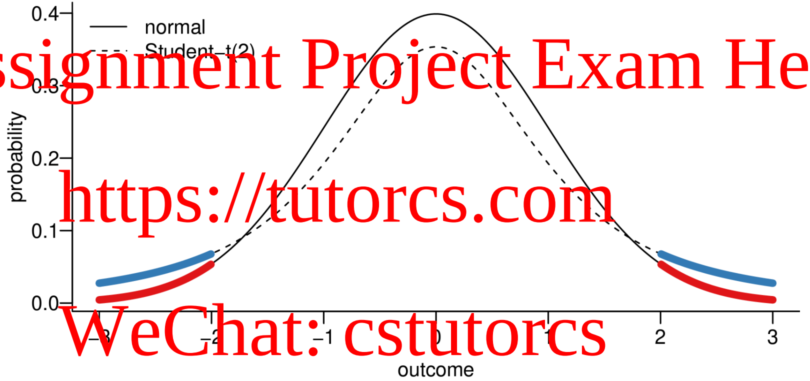
Student-t distribution

- The Student-t is convenient when we need a fat tailed distribution

- The degrees of freedom, (ν), of the Student–t distribution indicate how fat the tails are $\nu = \infty$ implies the normal

- $\nu < 2$ superfat tails

- For a typical stock $3 < \nu < 5$



A fat-tailed distribution implies that the probability of non-extreme outcomes is lower than that of the normal.

Test for non-normality

- One can use statistical methods to test for normality
- Jarque-Bera test: may point to fat tails if null of normality is rejected

$$JB = \frac{T}{6} \times \text{Skewness}^2 + \frac{T}{4}(\text{Kurtosis} - 3)^2 \sim \chi^2_2$$

- Tests only for an implication of normality
- For the S&P 500,
  - Kurtosis = 30.07 (fat tails)
  - Skewness = -1.01 (negatively skewed, or left-skewed)

  so $JB = 514530$, whereas the critical value at significance level 5% is approximately 5.98.

Visualize non-normality

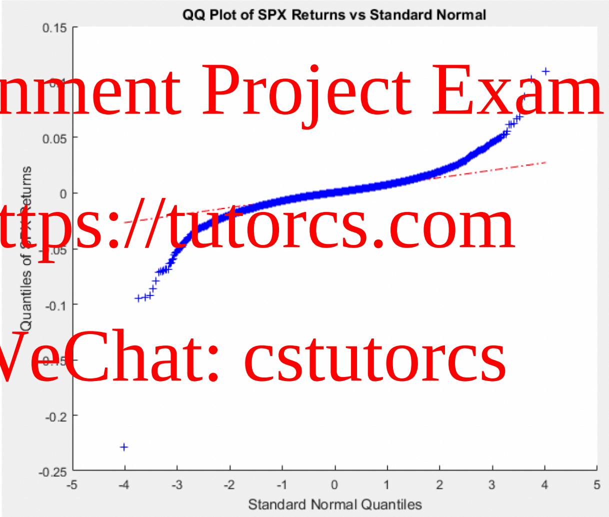
- One can also use graphical methods to inspect the data

- QQ-Plots plot quantiles of the target distribution (e.g., the empirical distribution of S&P500 returs) against quantiles of a reference distribution

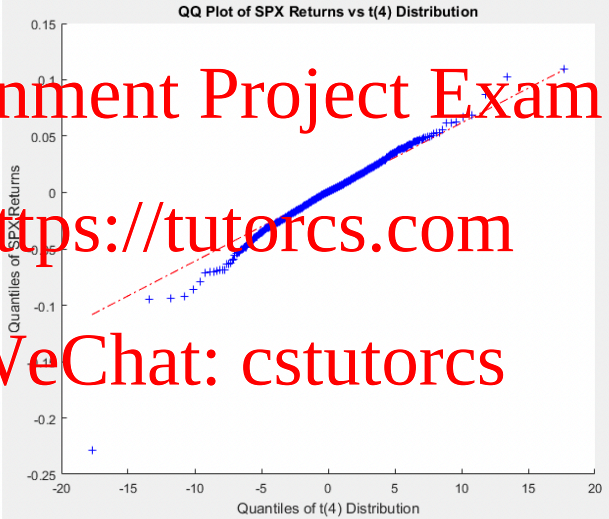
- Plot would be linear if the target distribution is a linear function of the reference one

- Deviations from linearity reveal the existence of heavy tails.

S&P 500 vs. normal distribution



S&P 500 vs. student-t(4) distribution



Implications of fat tails

- Extreme outcomes tend to have a disproportional influence on economic and financial matters, so the assumption of normality tends to lead to underestimation of risk . . .

- . . . but use of non-normal distributions is usually complicated, and tends to require more data

Stylized fact 3

Correlations tend to rise sharply at times of crisis.

Table 1.6. Return correlations and means for Microsoft (MSFT), Morgan Stanley (MS), Goldman Sachs (GS) and Bear Stearns (BSC)

(a) Daily return correlations (May 5, 1999–September 12, 2007)

	MSFT	MS	GS
MS	44%		
GS	44%	81%	
BSC	38%	74%	71%

(b) Daily return correlations (1 August 2007–15 August 2007) during the opening events of the 2007 crisis

	MSFT	MS	GS
MS	93%		
GS	82%	94%	
BSC	82%	92%	89%

- Many funds using quant strategies saw correlations in their holdings increase a lot, so riskiness of portfolios also increased

- Quote by David Viniar (Goldman Sachs's CFO) on the firms' Alpha Fund: "We were seeing things that were 25-standard deviation moves several days in a row."

- Under a normal distribution, the probability of an event of this magnitude is less than 3×10^{-138}; the age of the universe is about 5×1012 days, so these events should happen once in about 10125 universes.

- Maybe distribution of returns isn't normal after all.

Stylized facts

- The conditional volatility of financial returns exhibits cyclical patterns (market shocks tend to lead to periods of high volatility; the effect of shocks is persistent but eventually dies out)

- Financial returns are not normally distributed, and exhibit fat tails

- The correlations of returns series varies over time, and rises in times of crises

- We will develop models that help explain and analyze these patterns during the course