

# CHAPTER 6

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Distribution  
Models



work

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The models discussed in this chapter belong to a special class of linear programming problems called *network flow* problems. Five different problems are considered:

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- Transportation problem
- Assignment problem
- Transshipment problem
- Shortest route problem
- Minimax problem



A set of five problems is related to these problems because of the similarity in the problem structure. In each case, we will present a graphical representation of the problem as a *network*. We will then show how the problem can be formulated as a linear programming problem. In the last section of the chapter we present a production planning problem that is an interesting application of the transshipment problem.

### 6.1 TRANSPORTATION PROBLEM

The **Transportation problem** arises frequently in planning for the distribution of goods and services from several supply locations to several demand locations. Typically, the quantity of goods available at each supply location (origin) is limited, and the quantity of goods needed at each of several demand locations (destinations) is known. The usual objective in a transportation problem is to minimize the cost of shipping goods from the origins to the destinations.

Let us illustrate by considering a transportation problem faced by Foster Generators. This problem involves the transportation of a product from three plants to four distribution centers. Foster Generator operates plants in Cleveland, Ohio; Bedford, Indiana; and York, Pennsylvania. Production capacities over the next three-month planning period for one particular type of generator are as follows:

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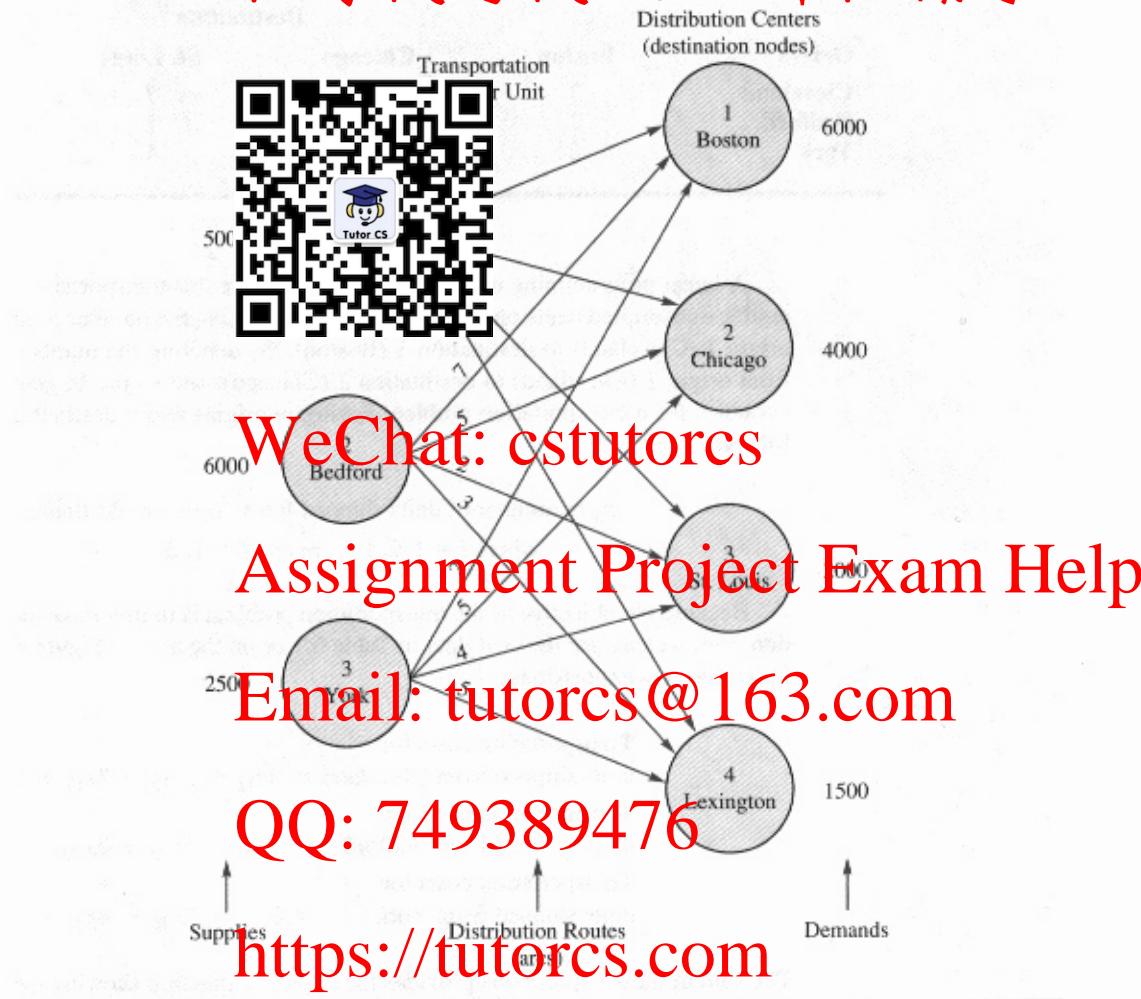
Origin	Plant	Three-Month Production Capacity (units)
1	Cleveland	5,000
2	Bedford	6,000
3	York	2,500
Total		13,500

The firm distributes its generators through four regional distribution centers located in Boston, Chicago, St. Louis, and Lexington; the three-month forecast of demand for the distribution centers is as follows:

Destination	Distribution Center	Three-Month Demand Forecast (units)
1	Boston	6,000
2	Chicago	4,000
3	St. Louis	2,000
4	Lexington	1,500
Total		13,500

**FIGURE 6.1 THE NETWORK REPRESENTATION OF THE FOSTER GENERATORS TRANSPORTATION PROBLEM**

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Management would like to determine how much of its production should be shipped from each plant to each distribution center. Figure 6.1 shows graphically the 12 distribution routes Foster can use. Such a graph is called a **network**; the circles are referred to as **nodes** and the lines connecting the nodes as **arcs**. Each origin and destination is represented by a node, and each possible shipping route is represented by an arc. The amount of the supply is written next to each origin node, and the amount of the demand is written next to each destination node. The goods shipped from the origins to the destinations represent the flow in the network. Note that the direction of flow (from origin to destination) is indicated by the arrows.

For Foster's transportation problem, the objective is to determine the routes to be used and the quantity to be shipped via each route that will provide the minimum total transportation cost. The cost for each unit shipped on each route is given in Table 6.1 and is shown on each arc in Figure 6.1.

Try Problem 1 for practice in developing a network model of a transportation problem.

The first subscript identifies the "from" node of the corresponding arc and the second subscript identifies the "to" node of the arc.

**TABLE 6.1 TRANSPORTATION COST PER UNIT FOR THE FOSTER GENERATORS TRANSPORTATION PROBLEM**

Origin	Destination			
	Boston	Chicago	St. Louis	Lexington
Cleveland	2		7	6
Bedford		5	2	3
York		5	4	5

odel can be used to solve this transportation problem. We use double variables, with  $x_{ij}$  denoting the number of units shipped from origin  $i$  (Cleveland) to destination  $j$  (Boston),  $x_{i2}$  denoting the number of units shipped from origin  $i$  (Cleveland) to destination 2 (Chicago), and so on. In general, the decision variables for a transportation problem having  $m$  origins and  $n$  destinations are written as follows:

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$x_{ij}$  = number of units shipped from origin  $i$  to destination  $j$ ,

where  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

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Because the objective of the transportation problem is to minimize the total transportation cost, we can use the cost data in Table 6.1 or on the arcs in Figure 6.1 to develop the following cost expressions:

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Transportation costs for

units shipped from Cleveland =  $3x_{11} + 2x_{12} + 7x_{13} + 6x_{14}$

Transportation costs for

units shipped from Bedford =  $7x_{21} + 5x_{22} + 2x_{23} + 3x_{24}$

Transportation costs for

units shipped from York =  $2x_{31} + 5x_{32} + 4x_{33} + 5x_{34}$

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The sum of these expressions provides the objective function showing the total transportation cost for Foster Generators.

Transportation problems need constraints because each origin has a limited supply and each destination has a demand requirement. We consider the supply constraints first. The capacity at the Cleveland plant is 5000 units. With the total number of units shipped from the Cleveland plant expressed as  $x_{11} + x_{12} + x_{13} + x_{14}$ , the supply constraint for the Cleveland plant is

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 5000 \quad \text{Cleveland supply}$$

With three origins (plants), the Foster transportation problem has three supply constraints. Given the capacity of 6000 units at the Bedford plant and 2500 units at the York plant, the two additional supply constraints are

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 6000 \quad \text{Bedford supply}$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 2500 \quad \text{York supply}$$

With the four distribution centers as the destinations, four demand constraints are needed to ensure that destination demands will be satisfied:

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$$\begin{aligned}x_{11} + x_{21} + x_{31} &= 6000 \quad \text{Boston demand} \\x_{12} + x_{22} + x_{32} &= 4000 \quad \text{Chicago demand} \\x_{13} + x_{23} + x_{33} &= 2000 \quad \text{St. Louis demand} \\+ x_{34} &= 1500 \quad \text{Lexington demand}\end{aligned}$$

*To obtain a feasible solution, the total supply must be greater than or equal to the total demand.*

Min  $3x_{11} + 2x_{12} + 7x_{13} + 2x_{14} + 2x_{23} + 3x_{24} + 2x_{31} + 5x_{32} + 4x_{33} + 5x_{34}$

s.t.

$$\begin{aligned}x_{11} + x_{12} + x_{13} + x_{14} &\leq 5000 \\x_{21} + x_{22} + x_{23} + x_{24} &\leq 6000 \\x_{31} + x_{32} + x_{33} + x_{34} &\leq 2500 \\x_{11} + x_{21} + x_{31} + x_{32} &= 6000 \\x_{12} + x_{22} + x_{32} &= 4000 \\x_{13} + x_{23} + x_{33} &= 2000 \\x_{ij} &\geq 0 \quad \text{for } i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4\end{aligned}$$

Comparing the linear programming formulation to the network in Figure 6.1 leads to several observations. All the information needed in the linear programming formulation is on the network. Each node has one constraint, and each arc has one variable. The sum of the variables corresponding to arcs from an origin node must be less than or equal to the origin's supply, and the sum of the variables corresponding to the arcs into a destination node must be equal to the destination's demand.

The solution to the Foster Generators problem (see Figure 6.2) shows that the minimum total transportation cost is \$39,500. The values for the decision variables show the

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**FIGURE 6.2 THE SOLUTION FOR THE FOSTER GENERATORS TRANSPORTATION PROBLEM**

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Optimal Objective Value = 39500.00000

Variable	Value	Reduced Costs
X11	3500.00000	0.00000
X12	1500.00000	0.00000
X13	0.00000	8.00000
X14	0.00000	6.00000
X21	0.00000	1.00000
X22	2500.00000	0.00000
X23	2000.00000	0.00000
X24	1500.00000	0.00000
X31	2500.00000	0.00000
X32	0.00000	4.00000
X33	0.00000	6.00000
X34	0.00000	6.00000

**TABLE 6.2 OPTIMAL SOLUTION TO THE FOSTER GENERATORS TRANSPORTATION PROBLEM**

From	To	Route	Units Shipped	Cost per Unit	Total Cost
Cleveland	Boston		3500	\$3	\$10,500
Cleveland	Chicago		1500	\$2	3,000
Bedford	Boston		2500	\$5	12,500
Bedford	Chicago		2000	\$2	4,000
Bedford	Detroit		1500	\$3	4,500
York	Boston		2500	\$2	5,000
					\$39,500

optimal amounts to ship over each route. For example, with  $x_{11} = 3500$ , 3500 units should be shipped from Cleveland to Boston, and with  $x_{12} = 1500$ , 1500 units should be shipped from Cleveland to Chicago. Other values of the decision variables indicate the remaining shipping quantities and routes. Table 6.2 shows the minimum cost transportation schedule and Figure 6.3 summarizes the optimal solution on the network.

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### Problem Variations

The Foster Generators problem illustrates use of the basic transportation model. Variations of the basic transportation model may involve one or more of the following situations:

1. Total supply not equal to total demand
2. Maximization objective function
3. Route capacities or lower bounds
4. Unacceptable routes

With slight modifications in the linear programming model, we can easily accommodate these situations.

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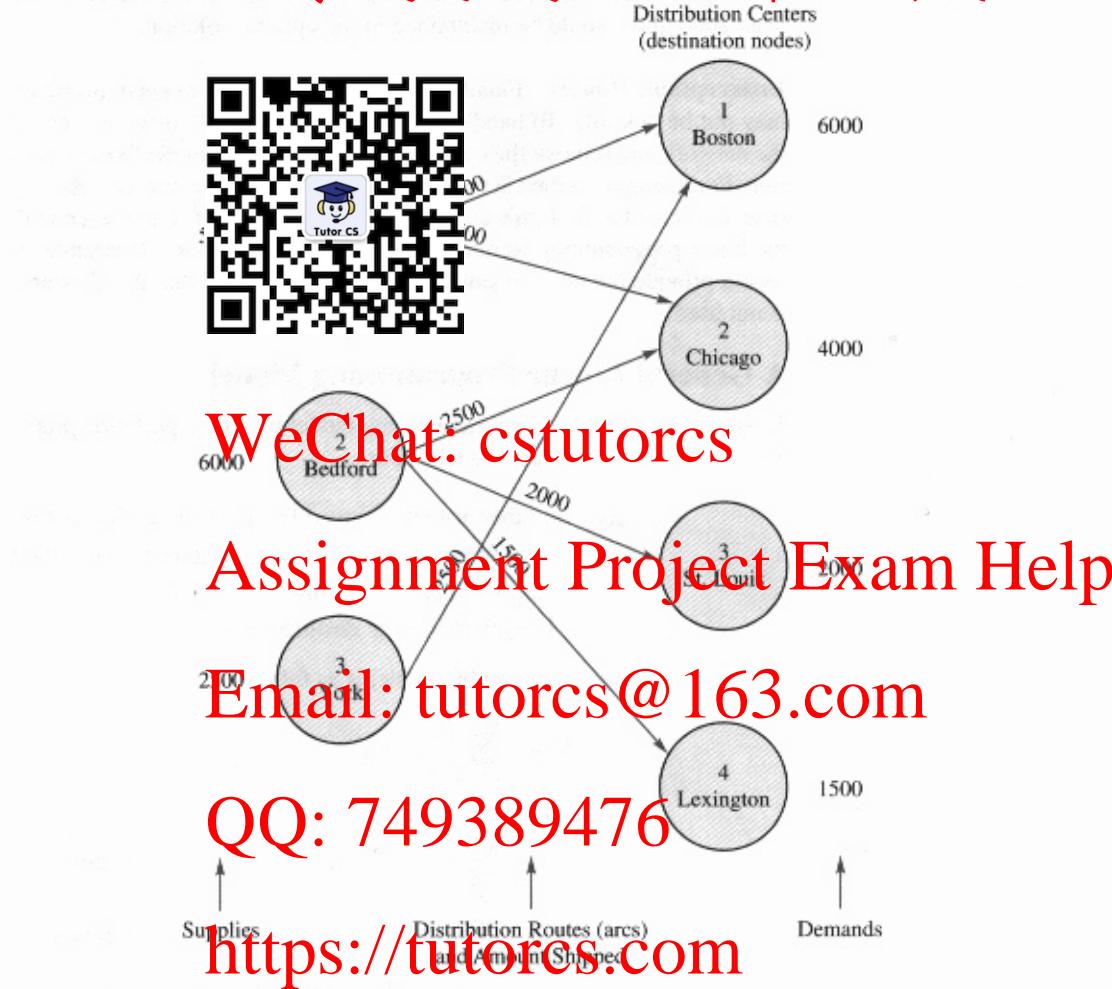
**Total Supply Not Equal to Total Demand** Often the *total supply is not equal to the total demand*. If total supply exceeds total demand, no modification in the linear programming formulation is necessary. Excess supply will appear as slack in the linear programming solution. Slack for any particular origin can be interpreted as the unused supply or amount not shipped from the origin.

If total supply is less than total demand, the linear programming model of a transportation problem will not have a feasible solution. In this case, we modify the network representation by adding a **dummy origin** with a supply equal to the difference between the total demand and the total supply. With the addition of the dummy origin, and an arc from the dummy origin to each destination, the linear programming model will have a feasible solution. A zero cost per unit is assigned to each arc leaving the dummy origin so that the value of the optimal solution for the revised problem will represent the shipping cost for the units actually shipped (no shipments actually will be made from the dummy origin). When the optimal solution is implemented, the destinations showing shipments being received from the dummy origin will be the destinations experiencing a shortfall, or unsatisfied demand.

*Whenever total supply is less than total demand, the model does not determine how the unsatisfied demand is handled (e.g., backorders). The manager must handle this aspect of the problem.*

**FIGURE 6.3 OPTIMAL SOLUTION TO THE FOSTER GENERATORS TRANSPORTATION PROBLEM**

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*Try Problem 6 for practice with a case in which demand is greater than supply with a maximization objective.*

**Maximization Objective Function** In some transportation problems, the objective is to find a solution that maximizes profit or revenue. Using the values for profit or revenue per unit as coefficients in the objective function, we simply solve a maximization rather than a minimization linear program. This change does not affect the constraints.

**Route Capacities or Route Minimums** The linear programming formulation of the transportation problem also can accommodate capacities or minimum quantities for one or more of the routes. For example, suppose that in the Foster Generators problem the York–Boston route (origin 3 to destination 1) had a capacity of 1000 units because of limited space availability on its normal mode of transportation. With  $x_{31}$  denoting the amount shipped from York to Boston, the route capacity constraint for the York–Boston route would be

$$x_{31} \leq 1000$$

Similarly, route minimums can be specified. For example,

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would guarantee that a previously committed order for a Cleveland–Chicago delivery of at least 2000 units would be maintained in the optimal solution.

Usually, establishing a route from every origin to every destination may not be necessary. In this situation, we simply drop the corresponding arc from the network and remove the corresponding variable from the linear programming formulation. If the Cleveland–St. Louis route were unacceptable or unusable, the arc from Cleveland to St. Louis could be dropped in Figure 6.1, and  $x_{13}$  could be removed from the formulation. Solving the resulting 11-variable, 7-constraint model would still result in an integer solution while guaranteeing that the Cleveland–St. Louis route is no longer used.



### A General Linear Programming Model

To show the general linear programming model for a transportation problem with  $m$  origins and  $n$  destinations, we use the notation:

$x_{ij}$  = number of units shipped from origin  $i$  to destination  $j$

$c_{ij}$  = cost per unit of shipping from origin  $i$  to destination  $j$

$s_i$  = supply or capacity in units at origin  $i$

$d_j$  = demand in units at destination  $j$

The general linear programming model is as follows:

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$$\text{Min } \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

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$$\sum_{j=1}^n x_{ij} \leq s_i \quad i = 1, 2, \dots, m \quad \text{Supply}$$

$$\sum_{i=1}^m x_{ij} = d_j \quad j = 1, 2, \dots, n \quad \text{Demand}$$

$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j$$

#### NOTES AND COMMENTS

- Transportation problems encountered in practice usually lead to large linear programs. Transportation problems with 100 origins and 100 destinations are not unusual. Such a problem would involve  $(100)(100) = 10,000$  variables.
- To handle a situation in which some routes may be unacceptable, we stated that you could drop the corresponding arc from the network and remove the corresponding variable from the linear programming formulation. Another approach often used is to assign an extremely large objective function cost coefficient to any unacceptable
- arc. If the problem has already been formulated, another option is to add a constraint to the formulation that sets the variable you want to remove equal to zero.
- The optimal solution to a transportation model will consist of integer values for the decision variables as long as all supply and demand values are integers. The reason is the special mathematical structure of the linear programming model. Each variable appears in exactly one supply and one demand constraint, and all coefficients in the constraint equations are 1 or 0.

## MANAGEMENT SCIENCE IN ACTION

### EMPTY FREIGHT CAR ASSIGNMENT AT UNION PACIFIC RAILROAD

Union Pacific Railroad is the largest railroad in North America. The company owns over 32,000 miles of railroad t~~o~~ oyes. It deploys over 100,000 freight cars for its customers.

When goods are transported, the freight car is left unloaded. Therefore, empty freight cars must be unloaded in time, but the North American rail network has a limit. At the same time, Union Pacific's customers require empty freight cars so they may be loaded for shipments. A critical decision for Union Pacific is how to assign empty freight cars to customers who are in need of cars. That is, given the location of the empty cars and the demand for empty cars at other locations, which cars should be sent to which demand points? The decision is further complicated by the fact that supply of freight cars is usually greater than the demand, there are a variety of types of freight cars (for example, box cars, auto transport cars, etc.) and service time expectations must be met while also controlling costs.



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Prior to the development of an optimization model for freight car assignment, car managers assigned cars to customers from pools of a particular type of car. These managers had no tool to assess how the assignments they were making impacted the overall rail network.

Union Pacific, working with faculty from Purdue University, developed a transportation model similar to the one discussed in this section. The model minimizes a linear weighted cost objective subject to supply and demand constraints. The objective function is a weighted combination of transportation costs, car-substitution costs (a penalty for assigning a feasible but different type of car than was demanded), penalties for deviating from the delivery schedule, customer priority, and corridor efficiency. The model was successfully implemented and resulted in 35% return on investment.

\*Based on A.K. Arnett, J.P. McHugh, D.R. Johnson, D. Murphy, G. Minks, and J. Fuller, "An Optimization Model for Empty Freight Car Assignment at Union Pacific Railroad," *Interfaces* (March/April: 2008): 89–100.

As mentioned previously, we can add constraints of the form  $x_{ij} \leq L_{ij}$  if the route from origin  $i$  to destination  $j$  has capacity  $L_{ij}$ . A transportation problem that includes constraints of this type is called a **capacitated transportation problem**. Similarly, we can add route minimum constraints of the form  $x_{ij} \geq M_{ij}$  if the route from origin  $i$  to destination  $j$  must handle at least  $M_{ij}$  units.

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## 6.2 ASSIGNMENT PROBLEM

The **assignment problem** arises in a variety of decision-making situations; typical assignment problems involve assigning jobs to machines, agents to tasks, sales personnel to sales territories, contracts to bidders, and so on. A distinguishing feature of the assignment problem is that *one* agent is assigned to *one and only one* task. Specifically, we look for the set of assignments that will optimize a stated objective, such as minimize cost, minimize time, or maximize profits.

To illustrate the assignment problem, let us consider the case of Fowle Marketing Research, which has just received requests for market research studies from three new clients. The company faces the task of assigning a project leader (agent) to each client (task). Currently, three individuals have no other commitments and are available for the project leader assignments. Fowle's management realizes, however, that the time required to complete each study will depend on the experience and ability of the project leader assigned. The three projects have approximately the same priority, and management wants to assign

project leaders to minimize the total number of days required to complete all three projects. If a project leader is to be assigned to one client only, what assignments should be made?

To answer the assignment question, Fowle's management consultant considers all possible project leader-client assignments and then estimates the corresponding project completion times. With three project leaders and three clients, nine assignment alternatives are possible. The alternatives and the estimated project completion times in days are summarized in Table 6.3.

*Try Problem 9 (part a) for practice in developing a network model for an assignment problem.*

The network representation of Fowle's assignment problem. The nodes represent project leaders and clients, and the arcs represent the possible assignments from project leaders to clients. The supply at each origin node and the demand at each destination node are both 1.

TABLE 6.3

COMPLETION TIMES (DAYS) FOR THE FOWLE MARKETING RESEARCH ASSIGNMENT PROBLEM

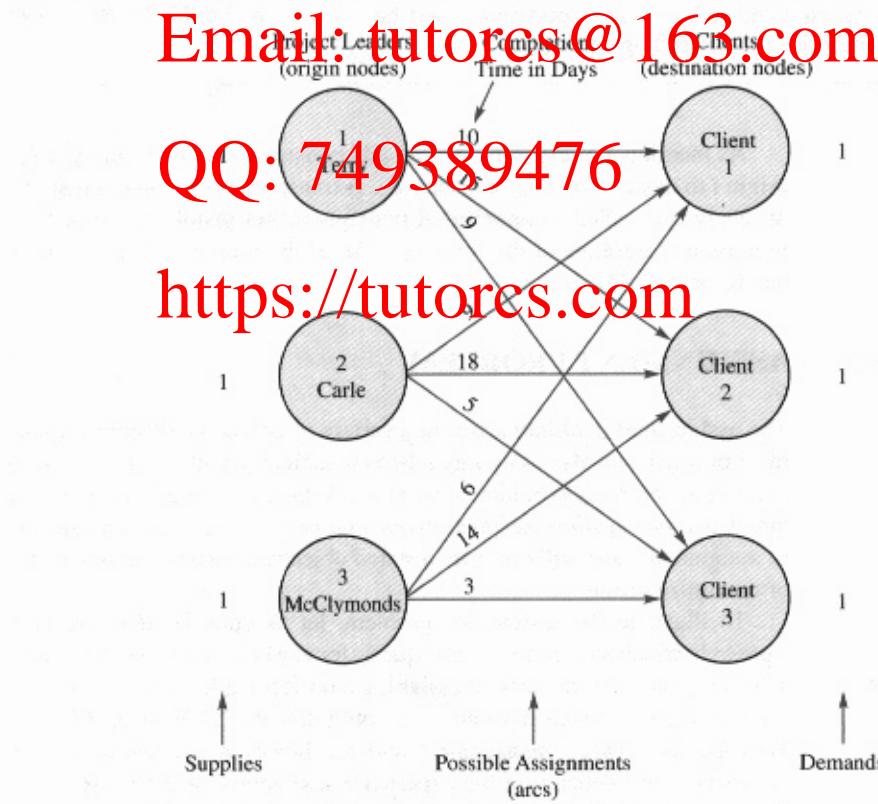
Project Leader	Client		
	1	2	3
1. Terry	10	15	9
2. Carle	9	18	5
3. McClymonds	6	14	3

FIGURE 6.4 A NETWORK MODEL OF THE FOWLE MARKETING RESEARCH ASSIGNMENT PROBLEM

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The assignment problem is a special case of the transportation problem.

each destination node are 1; the cost of assigning a project leader to a client is the time it takes that project leader to complete the client's task. Note the similarity between the network models of the assignment problem (Figure 6.1) and the assignment problem (Figure 6.1). The assignment problem is a special case of the transportation problem in which all supply and demand values equal 1, and the amount shipped over each arc is either 0 or 1.

Because the assignment problem is a special case of the transportation problem, a linear program can be developed. Again, we need a constraint for each node and a variable for each arc. In the transportation problem, we use double-subscripted decision variables for the assignment of project leader  $i$  (Terry) to client  $j$ ,  $x_{ij}$  denoting the assignment of project leader 1 (Terry) to client 2, and so on. Thus, we define the decision variables for the assignment problem as



$$\begin{cases} 1 & \text{if project leader } i \text{ is assigned to client } j \\ 0 & \text{otherwise} \end{cases}$$

where  $i = 1, 2, 3$ , and  $j = 1, 2, 3$

Using this notation and the completion time data in Table 6.3, we develop completion time expressions:

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$$\text{Days required for Terry's assignment} = 10x_{11} + 15x_{12} + 9x_{13}$$

$$\text{Days required for Carle's assignment} = 9x_{21} + 18x_{22} + 5x_{23}$$

$$\text{Days required for McClymonds's assignment} = 6x_{31} + 14x_{32} + 3x_{33}$$

The sum of the completion times for the three project leaders will provide the total days required to complete the three assignments. Thus, the objective function is

$$\text{Min } 10x_{11} + 15x_{12} + 9x_{13} + 9x_{21} + 18x_{22} + 5x_{23} + 6x_{31} + 14x_{32} + 3x_{33}$$

The constraints for the assignment problem reflect the conditions that each project leader can be assigned to at most one client and that each client must have one assigned project leader. These constraints are written as follows:

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$$x_{11} + x_{12} + x_{13} \leq 1 \quad \text{Terry's assignment}$$

$$x_{21} + x_{22} + x_{23} \leq 1 \quad \text{Carle's assignment}$$

$$x_{31} + x_{32} + x_{33} \leq 1 \quad \text{McClymonds's assignment}$$

$$x_{11} + x_{21} + x_{31} = 1 \quad \text{Client 1}$$

$$x_{12} + x_{22} + x_{32} = 1 \quad \text{Client 2}$$

$$x_{13} + x_{23} + x_{33} = 1 \quad \text{Client 3}$$

Note that each node in Figure 6.4 has one constraint.

Combining the objective function and constraints into one model provides the following 9-variable, 6-constraint linear programming model of the Fowle Marketing Research assignment problem:

$$\text{Min } 10x_{11} + 15x_{12} + 9x_{13} + 9x_{21} + 18x_{22} + 5x_{23} + 6x_{31} + 14x_{32} + 3x_{33}$$

s.t.

$$x_{11} + x_{12} + x_{13} \leq 1$$

$$x_{21} + x_{22} + x_{23} \leq 1$$

$$x_{31} + x_{32} + x_{33} \leq 1$$

$$x_{11} + x_{21} + x_{31} = 1$$

$$x_{12} + x_{22} + x_{32} = 1$$

$$x_{13} + x_{23} + x_{33} = 1$$

$$x_{ij} \geq 0 \quad \text{for } i = 1, 2, 3; j = 1, 2, 3$$

Because the number of project leaders equals the number of clients, all the constraints could be written as equalities. But when the number of project leaders exceeds the number of clients, less-than-or-equal-to constraints must be used for the project leader constraints.

Try Problem 9 (part b) for practice in formulating and solving a linear programming model for an assignment problem on the computer.

**FIGURE 6.5 THE SOLUTION FOR THE FOWLE MARKETING RESEARCH ASSIGNMENT PROBLEM**



Optimal Objective Value = 26.00000

Variables	Value	Reduced Costs
	0.00000	0.00000
	1.00000	0.00000
	0.00000	2.00000
	0.00000	1.00000
	0.00000	5.00000
	1.00000	0.00000
	1.00000	0.00000
X32	0.00000	3.00000
X33	0.00000	0.00000

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**TABLE 6.4 OPTIMAL PROJECT LEADER ASSIGNMENTS FOR THE FOWLE MARKETING RESEARCH ASSIGNMENT PROBLEM**

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Project Leader	Assigned Client	Days
Terry	2	15
Carle	3	6
McClymonds	1	6
Total		26

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Figure 6.5 shows the computer solution for this model. Terry is assigned to client 2 ( $x_{12} = 1$ ), Carle is assigned to client 3 ( $x_{23} = 1$ ), and McClymonds is assigned to client 1 ( $x_{31} = 1$ ). The total completion time required is 26 days. This solution is summarized in Table 6.4.

### Problem Variations

Because the assignment problem can be viewed as a special case of the transportation problem, the problem variations that may arise in an assignment problem parallel those for the transportation problem. Specifically, we can handle the following issues:

1. Total number of agents (supply) not equal to the total number of tasks (demand)
2. A maximization objective function
3. Unacceptable assignments

The situation in which the number of agents does not equal the number of tasks is analogous to total supply not equaling total demand in a transportation problem. If the number of agents exceeds the number of tasks, the extra agents simply remain unassigned in the linear programming solution. If the number of tasks exceeds the number of agents, the linear programming model will not have a feasible solution. In this situation, a simple modification is to add enough dummy agents to equalize the number of agents and the number of

*In the linear programming formulation of a problem with five clients and only three project leaders, we could get by with one dummy project leader by placing a 2 on the right-hand side of the constraint for the dummy project leader.*

tasks. For instance, in the Fowle problem we might have had five clients (tasks) and only three project leaders (agents). By adding two dummy project leaders, we can create a new assignment problem with the number of project leaders equal to the number of clients. The objective function coefficients for the assignment of dummy project leaders would be zero so that the value of the optimal solution would represent the total number of days required by the assignments actually made (no assignments will actually be made to the clients receiving no assignments).

If the assignment problem is evaluated in terms of revenue or profit rather than time or cost, the linear programming formulation can be solved as a maximization rather than a minimization problem. In addition, if one or more assignments are unacceptable, the corresponding rows or columns can be removed from the linear programming formulation. For example, if an agent did not have the experience necessary for one



### A General Linear Programming Model

To show the general linear programming model for an assignment problem with  $m$  agents and  $n$  tasks, we use the notation:

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$$x_{ij} = \begin{cases} 1 & \text{if agent } i \text{ is assigned to task } j \\ 0 & \text{otherwise} \end{cases}$$

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The general linear programming model is as follows:

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s.t.

$$\begin{aligned} \text{Min: } & \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \\ \text{s.t. } & \sum_{j=1}^n x_{ij} \leq 1 \quad i = 1, 2, \dots, m \quad \text{Agents} \\ & \sum_{i=1}^m x_{ij} = 1 \quad j = 1, 2, \dots, n \quad \text{Tasks} \\ & x_{ij} \geq 0 \quad \text{for all } i \text{ and } j \end{aligned}$$

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#### NOTES AND COMMENTS

- As noted, the assignment model is a special case of the transportation model. We stated in the notes and comments at the end of the preceding section that the optimal solution to the transportation problem will consist of integer values for the decision variables as long as the supplies and demands are integers. For the assignment problem, all supplies and demands equal 1; thus, the optimal solution must be integer valued and the integer values must be 0 or 1.
- Combining the method for handling multiple assignments with the notion of a dummy agent

- provides another means of dealing with situations when the number of tasks exceeds the number of agents. That is, we add one dummy agent, but provide the dummy agent with the capability to handle multiple tasks. The number of tasks the dummy agent can handle is equal to the difference between the number of tasks and the number of agents.
- The Management Science in Action, Assigning Project Managers at Heery International, describes how managers are assigned to construction projects. The application involves multiple assignments.

## MANAGEMENT SCIENCE IN ACTION

## ASSIGNING PROJECT MANAGERS AT HEERY INTERNATIONAL

Heery International contracts with the State of Tennessee and others for a variety of construction projects including roads, bridges, hotels, and park facilities. Heery typically has managers assigned to each project. Each of these projects requires one manager. With seven managers and 100 projects, there are  $7(100) = 700$  assignments. An outside consultant developed a mathematical model to assign construction managers to projects.

The assignment problem developed by Heery uses 0-1 decision variables for each manager/project pair, just as in the assignment problem discussed previously. The goal in assigning managers is to balance the workload among managers and, at the same time, to minimize travel cost from the manager's home to the construction site. Thus, an objective function coefficient for each possible assignment was developed that combined project intensity (a function of the size of the project budget) with the travel distance from the manager's home to the construction site. The objective function calls for minimizing the sum over all possible assignments of the product of these coefficients with the assignment variables.



With more construction projects than managers, it was necessary to consider a variation of the standard assignment problem involving multiple assignments. Of the two sets of constraints, one set enforces the requirement that each project receive one and only one manager. The other set of constraints limits the number of assignments each manager can accept by placing an upper bound on the total intensity that is acceptable over all projects assigned.

Heery International implemented this assignment model with considerable success. According to Emory F. Redden, a Heery vice president, "The optimization model . . . has been very helpful for assigning managers to projects. . . . We have been satisfied with the assignments chosen at the Nashville office. . . . We look forward to using the model in our Atlanta office and elsewhere in the Heery organization."

\*Based on Larry J. LeBlanc, Dale Randels, Jr., and T. K. Swann, "Heery International's Spreadsheet Optimization Model for Assigning Managers to Construction Projects," *Intervac* (November/December 2000): 95–106.

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At the beginning of this section, we indicated that a distinguishing feature of the assignment problem is that *one* agent is assigned to *one and only one* task. In generalizations of the assignment problem where one agent can be assigned to two or more tasks, the linear programming formulation of the problem can be easily modified. For example, let us assume that in the Fowl Marketing Research problem, Terry could be assigned up to two clients; in this case, the constraint representing Terry's assignment would be  $x_{11} + x_{12} + x_{13} \leq 2$ . In general, if  $a_i$  denotes the upper limit for the number of tasks to which agent  $i$  can be assigned, we write the agent constraints as

$$\sum_{j=1}^n x_{ij} \leq a_i \quad i = 1, 2, \dots, m$$

If some tasks require more than one agent, the linear programming formulation can also accommodate the situation. Use the number of agents required as the right-hand side of the appropriate task constraint.

### 6.3 TRANSSHIPMENT PROBLEM

The **transshipment problem** is an extension of the transportation problem in which intermediate nodes, referred to as *transshipment nodes*, are added to account for locations such as warehouses. In this more general type of distribution problem, shipments may be made between any pair of the three general types of nodes: origin nodes, transshipment nodes,

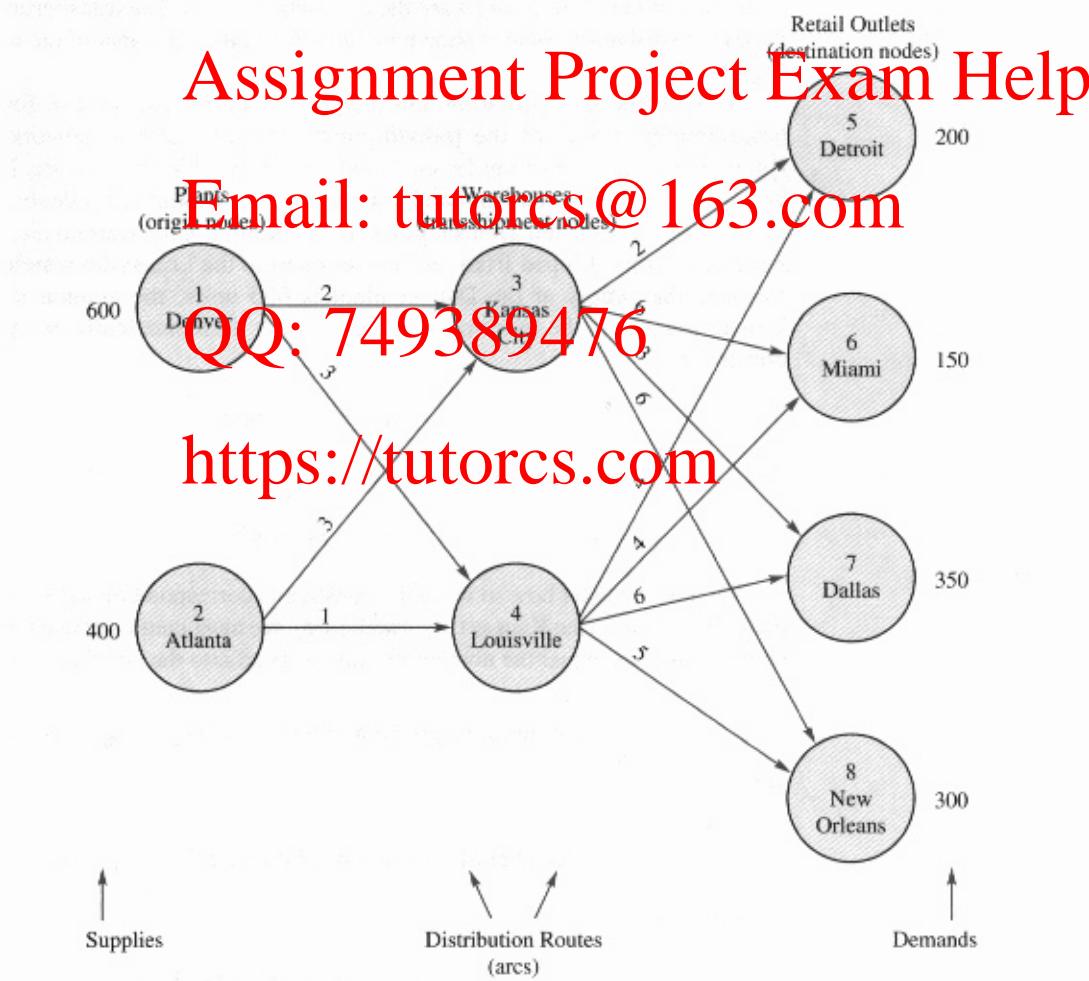
and destination nodes. For example, the transshipment problem permits shipments of goods from origins to intermediate nodes and on to destinations, from one origin to another origin, from one intermediate location to another, from one destination to another, and directly from origins to destination.

As was true for the transportation problem, the supply available at each origin is limited, and the demand at each destination is specified. The objective in the transshipment problem is to determine how many units should be shipped over each arc in the network so that all demands are satisfied with the minimum possible transportation cost.

*Try Problem 17 (part a) for practice in developing a network representation of a transshipment problem.*

Let's consider a transshipment problem faced by Ryan Electronics. Ryan is an electronics firm with two production facilities in Denver and Atlanta. Components produced at these facilities are shipped to either of the firm's regional warehouses, which are located in Kansas City and Louisville. From the regional warehouses, the firm supplies regional retail outlets in Detroit, Miami, Dallas, and New Orleans. The key features of the problem are summarized in the network model depicted in Figure 6.6. Note that the supply at

**FIGURE 6.6** NETWORK REPRESENTATION OF THE RYAN ELECTRONICS TRANSSHIPMENT PROBLEM



**TABLE 6.5** TRANSPORTATION COST PER UNIT FOR THE RYAN ELECTRONICS  
TRANSSHIPMENT PROBLEM

		Warehouse		Retail Outlet			
Plant		Kansas City	Louisville	Miami	Dallas	New Orleans	
Denver			3				
Atlanta			1				
Warehouse							
Kansas City				6	3	6	
Louisville				4	6	5	

each origin and demand at each destination are shown in the left and right margins, respectively. Nodes 1 and 2 are the origin nodes; nodes 3 and 4 are the transshipment nodes; and nodes 5, 6, 7, and 8 are the destination nodes. The transportation cost per unit for each distribution route is shown in Table 6.5 and on the arcs of the network model in Figure 6.6.

As with the transportation and assignment problems, we can formulate a linear programming model of the transshipment problem from a network representation. Again, we need a constraint for each node and a variable for each arc. Let  $x_{ij}$  denote the number of units shipped from node  $i$  to node  $j$ . For example,  $x_{13}$  denotes the number of units shipped from the Denver plant to the Kansas City warehouse,  $x_{14}$  denotes the number of units shipped from the Denver plant to the Louisville warehouse, and so on. Because the supply at the Denver plant is 600 units, the amount shipped from the Denver plant must be less than or equal to 600. Mathematically, we write this supply constraint as

$$x_{13} + x_{14} \leq 600$$

Similarly, for the Atlanta plant we have

$$x_{23} + x_{24} \leq 400$$

We now consider how to write the constraints corresponding to the two transshipment nodes. For node 3 (the Kansas City warehouse), we must guarantee that the number of units shipped out must equal the number of units shipped into the warehouse. If

$$\text{Number of units shipped out of node 3} = x_{35} + x_{36} + x_{37} + x_{38}$$

and

$$\text{Number of units shipped into node 3} = x_{13} + x_{23}$$

we obtain

$$x_{35} + x_{36} + x_{37} + x_{38} = x_{13} + x_{23}$$

**FIGURE 6.7 LINEAR PROGRAMMING FORMULATION OF THE RYAN ELECTRONICS TRANSSHIPMENT PROBLEM**

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$$\text{Min } 2x_{13} + 3x_{14} + 3x_{23} + 1x_{24} + 2x_{35} + 6x_{36} + 3x_{37} + 6x_{38} + 4x_{45} + 4x_{46} + 6x_{47} + 5x_{48}$$

s.t.

$$x_{13} + x_{14}$$

$$x_{23} + x_{24}$$



$$-x_{13} - x_{23}$$

$$-x_{14} - x_{24}$$

$$\begin{aligned} & \leq 600 \\ & \leq 400 \\ & = 0 \\ & x_{45} + x_{46} + x_{47} + x_{48} = 0 \\ & x_{45} + x_{46} = 200 \\ & x_{46} + x_{47} = 150 \\ & x_{47} + x_{48} = 350 \\ & x_{48} = 300 \end{aligned} \quad \begin{array}{l} \text{Origin node} \\ \text{constraints} \end{array} \quad \begin{array}{l} \text{Transshipment node} \\ \text{constraints} \end{array} \quad \begin{array}{l} \text{Destination node} \\ \text{constraints} \end{array}$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j$$

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Similarly, the constraint corresponding to node 4 is

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To develop the constraints associated with the destination nodes, we recognize that for each node the amount shipped to the destination must equal the demand. For example, to satisfy the demand for 200 units at node 5 (the Detroit retail outlet), we write

$$x_{35} + x_{45} = 200$$

Similarly, for nodes 6, 7, and 8, we have

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$$x_{36} + x_{46} = 150$$

$$x_{37} + x_{47} = 350$$

$$x_{38} + x_{48} = 300$$

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*Try Problem 17 (parts b and c) for practice in developing the linear programming model and in solving a transshipment problem on the computer.*

As usual, the objective function reflects the total shipping cost over the 12 shipping routes. Combining the objective function and constraints leads to a 12-variable, 8-constraint linear programming model of the Ryan Electronics transshipment problem (see Figure 6.7). Figure 6.8 shows the solution to the Ryan Electronics problem and Table 6.6 summarizes the optimal solution.

As mentioned at the beginning of this section, in the transshipment problem, arcs may connect any pair of nodes. All such shipping patterns are possible in a transshipment problem. We still require only one constraint per node, but the constraint must include a variable for every arc entering or leaving the node. For origin nodes, the sum of the shipments out minus the sum of the shipments in must be less than or equal to the origin supply. For destination nodes, the sum of the shipments in minus the sum of the shipments out must equal demand. For transshipment nodes, the sum of the shipments out must equal the sum of the shipments in, as before.

For an illustration of this more general type of transshipment problem, let us modify the Ryan Electronics problem. Suppose that it is possible to ship directly from Atlanta to New Orleans at \$4 per unit and from Dallas to New Orleans at \$1 per unit. The network

**FIGURE 6.8 THE SOLUTION FOR THE RYAN ELECTRONICS TRANSSHIPMENT PROBLEM**



Optimal Objective Value = 5200.00000

Value	Reduced Costs
550.00000	0.00000
50.00000	0.00000
0.00000	3.00000
400.00000	0.00000
200.00000	0.00000
0.00000	1.00000
350.00000	0.00000
0.00000	0.00000
0.00000	3.00000
100.00000	0.00000
0.00000	4.00000
300.00000	0.00000

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**TABLE 6.6 OPTIMAL SOLUTION TO THE RYAN ELECTRONICS**

TRANSSHIPMENT PROBLEM

From	To	Route	Units Shipped	Cost per Unit	Total Cost
Denver	Kansas City		550	\$2	\$1100
Denver	Louisville		50	\$3	150
Atlanta	Louisville		400	\$1	400
Kansas City	Detroit		200	\$2	400
Kansas City	Dallas		350	\$3	1050
Louisville	Miami		150	\$4	600
Louisville	New Orleans		300	\$5	1500
					\$5200

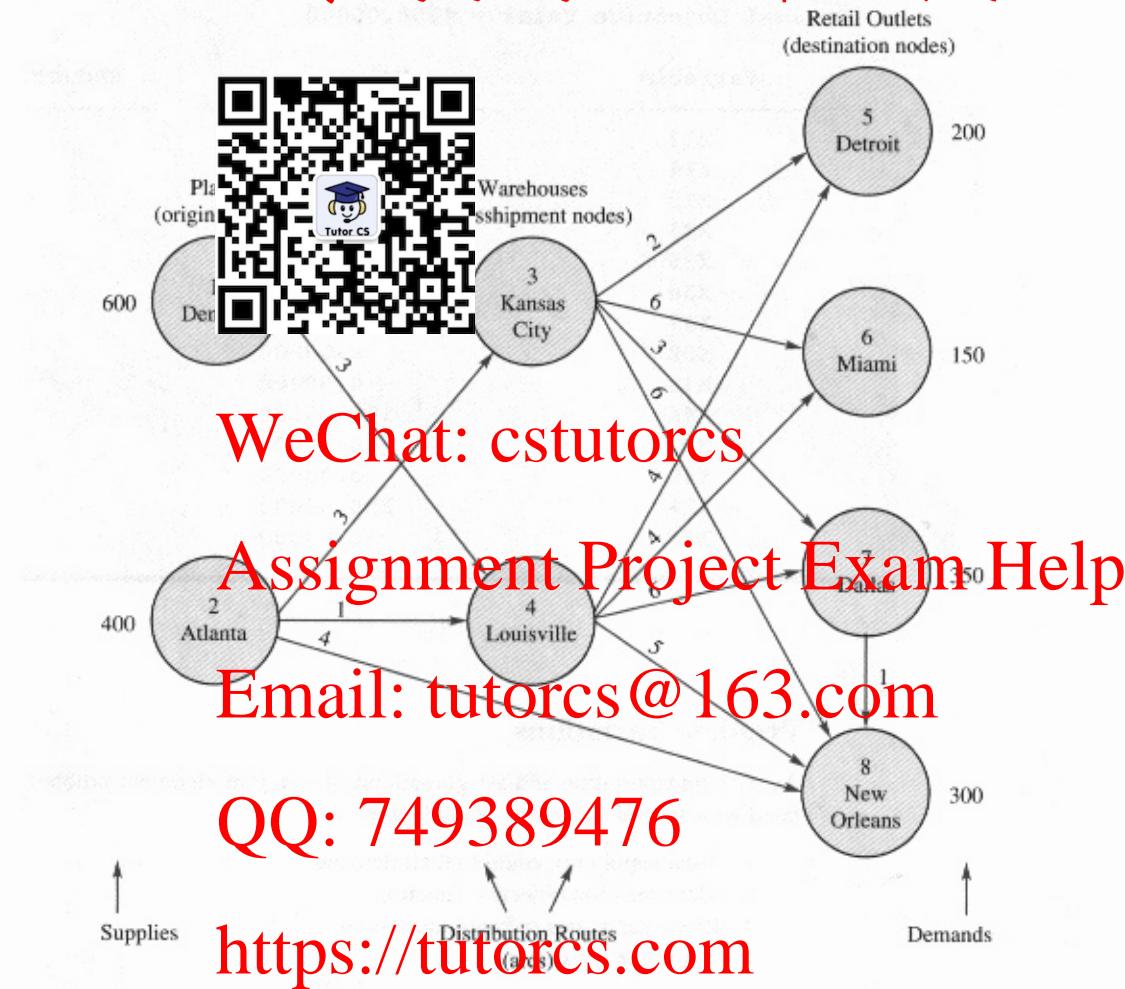
model corresponding to this modified Ryan Electronics problem is shown in Figure 6.9, the linear programming formulation is shown in Figure 6.10, and the computer solution is shown in Figure 6.11.

Try Problem 18 for practice working with transshipment problems with this more general structure.

In Figure 6.9 we added two new arcs to the network model. Thus, two new variables are necessary in the linear programming formulation. Figure 6.10 shows that the new variables  $x_{28}$  and  $x_{78}$  appear in the objective function and in the constraints corresponding to the nodes to which the new arcs are connected. Figure 6.11 shows that the value of the optimal solution has been reduced \$600 by adding the two new shipping routes;  $x_{28} = 250$  units are being shipped directly from Atlanta to New Orleans, and  $x_{78} = 50$  units are being shipped from Dallas to New Orleans.

**FIGURE 6.9** NETWORK REPRESENTATION OF THE MODIFIED RYAN ELECTRONICS TRANSSHIPMENT PROBLEM

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**FIGURE 6.10** LINEAR PROGRAMMING FORMULATION OF THE MODIFIED RYAN ELECTRONICS TRANSSHIPMENT PROBLEM

$$\text{Min } 2x_{13} + 3x_{14} + 3x_{23} + 1x_{24} + 2x_{35} + 6x_{36} + 3x_{37} + 6x_{38} + 4x_{45} + 4x_{46} + 6x_{47} + 5x_{48} + 4x_{28} + 1x_{78}$$

s.t.

$$\begin{aligned}
 & x_{13} + x_{14} && \leq 600 \\
 & -x_{13} - x_{23} + x_{35} + x_{36} + x_{37} + x_{38} && \leq 400 \\
 & -x_{14} - x_{24} + x_{35} + x_{36} + x_{37} + x_{38} && = 0 \\
 & x_{35} + x_{36} + x_{37} + x_{38} + x_{45} + x_{46} + x_{47} + x_{48} && = 200 \\
 & x_{36} + x_{46} + x_{47} + x_{48} - x_{78} && = 150 \\
 & x_{37} + x_{47} + x_{48} + x_{28} + x_{78} && = 350 \\
 & x_{38} + x_{48} + x_{28} + x_{78} && = 300
 \end{aligned}$$

} Origin node constraints  
 } Transshipment node constraints  
 } Destination node constraints

$x_{ij} \geq 0$  for all  $i$  and  $j$

**FIGURE 6.11 THE SOLUTION FOR THE MODIFIED RYAN ELECTRONICS TRANSSHIPMENT PROBLEM**

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Optimal Objective Value = 4600.00000

V	Value	Reduced Costs
	600.00000	0.00000
	0.00000	0.00000
	0.00000	3.00000
	150.00000	0.00000
	200.00000	0.00000
	0.00000	1.00000
	400.00000	0.00000
X38	0.00000	2.00000
X45	0.00000	3.00000
X46	150.00000	0.00000
X47	0.00000	4.00000
X48	0.00000	2.00000
X28	250.00000	0.00000
X78	50.00000	0.00000

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#### Problem Variations

As with transportation and assignment problems, transshipment problems may be formulated with several variations, including these:

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1. Total supply not equal to total demand
  2. Maximization objective function
  3. Route capacities or route minimums
  4. Unacceptable routes

The linear programming model modifications required to accommodate these variations are identical to the modifications required for the transportation problem described in Section 6.1. When we add one or more constraints of the form  $x_{ij} \leq L_{ij}$  to show that the route from node  $i$  to node  $j$  has capacity  $L_{ij}$ , we refer to the transshipment problem as a **capacitated transshipment problem**.

#### A General Linear Programming Model

To show the general linear programming model for the transshipment problem, we use the notation:

$x_{ij}$  = number of units shipped from the node  $i$  to node  $j$

$c_{ij}$  = cost per unit of shipping from node  $i$  to node  $j$

$s_i$  = supply at origin node  $i$

$d_j$  = demand at destination node  $j$

The general linear programming model for the transshipment problem is as follows:

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s.t.



$$\sum_{\substack{\text{arcs in} \\ \text{arcs out}}} x_{ij} = \sum_{\substack{\text{arcs in} \\ \text{arcs out}}} x_{ij} \leq s_i \quad \text{Origin node } i$$

$$= \sum_{\substack{\text{arcs in} \\ \text{arcs out}}} x_{ij} \quad \text{Transshipment nodes}$$

$$\sum_{\substack{\text{arcs out} \\ \text{arcs in}}} x_{ij} = d_j \quad \text{Destination node } j$$

0 for all  $i$  and  $j$

### NOTES AND COMMENTS

1. The Management Science in Action Product Sourcing Heuristic at Procter & Gamble, describes how Procter & Gamble used a transshipment model to redesign its North American distribution system. The constraints for the destination nodes are often written as
2. In more advanced treatments of linear programming and network flow problems, the capacitated transshipment problem is called the *pure network flow problem*. Efficient special-purpose solution procedures are available for network flow problems and their special cases.
3. In the general linear programming formulation of the transshipment problem, the

$$\sum_{\substack{\text{arcs out} \\ \text{arcs in}}} x_{ij} - \sum_{\substack{\text{arcs in} \\ \text{arcs out}}} x_{ij} = -d_j$$

The advantage of writing the constraints this way is that the left-hand side of each constraint then represents the flow out of the node minus the flow in, but such constraints would then have to be multiplied by  $-1$  to obtain nonnegative right-hand sides before solving the problem by many linear programming codes.

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### MANAGEMENT SCIENCE IN ACTION

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#### PRODUCT SOURCING HEURISTIC AT PROCTER & GAMBLE\*

A few years ago Procter & Gamble (P&G) embarked on a major strategic planning initiative called the North American Product Sourcing Study. P&G wanted to consolidate its product sources and optimize its distribution system design throughout North America. A decision support system used to aid in this project was called the Product Sourcing Heuristic (PSH) and was based on a transshipment model much like the ones described in this chapter.

In a preprocessing phase, the many P&G products were aggregated into groups that shared the same technology and could be made at the same plant. The PSH employing the transshipment model was then used by product strategy teams responsible for developing product sourcing options for these product groups. The various plants

that could produce the product group were the source nodes, the company's regional distribution centers were the transshipment nodes, and P&G's customer zones were the destinations. Direct shipments to customer zones as well as shipments through distribution centers were employed.

The product strategy teams used the heuristic interactively to explore a variety of questions concerning product sourcing and distribution. For instance, the team might be interested in the impact of closing two of five plants and consolidating production in the three remaining plants. The product sourcing heuristic would then delete the source nodes corresponding to the two closed plants,

*(continued)*

make any capacity modifications necessary to the sources corresponding to the remaining three plants, and re-solve the transshipment problem. The product strategy team could then examine the new solution, make some more modifications, solve again, and so on.

The Product Strategy Team viewed as a valuable decision

it. When P&G implemented the results of the study, it realized annual savings in the \$200-million range. The P&G project was successful in North America and has since been used in other markets around the world.

\*Based on information provided by Franz Dill and Tom Chorman of Procter & Gamble.



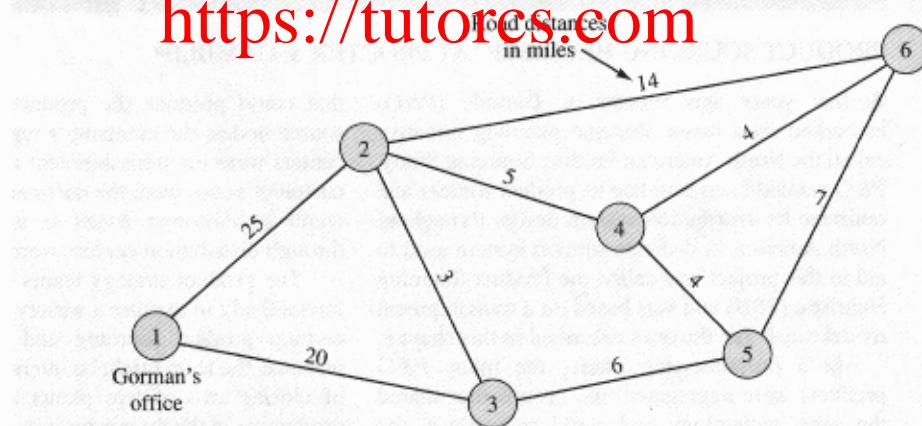
## 6.4 SHORTEST-ROUTE PROBLEM

In this problem in which the objective is to determine the **shortest route** between nodes in a network. We will demonstrate the shortest-route problem by considering the situation facing the Gorman Construction Company. Gorman has several construction sites located throughout a three-county area. With multiple daily trips carrying personnel, equipment, and supplies from Gorman's office to the construction sites, the costs associated with transportation activities are substantial. The travel alternatives between Gorman's office and each construction site can be described by the road network shown in Figure 6.12. The road distances in miles between the nodes are shown above the corresponding arcs. In this application, Gorman would like to determine the route that will minimize the total travel distance between Gorman's office (located at node 1) and the construction site located at node 6.

A key to developing a model for the shortest-route problem is to understand that the problem is a special case of the transshipment problem. Specifically, the Gorman shortest-route problem can be viewed as a transshipment problem with one origin node (node 1), one destination node (node 6), and four transshipment nodes (nodes 2, 3, 4, and 5). The

**FIGURE 6.12 ROAD NETWORK FOR THE GORMAN COMPANY SHORTEST-ROUTE PROBLEM**

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*Note:* (1) The length of each arc is not necessarily proportional to the travel distance it represents.

(2) All roads are two-way; thus, flow may be in either direction.

**FIGURE 6.13 TRANSSHIPMENT NETWORK FOR THE GORMAN SHORTEST-ROUTE PROBLEM**



transshipment network for the Gorman shortest-route problem is shown in Figure 6.13. Arrows added to the arcs show the direction of flow, which is always *out* of the origin node and *into* the destination node. Note also that two directed arcs are shown between the pairs of transshipment nodes. For example, one arc going from node 2 to node 3 indicates that the shortest route may go from node 2 to node 3; another arc going from node 3 to node 2 indicates that the shortest route may go from node 3 to node 2. The distance between two transshipment nodes is the same in either direction.

To find the shortest route between node 1 and node 6, think of node 1 as having a supply of 1 unit and node 6 as having a demand of 1 unit. Let  $x_{ij}$  denote the number of units that flow or are shipped from node  $i$  to node  $j$ . Because only 1 unit will be shipped from node 1 to node 6, the value of  $x_{ij}$  will be either 1 or 0. Thus, if  $x_{ij} = 1$ , the arc from node  $i$  to node  $j$  is on the shortest route from node 1 to node 6; if  $x_{ij} = 0$ , the arc from node  $i$  to node  $j$  is not on the shortest route. Because we are looking for the shortest route between node 1 and node 6, the objective function for the Gorman problem is

$$\begin{aligned} \text{Min } & 25x_{12} + 20x_{13} + 3x_{23} + 3x_{32} + 5x_{24} + 5x_{42} + 14x_{26} + 6x_{35} + 6x_{53} \\ & + 4x_{45} + 4x_{54} + 4x_{46} + 7x_{56} \end{aligned}$$

To develop the constraints for the model, we begin with node 1. Because the supply at node 1 is 1 unit, the flow out of node 1 must equal 1. Thus, the constraint for node 1 is written

$$x_{12} + x_{13} = 1$$

For transshipment nodes 2, 3, 4, and 5, the flow out of each node must equal the flow into each node; thus, the flow out minus the flow in must be 0. The constraints for the four transshipment nodes are as follows:

	<b>Flow Out</b>	<b>Flow In</b>
Node 2	$x_{23} + x_{24} + x_{26}$	$= x_{12} + x_{32} + x_{42}$
Node 3	$x_{32} + x_{35}$	$= x_{13} + x_{23} + x_{53}$
Node 4	$x_{42} + x_{45} + x_{46}$	$= x_{24} + x_{54}$
Node 5	$x_{53} + x_{54} + x_{56}$	$= x_{35} + x_{45}$

**FIGURE 6.14 LINEAR PROGRAMMING FORMULATION OF THE GORMAN SHORTEST-ROUTE PROBLEM**

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$$\begin{aligned}
 \text{Min } & 25x_{12} + 20x_{13} + 3x_{23} + 3x_{32} + 5x_{24} + 5x_{42} + 14x_{26} + 6x_{35} + 6x_{53} + 4x_{45} + 4x_{54} + 4x_{46} + 7x_{56} \\
 \text{s.t. } & \begin{aligned}
 x_{12} + x_{13} &= 1 && \text{Origin node} \\
 -x_{12} + x_{23} &= 0 \\
 -x_{13} - x_{23} &= 0 \\
 &+ x_{35} - x_{53} = 0 \\
 &+ x_{45} - x_{54} + x_{46} = 0 \\
 &- x_{35} + x_{53} - x_{45} + x_{54} + x_{56} = 0 \\
 &+ x_{46} + x_{56} = 1 && \text{Destination node}
 \end{aligned} \\
 & x_{ij} \geq 0 \text{ for all } i \text{ and } j
 \end{aligned}$$



Because node 6 is the destination node with a demand of one unit, the flow into node 6 must equal 1. Thus, the constraint for node 6 is written as

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$$x_{26} + x_{46} + x_{56} = 1$$

Including the negative constraints  $x_{ij} \geq 0$  for all  $i$  and  $j$ , the linear programming model for the Gorman shortest-route problem is shown in Figure 6.14.

The solution to the Gorman shortest-route problem is shown in Figure 6.15. The objective function value of 32 indicates that the shortest route between Gorman's office located at node 1 to the construction site located at node 6 is 32 miles. With  $x_{13} = 1$ ,  $x_{32} = 1$ ,  $x_{24} = 1$ , and  $x_{46} = 1$ , the shortest route from node 1 to node 6 is 1-3-2-4-6. In other words, the shortest route takes us from node 1 to node 3; then from node 3 to node 2; then from node 2 to node 4; and finally from node 4 to node 6.

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**FIGURE 6.15 THE SOLUTION OF THE GORMAN SHORTEST-ROUTE PROBLEM**

Try Problem 22 to practice solving a shortest-route problem.

WEB file  
Gorman

Variable	Value	Reduced Cost
X12	0.00000	2.00000
X13	1.00000	0.00000
X23	0.00000	6.00000
X32	1.00000	0.00000
X24	1.00000	0.00000
X42	0.00000	10.00000
X26	0.00000	5.00000
X35	0.00000	0.00000
X53	0.00000	12.00000
X45	0.00000	7.00000
X54	0.00000	1.00000
X46	1.00000	0.00000
X56	0.00000	0.00000

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## A General Linear Programming Model

To show the general linear programming model for the shortest-route problem, we use the notation:

$$x_{ij} = \begin{cases} 1 & \text{if the arc from node } i \text{ to node } j \text{ is on the shortest route} \\ 0 & \text{otherwise} \end{cases}$$

$c_{ij}$  = cost associated with the arc from node  $i$  to node  $j$

The general linear programming model for the shortest-route problem is as follows:



$$\sum_{\substack{\text{arcs out} \\ \text{from node } i}} x_{ij} = 1 \quad \text{Origin node } i$$

$$\sum_{\substack{\text{arcs out} \\ \text{from node } i}} x_{ij} = \sum_{\substack{\text{arcs in} \\ \text{to node } i}} x_{ij} \quad \text{Transshipment nodes}$$

$$\sum_{\substack{\text{arcs in} \\ \text{to node } j}} x_{ij} = 1 \quad \text{Destination node } j$$

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**NOTES AND COMMENTS**

In the Gorman problem, we assumed that all roads in the network are two-way. As a result, the road connecting nodes 2 and 3 in the road network resulted in the creation of two corresponding arcs in the transshipment network. Two decision variables,  $x_{23}$  and  $x_{32}$ , were required to show that the

shortest route might go from node 2 to node 3 or from node 3 to node 2. If the road connecting nodes 2 and 3 had been a one-way road allowing flow only from node 2 to node 3, decision variable  $x_{32}$  would not have been included in the model.

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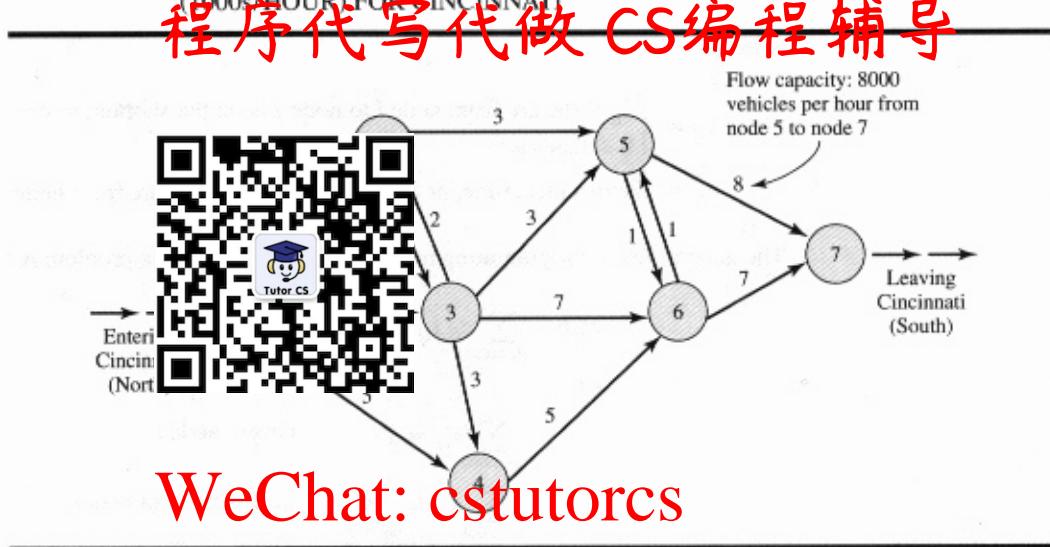
### 6.5 MAXIMAL FLOW PROBLEM

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The objective in a maximal flow problem is to determine the maximum amount of flow (vehicles, messages, fluid, etc.) that can enter and exit a network system in a given period of time. In this problem, we attempt to transmit flow through all arcs of the network as efficiently as possible. The amount of flow is limited due to capacity restrictions on the various arcs of the network. For example, highway types limit vehicle flow in a transportation system, whereas pipe sizes limit oil flow in an oil distribution system. The maximum or upper limit on the flow in an arc is referred to as the **flow capacity** of the arc. Even though we do not specify capacities for the nodes, we do assume that the flow out of a node is equal to the flow into the node.

As an example of the maximal flow problem, consider the north-south interstate highway system passing through Cincinnati, Ohio. The north-south vehicle flow reaches a level of 15,000 vehicles per hour at peak times. Due to a summer highway maintenance program, which calls for the temporary closing of lanes and lower speed limits, a network of alternate routes through Cincinnati has been proposed by a transportation planning committee. The alternate routes include other highways as well as city streets. Because of differences in speed limits and traffic patterns, flow capacities vary, depending on the

**FIGURE 6.16 NETWORK OF HIGHWAY SYSTEM AND FLOW CAPACITIES  
(1000s/HOUR) FOR CINCINNATI**



particular streets and roads used. The proposed network with arc flow capacities is shown in Figure 6.16.

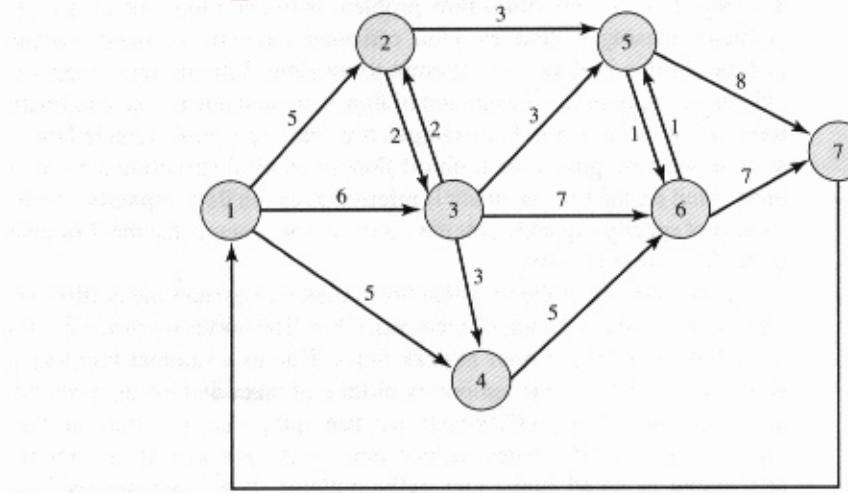
The direction of flow for each arc is indicated, and the arc capacity is shown next to each arc. Note that most of the streets are one-way. However, a two-way street can be found between nodes 3 and 4 and between nodes 5 and 6. In both cases, the capacity is the same in each direction.

We will show how to develop a capacitated transshipment model for the maximal flow problem. First, we will add an arc from node 7 back to node 1 to represent the total flow through the highway system. Figure 6.17 shows the modified network. The newly added

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**FIGURE 6.17 FLOW OVER ARC FROM NODE 7 TO NODE 1 TO REPRESENT TOTAL FLOW THROUGH THE CINCINNATI HIGHWAY SYSTEM**

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arc shows no capacity; indeed, we will want to maximize the flow over that arc. Maximizing the flow over the arc from node 7 to node 1 is equivalent to maximizing the number of cars that can get through the node, so in this system, passing through node 7 is the same as getting to node 1.

The decision variables are as follows:

$x_{ij}$  = amount of traffic flow from node  $i$  to node  $j$

The objective function that maximizes the flow over the highway system is

$$\text{Max } x_{71}$$

As with all linear programming problems, each arc generates a variable and each node generates a constraint. A conservation of flow constraint represents the requirement that the flow out equals the flow in. For node 1, the flow out is  $x_{12} + x_{13} + x_{14}$ , and the flow in is  $x_{71}$ . Therefore, the constraint for node 1 is

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The conservation of flow constraints for the other six nodes are developed in a similar fashion:

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Node 2	$x_{23} + x_{25}$
Node 3	$x_{32} + x_{34} + x_{35} + x_{36}$
Node 4	$x_{46}$
Node 5	$x_{56} + x_{57}$
Node 6	$x_{65} + x_{67}$
Node 7	$x_{71}$

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Additional constraints are needed to enforce the capacities on the arcs. These 14 simple upper-bound constraints are given:

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$$\begin{aligned}x_{23} &\leq 2 & x_{25} &\leq 3 \\x_{32} &\leq 2 & x_{34} &\leq 3 & x_{35} &\leq 3 & x_{36} &\leq 7 \\x_{46} &\leq 5 \\x_{56} &\leq 1 & x_{57} &\leq 8 \\x_{65} &\leq 1 & x_{67} &\leq 7\end{aligned}$$

Note that the only arc without a capacity is the one we added from node 7 to node 1.

The solution to this 15-variable, 21-constraint linear programming problem is shown in Figure 6.18. We note that the value of the optimal solution is 14. This result implies that the maximal flow over the highway system is 14,000 vehicles. Figure 6.19 shows how the vehicle flow is routed through the original highway network. We note, for instance, that 5000 vehicles per hour are routed between nodes 1 and 2, 2000 vehicles per hour are routed between nodes 2 and 3, and so on.

The results of the maximal flow analysis indicate that the planned highway network system will not handle the peak flow of 15,000 vehicles per hour. The transportation

Try Problem 29 for practice in solving a maximal flow problem.

**FIGURE 6.18 THE SOLUTION TO THE CINCINNATI HIGHWAY SYSTEM MAXIMAL FLOW PROBLEM**



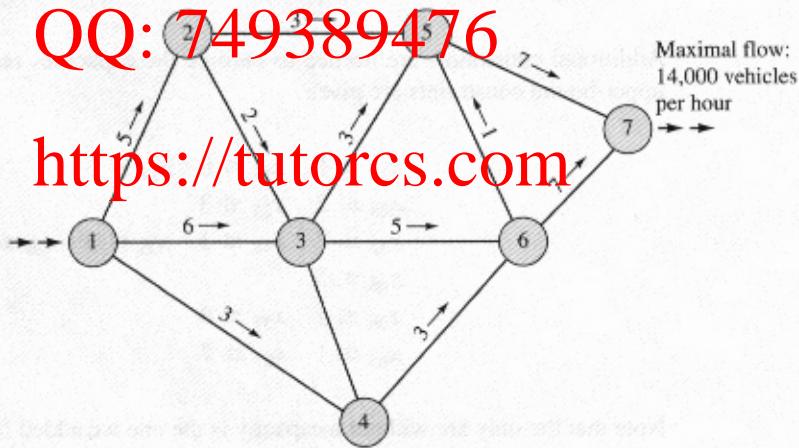
Optimal Objective Value = 14.00000



	Value	Reduced Cost
	5.00000	0.00000
	6.00000	0.00000
	3.00000	0.00000
	2.00000	0.00000
	3.00000	0.00000
	0.00000	0.00000
	3.00000	0.00000
X36	5.00000	0.00000
X32	0.00000	0.00000
X44	3.00000	0.00000
X56	1.00000	1.00000
X57	7.00000	0.00000
X65	1.00000	0.00000
X67	7.00000	0.00000
X7	14.00000	0.00000

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**FIGURE 6.19 MAXIMAL FLOW PATHS FOR THE CINCINNATI HIGHWAY SYSTEM NETWORK**



planners will have to expand the highway network, increase current arc flow capacities, or be prepared for serious traffic problems. If the network is extended or modified, another maximal flow analysis will determine the extent of any improved flow. The Management Science in Action, Optimizing Restoration Capacity at AT&T, notes that AT&T solved shortest-route and maximal flow problems in designing a transmission network.

## NOTES AND COMMENTS

1. The maximal flow problem of this section can also be solved with a slightly different formulation if the extra arc between nodes 7 and 1 is not used. The alternate approach is to maximize the flow into node 7, subject to the conservation of flow at nodes 1 and 7. However, the first formulation is most common in practice.



## MANAGEMENT



2. Network models can be used to describe a variety of management science problems. Unfortunately, no one network solution algorithm can be used to solve every network problem. It is important to recognize the specific type of problem being modeled in order to select the correct specialized solution algorithm.

## OPTIMIZING RESTORATION CAPACITY AT AT&T\*

AT&T is a global telecommunications company that provides long-distance voice and data, video, wireless, satellite, and Internet services. The company uses state-of-the-art switching and transmission equipment to provide service to more than 80 million customers. In the continental United States, AT&T's transmission network consists of more than 40,000 miles of fiber-optic cable. On peak days AT&T handles as many as 290 million calls of various types.

Power outages, natural disasters, cable cuts, and other events can disable a portion of the transmission network. When such events occur, spare capacity comprising the restoration network must be immediately employed so that service is not disrupted. Critical issues with respect to the restoration network are: How much capacity is necessary? and Where should it be located? In 1997, AT&T assembled a RestNet team to address these issues.

To optimize restoration capacity, the RestNet team developed a large-scale linear programming model. One subproblem in their model involves determining the shortest route connecting an origin and destination whenever a failure occurs in a span of the transmission network. Another subproblem solves a maximal flow problem to find the best restoration paths from each switch to a disaster recovery switch.

The RestNet team was successful, and their work is an example of how valuable management science methodology is to companies. According to C. Michael Armstrong, chair and CEO, "Last year the work of the RestNet team allowed us to reduce capital spending by tens of millions of dollars."

\*Based on Ken Ambs, Sebastian Cwilich, Mei Deng, David J. Houck, David F. Lynch, and Dicky Yan, "Optimizing Restoration Capacity in the AT&T Network," *Interfaces* (January/February 2000): 26–44.

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## 6.6 A PRODUCTION AND INVENTORY APPLICATION

The introduction to the transportation and transshipment problems in Sections 6.1 and 6.3 involved applications for the shipment of goods from several supply locations or origins to several demand sites or destinations. Although the shipment of goods is the subject of many transportation and transshipment problems, transportation or transshipment models can be developed for applications that have nothing to do with the physical shipment of goods from origins to destinations. In this section we show how to use a transshipment model to solve a production and inventory problem.

Contois Carpets is a small manufacturer of carpeting for home and office installations. Production capacity, demand, production cost per square yard, and inventory holding cost per square yard for the next four quarters are shown in Table 6.7. Note that production

**TABLE 6.7** PRODUCTION, DEMAND, AND COST ESTIMATES FOR CONTOIS CARPETS

Quarter	Production Capacity (square yards)	Demand (square yards)	Production Cost (\$/square yard)	Inventory Cost (\$/square yard)
1	600	400	2	0.25
2	500	500	5	0.25
3	400	400	3	0.25
4	400	400	3	0.25

capacity varies by quarter, whereas the cost of carrying inventory next is constant at \$0.25 per yard. Contois wants to determine how many yards of carpeting to manufacture each quarter to minimize the total production and inventory cost for the four-quarter period.

The network flows into and out of demand nodes are what make the model a transshipment model.

We begin by developing a network representation of the problem. First, we create four nodes corresponding to the production in each quarter and four nodes corresponding to the demand in each quarter. Each production node is connected by an outgoing arc to the demand node for the same period. The flow on the arc represents the number of square yards of carpet manufactured for the period. For each demand node, an outgoing arc represents the amount of inventory (square yards of carpet) carried over to the demand node for the next period. Figure 6.20 shows the network model. Note that nodes 1–4 represent the quarterly production capacities shown in the left margin, and the quarterly demands are shown in the right margin.

The objective is to determine a production scheduling and inventory policy that will minimize the total production and inventory cost for the four quarters. Constraints involve production capacity and demand in each quarter. As usual, a linear programming model can be developed from the network by establishing a constraint for each node and a variable for each arc.

Let  $x_{15}$  denote the number of square yards of carpet manufactured in quarter 1. The capacity of the facility is 600 square yards in quarter 1, so the production capacity constraint is

$$x_{15} \leq 600$$

Using similar decision variables, we obtain the production capacities for quarters 2–4:

$$x_{26} \leq 300$$

$$x_{37} \leq 500$$

$$x_{48} \leq 400$$

We now consider the development of the constraints for each of the demand nodes. For node 5, one arc enters the node, which represents the number of square yards of carpet produced in quarter 1, and one arc leaves the node, which represents the number of square yards of carpet that will not be sold in quarter 1 and will be carried over for possible sale in quarter 2. In general, for each quarter the beginning inventory plus the production minus the ending inventory must equal demand. However, because quarter 1 has no beginning inventory, the constraint for node 5 is

$$x_{15} - x_{56} = 400$$



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FIGURE 6.20 NETWORK REPRESENTATION OF THE CONTOIS CARPETS PROBLEM



The constraints associated with the demand nodes in quarters 2, 3, and 4 are

$$\begin{aligned}x_{56} + x_{26} - x_{67} &= 500 \\x_{67} + x_{37} - x_{78} &= 400 \\x_{78} + x_{48} &= 400\end{aligned}$$

Note that the constraint for node 8 (fourth-quarter demand) involves only two variables because no provision is made for holding inventory for a fifth quarter.

The objective is to minimize total production and inventory cost, so we write the objective function as

$$\text{Min } 2x_{15} + 5x_{26} + 3x_{37} + 3x_{48} + 0.25x_{56} + 0.25x_{67} + 0.25x_{78}$$

FIGURE 6.21 THE SOLUTION FOR THE CONTOIS CARPETS PROBLEM



Contois

Optimal Objective Value = 1150.000		
Variable	Value	Reduced Cost
	600.00000	0.00000
	300.00000	0.00000
	400.00000	0.00000
	400.00000	0.00000
	200.00000	0.00000
	0.00000	2.25000
	0.00000	0.00000

The complete linear programming formulation of the Contois Carpets problem is

$$\begin{aligned} \text{Min } & 2x_{15} + 5x_{24} + 3x_{37} + 3x_{48} + 0.25x_{56} + 0.25x_{67} + 0.25x_{78} \\ \text{s.t. } & \end{aligned}$$

$$x_{15} \leq 600$$

$$x_{26} \leq 300$$

$$x_{37} \leq 400$$

$$x_{48} \leq 400$$

$$x_{15} - x_{56} = 400$$

$$x_{26} - x_{56} = 500$$

$$x_{37} - x_{67} = 400$$

$$x_{48} + x_{67} - x_{78} = 400$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j$$

Figure 6.21 shows the solution to the Contois Carpets problem. Contois should manufacture 600 square yards of carpet in quarter 1, 300 square yards in quarter 2, 400 square yards in quarter 3, and 400 square yards in quarter 4. Note also that 200 square yards will be carried over from quarter 1 to quarter 2. The total production and inventory cost is \$1150.

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#### NOTES AND COMMENTS

For the network models presented in this chapter, the amount leaving the starting node for an arc is always equal to the amount entering the ending node for that arc. An extension of such a network model is the case where a gain or a loss occurs as an arc is traversed. The amount entering the destination node may be greater or smaller than the amount leaving

the origin node. For instance, if cash is the commodity flowing across an arc, the cash earns interest from one period to the next. Thus, the amount of cash entering the next period is greater than the amount leaving the previous period by the amount of interest earned. Networks with gains or losses are treated in more advanced texts on network flow programming.

#### SUMMARY

In this chapter we introduced transportation, assignment, transshipment, shortest-route, and maximal flow problems. All five types of problems belong to the special category of linear programs called *network flow problems*. In general, the network model for these

problems consists of nodes representing origins, destinations, and if necessary, transshipment points in the network system. Arcs are used to represent the routes for shipment, travel, or flow between the various nodes.

The general transportation problem has  $m$  origins and  $n$  destinations. Given the supply at each origin, the demand at each destination, and unit shipping cost between each origin and each destination, the transportation model determines the optimal amounts to ship from each

The assignment problem is a special case of the transportation problem in which all supply and demand are one unit. We represent each agent as an origin node and each task as a destination node. The assignment model determines the minimum cost or maximum profit for assigning agents to tasks.

The transshipment problem is an extension of the transportation problem involving transfer points or transshipment nodes. In this more general model, we allow arcs between nodes in the network. If desired, capacities can be specified for arcs, which makes it a capacitated transshipment problem.

The shortest-route problem finds the shortest route or path between two nodes of a network. Distance, time, and cost are often the criteria used for this model. The shortest-route problem can be expressed as a transshipment problem with one origin and one destination. By shipping one unit from the origin to the destination, the solution will determine the shortest route through the network.

The maximal flow problem can be used to allocate flow to the arcs of the network so that flow through the network system is maximized. Arc capacities determine the maximum amount of flow for each arc. With these flow capacity constraints, the maximal flow problem is expressed as a capacitated transshipment problem.

In the last section of the chapter, we showed how a variation of the transshipment problem could be used to solve a production and inventory problem. In the chapter appendix we show how to use Excel to solve three of the distribution and network problems presented in the chapter.

**GLOSSARY** QQ: 749389476

**Transportation problem** A network flow problem that often involves minimizing the cost of shipping goods from a set of origins to a set of destinations; it can be formulated and solved as a linear program by including a variable for each arc and a constraint for each node.

**Network** A graphical representation of a problem consisting of numbered circles (nodes) interconnected by a series of lines (arcs); arrowheads on the arcs show the direction of flow. Transportation, assignment, and transshipment problems are network flow problems.

**Nodes** The intersection or junction points of a network.

**Arcs** The lines connecting the nodes in a network.

**Dummy origin** An origin added to a transportation problem to make the total supply equal to the total demand. The supply assigned to the dummy origin is the difference between the total demand and the total supply.

**Capacitated transportation problem** A variation of the basic transportation problem in which some or all of the arcs are subject to capacity restrictions.

**Assignment problem** A network flow problem that often involves the assignment of agents to tasks; it can be formulated as a linear program and is a special case of the transportation problem.

**Transshipment problem** An extension of the transportation problem to distribution problems involving transfer points and possible shipments between any pair of nodes.

**Capacitated transshipment problem** A variation of the transshipment problem in which some or all of the arcs are subject to capacity restrictions.

**Shortest route** Shortest path between two nodes in a network.

**Maximal flow** The maximum amount of flow that can enter and exit a network system during a given period of time.

**Flow** Maximum flow for an arc of the network. The flow capacity in one direction is called the forward capacity; the flow capacity in the reverse direction is called the reverse capacity.

### PROBLEMS



1. A company ships at two ports: Philadelphia and New Orleans. Shipments of one product are made to four customers in Atlanta, Dallas, Columbus, and Boston. For the next month, the company has supplies at each port, customer demands, and shipping costs per case from each port to each customer are as follows:

Port	Customers				Port Supply
	Atlanta	Dallas	Columbus	Boston	
Philadelphia	2	6	6	2	5000
New Orleans	1	2	5	7	3000
Demands	400	3200	2000	1400	

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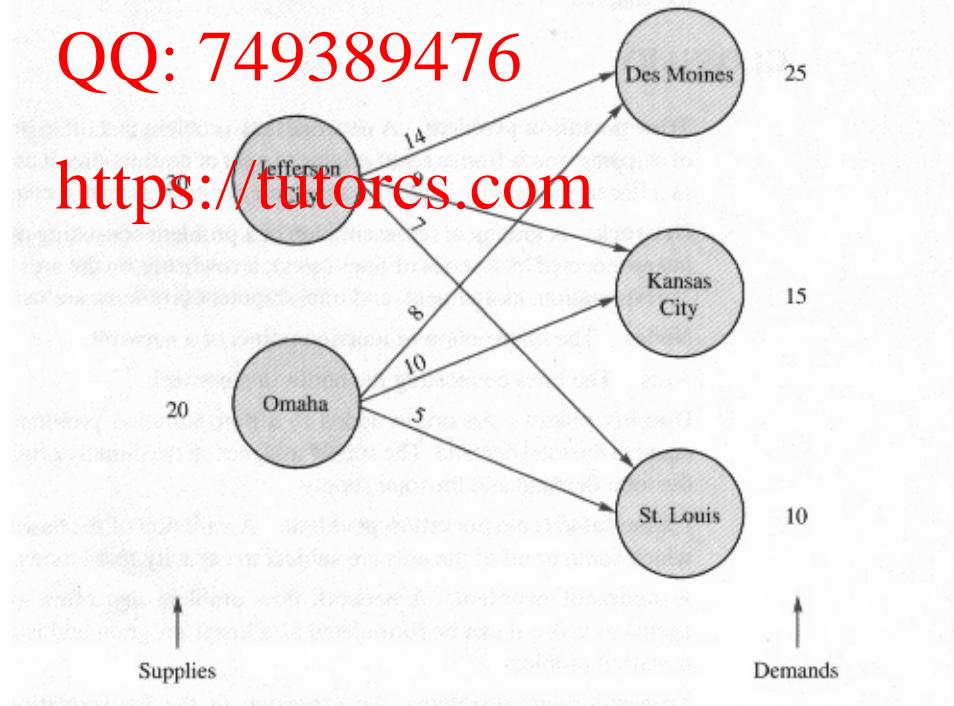
Develop a network representation of the distribution system (transportation problem).

2. Consider the following network representation of a transportation problem:



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The supplies, demands, and transportation costs per unit are shown on the network.

- Develop a linear programming model for this problem; be sure to define the variables in your model.
- Solve the linear program to determine the optimal solution.

- Tri-County Utilities, Inc., supplies natural gas to customers in a three-county area. The company purchases natural gas from two companies: Southern Gas and Northwest Gas. Demands during the coming winter season are Hamilton County, 400 units; Butler County, 300 units; and Clermont County, 300 units. Contracts to provide the following quantities are available: Southern Gas, 500 units; and Northwest Gas, 400 units. Distribution costs (in thousands of dollars) are as follows:



From	To		
	Hamilton	Butler	Clermont
Southern Gas	10	20	15
Northwest Gas	12	15	18

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- Develop a network representation of this problem.
- Develop a linear programming model that can be used to determine the plan that will minimize total distribution costs.
- Describe the distribution plan and show the total distribution cost.
- Recent residential and industrial growth in Butler County has the potential for increasing demand by as much as 100 units. Which supplier should Tri-County contract with to supply the additional capacity?

- Arnoff Enterprises manufactures the central processing unit (CPU) for a line of personal computers. The CPUs are manufactured in Seattle, Columbus, and New York and shipped to warehouses in Pittsburgh, Mobile, Denver, Los Angeles, and Washington, D.C., for further distribution. The following table shows the number of CPUs available at each plant, the number of CPUs required by each warehouse, and the shipping costs (dollars per unit):

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Plant	Warehouse					CPUs Available
	Pittsburgh	Mobile	Denver	Los Angeles	Washington	
Seattle	10	20	5	9	10	9000
Columbus	2	10	8	30	6	4000
New York	1	20	7	10	4	8000
CPUs Required	3000	5000	4000	6000	3000	21,000

- Develop a network representation of this problem.
- Determine the amount that should be shipped from each plant to each warehouse to minimize the total shipping cost.
- The Pittsburgh warehouse just increased its order by 1000 units, and Arnoff authorized the Columbus plant to increase its production by 1000 units. Will this production increase lead to an increase or decrease in total shipping costs? Solve for the new optimal solution.

5. Premier Consulting's two consultants, Avery and Baker, can be scheduled to work for clients up to a maximum of 160 hours each over the next four weeks. A third consultant, Campbell, has some administrative assignments already planned and is available for clients up to a maximum of 140 hours over the next four weeks. The company has four clients with projects in process. The estimated hourly requirements for each of the clients over the four-week period are



Client	Hours
A	180
B	75
C	100
D	85

Hourly rates vary for the consultant-client combination and are based on several factors, including project type and the consultant's experience. The rates (dollars per hour) for each consultant-client combination are as follows:

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Consultant	Client			
	A	B	C	D
Avery	100	125	115	100
Baker	120	135	115	120
Campbell	155	150	140	130

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- a. Develop a network representation of the problem.
- b. Formulate the problem as a linear program, with the optimal solution providing the hours each consultant should be scheduled for each client to maximize the consulting firm's billings. What is the schedule and what is the total billing?
- c. New information shows that Avery doesn't have the experience to be scheduled for client B. If this consulting assignment is not permitted, what impact does it have on total billings? What is the revised schedule?
6. Klein Chemicals, Inc., produces a special oil-based material that is currently in short supply. Four of Klein's customers have already placed orders that together exceed the combined capacity of Klein's two plants. Klein's management faces the problem of deciding how many units it should supply to each customer. Because the four customers are in different industries, different prices can be charged because of the various industry pricing structures. However, slightly different production costs at the two plants and varying transportation costs between the plants and customers make a "sell to the highest bidder" strategy unacceptable. After considering price, production costs, and transportation costs, Klein established the following profit per unit for each plant-customer alternative:

Plant	Customer			
	$D_1$	$D_2$	$D_3$	$D_4$
Clifton Springs	\$32	\$34	\$32	\$40
Danville	\$34	\$30	\$28	\$38

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The plant capacities and customer orders are as follows:

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Plant Capacity (units)	Distributed Orders (units)
Clifton Springs      5000	$D_1$ 2000 $D_2$ 5000 $D_3$ 3000 $D_4$ 2000
Fort Wayne      3000	

How many units should each plant produce for each customer in order to maximize profits? Will all customer demands be met? Show your network model and linear program.

7. Fort Wayne has a three-year contract to supply motors for all refrigerators produced by the Ice Age Corporation. Ice Age manufactures the refrigerators at four locations around the country: Boston, Dallas, Los Angeles, and St. Paul. Plans call for the following number (in thousands) of refrigerators to be produced at each location:

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Boston	50
Dallas	70
Los Angeles	60
St. Paul	20

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Forbelt's three plants are capable of producing the motors. The plants and production capacities (in thousands) are:

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Denver	100
Atlanta	100
Chicago	150

Because of varying production and transportation costs, the profit that Forbelt earns on each lot of 1000 units depends on which plant produced the lot and which destination it was shipped to. The following table gives the accounting department estimates of the profit per unit (shipments will be made in lots of 1000 units):

Produced At	Shipped To			
	Boston	Dallas	Los Angeles	St. Paul
Denver	7	11	8	13
Atlanta	20	17	12	10
Chicago	8	18	13	16

With profit maximization as a criterion, Forbelt's management wants to determine how many motors should be produced at each plant and how many motors should be shipped from each plant to each destination.

- a. Develop a network representation of this problem.
- b. Find the optimal solution.

8. The Ace Manufacturing Company has orders for three similar products:

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Product	Orders (units)
A	2000
B	500
C	1200



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able for the manufacturing operations. All three machines can produce the same production rate. However, due to varying defect percentages for each machine, the unit costs of the products vary depending on the machine used. Machine capacities for the next week, and the unit costs, are as follows:

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Machine	Capacity (units)	Machine	Product		
			A	B	C
1	1500	1	\$1.00	\$1.20	\$0.90
2	1500	2	\$1.10	\$1.40	\$1.10
3	1600	3	\$1.10	\$1.30	\$1.20

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Use the transportation model to develop the minimum cost production schedule for the products and machines. Show the linear programming formulation.

9. Scott and Associates, Inc., is an accounting firm that has three new clients. Project leaders will be assigned to the three clients. Based on the different backgrounds and experiences of the leaders, the various leader-client assignments differ in terms of projected completion times. The possible assignments and the estimated completion times in days are as follows:

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Project Leader	Client		
	1	2	3
Jackson	10	16	32
Ellis	14	22	40
Smith	22	24	34

- a. Develop a network representation of this problem.  
 b. Formulate the problem as a linear program, and solve. What is the total time required?
10. CarpetPlus sells and installs floor covering for commercial buildings. Brad Sweeney, a CarpetPlus account executive, was just awarded the contract for five jobs. Brad must now assign a CarpetPlus installation crew to each of the five jobs. Because the commission Brad will earn depends on the profit CarpetPlus makes, Brad would like to determine an assignment that will minimize total installation costs. Currently, five installation crews are available for assignment. Each crew is identified by a color code, which aids in tracking of

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job progress on a large white board. The following table shows the costs (in hundreds of dollars) for each crew to complete each of the five jobs:

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Crew	Job				
	1	2	3	4	5
	30	44	38	47	31
	25	32	45	44	25
	23	40	37	39	29
	26	38	37	45	28
	26	34	44	43	28



- a. presentation of the problem.  
 b. Formulate and solve a linear programming model to determine the minimum cost assignment.

11. A local television station plans to drop four Friday evening programs at the end of the season. Steve Botuchis, the station manager, developed a list of six potential replacement programs. Estimates of the advertising revenue (in dollars) that can be expected for each of the new programs in the four vacated time slots are as follows. Mr. Botuchis asked you to find the assignment of programs to time slots that will maximize total advertising revenue.

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	5:00– 5:30 P.M.	5:30– 6:00 P.M.	7:00– 7:30 P.M.	8:00– 8:30 P.M.
Home Improvement	5100	3000	6800	4000
World News	7500	8900	7600	5500
NASCAR Live	8500	5000	6500	8000
Wall Street Today	7000	6000	6500	5000
Hollywood Briefings	7900	8000	3000	6000
Ramund & Son	6000	4000	4500	7000

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12. The U.S. Cable Company uses a distribution system with five distribution centers and eight customer zones. Each customer zone is assigned a sole source supplier; each customer zone receives all of its cable products from the same distribution center. In an effort to balance demand and workload at the distribution centers, the company's vice president of logistics specified that distribution centers may not be assigned more than three customer zones. The following table shows the five distribution centers and cost of supplying each customer zone (in thousands of dollars):

Customer Zones

Distribution Centers	Los Angeles	Chicago	Columbus	Atlanta	Newark	Kansas City	Denver	Dallas
Plano	70	47	22	53	98	21	27	13
Nashville	75	38	19	58	90	34	40	26
Flagstaff	15	78	37	82	111	40	29	32
Springfield	60	23	8	39	82	36	32	45
Boulder	45	40	29	75	86	25	11	37

- a. Determine the assignment of customer zones to distribution centers that will minimize cost.

b. What distribution centers if any are needed?

Suppose that each distribution center is limited to a maximum of two customer zones.

How does this constraint change the assignment and the cost of supplying customer zones?

13.  (UES) uses large quantities of packaging materials at its four distribution hubs. After screening potential suppliers, UES identified six vendors that can supply the materials that will satisfy its quality standards. UES asked each of the vendors to submit bids to satisfy annual demand at each of its four distribution hubs. The following table lists the bids received (in thousands of dollars). Note that each of the distribution hubs is serviced by a different vendor. Which vendor's bid should be accepted, and which vendors should UES select to supply each distribution hub?

Buyer	Distribution Hub			
	1	2	3	4
Martin Products	190	175	125	230
Schmidt Materials	150	235	155	220
Miller Containers	210	225	135	260
DeVille Stoen	170	185	100	280
Larbes Furnishings	220	190	140	240
Lawler Depot	270	200	130	260

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14. The quantitative methods department head at a major midwestern university will be scheduling faculty to teach courses during the coming autumn term. Four core courses need to be covered. The four courses are at the UG, MBA, MS, and Ph.D. levels. Four professors will be assigned to the courses, with each professor receiving one of the courses. Student evaluations of professors are available from previous terms. Based on a rating scale of 4 (excellent), 3 (very good), 2 (average), 1 (fair), and 0 (poor), the average student evaluations for each professor are shown. Professor D does not have a Ph.D. and cannot be assigned to teach the Ph.D.-level course. If the department head makes teaching assignments based on maximizing the student evaluation ratings over all four courses, what staffing assignments should be made?

Professor	Course			
	UG	MBA	MS	Ph.D.
A	2.8	2.2	3.3	3.0
B	3.2	3.0	3.6	3.6
C	3.3	3.2	3.5	3.5
D	3.2	2.8	2.5	—

15. A market research firm's three clients each requested that the firm conduct a sample survey. Four available statisticians can be assigned to these three projects; however, all four statisticians are busy, and therefore each can handle only one client. The following data

show the number of hours required for each statistician to complete each job; the differences in time are based on experience and ability of the statisticians.

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Client



	A	B	C
150	210	270	
170	230	220	
180	230	225	
160	240	230	

- a. Formulate and solve a linear programming model for this problem.
- b. Suppose that the time statistician 4 needs to complete the job for client A is increased from 160 to 165 hours. What effect will this change have on the solution?
- c. Suppose that the time statistician 4 needs to complete the job for client A is decreased to 140 hours. What effect will this change have on the solution?
- d. Suppose that the time statistician 3 needs to complete the job for client B increases to 250 hours. What effect will this change have on the solution?

16. Hatcher Enterprises manufactures a chemical called Rbase in production operations at five divisions. Only six suppliers of Rbase meet Hatcher's quality control standards. All six suppliers can produce Rbase in sufficient quantities to accommodate the needs of each division. The quantity of Rbase needed by each Hatcher division and the price per gallon charged by each supplier are as follows:

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Division	Demand (000s of gallons)	Supplier	Price per Gallon (\$)
1	40	1	12.60
2	45	2	14.00
3	50	3	10.20
4	35	4	14.20
5	40	5	12.00
		6	13.00

The cost per gallon (in dollars) for shipping from each supplier to each division is provided in the following table:

Division	1	2	3	4	5	6
1	2.75	2.50	3.15	2.80	2.75	2.75
2	0.80	0.20	5.40	1.20	3.40	1.00
3	4.70	2.60	5.30	2.80	6.00	5.60
4	2.60	1.80	4.40	2.40	5.00	2.80
5	3.40	0.40	5.00	1.20	2.60	3.60

Hatcher believes in spreading its business among suppliers so that the company will be less affected by supplier problems (e.g., labor strikes or resource availability). Company policy requires that each division have a separate supplier.

- For each supplier–division combination, compute the total cost of supplying the division's demand.
- Determine the optimal assignment of suppliers to divisions.



17.  for the Herman Company consists of three plants, two warehouses, and two customers. Plant capacities and shipping costs per unit (in dollars) from each plant to each customer are as follows:

	Warehouse		Capacity
	1	2	
1	4	7	450
2	8	5	600
3	5	6	380

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Customer demand and shipping costs per unit (in dollars) from each warehouse to each customer are

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Warehouse	Customer			
	1	2	3	4
1	5	4	8	4
2	3	6	7	7
Demand	300	300	300	400

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- Develop a network representation of this problem.
- Formulate a linear programming model of the problem.
- Solve the linear program to determine the optimal shipping plan.

18. Refer to Problem 16. Suppose that shipments between the two warehouses are permitted at \$2 per unit and that direct shipments can be made from plant 3 to customer 4 at a cost of \$7 per unit.
- Develop a network representation of this problem.
  - Formulate a linear programming model of this problem.
  - Solve the linear program to determine the optimal shipping plan.
19. Adirondack Paper Mills, Inc., operates paper plants in Augusta, Maine, and Tupper Lake, New York. Warehouse facilities are located in Albany, New York, and Portsmouth, New Hampshire. Distributors are located in Boston, New York, and Philadelphia. The plant capacities and distributor demands for the next month are as follows:

Plant	Capacity (units)	Distributor	Demand (units)
Augusta	300	Boston	150
Tupper Lake	100	New York	100
		Philadelphia	150

The unit transportation costs (in dollars) for shipments from the two plants to the two warehouses and from the two warehouses to the three distributors are as follows:

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	Warehouse		
	Albany	Portsmouth	
Albany	7 3	5 4	
Portsmouth	5	6	
	Distributor		
	Boston	New York	Philadelphia
Albany	8	5	7
Portsmouth	5	6	10

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- a. Draw the network representation of the Adirondack Paper Mills problem.
- b. Formulate the Adirondack Paper Mills problem as a linear programming problem.
- c. Solve the linear program to determine the minimum cost shipping schedule for the problem.

20. The Moore & Harman Company is in the business of buying and selling grain. An important aspect of the company's business is arranging for the purchased grain to be shipped to customers. If the company can keep freight costs low, profitability will improve.

The company recently purchased three rail cars of grain at Muncie, Indiana; six rail cars at Brazil, Indiana; and five rail cars at Xenia, Ohio. Twelve carloads of grain have been sold. The locations and the amount sold at each location are as follows:

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Location	Number of Rail Car Loads
Muncie, IN	3
Greenwood, SC	4
Concord, SC	3
Chatham, NC	3

All shipments must be routed through either Louisville or Cincinnati. Shown are the shipping costs per bushel (in cents) from the origins to Louisville and Cincinnati and the costs per bushel to ship from Louisville and Cincinnati to the destinations.

From	To	
	Louisville	Cincinnati
Muncie	8	6
Brazil	3	8
Xenia	9	3

Cost per bushel  
from Muncie to  
Cincinnati is 6¢

	To				
	Macomb	Greenwood	Concord	Chatham	
From	44	34	34	32	
Louisville	57	35	28	24	
Cincinnati					



Cost per bushel from Cincinnati to Greenwood is 35¢

schedule that will minimize the freight costs necessary to satisfy rail cars of grain must be held at the origin until buyers can be

21. The following linear programming formulation is for a transshipment problem:

$$\text{Min } 11x_{13} + 12x_{14} + 10x_{21} + 8x_{31} + 10x_{35} + 11x_{42} + 9x_{45} + 12x_{52}$$

$$\text{s.t. } x_{13} + x_{14} - x_{21} \leq 5$$

$$x_{21} - x_{31} - x_{42} - x_{52} \leq 3$$

$$x_{31} - x_{14} - x_{34} - x_{35} + x_{42} + x_{45} \leq 2$$

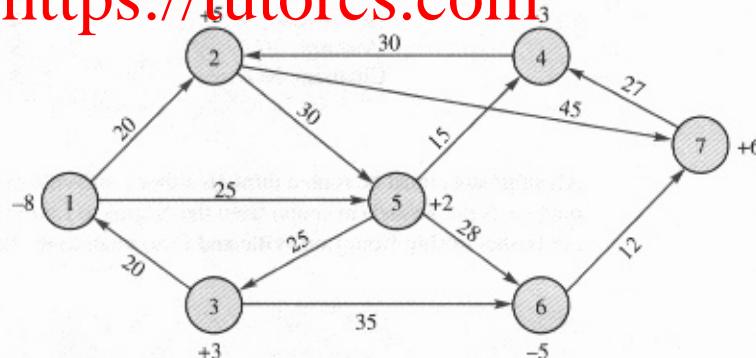
$$x_{35} + x_{45} - x_{52} = 4$$

$x_{ij} \geq 0$  for all  $i, j$

Show the network representation of this problem.

22. A rental car company has an imbalance of cars at seven of its locations. The following network shows the locations of concern (the nodes) and the cost to move a car between locations. A positive number by a node indicates an excess supply at the node, and a negative number indicates an excess demand.

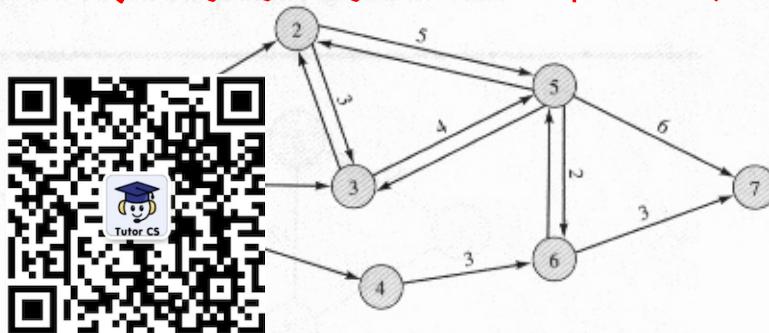
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- Develop a linear programming model of this problem.
- Solve the model formulated in part (a) to determine how the cars should be redistributed among the locations.

23. Find the shortest route from node 1 to node 7 in the network shown.

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24. In the original Gorman Construction Company problem, we found the shortest distance from the office (node 1) to the construction site located at node 6. Because some of the roads are highways and others are city streets, the shortest-distance routes between the office and the construction site may not necessarily provide the quickest or shortest-time route. Shown here is the Gorman road network with travel time rather than distance. Find the shortest route from the office to the construction site if the objective is to minimize travel time rather than distance.

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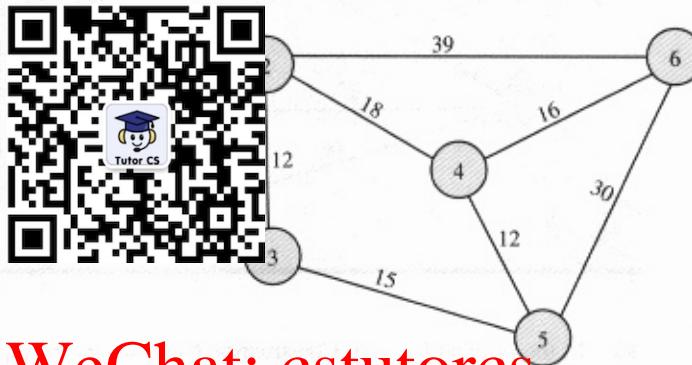
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25. CARD, Cleveland Area Rapid Delivery, operates a delivery service in the Cleveland metropolitan area. Most of CARD's business involves rapid delivery of documents and parcels between offices during the business day. CARD promotes its ability to make fast and on-time deliveries anywhere in the metropolitan area. When a customer calls with a delivery request, CARD quotes a guaranteed delivery time. The following network shows the street routes available. The numbers above each arc indicate the travel time in minutes between the two locations.
- Develop a linear programming model that can be used to find the minimum time required to make a delivery from location 1 to location 6.
  - How long does it take to make a delivery from location 1 to location 6?

- c. Assume that it is now 1:00 p.m. CARD just received a request for a pickup at location 1, and the closest CARD courier is 8 minutes away from location 1. If CARD provides a 20% safety margin in guaranteeing a delivery time, what is the guaranteed delivery time if the package picked up at location 1 is to be delivered to location 5?

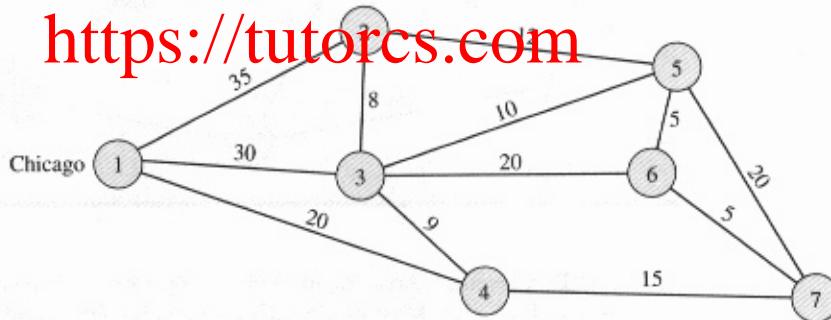


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26. Morgan Trucking Company operates a special pickup and delivery service between Chicago and six other cities located in a four-state area. When Morgan receives a request for service, it dispatches a truck from Chicago to the city requesting service as soon as possible. With both fast service and minimum travel costs as objectives for Morgan, it is important that the dispatched truck take the shortest route from Chicago to the specified city. Assume that the following network (not drawn to scale) with distances given in miles represents the highway network for this problem. Find the shortest-route distance from Chicago to node 6.

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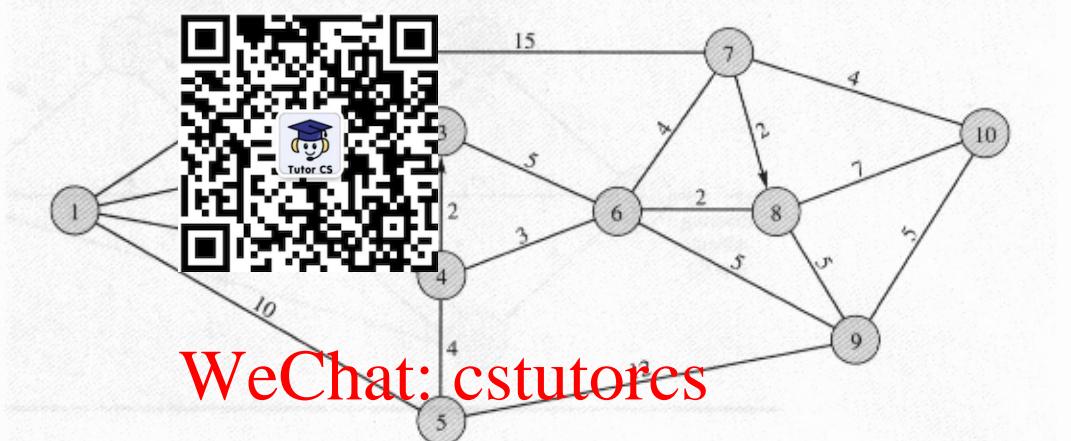
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27. City Cab Company identified 10 primary pickup and drop locations for cab riders in New York City. In an effort to minimize travel time and improve customer service and the utilization of the company's fleet of cabs, management would like the cab drivers to take the shortest route between locations whenever possible. Using the following network of roads

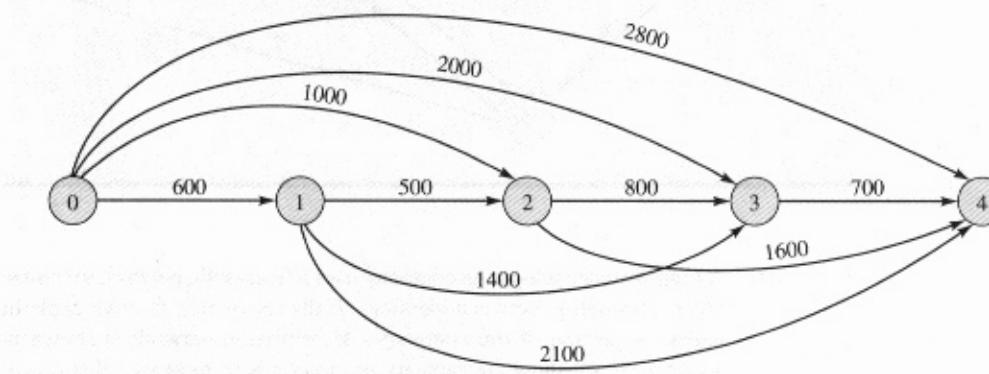
and streets, what is the route a driver beginning at location 1 should take to reach location 10? The travel times in minutes are shown on the arcs of the network. Note that there are two one-way streets with the direction shown by the arrows.

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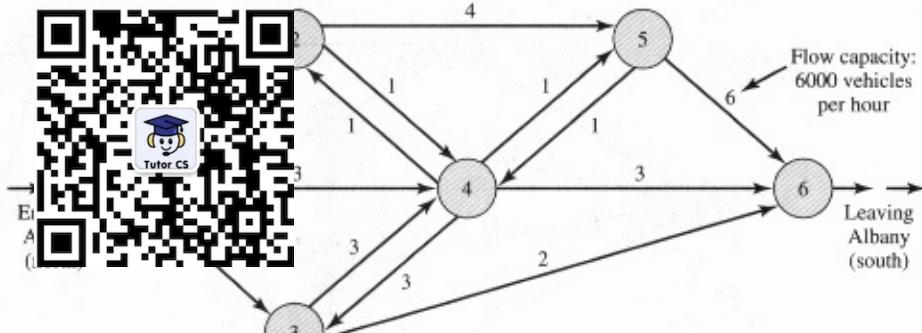
28. The five nodes in the following network represent points one year apart over a four-year period. Each node indicates a time when a decision is made to keep or replace a firm's computer equipment. If a decision is made to replace the equipment, a decision must also be made as to how long the new equipment will be used. The arc from node 0 to node 1 represents the decision to keep the current equipment one year and replace it at the end of the year. The arc from node 0 to node 2 represents the decision to keep the current equipment two years and replace it at the end of year. The numbers above the arcs indicate the total cost associated with the equipment replacement decisions. These costs include discounted purchase price, trade-in value, operating costs, and maintenance costs. Use a shortest-route model to determine the minimum cost equipment replacement policy for the four-year period.
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SELF test

29. The north-south highway system passing through Albany, New York, can accommodate the capacities shown:

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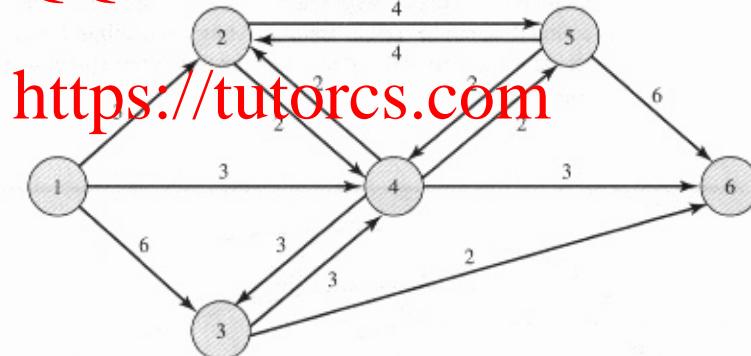


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30. On the highway system accommodate a north-south flow of 10,600 vehicles per hour? If the Albany highway system described in Problem 29 has revised flow capacities as shown in the following network, what is the maximal flow in vehicles per hour through the system? How many vehicles per hour must travel over each road (arc) to obtain this maximal flow?

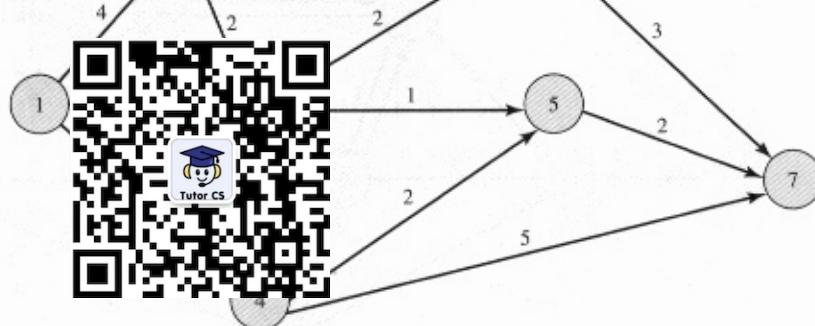
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31. A long-distance telephone company uses a fiber-optic network to transmit phone calls and other information between locations. Calls are carried through cable lines and switching nodes. A portion of the company's transmission network is shown here. The numbers above each arc show the capacity in thousands of messages that can be transmitted over that branch of the network.

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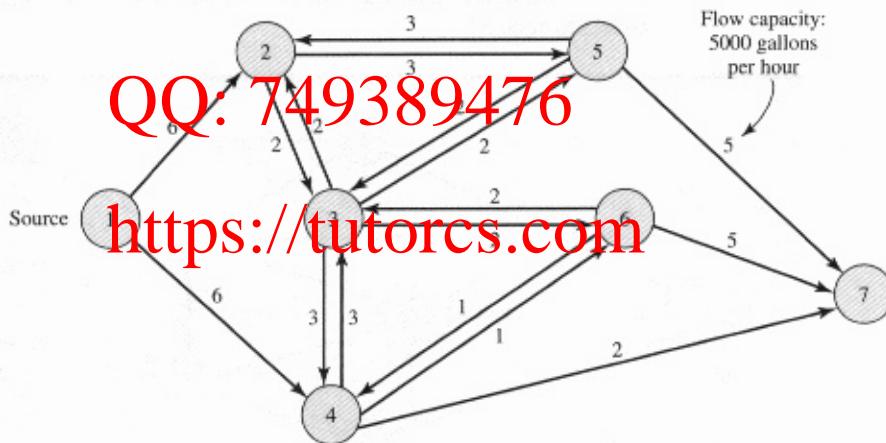


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To keep up with the volume of information transmitted between origin and destination points, use the network to determine the maximum number of messages that may be sent from a city located at node 1 to a city located at node 7.

32. The High Price Oil Company owns a pipeline network that is used to convey oil from its source to several storage locations. A portion of the network is as follows:

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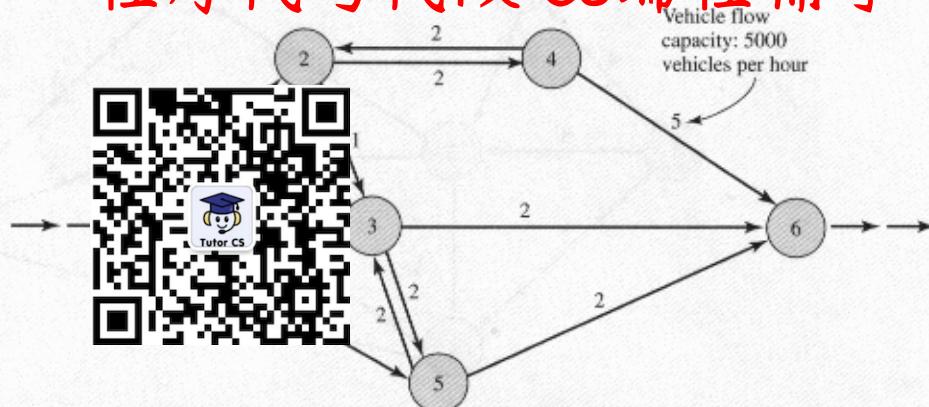


Due to the varying pipe sizes, the flow capacities vary. By selectively opening and closing sections of the pipeline network, the firm can supply any of the storage locations.

- If the firm wants to fully utilize the system capacity to supply storage location 7, how long will it take to satisfy a location 7 demand of 100,000 gallons? What is the maximal flow for this pipeline system?
- If a break occurs on line 2–3 and it is closed down, what is the maximal flow for the system? How long will it take to transmit 100,000 gallons to location 7?

33. For the following highway network system, determine the maximal flow in vehicles per hour.

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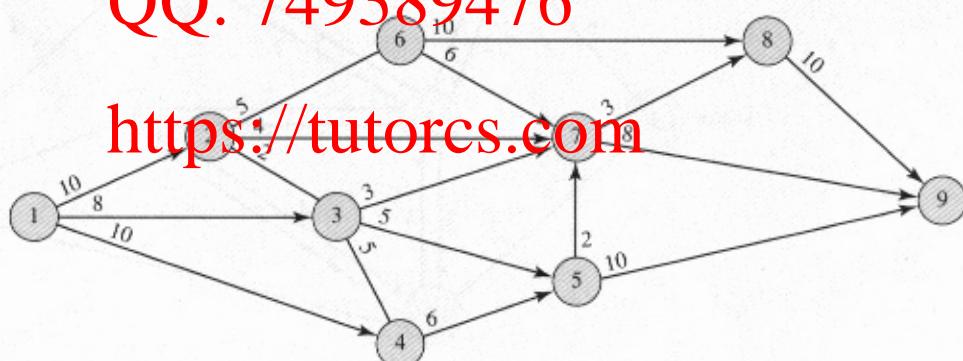
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The highway commission is considering adding highway section 3–4 to permit a flow of 2000 vehicles per hour or, at an additional cost, a flow of 3000 vehicles per hour. What is your recommendation for the 3–4 arc of the network?

34. A chemical processing plant has a network of pipes that are used to transfer liquid chemical products from one part of the plant to another. The following pipe network has pipe flow capacities in gallons per minute as shown. What is the maximum flow capacity for the system if the company wishes to transfer as much liquid chemical as possible from location 1 to location 9? How much of the chemical will flow through the section of pipe from node 3 to node 5?

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35. Refer to the Contois Carpets problem for which the network representation is shown in Figure 6.20. Suppose that Contois has a beginning inventory of 50 yards of carpet and requires an inventory of 100 yards at the end of quarter 4.
- Develop a network representation of this modified problem.
  - Develop a linear programming model and solve for the optimal solution.
36. Sanders Fishing Supply of Naples, Florida, manufactures a variety of fishing equipment that it sells throughout the United States. For the next three months, Sanders estimates demand for a particular product at 150, 250, and 300 units, respectively. Sanders can supply

this demand by producing on regular time or overtime. Because of other commitments and anticipated cost increases in month 3, the production capacities in units and the production cost per unit are as follows:

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Production	Capacity (units)	Cost per Unit
Month 1: Regular	275	\$ 50
	100	80
	200	50
	50	80
	100	60
	50	100

In month 2, regular production costs \$50 per unit, and overtime production costs \$80 per unit. In month 3, regular production costs \$50 per unit, and overtime production costs \$100 per unit. Overtime production from month 1 to month 2 would cost Sanders \$50 + \$20 = \$70 per unit. This same month 1 production used to meet demand in month 3 would cost Sanders \$50 + 2(\$20) = \$90 per unit.

- a. Develop a network representation of this production scheduling problem as a transportation problem. (*Hint:* Use four nodes; the supply for origin node 1 is the maximum that can be produced in month 1 on regular time, and so on.)
- b. Develop a linear programming model that can be used to schedule regular and overtime production for each of the three months.
- c. With the production schedule, how many units are carried in inventory each month, and what is the total cost?
- d. Is there any unused production capacity? If so, where?

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Case Problem 1 SOLUTIONS PLUS

Solutions Plus is an industrial chemicals company that produces specialized cleaning fluids and solvents for a wide variety of applications. Solutions Plus just received an invitation to submit a bid to supply Great North American railroad with a cleaning fluid for locomotives. Great North American needs the cleaning fluid at 11 locations (railway stations); it provided the following information to Solutions Plus regarding the number of gallons of cleaning fluid required at each location (see Table 6.8):

Solutions Plus can produce the cleaning fluid at its Cincinnati plant for \$1.20 per gallon. Even though the Cincinnati location is its only plant, Solutions Plus has negotiated with an industrial chemicals company located in Oakland, California, to produce and ship up to 50,000 gallons of the locomotive cleaning fluid to selected Solutions Plus customer locations. The Oakland company will charge Solutions Plus \$1.65 per gallon to produce the cleaning fluid, but Solutions Plus thinks that the lower shipping costs from Oakland to some customer locations may offset the added cost to produce the product.

**TABLE 6.8 GALLONS OF CLEANING FLUID REQUIRED AT EACH LOCATION**

Location	Gallons Required	Location	Gallons Required
Santa Ana	22,418	Glendale	33,689
El Paso	6,800	Jacksonville	68,486
Pendleton	80,290	Little Rock	148,586
Houston	100,447	Bridgeport	111,475
Kansas City	241,570	Sacramento	112,000
Los Angeles	64,761		

**TABLE 6.9 FREIGHT COST (\$ PER GALLON)**

	Cincinnati	Oakland
Santa Ana	—	0.22
El Paso	0.84	0.74
Bakersfield	0.83	0.49
Phoenix	0.45	—
Los Angeles	0.36	—
Portland	—	0.22
Seattle	—	0.22
Chicago	0.34	—
Detroit	0.34	—
St. Louis	0.34	—
Houston	—	0.15



The president of Solutions Plus, Charlie Weaver, contacted several trucking companies to negotiate shipping rates between the two production facilities (Cincinnati and Oakland) and the locations where the railroad locomotives are cleaned. Table 6.9 shows the quotes received in terms of dollars per gallon. The — entries in Table 6.9 identify shipping routes that will not be considered because of the large distances involved. These quotes for shipping rates are guaranteed for one year.

To submit a bid to the railroad company, Solutions Plus must determine the price per gallon they will charge. Solutions Plus usually sells its cleaning fluids for 15% more than its cost to produce and deliver the product. For this big contract, however, Fred Roedel, the director of marketing, suggested that maybe the company should consider a smaller profit margin. In addition, to ensure that if Solutions Plus wins the bid, they will have adequate capacity to satisfy existing orders as well as accept orders for other new business, the management team decided to limit the number of gallons of the locomotive cleaning fluid produced in the Cincinnati plant to 200,000 gallons at most.

### Managerial Report

You are asked to make recommendations that will help Solutions Plus prepare a bid. Your report should address, but not be limited to, the following issues:

1. If Solutions Plus wins the bid, which production facility (Cincinnati or Oakland) should supply the cleaning fluid to the locations where the railroad locomotives are cleaned? How much should be shipped from each facility to each location?
2. What is the breakeven point for Solutions Plus? That is, how low can the company go on its bid without losing money?
3. If Solutions Plus wants to use its standard 15% markup, how much should it bid?
4. Freight costs are significantly affected by the price of oil. The contract on which Solutions Plus is bidding is for two years. Discuss how fluctuation in freight costs might affect the bid Solutions Plus submits.

### Case Problem 2 DISTRIBUTION SYSTEM DESIGN

The Darby Company manufactures and distributes meters used to measure electric power consumption. The company started with a small production plant in El Paso and gradually built a customer base throughout Texas. A distribution center was established in Fort

**TABLE 6.10** SHIPPING COST PER UNIT FROM PRODUCTION PLANTS TO DISTRIBUTION CENTERS (IN \$)

Plant	Distribution Center		
	Fort Worth	Santa Fe	Las Vegas
El Paso	3.20	2.20	4.20
San Bernardino	—	3.90	1.20

Worth, Texas. As business expanded, a second distribution center was established in Santa Fe, New Mexico.

The company's distribution system expanded when the company began marketing its meters in Arizona, California, Nevada, and Utah. With the growth of the West Coast business, the Darby Company opened a third distribution center in Las Vegas and just two years ago opened a second production plant in San Bernardino, California.

Manufacturing costs differ between the company's production plants. The cost of each meter produced at the El Paso plant is \$10.50. The San Bernardino plant utilizes newer and more efficient equipment; as a result, manufacturing costs are \$0.50 per meter less than at the El Paso plant.

Due to the company's rapid growth, not much attention had been paid to the efficiency of the distribution system, but Darby's management decided that it is time to address this issue. The cost of shipping a meter from each of the two plants to each of the three distribution centers is shown in Table 6.10.

The quarterly production capacity is 30,000 meters at the older El Paso plant and 20,000 meters at the San Bernardino plant. Note that no shipments are allowed from the San Bernardino plant to the Fort Worth distribution center.

The company serves nine customer zones from the three distribution centers. The forecast of the number of meters needed in each customer zone for the next quarter is shown in Table 6.11.

The cost per unit of shipping from each distribution center to each customer zone is given in Table 6.12; note that some distribution centers cannot serve certain customer zones.

In the current distribution system, delivery of the Dallas, San Antonio, Wichita, and Kansas City customer zones is satisfied by shipments from the Fort Worth distribution center. In a similar manner, the Denver, Salt Lake City, and Phoenix customer zones are served by the Santa Fe distribution center, and the Los Angeles and San Diego customer zones are

**TABLE 6.11** QUARTERLY DEMAND FORECAST

Customer Zone	Demand (meters)
Dallas	6300
San Antonio	4880
Wichita	2130
Kansas City	1210
Denver	6120
Salt Lake City	4830
Phoenix	2750
Los Angeles	8580
San Diego	4460

**TABLE 6.12 SHIPPING COST FROM THE DISTRIBUTION CENTERS TO THE CUSTOMER ZONES**

Distribution Center	Dallas	Customer Zone							
		San Antonio	Wichita	Kansas City	Denver	Salt Lake City	Phoenix	Los Angeles	San Diego
Fort Worth	0.3	—	—	4	6.0	—	—	—	—
Santa Fe	5.2	—	—	0	2.7	4.7	3.4	3.3	2.7
Las Vegas	—	—	—	—	5.4	3.3	2.4	2.1	2.5

serve distribution center. To determine how many units to ship from each plant, demand forecasts are aggregated at the distribution centers, and a transportation model is used to minimize the cost of shipping from the production plants to the distribution centers.

## Managerial Report

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You are asked to make recommendations for improving the distribution system. Your report should address, but not be limited to, the following issues:

1. If the company does not change its current distribution strategy, what will its distribution costs be for the following quarter?
2. Suppose that the company is willing to consider dropping the distribution center limitations; that is, customers could be served by any of the distribution centers for which costs are available. Can costs be reduced? By how much?
3. The company wants to explore the possibility of satisfying some of the customer demand directly from the production plants. In particular, the shipping cost is \$0.30 per unit from San Bernardino to Los Angeles and \$0.70 from San Bernardino to San Diego. The cost for direct shipments from El Paso to San Antonio is \$3.50 per unit. Can distribution costs be further reduced by considering these direct plant-to-customer shipments?
4. Over the next five years, Darby is anticipating moderate growth (5000 meters) to the North and West. Would you recommend that they consider plant expansion at this time?

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## Appendix 6.1 EXCEL SOLUTION OF TRANSPORTATION, ASSIGNMENT, AND TRANSSHIPMENT PROBLEMS

In this appendix we will use an Excel Worksheet to solve transportation, assignment, and transshipment problems. We start with the Foster Generators transportation problem (see Section 6.1).

### Transportation Problem

The first step is to enter the data for the transportation costs, the origin supplies, and the destination demands in the top portion of the worksheet. Then the linear programming model is developed in the bottom portion of the worksheet. As with all linear programs, the worksheet model has four key elements: the decision variables, the objective function, the constraint left-hand sides, and the constraint right-hand sides. For a transportation problem, the decision variables are the amounts shipped from each origin to each destination; the

FIGURE 6.22 EXCEL SOLUTION OF THE FOSTER GENERATORS PROBLEM

**WEB file**  
Foster



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A	B	C	D	E	F	G	H	
1	<b>Foster Generators</b>							
2								
3		<b>Destination</b>						
4	<b>Origin</b>		St. Louis	Lexington	<b>Supply</b>			
5	Cleveland		7	6	5000			
6	Bedford		2	3	6000			
7	York		4	5	2500			
8	<b>Demand</b>		2000	1500				
9								
10								
11	<b>Model</b>							
12								
13								
14								
15		<b>Destination</b>						
16	<b>Origin</b>	Boston	Chicago	St. Louis	Lexington	<b>Total</b>		
17	Cleveland	3500	0	0	0	5000	$\leq$	5000
18	Bedford	0	2500	2000	1500	6000	$\leq$	6000
19	York	2500	0	0	0	2500	$\leq$	2500
20	<b>Total</b>	6000	4000	2000	1500			
21		=	=	=	=			
22		6000	4000	2000	1500			

objective function is the total transportation cost; the left-hand sides are the number of units shipped from each origin and the number of units shipped into each destination; and the right-hand sides are the origin supplies and the destination demands.

The formulation and solution of the Foster Generators problem are shown in Figure 6.22. The data are in the top portion of the worksheet. The model appears in the bottom portion of the worksheet; the key elements are screened.

### Formulation

The data and descriptive labels are contained in cells A1:F8. The transportation costs are in cells B5:F7. The origin supplies are in cells F5:F7, and the destination demands are in cells B8:E8. The key elements of the model required by the Excel Solver are the decision variables, the objective function, the constraint left-hand sides, and the constraint right-hand sides. These cells are screened in the bottom portion of the worksheet.

**Decision Variables** Cells B17:E19 are reserved for the decision variables. The optimal values are shown to be  $x_{11} = 3500$ ,  $x_{12} = 1500$ ,  $x_{22} = 2500$ ,  $x_{23} = 2000$ ,  $x_{24} = 1500$ , and  $x_{41} = 2500$ . All other decision variables equal zero, indicating nothing will be shipped over the corresponding routes.

**Objective Function** The formula =SUMPRODUCT(B5:E7,B17:E19) has been placed into cell C13 to compute the cost of the solution. The minimum cost solution is shown to have a value of \$39,500.

**Left-Hand Sides** Cells F17:F19 contain the left-hand sides for the supply constraints, and cells B20:E20 contain the left-hand sides for the demand constraints.

Cell F17 = SUM(B17:E17) (Copy to F18:F19)

Cell B20 = SUM(B17:B19) (Copy to C20:E20)

**FIGURE 6.23 SOLVER PARAMETERS DIALOG BOX FOR THE FOSTER GENERATORS PROBLEM**



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Right-Hand Sides

Cells H17:H19 contain the right-hand sides for the supply constraints and Cells B22:E22 contain the right-hand sides for the demand constraints.

Cell H17 = \$5 (Copy to H18:H19)

Cell B22 = B8 (Copy to C22:E22)

### Excel Solution

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The solution shown in Figure 6.22 can be obtained by selecting **Solver** from the **Analysis** group under the **Data** tab, entering the proper values into the **Solver Parameters** dialog box, and specifying the options, **Assume Linear Model** and **Assume Non-Negative**. Then click **Solve**. The information entered into the **Solver Parameters** dialog box is shown in Figure 6.22.

### Assignment Problem

The first step is to enter the data for the assignment costs in the top portion of the worksheet. Even though the assignment model is a special case of the transportation model, it is not necessary to enter values for origin supplies and destination demands because they are always equal to one.

The linear programming model is developed in the bottom portion of the worksheet. As with all linear programs the model has four key elements: the decision variables, the objective function, the constraint left-hand sides, and the constraint right-hand sides. For an assignment problem, the decision variables indicate whether an agent is assigned to a task (with a 1 for yes or 0 for no); the objective function is the total cost of all assignments; the constraint left-hand sides are the number of tasks that are assigned to each agent and the number of agents that are assigned to each task; and the right-hand sides are the number of tasks each agent can handle (1) and the number of agents each task requires (1). The worksheet formulation and solution for the Fowle Marketing Research Problem are shown in Figure 6.24.

**FIGURE 6.24 EXCEL SOLUTION OF THE FOWLE MARKETING RESEARCH PROBLEM**

**WEB file**  
Fowle

Fowle Marketing Research						
	Client					
Project Leader	2	3				
Terry	15	9				
Carle	18	5				
McClymonds	14	3				
Total	=	=	=	=	=	=
	1	1	1	1		

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Formulation  
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The data and descriptive labels are contained in cells A1:D7. Note that we have not inserted supply and demand values because they are always equal to 1 in an assignment problem. The model appears in the bottom portion of the worksheet with the key elements screened.

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#### Decision Variables

Cells B16:D18 are reserved for the decision variables. The optimal values are shown to be  $x_{12} = 1$ ,  $x_{23} = 1$ , and  $x_{31} = 1$  with all other variables = 0.

#### Objective Function

The formula =SUMPRODUCT(B5:D7,B16:D18) has been placed into cell C12 to compute the number of days required to complete all the jobs. The minimum time solution has a value of 26 days.

#### Left-Hand Sides

Cells E16:E18 contain the left-hand sides of the constraints for the number of clients each project leader can handle. Cells B19:D19 contain the left-hand sides of the constraints requiring that each client must be assigned a project leader.

Cell E16 = SUM(B16:D16) (Copy to E17:E18)

Cell B19 = SUM(B16:B18) (Copy to C19:D19)

#### Right-Hand Sides

Cells G16:G18 contain the right-hand sides for the project leader constraints and cells B21:D21 contain the right-hand sides for the client constraints. All right-hand side cell values are 1.

**FIGURE 6.25 SOLVER PARAMETERS DIALOG BOX FOR THE FOWLE MARKETING RESEARCH PROBLEM**



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### Excel Solution

The solution shown in Figure 6.24 can be obtained by selecting Solver from the Analysis group under the Data tab, entering the proper values into the Solver Parameters dialog box, and specifying the options Assume Linear Model and Assume Non-Negative. Then click Solve. The information entered into the Solver Parameters dialog box is shown in Figure 6.25.

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### Transshipment Problem

The worksheet model we present for the transshipment problem can be used for all the network flow problems (transportation, assignment, and transshipment) in this chapter. We organize the worksheet into two sections: an arc section and a node section. Let us illustrate by showing the worksheet formulation and solution of the Ryan Electronics transshipment problem. Refer to Figure 6.26 as we describe the steps involved. The key elements are screened.

### Formulation

The arc section uses cells A3:D16. For each arc, the start node and end node are identified in cells A5:B16. The arc costs are identified in cells C5:C16, and cells D5:D16 are reserved for the values of the decision variables (the amount shipped over the arcs).

The node section uses cells F5:K14. Each of the nodes is identified in cells F7:F14. The following formulas are entered into cells G7:H14 to represent the flow out and the flow in for each node:

Units shipped in:	Cell G9 =D5+D7
	Cell G10 =D6+D8
	Cell G11 =D9+D13
	Cell G12 =D10+D14

FIGURE 6.26 EXCEL SOLUTION FOR THE RYAN ELECTRONICS PROBLEM

WEB file  
Ryan



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A	B	C	D	E	F	G	H	I	J	K
1	Ryan Electronics Transshipment									
2										
3		Arc		Units						
4	Start Nod			ped						
5	Denver		550							
6	Denver		50							
7	Atlanta		0							
8	Atlanta		400							
9	Kansas C		200							
10	Kansas C		0							
11	Kansas C		350							
12	Kansas C		0							
13	Louisville		0							
14	Louisville	Miami	4	150						
15	Louisville	Dallas	6	0						
16	Louisville	New Orleans	5	300						
17										
18								Total Cost	5200	

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The net shipments in cells I7:I14 are the flows out minus the flows in for each node. For supply nodes, the flow out will exceed the flow in, resulting in positive net shipments. For demand nodes, the flow out will be less than the flow in, resulting in negative net shipments. The net supply figure in cells K7:K14. Note that the net supply is negative for demand nodes.

As in previous worksheet formulations, we screened the key elements required by the Excel Solver.

Decision Variables Cells D5:D16 are reserved for the decision variables. The optimal number of units to ship over each arc is shown.

Objective Function The formula =SUMPRODUCT(C5:C16,D5:D16) is placed into cell I18 to show the total cost associated with the solution. As shown, the minimum total cost is \$5200.

Left-Hand Sides The left-hand sides of the constraints represent the net shipments for each node. Cells I7:I14 are reserved for these constraints.

Cell I7 = H7-G7 (Copy to I8:I14)

Right-Hand Sides The right-hand sides of the constraints represent the supply at each node. Cells K7:K14 are reserved for these values. (Note the negative supply at the four demand nodes.)

FIGURE 6.27 SOLVER PARAMETERS DIALOG BOX FOR THE RYAN ELECTRONICS PROBLEM



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#### Excel Solution

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The solution can be obtained by selecting Solver from the Analysis group under the Data tab, entering the proper values into the Solver Parameters dialog box, and specifying the options Assume Linear Model and Assume Non-Negative. Then click Solve. The information entered into the Solver Parameters dialog box is shown in Figure 6.27.

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