M30242 Graphics and Computer Vision

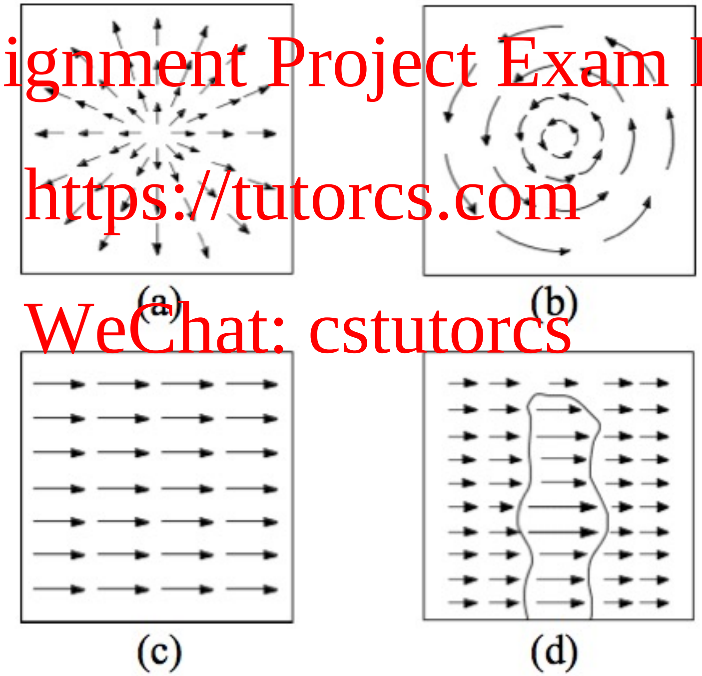
Lecture 10 Optical Flow

Motion Detection

- Image subtraction (image difference)
  - Find the differences between the consecutive frames.
  - Stationary camera.
- Feature tracking: tracking a small set of salient points/features
  - Camera and/or scene objects are moving.
- Optical flow
  - Relative motions (camera v.s. scene or vice versa).
  - Stationary or moving camera, where relative motion is important, e.g., in robotics.

Motion Detection By Optical Flow

- Motion is deduced by observing and tracking the changes of the brightness pattern of videos.



What motion do the patterns represent?

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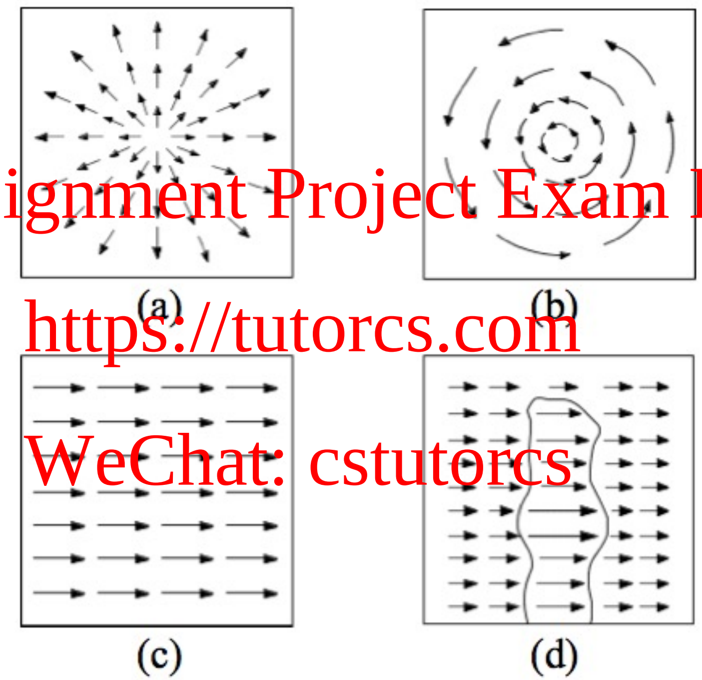


- (a) Motion in the direction perpendicular to the image plane of the camera.
- (b) Rotation about the axis perpendicular to image plane.
- (c) Motion of an object (or camera) at a fixed distance.
- (d) Motions of an object and a more distant background.

Cont'd

- To detect the actual motion from the apparent motion (optical flow), we need to answer the following questions:

  1. What is the relationship between the actual relative motion (3D) in the scene and the *apparent motion* (2D) in image?

  2. How can we calculate the apparent motion from the changes in brightness pattern?
  3. Given 1 and 2, how can we compute the actual motion from the apparent motion in images?

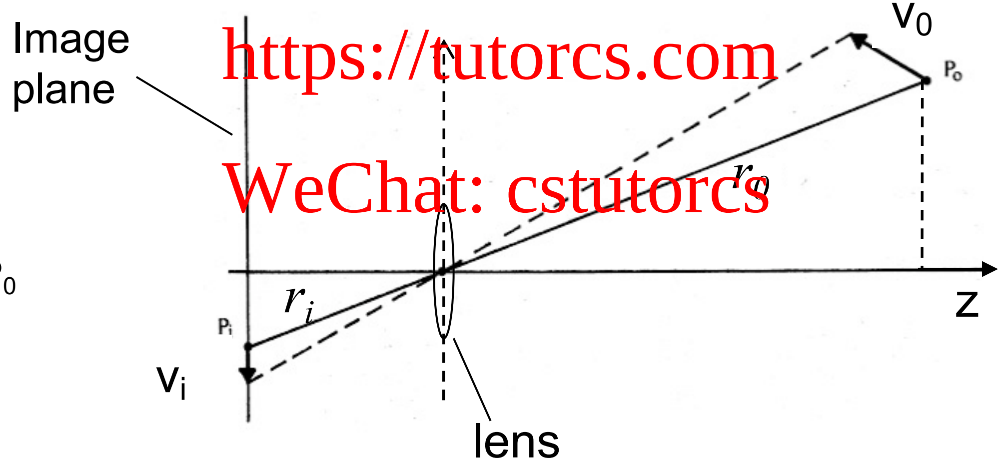
Solutions

- The answer to 1 is in the analysis of perspective projection.
- The answer to question 2 is provided by the optic flow equation.
- To solve the optical flow equation, the following quantities need to be evaluated at all image pixels (or at some salient features. Different approaches exist) :
  - the image *gradient* of each frame, and
  - *intensity change rate* between two consecutive frames.

Relationship Between Actual and Apparent Motions: Motion Field

- Motion field: projection of true 3D motions on image plane. For example, object point p_0 moving at velocity v_0 induces v_i on the image plane.

$$\mathbf{v}_0 = \frac{d\mathbf{r}_0}{dt}$$ Actual velocity of point P_o



velocity of the image of point P_0

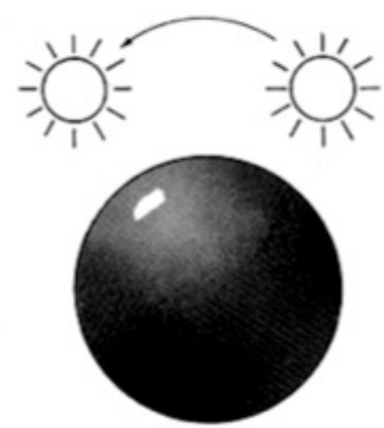
$$\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$$

$\mathbf{r}_0$ is related to $\mathbf{r}_i$ by $$\frac{\mathbf{r}_i}{f} = \frac{\mathbf{r}_0}{\mathbf{r}_0 \bullet \mathbf{z}}$$

where $\mathbf{z}$ is the unit vector along z-axis (the optical axis of the lens)

Optical Flow Field

- Optical flow is the apparent motion of an image feature.
- The velocity vectors of all image features form an optical flow field. Visually, the flow field is a change of brightness patterns.
- In most cases, optical flow reflects the motion field, but counter examples exist. For example, the change of position of light source induces non-zero optical flow (i.e., change of position of highlight), but the motion field is zero (the sphere is stationary).



Optical Flow Equation

- The optical flow equation states that *the rate of intensity change in time at an image point equals to the projection of velocity of apparent motion*, $\mathbf{v}=v_x\boldsymbol{i},+v_y\boldsymbol{j},$ *on the gradient vector at that point*:

$$\nabla f \bullet \mathbf{v} = -\frac{\partial f}{\partial t}$$

Where $\mathbf{v}$ is the vector of the apparent motion (of a feature/point), and

$$\nabla f = \frac{\partial f}{\partial y}\mathbf{i} + \frac{\partial f}{\partial x}\mathbf{j} \qquad \frac{\partial f}{\partial t} \approx \frac{I_{t+\delta t} - I_t}{\delta t}$$

∇f and $\partial f/\partial t$ are the (unit) *gradient vector* and *intensity change rate*, respectively.

- Solving the equation for $\mathbf{V}$ at every image point gives the optical flow field (the field of apparent motion).

Flow Field Calculation

- Given a sequence of images, we wish to calculate the optical flow field (i.e., solve the optical flow equation at each pixel), and from the flow field to evaluate the field of the real motion that induces the flow.
- In the optic flow equation, the image gradient and the intensity change rate can be readily calculated from the sequence of images.
- Different techniques exist for the solution of the optical equations, e.g., Lucas-Kanade method.

Image Gradient

- Because optical flow is a concept in time domain, so we now treat image intensity as a function of x, y and t:

$$I = f(x, y, t)$$

- At a fixed instant (or equivalently, at a frame), t, *image gradient vector* is the vector sum of the gradient in x- and y-directions:

$$\nabla f = \nabla I = \frac{\partial f}{\partial y}\mathbf{i} + \frac{\partial f}{\partial x}\mathbf{j} = I_x^{'}\mathbf{i} + I_y^{'}\mathbf{j}$$

Where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in x and y (vertical and horizontal) directions.

Compare it with the Laplacian operator $\nabla^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2}$

Intensity Change Rate

- The change rate of image intensity in time is the first derivative of intensity with respect to t:

$$\frac{\partial f}{\partial t} = I_t^{'} \approx \frac{I_{t+\delta t} - I_t}{\delta t}$$

  where I_t and $I_{t+\delta t}$ are the intensities of a pixel (or a feature point) at time t and $t+\delta t$ (two consecutive frames).

- Evaluating the dot product of the gradient vector and the apparent motion vector in the optical flow equation gives

$$\nabla f \bullet \mathbf{v} = -\frac{\partial f}{\partial t} \qquad I_x^{'} v_x + I_y^{'} v_y = -I_t^{'}$$

Dot Product and Projection

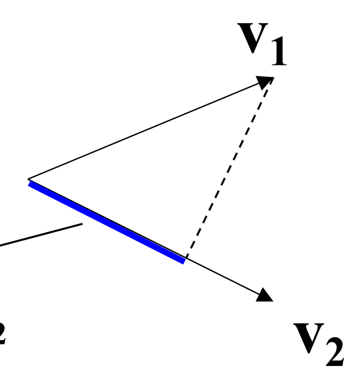
Dot (inner) product of two vectors:

$$\mathbf{v}_1 \bullet \mathbf{v}_2 = |\mathbf{v}_1||\mathbf{v}_2|\cos\theta = \begin{bmatrix} x_1 & y_1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Scalar projection of vector v_1 on vector v_2:

$$\mathbf{v}_{\mathbf{1}\,\mathbf{proj}} = \mathbf{v}_{\mathbf{1}} \bullet \hat{\mathbf{v}}_2 = \mathbf{v}_1 \bullet \left(\frac{\mathbf{v}_2}{|\mathbf{v}_2|}\right) = |\mathbf{v}_{\mathbf{1}}|\cos\theta = |\mathbf{v}_{\mathbf{1}}|\frac{(\mathbf{v}_1 \bullet \mathbf{v}_2)}{|\mathbf{v}_1||\mathbf{v}_2|}$$

Where $\hat{\mathbf{v}}_2$ is the unit vector representing the direction of $\mathbf{V}_2$ and θ is the angle between the two vectors



$\mathbf{V}_{\mathbf{1proj}}$: projection of $\mathbf{v}_1$ on $\mathbf{v}_2$

An example

In optical flow equation

$$I'_x v_x + I'_y v_y = -I'_t$$

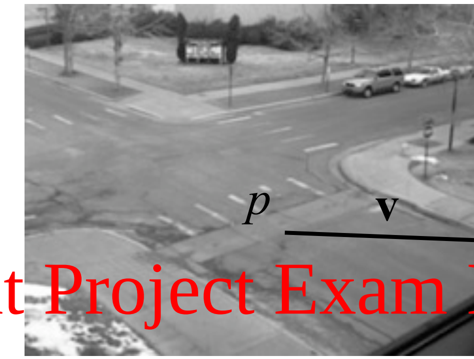
we can measure:

- gradients in x- and y-direction: I'_x and I'_y at p at time t
- image intensities I_t and $I_{t+\delta t}$ at p at time t and $t+\delta t$:

$$\frac{\partial f}{\partial t} = I'_t \approx \frac{I_{t+\delta t} - I_t}{\delta t}$$

- The apparent motion vector, $\mathbf{v}$, is the only unknown in the optical flow equation, so we can solve the equation for it..

Frame at t | Frame at $t+\delta t$



Suppose the scene involves apparent motion moves (caused by moving the camera) $\mathbf{v}$ at a point on an edge point p

$\mathbf{v} = \mathbf{v}_x\mathbf{i} + \mathbf{v}_y\mathbf{j}$

is the unknown to be solved

$$I'_t \approx \frac{I_{t+\delta t} - I_t}{\delta t}$$

$$\nabla f = I'_x\mathbf{i} + I'_y\mathbf{j}$$

Lucas-Kanade Methods

- The method solve the optical flow equation for v_x and v_y by dividing the original image into smaller blocks/sections and assumes a constant velocity in each block/section

- With each image block, the method minimises a weighted least-square fit of the optical flow equation:

$$\min\left(\sum_{\Omega} w^2 [I'_x v_x + I'_y v_y + I'_t]^2\right)$$

- This minimisation problem is equivalent to the linear equations in v_x and v_y (proof is omitted):

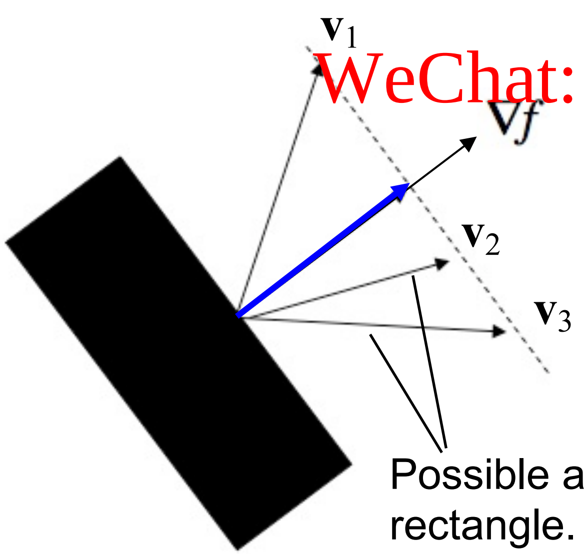
$$\begin{bmatrix} \Sigma w^2 I'^2_x & \Sigma w^2 I'_x I'_y \\ \Sigma w^2 I'_x I'_y & \Sigma w^2 I'^2_y \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \Sigma w^2 I'_x I'_t \\ \Sigma w^2 I'_y I'_t \end{bmatrix}$$

Cont'd

- The weight factor, w, is a window that gives more weight to the pixels at the centre of the region/section (like a Gaussian filter).
- The set of equations can be solved by any method for linear equations, e.g., Gaussian elimination.
- One normally needs to set a threshold to the calculated values of v_x and v_y to reduce the effect of noises.
- The approach is implemented in Matlab as **`opticalFlowLK`**

Limitations of Optical Flow Equation

- The existing algorithms allow us to solve the equation for $\mathbf{v}$. However, the solution is not unique.
- In fact, we can only determine the *projection* of the velocity vector on gradient vector (i.e., the motion perpendicular to the edge in the figure below), but not the velocity *itself*. This leads to the so-called aperture problem.

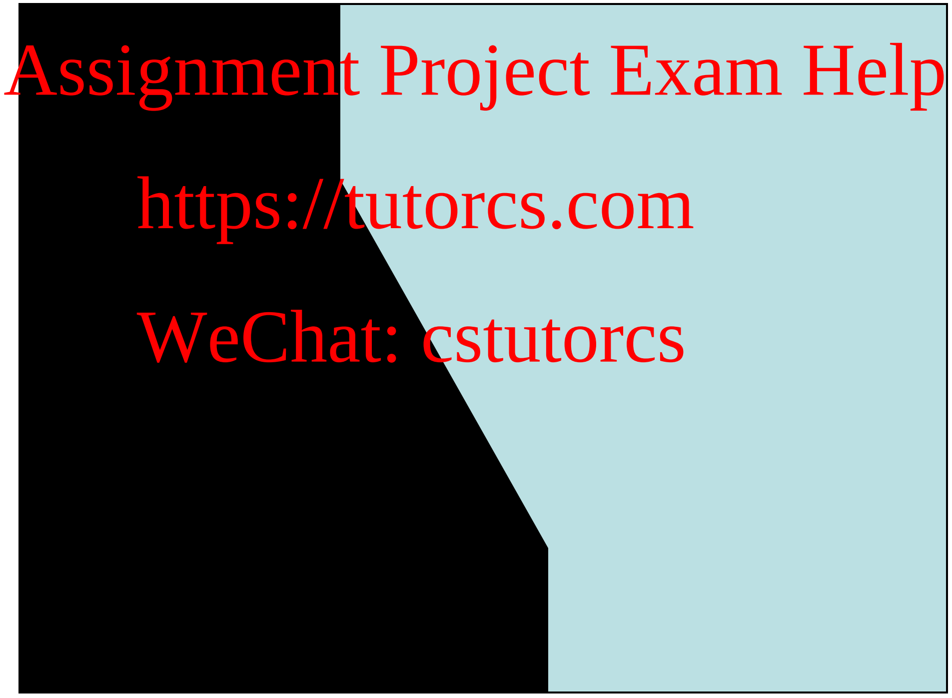


$$\nabla f \bullet \mathbf{v} = -\frac{\partial f}{\partial t}$$

Possible apparent motion vectors of the black rectangle. They all lead to the same projection on the gradient vector ∇f

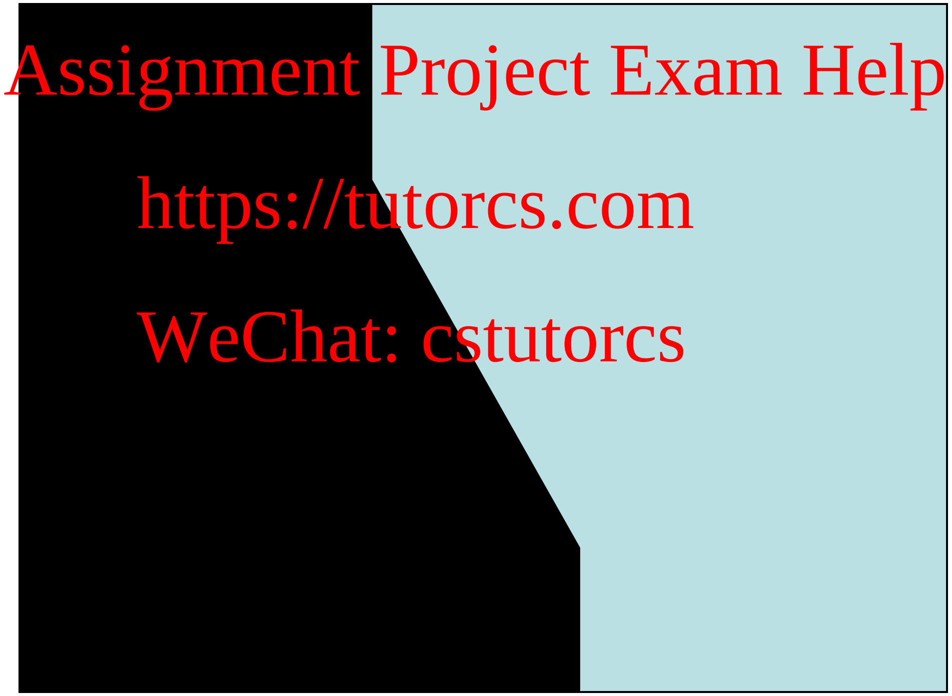
Aperture Problem

Where does the black object move?



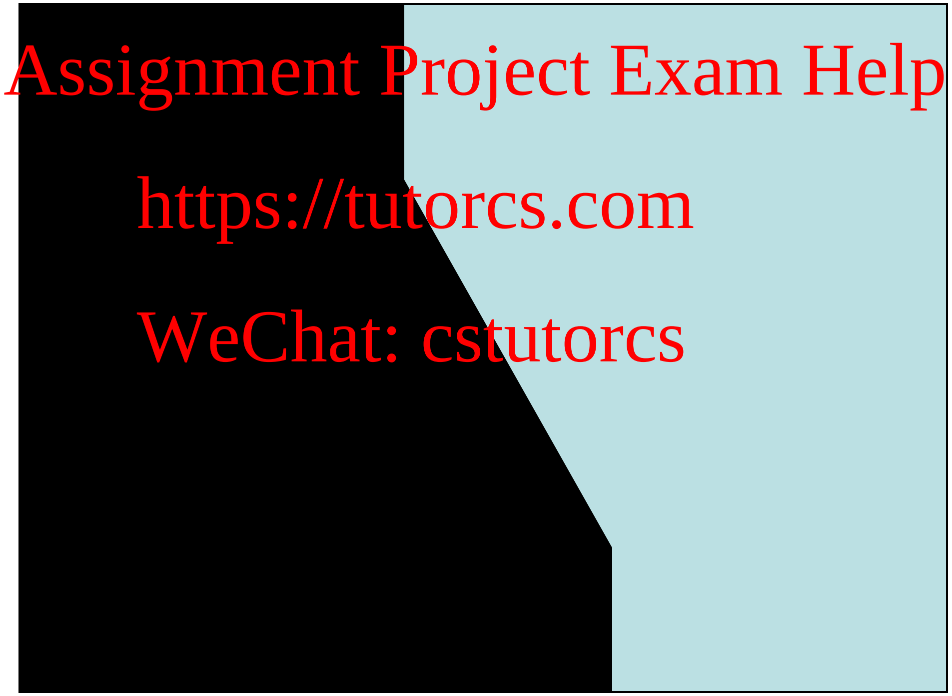
Aperture Problem

Where does the black object move?



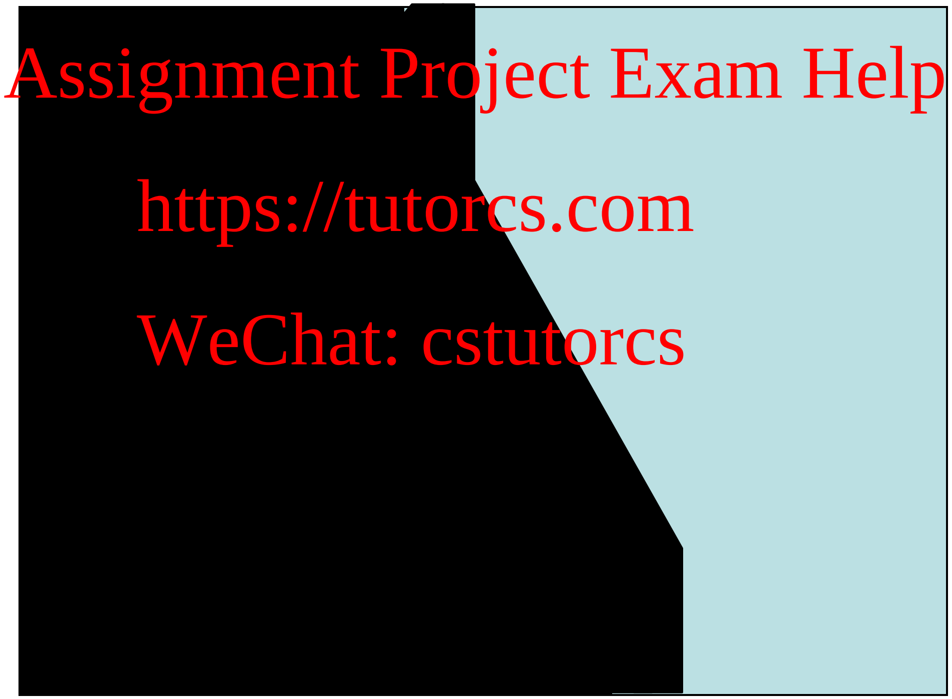
Aperture Problem

In which direction does the black object move?



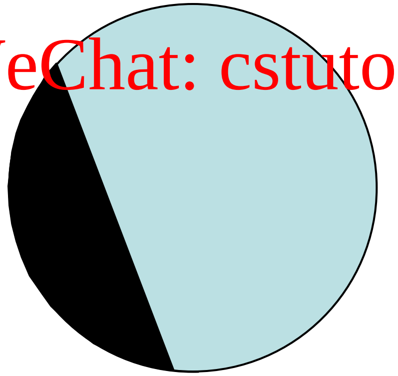
Aperture Problem

In which direction does the black object move?



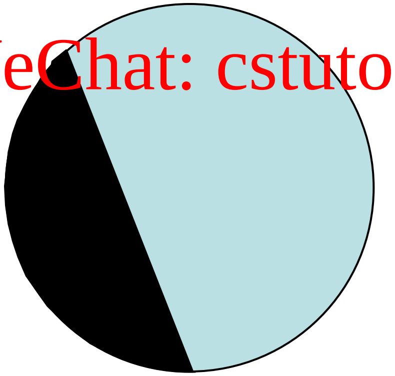
Aperture Problem

- However, if we observe the black object through a small aperture, we will see the object moves in the direction perpendicular to the edge – not the true motion of the object.



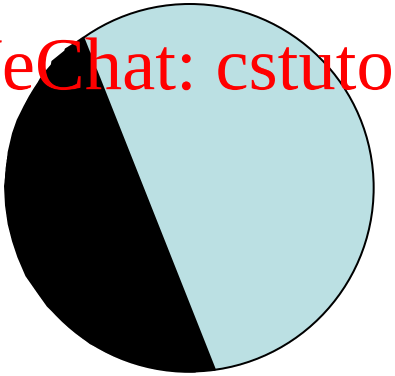
Where does the black edge move?

Aperture Problem



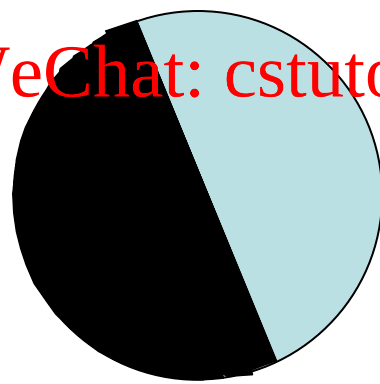
Where does the black edge move?

Aperture Problem



Now the direction of the motion of the black edge has changed

Aperture Problem

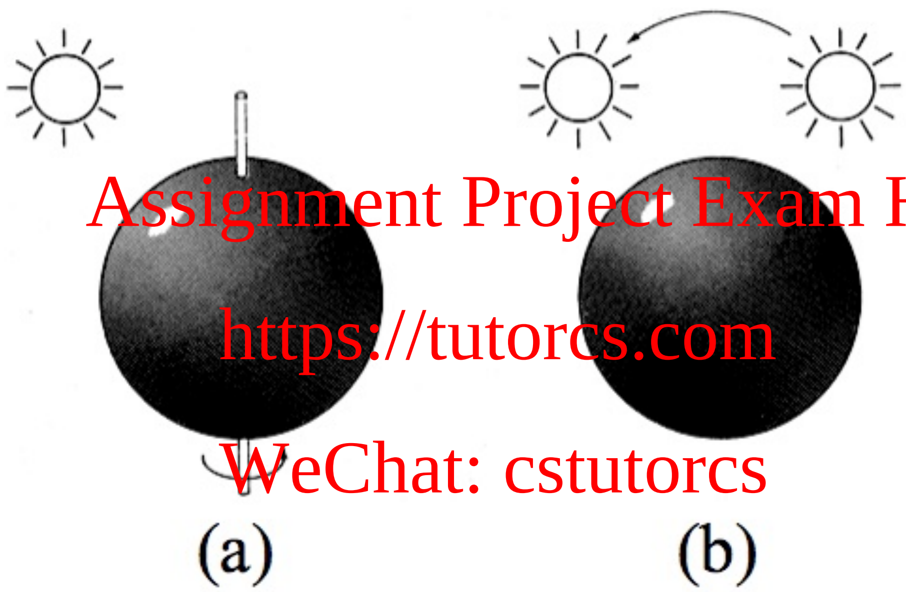


Now the direction of the motion of the black edge has changed

Evaluation of Motion from Optical Flow

- The existence of the aperture problem implies that in some cases we cannot accurately calculate the apparent motions from the optical flow equation
- When only local optical flow is measured, more conditions are needed to constrain the flow equation.
- Even if we can estimate the apparent motion field from the optical flow field, the apparent motions are not the same as the true 3D motions.
- If your application needs 3D motion info, then you may have to consider other approaches.

Cases Where Optical Flow Fails



- (a) A smooth sphere is rotating under constant illumination. Thus the optical flow field is zero, but the motion field is not.
- (b) A fixed sphere is illuminated by a moving light source. Thus the motion field is zero, but the optical flow field is not.

Further Reading

- Shapiro, L.G., Stockman, G.C., Computer Vision, Prentice-Hall, 2001, ISBN 0-13-030796-3
  – Chapter 9