CHAPTER 3

Linear Programming: Sensitivity Analysis and Interpretation of Solution

CONTENTS

Sensitivity analysis is the study of how the changes in the coefficients of an optimization model affect the optimal solution. Using sensitivity analysis, we can answer questions such as the following:

1. How will a change *in a coefficient of the objective function* affect the optimal solution?
2. [...]e *right-hand-side value for a constraint* affect the optimal

Becaus[...] concerned with how these changes affect the optimal solution, th[...]n until the optimal solution to the original linear programming p[...]d. For that reason, sensitivity analysis is often referred to as *postop*[...]

Ou[...]y analysis parallels the approach used to introduce linear progra[...] Chapter 2. We begin by showing how a graphical method can be used to perform sensitivity analysis for linear programming problems with two decision variables. Then, we show how optimization software provides sensitivity analysis information.

Finally, we extend the discussion of problem formulation started in Chapter 2 by formulating and solving three larger linear programming problems. In discussing the solution for each of these problems, we focus on managerial interpretation of the optimal solution and sensitivity analysis information.

MANAGEMENT SCIENCE IN ACTION

ASSIGNING PRODUCTS TO WORLDWIDE FACILITIES AT EASTMAN KODAK*

One of the major planning issues at Eastman Kodak involves the determination of which products should be manufactured at Kodak's facilities located throughout the world. The assignment of products to facilities is called the "world load." In determining the world load, Kodak faces a number of interesting trade-offs. For instance, not all manufacturing facilities are equally efficient for all products, and the margins by which some facilities are better varies from product to product. In addition to manufacturing costs, the transportation costs and the effects of duty and duty drawbacks can significantly affect the allocation decision.

To assist in determining the world load, Kodak developed a linear programming model that accounts for the physical nature of the distribution problem and the various costs (manufacturing, transportation, and duties) involved. The model's objective is to minimize the total cost subject to constraints such as satisfying demand and capacity constraints for each facility.

The linear programming model is a static representation of the problem situation, and the real world is always changing. Thus, the linear programming model must be used in a dynamic way. For instance, when demand expectations change, the model can be used to determine the effect the change will have on the world load. Suppose that the currency of country A rises compared to the currency of country B. How should the world load be modified? In addition to using the linear programming model in a "how-to-react" mode, the model is useful in a more active mode by considering questions such as the following: Is it worthwhile for facility F to spend *d* dollars to lower the unit manufacturing cost of product P from *x* to *y*? The linear programming model helps Kodak evaluate the overall effect of possible changes at any facility.

In the final analysis, managers recognize that they cannot use the model by simply turning it on, reading the results, and executing the solution. The model's recommendation combined with managerial judgment provide the final decision.

*Based on information provided by Greg Sampson of Eastman Kodak.

Sensitivity analysis and the interpretation of the optimal solution are important aspects of applying linear programming. The Management Science in Action, Assigning Products to Worldwide Facilities at Eastman Kodak, shows some of the sensitivity analysis and interpretation issues encountered at Kodak in determining the optimal product assignments. Later in the chapter, other Management Science in Action articles illustrate how Performance Analysis Corporation uses sensitivity analysis as part of an evaluation model for a chai[illegible]ow GE Plastics uses a linear programming model involving thou[illegible]nstraints to determine optimal production quantities, how the Nutr[illegible]er of the University of Minnesota uses a linear programming mod[illegible]nt amounts in new food products, and how Duncan Industries Lim[illegible]ng model for tea distribution convinced management of the bene[illegible] analysis techniques to support the decision-making process.

3.1 INTRODUCTION TO SENSITIVITY ANALYSIS

Sensitivity analysis is important to decision makers because real-world problems exist in a changing environment. Prices of raw materials change, product demand changes, companies purchase new machinery, stock prices fluctuate, employee turnover occurs, and so on. If a linear programming model has been used in such an environment, we can expect some of the coefficients to change over time. We will then want to determine how these changes affect the optimal solution to the original linear programming problem. Sensitivity analysis provides us with the information needed to respond to such changes without requiring the complete solution of a revised linear program.

Recall the Par, Inc., problem:

$$
\begin{aligned}
\text{Max} \quad & 10S + 9D \\
\text{s.t.} \quad & \\
& \tfrac{7}{10}S + 1D \le 630 \quad \text{Cutting and dyeing} \\
& \tfrac{1}{2}S + \tfrac{5}{6}D \le 600 \quad \text{Sewing} \\
& 1S + \tfrac{2}{3}D \le 708 \quad \text{Finishing} \\
& \tfrac{1}{10}S + \tfrac{1}{4}D \le 135 \quad \text{Inspection and packaging} \\
& S, D \ge 0
\end{aligned}
$$

The optimal solution, $S = 540$ standard bags and $D = 252$ deluxe bags, was based on profit contribution figures of \$10 per standard bag and \$9 per deluxe bag. Suppose we later learn that a price reduction causes the profit contribution for the standard bag to fall from \$10 to \$8.50. Sensitivity analysis can be used to determine whether the production schedule calling for 540 standard bags and 252 deluxe bags is still best. If it is, solving a modified linear programming problem with $8.50S + 9D$ as the new objective function will not be necessary.

Sensitivity analysis can also be used to determine which coefficients in a linear programming model are crucial. For example, suppose that management believes the \$9 profit contribution for the deluxe bag is only a rough estimate of the profit contribution that will actually be obtained. If sensitivity analysis shows that 540 standard bags and 252 deluxe bags will be the optimal solution as long as the profit contribution for the deluxe bag is between \$6.67 and \$14.29, management should feel comfortable with the \$9-per-bag estimate and the recommended production quantities. However, if sensitivity analysis shows that 540 standard bags and 252 deluxe bags will be the optimal solution only if the profit contribution for the deluxe bags is between \$8.90 and \$9.25, management may want to review the accuracy of the \$9-per-bag estimate. Management would especially want to consider how the optimal production quantities should be revised if the profit contribution per deluxe bag were to drop.

Another aspect of sensitivity analysis concerns changes in the right-hand-side values of the constraints. Recall that in the Par, Inc., problem the optimal solution used all available time in the cutting and dyeing department and the finishing department. What would happen to the optimal solution and total profit contribution if Par could obtain additional quantities of e[illegible]? Sensitivity analysis can help determine how much each additiona[illegible]ne is worth and how many hours can be added before diminishi[illegible]

3.2 GRAP[illegible]TY ANALYSIS

For linea[illegible], with two decision variables, graphical solution methods can be u[illegible]y analysis on the objective function coefficients and the right-hand-side values for the constraints.

Objective Function Coefficients

Let us consider how changes in the objective function coefficients might affect the optimal solution to the Par, Inc., problem. The current contribution to profit is $10 per unit for the standard bag and $9 per unit for the deluxe bag. It seems obvious that an increase in the profit contribution for one of the bags might lead management to increase production of that bag, and a decrease in the profit contribution for one of the bags might lead management to decrease production of that bag. It is not as obvious, however, how much the profit contribution would have to change before management would want to change the production quantities.

The current optimal solution to this problem calls for producing 540 standard golf bags and 252 deluxe golf bags. The **range of optimality** for each objective function coefficient provides the range of values over which the current solution will remain optimal. Managerial attention should be focused on those objective function coefficients that have a narrow range of optimality and coefficients near the end points of the range. With these coefficients, a small change can necessitate modifying the optimal solution. Let us now compute the ranges of optimality for this problem.

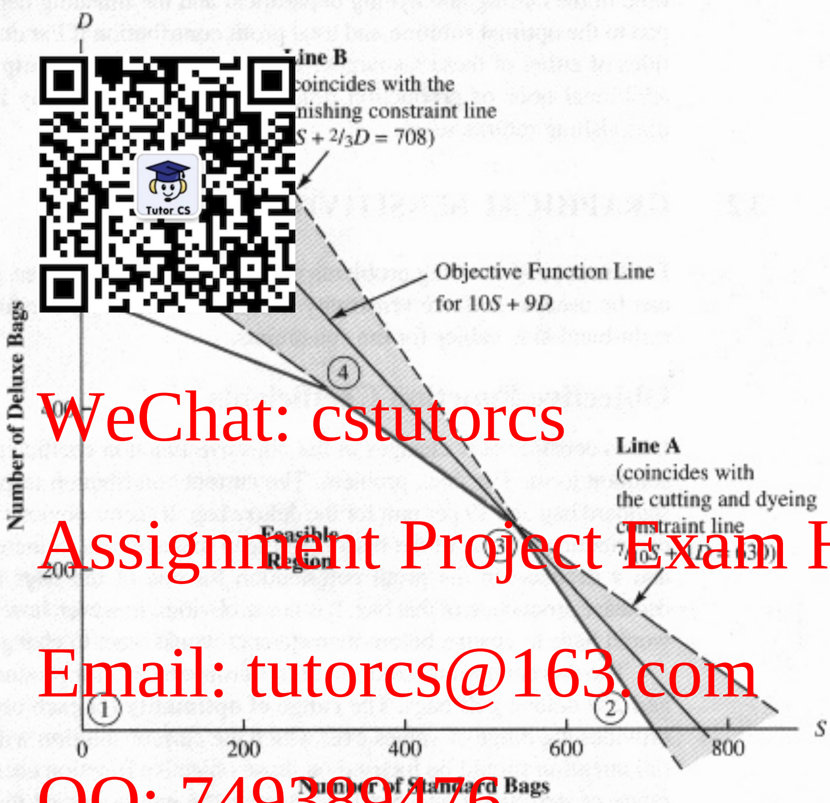
Figure 3.1 shows the graphical solution. A careful inspection of this graph shows that as long as the slope of the objective function line is between the slope of line A (which coincides with the cutting and dyeing constraint line) and the slope of line B (which coincides with the finishing constraint line), extreme point 3 with $S = 540$ and $D = 252$ will be optimal. Changing an objective function coefficient for S or D will cause the slope of the objective function line to change. In Figure 3.1 we see that such changes cause the objective function line to rotate around extreme point 3. However, as long as the objective function line stays within the shaded region, extreme point 3 will remain optimal.

Rotating the objective function line *counterclockwise* causes the slope to become less negative, and the slope increases. When the objective function line rotates counterclockwise (slope increased) enough to coincide with line A, we obtain alternative optimal solutions between extreme points 3 and 4. Any further counterclockwise rotation of the objective function line will cause extreme point 3 to be nonoptimal. Hence, the slope of line A provides an upper limit for the slope of the objective function line.

The slope of the objective function line usually is negative; hence, rotating the objective function line clockwise makes the line steeper even though the slope is getting smaller (more negative).

Rotating the objective function line *clockwise* causes the slope to become more negative, and the slope decreases. When the objective function line rotates clockwise (slope decreases) enough to coincide with line B, we obtain alternative optimal solutions between extreme points 3 and 2. Any further clockwise rotation of the objective function line

FIGURE 3.1 GRAPHICAL SOLUTION OF PAR, INC., PROBLEM WITH SLOPE OF OBJECTIVE FUNCTION LINE BETWEEN SLOPES OF LINES A AND B; EXTREME POINT (3) IS OPTIMAL



will cause extreme point (3) to be nonoptimal. Hence, the slope of line B provides a lower limit for the slope of the objective function line.

Thus, extreme point (3) will be the optimal solution as long as

Slope of line B ≤ slope of the objective function line ≤ slope of line A

In Figure 3.1 we see that the equation for line A, the cutting and dyeing constraint line, is as follows:

$$\tfrac{7}{10}S + 1D = 630$$

By solving this equation for D, we can write the equation for line A in its slope-intercept form, which yields

$$D = \underbrace{-\tfrac{7}{10}}_{\text{Slope of line A}}S + \underbrace{630}_{\text{Intercept of line A on } D \text{ axis}}$$

Thus, the slope for line A is $-\tfrac{7}{10}$, and its intercept on the D axis is 630.

The equation for line B in Figure 3.1 is

$$1S + \tfrac{2}{3}D = 708$$

Solving for D provides the slope-intercept form for line B. Doing so yields

$$\tfrac{2}{3}D = -1S + 708$$
$$D = -\tfrac{3}{2}S + 1062$$

Thus, the [illegible] and its intercept on the D axis is 1062.

Now [illegible] A and B have been computed, we see that for extreme point ③ [illegible] ust have

$$-\tfrac{3}{2} \leq \text{slope of objective function} \leq -\tfrac{7}{10} \tag{3.1}$$

Let us now consider the general form of the slope of the objective function line. Let C_S denote the profit of a standard bag, C_D denote the profit of a deluxe bag, and P denote the value of the objective function. Using this notation, the objective function line can be written as

$$P = C_S S + C_D D$$

Writing this equation in slope-intercept form, we obtain

$$C_D D = -C_S S + P$$

and

$$D = -\frac{C_S}{C_D} S + \frac{P}{C_D}$$

Thus, we see that the slope of the objective function line is given by $-C_S/C_D$. Substituting $-C_S/C_D$ into expression (3.1), we see that extreme point ③ will be optimal as long as the following expression is satisfied:

$$-\tfrac{3}{2} \leq -\frac{C_S}{C_D} \leq -\tfrac{7}{10} \tag{3.2}$$

To compute the range of optimality for the standard-bag profit contribution, we hold the profit contribution for the deluxe bag fixed at its initial value $C_D = 9$. Doing so in expression (3.2), we obtain

$$-\tfrac{3}{2} \leq -\frac{C_S}{9} \leq -\tfrac{7}{10}$$

From the left-hand inequality, we have

$$-\frac{3}{2} \leq -\frac{C_S}{9} \quad \text{or} \quad \frac{3}{2} \geq \frac{C_S}{9}$$

Thus,

$$\frac{27}{2} \geq C_S \quad \text{or} \quad C_S \leq \frac{27}{2} = 13.5$$

From the right-hand inequality, we have

$$-\frac{C_S}{9} \leq -\frac{7}{10} \quad \text{or} \quad \frac{C_S}{9} \geq \frac{7}{10}$$

Thus,

$$C_S \geq \frac{63}{10} \quad \text{or} \quad C_S \geq 6.3$$

Combining the calculated limits for C_S provides the following range of optimality for the standard-bag profit contribution:

$$6.3 \leq C_S \leq 13.5$$

In the original problem for Par, Inc., the standard bag had a profit contribution of \$10. The resulting optimal solution was 540 standard bags and 252 deluxe bags. The range of optimality for C_S tells Par's management that, with other coefficients unchanged, the profit contribution for the standard bag can be anywhere between \$6.30 and \$13.50 and the production quantities of 540 standard bags and 252 deluxe bags will remain optimal. Note, however, that even though the production quantities will not change, the total profit contribution (value of objective function) will change due to the change in profit contribution per standard bag.

These computations can be repeated, holding the profit contribution for standard bags constant at $C_S = 10$. In this case, the range of optimality for the deluxe-bag profit contribution can be determined. Check to see that this range is $6.67 \leq C_D \leq 14.29$.

In cases where the rotation of the objective function line about an optimal extreme point causes the objective function line to become *vertical*, there will be either no upper limit or no lower limit for the slope as it appears in the form of expression (3.2). To show how this special situation can occur, suppose that the objective function for the Par, Inc., problem is $18C_S + 9C_D$; in this case, extreme point (2) in Figure 3.2 provides the optimal solution. Rotating the objective function line counterclockwise around extreme point (2) provides an upper limit for the slope when the objective function line coincides with line B. We showed previously that the slope of line B is $-\frac{3}{2}$, so the upper limit for the slope of the objective function line must be $-\frac{3}{2}$. However, rotating the objective function line clockwise results in the slope becoming more and more negative, approaching a value of minus infinity as the objective function line becomes vertical; in this case, the slope of the objective function has no lower limit. Using the upper limit of $-\frac{3}{2}$, we can write

$$\underbrace{-\frac{C_S}{C_D}}_{\text{Slope of the objective function line}} \leq -\frac{3}{2}$$

FIGURE 3.2 GRAPHICAL SOLUTION OF PAR, INC., PROBLEM WITH AN OBJECTIVE FUNCTION OF $18S + 9D$; OPTIMAL SOLUTION AT EXTREME POINT (2)

Following the previous procedure of holding C_D constant at its original value, $C_D = 9$, we have

$$-\frac{C_S}{9} \le -\tfrac{3}{2} \quad \text{or} \quad \frac{C_S}{9} \ge \tfrac{3}{2}$$

Solving for C_S provides the following result:

$$C_S \ge \tfrac{27}{2} = 13.5$$

In reviewing Figure 3.2 we note that extreme point (2) remains optimal for all values of C_S above 13.5. Thus, we obtain the following range of optimality for C_S at extreme point (2):

$$13.5 \le C_S < \infty$$

Simultaneous Changes The range of optimality for objective function coefficients is only applicable for changes made to one coefficient at a time. All other coefficients are assumed to be fixed at their initial values. If two or more objective function coefficients are changed simultaneously, further analysis is necessary to determine whether the optimal solution will change. However, when solving two-variable problems graphically, expression (3.2) suggests an easy way to determine whether simultaneous changes in both objective function coefficients will cause a change in the optimal solution. Simply compute the slope of the objective function ($-C_S/C_D$) for the new coefficient values. If this ratio is greater than or equal to the lower limit on the slope of the objective function and less than or equal to the upper limit, then the changes made will not cause a change in the optimal solution.

Consider changes in both of the objective function coefficients for the Par, Inc., problem. Suppose the profit contribution per standard bag is increased to $13 and the profit contribution [illegible]neously reduced to $8. Recall that the ranges of optimality for C_S a[illegible] a one-at-a-time manner) are

$$6.3 \le C_S \le 13.5 \tag{3.3}$$

$$6.67 \le C_D \le 14.29 \tag{3.4}$$

For these ranges of optimality, we can conclude that changing either C_S to $13 or C_D to $8 (but not both) would not cause a change in the optimal solution of $S = 540$ and $D = 252$. But we cannot conclude from the ranges of optimality that changing both coefficients simultaneously would not result in a change in the optimal solution.

In expression (3.2) we showed that extreme point ③ remains optimal as long as

$$-3/2 \le -\frac{C_S}{C_D} \le -7/10$$

If C_S is changed to 13 and simultaneously C_D is changed to 8, the new objective function slope will be given by

$$-\frac{C_S}{C_D} = -\frac{13}{8} = -1.625$$

Because this value is less than the lower limit of $-3/2$, the current solution of $S = 540$ and $D = 252$ will no longer be optimal. By re-solving the problem with $C_S = 13$ and $C_D = 8$, we will find that extreme point ② is the new optimal solution.

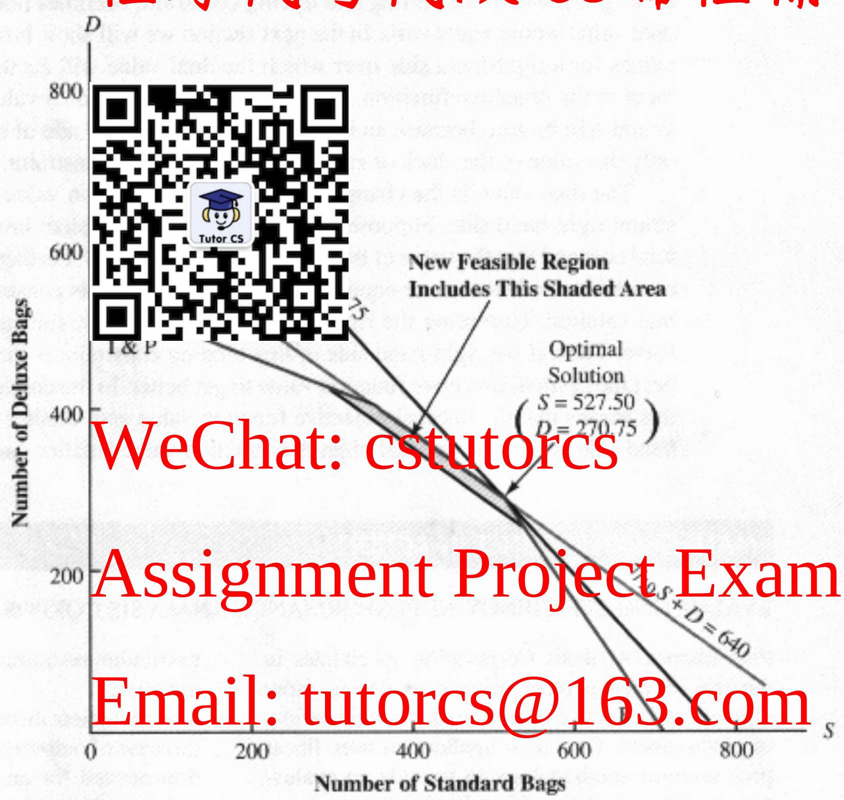
Looking at the ranges of optimality, we concluded that changing either C_S to $13 or C_D to $8 (but not both) would not cause a change in the optimal solution. But in recomputing the slope of the objective function with simultaneous changes for both C_S and C_D, we saw that the optimal solution did change. This result emphasizes the fact that a range of optimality, by itself, can only be used to draw a conclusion about changes made to *one objective function coefficient at a time.*

Right-Hand Sides

Let us now consider how a change in the right-hand side for a constraint may affect the feasible region and perhaps cause a change in the optimal solution to the problem. To illustrate this aspect of sensitivity analysis, let us consider what happens if an additional 10 hours of production time become available in the cutting and dyeing department of Par, Inc. The right-hand side of the cutting and dyeing constraint is changed from 630 to 640, and the constraint is rewritten as

$$\tfrac{7}{10}S + 1D \le 640$$

FIGURE 3.3 EFFECT OF A 10-UNIT CHANGE IN THE RIGHT-HAND SIDE OF THE CUTTING AND DYEING CONSTRAINT



By obtaining an additional 10 hours of cutting and dyeing time, we expand the feasible region for the problem, as shown in Figure 3.3. With an enlarged feasible region, we now want to determine whether one of the new feasible solutions provides an improvement in the value of the objective function. Application of the graphical solution procedure to the problem with the enlarged feasible region shows that the extreme point with $S = 527.5$ and $D = 270.75$ now provides the optimal solution. The new value for the objective function is $10(527.5) + 9(270.75) = \$7711.75$, with an increase in profit of $\$7711.75 - \$7668.00 = \$43.75$. Thus, the increased profit occurs at a rate of $43.75/10 hours = $4.375 per hour added.

The *change* in the value of the optimal solution per unit increase in the right-hand side of the constraint is called the **dual value.** Here, the dual value for the cutting and dyeing constraint is $4.375; in other words, if we increase the right-hand side of the cutting and dyeing constraint by 1 hour, the value of the objective function will increase by $4.375. Conversely, if the right-hand side of the cutting and dyeing constraint were to decrease by 1 hour, the objective function would go down by $4.375. The dual value can generally be used to determine what will happen to the value of the objective function when we make a one-unit change in the right-hand side of a constraint.

We caution here that the value of the dual value may be applicable only for small changes in the right-hand side. As more and more resources are obtained and the right-hand-side value continues to increase, other constraints will become binding and limit the change in the value of the objective function. For example, in the problem for Par, Inc., we would eventually reach a point where more cutting and dyeing time would be of no value; this would occur at the point where the cutting and dyeing constraint becomes nonbinding. At this point, the dual value would equal zero. In the next section we will show how to determine the range of values for a right-hand side over which the dual value will accurately predict the improvement in the objective function. Finally, we note that the dual value for any nonbinding constraint will be zero because an increase in the right-hand side of such a constraint will affect only the value of the slack or surplus variable for that constraint.

The dual value is the change in the objective function value per unit increase in a constraint's right-hand side. Suppose that we now solve a problem involving the minimization of total cost and that the value of the optimal solution is $100. Furthermore, suppose that the first constraint is a less-than-or-equal-to constraint and that this constraint is binding for the optimal solution. Increasing the right-hand side of this constraint makes the problem easier to solve. Thus, if the right-hand side of this binding constraint is increased by one unit, we expect the optimal objective function value to get better. In the case of a minimization problem, this means that the optimal objective function value gets smaller. If an increase in the right-hand side makes the optimal objective function value smaller, the dual value is negative.

MANAGEMENT SCIENCE IN ACTION

EVALUATING EFFICIENCY AT PERFORMANCE ANALYSIS CORPORATION*

Performance Analysis Corporation specializes in the use of management science to design more efficient and effective operations for a wide variety of chain stores. One such application uses linear programming methodology to provide an evaluation model for a chain of fast-food outlets.

According to the concept of Pareto optimality, a restaurant in a given chain is relatively inefficient if other restaurants in the same chain exhibit the following characteristics:

1. Operates in the same or worse environment
2. Produces at least the same level of *all* outputs
3. Utilizes no more of *any* resource and *less* of at least one of the resources

To determine which of the restaurants are Pareto inefficient, Performance Analysis Corporation developed and solved a linear programming model. Model constraints involve requirements concerning the minimum acceptable levels of output and conditions imposed by uncontrollable elements in the environment, and the objective function calls for the minimization of the resources necessary to produce the output. Solving the model produces the following output for each restaurant:

1. A score that assesses the level of so-called relative technical efficiency achieved by the particular restaurant over the time period in question
2. The reduction in controllable resources or the increase of outputs over the time period in question needed for an inefficient restaurant to be rated as efficient
3. A peer group of other restaurants with which each restaurant can be compared in the future

Sensitivity analysis provides important managerial information. For example, for each constraint concerning a minimum acceptable output level, the dual value tells the manager how much one more unit of output would change the efficiency measure.

The analysis typically identifies 40% to 50% of the restaurants as underperforming, given the previously stated conditions concerning the inputs available and outputs produced. Performance Analysis Corporation finds that if all the relative inefficiencies identified are eliminated simultaneously, corporate profits typically increase approximately 5% to 10%. This increase is truly substantial given the large scale of operations involved.

*Based on information provided by Richard C. Morey of Performance Analysis Corporation.

The Management Science in Action, Evaluating Efficiency at Performance Analysis Corporation, illustrates the use of dual values as part of an evaluation model for a chain of fast-food outlets. This type of model will be studied in more detail in Chapter 5 when we discuss an application referred to as data envelopment analysis.

NOTES AND COMMENTS

1. If two objecti[illegible] change simultaneously[illegible] their respective rang[illegible] affect the optimal sc[illegible] a two-variable linear [illegible] objective function will not change at all if both coefficients are changed by the same percentage.
2. Some textbooks and optimization solvers, for example Excel Solver, use the term *shadow price* rather than dual value.

3.3 SENSITIVITY ANALYSIS: COMPUTER SOLUTION

In Section 2.4 we showed how to interpret the output of a linear programming solver. In this section we continue that discussion and show how to interpret the sensitivity analysis output. We use the Par, Inc., problem restated below.

$$
\begin{aligned}
\text{Max} \quad & 10S + 9D \\
\text{s.t.} \quad & \\
& \tfrac{7}{10}S + 1D \le 630 \quad \text{Cutting and dyeing} \\
& \tfrac{1}{2}S + \tfrac{5}{6}D \le 600 \quad \text{Sewing} \\
& 1S + \tfrac{2}{3}D \le 708 \quad \text{Finishing} \\
& \tfrac{1}{10}S + \tfrac{1}{4}D \le 135 \quad \text{Inspection and packaging} \\
& S, D \ge 0
\end{aligned}
$$

Let us demonstrate interpreting the sensitivity analysis by considering the solution to the Par, Inc., linear program shown in Figure 3.4.

Interpretation of Computer Output

In Section 2.4 we discussed the output in the top portion of Figure 3.4. We see that the optimal solution is $S = 540$ standard bags and $D = 252$ deluxe bags; the value of the optimal solution is \$7668. Associated with each decision variable is reduced cost. We will interpret the reduced cost after our discussion on dual values.

Immediately following the optimal S and D values and the reduced cost information, the computer output provides information about the constraints. Recall that the Par, Inc., problem had four less-than-or-equal-to constraints corresponding to the hours available in each of four production departments. The information shown in the Slack/Surplus column provides the value of the slack variable for each of the departments. This information is summarized here:

Constraint Number	Constraint Name	Slack
1	Cutting and dyeing	0
2	Sewing	120
3	Finishing	0
4	Inspection and packaging	18

FIGURE 3.4 THE SOLUTION FOR THE PAR, INC., PROBLEM


```
Optimal Objective Value = [illegible]668.00000

     Variable             Value            Reduced Cost
  --------------     ---------------     -----------------
  [illegible]             540.00000              0.00000
  [illegible]             252.00000              0.00000

     Co[illegible]     Slack/Surplus         Dual Value
  --------------     ---------------     -----------------
  [illegible]               0.00000              4.37500
  [illegible]             120.00000              0.00000
  [illegible]               0.00000              6.93750
  [illegible]              18.00000              0.00000

                 Objective          Allowable        Allowable
    Variable     Coefficient        Increase         Decrease
  ----------   -------------      ----------       ----------
      S           10.00000          3.50000          3.70000
      D            9.00000          5.28571          2.33333

                    RHS             Allowable        Allowable
  Constraint       Value            Increase         Decrease
  ----------   -------------      ----------       ----------
      1          630.00000         52.36364        134.40000
      2          600.00000         Infinite        120.00000
      3          708.00000        192.00000        128.00000
      4          135.00000         Infinite         18.00000
```


WEB file

Par

From this information, we see that the binding constraints (the cutting and dyeing and the finishing constraints) have zero slack at the optimal solution. The sewing department has 120 hours of slack, or unused capacity, and the inspection and packaging department has 18 hours of slack, or unused capacity.

The Dual Value column contains information about the marginal value of each of the four resources at the optimal solution. In Section 3.2 we defined the *dual value* as follows:

> The dual value associated with a constraint is the *change* in the optimal value of the solution per unit increase in the right-hand side of the constraint.

Try Problem 5 to test your ability to use computer output to determine the optimal solution and to interpret the dual values.

Thus, the nonzero dual values of 4.37500 for constraint 1 (cutting and dyeing constraint) and 6.93750 for constraint 3 (finishing constraint) tell us that an additional hour of cutting and dyeing time increases the value of the optimal solution by $4.37, and an additional hour of finishing time increases the value of the optimal solution by $6.94. Thus, if the cutting and dyeing time were increased from 630 to 631 hours, with all other coefficients in the problem remaining the same, Par's profit would be increased by $4.37, from $7668 to $7668 + $4.37 = $7672.37. A similar interpretation for the finishing constraint implies that an increase from 708 to 709 hours of available finishing time, with all other coefficients in the problem remaining the same, would increase Par's profit to \$7668 + \$6.94 = \$7674.94. Because the sewing and the inspection and packaging constraints both have slack, or unused capacity, available, the dual values of zero show that additional hours of these two resources will not improve the value of the objective function.

Now that the concept of a dual value has been explained, we define the reduced cost associated with each variable. The **reduced cost** associated with a variable is equal to the dual valu[illegible]onstraint associated with the variable. From Figure 3.4, we see th[illegible]ariable S is zero and on variable D is zero. This makes sense. Co[illegible]nnegativity constraint is $S \geq 0$. The current value of S is 540, so c[illegible]y constraint to $S \geq 1$ has no effect on the optimal solution valu[illegible] right-hand side by one unit has no effect on the optimal objective [illegible]value (i.e., reduced cost) of this nonnegativity constraint is zero. A[illegible]es to variable D. In general, if a variable has a nonzero value in t[illegible]n it will have a reduced cost equal to zero. Later in this section we give an example where the reduced cost of a variable is nonzero, and this example provides more insight on why the term *reduced cost* is used for the nonnegativity constraint dual value.

Referring again to the computer output in Figure 3.4, we see that after providing the constraint information on slack/surplus variables and dual values, the solution output provides ranges for the objective function coefficients and the right-hand sides of the constraints.

Considering the objective function coefficient range analysis, we see that variable S, which has a current profit coefficient of 10, has an *allowable increase* of 3.5 and an *allowable decrease* of 3.7. Therefore, as long as the profit contribution associated with the standard bag is between \$10 – \$3.7 = \$6.30 and \$10 + \$3.5 = \$13.50, the production of S = 540 standard bags and D = 252 deluxe bags will remain the optimal solution. Therefore, the range of optimality for the objective function coefficient on variable S is from 6.3 to 13.5. Note that the range of optimality is the same as obtained by performing graphical sensitivity analysis for C_S in Section 3.2.

Using the objective function coefficient range information for deluxe bags, we see the following range of optimality (after rounding to two decimal places):

$$9 - 2.33 = 6.67 \leq C_D \leq 9 + 5.29 = 14.29$$

This result tells us that as long as the profit contribution associated with the deluxe bag is between \$6.67 and \$14.29, the production of S = 540 standard bags and D = 252 deluxe bags will remain the optimal solution.

Try Problem 6 to test your ability to use computer output to determine the ranges of optimality and the ranges of feasibility.

The final section of the computer output provides the allowable increase and allowable decrease in the right-hand sides of the constraints relative to the dual values holding. As long as the constraint right-hand side is not increased (decreased) by more than the allowable increase (decrease), the associated dual value gives the exact change in the value of the optimal solution per unit increase in the right-hand side. For example, let us consider the cutting and dyeing constraint with a current right-hand-side value of 630. Because the dual value for this constraint is \$4.37, we can conclude that additional hours will increase the objective function by \$4.37 per hour. It is also true that a reduction in the hours available will reduce the value of the objective function by \$4.37 per hour. From the range information given, we see that the dual value of \$4.37 has an allowable increase of 52.36364 and is therefore valid for right-hand side values up to 630 + 52.36364 = 682.363364. The allowable decrease is 134.4, so the dual value of \$4.37 is valid for right-hand side values down to 630 – 134.4 = 495.6. A similar interpretation for the finishing constraint's right-hand side (constraint 3) shows that the dual value of \$6.94 is applicable for increases up to 900 hours and decreases down to 580 hours.

As mentioned, the right-hand-side ranges provide limits within which the dual values give the exact change in the optimal objective function value. For changes outside the range, the problem must be re-solved to find the new optimal solution and the new dual value. We shall call the range over which the dual value is applicable the **range of feasibility.** The rang[illegible]ar, Inc., problem are summarized here:

	Min RHS	Max RHS
[illegible]	495.6	682.4
[illegible]	480.0	No upper limit
[illegible]	580.0	900.0
[illegible]ging	117.0	No upper limit

As long as the values of the right-hand sides are within these ranges, the dual values shown on the computer output will not change. Right-hand-side values outside these limits will result in changes in the dual value information.

Cautionary Note on the Interpretation of Dual Values

As stated previously, the dual value is the change in the value of the optimal solution per unit increase in the right-hand side of a constraint. When the right-hand side of the constraint represents the amount of a resource available, the associated dual value is often interpreted as the maximum amount one should be willing to pay for one additional unit of the resource. However, such an interpretation is not always correct. To see why, we need to understand the difference between sunk and relevant costs. A **sunk cost** is one that is not affected by the decision made. It will be incurred no matter what values the decision variables assume. A **relevant cost** is one that depends on the decision made. The amount of a relevant cost will vary depending on the values of the decision variables.

Let us reconsider the Par, Inc., problem. The amount of cutting and dyeing time available is 630 hours. The cost of the time available is a sunk cost if it must be paid regardless of the number of standard and deluxe golf bags produced. It would be a relevant cost if Par only had to pay for the number of hours of cutting and dyeing time actually used to produce golf bags. All relevant costs should be reflected in the objective function of a linear program. Sunk costs should not be reflected in the objective function. For Par, Inc., we have been assuming that the company must pay its employees' wages regardless of whether their time on the job is completely utilized. Therefore, the cost of the labor-hours resource for Par, Inc., is a sunk cost and has not been reflected in the objective function.

Only relevant costs should be included in the objective function.

When the cost of a resource is *sunk,* the dual value can be interpreted as the maximum amount the company should be willing to pay for one additional unit of the resource. When the cost of a resource used is relevant, the dual value can be interpreted as the amount by which the value of the resource exceeds its cost. Thus, when the resource cost is relevant, the dual value can be interpreted as the maximum premium over the normal cost that the company should be willing to pay for one unit of the resource.

The Modified Par, Inc., Problem

The graphical solution procedure is useful only for linear programs involving two decision variables. In practice, the problems solved using linear programming usually involve a large number of variables and constraints. For instance, the Management Science in Action, Determining Optimal Production Quantities at GE Plastics, describes how a linear programming model with 3100 variables and 1100 constraints was solved in less than 10 seconds to determine the optimal production quantities at GE Plastics. In this section we discuss the formulation and computer solution for two linear programs with three decision variables. In doing so, we will show how to interpret the reduced-cost portion of the computer output.

The original Par, Inc. problem is restated as follows:

$$
\begin{aligned}
\text{Max} \quad & 10S + 9D \\
\text{s.t.} \quad & \\
& \tfrac{7}{10}S + 1D \le 630 && \text{Cutting and dyeing} \\
& \tfrac{1}{2}S + \tfrac{5}{6}D \le 600 && \text{Sewing} \\
& 1S + \tfrac{2}{3}D \le 708 && \text{Finishing} \\
& \tfrac{1}{10}S + \tfrac{1}{4}D \le 135 && \text{Inspection and packaging} \\
& S, D \ge 0
\end{aligned}
$$

Recall that S is the number of standard golf bags produced and D is the number of deluxe golf bags produced. Suppose that management is also considering producing a lightweight model designed specifically for golfers who prefer to carry their bags. The design department estimates that each new lightweight model will require 0.8 hours for cutting and dyeing, 1 hour for sewing, 1 hour for finishing, and 0.25 hours for inspection and packaging. Because of the unique capabilities designed into the new model, Par's management feels they will realize a profit contribution of \$12.85 for each lightweight model produced during the current production period.

Let us consider the modifications in the original linear programming model that are needed to incorporate the effect of this additional decision variable. We will let L denote the number of lightweight bags produced. After adding L to the objective function and to each of the four constraints, we obtain the following linear program for the modified problem:

$$
\begin{aligned}
\text{Max} \quad & 10S + 9D + 12.85L \\
\text{s.t.} \quad & \\
& \tfrac{7}{10}S + 1D + 0.8L \le 630 && \text{Cutting and dyeing} \\
& \tfrac{1}{2}S + \tfrac{5}{6}D + 1L \le 600 && \text{Sewing} \\
& 1S + \tfrac{2}{3}D + 1L \le 708 && \text{Finishing} \\
& \tfrac{1}{10}S + \tfrac{1}{4}D + \tfrac{1}{4}L \le 135 && \text{Inspection and packaging} \\
& S, D, L \ge 0
\end{aligned}
$$

Figure 3.5 shows the solution to the modified problem. We see that the optimal solution calls for the production of 280 standard bags, 0 deluxe bags, and 428 of the new lightweight bags; the value of the optimal solution is \$8299.80.

Let us now look at the information contained in the Reduced Cost column. Recall that the reduced costs are the dual values of the corresponding nonnegativity constraints. As the computer output shows, the reduced costs for S and L are zero because these decision variables already have positive values in the optimal solution. However, the reduced cost for decision variable D is –1.15. The interpretation of this number is that if the production of deluxe bags is increased from the current level of 0 to 1, then the optimal objective function value will decrease by 1.15. Another interpretation is that if we "reduce the cost" of deluxe bags by 1.15 (i.e., increase the contribution margin by 1.15), then there is an optimal solution where we produce a nonzero number of deluxe bags.

FIGURE 3.5 SOLUTION FOR THE MODIFIED PAR, INC., PROBLEM

Optimal Objective Value = [illegible]9.80000

Variable	Value	Reduced Cost
[illegible]	280.00000	0.00000
[illegible]	0.00000	-1.15000
[illegible]	428.00000	0.00000

Co[illegible]	Slack/Surplus	Dual Value
[illegible]	91.60000	0.00000
[illegible]	32.00000	0.00000
[illegible]	0.00000	8.10000
4	0.00000	19.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
S	10.00000	2.07000	4.86000
D	9.00000	1.15000	Infinite
L	12.85000	12.15000	0.94091

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	630.00000	Infinite	91.60000
2	600.00000	Infinite	32.00000
3	708.00000	144.63158	168.00000
4	135.00000	9.60000	64.20000

Suppose we increase the coefficient of *D* by exactly \$1.15 so that the new value is \$9 + \$1.15 = \$10.15 and then re-solve. Figure 3.6 shows the new solution. Note that although *D* assumes a positive value in the new solution, the value of the optimal solution has not changed. In other words, increasing the profit contribution of *D* by *exactly* the amount of the reduced cost has resulted in alternative optimal solutions. Depending on the computer software package used to optimize this model, you may or may not see *D* assume a positive value if you re-solve the problem with an objective function coefficient of exactly 10.15 for *D*—that is, the software package may show a different alternative optimal solution. However, if the profit contribution of *D* is increased by *more than* \$1.15, then *D* will not remain at zero in the optimal solution.

We also note from Figure 3.6 that the dual values for constraints 3 and 4 are 8.1 and 19, respectively, indicating that these two constraints are binding in the optimal solution. Thus, each additional hour in the finishing department would increase the value of the optimal solution by \$8.10, and each additional hour in the inspection and packaging department would increase the value of the optimal solution by \$19.00. Because of a slack of 91.6 hours in the cutting and dyeing department and 32 hours in the sewing department (see Figure 3.6), management might want to consider the possibility of utilizing these unused labor-hours in the finishing or inspection and packaging departments. For example, some of the employees

FIGURE 3.6 SOLUTION FOR THE MODIFIED PAR, INC., PROBLEM WITH THE COEFFICIENT OF *D* INCREASED BY $1.1500

```
Optimal Objective Value =           8299.80000

      Variable              Value              Reduced Cost
    ------------      ---------------      -----------------
           S             403.78378               0.00000
           D             222.81081               0.00000
           L             155.67568               0.00000

     Constraint        Slack/Surplus            Dual Value
    ------------      ---------------      -----------------
 [illegible]               0.00000               0.00000
           2              56.75676               0.00000
           3               0.00000               8.10000
           4               0.00000              19.00000

                   Objective        Allowable        Allowable
     Variable      Coefficient       Increase         Decrease
    ----------    -------------    -----------    -----------
         S          10.00000         2.51071        0.00000
         D          10.15000         5.25790        0.00000
         L          12.85000         0.00000        2.19688

                       RHS           Allowable        Allowable
    Constraint        Value           Increase         Decrease
    ----------    -------------    -----------    -----------
         1         630.00000        52.36364       91.60000
         2         600.00000        Infinite       56.75676
         3         708.00000       447.63158      128.00000
         4         135.00000        16.15385       18.00000
```

in the cutting and dyeing department could be used to perform certain operations in either the finishing department or the inspection and packaging department. In the future, Par's management may want to explore the possibility of cross-training employees so that unused capacity in one department could be shifted to other departments. In the next chapter we will consider similar modeling situations.

NOTES AND COMMENTS

1. Computer software packages for solving linear programs are readily available. Most of these provide the optimal solution, dual or shadow price information, the range of optimality for the objective function coefficients, and the range of feasibility for the right-hand sides. The labels used for the ranges of optimality and feasibility may vary, but the meaning is the same as what we have described here.

2. Whenever one of the right-hand sides is at an end point of its range of feasibility, the dual and shadow prices only provide one-sided information. In this case, they only predict the change in the optimal value of the objective function for changes toward the interior of the range.

(*continued*)

3. A condition called *degeneracy* can cause a subtle difference in how we interpret changes in the objective function coefficients beyond the end points of the range of optimality. Degeneracy occurs when the dual value equals zero for one of the binding constraints. Degeneracy does not affect the in[illegible]toward the interior of [illegible] However, when dege[illegible]es beyond the end poi[illegible]ecessarily mean a diff[illegible]mal. From a practical point of view, changes beyond the end points of the range of optimality necessitate re-solving the problem.
4. Managers are frequently called on to provide an economic justification for new technology. Often the new technology is developed, or purchased, in order to conserve resources. The dual value can be helpful in such cases because it can be used to determine the savings attributable to the new technology by showing the savings per unit of resource conserved.

MANAGEMENT SCIENCE IN ACTION

DETERMINING OPTIMAL PRODUCTION QUANTITIES AT GE PLASTICS*

General Electric Plastics (GEP) is a $5 billion global materials supplier of plastics and raw materials to many industries (e.g., automotive, computer, and medical equipment). GEP has plants all over the globe. In the past, GEP followed a pole-centric manufacturing approach wherein each product was manufactured in the geographic area (Americas, Europe, or Pacific) where it was to be delivered. When many of GEP's customers started shifting their manufacturing operations to the Pacific, a geographic imbalance was created between GEP's capacity and demand in the form of overcapacity in the Americas and undercapacity in the Pacific.

Recognizing that a pole-centric approach was no longer effective, GEP adopted a global approach to its manufacturing operations. Initial work focused on the high-performance polymers (HPP) division. Using a linear programming model, GEP was able to determine the optimal production quantities at each HPP plant to maximize the total contribution margin for the division. The model included demand constraints, manufacturing capacity constraints, and constraints that modeled the flow of materials produced at resin plants to the finishing plants and on to warehouses in three geographical regions (Americas, Europe, and Pacific). The mathematical model for a one-year problem has 3100 variables and 1100 constraints, and can be solved in less than 10 seconds. The new system proved successful at the HPP division, and other GE Plastics divisions are adapting it for their supply chain planning.

*Based on R. Tyagi, P. Kalish, and K. Akbay, "GE Plastics Optimizes the Two-Echelon Global Fulfillment Network at Its High-Performance Polymers Division," *Interfaces* (September/October 2004): 359–366.

3.4 LIMITATIONS OF CLASSICAL SENSITIVITY ANALYSIS

As we have seen, classical sensitivity analysis obtained from computer output can provide useful information on the sensitivity of the solution to changes in the model input data. However, classical sensitivity analysis provided by most computer packages does have its limitations. In this section we discuss three such limitations: simultaneous changes in input data, changes in constraint coefficients, and nonintuitive dual values. We give examples of these three cases and discuss how to effectively deal with these through re-solving the model with changes. In fact, in our experience, it is rarely the case that one solves a model once and makes a recommendaiton. More often than not, a series of models are solved using a variety of input data sets before a final plan is adopted. With improved algorithms and more powerful computers, solving multiple runs of a model is extremely cost and time effective.

Simultaneous Changes

The sensitivity analysis information in computer output is based on the assumption that only one coefficient changes; it is assumed that all other coefficients will remain as stated in the original problem. Thus, the range analysis for the objective function coefficients and the constraint right-hand sides is only applicable for changes in a single coefficient. In many ca[illegible] [illegible]erested in what would happen if two or more coefficients are chan[illegible]

Let [illegible] modified Par, Inc., problem, whose solution appears in Figure 3[illegible] have solved the problem, we find a new supplier and can purchase[illegible] these bags at a lower cost. Leather is an important component [illegible]s of bags, but is used in different amounts in each type. After fa[illegible] [illegible]f leather, the profit margin per bag is found to be \$10.30 for a sta[illegible] [illegible]eluxe bag and \$12.97 for a lightweight bag. Does the current plan from Figure 3.5 remain optimal? We can easily answer this question by simply re-solving the model using the new profit margins as the objective function coefficients. That is, we use as our objective function: Maximize $10.3S + 11.4D + 12.97L$ with the same set of constraints as in the original model. The solution to this problem appears in Figure 3.7. The new optimal profit is \$8718.13. All three types of bags should be produced.

FIGURE 3.7 THE SOLUTION FOR THE MODIFIED PAR, INC., PROBLEM WITH REVISED OBJECTIVE FUNCTION COEFFICIENTS

Optimal Objective Value = 8718.12973

Variable	Value	Reduced Cost
S	403.78378	0.00000
D	222.81081	0.00000
L	155.67568	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	3.08919
2	56.75676	0.00000
3	0.00000	6.56351
4	0.00000	15.74054

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
S	10.30000	2.08000	2.28600
D	11.40000	4.26053	1.27000
L	12.97000	1.03909	1.82000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	630.00000	52.36364	91.60000
2	600.00000	Infinite	56.75676
3	708.00000	144.63158	128.00000
4	135.00000	16.15385	18.00000

Suppose we had not re-solved the model with the new objective function coefficients. We would have used the solution from the original model, the solution found in Figure 3.5. Our profit would have therefore been $[illegible](280) + $[illegible](0) + $[illegible](428) = $8435.16. By re-solving the model with the new information and using the revised plan in Figure 3.7, we have increased total profit by $8718.13 – $8435.16 = $282.97.

Ch[illegible]nt Coefficients

Clas[illegible] provides no information about changes resulting from a change in th[illegible] in a constraint. To illustrate such a case and how we may deal with [illegible] the Modified Par, Inc., problem discussed in Section 3.3.

[illegible]ng the adoption of a new technology that will allow us to more effic[illegible]gs. This technology is dedicated to standard bags and would decr[illegible] a standard bag from its current value of 1 to ½ of an hour. The techn[illegible] the finishing time of deluxe or lightweight bags. The finishing constraint under the new sceanario is:

$$\tfrac{1}{2}S + \tfrac{2}{3}D + H \le 708 \quad \text{Finishing with new technology}$$

Even though this is a single change in a coefficient in the model, there is no way to tell from classical sensitivity analysis what impact the change in the coefficient of S will have on the solution. Instead, we must simply change the coefficient and rerun the model. The solution appears in Figure 3.8. Note that the optimal number of standard bags has increased from 280 to 521.1 and the optimal number of lightweight bags decreased from 428 to 331.6. It remains optimal to produce no deluxe bags. Most importantly, with the new technology, the optimal profit increased from $8299.80 to $9471.32, an increase of $1171.52. Using this information with the cost of the new technology will provide an estimate for mangement as to how long it will take to pay off the new technology based on the increase in profits.

Nonintuitive Dual Values

Constraints with variables naturally on both the left-hand and right-hand sides often lead to dual values that have a nonintuitive explanation. To illustrate such a case and how we may deal with it, let us reconsider the Modified Par, Inc., problem discussed in Section 3.3.

Suppose that after reviewing the solution shown in Figure 3.5, management states that they will not consider any solution that does not include the production of some deluxe bags. Management then decides to add the requirement that the number of deluxe bags produced must be at least 30% of the number of standard bags produced. Writing this requirement using the decision variables S and D, we obtain

$$D \ge 0.3S$$

This new constraint is constraint 5 in the modified Par, Inc., linear program. Re-solving the problem with the new constraint 5 yields the optimal solution shown in Figure 3.9.

Let us consider the interpretation of the dual value for constraint 5, the requirement that the number of deluxe bags produced must be at least 30% of the number of standard bags produced. The dual value of –1.38 indicates that a one-unit increase in the right-hand side of the constraint will lower profits by $1.38. Thus, what the dual value of –1.38 is really

FIGURE 3.8 THE SOLUTION FOR THE MODIFIED PAR, INC., PROBLEM WITH NEW STANDARD BAG FINISHING TECHNOLOGY

Optimal Objective Value = 9471.31579

Variable	Value	Reduced Cost
S	521.05263	0.00000
D	0.00000	-6.40789
L	331.57895	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	12.78947
2	7.89474	0.00000
3	115.89474	0.00000
4	0.00000	10.47368

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
S	10.00000	[illegible]	4.86000
D	9.00000	6.40789	Infinite
L	12.85000	12.15000	1.42143

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	630.00000	30.00000	198.00000
2	600.00000	Infinite	7.89474
3	708.00000	Infinite	115.89474
4	135.00000	2.50000	45.00000

telling us is what will happen to the value of the optimal solution if the constraint is changed to

$$D \geq 0.3S + 1$$

The interpretation of the dual value of –1.38 is correctly stated as follows: If we are forced to produce one deluxe bag over and above the minimum 30% requirement, total profits will decrease by \$1.38. Conversely, if we relax the minimum 30% requirement by one bag ($D \geq 0.3S - 1$), total profits will increase by \$1.38.

We might instead be more interested in what happens when the percentage of 30% is increased to 31%. Note that dual value does *not* tell us what will happen in this case. Also, because 0.30 is the coefficient of a variable in a constraint rather than an objective function coefficient or right-hand side, no range analysis is given. Note that this is the case just discussed in the previous section. Because there is no way to get this information directly from classical sensitivity analysis, to answer such a question, we need to re-solve the problem

FIGURE 3.9 THE SOLUTION FOR THE MODIFIED PAR, INC., PROBLEM WITH THE DELUXE BAG REQUIREMENT

Optimal Objective Value = 8183.88000

Variable	Value	Reduced Cost
[illegible]	336.00000	0.00000
[illegible]	100.80000	0.00000
[illegible]	304.80000	0.00000

Constraint	Slack/Surplus	Dual Value
[illegible]	50.16000	0.00000
2	43.20000	0.00000
3	0.00000	7.41000
4	0.00000	21.76000
5	0.00000	-1.38000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
S	10.00000	2.07000	3.70500
D	9.00000	1.15000	12.35000
L	12.85000	5.29286	0.94091

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	630.00000	Infinite	50.16000
2	600.00000	Infinite	43.20000
3	708.00000	57.00000	168.00000
4	135.00000	12.00000	31.75000
5	0.00000	101.67568	84.00000

using the constraint $D \geq 0.31S$. To test the sensitivity of the solution to changes in the percentage required we can re-solve the model replacing 0.30 with any percentage of interest.

To get a feel for how the required percentage impacts total profit, we solved versions of the Par, Inc., model with the required percentage varying from 5% to 100% in increments of 5%. This resulted in 20 different versions of the model to be solved. The impact of changing this percentage on the total profit is shown in Figure 3.10, and results are shown in Table 3.1.

What have we learned from this analysis? Notice from Figure 3.10 that the slope of the graph becomes steeper for values larger than 55%. This indicates that there is a shift in the rate of deterioration in profit starting at 55%. Hence, we see that percentages less than or equal to 55% result in modest loss of profit. More pronounced loss of profit results from percentages larger than 55%. So, management now knows that 30% is a reasonable requirement from a profit point of view and that extending the requirement beyond 55% will lead to a more significant loss of profit. From Table 3.1, as we increase the percentage required, fewer lightweight bags are produced.

FIGURE 3.10 PROFIT FOR VARIOUS VALUES OF REQUIRED PERCENTAGE FOR DELUXE BAGS AS A PERECENTAGE OF STANDARD BAGS

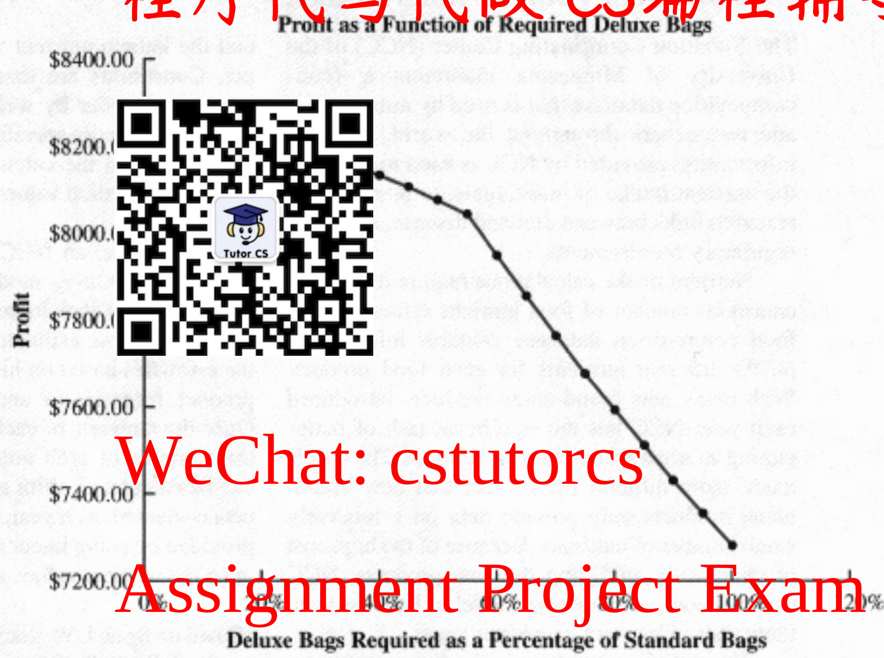


TABLE 3.1 SOLUTIONS VARIOUS VALUES OF REQUIRED PERCENTAGE FOR DELUXE BAGS AS A PERCENTAGE OF STANDARD BAGS

Percent	Profit	Standard	Deluxe	Lightweight
5%	$8283.24	287.9999	14.4000	410.4000
10%	$8265.71	296.4704	29.6470	391.7648
15%	$8247.11	305.4543	45.8181	372.0002
20%	$8227.35	314.9996	62.9999	351.0002
25%	$8206.31	325.1608	81.2902	328.6455
30%	$8183.88	335.9993	100.7998	304.8005
35%	$8159.89	347.5854	121.6549	279.3110
40%	$8134.20	359.9990	143.9996	252.0008
45%	$8106.60	373.3321	167.9994	222.6677
50%	$8076.87	387.6908	193.8454	191.0783
55%	$8044.77	403.1982	221.7590	156.9617
60%	$7948.80	396.0000	237.6000	144.0000
65%	$7854.27	388.2353	252.3529	132.3529
70%	$7763.37	380.7692	266.5385	121.1538
75%	$7675.90	373.5849	280.1887	110.3774
80%	$7591.67	366.6667	293.3333	100.0000
85%	$7510.50	360.0000	306.0000	90.0000
90%	$7432.23	353.5714	318.2143	80.3571
95%	$7356.71	347.3684	330.0000	71.0526
100%	$7283.79	341.3793	341.3793	62.0690

MANAGEMENT SCIENCE IN ACTION

ESTIMATION OF FOOD NUTRIENT VALUES*

The Nutrition Coordinating Center (NCC) of the University of Minnesota maintains a food-composition [illegible] utritionists and research[illegible]d. Nutrient information [illegible] to estimate the nutrient i[illegible]lan menus, research link[illegible]e, and meet regulatory re[illegible]

Nutrient i[illegible] data on an enormous nu[illegible]ues. NCC's food compos[illegible]nformation on 93 different nutrients for each food product. With many new brand-name products introduced each year, NCC has the significant task of maintaining an accurate and timely database. The task is made more difficult by the fact that new brand-name products only provide data on a relatively small number of nutrients. Because of the high cost of chemically analyzing the new products, NCC uses a linear programming model to help estimate thousands of nutrient values per year.

The decision variables in the linear programming model are the amounts of each ingredient in a food product. The objective is to minimize the differences between the estimated nutrient values and the known nutrient values for the food product. Constraints are that ingredients must be in descending order by weight, ingredients must be within nutritionist-specified bounds, and the differences between the calculated nutrient values and the known nutrient values must be within specified tolerances.

In practice, an NCC nutritionist employs the linear programming model to derive estimates of the amounts of each ingredient in a new food product. Given these estimates, the nutritionist refines the estimates based on his or her knowledge of the product formulation and the food composition. Once the amounts of each ingredient are obtained, the amounts of each nutrient in the food product can be calculated. With approximately 1000 products evaluated each year, the time and cost savings provided by using linear programming to help estimate the nutrient values are significant.

*Based on Brian J. Westrich, Michael A. Altmann, and Sandra J. Potthoff, "Minnesota's Nutrition Coordinating Center Uses Mathematical Optimization to Estimate Food Nutrient Values," *Interfaces* (September/October 1998): 86–99.

3.5 THE ELECTRONIC COMMUNICATIONS PROBLEM

The Electronic Communications problem introduced in this section is a maximization problem involving four decision variables, two less-than-or-equal-to constraints, one equality constraint, and one greater-than-or-equal-to constraint. Our objective is to provide a summary of the process of formulating a mathematical model, using software to obtain an optimal solution, and interpreting the solution and sensitivity report information. In the next chapter we will continue to illustrate how linear programming can be applied by showing additional examples from the areas of marketing, finance, and production management. Your ability to formulate, solve, and interpret the solution to problems like the Electronic Communications problem is critical to understanding how more complex problems can be modeled using linear programming.

Electronic Communications manufactures portable radio systems that can be used for two-way communications. The company's new product, which has a range of up to 25 miles, is particularly suitable for use in a variety of business and personal applications. The distribution channels for the new radio are as follows:

1. Marine equipment distributors
2. Business equipment distributors
3. National chain of retail stores
4. Direct mail

TABLE 3.2 PROFIT, ADVERTISING COST, AND PERSONAL SALES TIME DATA FOR THE ELECTRONIC COMMUNICATIONS PROBLEM

Distribution Channel	Profit per Unit Sold ($)	Advertising Cost per Unit Sold ($)	Personal Sales Effort per Unit Sold (hours)
Marine distri[illegible]	[illegible]	10	2
Business dist[illegible]	[illegible]	8	3
National retai[illegible]	[illegible]	9	3
Direct mail	[illegible]	15	None

Because of differing distribution and promotional costs, the profitability of the product will vary with the distribution channel. In addition, the advertising cost and the personal sales effort required will vary with the distribution channels. Table 3.2 summarizes the contribution to profit, advertising cost, and personal sales effort data pertaining to the Electronic Communications problem. The firm set the advertising budget at $5000, and a maximum of 1800 hours of salesforce time is available for allocation to the sales effort. Management also decided to produce exactly 600 units for the current production period. Finally, an ongoing contract with the national chain of retail stores requires that at least 150 units be distributed through this distribution channel.

Electronic Communications is now faced with the problem of establishing a strategy that will provide for the distribution of the radios in such a way that overall profitability of the new radio production will be maximized. Decisions must be made as to how many units should be allocated to each of the four distribution channels, as well as how to allocate the advertising budget and salesforce effort to each of the four distribution channels.

Problem Formulation

We will now write the objective function and the constraints for the Electronic Communications problem. For the objective function, we can write:

Objective function: Maximize profit

Four constraints are necessary for this problem. They are necessary because of (1) a limited advertising budget, (2) limited salesforce availability, (3) a production requirement, and (4) a retail stores distribution requirement.

Constraint 1: Advertising expenditures ≤ Budget
Constraint 2: Sales time used ≤ Time available
Constraint 3: Radios produced = Management requirement
Constraint 4: Retail distribution ≥ Contract requirement

These expressions provide descriptions of the objective function and the constraints. We are now ready to define the decision variables that will represent the decisions the manager must make.

For the Electronic Communications problem, we introduce the following four decision variables:

M = the number of units produced for the marine equipment distribution channel
B = the number of units produced for the business equipment distribution channel
R = the number of units produced for the national retail chain distribution channel
D [illegible] produced for the direct mail distribution channel

U[illegible], the objective function for maximizing the total contribution to pro[illegible]dios can be written as follows:

$$\text{Max } 90M + 84B + 70R + 60D$$

Let u[illegible]atical statement of the constraints for the problem. Because the advertising budget is set at $5000, the constraint that limits the amount of advertising expenditure can be written as follows:

$$10M + 8B + 9R + 15D \leq 5000$$

Similarly, because the sales time is limited to 1800 hours, we obtain the constraint

$$2M + 3B + 3R \leq 1800$$

Management's decision to produce exactly 600 units during the current production period is expressed as

$$M + B + R + D = 600$$

Finally, to account for the fact that the number of units distributed by the national chain of retail stores must be at least 150, we add the constraint

$$R \geq 150$$

Combining all of the constraints with the nonnegativity requirements enables us to write the complete linear programming model for the Electronic Communications problem as follows:

$$
\begin{array}{llr}
\text{Max} & 90M + 84B + 70R + 60D & \\
\text{s.t.} & & \\
& 10M + 8B + 9R + 15D \leq 5000 & \text{Advertising budget} \\
& 2M + 3B + 3R \leq 1800 & \text{Salesforce availability} \\
& M + B + R + D = 600 & \text{Production level} \\
& R \geq 150 & \text{Retail stores requirement} \\
& M, B, R, D \geq 0 &
\end{array}
$$

Computer Solution and Interpretation

This problem can be solved using either Excel Solver or LINGO. A portion of the standard solution output for the Electronic Communications problem is shown in Figure 3.11. The Objective Function Value section shows that the optimal solution to the problem will provide a maximum profit of $48,450. The optimal values of the decision variables are given by $M = 25$, $B = 425$, $R = 150$, and $D = 0$.

FIGURE 3.11 A PORTION OF THE COMPUTER OUTPUT FOR THE ELECTRONIC COMMUNICATIONS PROBLEM

WEB file
Electronic

```
Optimal Objective Value =           48450.00000

      Variable               Value              Reduced Cost
   --------------     ---------------      -----------------
         M                25.00000               0.00000
         B               425.00000               0.00000
         R               150.00000               0.00000
         D                 0.00000             -45.00000

     Constraint        Slack/Surplus            Dual Value
   --------------     ---------------      -----------------
         1                 0.00000               3.00000
         2                25.00000               0.00000
         3                 0.00000              60.00000
         4                 0.00000             -17.00000
```

Thus, the optimal strategy for Electronic Communications is to concentrate on the business equipment distribution channel with $B = 425$ units. In addition, the firm should allocate 25 units to the marine distribution channel ($M = 25$) and meet its 150-unit commitment to the national retail chain store distribution channel ($R = 150$). With $D = 0$, the optimal solution indicates that the firm should not use the direct mail distribution channel.

Now consider the information contained in the Reduced Cost column. Recall that the reduced cost of a variable is the dual value of the corresponding nonnegativity constraint. As the computer output shows, the first three reduced costs are zero because the corresponding decision variables already have positive values in the optimal solution. However, the reduced cost of −45 for decision variable D tells us that profit will decrease by \$45 for every unit produced for the direct mail channel. Stated another way, the profit for the new radios distributed via the direct mail channel would have to increase from its current value of \$60 per unit, by \$45 per unit, to at least \$60 + \$45 = \$105 per unit before it would be profitable to use the direct mail distribution channel.

The computer output information for the slack/surplus variables and the dual values is restated in Figure 3.12.

The advertising budget constraint has a slack of zero, indicating that the entire budget of \$5000 has been used. The corresponding dual value of 3 tells us that an additional dollar added to the advertising budget will increase the objective function (increase the profit) by \$3. Thus, the possibility of increasing the advertising budget should be seriously considered by the firm. The slack of 25 hours for the salesforce availability constraint shows that the allocated 1800 hours of sales time are adequate to distribute the radios produced and that 25 hours of sales time will remain unused. Because the production level constraint is an equality constraint, the zero slack/surplus shown on the output is expected. However, the dual value of 60 associated with this constraint shows that if the firm were to consider increasing the production level for the radios, the value of the objective function, or profit, would improve at the rate of \$60 per radio produced. Finally, the surplus of zero associated with the retail store distribution channel commitment is a result of this constraint being binding. The negative dual value indicates that increasing the commitment from 150 to 151 units will actually decrease the profit by \$17.

FIGURE 3.12 OBJECTIVE COEFFICIENT AND RIGHT-HAND-SIDE RANGES FOR THE ELECTRONIC COMMUNICATIONS PROBLEM


```
Optimal Objective Value =           48450.00000

     Variable                 Value              Reduced Cost
   ------------         ---------------        ---------------
   [illegible]              25.00000                 0.00000
   [illegible]             425.00000                 0.00000
   [illegible]             150.00000                 0.00000
   [illegible]               0.00000               -45.00000

    Con[illegible]      Slack/Surplus              Dual Value
   ------------         ---------------        ---------------
   [illegible]               0.00000                 3.00000
        2                   25.00000                 0.00000
        3                    0.00000                60.00000
        4                    0.00000               -17.00000

                    Objective        Allowable          Allowable
    Variable       Coefficient        Increase           Decrease
   ----------     ------------      ----------         ----------
      M             90.00000         Infinite            6.00000
      B             84.00000          6.00000           34.00000
      R             70.00000         17.00000           Infinite
      D             60.00000         45.00000           Infinite

                       RHS          Allowable          Allowable
   Constraint         Value          Increase           Decrease
   ----------     ------------      ----------         ----------
      1           5000.00000        850.00000           50.00000
      2           1800.00000         Infinite           25.00000
      3            600.00000          3.57143           85.00000
      4            150.00000         50.00000          150.00000
```

WEB file
Electronic

Thus, Electronic Communications may want to consider reducing its commitment to the retail store distribution channel. A *decrease* in the commitment will actually improve profit at the rate of $17 per unit.

We now consider the additional sensitivity analysis information provided by the computer output shown in Figure 3.12. The allowable increases and decreases for the objective function coefficients are as follows:

Objective Coefficient	Allowable Increase	Allowable Decrease
90.00000	Infinite	6.00000
84.00000	6.00000	34.00000
70.00000	17.00000	Infinite
60.00000	45.00000	Infinite

The current solution or strategy remains optimal, provided that the objective function coefficients do not increase or decrease by more than the allowed amount. Consider the allowable increase and decrease of the direct mail distribution channel coefficient. This information is consistent with the earlier observation for the Reduced Costs portion of the output. In both instances, we see that the per-unit profit would have to increase by \$45 to \$105 before the direct mail distribution channel could be in the optimal solution with a positive val[illegible]

Fina[illegible]lysis information on right-hand-side ranges, as shown in Figur[illegible]lowable increase and decrease for the right-hand-side values.

	Allowable Increase	Allowable Decrease
5000.00000	850.00000	50.00000
1800.00000	Infinite	25.00000
600.00000	3.57143	85.00000
150.00000	50.00000	150.00000

Try Problems 12 and 13 to test your ability at interpreting the computer output for problems involving more than two decision variables.

Several interpretations of these ranges are possible. In particular, recall that the dual value for the advertising budget enabled us to conclude that each \$1 increase in the budget would increase the profit by \$3. The current advertising budget is \$5000. The allowable increase in the advertising budget is \$850 and this implies that there is value in increasing the budget up to an advertising budget of \$5850. Increases above this level would not necessarily be beneficial. Also note that the dual value of −17 for the retail stores requirement suggested the desirability of reducing this commitment. The allowable decrease for this constraint is 150, and this implies that the commitment could be reduced to zero and the value of the reduction would be at the rate of \$17 per unit.

Again, the *sensitivity analysis* or *postoptimality analysis* provided by computer software packages for linear programming problems considers only *one change at a time*, with all other coefficients of the problem remaining as originally specified. As mentioned earlier, simultaneous changes are best handled by re-solving the problem.

Finally, recall that the complete solution to the Electronic Communications problem requested information not only on the number of units to be distributed over each channel, but also on the allocation of the advertising budget and the salesforce effort to each distribution channel. For the optimal solution of $M = 25$, $B = 425$, $R = 150$, and $D = 0$, we can simply evaluate each term in a given constraint to determine how much of the constraint resource is allocated to each distribution channel. For example, the advertising budget constraint of

$$10M + 8B + 9R + 15D \leq 5000$$

shows that $10M = 10(25) = \$250$, $8B = 8(425) = \$3400$, $9R = 9(150) = \$1350$, and $15D = 15(0) = \$0$. Thus, the advertising budget allocations are, respectively, \$250, \$3400, \$1350, and \$0 for each of the four distribution channels. Making similar calculations for the salesforce constraint results in the managerial summary of the Electronic Communications optimal solution as shown in Table 3.3.

TABLE 3.3 PROFIT-MAXIMIZING STRATEGY FOR THE ELECTRONIC COMMUNICATIONS PROBLEM

Distribution Channel	Volume	Advertising Allocation	Salesforce Allocation (hours)
Marine distr[illegible]	[illegible]	$ 250	50
Business dis[illegible]	[illegible]	3400	1275
National ret[illegible]	[illegible]	1350	450
Direct mail	[illegible]	0	0
Totals	[illegible]	$5000	1775
Projected [illegible]	[illegible]		

SUMMARY

We began the chapter with a discussion of sensitivity analysis: the study of how changes in the coefficients of a linear program affect the optimal solution. First, we showed how a graphical method can be used to determine how a change in one of the objective function coefficients or a change in the right-hand-side value for a constraint will affect the optimal solution to the problem. Because graphical sensitivity analysis is limited to linear programs with two decision variables, we showed how to use software to produce a sensitivity report containing the same information.

We continued our discussion of problem formulation, sensitivity analysis and its limitations, and the interpretation of the solution by introducing several modifications of the Par, Inc., problem. They involved an additional decision variable and several types of percentage, or ratio, constraints. Then, in order to provide additional practice in formulating and interpreting the solution for linear programs involving more than two decision variables, we introduced the Electronic Communications problem, a maximization problem with four decision variables, two less-than-or-equal-to constraints, one equality constraint, and one greater-than-or-equal-to constraint.

The Management Science in Action, Tea Production and Distribution in India, illustrates the diversity of problems in which linear programming can be applied and the importance of sensitivity analysis. In the next chapter we will see many more applications of linear programming.

MANAGEMENT SCIENCE IN ACTION

TEA PRODUCTION AND DISTRIBUTION IN INDIA*

In India, one of the largest tea producers in the world, approximately $1 billion of tea packets and loose tea are sold. Duncan Industries Limited (DIL), the third largest producer of tea in the Indian tea market, sells about $37.5 million of tea, almost all of which is sold in packets.

DIL has 16 tea gardens, three blending units, six packing units, and 22 depots. Tea from the gardens is sent to blending units, which then mix various grades of tea to produce blends such as Sargam, Double Diamond, and Runglee Rungliot. The blended tea is transported to packing units, where it is placed in packets of different sizes and shapes to produce about 120 different product lines. For example, one line is Sargam tea packed in 500-gram cartons, another line is Double Diamond packed in 100-gram pouches, and so on. The tea is then shipped to the depots that supply 11,500 distributors through whom the needs of approximately 325,000 retailers are satisfied.

For the coming month, sales managers provide estimates of the demand for each line of tea at each depot. Using these estimates, a team of senior managers would determine the amounts of loose tea of each blend to ship to each packing unit, the quantity of each line of tea to be packed at each packing unit, and the amounts of packed tea of each line to be transported from each packing unit to the various depots. This process requires two to three days each month and often results in stockouts of lines in demand at specific depots.

Consequently, [illegible] model involving approxim[illegible]es and 1500 constraints [illegible]ze the company's freight [illegible]mand, supply, and all operational constraints. The model was tested on past data and showed that stockouts could be prevented at little or no additional cost. Moreover, the model was able to provide management with the ability to perform various what-if types of exercises, convincing them of the potential benefits of using management science techniques to support the decision-making process.

*Based on Nilotpal Chakravarti, "Tea Company Steeped in OR," *OR/MS Today* (April 2000).

GLOSSARY

Sensitivity analysis The study of how changes in the coefficients of a linear programming problem affect the optimal solution.

Range of optimality The range of values over which an objective function coefficient may vary without causing any change in the values of the decision variables in the optimal solution.

Objective Function Allowable Increase (Decrease) The allowable increase/decrease of an objective function coefficient is the amount the coefficient may increase (decrease) without causing any change in the values of the decision variables in the optimal solution. The allowable increase/decrease for the objective function coefficients can be used to calculate the range of optimality.

Dual value The change in the value of the objective function per unit increase in the right-hand side of a constraint.

Reduced cost The reduced cost of a variable is equal to the dual value on the nonnegativity constraint for that variable.

Range of feasibility The range of values over which the dual value is applicable.

Right-Hand-Side Allowable Increase (Decrease) The allowable increase (decrease) of the right-hand side of a constraint is the amount the right-hand side may increase (decrease) without causing any change in the dual value for that constraint. The allowable increase (decrease) for the right-hand side can be used to calculate the range of feasibility for that constraint.

Sunk cost A cost that is not affected by the decision made. It will be incurred no matter what values the decision variables assume.

Relevant cost A cost that depends upon the decision made. The amount of a relevant cost will vary depending on the values of the decision variables.

PROBLEMS

SELF test

1. Consider the following linear program:

$$
\begin{aligned}
\text{Max} \quad & 3A + 2B \\
\text{s.t.} \quad & \\
& 1A + 1B \le 10 \\
& 3A + 1B \le 24 \\
& 1A + 2B \le 16 \\
& A, B \ge 0
\end{aligned}
$$

a. Use the graphical solution procedure to find the optimal solution.
b. Assume that the objective function coefficient for A changes from 3 to 5. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.

c. Assume that the objective function coefficient for A remains 3, but the objective function coefficient for B changes from 2 to 4. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.
d. The computer solution for the linear program in part (a) provides the following objective coefficient range information:

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
A	3.00000	3.00000	1.00000
B	2.00000	1.00000	1.00000

Use this objective coefficient range information to answer parts (b) and (c).

SELF test

2. Consider the linear program in Problem 1. The value of the optimal solution is 27. Suppose that the right-hand side for constraint 1 is increased from 10 to 11.
a. Use the graphical solution procedure to find the new optimal solution.
b. Use the solution to part a to determine the dual value for constraint 1.
c. The computer solution for the linear program in Problem 1 provides the following right-hand-side range information:

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	10.00000	1.20000	2.00000
2	24.00000	6.00000	6.00000
3	16.00000	Infinite	3.00000

What does the right-hand-side range information for constraint 1 tell you about the dual value for constraint 1?

d. The dual value for constraint 2 is 0.5. Using this dual value and the right-hand-side range information in part (c), what conclusion can be drawn about the effect of changes to the right-hand side of constraint 2?

3. Consider the following linear program:

$$
\begin{aligned}
\text{Min} \quad & 8X + 12Y \\
\text{s.t.} \quad & \\
& 1X + 3Y \geq 9 \\
& 2X + 2Y \geq 10 \\
& 6X + 2Y \geq 18 \\
& A, B \geq 0
\end{aligned}
$$

a. Use the graphical solution procedure to find the optimal solution.
b. Assume that the objective function coefficient for X changes from 8 to 6. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.

c. Assume that the objective function coefficient for X remains 8, but the objective function coefficient for Y changes from 12 to 6. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.

d. The computer solution for the linear program in part (a) provides the following objective coefficient range information:

Variab[illegible]	[illegible]e [illegible]t	Allowable Increase	Allowable Decrease
X	[illegible]	4.00000	4.00000
Y	[illegible]	12.00000	4.00000

[illegible] coefficient range information help you answer parts (b) and (c) prior to re-solving the problem?

4. Consider the linear program in Problem 3. The value of the optimal solution is 48. Suppose that the right-hand side for constraint 1 is increased from 9 to 10.

a. Use the graphical solution procedure to find the new optimal solution.

b. Use the solution to part (a) to determine the dual value for constraint 1.

c. The computer solution for the linear program in Problem 3 provides the following right-hand-side range information:

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	9.00000	2.00000	4.00000
2	10.00000	8.00000	1.00000
3	18.00000	4.00000	Infinite

What does the right-hand-side range information for constraint 1 tell you about the dual value for constraint 1?

d. The dual value for constraint 2 is 3. Using this dual value and the right-hand-side range information in part (c), what conclusion can be drawn about the effect of changes to the right-hand side of constraint 2?

SELF test

5. Refer to the Kelson Sporting Equipment problem (Chapter 2, Problem 24). Letting

$$R = \text{number of regular gloves}$$
$$C = \text{number of catcher's mitts}$$

leads to the following formulation:

$$
\begin{aligned}
\text{Max} \quad & 5R + 8C \\
\text{s.t.} \quad & \\
& R + \tfrac{3}{2}C \le 900 \quad \text{Cutting and sewing} \\
& \tfrac{1}{2}R + \tfrac{1}{3}C \le 300 \quad \text{Finishing} \\
& \tfrac{1}{8}R + \tfrac{1}{4}C \le 100 \quad \text{Packaging and shipping} \\
& R, C \ge 0
\end{aligned}
$$

FIGURE 3.13 THE SOLUTION FOR THE KELSON SPORTING EQUIPMENT PROBLEM

```
Optimal Objective Value = 7000.00000
```

Variable	Value	Reduced Cost
[illegible]	500.00000	0.00000
[illegible]	150.00000	0.00000

Constraint	Slack/Surplus	Dual Value
[illegible]	175.00000	0.00000
[illegible]	0.00000	3.00000
[illegible]	0.00000	28.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
R	5.00000	7.00000	1.00000
C	8.00000	2.00000	4.66667

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	900.00000	Infinite	175.00000
2	300.00000	100.00000	166.66667
3	100.00000	35.00000	25.00000

The computer solution is shown in Figure 3.13.

a. What is the optimal solution, and what is the value of the total profit contribution?
b. Which constraints are binding?
c. What are the dual values for the resources? Interpret each.
d. If overtime can be scheduled in one of the departments, where would you recommend doing so?

SELF test

6. Refer to the computer solution of the Kelson Sporting Equipment problem in Figure 3.13 (see Problem 5).
   a. Determine the objective coefficient ranges.
   b. Interpret the ranges in part (a).
   c. Interpret the right-hand-side ranges.
   d. How much will the value of the optimal solution improve if 20 extra hours of packaging and shipping time are made available?

7. Investment Advisors, Inc., is a brokerage firm that manages stock portfolios for a number of clients. A particular portfolio consists of U shares of U.S. Oil and H shares of Huber Steel. The annual return for U.S. Oil is \$3 per share and the annual return for Huber Steel is \$5 per share. U.S. Oil sells for \$25 per share and Huber Steel sells for \$50 per share. The portfolio has \$80,000 to be invested. The portfolio risk index (0.50 per share of U.S. Oil and 0.25 per share for Huber Steel) has a maximum of 700. In addition, the portfolio is limited to a maximum of 1000 shares of U.S. Oil. The linear

FIGURE 3.14 THE SOLUTION FOR THE INVESTMENT ADVISORS PROBLEM


```
Optimal Objective Value = 8400.00000

     Variable            Value            Reduced Cost
  --------------    ---------------    -----------------
        U              800.00000             0.00000
        H             1200.00000             0.00000

    Constraint      Slack/Surplus          Dual Value
  --------------    ---------------    -----------------
                         0.00000             0.09333
                         0.00000             1.33333
                       200.00000             0.00000

                 Objective        Allowable        Allowable
   Variable      Coefficient      Increase         Decrease
  ----------    ------------    ------------    ------------
      U            3.00000         7.00000         0.50000
      H            5.00000         1.00000         3.50000

                    RHS           Allowable        Allowable
  Constraint       Value          Increase         Decrease
  ----------    ------------    ------------    ------------
      1         80000.00000     60000.00000     15000.00000
      2           700.00000        75.00000       300.00000
      3          1000.00000       Infinite        200.00000
```


programming formulation that will maximize the total annual return of the portfolio is as follows:

Max	$3U + 5H$	Maximize total annual return
s.t.		
	$25U + 50H \le 80{,}000$	Funds available
	$0.50U + 0.25D \le 700$	Risk maximum
	$1U \le 1000$	U.S. Oil maximum
	$U, H \ge 0$	

The computer solution of this problem is shown in Figure 3.14.

a. What is the optimal solution, and what is the value of the total annual return?
b. Which constraints are binding? What is your interpretation of these constraints in terms of the problem?
c. What are the dual values for the constraints? Interpret each.
d. Would it be beneficial to increase the maximum amount invested in U.S. Oil? Why or why not?

8. Refer to Figure 3.14, which shows the computer solution of Problem 7.
   a. How much would the return for U.S. Oil have to increase before it would be beneficial to increase the investment in this stock?

b. How much would the return for Huber Steel have to decrease before it would be beneficial to reduce the investment in this stock?

c. How much would the total annual return be reduced if the U.S. Oil maximum were reduced to 900 shares?

9. Recall the Tom's, Inc., problem (Chapter 2, Problem 28). Letting

W = jars of Western Foods Salsa
M = jars of Mexico City Salsa

$1W + 1.25M$

$5W + 7M \leq 4480$ Whole tomatoes
$3W + 1M \leq 2080$ Tomato sauce
$2W + 2M \leq 1600$ Tomato paste
$W, M \geq 0$

The computer solution is shown in Figure 3.15.

a. What is the optimal solution, and what are the optimal production quantities?

b. Specify the objective function ranges.

FIGURE 3.15 THE SOLUTION FOR THE TOM'S, INC., PROBLEM

Optimal Objective Value = 860.00000

Variable	Value	Reduced Cost
W	560.00000	0.00000
M	240.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	0.12500
2	160.00000	0.00000
3	0.00000	0.18750

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
W	1.00000	0.25000	0.10714
M	1.25000	0.15000	0.25000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	4480.00000	1120.00000	160.00000
2	2080.00000	Infinite	160.00000
3	1600.00000	40.00000	320.00000

c. What are the dual values for each constraint? Interpret each.
d. Identify each of the right-hand-side ranges.

SELF test

10. Recall the Innis Investments problem (Chapter 2, Problem 1[...]). Letting

S = units purchased in the stock fund
M = units purchased in the money market fund

leads [...]tion:

$$[...]3M$$

$[...]0M \leq$	1,200,000	Funds available
$[...]4M \geq$	60,000	Annual income
$M \geq$	3,000	Units in money market

$$S, M \geq 0$$

The computer solution is shown in Figure 3.16.
a. What is the optimal solution, and what is the minimum total risk?
b. Specify the objective coefficient ranges.
c. How much annual income will be earned by the portfolio?

FIGURE 3.16 THE SOLUTION FOR THE INNIS INVESTMENTS PROBLEM


```
Optimal Objective Value = 62000.00000

    Variable              Value              Reduced Cost
  --------------     --------------       -----------------
       S              4000.00000                0.00000
       M             10000.00000                0.00000

   Constraint        Slack/Surplus           Dual Value
  --------------     --------------       -----------------
        1                0.00000               -0.05667
        2                0.00000                2.16667
        3             7000.00000                0.00000

                   Objective      Allowable          Allowable
    Variable       Coefficient    Increase           Decrease
   ----------     ------------   ----------         ----------
        S            8.00000      Infinite            4.25000
        M            3.00000      3.40000             Infinite

                      RHS         Allowable          Allowable
   Constraint        Value        Increase           Decrease
   ----------     ------------   ----------         ----------
        1        1200000.00000  300000.00000      420000.00000
        2          60000.00000   42000.00000       12000.00000
        3           3000.00000    7000.00000          Infinite
```

d. What is the rate of return for the portfolio?
e. What is the dual value for the funds available constraint?
f. What is the marginal rate of return on extra funds added to the portfolio?

11. Refer to Problem 10 and the computer solution shown in Figure 3.16.
   a. Suppose the risk index for the stock fund (the value of C_S) increases from its current value of 8 to 12. How does the optimal solution change, if at all?
   b. ...x for the money market fund (the value of C_M) increases from its ...3.5. How does the optimal solution change, if at all?
   c. ...s to 12 and C_M increases to 3.5. How does the optimal solution ...

SELF test

12. ... manufactures three home air conditioners: an economy model, a ...luxe model. The profits per unit are \$63, \$95, and \$135, respec-... ...quirements per unit are as follows:

	Number of Fans	Number of Cooling Coils	Manufacturing Time (hours)
Economy	1	1	8
Standard	1	2	12
Deluxe	1	4	14

For the coming production period, the company has 200 fan motors, 320 cooling coils, and 2400 hours of manufacturing time available. How many economy models (E), standard models (S), and deluxe models (D) should the company produce in order to maximize profit? The linear programming model for the problem is as follows:

$$
\begin{array}{llr}
\text{Max} & 63E + 95S + 135D & \\
\text{s.t.} & & \\
& 1E + 1S + 1D \le 200 & \text{Fan motors} \\
& 1E + 2S + 4D \le 320 & \text{Cooling coils} \\
& 8E + 12S + 14D \le 2400 & \text{Manufacturing time} \\
& E, S, D \ge 0 &
\end{array}
$$

The computer solution is shown in Figure 3.17.
   a. What is the optimal solution, and what is the value of the objective function?
   b. Which constraints are binding?
   c. Which constraint shows extra capacity? How much?
   d. If the profit for the deluxe model were increased to \$150 per unit, would the optimal solution change? Use the information in Figure 3.17 to answer this question.

SELF test

13. Refer to the computer solution of Problem 12 in Figure 3.17.
   a. Identify the range of optimality for each objective function coefficient.
   b. Suppose the profit for the economy model is increased by \$6 per unit, the profit for the standard model is decreased by \$2 per unit, and the profit for the deluxe model is increased by \$4 per unit. What will the new optimal solution be?
   c. Identify the range of feasibility for the right-hand-side values.
   d. If the number of fan motors available for production is increased by 100, will the dual value for that constraint change? Explain.

FIGURE 3.17 THE SOLUTION FOR THE QUALITY AIR CONDITIONING PROBLEM

Optimal Objective Value = 16440.00000

Variable	Value	Reduced Cost
E	80.00000	0.00000
S	120.00000	0.00000
D	0.00000	-24.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	31.00000
2	0.00000	32.00000
3	320.00000	0.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
E	63.00000	12.00000	15.50000
S	95.00000	31.00000	8.00000
D	135.00000	24.00000	Infinite

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	200.00000	80.00000	40.00000
2	320.00000	80.00000	120.00000
3	2400.00000	Infinite	320.00000

14. Digital Controls, Inc. (DCI), manufactures two models of a radar gun used by police to monitor the speed of automobiles. Model A has an accuracy of plus or minus 1 mile per hour, whereas the smaller model B has an accuracy of plus or minus 3 miles per hour. For the next week, the company has orders for 100 units of model A and 150 units of model B. Although DCI purchases all the electronic components used in both models, the plastic cases for both models are manufactured at a DCI plant in Newark, New Jersey. Each model A case requires 4 minutes of injection-molding time and 6 minutes of assembly time. Each model B case requires 3 minutes of injection-molding time and 8 minutes of assembly time. For next week, the Newark plant has 600 minutes of injection-molding time available and 1080 minutes of assembly time available. The manufacturing cost is \$10 per case for model A and \$6 per case for model B. Depending upon demand and the time available at the Newark plant, DCI occasionally purchases cases for one or both models from an outside supplier in order to fill customer orders that could not be filled otherwise. The purchase cost is \$14 for each model A case and \$9 for each model B case. Management wants to develop a minimum cost plan that will determine how many cases of each model should be produced at the Newark plant and how many cases of each model

should be purchased. The following decision variables were used to formulate a linear programming model for this problem:

$$AM = \text{number of cases of model A manufactured}$$
$$BM = \text{number of cases of model B manufactured}$$
$$AP = \text{number of cases of model A purchased}$$
$$BP = \text{number of cases of model B purchased}$$

[illegible] model that can be used to solve this problem is as follows:

[illegible] $BM + 14AP + 9BP$

[illegible] $+ 1AP +$	$=$	100	Demand for model A
$1BM + 1BP$	$=$	150	Demand for model B
$4AM + 3BM$	$\leq$	600	Injection molding time
$6AM + 8BM$	$\leq$	1080	Assembly time

$$AM, BM, AP, BP \geq 0$$

The computer solution is shown in Figure 3.18.

a. What is the optimal solution and what is the optimal value of the objective function?
b. Which constraints are binding?
c. What are the dual values? Interpret each.
d. If you could change the right-hand side of one constraint by one unit, which one would you choose? Why?

15. Refer to the computer solution to Problem 14 in Figure 3.18.
a. Interpret the ranges of optimality for the objective function coefficients.
b. Suppose that the manufacturing cost increases to $11.20 per case for model A. What is the new optimal solution?
c. Suppose that the manufacturing cost increases to $11.20 per case for model A and the manufacturing cost for model B decreases to $5 per unit. Would the optimal solution change?

16. Tucker Inc. produces high-quality suits and sport coats for men. Each suit requires 1.2 hours of cutting time and 0.7 hours of sewing time, uses 6 yards of material, and provides a profit contribution of $190. Each sport coat requires 0.8 hours of cutting time and 0.6 hours of sewing time, uses 4 yards of material, and provides a profit contribution of $150. For the coming week, 200 hours of cutting time, 180 hours of sewing time, and 1200 yards of fabric are available. Additional cutting and sewing time can be obtained by scheduling overtime for these operations. Each hour of overtime for the cutting operation increases the hourly cost by $15, and each hour of overtime for the sewing operation increases the hourly cost by $10. A maximum of 100 hours of overtime can be scheduled. Marketing requirements specify a minimum production of 100 suits and 75 sport coats. Let

$$S = \text{number of suits produced}$$
$$SC = \text{number of sport coats produced}$$
$$D1 = \text{hours of overtime for the cutting operation}$$
$$D2 = \text{hours of overtime for the sewing operation}$$

FIGURE 3.18 THE SOLUTION FOR THE DIGITAL CONTROLS, INC., PROBLEM

Optimal Objective Value = 2[illegible].00000

Variable	Value	Reduced Cost
A[illegible]	100.00000	0.00000
B[illegible]	60.00000	0.00000
A[illegible]	0.00000	1.75000
B[illegible]	90.00000	0.00000

Cons[illegible]	[illegible]lack/Surplus	Dual Value
[illegible]	0.00000	12.25000
2	0.00000	9.00000
3	20.00000	0.00000
4	0.00000	-0.37500

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
AB	10.00000	1.75000	Infinite
BM	6.00000	3.00000	2.33333
AP	14.00000	Infinite	1.75000
BP	9.00000	2.33333	3.00000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	100.00000	11.42857	100.00000
2	150.00000	Infinite	90.00000
3	600.00000	Infinite	20.00000
4	1080.00000	53.33333	480.00000

The computer solution is shown in Figure 3.19.

a. What is the optimal solution, and what is the total profit? What is the plan for the use of overtime?

b. A price increase for suits is being considered that would result in a profit contribution of \$210 per suit. If this price increase is undertaken, how will the optimal solution change?

c. Discuss the need for additional material during the coming week. If a rush order for material can be placed at the usual price plus an extra \$8 per yard for handling, would you recommend the company consider placing a rush order for material? What is the maximum price Tucker would be willing to pay for an additional yard of material? How many additional yards of material should Tucker consider ordering?

d. Suppose the minimum production requirement for suits is lowered to 75. Would this change help or hurt profit? Explain.

FIGURE 3.19 THE SOLUTION FOR THE TUCKER INC. PROBLEM

Optimal Objective Value = [illegible]900.00000

Variable	Value	Reduced Cost
[illegible]	100.00000	0.00000
[illegible]	150.00000	0.00000
[illegible]	40.00000	0.00000
[illegible]	0.00000	-10.00000

Constraint	Slack/Surplus	Dual Value
[illegible]	0.00000	15.00000
2	20.00000	0.00000
3	0.00000	34.50000
4	60.00000	0.00000
5	[illegible]	-35.00000
6	75.00000	0.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
S	190.00000	35.00000	Infinite
SC	150.00000	Infinite	23.33333
D1	-15.00000	15.00000	12.50000
D2	-10.00000	10.00000	Infinite

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	200.00000	40.00000	60.00000
2	180.00000	Infinite	20.00000
3	1200.00000	133.33333	200.00000
4	100.00000	Infinite	60.00000
5	100.00000	50.00000	100.00000
6	75.00000	75.00000	Infinite

17. The Porsche Club of America sponsors driver education events that provide high-performance driving instruction on actual race tracks. Because safety is a primary consideration at such events, many owners elect to install roll bars in their cars. Deegan Industries manufactures two types of roll bars for Porsches. Model DRB is bolted to the car using existing holes in the car's frame. Model DRW is a heavier roll bar that must be welded to the car's frame. Model DRB requires 20 pounds of a special high alloy steel, 40 minutes of manufacturing time, and 60 minutes of assembly time. Model DRW requires 25 pounds of the special high alloy steel, 100 minutes of manufacturing time, and 40 minutes of assembly time. Deegan's steel supplier indicated that at most 40,000 pounds of the high-alloy steel will be available next quarter. In addition, Deegan estimates that 2000 hours of manufacturing time and 1600 hours of assembly time will be available next quarter. The profit

contributions are \$200 per unit for model DRB and \$280 per unit for model DRW. The linear programming model for this problem is as follows:

$$\text{Max } 200DRB + 280DRW$$

s.t.

$20DRB + 25DRW \leq 40{,}000$		Steel available
[illegible] $100DRW \leq 120{,}000$		Manufacturing minutes
[illegible] $40DRW \leq 96{,}000$		Assembly minutes
[illegible] $W \geq 0$		

The [illegible] n in Figure 3.20.

a. [illegible] ion and the total profit contribution?

b. [illegible] provide Deegan Industries with an additional 500 pounds [illegible] pound. Should Deegan purchase the additional pounds of [illegible] Explain.

c. Deegan is considering using overtime to increase the available assembly time. What would you advise Deegan to do regarding this option? Explain.

d. Because of increased competition, Deegan is considering reducing the price of model DRB such that the new contribution to profit is \$175 per unit. How would this change in price affect the optimal solution? Explain.

e. If the available manufacturing time is increased by 500 hours, will the dual value for the manufacturing time constraint change? Explain.

FIGURE 3.20 THE SOLUTION FOR THE DEEGAN INDUSTRIES PROBLEM

Optimal Objective Value = 424000.00000

Variable	Value	Reduced Cost
DRB	1000.00000	0.00000
DRW	800.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	8.80000
2	0.00000	0.60000
3	4000.00000	0.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
DRB	200.00000	24.00000	88.00000
DRW	280.00000	220.00000	30.00000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	40000.00000	909.09091	10000.00000
2	120000.00000	40000.00000	5714.28571
3	96000.00000	Infinite	4000.00000

18. Davison Electronics manufactures two LCD television monitors, identified as model A and model B. Each model has its lowest possible production cost when produced on Davison's new production line. However, the new production line does not have the capacity to handle the total production of both models. As a result, at least some of the production must be routed to a higher-cost, old production line. The following table shows the minimum production requirements for next month, the production line [illegible] month, and the production cost per unit for each production [illegible]

Model	Production Cost per Unit New Line	Production Cost per Unit Old Line	Minimum Production Requirements
A	\$30	\$50	50,000
B	\$25	\$40	70,000
Production Line Capacity	80,000	60,000	

Let

AN = Units of model A produced on the new production line
AO = Units of model A produced on the old production line
BN = Units of model B produced on the new production line
BO = Units of model B produced on the old production line

Davison's objective is to determine the minimum cost production plan. The computer solution is shown in Figure 3.21.

a. Formulate the linear programming model for this problem using the following four constraints:

Constraint 1: Minimum production for model A
Constraint 2: Minimum production for model B
Constraint 3: Capacity of the new production line
Constraint 4: Capacity of the old production line

b. Using computer solution in Figure 3.21, what is the optimal solution, and what is the total production cost associated with this solution?
c. Which constraints are binding? Explain.
d. The production manager noted that the only constraint with a positive dual value is the constraint on the capacity of the new production line. The manager's interpretation of the dual value was that a one-unit increase in the right-hand side of this constraint would actually increase the total production cost by \$15 per unit. Do you agree with this interpretation? Would an increase in capacity for the new production line be desirable? Explain.
e. Would you recommend increasing the capacity of the old production line? Explain.
f. The production cost for model A on the old production line is \$50 per unit. How much would this cost have to change to make it worthwhile to produce model A on the old production line? Explain.
g. Suppose that the minimum production requirement for model B is reduced from 70,000 units to 60,000 units. What effect would this change have on the total production cost? Explain.

FIGURE 3.21 THE SOLUTION FOR THE DAVISON INDUSTRIES PROBLEM

Optimal Objective Value = 38[illegible]00.00000

Variable	Value	Reduced Cost
AN	[illegible]0000.00000	0.00000
AO	0.00000	5.00000
BN	[illegible]0000.00000	0.00000
BO	[illegible]0000.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	45.00000
2	0.00000	40.00000
3	0.00000	-15.00000
4	20000.00000	0.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
AN	30.00000	5.00000	Infinite
AO	50.00000	Infinite	5.00000
BN	25.00000	15.00000	5.00000
BO	40.00000	5.00000	15.00000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	50000.00000	20000.00000	40000.00000
2	70000.00000	20000.00000	40000.00000
3	80000.00000	40000.00000	20000.00000
4	60000.00000	Infinite	20000.00000

Problems 19–32 require formulation and computer solution.

19. Better Products, Inc., manufactures three products on two machines. In a typical week, 40 hours are available on each machine. The profit contribution and production time in hours per unit are as follows:

Category	Product 1	Product 2	Product 3
Profit/unit	\$30	\$50	\$20
Machine 1 time/unit	0.5	2.0	0.75
Machine 2 time/unit	1.0	1.0	0.5

Two operators are required for machine 1; thus, 2 hours of labor must be scheduled for each hour of machine 1 time. Only one operator is required for machine 2. A maximum of 100 labor-hours is available for assignment to the machines during the coming week.

Other production requirements are that product 1 cannot account for more than 50% of the units produced and that product 3 must account for at least 20% of the units produced.

a. How many units of each product should be produced to maximize the total profit contribution? What is the projected weekly profit associated with your solution?

b. How many hours of production time will be scheduled on each machine?

c. What is the value of an additional hour of labor?

d. [illegible] pacity can be increased to 120 hours. Would you be interested in [illegible] 0 hours available for this resource? Develop the optimal product [illegible] ra hours are made available.

20. [illegible] k (ASB) has $1 million in new funds that must be allocated to [illegible] s, and automobile loans. The annual rates of return for the three [illegible] r home loans, 12% for personal loans, and 9% for automobile [illegible] g committee has decided that at least 40% of the new funds must [illegible] ns. In addition, the planning committee has specified that the amount allocated to personal loans cannot exceed 60% of the amount allocated to automobile loans.

a. Formulate a linear programming model that can be used to determine the amount of funds ASB should allocate to each type of loan in order to maximize the total annual return for the new funds.

b. How much should be allocated to each type of loan? What is the total annual return? What is the annual percentage return?

c. If the interest rate on home loans increased to 9%, would the amount allocated to each type of loan change? Explain.

d. Suppose the total amount of new funds available was increased by $10,000. What effect would this have on the total annual return? Explain.

e. Assume that ASB has the original $1 million in new funds available and that the planning committee has agreed to relax the requirement that at least 40% of the new funds must be allocated to home loans by 1%. How much would the annual return change? How much would the annual percentage return change?

21. Round Tree Manor is a hotel that provides two types of rooms with three rental classes: Super Saver, Deluxe, and Business. The profit per night for each type of room and rental class is as follows:

		Rental Class		
		Super Saver	Deluxe	Business
Room	Type I	$30	$35	—
	Type II	$20	$30	$40

Type I rooms do not have Internet access and are not available for the Business rental class.

Round Tree's management makes a forecast of the demand by rental class for each night in the future. A linear programming model developed to maximize profit is used to determine how many reservations to accept for each rental class. The demand forecast for a particular night is 130 rentals in the Super Saver class, 60 rentals in the Deluxe class, and 50 rentals in the Business class. Round Tree has 100 Type I rooms and 120 Type II rooms.

a. Use linear programming to determine how many reservations to accept in each rental class and how the reservations should be allocated to room types. Is the demand by any rental class not satisfied? Explain.

b. How many reservations can be accommodated in each rental class?

c. Management is considering offering a free breakfast to anyone upgrading from a Super Saver reservation to Deluxe class. If the cost of the breakfast to Round Tree is $5, should this incentive be offered?

d. With a little work, an unused office area could be converted to a rental room. If the conversion cost is the same for both types of rooms, would you recommend converting the office to a Type I or a Type II room? Why?

e. C[illegible]ming model be modified to plan for the allocation of rental [illegible]t? What information would be needed and how would the [illegible]

22. Indu[illegible]arded a contract to design a label for a new wine produced by L[illegible]mpany estimates that 150 hours will be required to complete the p[illegible]aphics designers available for assignment to this project are Lisa[illegible]eam leader; David, a senior designer; and Sarah, a junior desi[illegible]rked on several projects for Lake View Winery, management spec[illegible]igned at least 40% of the total number of hours assigned to the two senior designers. To provide label-designing experience for Sarah, Sarah must be assigned at least 15% of the total project time. However, the number of hours assigned to Sarah must not exceed 25% of the total number of hours assigned to the two senior designers. Due to other project commitments, Lisa has a maximum of 50 hours available to work on this project. Hourly wage rates are $30 for Lisa, $25 for David, and $18 for Sarah.

a. Formulate a linear program that can be used to determine the number of hours each graphic designer should be assigned to the project in order to minimize total cost.

b. How many hours should each graphic designer be assigned to the project? What is the total cost?

c. Suppose Lisa could be assigned more than 50 hours. What effect would this have on the optimal solution? Explain.

d. If Sarah were not required to work a minimum number of hours on this project, would the optimal solution change? Explain.

23. Vollmer Manufacturing makes three components for sale to refrigeration companies. The components are processed on two machines: a shaper and a grinder. The times (in minutes) required on each machine are as follows:

	Machine	
Component	**Shaper**	**Grinder**
1	6	4
2	4	5
3	4	2

The shaper is available for 120 hours, and the grinder is available for 110 hours. No more than 200 units of component 3 can be sold, but up to 1000 units of each of the other components can be sold. In fact, the company already has orders for 600 units of component 1 that must be satisfied. The profit contributions for components 1, 2, and 3 are $8, $6, and $9, respectively.

a. Formulate and solve for the recommended production quantities.

b. What are the objective coefficient ranges for the three components? Interpret these ranges for company management.

c. What are the right-hand-side ranges? Interpret these ranges for company management.

d. If more time could be made available on the grinder, how much would it be worth?

e. If more units of component 3 can be sold by reducing the sales price by $4, should the company reduce the price?

24. National Insurance Associates carries an investment portfolio of stocks, bonds, and other investment alternatives. Currently $200,000 of funds are available and must be considered for new investment opportunities. The four stock options National is considering and the relevant financial data are as follows:

	Stock A	Stock B	Stock C	Stock D
P[illegible]	$100	$50	$80	$40
A[illegible]	0.12	0.08	0.06	0.10
R[illegible]ested	0.10	0.07	0.05	0.08

[illegible]s the relative uncertainty associated with the stock in terms of its realizing the projected annual return; higher values indicate greater risk. The risk measures are provided by the firm's top financial advisor.

National's top management has stipulated the following investment guidelines: The annual rate of return for the portfolio must be at least 9% and no one stock can account for more than 50% of the total dollar investment.

a. Use linear programming to develop an investment portfolio that minimizes risk.

b. If the firm ignores risk and uses a maximum return-on-investment strategy, what is the investment portfolio?

c. What is the dollar difference between the portfolios in parts (a) and (b)? Why might the company prefer the solution developed in part (a)?

25. Georgia Cabinets manufactures kitchen cabinets that are sold to local dealers throughout the Southeast. Because of a large backlog of orders for oak and cherry cabinets, the company decided to contract with three smaller cabinetmakers to do the final finishing operation. For the three cabinetmakers, the number of hours required to complete all the oak cabinets, the number of hours required to complete all the cherry cabinets, the number of hours available for the final finishing operation, and the cost per hour to perform the work are shown here.

	Cabinetmaker 1	Cabinetmaker 2	Cabinetmaker 3
Hours required to complete all the oak cabinets	50	42	30
Hours required to complete all the cherry cabinets	60	48	35
Hours available	40	30	35
Cost per hour	$36	$42	$55

For example, Cabinetmaker 1 estimates it will take 50 hours to complete all the oak cabinets and 60 hours to complete all the cherry cabinets. However, Cabinetmaker 1 only has 40 hours available for the final finishing operation. Thus, Cabinetmaker 1 can only complete 40/50 = 0.80, or 80%, of the oak cabinets if it worked only on oak cabinets. Similarly, Cabinetmaker 1 can only complete 40/60 = 0.67, or 67%, of the cherry cabinets if it worked only on cherry cabinets.

a. Formulate a linear programming model that can be used to determine the percentage of the oak cabinets and the percentage of the cherry cabinets that should be given to each of the three cabinetmakers in order to minimize the total cost of completing both projects.

b. Solve the model formulated in part (a). What percentage of the oak cabinets and what percentage of the cherry cabinets should be assigned to each cabinetmaker? What is the total cost of completing both projects?

c. If Cabinetmaker 1 has additional hours available, would the optimal solution change? E[illegible]

d. I[illegible]tional hours available, would the optimal solution change? E[illegible]

e. S[illegible] reduced its cost to $38 per hour. What effect would this c[illegible]l solution? Explain.

26. Bens[illegible]res three components used to produce cell telephones and other [illegible] In a given production period, demand for the three components [illegible]manufacturing capacity. In this case, the company meets demand by purchasing the components from another manufacturer at an increased cost per unit. Benson's manufacturing cost per unit and purchasing cost per unit for the three components are as follows:

Source	Component 1	Component 2	Component 3
Manufacture	$4.50	$5.00	$2.75
Purchase	$6.50	$8.80	$7.00

Manufacturing times in minutes per unit for Benson's three departments are as follows:

Department	Component 1	Component 2	Component 3
Production	2	3	4
Assembly	1	1.5	3
Testing & Packaging	1.5	2	5

For instance, each unit of component 1 that Benson manufactures requires 2 minutes of production time, 1 minute of assembly time, and 1.5 minutes of testing and packaging time. For the next production period, Benson has capacities of 360 hours in the production department, 250 hours in the assembly department, and 300 hours in the testing and packaging department.

a. Formulate a linear programming model that can be used to determine how many units of each component to manufacture and how many units of each component to purchase. Assume that component demands that must be satisfied are 6000 units for component 1, 4000 units for component 2, and 3500 units for component 3. The objective is to minimize the total manufacturing and purchasing costs.

b. What is the optimal solution? How many units of each component should be manufactured and how many units of each component should be purchased?

c. Which departments are limiting Benson's manufacturing quantities? Use the dual value to determine the value of an *extra hour* in each of these departments.

d. Suppose that Benson had to obtain one additional unit of component 2. Discuss what the dual value for the component 2 constraint tells us about the cost to obtain the additional unit.

27. Golf Shafts, Inc. (GSI), produces graphite shafts for several manufacturers of golf clubs. Two GSI manufacturing facilities, one located in San Diego and the other in Tampa, have the capability to produce shafts in varying degrees of stiffness, ranging from regular models used primarily by average golfers to extra stiff models used primarily by low-handicap and professional golfers. GSI just received a contract for the production of 200,000 regular shafts and 75,000 stiff shafts. Because both plants are currently producing shafts for [illegible] plant has sufficient capacity by itself to fill the new order. The San [illegible] up to a total of 120,000 shafts, and the Tampa plant can produce [illegible] shafts. Because of equipment differences at each of the plants and [illegible] per-unit production costs vary as shown here:

	San Diego Cost	Tampa Cost
[illegible]	$5.25	$4.95
[illegible]	$5.45	$5.70

a. Formulate a linear programming model to determine how GSI should schedule production for the new order in order to minimize the total production cost.
b. Solve the model that you developed in part (a).
c. Suppose that some of the previous orders at the Tampa plant could be rescheduled in order to free up additional capacity for the new order. Would this option be worthwhile? Explain.
d. Suppose that the cost to produce a stiff shaft in Tampa had been incorrectly computed, and that the correct cost is $5.30 per shaft. What effect, if any, would the correct cost have on the optimal solution developed in part (b)? What effect would it have on total production cost?

28. The Pfeiffer Company manages approximately $15 million for clients. For each client, Pfeiffer chooses a mix of three investment vehicles: a growth stock fund, an income fund, and a money market fund. Each client has different investment objectives and different tolerances for risk. To accommodate these differences, Pfeiffer places limits on the percentage of each portfolio that may be invested in the three funds and assigns a portfolio risk index to each client.

Here's how the system works for Dennis Hartmann, one of Pfeiffer's clients. Based on an evaluation of Hartmann's risk tolerance, Pfeiffer has assigned Hartmann's portfolio a risk index of 0.05. Furthermore, to maintain diversity, the fraction of Hartmann's portfolio invested in the growth and income funds must be at least 10% for each, and at least 20% must be in the money market fund.

The risk ratings for the growth, income, and money market funds are 0.10, 0.05, and 0.01, respectively. A portfolio risk index is computed as a weighted average of the risk ratings for the three funds where the weights are the fraction of the portfolio invested in each of the funds. Hartmann has given Pfeiffer $300,000 to manage. Pfeiffer is currently forecasting a yield of 20% on the growth fund, 10% on the income fund, and 6% on the money market fund.

a. Develop a linear programming model to select the best mix of investments for Hartmann's portfolio.
b. Solve the model you developed in part (a).
c. How much may the yields on the three funds vary before it will be necessary for Pfeiffer to modify Hartmann's portfolio?
d. If Hartmann were more risk tolerant, how much of a yield increase could he expect? For instance, what if his portfolio risk index is increased to 0.06?
e. If Pfeiffer revised the yield estimate for the growth fund downward to 0.10, how would you recommend modifying Hartmann's portfolio?

f. What information must Pfeiffer maintain on each client in order to use this system to manage client portfolios?

g. On a weekly basis Pfeiffer revises the yield estimates for the three funds. Suppose Pfeiffer has 50 clients. Describe how you would envision Pfeiffer making weekly modifications in each client's portfolio and allocating the total funds managed among the three investment funds.

29. La Jo[illegible]s considering producing a wine cooler that would be a blend[illegible]ine, and fruit juice. To meet taste specifications, the wine coole[illegible]0% white wine, at least 20% and no more than 30% rosé, and e[illegible] Jolla purchases the wine from local wineries and the fruit juice [illegible] in San Francisco. For the current production period, 10,00[illegible]nd 8000 gallons of rosé wine can be purchased; an unlimited a[illegible]e ordered. The costs for the wine are $1.00 per gallon for the w[illegible] for the rosé; the fruit juice can be purchased for $0.50 per gallon. La Jolla Beverage Products can sell all of the wine cooler they can produce for $2.50 per gallon.

a. Is the cost of the wine and fruit juice a sunk cost or a relevant cost in this situation? Explain.

b. Formulate a linear program to determine the blend of the three ingredients that will maximize the total profit contribution. Solve the linear program to determine the number of gallons of each ingredient La Jolla should purchase and the total profit contribution they will realize from this blend.

c. If La Jolla could obtain additional amounts of the white wine, should they do so? If so, how much should they be willing to pay for each additional gallon, and how many additional gallons would they want to purchase?

d. If La Jolla Beverage Products could obtain additional amounts of the rosé wine, should they do so? If so, how much should they be willing to pay for each additional gallon, and how many additional gallons would they want to purchase?

e. Interpret the dual value for the constraint corresponding to the requirement that the wine cooler must contain at least 50% white wine. What is your advice to management given this dual value?

f. Interpret the dual value for the constraint corresponding to the requirement that the wine cooler must contain exactly 20% fruit juice. What is your advice to management given this dual value?

30. The program manager for Channel 10 would like to determine the best way to allocate the time for the 11:00–11:30 evening news broadcast. Specifically, she would like to determine the number of minutes of broadcast time to devote to local news, national news, weather, and sports. Over the 30-minute broadcast, 10 minutes are set aside for advertising. The station's broadcast policy states that at least 15% of the time available should be devoted to local news coverage; the time devoted to local news or national news must be at least 50% of the total broadcast time; the time devoted to the weather segment must be less than or equal to the time devoted to the sports segment; the time devoted to the sports segment should be no longer than the total time spent on the local and national news; and at least 20% of the time should be devoted to the weather segment. The production costs per minute are $300 for local news, $200 for national news, $100 for weather, and $100 for sports.

a. Formulate and solve a linear program that can determine how the 20 available minutes should be used to minimize the total cost of producing the program.

b. Interpret the dual value for the constraint corresponding to the available time. What advice would you give the station manager given this dual value?

c. Interpret the dual value for the constraint corresponding to the requirement that at least 15% of the available time should be devoted to local coverage. What advice would you give the station manager given this dual value?

d. Interpret the dual value for the constraint corresponding to the requirement that the time devoted to the local and the national news must be at least 50% of the total broadcast time. What advice would you give the station manager given this dual value?

e. Interpret the dual value for the constraint corresponding to the requirement that the time devoted to the weather segment must be less than or equal to the time devoted to the sports segment. What advice would you give the station manager given this dual value?

31. [illegible] ready to award contracts for printing their annual report. For the [illegible]ur-color annual report has been printed by Johnson Printing and [illegible]rm, Benson Printing, inquired into the possibility of doing a por- [illegible] quality and service level provided by Lakeside Litho has been [illegible] only 0.5% of their reports have had to be discarded because of [illegible]n Printing has also had a high quality level historically, produc- [illegible]% unacceptable reports. Because Gulf Coast Electronics has had [illegible]on Printing, they estimated their defective rate to be 10%. Gulf [illegible]mine how many reports should be printed by each firm to obtain 75,000 acceptable-quality reports. To ensure that Benson Printing will receive some of the contract, management specified that the number of reports awarded to Benson Printing must be at least 10% of the volume given to Johnson Printing. In addition, the total volume assigned to Benson Printing, Johnson Printing, and Lakeside Litho should not exceed 30,000, 50,000, and 50,000 copies, respectively. Because of the long-term relationship with Lakeside Litho, management also specified that at least 30,000 reports should be awarded to Lakeside Litho. The cost per copy is $2.45 for Benson Printing, $2.50 for Johnson Printing, and $2.75 for Lakeside Litho.

a. Formulate and solve a linear program for determining how many copies should be assigned to each printing firm to minimize the total cost of obtaining 75,000 acceptable-quality reports.

b. Suppose that the quality level for Benson Printing is much better than estimated. What effect, if any, would this quality level have?

c. Suppose that management is willing to reconsider their requirement that Lakeside Litho be awarded at least 30,000 reports. What effect, if any, would this consideration have?

32. PhotoTech, Inc., a manufacturer of rechargeable batteries for digital cameras, signed a contract with a digital photography company to produce three different lithium-ion battery packs for a new line of digital cameras. The contract calls for the following:

Battery Pack	Production Quantity
PT-100	200,000
PT-200	100,000
PT-300	150,000

PhotoTech can manufacture the battery packs at manufacturing plants located in the Philippines and Mexico. The unit cost of the battery packs differs at the two plants because of differences in production equipment and wage rates. The unit costs for each battery pack at each manufacturing plant are as follows:

	Plant	
Product	**Philippines**	**Mexico**
PT-100	$0.95	$0.98
PT-200	$0.98	$1.06
PT-300	$1.34	$1.15

The PT-100 and PT-200 battery packs are produced using similar production equipment available at both plants. However, each plant has a limited capacity for the total number of PT-100 and PT-200 battery packs produced. The combined PT-100 and PT-200 production capacities are 175,000 units at the Philippines plant and 160,000 units at the Mexico plant. The PT-300 production capacities are 75,000 units at the Philippines plant and 100,000 units at the Mexico plant. The cost of shipping from the Philippines plant is $0.18 per unit, and th[illegible]e Mexico plant is $0.10 per unit.

a. D[illegible] that PhotoTech can use to determine how many units of ea[illegible]ce at each plant in order to minimize the total production ar[illegible]d with the new contract.

b. S[illegible]eveloped in part (a) to determine the optimal production pl[illegible]

c. U[illegible]determine how much the production and/or shipping cost pe[illegible]ge in order to produce additional units of the PT-100 in the Pl[illegible]

d. Use sensitivity analysis to determine how much the production and/or shipping cost per unit would have to change in order to produce additional units of the PT-200 in the Mexico plant.

Case Problem 1 PRODUCT MIX

TJ's, Inc., makes three nut mixes for sale to grocery chains located in the Southeast. The three mixes, referred to as the Regular Mix, the Deluxe Mix, and the Holiday Mix, are made by mixing different percentages of five types of nuts.

In preparation for the fall season, TJ's has just purchased the following shipments of nuts at the prices shown:

Type of Nut	Shipment Amount (pounds)	Cost per Shipment ($)
Almond	6000	7500
Brazil	7500	7125
Filbert	7500	6750
Pecan	6000	7200
Walnut	7500	7875

The Regular Mix consists of 15% almonds, 25% Brazil nuts, 25% filberts, 10% pecans, and 25% walnuts. The Deluxe Mix consists of 20% of each type of nut, and the Holiday Mix consists of 25% almonds, 15% Brazil nuts, 15% filberts, 25% pecans, and 20% walnuts.

TJ's accountant analyzed the cost of packaging materials, sales price per pound, and so forth, and determined that the profit contribution per pound is $1.65 for the Regular Mix, $2.00 for the Deluxe Mix, and $2.25 for the Holiday Mix. These figures do not include the cost of specific types of nuts in the different mixes because that cost can vary greatly in the commodity markets.

Customer orders already received are summarized here:

Type of Mix	Orders (pounds)
Regular	10,000
Deluxe	3,000
Holiday	5,000

Because demand is running high, it is expected that TJ's will receive many more orders than can be satisfied.

TJ's is committed to using the available nuts to maximize profit over the fall season; nuts not used will be given to a local charity. Even if it is not profitable to do so, TJ's president indicated that the orders already received must be satisfied.

Managerial Report

Perform an analysis of TJ's product-mix problem, and prepare a report for TJ's president that summarizes your findings. Be sure to include information and analysis on the following:

1. The cost per pound of the nuts included in the Regular, Deluxe, and Holiday mixes
2. The optimal product mix and the total profit contribution
3. Recommendations regarding how the total profit contribution can be increased if additional quantities of nuts can be purchased
4. A recommendation as to whether TJ's should purchase an additional 1000 pounds of almonds for $1000 from a supplier who overbought
5. Recommendations on how profit contribution could be increased (if at all) if TJ's does not satisfy all existing orders

Case Problem 2 INVESTMENT STRATEGY

J. D. Williams, Inc., is an investment advisory firm that manages more than $120 million in funds for its numerous clients. The company uses an asset allocation model that recommends the portion of each client's portfolio to be invested in a growth stock fund, an income fund, and a money market fund. To maintain diversity in each client's portfolio, the firm places limits on the percentage of each portfolio that may be invested in each of the three funds. General guidelines indicate that the amount invested in the growth fund must be between 20% and 40% of the total portfolio value. Similar percentages for the other two funds stipulate that between 20% and 50% of the total portfolio value must be in the income fund and at least 30% of the total portfolio value must be in the money market fund.

In addition, the company attempts to assess the risk tolerance of each client and adjust the portfolio to meet the needs of the individual investor. For example, Williams just contracted with a new client who has $800,000 to invest. Based on an evaluation of the client's risk tolerance, Williams assigned a maximum risk index of 0.05 for the client. The firm's risk indicators show the risk of the growth fund at 0.10, the income fund at 0.07, and the money market fund at 0.01. An overall portfolio risk index is computed as a weighted average of the risk rating for the three funds where the weights are the fraction of the client's portfolio invested in each of the funds.

Additionally, Williams is currently forecasting annual yields of 18% for the growth fund, 12.5% for the income fund, and 7.5% for the money market fund. Based on the information provided, how should the new client be advised to allocate the $800,000 among the growth, income, and money market funds? Develop a linear programming model that will provide the maximum yield for the portfolio. Use your model to develop a managerial report.

Managerial Report

1. Recommend how much of the $800,000 should be invested in each of the three funds. What is the annual yield you anticipate for the investment recommendation?
2. Assume that the client's risk index could be increased to 0.055. How much would the yield increase and how would the investment recommendation change?

3. Refer again to the original situation where the client's risk index was assessed to be 0.05. How would your investment recommendation change if the annual yield for the growth fund were revised downward to 16% or even to 14%?
4. Assume that the client expressed some concern about having too much money in the growth fund. How would the original recommendation change if the amount invested in the growth fund is not allowed to exceed the amount invested in the in
5. T l you developed may be useful in modifying the portfoli ients whenever the anticipated yields for the three funds a What is your recommendation as to whether use of this m

Case Problem ING STRATEGY

Reep Construction recently won a contract for the excavation and site preparation of a new rest area on the Pennsylvania Turnpike. In preparing his bid for the job, Bob Reep, founder and president of Reep Construction, estimated that it would take four months to perform the work and that 10, 12, 14, and 8 trucks would be needed in months 1 through 4, respectively.

The firm currently has 20 trucks of the type needed to perform the work on the new project. These trucks were obtained last year when Bob signed a long-term lease with PennState Leasing. Although most of these trucks are currently being used on existing jobs, Bob estimates that one truck will be available for use on the new project in month 1, two trucks will be available in month 2, three trucks will be available in month 3, and one truck will be available in month 4. Thus, to complete the project, Bob will have to lease additional trucks.

The long-term leasing contract with PennState has a monthly cost of $600 per truck. Reep Construction pays its truck drivers $20 an hour, and daily fuel costs are approximately $100 per truck. All maintenance costs are paid by PennState Leasing. For planning purposes, Bob estimates that each truck used on the new project will be operating eight hours a day, five days a week for approximately four weeks each month.

Bob does not believe that current business conditions justify committing the firm to additional long-term leases. In discussing the short-term leasing possibilities with PennState Leasing, Bob learned that he can obtain short-term leases of 1–4 months. Short-term leases differ from long-term leases in that the short-term leasing plans include the cost of both a truck and a driver. Maintenance costs for short-term leases also are paid by PennState Leasing. The following costs for each of the four months cover the lease of a truck and driver:

Length of Lease	Cost per Month ($)
1	4000
2	3700
3	3225
4	3040

Bob Reep would like to acquire a lease that would minimize the cost of meeting the monthly trucking requirements for his new project, but he also takes great pride in the fact that his company has never laid off employees. Bob is committed to maintaining his no-layoff policy; that is, he will use his own drivers even if costs are higher.

Managerial Report

Perform an analysis of Reep Construction's leasing problem and prepare a report for Bob Reep that summarizes your findings. Be sure to include information on and analysis of the following items:

1. The optimal leasing plan
2. ... with the optimal leasing plan
3. ... nstruction to maintain its current policy of no layoffs

Appendix 3 ... ANALYSIS WITH EXCEL

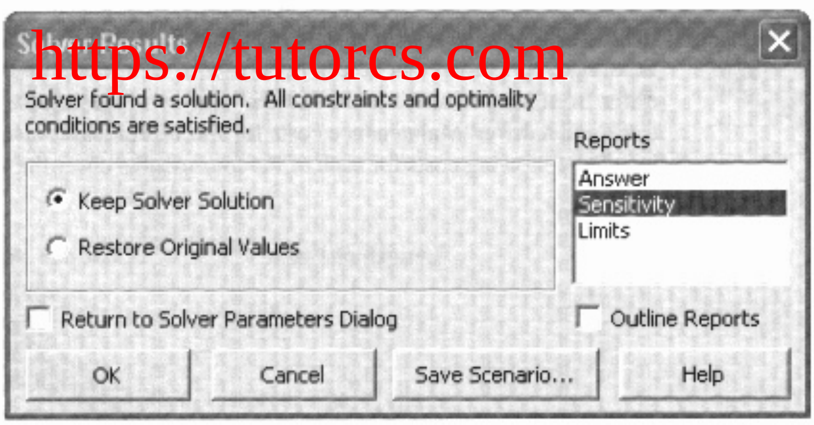
In A... how Excel Solver can be used to solve a linear program by using ... problem. Let us now see how it can be used to provide sensitivity ...

When Solver finds the optimal solution to a linear program, the **Solver Results** dialog box (see Figure 3.22) will appear on the screen. If only the solution is desired, you simply click **OK**. To obtain the optimal solution and the sensitivity analysis output, you must select **Sensitivity** in the **Reports** box before clicking **OK**; the sensitivity report is created on another worksheet in the same Excel workbook. Using this procedure for the Par problem, we obtained the optimal solution shown in Figure 3.23 and the sensitivity report shown in Figure 3.24.

Interpretation of Excel Sensitivity Report

In the Adjustable Cells section of the Sensitivity Report, the column labeled Final Value contains the optimal values of the decision variables. For the Par, Inc., problem the optimal solution is 540 standard bags and 252 deluxe bags. Next, let us consider the values in the Reduced Cost column.[1] For the Par, Inc., problem the reduced costs for both decision variables are zero; they are at their optimal values.

FIGURE 3.22 EXCEL SOLVER RESULTS DIALOG BOX



[1] In Excel, if the value of a variable in an optimal solution is equal to the upper bound of the variable, then reduced cost will be the dual value of this upper bound constraint.

FIGURE 3.23 EXCEL SOLUTION FOR THE PAR, INC., PROBLEM

WEB file
Par

	A	B	C	D
1	Par, Inc.			
2				
3		Production Time		
4	Ope	Standard	Deluxe	Time Available
5	Cutti	0.7	1	630
6	Sewi	0.5	0.83333	600
7	Finis	1	0.66667	708
8	Insp	0.1	0.25	135
9	Profi	10	9	
10				
11				
12	Model			
13				
14		Decision Variables		
15		Standard	Deluxe	
16	Bags Produced	539.99842	252.00110	
17				
18	Maximize Total Profit	7668		
19				
20	Constraints	Hours Used (LHS)		Hours Available (RHS)
21	Cutting and Dyeing	630	<=	630
22	Sewing	479.99929	<=	600
23	Finishing	708	<=	708
24	Inspection and Packaging	117.00012	<=	135

FIGURE 3.24 EXCEL SENSITIVITY REPORT FOR THE PAR, INC., PROBLEM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B16	Bags Produced Standard	539.99842	0.00000	10	3.4999325	3.7
C16	Bags Produced Deluxe	252.00110	0.00000	9	5.285714286	2.3333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B21	Cutting and Dyeing Hours Used (LHS)	630	4.37495664	630	52.36315884	134.4
B22	Sewing Hours Used (LHS)	479.99929	0.00000	600	1E+30	120.0007088
B23	Finishing Hours Used (LHS)	708	6.937530352	708	192	127.9986
B24	Inspection and Packaging Hours Used (LHS)	117.00012	0.00000	135	1E+30	17.99988187

To the right of the Reduced Cost column in Figure 3.24, we find three columns labeled Objective Coefficient, Allowable Increase, and Allowable Decrease. Note that for the standard bag decision variable, the objective function coefficient value is 10, the allowable increase is 3.5, and the allowable decrease is 3.7. Adding 3.5 to and subtracting 3.7 from the current coefficient of 10 provides the range of optimality for C_S.

$$6.3 \leq C_S \leq 13.5$$

Simi[illegible]ality for C_D is

$$6.67 \leq C_D \leq 14.29$$

N[illegible]nation in the Constraints section of the report. The entries in the F[illegible]e number of hours needed in each department to produce the optim[illegible] of 540 standard bags and 252 deluxe bags. Thus, at the optimal s[illegible]tting and dyeing time, 480 hours of sewing time, 708 hours of finishing time, and 117 hours of inspection and packaging time are required. The values in the Constraint R.H. Side column are just the original right-hand-side values: 630 hours of cutting and dyeing time, 600 hours of sewing time, 708 hours of finishing time, and 135 hours of inspection and packaging time. Note that for the Par, Inc., problem, the values of the slack variables for each constraint are simply the differences between the entries in the Constraint R.H. Side column and the corresponding entries in the Final Value column.

The sensitivity analysis interpretations provided in this appendix are based on the assumption that only one objective function coefficient or only one right-hand-side change occurs at a time.

The entries in the Shadow Price column provide the *shadow price* for each constraint. The shadow price is another, often-used term for the dual value. The last two columns of the Sensitivity Report contain the range of feasibility information for the constraint right-hand sides. For example, consider the cutting and dyeing constraint with an allowable increase value of 52.4 and an allowable decrease value of 134.4. The values in the Allowable Increase and Allowable Decrease columns indicate that the shadow price of \$4.375 is valid for increases up to 52.4 hours and decreases to 134.4 hours. Thus, the shadow price of \$4.375 is applicable for increases up to 630 + 52.4 = 682.4 and decreases down to 630 − 134.4 = 495.6 hours.

In summary, the range of feasibility information provides the limits where the shadow prices are applicable. For changes outside the range, the problem must be re-solved to find the new optimal solution and the new shadow price.

Appendix 3.2 SENSITIVITY ANALYSIS WITH LINGO

In Appendix 2.1 we showed how LINGO can be used to solve a linear program by using it to solve the Par, Inc., problem. A copy of the Solution Report is shown in Figure 3.25. As we discussed previously, the value of the objective function is 7668, the optimal solution is $S = 540$ and $D = 252$, and the values of the slack variables corresponding to the four constraints (rows 2–5) are 0, 120, 0, and 18. Now let us consider the information in the Reduced Cost column and the Dual Price column.

LINGO always takes the absolute value of the reduced cost.

For the Par, Inc., problem, the reduced costs for both decision variables are zero because both variables are at a positive value. LINGO reports a **dual price** rather than a dual value. For a maximization problem, the dual value and dual price are identical. For a minimization problem, the dual price is equal to the negative of the dual value. There are historical reasons for this oddity that are beyond the scope of the book. When interpreting the LINGO output for a minimization problem, multiply the dual prices by −1, treat the resulting number as the dual value, and interpret the number as described in Section 3.2. The nonzero dual prices of 4.374957 for constraint 1 (cutting and dyeing constraint in row 2)

FIGURE 3.25 PAR, INC., SOLUTION REPORT USING LINGO

```
Global optimal solution found.
  Objective value:                       7668.000
  Total solver iterations:                      2

      Variable           Value         Reduced Cost
      --------------------------      ------------------
             S          540.0000            0.000000
             D          252.0000            0.000000

           Row    Slack or Surplus         Dual Price
      --------------------------      ------------------
             1          7668.000            1.000000
             2          0.000000            4.375000
             3          120.0000            0.000000
             4          0.000000            6.937500
             5          18.00000            0.000000
```

and 6.937530 for constraint 3 (finishing constraint in row 4) tell us that an additional hour of cutting and dyeing time improves (increases) the value of the optimal solution by $4.37 and an additional hour of finishing time improves (increases) the value of the optimal solution by $6.94.

Next, let us consider how LINGO can be used to compute the range of optimality for each objective function coefficient and the range of feasibility for each of the dual prices. By default, range computations are not enabled in LINGO. To enable range computations, perform the following steps:

Step 1. Choose the **LINGO** menu
Step 2. Select **Options**
Step 3. When the LINGO Options dialog box appears:
  Select the **General Solver** tab
  Choose **Prices and Ranges** in the **Dual Computations** box
  Click **Apply**
  Click **OK**

You will now have to re-solve the Par, Inc., problem in order for LINGO to perform the range computations. After re-solving the problem, close or minimize the Solution Report window. To display the range information, select the Range command from the LINGO menu. LINGO displays the range information in a new window titled Range Report. The output that appears in the Range Report window for the Par, Inc., problem is shown in Figure 3.26.

We will use the information in the Objective Coefficient Ranges section of the range report to compute the range of optimality for the objective function coefficients. For example, the current objective function coefficient for S is 10. Note that the corresponding allowable increase is 3.5 and the corresponding allowable decrease is 3.700000. Thus, the range of optimality for C_S, the objective function coefficient for S, is $10 - 3.700000 = 6.300000$ to $10 + 3.5 = 13.5$. After rounding, the range of optimality for C_S is $6.30 \le C_S \le 13.50$.

FIGURE 3.26 PAR, INC., SENSITIVITY REPORT USING LINGO

Ranges in which the basis is unchanged:

OBJECTIVE COEFFICIENT RANGES

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
S	[illegible]	3.500000	3.700000
D	[illegible]	5.285714	2.333333

RIGHTHAND SIDE RANGES

Row	Current RHS	Allowable Increase	Allowable Decrease
2	630.0000	52.36364	134.4000
3	600.0000	INFINITY	120.0000
4	708.0000	192.0000	128.0000
5	135.0000	INFINITY	18.00000

Similarly, with an allowable increase of 5.285714 and an allowable decrease of 2.333300, the range of optimality for C_D is $6.67 \leq C_S \leq 14.29$.

To compute the range of feasibility for each dual price, we will use the information in the Right-Hand-Side Ranges section of the range report. For example, the current right-hand-side value for the cutting and dyeing constraint (row 2) is 630, the allowable increase is 52.36316, and the allowable decrease is 134.40000. Because the dual price for this constraint is 4.375 (shown in the LINGO solution report), we can conclude that additional hours will increase the objective function by \$4.37 per hour. From the range information given, we see that after rounding the dual price of \$4.37 is valid for increases up to 630 + 52.36 = 682.4 and decreases to 630 – 134.4 = 495.6. Thus, the range of feasibility for the cutting and dyeing constraint is 495.6 to 682.4. The ranges of feasibility for the other constraints can be determined in a similar manner.