CHAPTER 7

Integer Linear Programming

CONTENTS

In this chapter we discuss a class of problems that are modeled as linear programs with the additional requirement that one or more variables must be integer. Such problems are called **integer linear programs**. If all variables must be integer, we have an all-integer linear program. If some, but not all, variables must be integer, we have a mixed-integer linear program. In many applications of integer linear programming, one or more integer variables are required to equal either 0 or 1. Such variables are called 0-1 or *binary variables*. If al[illegible]les, we have a 0-1 integer linear program.

[illegible]ially 0-1 variables—provide substantial modeling flexibility. As [illegible] applications that can be addressed with linear programming met[illegible] For instance, the Management Science in Action, Crew Sch[illegible]land, describes how that airline company employs 0-1 integer pro[illegible]chedule its pilots and flight attendants. Later Management Sci[illegible]how Valley Metal Containers uses a mixed-integer program for scheduling aluminum can production for Coors beer, and how the modeling flexibility provided by 0-1 variables helped Ketron build a customer order allocation model for a sporting-goods company. Many other applications of integer programming are described throughout the chapter.

The objective of this chapter is to provide an applications-oriented introduction to integer linear programming. First, we discuss the different types of integer linear programming models. Then we show the formulation, graphical solution, and computer solution of an all-integer linear program. In Section 7.3 we discuss five applications of integer linear programming that make use of 0-1 variables: capital budgeting, fixed cost, distribution system design, bank location, and market share optimization problems. In Section 7.4 we provide additional illustrations of the modeling flexibility provided by 0-1 variables. Chapter appendices illustrate the use of Excel and LINGO for solving integer programs.

The cost of the added modeling flexibility provided by integer programming is that problems involving integer variables are often much more difficult to solve. A linear programming problem with several thousand continuous variables can be solved with any of several commercial linear programming solvers. However, an all-integer linear programming problem with fewer than 100 variables can be extremely difficult to solve. Experienced management scientists can help identify the types of integer linear programs that are easy, or at least reasonable, to solve. Commercial computer software packages, such as LINGO, CPLEX, Xpress-MP, and the commercial version of Solver have extensive integer programming capability, and very robust open-source software packages for integer programming are also available.

Information about open-source software can be found at the COIN-OR foundation website.

MANAGEMENT SCIENCE IN ACTION

CREW SCHEDULING AT AIR NEW ZEALAND*

As noted in Chapter 1, airlines make extensive use of management science (see Management Science in Action, Revenue Management at American Airlines). Air New Zealand is the largest national and international airline based in New Zealand. Over the past 15 years, Air New Zealand developed integer programming models for crew scheduling.

Air New Zealand finalizes flight schedules at least 12 weeks in advance of when the flights are to take place. At that point the process of assigning crews to implement the flight schedule begins. The crew-scheduling problem involves staffing the flight schedule with pilots and flight attendants. It is solved in two phases. In the first phase, tours of duty (ToD) are generated that will permit constructing sequences of flights for pilots and flight attendants that will allow the airline's flight schedule to be implemented. A tour of duty is a one-day or multiday alternating sequence of duty periods (flight legs, training, etc.) and rest periods (layovers). In the ToD problem, no consideration is given to which individual crew members will perform the tours of duty. In the second phase, individual crew members are assigned to the tours of duty, which is called the rostering problem.

Air New Zealand employs integer programming models to solve both the ToD problem and the rostering problem. In the integer programming model of the T[illegible]le is a 0-1 variable that c[illegible]ur of duty that could be f[illegible] (e.g., pilot or flight attend[illegible]esponds to a particular fli[illegible]light is included in exact[illegible]ost of variable j reflects [illegible]th tour of duty, and the [illegible] total cost. Air New Zeal[illegible]D problem for each crew type (pilot type or flight attendant type).

In the rostering problem, the tours of duty from the solution to the ToD problem are used to construct lines of work (LoW) for each crew member. In the integer programming model of the rostering problem, a 0-1 variable represents the possible LoWs for each crew member. A separate constraint for each crew member guarantees that each will be assigned a single LoW. Other constraints correspond to the ToDs that must be covered by any feasible solution to the rostering problem.

The crew-scheduling optimizers developed by Air New Zealand showed a significant impact on profitability. Over the 15 years it took to develop these systems, the estimated development costs were approximately NZ\$2 million. The estimated savings are NZ\$15.6 million per year. In 1999 the savings from employing these integer programming models represented 11% of Air New Zealand's net operating profit. In addition to the direct dollar savings, the optimization systems provided many intangible benefits such as higher-quality solutions in less time, less dependence on a small number of highly skilled schedulers, flexibility to accommodate small changes in the schedule, and a guarantee that the airline satisfies legislative and contractual rules.

*Based on E. Rod Butchers et al., "Optimized Crew Scheduling at Air New Zealand," *Interfaces* (January/February 2001): 30–56.

NOTES AND COMMENTS

1. Because integer linear programs are harder to solve than linear programs, one should not try to solve a problem as an integer program if simply rounding the linear programming solution is adequate. In many linear programming problems, such as those in previous chapters, rounding has little economic consequence on the objective function, and feasibility is not an issue. But, in problems such as determining how many jet engines to manufacture, the consequences of rounding can be substantial and integer programming methodology should be employed.
2. Some linear programming problems have a special structure, which guarantees that the variables will have integer values. The assignment, transportation, and transshipment problems of Chapter 6 have such structures. If the supply and the demand for transportation and transshipment problems are integer, the optimal linear programming solution will provide integer amounts shipped. For the assignment problem, the optimal linear programming solution will consist of 0s and 1s. So, for these specially structured problems, linear programming methodology can be used to find optimal integer solutions. Integer linear programming algorithms are not necessary.

7.1 TYPES OF INTEGER LINEAR PROGRAMMING MODELS

The only difference between the problems studied in this chapter and the ones studied in earlier chapters on linear programming is that one or more variables are required to be integer. If all variables are required to be integer, we have an **all-integer linear program.** The following is a two-variable, all-integer linear programming model:

$$
\begin{aligned}
\text{Max} \quad & 2x_1 + 3x_2 \\
\text{s.t.} \quad & \\
& 3x_1 + 3x_2 \leq 12 \\
& {}^2\!/_3 x_1 + 1x_2 \leq 4 \\
& 1x_1 + 2x_2 \leq 6 \\
& x_1, x_2 \geq 0 \text{ and integer}
\end{aligned}
$$

If [illegible] integer" from the last line of this model, we have the familiar tw[illegible]m. The linear program that results from dropping the integer requi[illegible] **Relaxation** of the integer linear program.

If [illegible]ly all, variables are required to be integer, we have a **mixed-integ**[illegible] following is a two-variable, mixed-integer linear program:

$$
\begin{aligned}
\text{Max} \quad & 3x_1 + 4x_2 \\
\text{s.t.} \quad & \\
& -1x_1 + 2x_2 \leq 8 \\
& 1x_1 + 2x_2 \leq 12 \\
& 2x_1 + 1x_2 \leq 16 \\
& x_1, x_2 \geq 0 \textit{ and } x_2 \text{ integer}
\end{aligned}
$$

We obtain the LP Relaxation of this mixed-integer linear program by dropping the requirement that x_2 be integer.

In some applications, the integer variables may only take on the values 0 or 1. Then we have a **0-1 linear integer program**. As we see later in the chapter, 0-1 variables provide additional modeling capability. The Management Science in Action, Aluminum Can Production at Valley Metal Container, describes how a mixed-integer linear program involving 0-1 integer variables is used to schedule production of aluminum beer cans for Coors breweries. The 0-1 variables are used to model production line changeovers; the continuous variables model production quantities.

MANAGEMENT SCIENCE IN ACTION

ALUMINUM CAN PRODUCTION AT VALLEY METAL CONTAINER*

Valley Metal Container (VMC) produces cans for the seven brands of beer produced by the Coors breweries: Coors Extra Gold, Coors Light, Coors Original, Keystone Ale, Keystone Ice, Keystone Light, and Keystone Premium. VMC produces these cans on six production lines and stores them in three separate inventory storage areas from which they are shipped on to the Coors breweries in Golden, Colorado; Memphis, Tennessee; and Shenandoah, Virginia.

Two important issues face production scheduling at the VMC facility. First, each time a production line must be changed over from producing one type of can to another (label change), it takes time to get the color just right for the new label. As a result, downtime is incurred and scrap is generated. Second, proper scheduling can reduce the amount of inventory that must be transferred from long-term to short-term storage. Thus, two costs are critical in determining the best production schedule at the VMC facility: the label-change cost and the cost of transferring inventory from one type of storage to another. To determine a production schedule that will minimize these two costs, VMC developed a mixed-integer linear programming model of its production process.

The model's objective function calls for minimizing the sum of the weekly cost of changing labels and the cost of transferring inventory from long-term to short-term storage. Binary (0-1) variables are used to represent a label change in the production process. Continuous variables are used to represent the size of the production run for each type of label on each line during each shift; analogous variables are used to represent inventories for each type of can produced. Additional continuous variables are used to represent the amount of inventory transferred to short-term storage during the week.

The VMC ... problem is solved weekly ... computer. Excel worksheets are ... ration and for storing the ... MS mathematical programming system is used to solve the mixed-integer linear program. Susan Schultz, manager of Logistics for Coors Container Operations, reports that using the system resulted in documented annual savings of $169,230.

*Based on Elena Katok and Dennis Ott, "Using Mixed-Integer Programming to Reduce Label Changes in the Coors Aluminum Can Plant," *Interfaces* (March/April 2000): 1–12.

7.2 GRAPHICAL AND COMPUTER SOLUTIONS FOR AN ALL-INTEGER LINEAR PROGRAM

Eastborne Realty has $2 million available for the purchase of new rental property. After an initial screening, Eastborne reduced the investment alternatives to townhouses and apartment buildings. Each townhouse can be purchased for $282,000, and five are available. Each apartment building can be purchased for $400,000, and the developer will construct as many buildings as Eastborne wants to purchase.

Eastborne's property manager can devote up to 140 hours per month to these new properties; each townhouse is expected to require 4 hours per month, and each apartment building is expected to require 40 hours per month. The annual cash flow, after deducting mortgage payments and operating expenses, is estimated to be $10,000 per townhouse and $15,000 per apartment building. Eastborne's owner would like to determine the number of townhouses and the number of apartment buildings to purchase to maximize annual cash flow.

We begin by defining the decision variables as follows:

$$T = \text{number of townhouses}$$
$$A = \text{number of apartment buildings}$$

The objective function for cash flow ($1000s) is

$$\text{Max} \quad 10T + 15A$$

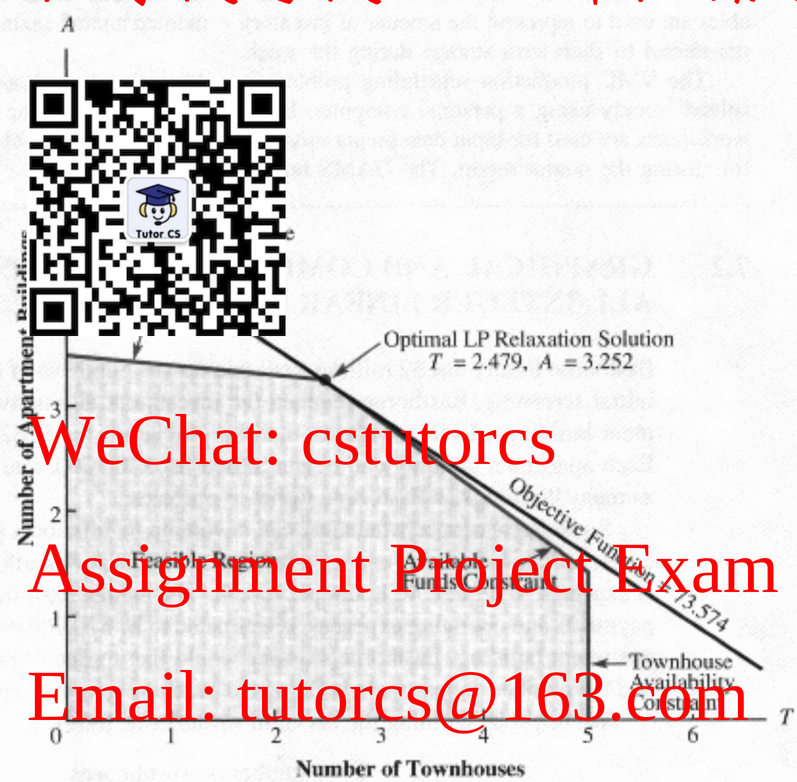
Three constraints must be satisfied:

$$\begin{aligned} 282T + 400A &\leq 2000 && \text{Funds available (\$1000s)} \\ 4T + 40A &\leq 140 && \text{Manager's time (hours)} \\ T &\leq 5 && \text{Townhouses available} \end{aligned}$$

The variables T and A must be nonnegative. In addition, the purchase of a fractional number of townhouses and/or a fractional number of apartment buildings is unacceptable. Thus, T and A must be integer. The model for the Eastborne Realty problem is the following all-integer linear program:

$$\begin{aligned} \text{Max} \quad & 10T + 15A \\ \text{s.t.} \quad & \\ & 282T + 400A \leq 2000 \\ & 4T + 40A \leq 140 \\ & T \leq 5 \\ & T, A \geq 0 \text{ and integer} \end{aligned}$$

FIGURE 7.1 GRAPHICAL SOLUTION TO THE LP RELAXATION OF THE EASTBORNE REALTY PROBLEM



Graphical Solution of the LP Relaxation

Suppose that we drop the integer requirements for T and A and solve the LP Relaxation of the Eastborne Realty problem. Using the graphical solution procedure, as presented in Chapter 2, the optimal linear programming solution is shown in Figure 7.1. It is $T = 2.479$ townhouses and $A = 3.252$ apartment buildings. The optimal value of the objective function is 73.574, which indicates an annual cash flow of $73,574. Unfortunately, Eastborne cannot purchase fractional numbers of townhouses and apartment buildings; further analysis is necessary.

Rounding to Obtain an Integer Solution

In many cases, a noninteger solution can be rounded to obtain an acceptable integer solution. For instance, a linear programming solution to a production scheduling problem might call for the production of 15,132.4 cases of breakfast cereal. The rounded integer solution of 15,132 cases would probably have minimal impact on the value of the objective function and the feasibility of the solution. Rounding would be a sensible approach. Indeed, whenever rounding has a minimal impact on the objective function and constraints, most managers find it acceptable. A near-optimal solution is fine.

However, rounding may not always be a good strategy. When the decision variables take on small values that have a major impact on the value of the objective function or feasibility, an optimal integer solution is needed. Let us return to the Eastborne Realty problem and examine the impact of rounding. The optimal solution to the LP Relaxation for Eastborne Realty resulted in $T = 2.479$ townhouses and $A = 3.252$ apartment buildings. Because each townhouse costs \$282,000 and each apartment building costs \$400,000, rou[illegible]ion can be expected to have a significant economic impact on the [illegible]

If a problem has only less-than-or-equal-to constraints with nonnegative coefficients for the variables, rounding down will always provide a feasible integer solution.

[illegible]he solution to the LP Relaxation to obtain the integer solution $T =$ [illegible]jective function value of $10(2) + 15(3) = 65$. The annual cash flow [illegible]ally less than the annual cash flow of \$73,574 provided by the solu[illegible]ion. Do other rounding possibilities exist? Exploring other rou[illegible] that the integer solution $T = 3$ and $A = 3$ is infeasible because it re[illegible] the \$2,000,000 Eastborne has available. The rounded solution of $T = 2$ and $A = 4$ is also infeasible for the same reason. At this point, rounding has led to two townhouses and three apartment buildings with an annual cash flow of \$65,000 as the best feasible integer solution to the problem. Unfortunately, we don't know whether this solution is the best integer solution to the problem.

Rounding to an integer solution is a trial-and-error approach. Each rounded solution must be evaluated for feasibility as well as for its impact on the value of the objective function. Even in cases where a rounded solution is feasible, we do not have a guarantee that we have found the optimal integer solution. We will see shortly that the rounded solution ($T = 2$ and $A = 3$) is not optimal for Eastborne Realty.

Graphical Solution of the All-Integer Problem

Figure 7.2 shows the changes in the linear programming graphical solution procedure required to solve the Eastborne Realty integer linear programming problem. First, the graph of the feasible region is drawn exactly as in the LP Relaxation of the problem. Then, because the optimal solution must have integer values, we identify the feasible integer solutions with the dots shown in Figure 7.2. Finally, instead of moving the objective function line to the best extreme point in the feasible region, we move it in an improving direction as far as possible until reaching the dot (feasible integer point) providing the best value for the objective function. Viewing Figure 7.2, we see that the optimal integer solution occurs at $T = 4$ townhouses and $A = 2$ apartment buildings. The objective function value is $10(4) + 15(2) = 70$, providing an annual cash flow of \$70,000. This solution is significantly better than the best solution found by rounding: $T = 2$, $A = 3$, with an annual cash flow of \$65,000. Thus, we see that rounding would not have been the best strategy for Eastborne Realty.

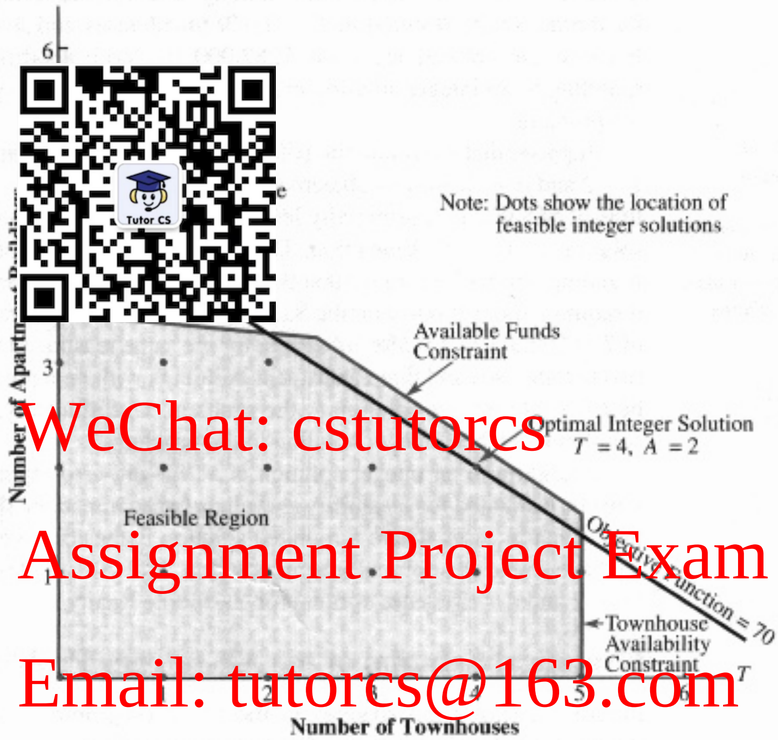
Try Problem 2 for practice with the graphical solution of an integer program.

Using the LP Relaxation to Establish Bounds

An important observation can be made from the analysis of the Eastborne Realty problem. It has to do with the relationship between the value of the optimal integer solution and the value of the optimal solution to the LP Relaxation.

> For integer linear programs involving maximization, the value of the optimal solution to the LP Relaxation provides an upper bound on the value of the optimal integer solution. For integer linear programs involving minimization, the value of the optimal solution to the LP Relaxation provides a lower bound on the value of the optimal integer solution.

FIGURE 7.2 GRAPHICAL SOLUTION OF THE EASTBORNE REALTY INTEGER PROBLEM



This observation is valid for the Eastborne Realty problem. The value of the optimal integer solution is \$70,000, and the value of the optimal solution to the LP Relaxation is \$73,574. Thus, we know from the LP Relaxation solution that the upper bound for the value of the objective function is \$73,574.

Try Problem 5 for the graphical solution of a mixed-integer program.

The bounding property of the LP Relaxation allows us to conclude that if, by chance, the solution to an LP Relaxation turns out to be an integer solution, it is also optimal for the integer linear program. This bounding property can also be helpful in determining whether a rounded solution is "good enough." If a rounded LP Relaxation solution is feasible and provides a value of the objective function that is "almost as good as" the value of the objective function for the LP Relaxation, we know the rounded solution is a near-optimal integer solution. In this case, we can avoid having to solve the problem as an integer linear program.

Computer Solution

LINGO or Frontline Systems' Solver can be used to solve most of the integer linear programs in this chapter. In the appendices at the end of this chapter, we discuss how to solve integer linear programs using Solver and LINGO.

Specifying both T and A as integers provides the optimal integer solution shown in Figure 7.3. The solution of $T = 4$ townhouses and $A = 2$ apartment buildings has a maximum

FIGURE 7.3 THE SOLUTION FOR THE EASTBORNE REALTY PROBLEM

WEB file
Eastborne

```
Optimal Objective Value = 70.00000

     Variable            Value
   --------------    ---------------
                          4.00000
                          2.00000

    Constraint       Slack/Surplus
   --------------    ---------------
                         72.00000
                         44.00000
                          1.00000
```


annual cash flow of $70,000. The values of the slack variables tell us that the optimal solution has $72,000 of available funds unused, 44 hours of the manager's time still available, and 1 of the available townhouses not purchased.

NOTES AND COMMENTS

The computer output we show in this chapter for integer programs does not include reduced costs, dual values, or sensitivity ranges because these are not meaningful for integer programs.

7.3 APPLICATIONS INVOLVING 0-1 VARIABLES

Much of the modeling flexibility provided by integer linear programming is due to the use of 0-1 variables. In many applications, 0-1 variables provide selections or choices with the value of the variable equal to 1 if a corresponding activity is undertaken and equal to 0 if the corresponding activity is not undertaken. The capital budgeting, fixed cost, distribution system design, bank location, and product design/market share applications presented in this section make use of 0-1 variables.

Capital Budgeting

The Ice-Cold Refrigerator Company is considering investing in several projects that have varying capital requirements over the next four years. Faced with limited capital each year, management would like to select the most profitable projects. The estimated net present value for each project,[1] the capital requirements, and the available capital over the four-year period are shown in Table 7.1.

[1] The estimated net present value is the net cash flow discounted back to the beginning of year 1.

TABLE 7.1 PROJECT NET PRESENT VALUE, CAPITAL REQUIREMENTS, AND AVAILABLE CAPITAL FOR THE ICE-COLD REFRIGERATOR COMPANY

	Project				
	Plant Ex[illegible]	Warehouse [illegible]	New Machinery	New Product Research	Total Capital Available
Present Value	$9[illegible]	[illegible]	$10,000	$37,000	
Year 1 Cap Rqmt	$1[illegible]	[illegible]	$10,000	$15,000	$40,000
Year 2 Cap Rqmt	$2[illegible]	[illegible]		$10,000	$50,000
Year 3 Cap Rqmt	$2[illegible]	[illegible]		$10,000	$40,000
Year 4 Cap Rqmt	$1[illegible]	[illegible]	$ 4,000	$10,000	$35,000

The four 0-1 decision variables are as follows:

P = 1 if the plant expansion project is accepted; 0 if rejected
W = 1 if the warehouse expansion project is accepted; 0 if rejected
M = 1 if the new machinery project is accepted; 0 if rejected
R = 1 if the new product research project is accepted; 0 if rejected

In a **capital budgeting problem**, the company's objective function is to maximize the net present value of the capital budgeting projects. This problem has four constraints: one for the funds available in each of the next four years.

A 0-1 integer linear programming model with dollars in thousands is as follows:

$$\text{Max} \quad 90P + 40W + 10M + 37R$$

s.t.

$$
\begin{aligned}
15P + 10W + 10M + 15R &\leq 40 \quad \text{(Year 1 capital available)}\\
20P + 15W + 10R &\leq 50 \quad \text{(Year 2 capital available)}\\
20P + 20W + 10R &\leq 40 \quad \text{(Year 3 capital available)}\\
15P + 5W + 4M + 10R &\leq 35 \quad \text{(Year 4 capital available)}\\
P, W, M, R &= 0, 1
\end{aligned}
$$

The integer programming solution is shown in Figure 7.4. The optimal solution is $P = 1$, $W = 1$, $M = 1$, $R = 0$, with a total estimated net present value of $140,000. Thus, the company should fund the plant expansion, the warehouse expansion, and the new machinery projects. The new product research project should be put on hold unless additional capital funds become available. The values of the slack variables (see Figure 7.4) show that the company will have $5,000 remaining in year 1, $15,000 remaining in year 2, and $11,000 remaining in year 4. Checking the capital requirements for the new product research project, we see that enough funds are available for this project in year 2 and year 4. However, the company would have to find additional capital funds of $10,000 in year 1 and $10,000 in year 3 to fund the new product research project.

Fixed Cost

In many applications, the cost of production has two components: a setup cost, which is a fixed cost, and a variable cost, which is directly related to the production quantity. The use of 0-1 variables makes including the setup cost possible in a model for a production application.

FIGURE 7.4 THE SOLUTION FOR THE ICE-COLD REFRIGERATOR COMPANY PROBLEM

WEB file

Ice-Cold

Optimal Objective Value = 140.00000

Variable	Value
[illegible]	1.00000
[illegible]	1.00000
[illegible]	1.00000
[illegible]	0.00000

Constraint	Slack/Surplus
[illegible]	5.00000
2	15.00000
3	0.00000
4	11.00000

As an example of a **fixed cost problem**, consider the RMC problem. Three raw materials are used to produce three products: a fuel additive, a solvent base, and a carpet cleaning fluid. The following decision variables are used:

F = tons of fuel additive produced
S = tons of solvent base produced
C = tons of carpet cleaning fluid produced

The profit contributions are $40 per ton for the fuel additive, $30 per ton for the solvent base, and $50 per ton for the carpet cleaning fluid. Each ton of fuel additive is a blend of 0.4 tons of material 1 and 0.6 tons of material 3. Each ton of solvent base requires 0.5 tons of material 1, 0.2 tons of material 2, and 0.3 tons of material 3. Each ton of carpet cleaning fluid is a blend of 0.6 tons of material 1, 0.1 tons of material 2, and 0.3 tons of material 3. RMC has 20 tons of material 1, 5 tons of material 2, and 21 tons of material 3 and is interested in determining the optimal production quantities for the upcoming planning period.

A linear programming model of the RMC problem is shown:

$$
\begin{array}{lrl}
\text{Max} & 40F + 30S + 50C & \\
\text{s.t.} & & \\
& 0.4F + 0.5S + 0.6C \leq 20 & \text{Material 1} \\
& 0.2S + 0.1C \leq 5 & \text{Material 2} \\
& 0.6F + 0.3S + 0.3C \leq 21 & \text{Material 3} \\
& F, S, C \geq 0 &
\end{array}
$$

The optimal solution consists of 27.5 tons of fuel additive, 0 tons of solvent base, and 15 tons of carpet cleaning fluid, with a value of $1850, as shown in Figure 7.5.

This linear programming formulation of the RMC problem does not include a fixed cost for production setup of the products. Suppose that the following data are available

FIGURE 7.5 THE SOLUTION TO THE RMC PROBLEM

WEB file RMC

```
Optimal Objective Value = 1850.00000

   Variable              Value              Reduced Costs
  --------------    ---------------      ----------------
  [illegible]           27.50000               0.00000
  [illegible]            0.00000             -12.50000
  [illegible]           15.00000               0.00000
```

conc[illegible]nd the maximum production quantity for each of the three products.

Product	Setup Cost	Maximum Production
Fuel additive	\$200	50 tons
Solvent base	\$ 50	25 tons
Carpet cleaning fluid	\$400	40 tons

The modeling flexibility provided by 0-1 variables can now be used to incorporate the fixed setup costs into the production model. The 0-1 variables are defined as follows:

SF = 1 if the fuel additive is produced; 0 if not

SS = 1 if the solvent base is produced; 0 if not

SC = 1 if the carpet cleaning fluid is produced; 0 if not

Using these setup variables, the total setup cost is

$$200SF + 50SS + 400SC$$

We can now rewrite the objective function to include the setup cost. Thus, the net profit objective function becomes

$$\text{Max} \quad 40F + 30S + 50C - 200SF - 50SS - 400SC$$

Next, we must write production capacity constraints so that if a setup variable equals 0, production of the corresponding product is not permitted and, if a setup variable equals 1, production is permitted up to the maximum quantity. For the fuel additive, we do so by adding the following constraint:

$$F \leq 50SF$$

Note that, with this constraint present, production of the fuel additive is not permitted when $SF = 0$. When $SF = 1$, production of up to 50 tons of fuel additive is permitted. We can think of the setup variable as a switch. When it is off ($SF = 0$), production is not permitted; when it is on ($SF = 1$), production is permitted.

FIGURE 7.6 THE SOLUTION TO THE RMC PROBLEM WITH SETUP COSTS


```
Optimal Objective Value = 1350.00000

       Variable              Value
  ---------------   ---------------
    [illegible]           25.00000
    [illegible]           20.00000
    [illegible]            0.00000
    [illegible]F           1.00000
    [illegible]S           1.00000
    [illegible]C           0.00000
```

WEB file
RMC Setup

Similar production capacity constraints, using 0-1 variables, are added for the solvent base and carpet cleaning products:

$$S \le 25SS$$
$$C \le 40SC$$

We have then the following fixed cost model for the RMC problem:

$$
\begin{array}{lllll}
\text{Max} & 40F + 30S + 50C - 200SF - 50SS - 400SC & & & \\
\text{s.t.} & & & & \\
 & 0.4F + 0.5S + 0.6C & \le & 20 & \text{Material 1} \\
 & \phantom{0.4F +{}} 0.1S + 0.1C & \le & 5 & \text{Material 2} \\
 & 0.6F + 0.3S + 0.3C & \le & 21 & \text{Material 3} \\
 & F & \le & 50SF & \text{Maximum } F \\
 & \phantom{0.4F +{}} S & \le & 25SS & \text{Maximum } S \\
 & \phantom{0.4F + 0.5S +{}} C & \le & 40SC & \text{Maximum } C \\
 & F, S, C \ge 0;\ SF, SS, SC = 0, 1 & & &
\end{array}
$$

The solution to the RMC problem with setup costs is shown in Figure 7.6. The optimal solution shows 25 tons of fuel additive and 20 tons of solvent base. The value of the objective function after deducting the setup cost is \$1350. The setup cost for the fuel additive and the solvent base is \$200 + \$50 = \$250. The optimal solution shows $SC = 0$, which indicates that the more expensive \$400 setup cost for the carpet cleaning fluid should be avoided. Thus, the carpet cleaning fluid is not produced.

The Management Science in Action, Aluminum Can Production at Valley Metal Containers (see Section 7.1), employs 0-1 fixed cost variables for production line changeovers.

The key to developing a fixed cost model is the introduction of a 0-1 variable for each fixed cost and the specification of an upper bound for the corresponding production variable. For a production quantity x, a constraint of the form $x \le My$ can then be used to allow production when the setup variable $y = 1$ and not to allow production when the setup variable $y = 0$. The value of the maximum production quantity M should be large enough to allow for all reasonable levels of production. But research has shown that choosing values of M excessively large will slow the solution procedure.

Distribution System Design

The Martin-Beck Company operates a plant in St. Louis with an annual capacity of 30,000 units. Product is shipped to regional distribution centers located in Boston, Atlanta, and

Houston. Because of an anticipated increase in demand, Martin-Beck plans to increase capacity by constructing a new plant in one or more of the following cities: Detroit, Toledo, Denver, or Kansas City. The estimated annual fixed cost and the annual capacity for the four proposed plants are as follows:

Annual Fixed Cost	Annual Capacity
$175,000	10,000
$300,000	20,000
$375,000	30,000
$500,000	40,000

The long-range planning group developed forecasts of the anticipated annual demand at the distribution centers as follows:

Distribution Center	Annual Demand
Boston	30,000
Atlanta	20,000
Houston	20,000

The shipping cost per unit from each plant to each distribution center is shown in Table 7.2. A network representation of the potential Martin-Beck distribution system is shown in Figure 7.7. Each potential plant location is shown; capacities and demands are shown in thousands of units. This network representation is for a transportation problem with a plant at St. Louis and at all four proposed sites. However, the decision has not yet been made as to which new plant or plants will be constructed.

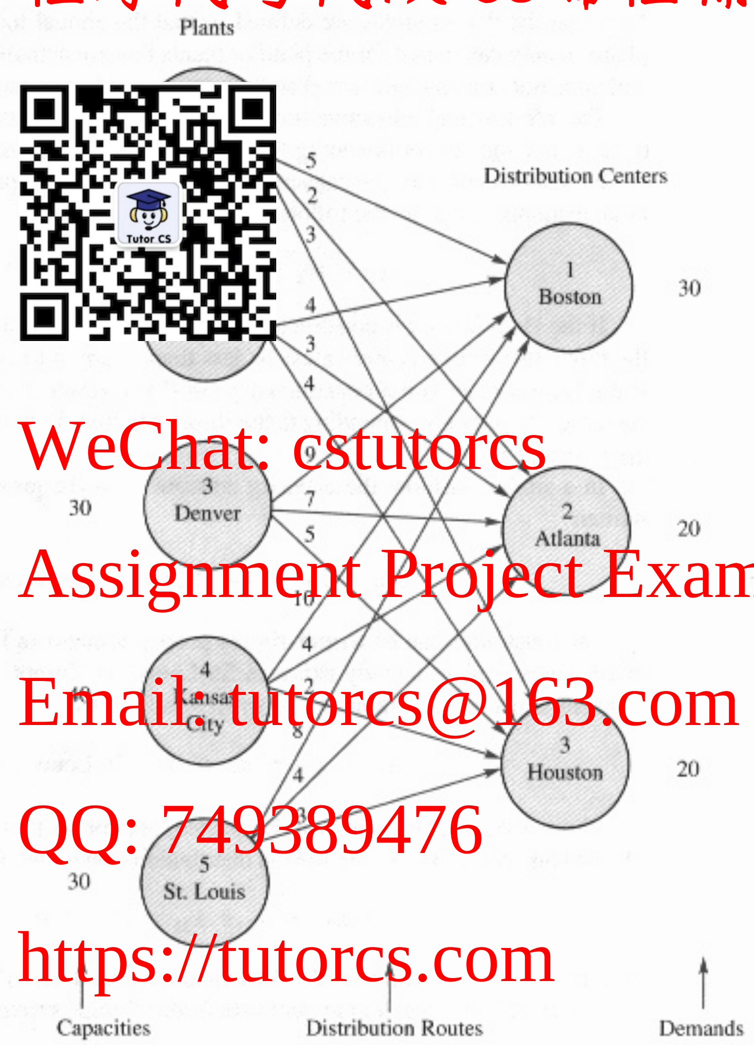
Let us now show how 0-1 variables can be used in this **distribution system design problem** to develop a model for choosing the best plant locations and for determining how much to ship from each plant to each distribution center. We can use the following 0-1 variables to represent the plant construction decision:

y_1 = 1 if a plant is constructed in Detroit; 0 if not
y_2 = 1 if a plant is constructed in Toledo; 0 if not
y_3 = 1 if a plant is constructed in Denver; 0 if not
y_4 = 1 if a plant is constructed in Kansas City; 0 if not

TABLE 7.2 SHIPPING COST PER UNIT FOR THE MARTIN-BECK DISTRIBUTION SYSTEM

	Distribution Centers		
Plant Site	Boston	Atlanta	Houston
Detroit	5	2	3
Toledo	4	3	4
Denver	9	7	5
Kansas City	10	4	2
St. Louis	8	4	3

FIGURE 7.7 THE NETWORK REPRESENTATION OF THE MARTIN-BECK COMPANY DISTRIBUTION SYSTEM PROBLEM



The variables representing the amount shipped from each plant site to each distribution center are defined just as for a transportation problem.

$$x_{ij} = \text{the units shipped in thousands from plant } i \text{ to distribution center } j$$
$$i = 1, 2, 3, 4, 5 \quad \text{and} \quad j = 1, 2, 3$$

Using the shipping cost data in Table 7.2, the annual transportation cost in thousands of dollars is written

$$5x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + 3x_{22} + 4x_{23} + 9x_{31} + 7x_{32} + 5x_{33}$$
$$+ 10x_{41} + 4x_{42} + 2x_{43} + 8x_{51} + 4x_{52} + 3x_{53}$$

The annual fixed cost of operating the new plant or plants in thousands of dollars is written as

$$175y_1 + 300y_2 + 375y_3 + 500y_4$$

Note that the 0-1 variables are defined so that the annual fixed cost of operating the new plants is only calculated for the plant or plants that are actually constructed (i.e., $y_i = 1$). If a pl[illegible] = 0 and the corresponding annual fixed cost is \$0.

[illegible]ive function is the sum of the annual transportation cost plus the a[illegible]ating the newly constructed plants.

[illegible]capacity constraints at the four proposed plants. Using Detroit as a[illegible]following constraint:

$$[illegible] + x_{12} + x_{13} \le 10y_1 \quad \text{Detroit capacity}$$

[illegible]nstructed, $y_1 = 1$ and the total amount shipped from Detroit to the three distribution centers must be less than or equal to Detroit's 10,000-unit capacity. If the Detroit plant is not constructed, $y_1 = 0$ will result in a 0 capacity at Detroit. In this case, the variables corresponding to the shipments from Detroit must all equal zero: $x_{11} = 0$, $x_{12} = 0$, and $x_{13} = 0$.

In a similar fashion, the capacity constraint for the proposed plant in Toledo can be written

$$x_{21} + x_{22} + x_{23} \le 20y_2 \quad \text{Toledo capacity}$$

Similar constraints can be written for the proposed plants in Denver and Kansas City. Note that because a plant already exists in St. Louis, we do not define a 0-1 variable for this plant. Its capacity constraint can be written as follows:

$$x_{51} + x_{52} + x_{53} \le 30 \quad \text{St. Louis capacity}$$

Three demand constraints will be needed, one for each of the three distribution centers. The demand constraint for the Boston distribution center with units in thousands is written as

$$x_{11} + x_{21} + x_{31} + x_{51} = 30 \quad \text{Boston demand}$$

Similar constraints appear for the Atlanta and Houston distribution centers.

The complete model for the Martin-Beck distribution system design problem is as follows:

$$\begin{aligned}
\text{Min} \quad & 5x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + 3x_{22} + 4x_{23} + 9x_{31} + 7x_{32} + 5x_{33} + 10x_{41} + 4x_{42} \\
& + 2x_{43} + 8x_{51} + 4x_{52} + 3x_{53} + 175y_1 + 300y_2 + 375y_3 + 500y_4
\end{aligned}$$

s.t.

$$\begin{array}{lll}
x_{11} + x_{12} + x_{13} & \le 10y_1 & \text{Detroit capacity} \\
x_{21} + x_{22} + x_{23} & \le 20y_2 & \text{Toledo capacity} \\
x_{31} + x_{32} + x_{33} & \le 30y_3 & \text{Denver capacity} \\
x_{41} + x_{42} + x_{43} & \le 40y_4 & \text{Kansas City capacity} \\
x_{51} + x_{52} + x_{53} & \le 30 & \text{St. Louis capacity} \\
x_{11} + x_{21} + x_{31} + x_{41} + x_{51} & = 30 & \text{Boston demand} \\
x_{12} + x_{22} + x_{32} + x_{42} + x_{52} & = 20 & \text{Atlanta demand} \\
x_{13} + x_{23} + x_{33} + x_{43} + x_{53} & = 20 & \text{Houston demand} \\
\end{array}$$

$x_{ij} \ge 0$ for all i and j; $y_1, y_2, y_3, y_4 = 0, 1$

FIGURE 7.8 THE SOLUTION FOR THE MARTIN-BECK COMPANY DISTRIBUTION SYSTEM PROBLEM

WEB file
Martin-Beck

Optimal Objective Value = 860.00000

Variable	Value
X11	0.00000
X12	0.00000
X13	0.00000
X21	0.00000
X22	0.00000
X23	0.00000
X31	0.00000
X32	0.00000
X33	0.00000
X41	0.00000
X42	20.00000
X43	20.00000
X51	30.00000
X52	0.00000
X53	0.00000
Y1	0.00000
Y2	0.00000
Y3	0.00000
Y4	1.00000

Constraint	Slack/Surplus
1	0.00000
2	0.00000
3	0.00000
4	0.00000
5	0.00000
6	0.00000
7	0.00000
8	0.00000

The solution for the Martin-Beck problem is shown in Figure 7.8. The optimal solution calls for the construction of a plant in Kansas City ($y_4 = 1$); 20,000 units will be shipped from Kansas City to Atlanta ($x_{42} = 20$), 20,000 units will be shipped from Kansas City to Houston ($x_{43} = 20$), and 30,000 units will be shipped from St. Louis to Boston ($x_{51} = 30$). Note that the total cost of this solution including the fixed cost of \$500,000 for the plant in Kansas City is \$860,000.

This basic model can be expanded to accommodate distribution systems involving direct shipments from plants to warehouses, from plants to retail outlets, and multiple products.[2] Using the special properties of 0-1 variables, the model can also be expanded to accommodate a variety of configuration constraints on the plant locations. For example, suppose in another problem site 1 were in Dallas and site 2 were in Fort Worth. A company might not want to locate plants in both Dallas and Fort Worth because the cities are so close together. To prevent this result from happening, the following constraint can be added to the model:

Problem 13, which is based on the Martin-Beck distribution system problem, provides additional practice involving 0-1 variables.

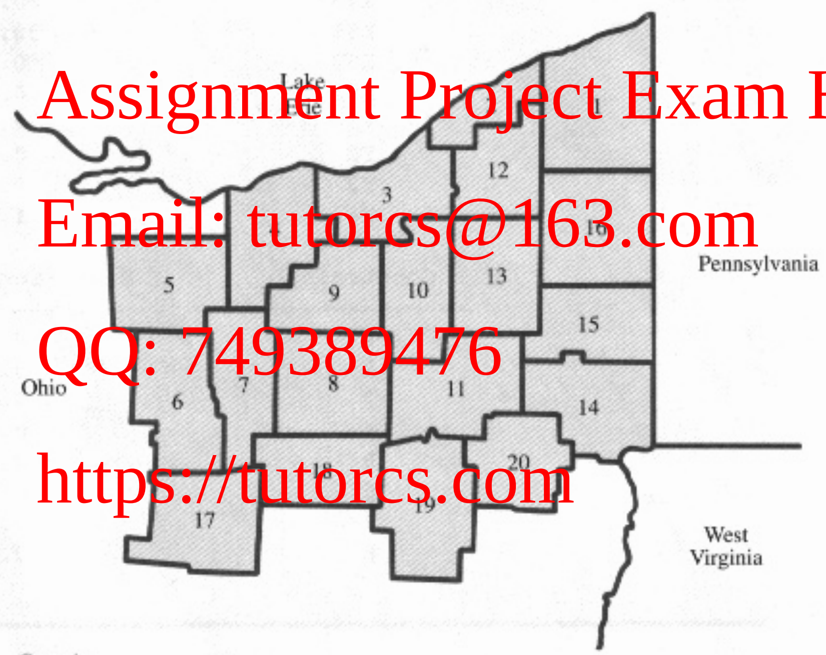
$$y_1 + y_2 \leq 1$$

This ... y_1 or y_2 to equal 1, but not both. If we had written the constraint ... d require that a plant be located in either Dallas or Fort Worth.

Bar...

The ... artment for the Ohio Trust Company is considering expanding its operation into a 20-county region in northeastern Ohio (see Figure 7.9). Currently, Ohio

FIGURE 7.9 THE 20-COUNTY REGION IN NORTHEASTERN OHIO



Counties

1. Ashtabula	6. Richland	11. Stark	16. Trumbull
2. Lake	7. Ashland	12. Geauga	17. Knox
3. Cuyahoga	8. Wayne	13. Portage	18. Holmes
4. Lorain	9. Medina	14. Columbiana	19. Tuscarawas
5. Huron	10. Summit	15. Mahoning	20. Carroll

[2]For computational reasons, it is usually preferable to replace the m plant capacity constraints with mn shipping route capacity constraints of the form $x_{ij} \leq \text{Min}\{s_i, d_j\}\, y_i$ for $i = 1, \ldots, m$, and $j = 1, \ldots, n$. The coefficient for y_i in each of these constraints is the smaller of the origin capacity (s_i) or the destination demand (d_j). These additional constraints often cause the solution of the LP Relaxation to be integer.

TABLE 7.3 COUNTIES IN THE OHIO TRUST EXPANSION REGION

Counties Under Consideration	Adjacent Counties (by Number)
1. Ashtabula	2, 12, 16
2. Lake	1, 3, 12
3. [illegible]	2, 4, 9, 10, 12, 13
4. [illegible]	3, 5, 7, 9
5. [illegible]	4, 6, 7
6. [illegible]	5, 7, 17
7. [illegible]	4, 5, 6, 8, 9, 17, 18
8. [illegible]	7, 9, 10, 11, 18
9. [illegible]	3, 4, 7, 8, 10
10. [illegible]	3, 8, 9, 11, 12, 13
11. Stark	8, 10, 13, 14, 15, 18, 19, 20
12. Geauga	1, 2, 3, 10, 13, 16
13. Portage	3, 10, 11, 12, 15, 16
14. Columbiana	11, 15, 20
15. Mahoning	11, 13, 14, 16
16. Trumbull	1, 12, 13, 15
17. Knox	6, 7, 18
18. Holmes	7, 8, 11, 17, 19
19. Tuscarawas	11, 18, 20
20. Carroll	11, 14, 19

Trust does not have a principal place of business in any of the 20 counties. According to the banking laws in Ohio, if a bank establishes a principal place of business (PPB) in any county, branch banks can be established in that county and in any adjacent county. However, to establish a new principal place of business, Ohio Trust must either obtain approval for a new bank from the state's superintendent of banks or purchase an existing bank.

Table 7.3 lists the 20 counties in the region and adjacent counties. For example, Ashtabula County is adjacent to Lake, Geauga, and Trumbull counties; Lake County is adjacent to Ashtabula, Cuyahoga, and Geauga counties; and so on.

As an initial step in its planning, Ohio Trust would like to determine the minimum number of PPBs necessary to do business throughout the 20-county region. A 0-1 integer programming model can be used to solve this **location problem** for Ohio Trust. We define the variables as

$$x_i = 1 \text{ if a PPB is established in county } i\text{; 0 otherwise}$$

To minimize the number of PPBs needed, we write the objective function as

$$\text{Min} \quad x_1 + x_2 + \cdots + x_{20}$$

The bank may locate branches in a county if the county contains a PPB or is adjacent to another county with a PPB. Thus, the linear program will need one constraint for each county. For example, the constraint for Ashtabula County is

$$x_1 + x_2 + x_{12} + x_{16} \geq 1 \quad \text{Ashtabula}$$

Note that satisfaction of this constraint ensures that a PPB will be placed in Ashtabula County *or* in one or more of the adjacent counties. This constraint thus guarantees that Ohio Trust will be able to place branch banks in Ashtabula County.

The complete statement of the bank location problem is

$$
\begin{array}{llr}
\text{Min} & x_1 + x_2 + \cdots + x_{20} & \\
\text{s.t.} & & \\
& x_1 + x_2 + x_{12} + x_{16} \geq 1 & \text{Ashtabula} \\
& x_1 + x_2 + x_3 + x_{12} \geq 1 & \text{Lake} \\
& \vdots & \\
& x_{11} + x_{14} + x_{19} + x_{20} \geq 1 & \text{Carroll} \\
& x_i = 0, 1 \quad i = 1, 2, \ldots, 20 &
\end{array}
$$

In Figure 7.10 we show the solution to the Ohio Trust problem. Using the output, we see that the optimal solution calls for principal places of business in Ashland, Stark, and Geauga counties. With PPBs in these three counties, Ohio Trust can place branch banks in all 20 counties (see Figure 7.11). All other decision variables have an optimal value of zero, indicating that a PPB should not be placed in these counties. Clearly the integer

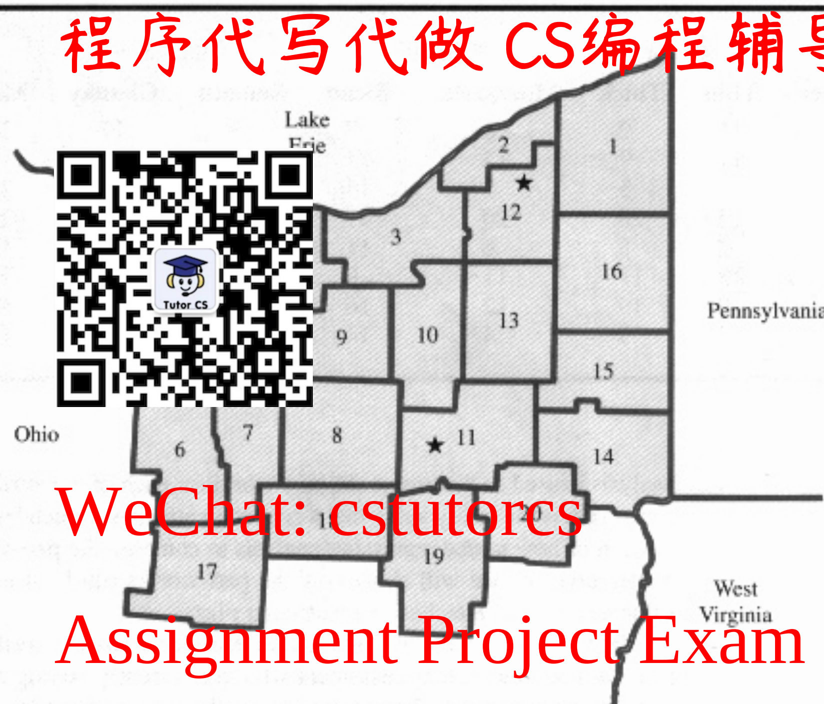
FIGURE 7.10 THE SOLUTION FOR THE OHIO TRUST PPB LOCATION PROBLEM

WEB file Ohio Trust

Optimal Objective Value = 3.00000

Variable	Value
X1	0.00000
X2	0.00000
X3	0.00000
X4	0.00000
X5	0.00000
X6	0.00000
X7	1.00000
X8	0.00000
X9	0.00000
X10	0.00000
X11	1.00000
X12	1.00000
X13	0.00000
X14	0.00000
X15	0.00000
X16	0.00000
X17	0.00000
X18	0.00000
X19	0.00000
X20	0.00000

FIGURE 7.11 PRINCIPAL PLACE OF BUSINESS COUNTIES FOR OHIO TRUST



Counties

1. Ashtabula	6. Richland	11. Stark	16. Trumbull	★ A principal place of business should be located in these counties.
2. Lake	7. Ashland	12. Geauga	17. Knox	
3. Cuyahoga	8. Wayne	13. Portage	18. Holmes	
4. Lorain	9. Medina	14. Columbiana	19. Tuscarawas	
5. Huron	10. Summit	15. Mahoning	20. Carroll	

programming model could be enlarged to allow for expansion into a larger area or throughout the entire state.

Product Design and Market Share Optimization

Conjoint analysis is a market research technique that can be used to learn how prospective buyers of a product value the product's attributes. In this section we will show how the results of conjoint analysis can be used in an integer programming model of a **product design and market share optimization problem.** We illustrate the approach by considering a problem facing Salem Foods, a major producer of frozen foods.

Salem Foods is planning to enter the frozen pizza market. Currently, two existing brands, Antonio's and King's, have the major share of the market. In trying to develop a sausage pizza that will capture a significant share of the market, Salem determined that the four most important attributes when consumers purchase a frozen sausage pizza are crust, cheese, sauce, and sausage flavor. The crust attribute has two levels (thin and thick); the cheese attribute has two levels (mozzarella and blend); the sauce attribute has two levels (smooth and chunky); and the sausage flavor attribute has three levels (mild, medium, and hot).

In a typical conjoint analysis, a sample of consumers is asked to express their preference for specially prepared pizzas with chosen levels for the attributes. Then regression

TABLE 7.4 PART-WORTHS FOR THE SALEM FOODS PROBLEM

	Crust		Cheese		Sauce		Sausage Flavor		
Consumer	Thin	Thick	Mozzarella	Blend	Smooth	Chunky	Mild	Medium	Hot
1	11	2	6	7	3	17	26	27	8
2	11	[illegible]	[illegible]	17	16	26	14	1	10
3	7	[illegible]	[illegible]	14	16	7	29	16	19
4	13	[illegible]	[illegible]	17	17	14	25	29	10
5	2	[illegible]	[illegible]	11	30	20	15	5	12
6	12	[illegible]	[illegible]	9	2	30	22	12	20
7	9	[illegible]	[illegible]	16	16	25	30	23	19
8	5	[illegible]	[illegible]	14	23	16	16	30	3

analysis is used to determine the part-worth for each of the attribute levels. In essence, the part-worth is the utility value that a consumer attaches to each level of each attribute. A discussion of how to use regression analysis to compute the part-worths is beyond the scope of this text, but we will show how the part-worths can be used to determine the overall value a consumer attaches to a particular pizza.

Table 7.4 shows the part-worths for each level of each attribute provided by a sample of eight potential Salem customers who are currently buying either King's or Antonio's pizza. For consumer 1, the part-worths for the crust attribute are 11 for thin crust and 2 for thick crust, indicating a preference for thin crust. For the cheese attribute, the part-worths are 6 for the mozzarella cheese and 7 for the cheese blend; thus, consumer 1 has a slight preference for the cheese blend. From the other part-worths, we see that consumer 1 shows a strong preference for the chunky sauce over the smooth sauce (17 to 3) and has a slight preference for the medium-flavored sausage. Note that consumer 2 shows a preference for the thin crust, the cheese blend, the chunky sauce, and mild-flavored sausage. The part-worths for the other consumers are interpreted in a similar manner.

The part-worths can be used to determine the overall value (utility) each consumer attaches to a particular type of pizza. For instance, consumer 1's current favorite pizza is the Antonio's brand, which has a thick crust, mozzarella cheese, chunky sauce, and medium-flavored sausage. We can determine consumer 1's utility for this particular type of pizza using the part-worths in Table 7.4. For consumer 1, the part-worths are 2 for thick crust, 6 for mozzarella cheese, 17 for chunky sauce, and 27 for medium-flavored sausage. Thus, consumer 1's utility for the Antonio's brand pizza is $2 + 6 + 17 + 27 = 52$. We can compute consumer 1's utility for a King's brand pizza in a similar manner. The King's brand pizza has a thin crust, a cheese blend, smooth sauce, and mild-flavored sausage. Because the part-worths for consumer 1 are 11 for thin crust, 7 for cheese blend, 3 for smooth sauce, and 26 for mild-flavored sausage, consumer 1's utility for the King's brand pizza is $11 + 7 + 3 + 26 = 47$. In general, each consumer's utility for a particular type of pizza is just the sum of the appropriate part-worths.

In order to be successful with its brand, Salem Foods realizes that it must entice consumers in the marketplace to switch from their current favorite brand of pizza to the Salem product. That is, Salem must design a pizza (choose the type of crust, cheese, sauce, and sausage flavor) that will have the highest utility for enough people to ensure sufficient sales to justify making the product. Assuming the sample of eight consumers in the current study is representative of the marketplace for frozen sausage pizza, we can formulate and solve an integer programming model that can help Salem come up with such a design. In marketing literature, the problem being solved is called the *share of choices* problem.

The decision variables are defined as follows:

$$l_{ij} = 1 \text{ if Salem chooses level } i \text{ for attribute } j\text{; 0 otherwise}$$
$$y_k = 1 \text{ if consumer } k \text{ chooses the Salem brand; 0 otherwise}$$

The objective is to choose the levels of each attribute that will maximize the number of consumers preferring the Salem brand pizza. Because the number of customers preferring the Salem brand pizza is the sum of the y_k variables, the objective function is

$$\text{Max} \quad y_1 + y_2 + \cdots + y_8$$

One constraint is needed for each consumer in the sample. To illustrate how the constraints are formulated, let us consider the constraint corresponding to consumer 1. For consumer 1, the utility of a particular type of pizza can be expressed as the sum of the part-worths:

$$\begin{aligned}\text{Utility for Consumer 1} = {} & 11l_{11} + 2l_{21} + 6l_{12} + 7l_{22} + 3l_{13} + 17l_{23} \\ & + 26l_{14} + 27l_{24} + 8l_{34}\end{aligned}$$

In order for consumer 1 to prefer the Salem pizza, the utility for the Salem pizza must be greater than the utility for consumer 1's current favorite. Recall that consumer 1's current favorite brand of pizza is Antonio's, with a utility of 52. Thus, consumer 1 will only purchase the Salem brand if the levels of the attributes for the Salem brand are chosen such that

$$11l_{11} + 2l_{21} + 6l_{12} + 7l_{22} + 3l_{13} + 17l_{23} + 26l_{14} + 27l_{24} + 8l_{34} > 52$$

Given the definitions of the y_k decision variables, we want $y_1 = 1$ when the consumer prefers the Salem brand and $y_1 = 0$ when the consumer does not prefer the Salem brand. Thus, we write the constraint for consumer 1 as follows:

$$11l_{11} + 2l_{21} + 6l_{12} + 7l_{22} + 3l_{13} + 17l_{23} + 26l_{14} + 27l_{24} + 8l_{34} \geq 1 + 52y_1$$

With this constraint, y_1 cannot equal 1 unless the utility for the Salem design (the left-hand side of the constraint) exceeds the utility for consumer 1's current favorite by at least 1. Because the objective function is to maximize the sum of the y_k variables, the optimization will seek a product design that will allow as many y_k as possible to equal 1.

A similar constraint is written for each consumer in the sample. The coefficients for the l_{ij} variables in the utility functions are taken from Table 7.4 and the coefficients for the y_k variables are obtained by computing the overall utility of the consumer's current favorite brand of pizza. The following constraints correspond to the eight consumers in the study:

Antonio's brand is the current favorite pizza for consumers 1, 4, 6, 7, and 8. King's brand is the current favorite pizza for consumers 2, 3, and 5.

$$\begin{aligned}
11l_{11} + 2l_{21} + 6l_{12} + 7l_{22} + 3l_{13} + 17l_{23} + 26l_{14} + 27l_{24} + 8l_{34} &\geq 1 + 52y_1 \\
11l_{11} + 7l_{21} + 15l_{12} + 17l_{22} + 16l_{13} + 26l_{23} + 14l_{14} + 1l_{24} + 10l_{34} &\geq 1 + 58y_2 \\
7l_{11} + 5l_{21} + 8l_{12} + 14l_{22} + 16l_{13} + 7l_{23} + 29l_{14} + 16l_{24} + 19l_{34} &\geq 1 + 66y_3 \\
13l_{11} + 20l_{21} + 20l_{12} + 17l_{22} + 17l_{13} + 14l_{23} + 25l_{14} + 29l_{24} + 10l_{34} &\geq 1 + 83y_4 \\
2l_{11} + 8l_{21} + 6l_{12} + 11l_{22} + 30l_{13} + 20l_{23} + 15l_{14} + 5l_{24} + 12l_{34} &\geq 1 + 58y_5 \\
12l_{11} + 17l_{21} + 11l_{12} + 9l_{22} + 2l_{13} + 30l_{23} + 22l_{14} + 12l_{24} + 20l_{34} &\geq 1 + 70y_6 \\
9l_{11} + 19l_{21} + 12l_{12} + 16l_{22} + 16l_{13} + 25l_{23} + 30l_{14} + 23l_{24} + 19l_{34} &\geq 1 + 79y_7 \\
5l_{11} + 9l_{21} + 4l_{12} + 14l_{22} + 23l_{13} + 16l_{23} + 16l_{14} + 30l_{24} + 3l_{34} &\geq 1 + 59y_8
\end{aligned}$$

Four more constraints must be added, one for each attribute. These constraints are necessary to ensure that one and only one level is selected for each attribute. For attribute 1 (crust), we must add the constraint

$$l_{11} + l_{21} = 1$$

Beca[illegible]-1 variables, this constraint requires that one of the two variables[illegible] equals 0. The following three constraints ensure that one and only [illegible] each of the other three attributes:

$$l_{12} + l_{22} = 1$$
$$l_{13} + l_{23} = 1$$
$$l_{14} + l_{24} + l_{34} = 1$$

WEB file Salem

The optimal solution to this integer linear program is $l_{11} = l_{22} = l_{23} = l_{14} = 1$ and $y_1 = y_2 = y_6 = y_7 = 1$. The value of the optimal solution is 4, indicating that if Salem makes this type of pizza it will be preferable to the current favorite for four of the eight consumers. With $l_{11} = l_{22} = l_{23} = l_{14} = 1$, the pizza design that obtains the largest market share for Salem has a thin crust, a cheese blend, a chunky sauce, and mild-flavored sausage. Note also that with $y_1 = y_2 = y_6 = y_7 = 1$, consumers 1, 2, 6, and 7 will prefer the Salem pizza. With this information Salem may choose to market this type of pizza.

NOTES AND COMMENTS

1. Most practical applications of integer linear programming involve only 0-1 integer variables. Indeed, some mixed-integer computer codes are designed to handle only integer variables with binary values. However, if a clever mathematical trick is employed, these codes can still be used for problems involving general integer variables. The trick is called *binary expansion* and requires that an upper bound be established for each integer variable. More advanced texts on integer programming show how it can be done.
2. The Management Science in Action, Volunteer Scheduling for the Edmonton Folk Festival, describes how a series of three integer programming models was used to schedule volunteers. Two of the models employ 0-1 variables.
3. General-purpose mixed-integer linear programming codes and some spreadsheet packages can be used for linear programming problems, all-integer problems, and problems involving some continuous and some integer variables. General-purpose codes are seldom the fastest for solving problems with special structure (such as the transportation, assignment, and transshipment problems); however, unless the problems are very large, speed is usually not a critical issue. Thus, most practitioners prefer to use one general-purpose computer package that can be used on a variety of problems rather than to maintain a variety of computer programs designed for special problems.

MANAGEMENT SCIENCE IN ACTION

VOLUNTEER SCHEDULING FOR THE EDMONTON FOLK FESTIVAL*

The Edmonton Folk Festival is a four-day outdoor event that is run almost entirely by volunteers. In 2002, 1800 volunteers worked on 35 different crews and contributed more than 50,000 volunteer hours. With this many volunteers, coordination requires a major effort. For instance, in 2002, two volunteer coordinators used a trial-and-error procedure to develop schedules for the volunteers in the two gate crews. However, developing these schedules proved to be time consuming and frustrating; the coordinators spent as much time scheduling as they did supervising volunteers during the festival. To reduce the time spent on gate-crew scheduling, one of the coordinators asked the Centre for Excellence in Operations at the University of Alberta [illegible] r help in automating the [illegible] he Centre agreed to help.

The schedu[illegible] onsists of three integer p[illegible] odel 1 is used to determi[illegible] his model determines the [illegible] umber of hours) and ho[illegible] eeded for each shift to meet the peaks and valleys in demand. Model 2 is a binary integer program used to assign volunteers to shifts. The objective is to maximize volunteer preferences subject to several constraints, such as number of hours worked, balance between morning and afternoon shifts, a mix of experienced and inexperienced volunteers on each shift, no conflicting shifts, and so on. Model 3 is used to allocate volunteers between the two gates.

The coordinators of the gate crews were pleased with the results provided by the models and learned to use them effectively. Vicki Fannon, the manager of volunteers for the festival, now has plans to expand the use of the integer programming models to the scheduling of other crews in the future.

*Based on L. Gordon and E. Erkut, "Improving Volunteer Scheduling for the Edmonton Folk Festival," *Interfaces* (September/October 2004): 367–376.

7.4 MODELING FLEXIBILITY PROVIDED BY 0-1 INTEGER VARIABLES

In Section 7.3 we presented four applications involving 0-1 integer variables. In this section we continue the discussion of the use of 0-1 integer variables in modeling. First, we show how 0-1 integer variables can be used to model multiple-choice and mutually exclusive constraints. Then, we show how 0-1 integer variables can be used to model situations in which k projects out of a set of n projects must be selected, as well as situations in which the acceptance of one project is conditional on the acceptance of another. We close the section with a cautionary note on the role of sensitivity analysis in integer linear programming.

Multiple-Choice and Mutually Exclusive Constraints

Recall the Ice-Cold Refrigerator capital budgeting problem introduced in Section 7.3. The decision variables were defined as

P = 1 if the plant expansion project is accepted; 0 if rejected
W = 1 if the warehouse expansion project is accepted; 0 if rejected
M = 1 if the new machinery project is accepted; 0 if rejected
R = 1 if the new product research project is accepted; 0 if rejected

Suppose that, instead of one warehouse expansion project, the Ice-Cold Refrigerator Company actually has three warehouse expansion projects under consideration. One of the warehouses *must* be expanded because of increasing product demand, but new demand isn't sufficient to make expansion of more than one warehouse necessary. The following variable definitions and **multiple-choice constraint** could be incorporated into the previous 0-1 integer linear programming model to reflect this situation. Let

W_1 = 1 if the original warehouse expansion project is accepted; 0 if rejected
W_2 = 1 if the second warehouse expansion project is accepted; 0 if rejected
W_3 = 1 if the third warehouse expansion project is accepted; 0 if rejected

The following multiple-choice constraint reflects the requirement that exactly one of these projects must be selected:

$$W_1 + W_2 + W_3 = 1$$

If W_1, W_2, and W_3 are allowed to assume only the values 0 or 1, then one and only one of these projects will be selected from among the three choices.

I[illegible]ne warehouse must be expanded did not exist, the multiple-choi[illegible]odified as follows:

$$W_1 + W_2 + W_3 \leq 1$$

This [illegible] the case of no warehouse expansion ($W_1 = W_2 = W_3 = 0$) but does [illegible]ne warehouse to be expanded. This type of constraint is often calle[illegible] **constraint.**

k out of *n* Alternatives Constraint

An extension of the notion of a multiple-choice constraint can be used to model situations in which *k out of a set of n* projects must be selected—a ***k* out of *n* alternatives constraint.** Suppose that W_1, W_2, W_3, W_4, and W_5 represent five potential warehouse expansion projects and that two of the five projects must be accepted. The constraint that satisfies this new requirement is

$$W_1 + W_2 + W_3 + W_4 + W_5 = 2$$

If no more than two of the projects are to be selected, we would use the following less-than-or-equal-to constraint:

$$W_1 + W_2 + W_3 + W_4 + W_5 \leq 2$$

Again, each of these variables must be restricted to 0-1 values.

Conditional and Corequisite Constraints

Sometimes the acceptance of one project is conditional on the acceptance of another. For example, suppose for the Ice-Cold Refrigerator Company that the warehouse expansion project was conditional on the plant expansion project. That is, management will not consider expanding the warehouse unless the plant is expanded. With P representing plant expansion and W representing warehouse expansion, a **conditional constraint** could be introduced to enforce this requirement:

$$W \leq P$$

Both P and W must be 0 or 1; whenever P is 0, W will be forced to 0. When P is 1, W is also allowed to be 1; thus, both the plant and the warehouse can be expanded. However, we note that the preceding constraint does not force the warehouse expansion project (W) to be accepted if the plant expansion project (P) is accepted.

If the warehouse expansion project had to be accepted whenever the plant expansion project was, and vice versa, we would say that P and W represented **corequisite constraint** projects. To model such a situation, we simply write the preceding constraint as an equality:

$$W = P$$

The constraint forces P and W to take on the same value.

Try Problem 7 for practice with the modeling flexibility provided by 0-1 variables.

The Management Science in Action, Customer Order Allocation Model at Ketron, describes how the modeling flexibility provided by 0-1 variables helped Ketron build a customer order allocation model for a sporting goods company.

CUSTOMER ORDER ALLOCATION MODEL AT KETRON*

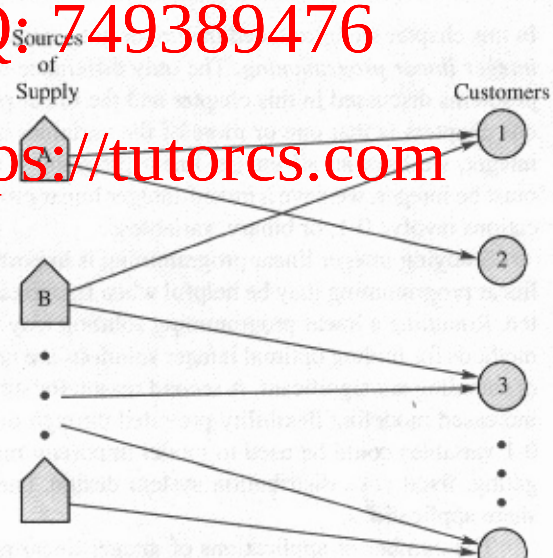
Ketron Management Science provides consulting services for the design and implementation of mathematical programming applications. One such application involved the development of a mixed-integer programming model of the customer order allocation probl[illegible]ods company. The spo[illegible]rkets approximately 30[illegible]0 sources of supply (fact[illegible]ons). The problem is to d[illegible]ocate customer orders to[illegible]pply such that the total m[illegible] products ordered is mi[illegible]rovides a graphical repre[illegible]. Note in the figure that each customer can receive shipments from only a few of the various sources of supply. For example, we see that customer 1 may be supplied by source A or B, customer 2 may be supplied only by source A, and so on.

The sporting equipment company classifies each customer order as either a "guaranteed" or "secondary" order. Guaranteed orders are single-source orders in that they must be filled by a single supplier to ensure that the complete order will be delivered to the customer at one time. This single-source requirement necessitates the use of 0-1 integer variables in the model. Approximately 80% of the company's orders are guaranteed orders. Secondary orders can be split among the various sources of supply. These orders are made by customers restocking inventory, and receiving partial shipments from different sources at different times is not a problem. The 0-1 variables are used to represent the assignment of a guaranteed order to a supplier and continuous variables are used to represent the secondary orders.

Constraints for the problem involve raw material capacities, manufacturing capacities, and individual product capacities. A fairly typical problem has about 800 constraints, 2000 0-1 assignment variables, and 500 continuous variables associated with the secondary orders. The customer order allocation problem is solved periodically as orders are received. In a typical period, between 20 and 40 customers are to be supplied. Because most customers require several products, usually between 600 and 800 orders must be assigned to the sources of supply.

*Based on information provided by J. A. Tomlin of Ketron Management Science.

FIGURE 7.12 GRAPHICAL REPRESENTATION OF THE CUSTOMER ORDER ALLOCATION PROBLEM



Note: Arrows show possible sources of supply for each customer.

A Cautionary Note About Sensitivity Analysis

Sensitivity analysis often is more crucial for integer linear programming problems than for linear programming problems. A small change in one of the coefficients in the constraints can cause a relatively large change in the value of the optimal solution. To understand why, consider the following integer programming model of a simple capital budgeting problem invol[illegible] budgetary constraint for a single time period:

$$40x_1 + 60x_2 + 70x_3 + 160x_4$$

$$16x_1 + 35x_2 + 45x_3 + 85x_4 \le 100$$
$$x_1, x_2, x_3, x_4 = 0, 1$$

We can obtain the optimal solution to this problem by enumerating the alternatives. It is $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, and $x_4 = 0$, with an objective function value of \$170. However, note that if the budget available is increased by \$1 (from \$100 to \$101), the optimal solution changes to $x_1 = 1$, $x_2 = 0$, $x_3 = 0$, and $x_4 = 1$, with an objective function value of \$200. That is, one additional dollar in the budget would lead to a \$30 increase in the return. Surely management, when faced with such a situation, would increase the budget by \$1. Because of the extreme sensitivity of the value of the optimal solution to the constraint coefficients, practitioners usually recommend re-solving the integer linear program several times with slight variations in the coefficients before attempting to choose the best solution for implementation.

SUMMARY

In this chapter we introduced the important extension of linear programming referred to as *integer linear programming*. The only difference between the integer linear programming problems discussed in this chapter and the linear programming problems studied in previous chapters is that one or more of the variables must be integer. If all variables must be integer, we have an all-integer linear program. If some, but not necessarily all, variables must be integer, we have a mixed-integer linear program. Most integer programming applications involve 0-1, or binary, variables.

Studying integer linear programming is important for two major reasons. First, integer linear programming may be helpful when fractional values for the variables are not permitted. Rounding a linear programming solution may not provide an optimal integer solution; methods for finding optimal integer solutions are needed when the economic consequences of rounding are significant. A second reason for studying integer linear programming is the increased modeling flexibility provided through the use of 0-1 variables. We showed how 0-1 variables could be used to model important managerial considerations in capital budgeting, fixed cost, distribution system design, bank location, and product design/market share applications.

The number of applications of integer linear programming continues to grow rapidly. This growth is due in part to the availability of good integer linear programming software packages. As researchers develop solution procedures capable of solving larger integer linear programs and as computer speed increases, a continuation of the growth of integer programming applications is expected.

GLOSSARY

Integ[illegible] [illegible] linear program with the additional requirement that one or more [illegible] integer.

All-i[illegible] An integer linear program in which all variables are required to be [illegible]

LP R[illegible] [illegible] program that results from dropping the integer requirements for th[illegible] linear program.

Mixed-integer linear program An integer linear program in which some, but not necessarily all, variables are required to be integer.

0-1 integer linear program An all-integer or mixed-integer linear program in which the integer variables are only permitted to assume the values 0 or 1. Also called *binary integer program.*

Capital budgeting problem A 0-1 integer programming problem that involves choosing which projects or activities provide the best investment return.

Fixed-cost problem A 0-1 mixed-integer programming problem in which the binary variables represent whether an activity, such as a production run, is undertaken (variable = 1) or not (variable = 0).

Distribution system design problem A mixed-integer linear program in which the binary integer variables usually represent sites selected for warehouses or plants and continuous variables represent the amount shipped over arcs in the distribution network.

Location problem A 0-1 integer programming problem in which the objective is to select the best locations to meet a stated objective. Variations of this problem (see the bank location problem in Section 7.3) are known as covering problems.

Product design and market share optimization problem Sometimes called the share of choices problem, it involves choosing a product design that maximizes the number of consumers preferring it.

Multiple-choice constraint A constraint requiring that the sum of two or more 0-1 variables equal 1. Thus, any feasible solution makes a choice of which variable to set equal to 1.

Mutually exclusive constraint A constraint requiring that the sum of two or more 0-1 variables be less than or equal to 1. Thus, if one of the variables equals 1, the others must equal 0. However, all variables could equal 0.

***k* out of *n* alternatives constraint** An extension of the multiple-choice constraint. This constraint requires that the sum of n 0-1 variables equal k.

Conditional constraint A constraint involving 0-1 variables that does not allow certain variables to equal 1 unless certain other variables are equal to 1.

Corequisite constraint A constraint requiring that two 0-1 variables be equal. Thus, they are both in or out of solution together.

PROBLEMS

1. Indicate which of the following is an all-integer linear program and which is a mixed-integer linear program. Write the LP Relaxation for the problem but do not attempt to solve.

a. Max $30x_1 + 25x_2$

[illegible] ≤ 400

[illegible] ≤ 250

[illegible] ≤ 150

[illegible] and x_2 integer

b. [illegible]

$2x_1 + 4x_2 \geq 8$

$2x_1 + 6x_2 \geq 12$

$x_1, x_2 \geq 0$ and integer

SELF test

2. Consider the following all-integer linear program:

$$
\begin{aligned}
\text{Max} \quad & 5x_1 + 8x_2 \\
\text{s.t.} \quad & \\
& 6x_1 + 5x_2 \leq 30 \\
& 9x_1 + 4x_2 \leq 36 \\
& 1x_1 + 2x_2 \leq 10 \\
& x_1, x_2 \geq 0 \text{ and integer}
\end{aligned}
$$

a. Graph the constraints for this problem. Use dots to indicate all feasible integer solutions.
b. Find the optimal solution to the LP Relaxation. Round down to find a feasible integer solution.
c. Find the optimal integer solution. Is it the same as the solution obtained in part (b) by rounding down?

3. Consider the following all-integer linear program:

$$
\begin{aligned}
\text{Max} \quad & 1x_1 + 1x_2 \\
\text{s.t.} \quad & \\
& 4x_1 + 6x_2 \leq 22 \\
& 1x_1 + 5x_2 \leq 15 \\
& 2x_1 + 1x_2 \leq 9 \\
& x_1, x_2 \geq 0 \text{ and integer}
\end{aligned}
$$

a. Graph the constraints for this problem. Use dots to indicate all feasible integer solutions.
b. Solve the LP Relaxation of this problem.
c. Find the optimal integer solution.

4. Consider the following all-integer linear program:

$$
\begin{aligned}
\text{Max} \quad & 10x_1 + 3x_2 \\
\text{s.t.} \quad & \\
& 6x_1 + 7x_2 \leq 40 \\
& 3x_1 + 1x_2 \leq 11 \\
& x_1, x_2 \geq 0 \text{ and integer}
\end{aligned}
$$

a. Formulate and solve the LP Relaxation of the problem. Solve it graphically, and round down to find a feasible solution. Specify an upper bound on the value of the optimal solution.
b. Solve the integer linear program graphically. Compare the value of this solution with the solution obtained in part (a).
c. Suppose the objective function changes to Max $3x_1 + 6x_2$. Repeat parts (a) and (b).

SELF test

5. Consider the following mixed-integer linear program:

$$\begin{aligned} \text{Max} \quad & 2x_1 + 3x_2 \\ \text{s.t.} \quad & \\ & 4x_1 + 9x_2 \le 36 \\ & 7x_1 + 5x_2 \le 35 \\ & x_1, x_2 \ge 0 \text{ and } x_1 \text{ integer} \end{aligned}$$

a. Graph the constraints for this problem. Indicate on your graph all feasible mixed-integer solutions.
b. Find the optimal solution to the LP Relaxation. Round the value of x_1 down to find a feasible mixed-integer solution. Is this solution optimal? Why or why not?
c. Find the optimal solution for the mixed-integer linear program.

6. Consider the following mixed-integer linear program:

$$\begin{aligned} \text{Max} \quad & 1x_1 + 1x_2 \\ \text{s.t.} \quad & \\ & 7x_1 + 9x_2 \le 63 \\ & 9x_1 + 5x_2 \le 45 \\ & 3x_1 + 1x_2 \le 12 \\ & x_1, x_2 \ge 0 \text{ and } x_2 \text{ integer} \end{aligned}$$

a. Graph the constraints for this problem. Indicate on your graph all feasible mixed-integer solutions.
b. Find the optimal solution to the LP Relaxation. Round the value of x_2 down to find a feasible mixed-integer solution. Specify upper and lower bounds on the value of the optimal solution to the mixed-integer linear program.
c. Find the optimal solution to the mixed-integer linear program.

SELF test

7. The following questions refer to a capital budgeting problem with six projects represented by 0-1 variables x_1, x_2, x_3, x_4, x_5, and x_6:
a. Write a constraint modeling a situation in which two of the projects 1, 3, 5, and 6 must be undertaken.
b. Write a constraint modeling a situation in which, if projects 3 and 5 must be undertaken, they must be undertaken simultaneously.
c. Write a constraint modeling a situation in which project 1 or 4 must be undertaken, but not both.
d. Write constraints modeling a situation where project 4 cannot be undertaken unless projects 1 and 3 also are undertaken.
e. In addition to the requirement in part (d), assume that when projects 1 and 3 are undertaken, project 4 also must be undertaken.

8. Spencer Enterprises must choose among a series of new investment alternatives. The potential investment alternatives, the net present value of the future stream of returns,

the capital requirements, and the available capital funds over the next three years are summarized as follows:

Alternative	Net Present Value ($)	Capital Requirements ($) Year 1	Year 2	Year 3
Li[illegible]on	4,000	3,000	1,000	4,000
E[illegible]sion	6,000	2,500	3,500	3,500
T[illegible]	10,500	6,000	4,000	5,000
A[illegible]	4,000	2,000	1,500	1,800
B[illegible]	8,000	5,000	1,000	4,000
P[illegible]	3,000	1,000	500	900
[illegible]		10,500	7,000	8,750

a. Develop and solve an integer programming model for maximizing the net present value.

b. Assume that only one of the warehouse expansion projects can be implemented. Modify your model of part (a).

c. Suppose that, if test marketing of the new product is carried out, the advertising campaign also must be conducted. Modify your formulation of part (b) to reflect this new situation.

9. Hawkins Manufacturing Company produces connecting rods for 4- and 6-cylinder automobile engines using the same production line. The cost required to set up the production line to produce the 4-cylinder connecting rods is $2000, and the cost required to set up the production line for the 6-cylinder connecting rods is $3500. Manufacturing costs are $15 for each 4-cylinder connecting rod and $18 for each 6-cylinder connecting rod. Hawkins makes a decision at the end of each week as to which product will be manufactured the following week. If a production changeover is necessary from one week to the next, the weekend is used to reconfigure the production line. Once the line has been set up, the weekly production capacities are 6000 6-cylinder connecting rods and 8000 4-cylinder connecting rods. Let

x_4 = the number of 4-cylinder connecting rods produced next week

x_6 = the number of 6-cylinder connecting rods produced next week

s_4 = 1 if the production line is set up to produce the 4-cylinder connecting rods; 0 if otherwise

s_6 = 1 if the production line is set up to produce the 6-cylinder connecting rods; 0 if otherwise

a. Using the decision variables x_4 and s_4, write a constraint that limits next week's production of the 4-cylinder connecting rods to either 0 or 8000 units.

b. Using the decision variables x_6 and s_6, write a constraint that limits next week's production of the 6-cylinder connecting rods to either 0 or 6000 units.

c. Write three constraints that, taken together, limit the production of connecting rods for next week.

d. Write an objective function for minimizing the cost of production for next week.

10. Grave City is considering the relocation of several police substations to obtain better enforcement in high-crime areas. The locations under consideration together with the areas that can be covered from these locations are given in the following table:

Potential Locations for Substations	Areas Covered
A	1, 5, 7
B	1, 2, 5, 7
C	1, 3, 5
D	2, 4, 5
E	3, 4, 6
F	4, 5, 6
G	1, 5, 6, 7

a. Formulate an integer programming model that could be used to find the minimum number of locations necessary to provide coverage to all areas.
b. Solve the problem in part (a).

11. Hart Manufacturing makes three products. Each product requires manufacturing operations in three departments: A, B, and C. The labor-hour requirements, by department, are as follows:

Department	Product 1	Product 2	Product 3
A	1.50	3.00	2.00
B	2.00	1.00	2.50
C	0.25	0.25	0.25

During the next production period, the labor-hours available are 450 in department A, 350 in department B, and 50 in department C. The profit contributions per unit are $25 for product 1, $28 for product 2, and $30 for product 3.

a. Formulate a linear programming model for maximizing total profit contribution.
b. Solve the linear program formulated in part (a). How much of each product should be produced, and what is the projected total profit contribution?
c. After evaluating the solution obtained in part (b), one of the production supervisors noted that production setup costs had not been taken into account. She noted that setup costs are $400 for product 1, $550 for product 2, and $600 for product 3. If the solution developed in part (b) is to be used, what is the total profit contribution after taking into account the setup costs?

d. Management realized that the optimal product mix, taking setup costs into account, might be different from the one recommended in part (b). Formulate a mixed-integer linear program that takes setup costs into account. Management also stated that we should not consider making more than 175 units of product 1, 150 units of product 2, or 140 units of product 3.
e. Solve the mixed-integer linear program formulated in part (d). How much of each product should be produced, and what is the projected total profit contribution? Compare this profit contribution to that obtained in part (c).

12. Yates Company supplies road salt to county highway departments. The company has three trucks, and the dispatcher is trying to schedule tomorrow's deliveries to Polk, Dallas, and Jasper counties. Two trucks have 15-ton capacities, and the third truck has a 30-ton capacity. Based on these truck capacities, two counties will receive 15 tons and the third will

receive 30 tons of road salt. The dispatcher wants to determine how much to ship to each county. Let

P = amount shipped to Polk County
D = amount shipped to Dallas County
J = amount shipped to Jasper County

[illegible] 1 if the 30-ton truck is assigned to county i
[illegible] 0 otherwise

[illegible] definitions and write constraints that appropriately restrict the [illegible]ch county.

[illegible] the 30-ton truck to the three counties is \$100 to Polk, \$85 to [illegible]sper. Formulate and solve a mixed-integer linear program to determine how much to ship to each county.

SELF test

13. Recall the Martin-Beck Company distribution system problem in Section 7.3.
    a. Modify the formulation shown in Section 7.3 to account for the policy restriction that one plant, but not two, must be located either in Detroit or in Toledo.
    b. Modify the formulation shown in Section 7.3 to account for the policy restriction that no more than two plants can be located in Denver, Kansas City, and St. Louis.

14. An automobile manufacturer has five outdated plants: one each in Michigan, Ohio, and California and two in New York. Management is considering modernizing these plants to manufacture engine blocks and transmissions for a new model car. The cost to modernize each plant and the manufacturing capacity after modernization are as follows:

Plant	Cost ($ millions)	Engine Blocks (1000s)	Transmissions (1000s)
Michigan	25	500	300
New York	35	800	400
New York	35	400	800
Ohio	40	900	600
California	20	200	300

The projected needs are for total capacities of 900,000 engine blocks and 900,000 transmissions. Management wants to determine which plants to modernize to meet projected manufacturing needs and, at the same time, minimize the total cost of modernization.
   a. Develop a table that lists every possible option available to management. As part of your table, indicate the total engine block capacity and transmission capacity for each possible option, whether the option is feasible based on the projected needs, and the total modernization cost for each option.
   b. Based on your analysis in part (a), what recommendation would you provide management?
   c. Formulate a 0-1 integer programming model that could be used to determine the optimal solution to the modernization question facing management.
   d. Solve the model formulated in part (c) to provide a recommendation for management.

15. CHB, Inc., is a bank holding company that is evaluating the potential for expanding into a 13-county region in the southwestern part of the state. State law permits establishing branches in any county that is adjacent to a county in which a PPB (principal place of business) is located. The following map shows the 13-county region with the population of each county indicated.

a. Assume that only one PPB can be established in the region. Where should it be located to maximize the population served? (*Hint:* Review the Ohio Trust formulation in Section 7.3. Consider minimizing the population not served, and introduce variable $y_i = 1$ if it is not possible to establish a branch in county i, and $y_i = 0$ otherwise.)

b. Suppose that two PPBs can be established in the region. Where should they be located to maximize the population served?

c. Management learned that a bank located in county 5 is considering selling. If CHB purchases this bank, the requisite PPB will be established in county 5, and a base for beginning expansion in the region will also be established. What advice would you give the management of CHB?

16. The Northshore Bank is working to develop an efficient work schedule for full-time and part-time tellers. The schedule must provide for efficient operation of the bank including adequate customer service, employee breaks, and so on. On Fridays the bank is open from 9:00 A.M. to 7:00 P.M. The number of tellers necessary to provide adequate customer service during each hour of operation is summarized here.

Time	Number of Tellers	Time	Number of Tellers
9:00 A.M.–10:00 A.M.	6	2:00 P.M.–3:00 P.M.	6
10:00 A.M.–11:00 A.M.	4	3:00 P.M.–4:00 P.M.	4
11:00 A.M.–Noon	8	4:00 P.M.–5:00 P.M.	7
Noon–1:00 P.M.	10	5:00 P.M.–6:00 P.M.	6
1:00 P.M.–2:00 P.M.	9	6:00 P.M.–7:00 P.M.	6

Each full-time employee starts on the hour and works a 4-hour shift, followed by 1 hour for lunch and then a 3-hour shift. Part-time employees work one 4-hour shift beginning on the hour. Considering salary and fringe benefits, full-time employees cost the bank \$15 per hour (\$105 a day), and part-time employees cost the bank \$8 per hour (\$32 per day).

a. Formulate an integer programming model that can be used to develop a schedule that will satisfy customer service needs at a minimum employee cost. (*Hint:* Let x_i = num- ...loyees coming on duty at the beginning of hour i and y_i = num- ...loyees coming on duty at the beginning of hour i.)

...ion of your model in part (a).

... schedule of tellers. Comment on the solution.

...solution to part (c), the bank manager realized that some addi- ...must be specified. Specifically, she wants to ensure that one full- ... duty at all times and that there is a staff of at least five full-time ...our model to incorporate these additional requirements and solve for the optimal solution.

17. Refer to the Ohio Trust bank location problem introduced in Section 7.3. Table 7.3 shows the counties under consideration and the adjacent counties.

a. Write the complete integer programming model for expansion into the following counties only: Lorain, Huron, Richland, Ashland, Wayne, Medina, and Knox.
b. Use trial and error to solve the problem in part (a).
c. Use a computer program for integer programs to solve the problem.

18. Refer to the Salem Foods share of choices problem in Section 7.3 and address the following issues. It is rumored that King's is getting out of the frozen pizza business. If so, the major competitor for Salem Foods will be the Antonio's brand pizza.

a. Compute the overall utility for the Antonio's brand pizza for each of the consumers in Table 7.4.
b. Assume that Salem's only competitor is the Antonio's brand pizza. Formulate and solve the share of choices problem that will maximize market share. What is the best product design and what share of the market can be expected?

19. Burnside Marketing Research conducted a study for Barker Foods on some designs for a new dry cereal. Three attributes were found to be most influential in determining which cereal had the best taste: ratio of wheat to corn in the cereal flake, type of sweetener (sugar, honey, or artificial), and the presence or absence of flavor bits. Seven children participated in taste tests and provided the following part-worths for the attributes:

	Wheat/Corn		Sweetener			Flavor Bits	
Child	**Low**	**High**	**Sugar**	**Honey**	**Artificial**	**Present**	**Absent**
1	15	35	30	40	25	15	9
2	30	20	40	35	35	8	11
3	40	25	20	40	10	7	14
4	35	30	25	20	30	15	18
5	25	40	40	20	35	18	14
6	20	25	20	35	30	9	16
7	30	15	25	40	40	20	11

a. Suppose the overall utility (sum of part-worths) of the current favorite cereal is 75 for each child. What is the product design that will maximize the share of choices for the seven children in the sample?
b. Assume the overall utility of the current favorite cereal for the first four children in the group is 70, and the overall utility of the current favorite cereal for the last three children

in the group is 80. What is the product design that will maximize the share of choices for the seven children in the sample?

20. Refer to Problem 14. Suppose that management determined that its cost estimates to modernize the New York plants were too low. Specifically, suppose that the actual cost is $40 million to modernize each plant.

a. What changes in your previous 0-1 integer linear programming model are needed to [illegible]ges in costs?

b. [illegible]s, what recommendations would you now provide management [illegible]zation plan?

c. [illegible]n obtained using the revised cost figures. Suppose that management [illegible]sing two plants in the same state is not acceptable. How could [illegible] be added to your 0-1 integer programming model?

d. [illegible]sion and the policy restriction presented in part (c), what recom[illegible]u now provide management regarding the modernization plan?

21. The Bayside Art Gallery is considering installing a video camera security system to reduce its insurance premiums. A diagram of the eight display rooms that Bayside uses for exhibitions is shown in Figure 7.13; the openings between the rooms are numbered 1 through 13. A security firm proposed that two-way cameras be installed at some room openings. Each camera has the ability to monitor the two rooms between which the camera is located. For example, if a camera were located at opening number 4, rooms 1 and 4 would be covered; if a camera were located at opening 11, rooms 7 and 8 would be covered; and so on. Management decided not to locate a camera system at the entrance to the display rooms. The objective is to provide security coverage for all eight rooms using the minimum number of two-way cameras.

a. Formulate a 0-1 integer linear programming model that will enable Bayside's management to determine the locations for the camera systems.

b. Solve the model formulated in part (a) to determine how many two-way cameras to purchase and where they should be located.

c. Suppose that management wants to provide additional security coverage for room 7. Specifically, management wants room 7 to be covered by two cameras. How would your model formulated in part (a) have to change to accommodate this policy restriction?

d. With the policy restriction specified in part (c), determine how many two-way camera systems will need to be purchased and where they will be located.

22. The Delta Group is a management consulting firm specializing in the health care industry. A team is being formed to study possible new markets, and a linear programming model has been developed for selecting team members. However, one constraint the president imposed is a team size of three, five, or seven members. The staff cannot figure out how to incorporate this requirement in the model. The current model requires that team members be selected from three departments and uses the following variable definitions:

$$x_1 = \text{the number of employees selected from department 1}$$
$$x_2 = \text{the number of employees selected from department 2}$$
$$x_3 = \text{the number of employees selected from department 3}$$

Show the staff how to write constraints that will ensure that the team will consist of three, five, or seven employees. The following integer variables should be helpful:

$$y_1 = \begin{cases} 1 & \text{if team size is 3} \\ 0 & \text{otherwise} \end{cases}$$

$$y_2 = \begin{cases} 1 & \text{if team size is 5} \\ 0 & \text{otherwise} \end{cases}$$

$$y_3 = \begin{cases} 1 & \text{if team size is 7} \\ 0 & \text{otherwise} \end{cases}$$

FIGURE 7.13 DIAGRAM OF DISPLAY ROOMS FOR BAYSIDE ART GALLERY

23. Roedel Electronics produces a variety of electrical components, including a remote controller for televisions and a remote controller for VCRs. Each controller consists of three subassemblies that are manufactured by Roedel: a base, a cartridge, and a keypad. Both controllers use the same base subassembly, but different cartridge and keypad subassemblies.

Roedel's sales forecast indicates that 7000 TV controllers and 5000 VCR controllers will be needed to satisfy demand during the upcoming Christmas season. Because only 500 hours of in-house manufacturing time are available, Roedel is considering purchasing some, or all, of the subassemblies from outside suppliers. If Roedel manufactures a subassembly in-house, it incurs a fixed setup cost as well as a variable manufacturing cost. The following table shows the setup cost, the manufacturing time per subassembly, the manufacturing cost per subassembly, and the cost to purchase each of the subassemblies from an outside supplier:

Subassembly	Setup Cost ($)	Manufacturing Time per Unit (min.)	Manufacturing Cost per Unit ($)	Purchase Cost per Unit ($)
Base	[illegible]	0.9	0.40	0.65
TV cartridge	[illegible]	2.2	2.90	3.45
VCR cartridge	[illegible]	3.0	3.15	3.70
TV keypad	[illegible]	0.8	0.30	0.50
VCR keypad	[illegible]	1.0	0.55	0.70

a. Determine how many units of each subassembly Roedel should manufacture and how many units Roedel should purchase. What is the total manufacturing and purchase cost associated with your recommendation?

b. Suppose Roedel is considering purchasing new machinery to produce VCR cartridges. For the new machinery, the setup cost is $3000; the manufacturing time is 2.5 minutes per cartridge, and the manufacturing cost is $2.60 per cartridge. Assuming that the new machinery is purchased, determine how many units of each subassembly Roedel should manufacture and how many units of each subassembly Roedel should purchase? What is the total manufacturing and purchase cost associated with your recommendation? Do you think the new machinery should be purchased? Explain.

24. A mathematical programming system named SilverScreener uses a 0-1 integer programming model to help theater managers decide which movies to show on a weekly basis in a multiple-screen theater (*Interfaces*, May/June 2001). Suppose that management of Valley Cinemas would like to investigate the potential of using a similar scheduling system for their chain of multiple-screen theaters. Valley selected a small two-screen movie theater for the pilot testing and would like to develop an integer programming model to help schedule movies for the next four weeks. Six movies are available. The first week each movie is available, the last week each movie can be shown, and the maximum number of weeks that each movie can run are shown here:

Movie	First Week Available	Last Week Available	Max. Run (weeks)
1	1	2	2
2	1	3	2
3	1	1	2
4	2	4	2
5	3	6	3
6	3	5	3

The overall viewing schedule for the theater is composed of the individual schedules for each of the six movies. For each movie, a schedule must be developed that specifies the week the movie starts and the number of consecutive weeks it will run. For instance, one possible schedule for movie 2 is for it to start in week 1 and run for two weeks. Theater policy requires that once a movie is started it must be shown in consecutive weeks. It cannot be stopped and restarted again. To represent the schedule possibilities for each movie, the following decision variables were developed:

$$x_{ijw} = \begin{cases} 1 & \text{if movie } i \text{ is scheduled to start in week } j \text{ and run for } w \text{ weeks} \\ 0 & \text{otherwise} \end{cases}$$

...ans that the schedule selected for movie 5 is to begin in week 3 ... For each movie, a separate variable is given for each possible ...

... associated with movie 1. List the variables that represent these ...

... uiring that only one schedule be selected for movie 1.
... uiring that only one schedule be selected for movie 5.
... mber of movies that can be shown in week 1? Write a constraint that restricts the number of movies selected for viewing in week 1.

e. Write a constraint that restricts the number of movies selected for viewing in week 3.

25. East Coast Trucking provides service from Boston to Miami using regional offices located in Boston, New York, Philadelphia, Baltimore, Washington, Richmond, Raleigh, Florence, Savannah, Jacksonville, and Tampa. The number of miles between each of the regional offices is provided in the following table:

	New York	Philadelphia	Baltimore	Washington	Richmond	Raleigh	Florence	Savannah	Jacksonville	Tampa	Miami
Boston	211	320	424	459	565	713	884	1056	1196	1399	1669
New York		109	[illegible]	248	354	[illegible]	673	845	985	1188	1458
Philadelphia			104	139	245	393	564	736	876	1079	1349
Baltimore				35	141	289	460	632	772	975	1245
Washington					106	254	425	597	737	940	1210
Richmond						148	319	491	631	834	1104
Raleigh							171	343	483	686	956
Florence								172	312	515	785
Savannah									140	343	613
Jacksonville										203	473
Tampa											270

The company's expansion plans involve constructing service facilities in some of the cities where a regional office is located. Each regional office must be within 400 miles of a service facility. For instance, if a service facility is constructed in Richmond, it can provide service to regional offices located in New York, Philadelphia, Baltimore, Washington, Richmond, Raleigh, and Florence. Management would like to determine the minimum number of service facilities needed and where they should be located.

a. Formulate an integer linear program that can be used to determine the minimum number of service facilities needed and their location.
b. Solve the linear program formulated in part (a). How many service facilities are required, and where should they be located?
c. Suppose that each service facility can only provide service to regional offices within 300 miles. How many service facilities are required, and where should they be located?

Case Problem 1 TEXTBOOK PUBLISHING

ASW Publishing, Inc., a small publisher of college textbooks, must make a decision regarding which books to publish next year. The books under consideration are listed in the following table, along with the projected three-year sales expected from each book:

Bo[illegible]	Type of Book	Projected Sales ($1000s)
Bu[illegible]	New	20
Fi[illegible]	Revision	30
Ge[illegible]	New	15
M[illegible]	New	10
Bu[illegible]	Revision	25
Fi[illegible]	New	18
Financial accounting	New	25
Managerial accounting	Revision	50
English literature	New	20
German	New	30

The books listed as revisions are texts that ASW already has under contract; these texts are being considered for publication as new editions. The books that are listed as new have been reviewed by the company, but contracts have not yet been signed.

Three individuals in the company can be assigned to these projects, all of whom have varying amounts of time available; John has 60 days available, and Susan and Monica both have 40 days available. The days required by each person to complete each project are shown in the following table. For instance, if the business calculus book is published, it will require 30 days of John's time and 40 days of Susan's time. An "X" indicates that the person will not be used on the project. Note that at least two staff members will be assigned to each project except the finance book.

Book Subject	John	Susan	Monica
Business calculus	30	40	X
Finite mathematics	16	24	X
General statistics	24	X	30
Mathematical statistics	20	X	24
Business statistics	10	X	16
Finance	X	X	14
Financial accounting	X	24	26
Managerial accounting	X	28	30
English literature	40	34	30
German	X	50	36

ASW will not publish more than two statistics books or more than one accounting text in a single year. In addition, management decided that one of the mathematics books (business calculus or finite math) must be published, but not both.

Managerial Report

Prepare a report for the managing editor of ASW that describes your findings and recommendations regarding the best publication strategy for next year. In carrying out your analysis, assume that the fixed costs and the sales revenues per unit are approximately equal for all books; management is interested primarily in maximizing the total unit sales volume.

The managing editor also asked that you include recommendations regarding the following possible changes:

1. [illegible] geous to do so, Susan can be moved off another project to [illegible] more days.
2. [illegible] eous to do so, Monica can also be made available for another [illegible]
3. [illegible] visions could be postponed for another year, should they be? [illegible] will risk losing market share by postponing a revision.

Inclu[illegible] s in an appendix to your report.

Case Problem 2 YEAGER NATIONAL BANK

Using aggressive mail promotion with low introductory interest rates, Yeager National Bank (YNB) built a large base of credit card customers throughout the continental United States. Currently, all customers send their regular payments to the bank's corporate office located in Charlotte, North Carolina. Daily collections from customers making their regular payments are substantial, with an average of approximately $600,000. YNB estimates that it makes about 15 percent on its funds and would like to ensure that customer payments are credited to the bank's account as soon as possible. For instance, if it takes five days for a customer's payment to be sent through the mail, processed, and credited to the bank's account, YNB has potentially lost five days' worth of interest income. Although the time needed for this collection process cannot be completely eliminated, reducing it can be beneficial given the large amounts of money involved.

Instead of having all its credit card customers send their payments to Charlotte, YNB is considering having customers send their payments to one or more regional collection centers, referred to in the banking industry as lockboxes. Four lockbox locations have been proposed: Phoenix, Salt Lake City, Atlanta, and Boston. To determine which lockboxes to open and where lockbox customers should send their payments, YNB divided its customer base into five geographical regions: Northwest, Southwest, Central, Northeast, and Southeast. Every customer in the same region will be instructed to send his or her payment to the same lockbox. The following table shows the average number of days it takes before a customer's payment is credited to the bank's account when the payment is sent from each of the regions to each of the potential lockboxes:

Customer Zone	Location of Lockbox: Phoenix	Salt Lake City	Atlanta	Boston	Daily Collection ($1000s)
Northwest	4	2	4	4	80
Southwest	2	3	4	6	90
Central	5	3	3	4	150
Northeast	5	4	3	2	180
Southeast	4	6	2	3	100

Managerial Report

Dave Wolff, the vice president for cash management, asked you to prepare a report containing your recommendations for the number of lockboxes and the best lockbox locations. Mr. Wolff is primarily concerned with minimizing lost interest income, but he wants you to also consider the effect of an annual fee charged for maintaining a lockbox at any location. Althou[illegible]e is unknown at this time, we can assume that the fees will be in t[illegible]30,000 per location. Once good potential locations have been select[illegible]e as to the annual fees.

Case Problem 3 PRODUCTION SCHEDULING WITH CHANGEOVER COSTS

Buckeye Manufacturing produces heads for engines used in the manufacture of trucks. The production line is highly complex, and it measures 900 feet in length. Two types of engine heads are produced on this line: the P-Head and the H-Head. The P-Head is used in heavy-duty trucks and the H-Head is used in smaller trucks. Because only one type of head can be produced at a time, the line is set up to manufacture either the P-Head or the H-Head, but not both. Changeovers are made over a weekend; costs are $500 in going from a setup for the P-Head to a setup for the H-Head, and vice versa. When set up for the P-Head, the maximum production rate is 100 units per week and when set up for the H-Head, the maximum production rate is 80 units per week.

Buckeye just shut down for the week after using the line to produce the P-Head. The manager wants to plan production and changeovers for the next eight weeks. Currently, Buckeye's inventory consists of 125 P-Heads and 143 H-Heads. Inventory carrying costs are charged at an annual rate of 19.5 percent of the value of inventory. The production cost for the P-Head is $225, and the production cost for the H-Head is $310. The objective in developing a production schedule is to minimize the sum of production cost, plus inventory carrying cost, plus changeover cost.

Buckeye received the following requirements schedule from its customer (an engine assembly plant) for the next nine weeks:

	Product Demand	
Week	P-Head	H-Head
1	55	38
2	55	38
3	44	30
4	0	0
5	45	48
6	45	48
7	36	58
8	35	57
9	35	58

Safety stock requirements are such that week-ending inventory must provide for at least 80 percent of the next week's demand.

Managerial Report

Prepare a report for Buckeye's management with a production and changeover schedule for the next eight weeks. Be sure to note how much of the total cost is due to production, how much is due to inventory, and how much is due to changeover.

Appendix 7 [illegible] TION OF INTEGER LINEAR PROGRAMS

Work[illegible] olution for integer linear programs are similar to that for linear p[illegible] Actually the worksheet formulation is exactly the same, but some[illegible] must be provided when setting up the **Solver Parameters** and **Solve**[illegible]. First, constraints must be added in the **Solver Parameters** dialo[illegible] teger variables. In addition, the value for **Tolerance** in the **Options** dialog box will need to be adjusted to obtain an optimal solution.

Let us demonstrate the Excel solution of an integer linear program by showing how Excel can be used to solve the Eastborne Realty problem. The worksheet with the optimal solution is shown in Figure 7.14. We will describe the key elements of the worksheet and how to obtain the solution, and then interpret the solution.

Formulation

The data and descriptive labels appear in cells A1:G7 of the worksheet in Figure 7.14. The screened cells in the lower portion of the worksheet contain the information required by the

FIGURE 7.14 EXCEL SOLUTION FOR THE EASTBORNE REALTY PROBLEM

WEB file
Eastborne

	A	B	C	D	E	F	G	H
1	**Eastborne Realty Problem**							
2								
3		Townhouse	Apt. Bldg.					
4	**Price($1000s)**	282	400			**Funds Avl.($1000s)**	2000	
5	**Mgr. Time**	4	40			**Mgr. Time Avl.**	140	
6						**Townhouses Avl.**	5	
7	**Ann. Cash Flow ($1000s)**	10	15					
8								
9								
10	**Model**							
11								
12								
13	**Max Cash Flow**	70						
14					**Constraints**	**LHS**		**RHS**
15		**Number of**			Funds	1928	<=	2000
16		Townhouses	Apt. Bldgs.		Time	96	<=	140
17	**Purchase Plan**	4	2		Townhouses	4	<=	5
18								

Excel Solver (decision variables, objective function, constraint left-hand sides, and constraint right-hand sides).

Decision Variables — Cells B17:C17 are reserved for the decision variables. The optimal solution is to purchase four townhouses and two apartment buildings.

Objective Function — The formula =SUMPRODUCT(B7:C7,B17:C17) has been ...ed into cell B13 to reflect the annual cash flow associated ... the solution. The optimal solution provides an annual cash ... of $70,000.

Le... — ...left-hand sides for the three constraints are placed into cells ...F17.

...ell F15 =SUMPRODUCT(B4:C4,$B$17:$C$17)

(Copy to sell F16)

...ell F17 =B17

Right-Hand Sides — The right-hand sides for the three constraints are placed into cells H15:H17.

Cell H15 =G4 (Copy to cells H16:H17)

Excel Solution

Begin the solution procedure by selecting the Data tab and Solver from the Analysis group, and entering the proper values into the **Solver Parameters** dialog box as shown in Figure 7.15. The first constraint shown is $B$17:$C$17 = integer. This constraint tells Solver that the decision variables in cell B17 and cell C17 must be integer. The integer requirement is created by using the **Add-Constraint** procedure. $B$17:$C$17 is entered in the left-hand box of the **Cell Reference** area and **"int"** rather than <=, =, or >= is selected as the form of the constraint. When **"int"** is selected, the term integer automatically appears as the right-hand side of the constraint. Figure 7.15 shows the additional information required to complete the **Solver Parameters** dialog box.

FIGURE 7.15 SOLVER PARAMETERS DIALOG BOX FOR THE EASTBORNE REALTY PROBLEM

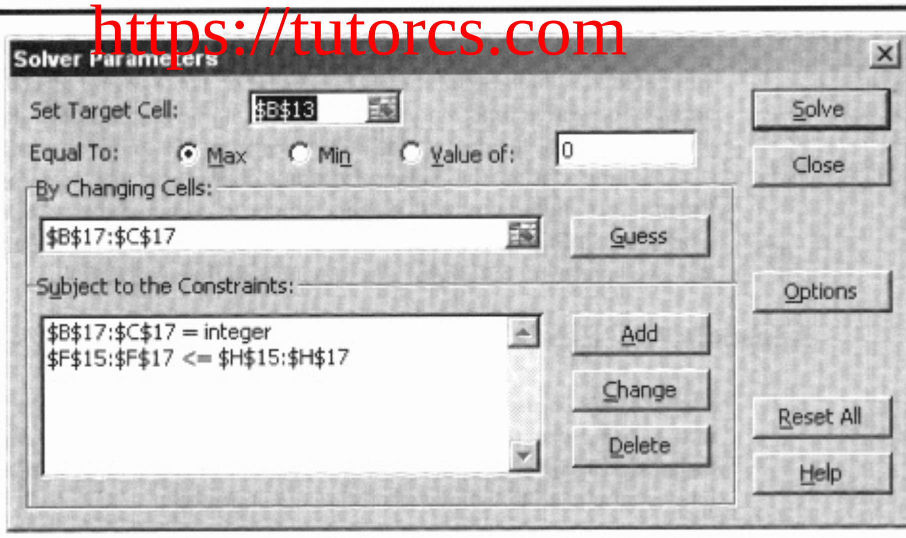


FIGURE 7.16 SOLVER OPTIONS DIALOG BOX FOR THE EASTBORNE REALTY PROBLEM

Next the **Options** button must be selected. The options **Assume Non-Negative** and **Assume Linear Model** must be checked. Figure 7.16 shows the **Solver Options** dialog box for the Eastborne Realty problem after completing this step.

*0-1 variables are identified with the "bin" designation in the **Solver Parameters** dialog box.*

If binary variables are present in an integer linear programming problem, you must select the designation "bin" instead of "int" when setting up the constraints in the **Solver Parameters** dialog box.

To ensure you will find the optimal solution to an integer program using Excel Solver, be sure to set the Tolerance parameter in the Solver Options dialog box to 0%.

The time required to obtain an optimal solution can be highly variable for integer linear programs. As shown in Figure 7.16, the **Tolerance** parameter in the **Solver Options** dialog box has a default value of 5%. This means **Solver** will stop its search when it can guarantee that the best solution it has found so far is within 5% of the optimal solution in terms of objective function value. Therefore, to be assured of finding an optimal solution, you must change the default for **Tolerance** from 5% to 0% as shown in Figure 7.17.

Clicking **OK** in the **Solver Options** dialog box and selecting **Solve** in the **Solver Parameters** dialog box will instruct Solver to compute the optimal integer solution. The worksheet in Figure 7.14 shows that the optimal solution is to purchase four townhouses and two apartment buildings. The annual cash flow is $70,000.

Appendix 7.2 LINGO SOLUTION OF INTEGER LINEAR PROGRAMS

LINGO may be used to solve linear integer programs. An integer linear model is entered into LINGO exactly as described in Appendix 7.2, but with additional statements for declaring variables as either general integer or binary. For example, to declare a variable x integer, you need to include the following statement:

```
@GIN(x);
```

FIGURE 7.17 SOLVER OPTIONS DIALOG BOX WITH TOLERANCE SET TO 0% FOR THE EASTBORNE REALTY PROBLEM

Note the use of the semicolon to end the statement. GIN stands for "general integer." Likewise to declare a variable y a binary variable, the following statement is required:

@BIN(y);

BIN stands for "binary."

To illustrate the use of integer variables, the following statements are used to model the Eastborne Realty problem discussed in this chapter.

First, we enter the following:

```
MODEL:
TITLE   EASTBORNE REALTY;
```

This statement gives the LINGO model the title Eastborne Realty.

Next, we enter the following two lines to document the definition of our decision variables (recall that ! denotes a comment, and each comment ends with a semicolon).

```
! T = NUMBER OF TOWNHOUSES PURCHASED;
! A = NUMBER OF APARTMENT BUILDINGS PURCHASED;
```

Next, we enter the objective function and constraints, each with a descriptive comment.

```
! MAXIMIZE THE CASH FLOW;
MAX = 10*T + 15*A;
```

```
! FUNDS AVAILABLE ($1000);
282*T + 400*A <= 2000;

! TIME AVAILABILITY;
4*T + 40*A <= 140;

! ...ABLE;
T...
```

Final... ariables T and A as general integer variables. Again, to document... th a descriptive comment and then declare each variable as a gener...

```
! ...ABLES TO BE GENERAL INTEGER VARIABLES;
@GIN(T);
@GIN(A);
```

The complete LINGO model is available on the Web.