Linear Programming and its Applications

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MBA 8419 - Decision Making Technology

Overview of the presentation

- What is a linea[illegible] (LP) ?
    - Problem $\Rightarrow$ [illegible]on model
    - General Form
- Applications and the use of EXCEL's Solver
    - Marketing
    - Finance
    - Operations management
- How are these models solved ?
    - Graphical solution
    - Sensitivity analysis

What is a linear program ?

Problem ⇒ optimization model

Here are some typic[illegible]ations :

1. A manufacturer wa[illegible] production schedule and an inventory policy that will satisfy sale[illegible]re periods. Ideally, the schedule and policy will enable the company to satisfy demand and at the same time *minimize* the total production and inventory costs.
2. A financial analyst must select an investment portfolio from a variety of stock and bond investment alternatives. The analyst would like to establish the portfolio that *maximizes* the return on investment.
3. A marketing manager wants to determine how best to allocate a fixed advertising budget among alternative advertising media such as radio, television, newspaper, and magazine. The manager would like to determine the media mix that *maximizes* advertising effectiveness.
4. A company has warehouses in a number of locations throughout the United States. For a set of customer demands, the company would like to determine how much each warehouse should ship to each customer so that total transportation costs are *minimized*.

FIGURE – Taken from Anderson et. al. (2012), Chap.2

What is a linear program ?

Problem ⇒ optimization model

General characteris

- A series of Dec be made
- Desire to max o ne quantity
  - **Objective** of the LP
- Presence of restrictions, or **Constraints**
  - limit the values the decisions can take
  - implicitly limit the degree to which the objective can be pursued
  - Examples :
    - Satisfying customer demands
    - Budgets
    - Limited supplies
    - Limited capacities (space, time, employees, etc.)
    - etc.

What is a linear program ?

Problem ⇒ optimization model

Definition: what decisions need to be made and how do they influence the state of the system under study.

Characteristics:

- Varying impacts on the system
- Can be grouped by category
- Can be made over multiple time periods

Definition: what is the objective pursued in solving the problem or in modifying the system (i.e., what criteria is used to evaluate the decisions).

Characteristics:

- Evaluate the quality of decisions

Definition: set of obligations and limits that need to be enforced and that define admissible/feasible decisions.

Characteristics:

- Technological:
  - Hard
  - Soft
- Non-negativity
- Integrity

FIGURE – Process to formulate a problem

What is a linear program ?

General form

Decision variables

$x_1, x_2, \ldots, x_n$

Objective Function

$$\max \ \textbf{or} \ \min z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \left(\begin{array}{c}\leq\\ \geq\\ =\end{array}\right) b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \left(\begin{array}{c}\leq\\ \geq\\ =\end{array}\right) b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \left(\begin{array}{c}\leq\\ \geq\\ =\end{array}\right) b_m$$

$x_1, x_2, \ldots, x_n \geq 0$ and x_j is integer, $\forall j \in E$ and given $E \subseteq \{1, 2, \ldots, n\}$

Applications and the use of EXCEL's Solver

Marketing

Types of problems

- Promotional pla[illegible]

  Description : Ch[illegible]en a set of available media options such as to maximize the promotional effort for a given set of products or services, targeted at specific segments of a given population

- Sales territory coverage

  Description : Assign a set of salespersons to a set of existing or potential customers such as to minimize costs, or, ensure that the workload (or value) among the salespersons are uniformly distributed

- Marketing research

  Description : To understand the composition and nature of a targeted market, establish the number and types of studies that need to be performed to obtain the desired information while minimizing the costs

Applications and the use of EXCEL's Solver

Marketing

Marketing research

Context : Market Survey Inc. (MSI), specializes in evaluation of consumer reaction to new products, services, and advertising campaigns. A client firm requested that MSI's assistances in ascertaining consumer reaction to a recently marketed household product.

Strategy : door-to-door personal interviews with families (i.e., households) that either have, or don't have, children.

Contract : MSI must conduct 1,000 interviews

Applications and the use of EXCEL's Solver

Marketing

Marketing research (cont'd)

Quota guidelines :

1. Interview at least 4[illegible]olds with children
2. Interview at least 4[illegible]olds without children
3. The total number of households interviewed during the evening must be at least as great as the number of households interviewed during the day
4. At least 40% of the interviews for households with children must be conducted during the evening
5. At least 60% of the interviews for households without children must be conducted during the evening

	Interview cost	
Household	Day	Evening
Children	20$	25$
No Children	18$	20$

TABLE – Unitary costs per interview type

Applications and the use of EXCEL's Solver

Marketing

Marketing research (cont'd)

Model :

Decision variables :

DC = the number of daytime interviews of households with children,

EC = the number of evening interviews of households with children,

DNC = the number of daytime interviews of households without children,

ENC = the number of evening interviews of households without children.

Objective Function : $\min 20DC + 25EC + 18DNC + 20ENC$

Subject to :

$DC + EC + DNC + ENC = 1\,000$

$DC + EC \geq 400$

$DNC + ENC \geq 400$

$EC + ENC \geq DC + DNC$

$EC \geq 0,4(DC + EC)$

$ENC \geq 0,6(DNC + ENC)$

$DC, EC, DNC, ENC \geq 0$ and integer.

Applications and the use of EXCEL's Solver

Marketing

Solving the proble[illegible] EXCEL

1. Build the sprea[illegible] Standard Form[illegible]
   - Each column is associated with a specific decision variable
   - Each line is associated to a linear function (i.e., **objective** and **constraints**)
2. Use of the *Solver* function
   - Define the variable cells
   - Define the objective cell and max or min
   - Add the different constraints
   - *Make Unconstrained Variables Non-Negative*
   - Select solving method
     - GRG Nonlinear ⇒ for nonlinear optimization models
     - **Simplex LP** ⇒ exact method for linear optimization models
     - Evolutionary ⇒ heuristic method for optimization models

Applications and the use of EXCEL's Solver

Marketing

Optimal solution to t[illegible]roblem

	Number of Interviews		
Household	Day	Evening	Total
Children	240	160	400
No Children	240	360	600
Total	480	520	1000

TABLE – Optimal Solution

Applications and the use of EXCEL's Solver

Finance

Types of problems

- Portfolio theory

  Description : Cor[illegible]set of available stocks and bonds, determine the amount of investment that a company (or particular) should make in each financial instrument with the objective of minimizing risk, while also maximizing the returns

- Valuation of financial instruments

  Description : In the context of trading within financial markets, determine what is the value of the assets that are being traded

- Financial planning

  Description : Figure out what funding decisions should be made to best raise the necessary capital from the financial markets to finance an organization's activities

Applications and the use of EXCEL's Solver

Finance

Designing Portfoli[illegible]ual Funds

Description : Portf[illegible]els are used to determine the % of the investment fu[illegible] should be made in each available assets.

Goal : provide the best balance between risk and return.

Context : Hauck Investment Services designs annuities and long term investment plans for investors with a variety of risk tolerances. Hauck would like to develop a portfolio model that can be used to determine an optimal portfolio involving a mix of six mutual funds. A variety of measures can be used to indicate risk, but for portfolios of financial assets all are related to variability in return.

Applications and the use of EXCEL's Solver

Finance

Designing Portfolio of Mutual Funds (cont'd)

Managers at Hauck Financial Services think that the returns of past years can be used to represent the possibilities (i.e., scenarios) for the next year. Therefore, the following information will be used as planning scenarios for the next 12 months :

	Annual Return (%)				
Mutual Fund	Year 1	Year 2	Year 3	Year 4	Year 5
Foreign Stock	10,06	13,12	13,47	45,42	-21,93
Intermediate-Term Bond	17,64	3,25	7,51	-1,33	7,36
Large-Cap Growth	32,41	18,71	33,28	41,46	-23,26
Large-Cap Value	32,36	20,61	12,93	7,06	-5,37
Small-Cap Growth	33,44	19,40	3,85	58,68	-9,02
Small-Cap Value	24,56	25,32	-6,70	5,43	17,31

TABLE – Mutual fund performance in 5 selected years

Applications and the use of EXCEL's Solver

Finance

Conservative Port[illegible]

Idea : Design a portfoli[illegible]ervative client that has a strong aversion to risk. Determine the p[illegible] the portfolio to invest in each of the six mutual funds so that the portfolio provides the best return possible with a minimum of risk.

Decision variables

FS = proportion of portfolio invested in the Foreign Stock mutual fund

IB = proportion of portfolio invested in the Intermediate-Term Bond fund

LG = proportion of portfolio invested in the Large-Cap Growth fund

LV = proportion of portfolio invested in the Large-Cap Value fund

SG = proportion of portfolio invested in the Small-Cap Growth fund

SV = proportion of portfolio invested in the Small-Cap Value fund

Applications and the use of EXCEL's Solver

Finance

Given the definition of the variables the following constraint needs to be imposed :

$$FS + IB + LG + LV + SG + SV = 1$$

The portfolio return over the next year will depend on which scenario will occur :

$$\begin{aligned}
R_1 &= 10,06FS + 17,64IB + 32,41LG + 32,36LV + 33,44SG + 24,56SV \\
R_2 &= 13,12FS + 3,25IB + 18,71LG + 20,61LV + 19,40SG + 25,32SV \\
R_3 &= 13,47FS + 7,51IB + 33,28LG + 12,93LV + 3,85SG - 6,70SV \\
R_4 &= 45,42FS + 1,33IB + 41,46LG + 7,06LV + 58,68SG + 5,43SV \\
R_5 &= -21,93FS + 7,36IB - 23,26LG - 5,37LV - 9,02SG + 17,31SV
\end{aligned}$$

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Finance

Decision variables

M = minimum return of the portfolio

To define M, we need to add the following minimum-return constraints

$$
\begin{array}{ll}
R_1 \geq M & \text{Scenario 1 minimum return} \\
R_2 \geq M & \text{Scenario 2 minimum return} \\
R_3 \geq M & \text{Scenario 3 minimum return} \\
R_4 \geq M & \text{Scenario 4 minimum return} \\
R_5 \geq M & \text{Scenario 5 minimum return}
\end{array}
$$

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Finance

Substituting the values previously define for R_1, R_2, and so on, provides the following minimum-return constraints :

$$
\begin{array}{rl}
10,06FS + 17,64IB + 32,41LG + 32,36LV + 33,44SG + 24,56SV \geq M & \text{Scenario 1} \\
13,12FS + 3,25IB + 18,71LG + 20,61LV + 19,40SG + 25,32SV \geq M & \text{Scenario 2} \\
13,47FS + 7,51IB + 33,28LG + 12,93LV + 3,85SG - 6,70SV \geq M & \text{Scenario 3} \\
45,42FS + 1,33IB + 41,46LG + 7,06LV + 58,68SG + 5,43SV \geq M & \text{Scenario 4} \\
-21,93FS + 7,36IB - 23,26LG - 5,37LV - 9,02SG + 17,31SV \geq M & \text{Scenario 5}
\end{array}
$$

Objective Function

Apply a *maximin* approach

$$\max M$$

Applications and the use of EXCEL's Solver

Finance

Model

$$\max \quad M$$

s.t.

$$10,06FS + 17,64IB + 32,41LG + 32,36LV + 33,44SG + 24,56SV \geq M$$

$$13,12FS + 3,25IB + 18,71LG + 20,61LV + 19,40SG + 25,32SV \geq M$$

$$13,47FS + 7,51IB + 33,28LG + 12,93LV + 3,85SG - 6,70SV \geq M$$

$$45,42FS + 1,33IB + 41,46LG + 7,06LV + 58,68SG + 5,43SV \geq M$$

$$-21,93FS + 7,36IB - 23,28LG - 5,37LV - 9,02SG + 17,31SV \geq M$$

$$FS + IB + LG + LV + SG + SV = 1$$

$$FS, IB, LG, LV, SG, SV \geq 0$$

Applications and the use of EXCEL's Solver

Finance

Moderate Risk Portfolio

Note : certain clients are willing to accept a moderate amount of risk in order to attempt to achieve better returns.

Assumption

Clients in this category are willing to accept some risks but do not want the annual return for the portfolio to drop below 2%

Therefore, $M = 2$

$$
\begin{aligned}
R_1 &\geq 2 && \text{Scenario 1 minimum return}\\
R_2 &\geq 2 && \text{Scenario 2 minimum return}\\
R_3 &\geq 2 && \text{Scenario 3 minimum return}\\
R_4 &\geq 2 && \text{Scenario 4 minimum return}\\
R_5 &\geq 2 && \text{Scenario 5 minimum return}
\end{aligned}
$$

Applications and the use of EXCEL's Solver

Finance

Objective Function

A different objective is used here ⇒ Maximize the expected return for the portfolio.

Assumption

Assuming that p_i, for $i = 1, \dots, 5$, are the probabilities of observing the scenarios i.

Then, $\overline{R} = p_1 R_1 + p_2 R_2 + p_3 R_3 + p_4 R_4 + p_5 R_5$, defines an estimator of the expected value of the return

Therefore

$$\max p_1 R_1 + p_2 R_2 + p_3 R_3 + p_4 R_4 + p_5 R_5$$

If all scenarios are equiprobable, then

$$\max \frac{1}{5} R_1 + \frac{1}{5} R_2 + \frac{1}{5} R_3 + \frac{1}{5} R_4 + \frac{1}{5} R_5$$

Applications and the use of EXCEL's Solver

Finance

Model

max $\ldots 2R_2 + 0.2R_3 + 0.2R_4 + 0.2R_5$

s.t.

$$10,06FS + 17,64IB + 32,41LG + 32,36LV + 33,44SG + 24,56SV = R_1,$$

$$13,12FS + 3,25IB + 18,71LG + 20,61LV + 19,40SG + 25,32SV = R_2,$$

$$13,47FS + 7,51IB + 33,28LG + 12,93LV + 8,85SG - 6,70SV = R_3,$$

$$45,42FS + 1,33IB + 41,46LG + 7,06LV + 58,68SG + 5,43SV = R_4,$$

$$-21,93FS + 7,36IB - 23,26LG - 5,37LV - 9,02SG + 17,31SV = R_5,$$

$$R_1 \geq 2, R_2 \geq 2, R_3 \geq 2, R_4 \geq 2, R_5 \geq 2,$$

$$FS + IB + LG + LV + SG + SV = 1,$$

$$FS, IB, LG, LV, SG, SV \geq 0$$

Applications and the use of EXCEL's Solver

Operations management

Types of problems

- Distribution Management

  Description : Plan[illegible]executing the various distribution operations of a company to serve its clients in a timely manner, while minimizing the costs.

- Production Planning

  Description : Planning and scheduling the production operations of a company, which may include the procurement processes, the management of inventory, establishing the production levels through time, assigning ressources, etc., while minimizing the overall costs.

- Logistics Network Design

  Description : Design, manage and coordinate a logistics network such as to perform the necessary operations while minimizing costs.

Applications and the use of EXCEL's Solver

Operations management

Production Scheduling

Description : Bollinger Electronics Company (BEC) produces two different electronic components for a major airplane engine manufacturer. The airplane engine manufacturer notifies the Bollinger sales office each quarter of its monthly requirements for components for each of the next three months. The requirements may vary considerably, depending on the type of engine the manufacturer is producing.

Component	April	May	June
322A	1 000	3 000	5 000
802B	1 000	500	3 000

TABLE – Three-month demand schedule for BEC

Production Scheduling (cont'd)

Once the order is processed, a demand statement is sent to the production control department. The production control department then develops a three-month production plan for the components. The production manager will want to identify the following :

1. Total production cost
2. Inventory holding cost
3. Change-in-production-level costs

Costs :

- 322A costs 20$ per unit produced while 802B costs 10$ per unit produced
- Inventory holding costs are 1.5% of the cost of the product (monthly)
- Cost associated with $\uparrow$ the production level for any month is 0.50$ per unit increase
- Cost associated with $\downarrow$ the production level for any month is 0.20$ per unit decrease

Production Scheduling (cont'd)

Month	Machine C[illegible] (hou[illegible]	Labor Capacity (hours)	Storage Capacity (square feet)
April	400	300	10 000
May	500	300	10 000
June	600	300	10 000

TABLE – Machine, Labor and Storage Capacities for BEC

Component	Machine (hours/unit)	Labor (hours/unit)	Storage (square feet/unit)
322A	0.10	0.05	2
802B	0.08	0.07	3

TABLE – Machine, Labor and Storage requirements for components 322A and 802B

Applications and the use of EXCEL's Solver

Operations management

Decision Variables

- x_{im} = production level in units for product i in month m
- s_{im} = inventory level for product i at the end of month m
- I_m = increase in the total production level necessary during month m
- D_m = decrease in the total production level necessary during month m

Where,

$i = 1 \Rightarrow$ 322A and $i = 2 \Rightarrow$ 802B

$m = 1 \Rightarrow$ April, $m = 2 \Rightarrow$ May and $m = 3 \Rightarrow$ June

Applications and the use of EXCEL's Solver

Operations management

Objective function

min Total Cost = Total production cost + Inventory holding cost + Change-in-production-level costs

$$\text{Total production cost} = 20x_{11} + 20x_{12} + 20x_{13} + 10x_{21} + 10x_{22} + 10x_{23}$$

$$\text{Inventory holding cost} = (0.015 \times 20)(s_{11}+s_{12}+s_{13}) + (0.015 \times 10)(s_{21}+s_{22}+s_{23})$$

$$\text{Change-in-production-level costs} = 0.50(I_1+I_2+I_3) + 0.20(D_1+D_2+D_3)$$

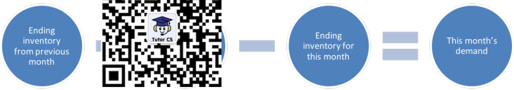
Therefore,

$$\min\ 20x_{11} + 20x_{12} + 20x_{13} + 10x_{21} + 10x_{22} + 10x_{23} + 0.30s_{11} + 0.30s_{12} + 0.30s_{13} + 0.15s_{21} + 0.15s_{22} + 0.15s_{23} + 0.50I_1 + 0.50I_2 + 0.50I_3 + 0.20D_1 + 0.20D_2 + 0.20D_3$$

Applications and the use of EXCEL's Solver

Operations management

Subject to :



Inventories at the beginning of the three-month scheduling period were 500 units for component 322A and 200 units for component 802B. The company would like to have 400 and 200 units, respectively for each product, in the inventory at the end of the planning.

$$\text{Month 1}: \begin{aligned} 500 + x_{11} - s_{11} &= 1000 \\ 200 + x_{21} - s_{21} &= 1000 \end{aligned}$$

$$\text{Month 2}: \begin{aligned} s_{11} + x_{12} - s_{12} &= 3000 \\ s_{21} + x_{22} - s_{22} &= 500 \end{aligned}$$

$$\text{Month 3}: \begin{aligned} s_{12} + x_{13} - s_{13} &= 5000 \\ s_{22} + x_{23} - s_{23} &= 3000 \end{aligned}$$

Ending inventory :

$$s_{13} \geq 400$$
$$s_{23} \geq 200$$

Applications and the use of EXCEL's Solver

Operations management

Subject to (cont'd)

Capacities :

Machines :

$$\text{Month 1} : 0.10x_{11} + 0.08x_{21} \leq 400$$
$$\text{Month 2} : 0.10x_{12} + 0.08x_{22} \leq 500$$
$$\text{Month 3} : 0.10x_{13} + 0.08x_{23} \leq 600$$

Labor :

$$\text{Month 1} : 0.05x_{11} + 0.07x_{21} \leq 300$$
$$\text{Month 2} : 0.05x_{12} + 0.07x_{22} \leq 300$$
$$\text{Month 3} : 0.05x_{13} + 0.07x_{23} \leq 300$$

Storage :

$$\text{Month 1} : 2s_{11} + 3s_{21} \leq 10\,000$$
$$\text{Month 2} : 2s_{12} + 3s_{22} \leq 10\,000$$
$$\text{Month 3} : 2s_{13} + 3s_{23} \leq 10\,000$$

Applications and the use of EXCEL's Solver

Operations management

Subject to (cont'd)

Δ production volumes :

Production levels during [illegible] of March were 1 500 units of 322A and 1 000 units of 802B

Month 1 : $(x_{11} + x_{21}) - 2\,500 = I_1 - D_1$

Note :

There are three possible cases :

- If $(x_{11} + x_{21}) - 2\,500 > 0$ then $I_1 > 0$ and $D_1 = 0$
- If $(x_{11} + x_{21}) - 2\,500 < 0$ then $I_1 = 0$ and $D_1 > 0$
- If $(x_{11} + x_{21}) - 2\,500 = 0$ then $I_1 = 0$ and $D_1 = 0$

Month 2 : $(x_{12} + x_{22}) - (x_{11} + x_{21}) = I_2 - D_2$

Month 3 : $(x_{13} + x_{23}) - (x_{12} + x_{22}) = I_3 - D_3$

Non-negativity and integrality :

$x_{im}, s_{im}, I_m, D_m \geq 0$ and integer, $\forall i, m$.

How are these models solved ?

Graphical solution

Consider the optimization problem (P) :

$\max z = 1000x_1 + 1$

subject to

(C1) $10x_1 + 5x_2 \leq 200$

(C2) $2x_1 + 3x_2 \leq 60$

(C3) $x_1 \leq 34$

(C4) $x_2 \leq 14$

(C5) $x_1, x_2 \geq 0$

Dimensions of the model :

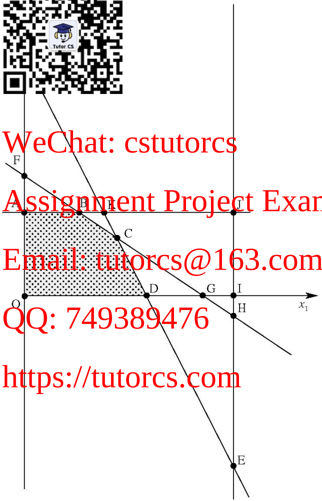
- 2 decision variables
- 4 technological constraints
- 2 non-negativity constraints

The feasible region of the model can be graphically represented

How are these models solved?

Graphical solution

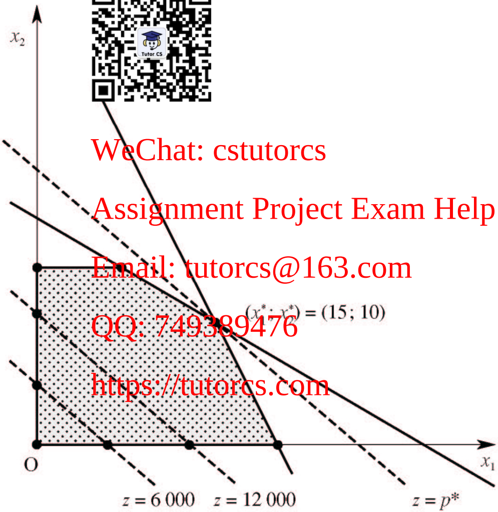
Graphical representation of the feasible region



How are these models solved ?

Graphical solution

Solving the problem graphically



How are these models solved ?

Graphical solution

Extreme points and optimal solution

Categories of feasible points for the previous example :

- Interior : none of the constraints is satisfied as an equation
- Border : at least one constraint is satisfied as an equation
- Extreme : located at the intersection of two constraints (two constraints are satisfied as equations)

Theorem

If an optimal solution to a specific linear programming model exists, then such a solution can be found at an extreme point of the feasible region.

Note : This result means that if you are looking for the optimal solution to a linear program, one does not need to evaluate all feasible solution points.

Only the feasible solutions that occur at the extreme points of the feasible region need to be evaluated.

How are these models solved ?

Graphical solution

Evaluation of z at the extreme points

Points	Coordinates	Values of z
O	$(x_1 = 0; x_2 = 0)$	$z = 0$
A	$(x_1 = 0; x_2 = 14)$	$z = 16\,800$
B	$(x_1 = 9; x_2 = 14)$	$z = 25\,800$
C	$(x_1 = 15; x_2 = 10)$	$z = 27\,000^{\star}$
D	$(x_1 = 20; x_2 = 0)$	$z = 20\,000$

How are these models solved ?

Sensitivity analysis

Questions :

To what extent is the optimal solution obtained for a linear programming model sensitive to modifications to the values of the parameters of the model ?

When the value of the coefficient of a decision variable in the objective function changes does that necessarily entail a change in the optimal solution obtained ?

What is the impact of a change in the right-hand side value of a constraint with respect to the optimal solution found ?

Types of analyses

We will consider two types of modifications in the model :

- Modification of the value of a c_j (a coefficient in the objective function)
- Modification of the value of a b_i (the right-hand side of a constraint)

How are these models solved ?

Sensitivity analysis

Modifying a c_j

Le us consider the following nction :

$\max z = c_1 x_1 + c_2 x_2$

In the standard form, the previous function can be expressed as : $x_2 = \frac{-c_1}{c_2} x_1 + \frac{z}{c_2}$

Therefore :

- When $c_1 \uparrow$ or $c_2 \downarrow \Rightarrow$ the value of the slope of the objective function $\downarrow$
- When $c_1 \downarrow$ or $c_2 \uparrow \Rightarrow$ the value of the slope of the objective function $\uparrow$

Observation : When the modification brings the objective function to cross the feasible region of the model then the optimal solution changes.

Conclusion :

As long as :

$$\text{slope of (C1)} \leq \frac{-c_1}{c_2} \leq \text{slope of (C2)}$$

The optimal solution remains the same

How are these models solved ?

Sensitivity analysis

Modifying a b_i

The optimal solution, extre $(x_1 = 15, x_2 = 10)$, is at the intersection of constraints (C1) and (C2). T aints are said to be active at the optimum

Definitions :

- A constraint is **active** at a given solution if the solution satisfies the constraint as an **equation**
- A constraint is **inactive** at a given solution if the solution satisfies the constraint as an **inequality**

Therefore

- If $b_1 \uparrow 1$ (i.e., $200 \rightarrow 201$) and b_2 remains the same

  Impact :

  Optimal solution becomes $C' = (x_1 = 15.15, x_2 = 9.9)$

  Value of the solution : $z' = 27\,030$ (i.e., $\Delta = +30$)
- If $b_2 \uparrow 1$ (i.e., $60 \rightarrow 61$) and b_1 remains the same

  Impact :

  Optimal solution becomes $C'' = (x_1 = 14.75, x_2 = 10.5)$

  Value of the solution : $z'' = 27\,350$ (i.e., $\Delta = +350$)

How are these models solved ?

Sensitivity analysis

EXCEL Model

Parameters	**X1**	**X2**				**Model**					
Values cj	**1000**	**1200**				**Variables**	**X1**	**X2**			
						Values	**15**	**10**			
	b1	**b2**	**b3**	**b4**					**R.H.S**		**L.H.S**
Values bj	**200**	**60**	**34**	**14**		**(C1)**	10	5	200	<=	200
						(C2)	2	3	60	<=	60
						(C3)	1	0	15	<=	34
						(C4)	0	1	10	<=	14
									Val. Z		
						cj	1000	1200	**27000**		

How are these models solved ?

Sensitivity analysis

Answer Report

Summarizes the values ctive function, the decision variables, and the LHS of constraints.

Objective Cel

Cell	Name	Original Value	Final Value
$J$11	cj Val. Z	27000	27000

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$H$3	Values X1	15	15	Contin
$I$3	Values X2	10	10	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$J$5	(C1) R.H.S	200	$J$5<=$L$5	Binding	0
$J$6	(C2) R.H.S	60	$J$6<=$L$6	Binding	0
$J$7	(C3) R.H.S	15	$J$7<=$L$7	Not Binding	19
$J$8	(C4) R.H.S	10	$J$8<=$L$8	Not Binding	4

How are these models solved ?

Sensitivity analysis

Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$H$3	Values X1	15	0	1000	1400	200
$I$3	Values X2	10	0	1200	300	700

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$J$5	(C1) R.H.S	200	30	200	100	40
$J$6	(C2) R.H.S	60	350	60	8	20
$J$7	(C3) R.H.S	15	0	34	1E+30	19
$J$8	(C4) R.H.S	10	0	14	1E+30	4

Variables : if reduced cost is 0, the current objective coefficient can be increased or decreased by the indicated “allowable” values without changing the current optimal solution.

Constraints : the RHS of the constraint can be increased or decreased by up to the indicated “allowable” amount, and for any unit increase/decrease the objective function will increase/decrease by the amount indicated as shadow price.