8. a. Let $x_i = \begin{cases} 1 \text{ if investment alternative } i \text{ is selected} \\ 0 \text{ otherwise} \end{cases}$



$$\max \quad 4000x_1 + 6000x_2 + 10500x_3 + 4000x_4 + 8000x_5 + 3000x_6$$

s.t.

$$\begin{aligned} + \; 6000x_3 + 2000x_4 + 5000x_5 + 1000x_6 &\le 10{,}500 \\ + \; 4000x_3 + 1500x_4 + 1000x_5 + 500x_6 &\le 7{,}000 \\ + \; 5000x_3 + 1800x_4 + 4000x_5 + 900x_6 &\le 8{,}750 \end{aligned}$$

$$x_2, x_3, x_4, x_5, x_6 = 0, 1$$

The optimal solution is

$x_3 = 1$
$x_4 = 1$
$x_6 = 1$

Value = 17,500



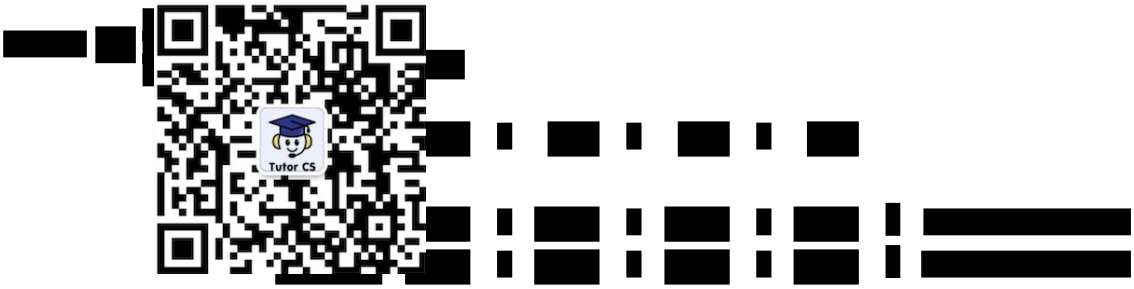
b. The following mutually exclusive constraint must be added to the model.

$x_1 + x_2 \le 1$ No change in optimal solution.



c. The following co-requisite constraint must be added to the model in b.

$x_3 - x_4 = 0$ No change in optimal solution.





15. a. Let $x_i = \begin{cases} 1 \text{ if a principal place of business in in county } i \\ 0 \text{ otherwise} \end{cases}$

$y_i = \begin{cases} 1 \text{ if county } i \text{ is not served} \\ 0 \text{ if county } i \text{ is served} \end{cases}$



The objective function for an integer programming model calls for minimizing the population not served.



$$\min \quad 195y_1 + 96y_2 + \bullet\bullet\bullet + 175\,y_{13}$$

There are 13 constraints needed; each is written so that $y_i$ will be forced to equal one whenever it is not possible to do business in county $i$.



$$\begin{array}{lrcrcrcrcrcrcrcl}
\text{Constraint 1:} & x_1 & + & x_2 & + & x_3 & & & & & & & + & y_1 & \geq & 1 \\
\text{Constraint 2:} & x_1 & + & x_2 & + & x_3 & + & x_4 & + & x_6 & + & x_7 & + & y_2 & \geq & 1 \\
\bullet & & & & & & & & & \bullet & & & & \bullet & & \\
\bullet & & & & & & & & & \bullet & & & & \bullet & & \\
\bullet & & & & & & & & & \bullet & & & & \bullet & &
\end{array}$$



$$\text{Constraint 13:} \quad x_{11} + x_{12} + x_{13} + y_{13} \geq 1$$

One more constraint must be added to reflect the requirement that only one principal place of business may be established.

$$x_1 + x_2 + \bullet\bullet\bullet + x_{13} = 1$$

The optimal solution has a principal place of business in County 11 with an optimal value of 739,000. A population of 739,000 cannot be served by this solution. Counties 1-5 and 10 will not be served.

b. The only change necessary in the integer programming model for part a is that the right-hand side of the last constraint is increased from 1 to 2.

$x_1 + x_2 + \bullet\bullet\bullet + x_{12} = 2.$



The optimal solution has principal places of business in counties 3 and 11 with an optimal value of 76,000. Only County 10 with a population of 76,000 is not served.

c. It is not t[illegible]e principal place of business can be established; 1,058,000 customers [illegible]erved. However, 642,000 can be served and if there is no opportuni[illegible]ace of business in County 11, this may be a good start. Perhaps later there[illegible] County 11.