CW9

28 November, 2022

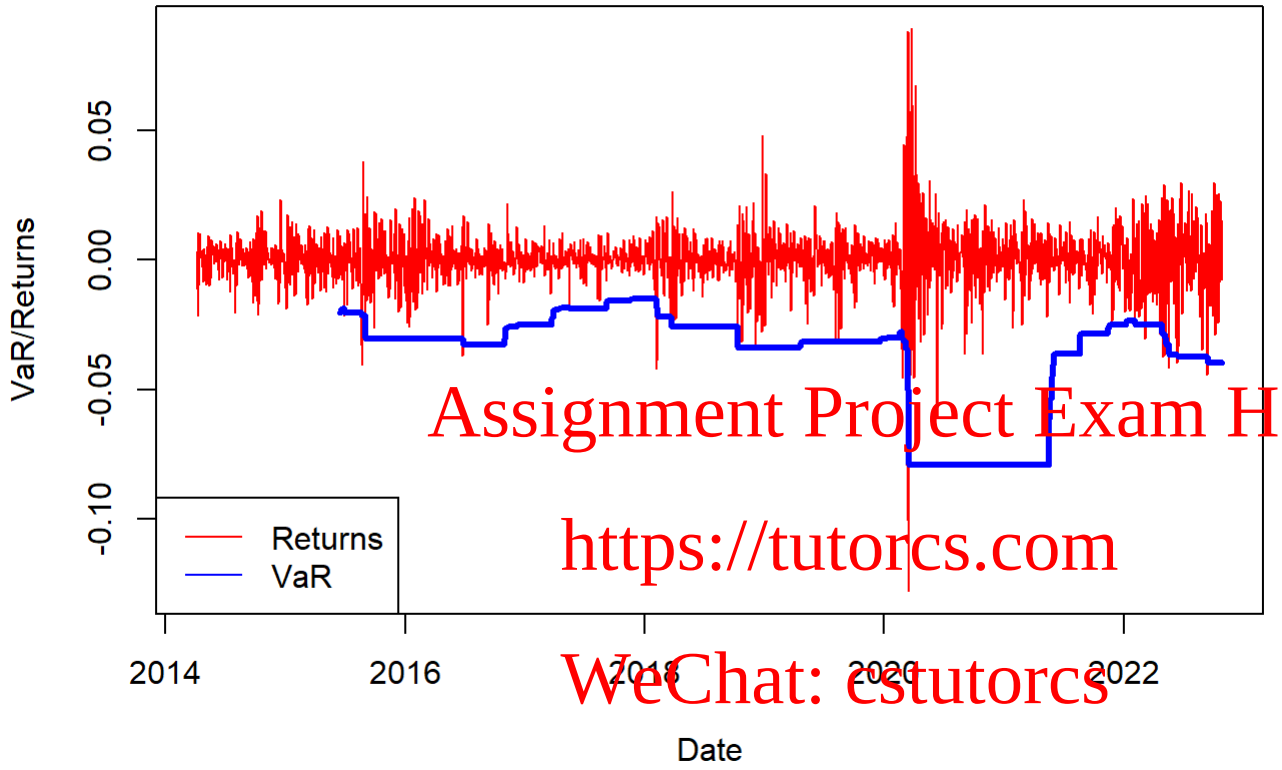
Backtesting HS VaR

VaR and ES with Historical Simulation

```
Ts <- length(index(log_returns_demean)) # Number of trading days in sample - Ts
VaR_HS <- xts(matrix(nrow = Ts, ncol = 1), order.by = index(log_returns_demean))
ES_HS <- xts(matrix(nrow = Ts, ncol = 1), order.by = index(log_returns_demean))
p = 0.01
WE <- 3 / p
VaR_HS <- rollapply(data = log_returns_demean, width = WE, FUN = function(x) - sort(coredata(x))
        [3])
ES_HS <- rollapply(data = log_returns_demean, width = WE, FUN = function(x) - mean(sort(coredata
        (x))[1:3]))
VaR_HS <- lag(VaR_HS, k = 1, na.pad = TRUE)
ES_HS <- lag(ES_HS, k = 1, na.pad = TRUE)
```



```
plot(x = index(log_returns_demean), y = log_returns_demean, ylab = "VaR/Returns", lwd = 1,
       xlab = "Date", type = "l", col = "red")
  lines(x = index(log_returns_demean), y = -VaR_HS , col = "blue", lwd = 3)
  legend("bottomleft", legend = c('Returns', 'VaR'), lty = 1, col = c("red", "blue"))
```



Calculate the number of violations

```
WT <- Ts - WE # Size of testing window
v <- sum(log_returns_demean < - VaR_HS, na.rm = TRUE) # na.rm  = remove missing values (NA)
v0 <- WT - v
EV <- p*WT # expected number of violations

print(paste('Number of HS violations is',v))
```

```
## [1] "Number of HS violations is 30"
```

```
print(paste('Number of no violations is',v0))
```

```
## [1] "Number of no violations is 1827"
```

```
print(paste('Expected number of violations is',EV))
```

```
## [1] "Expected number of violations is 18.57"
```



```
VR = v / EV # observed number of violations/expected number of violations
print(paste('Violation ratio',VR))
```



```
## [1] "Violation ratio 1.61550883520887"
```



```
if (v > EV) {
  print('You have underforecasted VaR')
} else {
  print('You have overforecasted VaR')
}
```

```
## [1] "You have underforecasted VaR"
```

Coverage tests

Bernoulli Test - Unconditional coverage test

```
ra <- log_returns_demean[(WE + 1):Ts]
VaRa <- VaR_HS[(WE + 1):Ts]
eta <- ra < - VaRa
v1 <- sum(eta)
v0 <- length(eta) - v1
picap <- v1 / (v1 + v0)

a <- (1 - p)^v0 * p^v1 # likelihood of restricted model
b <- (1 - picap)^v0 * picap^v1 # likelihood of unrestricted model

LR <- 2 * (log(b / a))
if (LR > qchisq(p = 1 - p, df = 1)) {
  print('null hypothesis H0 is rejected')
} else {
  print('We cannot reject the null')
}
```



```
## [1] "We cannot reject the null"
```

Independence Test

```
logical <- matrix(nrow = WT, ncol = 4)

for (i in 2:WT) {
  logical[i,1] <- coredata(eta)[i-1] == 0 & coredata(eta)[i] == 0
  logical[i,2] <- coredata(eta)[i-1] == 0 & coredata(eta)[i] == 1
  logical[i,3] <- coredata(eta)[i-1] == 1 & coredata(eta)[i] == 0
  logical[i,4] <- coredata(eta)[i-1] == 1 & coredata(eta)[i] == 1
}

eta_00 = sum(logical[, 1], na.rm = TRUE)
eta_01 = sum(logical[, 2], na.rm = TRUE)
eta_10 = sum(logical[, 3], na.rm = TRUE)
eta_11 = sum(logical[, 4], na.rm = TRUE)

P_00 = eta_00 / (eta_00 + eta_01)
P_01 = eta_01 / (eta_00 + eta_01)
P_10 = eta_10 / (eta_10 + eta_11)
P_11 = eta_11 / (eta_10 + eta_11)

hat_p = (eta_01 + eta_11) / (eta_00 + eta_01 + eta_10 + eta_11)

b1 = P_00^(eta_00) * P_01^(eta_01) * P_10^(eta_10) * P_11^(eta_11)
a1 = (1 - hat_p)^(eta_00 + eta_10) * hat_p^(eta_01 + eta_11)

LR1 = 2 * log(b1 / a1)

if (LR1 > qchisq(p = 1 - p, df = 1)) {
  print('Null hypothesis H0 is rejected')
} else {
  print('We cannot reject the null')
}
```

```
## [1] "Null hypothesis H0 is rejected"
```

Backtest Expected Shortfall

```
ESa = ES_HS[(WE + 1):Ts]

NS <- ra[ra < - VaRa] / - ESa[ra < - VaRa]
avNS = mean(NS)
avNS
```

```
## [1] 1.2171
```