CHAPTER 4

Linear Programming Applications in Marketing, Finance, and Operations Management

CONTENTS

Linear programming has proven to be one of the most successful quantitative approaches to decision making. Applications have been reported in almost every industry. These applications include production scheduling, media selection, financial planning, capital budgeting, transportation, distribution system design, product mix, staffing, and blending.

The wide variety of Management Science in Actions presented in Chapters 2 and 3 illustrated the use of linear programming as a flexible problem-solving tool. The Management Science in Action, A Marketing Planning Model at Marathon Oil Company, provides another example of the use of linear programming by showing how Marathon uses a large-scale linear programming model to solve a wide variety of planning problems. Later in the chapter, other Management Science in Action features illustrate how GE Capital uses linear programming for optimal lease structuring; how Jeppesen Sanderson uses linear programming for optimal production of flight manuals; and how the Kellogg Company uses a large-scale linear programming model to integrate production, distribution, and inventory planning.

In this chapter we present a variety of applications from the traditional business areas of marketing, finance, and operations management. Modeling, computer solution, and interpretation of output are emphasized. A mathematical model is developed for each problem studied, and solutions are presented for most of the applications. In the chapter appendix we illustrate the use of Excel Solver by solving a financial planning problem.

MANAGEMENT SCIENCE IN ACTION

A MARKETING PLANNING MODEL AT MARATHON OIL COMPANY*

Marathon Oil Company has four refineries within the United States, operates 50 light products terminals, and has product demand at more than 95 locations. The Supply and Transportation Division faces the problem of determining which refinery should supply which terminal and, at the same time, determining which products should be transported via pipeline, barge, or tanker to minimize cost. Product demand must be satisfied, and the supply capability of each refinery must not be exceeded. To help solve this difficult problem, Marathon Oil developed a marketing planning model.

The marketing planning model is a large-scale linear programming model that takes into account sales not only at Marathon product terminals but also at all exchange locations. An exchange contract is an agreement with other oil product marketers that involves exchanging or trading Marathon's products for theirs at different locations. All pipelines, barges, and tankers within Marathon's marketing area are also represented in the linear programming model. The objective of the model is to minimize the cost of meeting a given demand structure, taking into account sales price, pipeline tariffs, exchange contract costs, product demand, terminal operating costs, refining costs, and product purchases.

The marketing planning model is used to solve a wide variety of planning problems that vary from evaluating gasoline blending economics to analyzing the economics of a new terminal or pipeline. With daily sales of about 10 million gallons of refined light product, a savings of even one-thousandth of a cent per gallon can result in significant long-term savings. At the same time, what may appear to be a savings in one area, such as refining or transportation, may actually add to overall costs when the effects are fully realized throughout the system. The marketing planning model allows a simultaneous examination of this total effect.

*Based on information provided by Robert W. Wernert at Marathon Oil Company, Findlay, Ohio.

4.1 MARKETING APPLICATIONS

Applications of linear programming in marketing are numerous. In this section we discuss applications in media selection and marketing research.

Media Selection

In Section 2.1 we provided some general guidelines for modeling linear programming problems. You may want to review Section 2.1 before proceeding with the linear programming applications in this chapter.

Media selection applications of linear programming are designed to help marketing managers allocate a fixed advertising budget to various advertising media. Potential media include newspapers, magazines, radio, television, and direct mail. In these applications, the objective is to maximize reach, frequency, and quality of exposure. Restrictions on the allowable all[illegible]ring consideration of company policy, contract requirements, and [illegible] the application that follows, we illustrate how a media selection p[illegible]ated and solved using a linear programming model.

Relax-[illegible]pment Corporation is developing a lakeside community at a private[illegible]ary market for the lakeside lots and homes includes all middle- an[illegible]s within approximately 100 miles of the development. Relax-and-[illegible]ertising firm of Boone, Phillips, and Jackson (BP&J) to design the [illegible]

After considering possible advertising media and the market to be covered, BP&J recommended that the first month's advertising be restricted to five media. At the end of the month, BP&J will then reevaluate its strategy based on the month's results. BP&J collected data on the number of potential customers reached, the cost per advertisement, the maximum number of times each medium is available, and the exposure quality rating for each of the five media. The quality rating is measured in terms of an exposure quality unit, a measure of the relative value of one advertisement in each of the media. This measure, based on BP&J's experience in the advertising business, takes into account factors such as audience demographics (age, income, and education of the audience reached), image presented, and quality of the advertisement. The information collected is presented in Table 4.1.

Relax-and-Enjoy provided BP&J with an advertising budget of $30,000 for the first month's campaign. In addition, Relax-and-Enjoy imposed the following restrictions on how BP&J may allocate these funds: At least 10 television commercials must be used, at least 50,000 potential customers must be reached, and no more than $18,000 may be spent on television advertisements. What advertising media selection plan should be recommended?

TABLE 4.1 ADVERTISING MEDIA ALTERNATIVES FOR THE RELAX-AND-ENJOY LAKE DEVELOPMENT CORPORATION

Advertising Media	Number of Potential Customers Reached	Cost ($) per Advertisement	Maximum Times Available per Month*	Exposure Quality Units
1. Daytime TV (1 min), station WKLA	1000	1500	15	65
2. Evening TV (30 sec), station WKLA	2000	3000	10	90
3. Daily newspaper (full page), *The Morning Journal*	1500	400	25	40
4. Sunday newspaper magazine (½ page color), *The Sunday Press*	2500	1000	4	60
5. Radio, 8:00 A.M. or 5:00 P.M. news (30 sec), station KNOP	300	100	30	20

*The maximum number of times the medium is available is either the maximum number of times the advertising medium occurs (e.g., four Sundays per month) or the maximum number of times BP&J recommends that the medium be used.

The decision to be made is how many times to use each medium. We begin by defining the decision variables:

$$\begin{aligned}
DTV &= \text{number of times daytime TV is used}\\
ETV &= \text{number of times evening TV is used}\\
DN &= \text{number of times daily newspaper is used}\\
SN &= \text{number of times Sunday newspaper is used}\\
R &= \text{number of times radio is used}
\end{aligned}$$

T[illegible]posure in Table 4.1 show that each daytime TV (*DTV*) advertisem[illegible]re quality units. Thus, an advertising plan with *DTV* advertisem[illegible] of 65*DTV* exposure quality units. Continuing with the data in Table [illegible]V (*ETV*) rated at 90 exposure quality units, daily newspaper (*DN*) rated at 40 exposure quality units, Sunday newspaper (*SN*) rated at 60 exposure quality units, and radio (*R*) rated at 20 exposure quality units. With the objective of maximizing the total exposure quality units for the overall media selection plan, the objective function becomes

Care must be taken to ensure the linear programming model accurately reflects the real problem. Always review your formulation thoroughly before attempting to solve the model.

$$\text{Max} \quad 65DTV + 90ETV + 40DN + 60SN + 20R \qquad \text{Exposure quality}$$

We now formulate the constraints for the model from the information given:

$$\begin{aligned}
DTV &\le 15\\
ETV &\le 10\\
DN &\le 25\\
SN &\le 4\\
R &\le 30
\end{aligned} \quad \Bigg\} \text{ Availability of media}$$

$$1500DTV + 3000ETV + 400DN + 1000SN + 100R \le 30{,}000 \qquad \text{Budget}$$

$$\left.\begin{aligned}
DTV + ETV &\ge 10\\
1500DTV + 3000ETV &\le 18{,}000
\end{aligned}\right\} \text{ Television restrictions}$$

$$1000DTV + 2000ETV + 1500DN + 2500SN + 300R \ge 50{,}000 \qquad \text{Customers reached}$$

$$DTV, ETV, DN, SN, R \ge 0$$

Problem 1 provides practice at formulating a similar media selection model.

The optimal solution to this five-variable, nine-constraint linear programming model is shown in Figure 4.1; a summary is presented in Table 4.2.

The optimal solution calls for advertisements to be distributed among daytime TV, daily newspaper, Sunday newspaper, and radio. The maximum number of exposure quality units is 2370, and the total number of customers reached is 61,500. The Reduced Costs column in Figure 4.1 indicates that the number of exposure quality units for evening TV would have to increase by at least 65 before this media alternative could appear in the optimal solution. Note that the budget constraint (constraint 6) has a dual value of 0.060. Therefore, a $1.00 increase in the advertising budget will lead to an increase of 0.06 exposure quality units. The dual value of –25.000 for constraint 7 indicates that increasing the number of television commercials by 1 will decrease the exposure quality of the advertising plan by 25 units. Alternatively, decreasing the number of television commercials by 1 will increase the exposure quality of the advertising plan by 25 units. Thus,

FIGURE 4.1 THE SOLUTION FOR THE RELAX-AND-ENJOY LAKE DEVELOPMENT CORPORATION PROBLEM

```
Optimal Objective Value =        2370.00000

   Variable          Value          Reduced Cost
 --------------  ---------------  ---------------
     DTV              [illegible]00000          0.00000
     ETV              [illegible]00000        -65.00000
     DN               [illegible]00000          0.00000
     SN               [illegible]00000          0.00000
     R                [illegible]00000          0.00000

  Constraint     Slack/Surplus      Dual Value
 --------------  ---------------  ---------------
      1               5.00000          0.00000
      2              10.00000          0.00000
      3               0.00000         16.00000
      4               2.00000          0.00000
      5               0.00000         14.00000
      6               0.00000          0.06000
      7               0.00000          [illegible]
      8            3000.00000          0.00000
      9           11500.00000          0.00000
```

Media Availability

Budget

Television Restrictions

Audience Coverage

WEB file

Relax

TABLE 4.2 ADVERTISING PLAN FOR THE RELAX-AND-ENJOY LAKE DEVELOPMENT CORPORATION

Media	Frequency	Budget
Daytime TV	10	$15,000
Daily newspaper	25	10,000
Sunday newspaper	2	2,000
Radio	30	3,000
		$30,000

Exposure quality units = 2370
Total customers reached = 61,500

Relax-and-Enjoy should consider reducing the requirement of having at least 10 television commercials.

A possible shortcoming of this model is that, even if the exposure quality measure were not subject to error, it offers no guarantee that maximization of total exposure quality will lead to maximization of profit or of sales (a common surrogate for profit). However, this issue is not a shortcoming of linear programming; rather, it is a shortcoming of the use of exposure quality as a criterion. If we could directly measure the effect of an advertisement on profit, we could use total profit as the objective to be maximized.

NOTES AND COMMENTS

1. The media selection model required subjective evaluations of the exposure quality for the media alternatives. Marketing managers may have substantial data concerning exposure quality, but the [illegible] the objective functi[illegible] siderations based prim[illegible] ment. Judgment is an [illegible] g input for a linear pro[illegible]

2. The media selection model presented in this section uses exposure quality as the objective function and places a constraint on the number of customers reached. An alternative formulation of this problem would be to use the number of customers reached as the objective function and add a constraint indicating the minimum total exposure quality required for the media plan.

Marketing Research

An organization conducts marketing research to learn about consumer characteristics, attitudes, and preferences. Marketing research firms that specialize in providing such information often do the actual research for client organizations. Typical services offered by a marketing research firm include designing the study, conducting market surveys, analyzing the data collected, and providing summary reports and recommendations for the client. In the research design phase, targets or quotas may be established for the number and types of respondents to be surveyed. The marketing research firm's objective is to conduct the survey so as to meet the client's needs at a minimum cost.

Market Survey, Inc. (MSI), specializes in evaluating consumer reaction to new products, services, and advertising campaigns. A client firm requested MSI's assistance in ascertaining consumer reaction to a recently marketed household product. During meetings with the client, MSI agreed to conduct door-to-door personal interviews to obtain responses from households with children and households without children. In addition, MSI agreed to conduct both day and evening interviews. Specifically, the client's contract called for MSI to conduct 1000 interviews under the following quota guidelines.

1. Interview at least 400 households with children.
2. Interview at least 400 households without children.
3. The total number of households interviewed during the evening must be at least as great as the number of households interviewed during the day.
4. At least 40% of the interviews for households with children must be conducted during the evening.
5. At least 60% of the interviews for households without children must be conducted during the evening.

Because the interviews for households with children take additional interviewer time and because evening interviewers are paid more than daytime interviewers, the cost varies with the type of interview. Based on previous research studies, estimates of the interview costs are as follows:

	Interview Cost	
Household	**Day**	**Evening**
Children	$20	$25
No children	$18	$20

What is the household, time-of-day interview plan that will satisfy the contract requirements at a minimum total interviewing cost?

In formulating the linear programming model for the MSI problem, we utilize the following decision-variable notation:

DC = the number of daytime interviews of households with children
EC = the number of evening interviews of households with children
DNC = the number of daytime interviews of households without children
ENC = the number of evening interviews of households without children

We begin the linear programming model formulation by using the cost-per-interview data to develop the objective function:

$$\text{Min } 20DC + 25EC + 18DNC + 20ENC$$

The constraint requiring a total of 1000 interviews is

$$DC + EC + DNC + ENC = 1000$$

The five specifications concerning the types of interviews are as follows.

- Households with children:

$$DC + EC \geq 400$$

- Households without children:

$$DNC + ENC \geq 400$$

- At least as many evening interviews as day interviews:

$$EC + ENC \geq DC + DNC$$

- At least 40% of interviews of households with children during the evening:

$$EC \geq 0.4(DC + EC)$$

- At least 60% of interviews of households without children during the evening:

$$ENC \geq 0.6(DNC + ENC)$$

When we add the nonnegativity requirements, the four-variable and six-constraint linear programming model becomes

Min $\quad 20DC + 25EC + 18DNC + 20ENC$

s.t.

$DC + EC + DNC + ENC = 1000$	Total interviews
$DC + EC \geq 400$	Households with children
$DNC + ENC \geq 400$	Households without children
$EC + ENC \geq DC + DNC$	Evening interviews
$EC \geq 0.4(DC + EC)$	Evening interviews in households with children
$ENC \geq 0.6(DNC + ENC)$	Evening interviews in households without children

$$DC, EC, DNC, ENC \geq 0$$

The optimal solution to this linear program is shown in Figure 4.2. The solution reveals that the minimum cost of $20,320 occurs with the following interview schedule.

	Number of Interviews		
Household	Day	Evening	Totals
[illegible]	240	160	400
[illegible]	240	360	600
[illegible]	480	520	1000

Henc[illegible] be scheduled during the day and 520 during the evening. Hous[illegible] be covered by 400 interviews, and households without children [illegible] interviews.

Selected sensitivity analysis information from Figure 4.2 shows a dual value of 19.200 for constraint 1. In other words, the value of the optimal solution will increase by $19.20 if the number of interviews is increased from 1000 to 1001. Thus, $19.20 is the incremental cost of obtaining additional interviews. It also is the savings that could be realized by reducing the number of interviews from 1000 to 999.

The surplus variable, with a value of 200.000, for constraint 3 shows that 200 more households without children will be interviewed than required. Similarly, the surplus variable, with a value of 40.000, for constraint 4 shows that the number of evening interviews exceeds the number of daytime interviews by 40. The zero values for the surplus variables in constraints 5 and 6 indicate that the more expensive evening interviews are being held at a minimum. Indeed, the dual value of 5.000 for constraint 5 indicates that if one more household (with children) than the minimum requirement must be interviewed during the evening, the total interviewing cost will go up by $5.00. Similarly, constraint 6 shows that requiring one more household (without children) to be interviewed during the evening will increase costs by $2.00.

FIGURE 4.2 THE SOLUTION FOR THE MARKET SURVEY PROBLEM

WEB file

Market

```
Optimal Objective Value =              20320.00000

    Variable              Value              Reduced Cost
 ---------------     ---------------     ------------------
        DC              240.00000               0.00000
        EC              160.00000               0.00000
        DNC             240.00000               0.00000
        ENC             360.00000               0.00000

   Constraint         Slack/Surplus           Dual Value
 ---------------     ---------------     ------------------
        1                 0.00000              19.20000
        2                 0.00000               2.80000
        3               200.00000               0.00000
        4                40.00000               0.00000
        5                 0.00000               5.00000
        6                 0.00000               2.00000
```

4.2 FINANCIAL APPLICATIONS

In finance, linear programming can be applied in problem situations involving capital budgeting, make-or-buy decisions, asset allocation, portfolio selection, financial planning, and many more. In this section, we describe a portfolio selection problem and a problem involving funding of an early retirement program.

Portfoli[illegible]

Portfolio [illegible]lve situations in which a financial manager must select specific i[illegible]le, stocks and bonds—from a variety of investment alternativ[illegible] funds, credit unions, insurance companies, and banks frequently [illegible] problem. The objective function for portfolio selection problems [illegible] of expected return or minimization of risk. The constraints usually take the form of restrictions on the type of permissible investments, state laws, company policy, maximum permissible risk, and so on. Problems of this type have been formulated and solved using a variety of mathematical programming techniques. In this section we formulate and solve a portfolio selection problem as a linear program.

Consider the case of Welte Mutual Funds, Inc., located in New York City. Welte just obtained $100,000 by converting industrial bonds to cash and is now looking for other investment opportunities for these funds. Based on Welte's current investments, the firm's top financial analyst recommends that all new investments be made in the oil industry, steel industry, or in government bonds. Specifically, the analyst identified five investment opportunities and projected their annual rates of return. The investments and rates of return are shown in Table 4.3.

Management of Welte imposed the following investment guidelines.

1. Neither industry (oil or steel) should receive more than $50,000.
2. Government bonds should be at least 25% of the steel industry investments.
3. The investment in Pacific Oil, the high-return but high-risk investment, cannot be more than 60% of the total oil industry investment.

What portfolio recommendations—investments and amounts—should be made for the available $100,000? Given the objective of maximizing projected return subject to the budgetary and managerially imposed constraints, we can answer this question by formulating and solving a linear programming model of the problem. The solution will provide investment recommendations for the management of Welte Mutual Funds.

TABLE 4.3 INVESTMENT OPPORTUNITIES FOR WELTE MUTUAL FUNDS

Investment	Projected Rate of Return (%)
Atlantic Oil	7.3
Pacific Oil	10.3
Midwest Steel	6.4
Huber Steel	7.5
Government bonds	4.5

Let

A = dollars invested in Atlantic Oil
P = dollars invested in Pacific Oil
M = dollars invested in Midwest Steel
H = dollars invested in Huber Steel
G = dollars invested in government bonds

Using the projected annual rates of return shown in Table 4.3, we write the objective function for maximizing the total annual return for the portfolio as

$$\text{Max } 0.073A + 0.103P + 0.064M + 0.075H + 0.045G$$

The constraint specifying investment of the available \$100,000 is

$$A + P + M + H + G = 100{,}000$$

The requirements that neither the oil nor the steel industry should receive more than \$50,000 are

$$A + P \leq 50{,}000$$
$$M + H \leq 50{,}000$$

The requirement that government bonds be at least 25% of the steel industry investment is expressed as

$$G \geq 0.25(M + H)$$

Finally, the constraint that Pacific Oil cannot be more than 60% of the total oil industry investment is

$$P \leq 0.60(A + P)$$

By adding the nonnegativity restrictions, we obtain the complete linear programming model for the Welte Mutual Funds investment problem:

Max $0.073A + 0.103P + 0.064M + 0.075H + 0.045G$

s.t.

$A + P + M + H + G = 100{,}000$	Available funds
$A + P \leq 50{,}000$	Oil industry maximum
$M + H \leq 50{,}000$	Steel industry maximum
$G \geq 0.25\,(M + H)$	Government bonds minimum
$P \leq 0.60\,(A + P)$	Pacific Oil restriction

$$A, P, M, H, G \geq 0$$

The optimal solution to this linear program is shown in Figure 4.3. Table 4.4 shows how the funds are divided among the securities. Note that the optimal solution indicates that the portfolio should be diversified among all the investment opportunities except

FIGURE 4.3 THE SOLUTION FOR THE WELTE MUTUAL FUNDS PROBLEM


```
Optimal Objective Value = 8000.00000

   Variable              Value           Reduced Costs
 --------------     ---------------     ---------------
        A              20000.00000              0.00000
        P              30000.00000              0.00000
        M                  0.00000             -0.01100
        H              40000.00000              0.00000
        G              10000.00000              0.00000

   Constraint        Slack/Surplus          Dual Value
 --------------     ---------------     ---------------
        1                  0.00000              0.06900
        2                  0.00000              0.02200
        3              10000.00000              0.00000
        4                  0.00000             -0.02400
        5                  0.00000              0.03000
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WEB file

Welte

Midwest Steel. The projected annual return for this portfolio is $8000, which is an overall return of 8%.

The optimal solution shows the dual value for constraint 3 is zero. The reason is that the steel industry maximum isn't a binding constraint; increases in the steel industry limit of $50,000 will not improve the value of the optimal solution. Indeed, the slack variable for this constraint shows that the current steel industry investment is $10,000 below its limit of $50,000. The dual values for the other constraints are nonzero, indicating that these constraints are binding.

The dual value for the available funds constraint provides information on the rate of return from additional investment funds.

The dual value of 0.069 for constraint 1 shows that the value of the optimal solution can be increased by 0.069 if one more dollar can be made available for the portfolio investment. If more funds can be obtained at a cost of less than 6.9%, management should consider obtaining them. However, if a return in excess of 6.9% can be obtained by investing funds elsewhere (other than in these five securities), management should question the wisdom of investing the entire $100,000 in this portfolio.

Similar interpretations can be given to the other dual values. Note that the dual value for constraint 4 is negative at –0.024. This result indicates that increasing the value on the

TABLE 4.4 OPTIMAL PORTFOLIO SELECTION FOR WELTE MUTUAL FUNDS

Investment	Amount	Expected Annual Return
Atlantic Oil	$ 20,000	$1460
Pacific Oil	30,000	3090
Huber Steel	40,000	3000
Government bonds	10,000	450
Totals	$100,000	$8000

Expected annual return of $8000
Overall rate of return = 8%

right-hand side of the constraint by one unit can be expected to decrease the objective function value of the optimal solution by 0.024. In terms of the optimal portfolio, then, if Welte invests one more dollar in government bonds (beyond the minimum requirement), the total return will decrease by $0.024. To see why this decrease occurs, note again from the dual value for constraint 1 that the marginal return on the funds invested in the portfolio is 6.9% (the average return is 8%). The rate of return on government bonds is 4.5%. Thus, the cost of in[illegible] in government bonds is the difference between the marginal retur[illegible] marginal return on government bonds: 6.9% – 4.5% = 2.4%.

Practice formulating a variation of the Welte problem by working Problem 9.

N[illegible]tion shows that Midwest Steel should not be included in the portf[illegible]iated reduced cost for *M* of –0.011 tells us that the objective funct[illegible]est Steel would have to increase by 0.011 before considering the M[illegible]t alternative would be advisable. With such an increase the Midw[illegible]e 0.064 + 0.011 = 0.075, making this investment just as desirab[illegible]Huber Steel investment alternative.

Finally, a simple modification of the Welte linear programming model permits determining the fraction of available funds invested in each security. That is, we divide each of the right-hand-side values by 100,000. Then the optimal values for the variables will give the fraction of funds that should be invested in each security for a portfolio of any size.

NOTES AND COMMENTS

1. The optimal solution to the Welte Mutual Funds problem indicates that $20,000 is to be spent on the Atlantic Oil stock. If Atlantic Oil sells for $75 per share, we would have to purchase exactly 266⅔ shares in order to spend exactly $20,000. The difficulty of purchasing fractional shares can be handled by purchasing the largest possible integer number of shares with the allotted funds (e.g., 266 shares of Atlantic Oil). This approach guarantees that the budget constraint will not be violated. This approach, of course, introduces the possibility that the solution will no longer be optimal, but the danger is slight if a large number of securities are involved. In cases where the analyst believes that the decision variables *must* have integer values, the problem must be formulated as an integer linear programming model. Integer linear programming is the topic of Chapter 7.
2. Financial portfolio theory stresses obtaining a proper balance between risk and return. In the Welte problem, we explicitly considered return in the objective function. Risk is controlled by choosing constraints that ensure diversity among oil and steel stocks and a balance between government bonds and the steel industry investment.

Financial Planning

Linear programming has been used for a variety of financial planning applications. The Management Science in Action, Optimal Lease Structuring at GE Capital, describes how linear programming is used to optimize the structure of a leveraged lease.

MANAGEMENT SCIENCE IN ACTION

OPTIMAL LEASE STRUCTURING AT GE CAPITAL*

GE Capital is a $70 billion subsidiary of General Electric. As one of the nation's largest and most diverse financial services companies, GE Capital arranges leases in both domestic and international markets, including leases for telecommunications; data processing; construction; and fleets of cars, trucks, and commercial aircraft. To help allocate and schedule the rental and debt payments of a leveraged lease, GE Capital analysts developed an optimization model, which is available as an optional component of the company's lease analysis proprietary software.

Leveraged leases are designed to provide financing for assets with economic lives of at least five years, which require large capital outlays. A leveraged lease represents an agreement among the lessor (the owner of the asset), the lessee (the user of the asset), and th[illegible] nonrecourse loan of 50% [illegible]chase price. In a nonrec[illegible]annot turn to the lessor f[illegible] of default. As the lessor [illegible] Capital is able to claim [illegible]come tax benefits such a[illegible]st deductions. These d[illegible]e tax losses during the early years of the lease, which reduces the total tax liability. Approximately 85% of all financial leases in the United States are leveraged leases.

In its simplest form the leveraged lease structuring problem can be formulated as a linear program. The linear program models the after-tax cash flow for the lessor, taking into consideration rental receipts, borrowing and repaying of the loan, and income taxes. Constraints are formulated to ensure compliance with IRS guidelines and to enable customizing of leases to meet lessee and lessor requirements. The objective function can be entered in a custom fashion or selected from a predefined list. Typically, the objective is to minimize the lessee's cost, expressed as the net present value of rental payments, or to maximize the lessor's after-tax yield.

GE Capital developed an optimization approach that could be applied to single-investor lease structuring. In a study with the department most involved with these transactions, the optimization approach yielded substantial benefits. The approach helped GE Capital win some single-investor transactions ranging in size from $1 million to $20 million.

*Based on C. J. Litty, "Optimal Lease Structuring at GE Capital," *Interfaces* (May/June 1994): 34–45.

Hewlitt Corporation established an early retirement program as part of its corporate restructuring. At the close of the voluntary sign-up period, 68 employees had elected early retirement. As a result of these early retirements, the company incurs the following obligations over the next eight years:

Year	1	2	3	4	5	6	7	8
Cash Requirement	430	210	222	231	240	195	225	255

The cash requirements (in thousands of dollars) are due at the beginning of each year.

The corporate treasurer must determine how much money must be set aside today to meet the eight yearly financial obligations as they come due. The financing plan for the retirement program includes investments in government bonds as well as savings. The investments in government bonds are limited to three choices:

Bond	Price	Rate (%)	Years to Maturity
1	$1150	8.875	5
2	1000	5.500	6
3	1350	11.750	7

The government bonds have a par value of $1000, which means that even with different prices each bond pays $1000 at maturity. The rates shown are based on the par value. For purposes of planning, the treasurer assumed that any funds not invested in bonds will be placed in savings and earn interest at an annual rate of 4%.

We define the decision variables as follows:

F = total dollars required to meet the retirement plan's eight-year obligation
B_1 = units of bond 1 purchased at the beginning of year 1
B_2 = units of bond 2 purchased at the beginning of year 1
B_3 = units of bond 3 purchased at the beginning of year 1
S_i = amount placed in savings at the beginning of year i for $i = 1, \ldots, 8$

The objective function is to minimize the total dollars needed to meet the retirement plan's eight-year obligation, or

$$\text{Min} \quad F$$

A key feature of this type of financial planning problem is that a constraint must be formulated for each year of the planning horizon. In general, each constraint takes the form:

$$\begin{pmatrix}\text{Funds available at} \\ \text{the beginning of the year}\end{pmatrix} - \begin{pmatrix}\text{Funds invested in bonds} \\ \text{and placed in savings}\end{pmatrix} = \begin{pmatrix}\text{Cash obligation for} \\ \text{the current year}\end{pmatrix}$$

The funds available at the beginning of year 1 are given by F. With a current price of \$1150 for bond 1 and investments expressed in thousands of dollars, the total investment for B_1 units of bond 1 would be $1.15B_1$. Similarly, the total investment in bonds 2 and 3 would be $1B_2$ and $1.35B_3$, respectively. The investment in savings for year 1 is S_1. Using these results and the first-year obligation of 430, we obtain the constraint for year 1:

$$F - 1.15B_1 - 1B_2 - 1.35B_3 - S_1 = 430 \quad \text{Year 1}$$

We do not consider future investments in bonds because the future price of bonds depends on interest rates and cannot be known in advance.

Investments in bonds can take place only in this first year, and the bonds will be held until maturity.

The funds available at the beginning of year 2 include the investment returns of 8.875% on the par value of bond 1, 5.5% on the par value of bond 2, 11.75% on the par value of bond 3, and 4% on savings. The new amount to be invested in savings for year 2 is S_2. With an obligation of 210, the constraint for year 2 is

$$0.08875B_1 + 0.055B_2 + 0.1175B_3 + 1.04S_1 - S_2 = 210 \quad \text{Year 2}$$

Similarly, the constraints for years 3 to 8 are:

$$\begin{aligned}
0.08875B_1 + 0.055B_2 + 0.1175B_3 + 1.04S_2 - S_3 &= 222 \quad \text{Year 3} \\
0.08875B_1 + 0.055B_2 + 0.1175B_3 + 1.04S_3 - S_4 &= 231 \quad \text{Year 4} \\
0.08875B_1 + 0.055B_2 + 0.1175B_3 + 1.04S_4 - S_5 &= 240 \quad \text{Year 5} \\
1.08875B_1 + 0.055B_2 + 0.1175B_3 + 1.04S_5 - S_6 &= 195 \quad \text{Year 6} \\
1.055B_2 + 0.1175B_3 + 1.04S_6 - S_7 &= 225 \quad \text{Year 7} \\
1.1175B_3 + 1.04S_7 - S_8 &= 255 \quad \text{Year 8}
\end{aligned}$$

Note that the constraint for year 6 shows that funds available from bond 1 are $1.08875B_1$. The coefficient of 1.08875 reflects the fact that bond 1 matures at the end of year 5. As a result, the par value plus the interest from bond 1 during year 5 is available at the beginning of year 6. Also, because bond 1 matures in year 5 and becomes available for use at the beginning of year 6, the variable B_1 does not appear in the constraints for years 7 and 8. Note the similar interpretation for bond 2, which matures at the end of year 6 and has the par value plus interest available at the beginning of year 7. In addition, bond 3 matures at the end of year 7 and has the par value plus interest available at the beginning of year 8.

Finally, note that a variable S_8 appears in the constraint for year 8. The retirement fund obligation will be completed at the beginning of year 8, so we anticipate that S_8 will be zero and no funds will be put into savings. However, the formulation includes S_8 in the event that the bond income plus interest from the savings in year 7 exceed the 255 cash requirement for year 8. Thus, S_8 is a surplus variable that shows any funds remaining after the eight-year cash requirements have been satisfied.

The o[illegible]-variable, 8-constraint linear program is shown in Figure 4.4. W[illegible] value of 1728.79385, the total investment required to meet the r[illegible]ar obligation is \$1,728,794. Using the current prices of \$1150, \$1[illegible] of the bonds, respectively, we can summarize the initial investmen[illegible]ollows:

Bo[illegible]	[illegible]chased	Investment Amount
1	$B_1 = 144.988$	\$1150(144.988) = \$166,736
2	$B_2 = 187.856$	\$1000(187.856) = \$187,856
3	$B_3 = 228.188$	\$1350(228.188) = \$308,054

FIGURE 4.4 THE SOLUTION FOR THE HEWLITT CORPORATION CASH REQUIREMENTS PROBLEM

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Hewlitt

Optimal Objective Value = 1728.79385

Variable	Value	Reduced Cost
F	1728.79385	0.00000
B1	144.98815	0.00000
B2	187.85586	0.00000
B3	228.18792	0.00000
S1	636.14794	0.00000
S2	501.60571	0.00000
S3	349.68179	0.00000
S4	182.68091	0.00000
S5	0.00000	0.06403
S6	0.00000	0.01261
S7	0.00000	0.02132
S8	0.00000	0.67084

Constraint	Slack/Surplus	Dual Value
1	0.00000	1.00000
2	0.00000	0.96154
3	0.00000	0.92456
4	0.00000	0.88900
5	0.00000	0.85480
6	0.00000	0.76036
7	0.00000	0.71899
8	0.00000	0.67084

The solution also shows that \$636,148 (see S_1) will be placed in savings at the beginning of the first year. By starting with \$1,728,794, the company can make the specified bond and savings investments and have enough left over to meet the retirement program's first-year cash requirement of \$430,000.

The optimal solution in Figure 4.4 shows that the decision variables S_1, S_2, S_3, and S_4 all are greater than zero, indicating investments in savings are required in each of the first four y[illegible] from the bonds plus the bond maturity incomes will be sufficient t[illegible] program's cash requirements in years 5 through 8.

In this application, the dual value can be thought of as the present value of each dollar in the cash requirement. For example, each dollar that must be paid in year 8 has a present value of \$0.67084.

T[illegible] interesting interpretation in this application. Each right-hand-side v[illegible] payment that must be made in that year. Note that the dual values [illegible] that increasing the required payment in any year by \$1,000 would [illegible] required for the retirement program's obligation by \$1,000 times [illegible] e that the dual values show that increases in required payments [illegible] he largest impact. This makes sense in that there is little time to build up investment income in the early years versus the subsequent years. This suggests that if Hewlitt faces increases in required payments it would benefit by deferring those increases to later years if possible.

NOTES AND COMMENTS

1. The optimal solution for the Hewlitt Corporation problem shows fractional numbers of government bonds at 144.988, 187.856, and 228.188 units, respectively. However, fractional bond units usually are not available. If we were conservative and rounded up to 145, 188, and 229 units, respectively, the total funds required for the eight-year retirement program obligation would be approximately \$1254 more than the total funds indicated by the objective function. Because of the magnitude of the funds involved, rounding up probably would provide a workable solution. If an optimal integer solution were required, the methods of integer linear programming covered in Chapter 7 would have to be used.
2. We implicitly assumed that interest from the government bonds is paid annually. Investments such as treasury notes actually provide interest payments every six months. In such cases, the model can be reformulated with six-month periods, with interest and/or cash payments occurring every six months.

4.3 OPERATIONS MANAGEMENT APPLICATIONS

Linear programming applications developed for production and operations management include scheduling, staffing, inventory control, and capacity planning. In this section we describe examples with make-or-buy decisions, production scheduling, and workforce assignments.

A Make-or-Buy Decision

We illustrate the use of a linear programming model to determine how much of each of several component parts a company should manufacture and how much it should purchase from an outside supplier. Such a decision is referred to as a make-or-buy decision.

The Janders Company markets various business and engineering products. Currently, Janders is preparing to introduce two new calculators: one for the business market called the Financial Manager and one for the engineering market called the Technician. Each calculator has three components: a base, an electronic cartridge, and a faceplate or top. The same base is used for both calculators, but the cartridges and tops are different. All components can be manufactured by the company or purchased from outside suppliers. The manufacturing costs and purchase prices for the components are summarized in Table 4.5.

TABLE 4.5 MANUFACTURING COSTS AND PURCHASE PRICES FOR JANDERS CALCULATOR COMPONENTS

	Cost per Unit	
Component	**Manufacture (regular time)**	**Purchase**
Base	\$0.50	\$0.60
Financial cartridge	\$3.75	\$4.00
Technician cartridge	\$3.30	\$3.90
Financial top	\$0.60	\$0.65
Technician top	\$0.75	\$0.78

Company forecasters indicate that 3000 Financial Manager calculators and 2000 Technician calculators will be needed. However, manufacturing capacity is limited. The company has 200 hours of regular manufacturing time and 50 hours of overtime that can be scheduled for the calculators. Overtime involves a premium at the additional cost of \$9 per hour. Table 4.6 shows manufacturing times (in minutes) for the components.

The problem for Janders is to determine how many units of each component to manufacture and how many units of each component to purchase. We define the decision variables as follows:

BM = number of bases manufactured
BP = number of bases purchased
FCM = number of Financial cartridges manufactured
FCP = number of Financial cartridges purchased
TCM = number of Technician cartridges manufactured
TCP = number of Technician cartridges purchased
FTM = number of Financial tops manufactured
FTP = number of Financial tops purchased
TTM = number of Technician tops manufactured
TTP = number of Technician tops purchased

One additional decision variable is needed to determine the hours of overtime that must be scheduled:

OT = number of hours of overtime to be scheduled

TABLE 4.6 MANUFACTURING TIMES IN MINUTES PER UNIT FOR JANDERS CALCULATOR COMPONENTS

Component	Manufacturing Time
Base	1.0
Financial cartridge	3.0
Technician cartridge	2.5
Financial top	1.0
Technician top	1.5

The objective function is to minimize the total cost, including manufacturing costs, purchase costs, and overtime costs. Using the cost-per-unit data in Table 4.5 and the overtime premium cost rate of $9 per hour, we write the objective function as

$$\text{Min} \quad 0.5BM + 0.6BP + 3.75FCM + 4FCP + 3.3TCM + 3.9TCP + 0.6FTM + 0.65FTP + 0.75TTM + 0.78TTP + 9OT$$

T[illegible] specify the number of each component needed to satisfy the dema[illegible]Manager calculators and 2000 Technician calculators. A total of 50[illegible] needed, with the number of other components depending on the d[illegible] calculator. The five demand constraints are

$$\begin{aligned}
[illegible] + BP &= 5000 \quad \text{Bases}\\
[illegible] + FCP &= 3000 \quad \text{Financial cartridges}\\
[illegible] + TCP &= 2000 \quad \text{Technician cartridges}\\
FTM + FTP &= 3000 \quad \text{Financial tops}\\
TTM + TTP &= 2000 \quad \text{Technician tops}
\end{aligned}$$

Two constraints are needed to guarantee that manufacturing capacities for regular time and overtime cannot be exceeded. The first constraint limits overtime capacity to 50 hours, or

$$OT \le 50$$

The same units of measure must be used for both the left-hand side and right-hand side of the constraint. In this case, minutes are used.

The second constraint states that the total manufacturing time required for all components must be less than or equal to the total manufacturing capacity, including regular time plus overtime. The manufacturing times for the components are expressed in minutes, so we state the total manufacturing capacity constraint in minutes, with the 200 hours of regular time capacity becoming 60(200) = 12,000 minutes. The actual overtime required is unknown at this point, so we write the overtime as $60OT$ minutes. Using the manufacturing times from Table 4.6, we have

$$BM + 3FCM + 2.5TCM + FTM + 1.5TTM \le 12{,}000 + 60OT$$

The complete formulation of the Janders make-or-buy problem with all decision variables greater than or equal to zero is

$$\text{Min} \quad 0.5BM + 0.6BP + 3.75FCM + 4FCP + 3.3TCM + 3.9TCP + 0.6FTM + 0.65FTP + 0.75TTM + 0.78TTP + 9OT$$

s.t.

$$\begin{aligned}
BM + BP &= 5000 &&\text{Bases}\\
FCM + FCP &= 3000 &&\text{Financial cartridges}\\
TCM + TCP &= 2000 &&\text{Technician cartridges}\\
FTM + FTP &= 3000 &&\text{Financial tops}\\
TTM + TTP &= 2000 &&\text{Technician tops}\\
OT &\le 50 &&\text{Overtime hours}\\
BM + 3FCM + 2.5TCM + FTM + 1.5TTM &\le 12{,}000 + 60OT &&\text{Manufacturing capacity}
\end{aligned}$$

The optimal solution to this 11-variable, 7-constraint linear program is shown in Figure 4.5. The optimal solution indicates that all 5000 bases (BM), 667 Financial Manager cartridges (FCM), and 2000 Technician cartridges (TCM) should be manufactured. The remaining 2333 Financial Manager cartridges (FCP), all the Financial Manager tops (FTP),

FIGURE 4.5 THE SOLUTION FOR THE JANDERS MAKE-OR-BUY PROBLEM

Optimal Objective Value = 24443.33333

Variable	Value	Reduced Cost
BM	5000.00000	0.00000
BP	0.00000	0.01667
FCM	666.66667	0.00000
FCP	2333.33333	0.00000
TCM	2000.00000	0.00000
TCP	0.00000	0.39167
FTM	0.00000	0.03333
FTP	3000.00000	0.00000
TTM	0.00000	0.09500
TTP	2000.00000	0.00000
OT	0.00000	4.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	0.58333
2	0.00000	4.00000
3	0.00000	3.08333
4	0.00000	0.65000
5	0.00000	0.78000
6	50.00000	0.00000
7	0.00000	-0.08333

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
BM	0.50000	0.01667	Infinite
BP	0.60000	Infinite	0.01667
FCM	3.75000	0.10000	0.05000
FCP	4.00000	0.05000	0.10000
TCM	3.30000	0.39167	Infinite
TCP	3.90000	Infinite	0.39167
FTM	0.60000	Infinite	0.03333
FTP	0.65000	0.03333	Infinite
TTM	0.75000	Infinite	0.09500
TTP	0.78000	0.09500	Infinite
OT	9.00000	Infinite	4.00000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	5000.00000	2000.00000	5000.00000
2	3000.00000	Infinite	2333.33333
3	2000.00000	800.00000	2000.00000
4	3000.00000	Infinite	3000.00000
5	2000.00000	Infinite	2000.00000
6	50.00000	Infinite	50.00000
7	12000.00000	7000.00000	2000.00000

WEB file

Janders

and all Technician tops (*TTP*) should be purchased. No overtime manufacturing is necessary, and the total cost associated with the optimal make-or-buy plan is $24,443.33.

Sensitivity analysis provides some additional information about the unused overtime capacity. The Reduced Costs column shows that the overtime (*OT*) premium would have to decrease by $4 per hour before overtime production should be considered. That is, if the overtime premium is $9 − $4 = $5 or less, Janders may want to replace some of the purchas[illegible]ponents manufactured on overtime.

T[illegible]nufacturing capacity constraint 7 is −0.083. This value indicates [illegible] of manufacturing capacity is worth $0.083 per minute or ($0.0[illegible]he right-hand-side range for constraint 7 shows that this conclusion i[illegible]of regular time increases to 19,000 minutes, or 316.7 hours.

S[illegible]ndicates that a change in prices charged by the outside suppliers c[illegible]ution. For instance, the objective coefficient range for *BP* is 0.58[illegible]upper limit. If the purchase price for bases remains at $0.583 or more, the number of bases purchased (*BP*) will remain at zero. However, if the purchase price drops below $0.583, Janders should begin to purchase rather than manufacture the base component. Similar sensitivity analysis conclusions about the purchase price ranges can be drawn for the other components.

NOTES AND COMMENTS

The proper interpretation of the dual value for manufacturing capacity (constraint 7) in the Janders problem is that an additional hour of manufacturing capacity is worth ($0.083)(60) = $5 per hour. Thus, the company should be willing to pay a premium of $5 per hour over and above the current regular time cost per hour, which is already included in the manufacturing cost of the product. Thus, if the regular time cost is $18 per hour, Janders should be willing to pay up to $18 + $5 = $23 per hour to obtain additional labor capacity.

Production Scheduling

One of the most important applications of linear programming deals with multiperiod planning such as production scheduling. The solution to a production scheduling problem enables the manager to establish an efficient low-cost production schedule for one or more products over several time periods (weeks or months). Essentially, a production scheduling problem can be viewed as a product-mix problem for each of several periods in the future. The manager must determine the production levels that will allow the company to meet product demand requirements, given limitations on production capacity, labor capacity, and storage space, while minimizing total production costs.

One advantage of using linear programming for production scheduling problems is that they recur. A production schedule must be established for the current month, then again for the next month, for the month after that, and so on. When looking at the problem each month, the production manager will find that, although demand for the products has changed, production times, production capacities, storage space limitations, and so on are roughly the same. Thus, the production manager is basically re-solving the same problem handled in previous months, and a general linear programming model of the production scheduling procedure may be applied frequently. Once the model has been formulated, the manager can simply supply the data—demand, capacities, and so on—for the given production period and use the linear programming model repeatedly to develop the production schedule. The Management Science in Action, Optimizing Production of Flight Manuals at

TABLE 4.7 THREE-MONTH DEMAND SCHEDULE FOR BOLLINGER ELECTRONICS COMPANY

Component	April	May	June
322A	1000	3000	5000
802B	1000	500	3000

Jeppesen S[illegible] how linear programming is used to minimize the cost of producing [illegible]t manuals.

Let us [illegible] Bollinger Electronics Company, which produces two different e[illegible] a major airplane engine manufacturer. The airplane engine manu[illegible]linger sales office each quarter of its monthly requirements for components for each of the next three months. The monthly requirements for the components may vary considerably, depending on the type of engine the airplane engine manufacturer is producing. The order shown in Table 4.7 has just been received for the next three-month period.

After the order is processed, a demand statement is sent to the production control department. The production control department must then develop a three-month production plan for the components. In arriving at the desired schedule, the production manager will want to identify the following:

1. Total production cost
2. Inventory holding cost
3. Change-in-production-level costs

In the remainder of this section, we show how to formulate a linear programming model of the production and inventory process for Bollinger Electronics to minimize the total cost.

MANAGEMENT SCIENCE IN ACTION

OPTIMIZING PRODUCTION OF FLIGHT MANUALS AT JEPPESEN SANDERSON, INC.*

Jeppesen Sanderson, Inc., manufactures and distributes flight manuals that contain safety information to more than 300,000 pilots and 4000 airlines. Every week Jeppesen mails between 5 and 30 million pages of chart revisions to 200,000 customers worldwide, and the company receives about 1500 new orders each week. In the late 1990s, its customer service deteriorated as its existing production and supporting systems failed to keep up with this level of activity. To meet customer service goals, Jeppesen turned to optimization-based decision support tools for production planning.

Jeppesen developed a large-scale linear program called Scheduler to minimize the cost of producing the weekly revisions. Model constraints included capacity constraints and numerous internal business rules. The model includes 250,000 variables and 40,000 to 50,000 constraints. Immediately after introducing the model, Jeppesen established a new record for the number of consecutive weeks with 100% on-time revisions. Scheduler decreased tardiness of revisions from approximately 9% to 3% and dramatically improved customer satisfaction. Even more importantly, Scheduler provided a model of the production system for Jeppesen to use in strategic economic analysis. Overall, the use of optimization techniques at Jeppesen resulted in cost reductions of nearly 10% and a 24% increase in profit.

*Based on E. Katok, W. Tarantino, and R. Tiedman, "Improving Performance and Flexibility at Jeppesen: The World's Leading Aviation-Information Company," *Interfaces* (January/February 2001): 7–29.

To develop the model, we let x_{im} denote the production volume in units for product i in month m. Here $i = 1, 2$, and $m = 1, 2, 3$; $i = 1$ refers to component 322A, $i = 2$ refers to component 802B, $m = 1$ refers to April, $m = 2$ refers to May, and $m = 3$ refers to June. The purpose of the double subscript is to provide a more descriptive notation. We could simply use x_6 to represent the number of units of product 2 produced in month 3, but x_{23} is more descriptive, identifying directly the product and month represented by the variable.

If [illegible] $20 per unit produced and component 802B costs $10 per unit p[illegible]ction cost part of the objective function is

$$[illegible] = 20x_{11} + 20x_{12} + 20x_{13} + 10x_{21} + 10x_{22} + 10x_{23}$$

Becau[illegible]er unit is the same each month, we don't need to include the produ[illegible]ve function; that is, regardless of the production schedule selecte[illegible]st will remain the same. In other words, production costs are not re[illegible]duction scheduling decision under consideration. In cases in which the production cost per unit is expected to change each month, the variable production costs per unit per month must be included in the objective function. The solution for the Bollinger Electronics problem will be the same regardless of whether these costs are included; therefore, we included them so that the value of the linear programming objective function will include all the costs associated with the problem.

To incorporate the relevant inventory holding costs into the model, we let s_{im} denote the inventory level for product i at the end of month m. Bollinger determined that on a monthly basis inventory holding costs are 1.5% of the cost of the product; that is, (0.015)($20) = $0.30 per unit for component 322A and (0.015)($10) = $0.15 per unit for component 802B. A common assumption made in using the linear programming approach to production scheduling is that monthly ending inventories are an acceptable approximation to the average inventory levels throughout the month. Making this assumption, we write the inventory holding cost portion of the objective function as

$$\text{Inventory holding cost} = 0.30s_{11} + 0.30s_{12} + 0.30s_{13} + 0.15s_{21} + 0.15s_{22} + 0.15s_{23}$$

To incorporate the costs of fluctuations in production levels from month to month, we need to define two additional variables:

I_m = increase in the total production level necessary during month m
D_m = decrease in the total production level necessary during month m

After estimating the effects of employee layoffs, turnovers, reassignment training costs, and other costs associated with fluctuating production levels, Bollinger estimates that the cost associated with increasing the production level for any month is $0.50 per unit increase. A similar cost associated with decreasing the production level for any month is $0.20 per unit. Thus, we write the third portion of the objective function as

$$\begin{aligned}\text{Change-in-production-level costs} &= 0.50I_1 + 0.50I_2 + 0.50I_3 \\ &\quad + 0.20D_1 + 0.20D_2 + 0.20D_3\end{aligned}$$

Note that the cost associated with changes in production level is a function of the change in the total number of units produced in month m compared to the total number of units produced in month $m - 1$. In other production scheduling applications, fluctuations in production level might be measured in terms of machine hours or labor-hours required rather than in terms of the total number of units produced.

Combining all three costs, the complete objective function becomes

$$
\begin{aligned}
\text{Min } & 20x_{11} + 20x_{12} + 20x_{13} + 10x_{21} + 10x_{22} + 10x_{23} + 0.30s_{11} \\
& + 0.30s_{12} + 0.30s_{13} + 0.15s_{21} + 0.15s_{22} + 0.15s_{23} + 0.50I_1 \\
& + 0.50I_2 + 0.50I_3 + 0.20D_1 + 0.20D_2 + 0.20D_3
\end{aligned}
$$

We now [illegible]. First, we must guarantee that the schedule meets customer dema[illegible]pped can come from the current month's production or from invent[illegible]vious months, the demand requirement takes the form

$$
\left(\text{fr[illegible]}\right) + \left(\text{[illegible]rrent}\ \text{[illegible]uction}\right) - \left(\begin{array}{c}\text{Ending} \\ \text{inventory} \\ \text{for this} \\ \text{month}\end{array}\right) = \left(\begin{array}{c}\text{This month's} \\ \text{demand}\end{array}\right)
$$

Suppose that the inventories at the beginning of the three-month scheduling period were 500 units for component 322A and 200 units for component 802B. The demand for both products in the first month (April) was 1000 units, so the constraints for meeting demand in the first month become

$$
\begin{aligned}
500 + x_{11} - s_{11} &= 1000 \\
200 + x_{21} - s_{21} &= 1000
\end{aligned}
$$

Moving the constants to the right-hand side, we have

$$
\begin{aligned}
x_{11} - s_{11} &= 500 \\
x_{21} - s_{21} &= 800
\end{aligned}
$$

Similarly, we need demand constraints for both products in the second and third months. We write them as follows:

Month 2

$$
\begin{aligned}
s_{11} + x_{12} - s_{12} &= 3000 \\
s_{21} + x_{22} - s_{22} &= 500
\end{aligned}
$$

Month 3

$$
\begin{aligned}
s_{12} + x_{13} - s_{13} &= 5000 \\
s_{22} + x_{23} - s_{23} &= 3000
\end{aligned}
$$

If the company specifies a minimum inventory level at the end of the three-month period of at least 400 units of component 322A and at least 200 units of component 802B, we can add the constraints

$$
\begin{aligned}
s_{13} &\geq 400 \\
s_{23} &\geq 200
\end{aligned}
$$

TABLE 4.8 MACHINE, LABOR, AND STORAGE CAPACITIES FOR BOLLINGER ELECTRONICS

Month	Machine Capacity (hours)	Labor Capacity (hours)	Storage Capacity (square feet)
April	[illegible]	300	10,000
May	[illegible]	300	10,000
June	[illegible]	300	10,000

TABLE 4.9 M[illegible]ND STORAGE REQUIREMENTS FOR COMPONENTS 3[illegible]

Component	Machine (hours/unit)	Labor (hours/unit)	Storage (square feet/unit)
322A	0.10	0.05	2
802B	0.08	0.07	3

Suppose that we have the additional information on machine, labor, and storage capacity shown in Table 4.8. Machine, labor, and storage space requirements are given in Table 4.9. To reflect these limitations, the following constraints are necessary:

Machine Capacity

$$0.10x_{11} + 0.08x_{21} \leq 400 \quad \text{Month 1}$$
$$0.10x_{12} + 0.08x_{22} \leq 500 \quad \text{Month 2}$$
$$0.10x_{13} + 0.08x_{23} \leq 600 \quad \text{Month 3}$$

Labor Capacity

$$0.05x_{11} + 0.07x_{21} \leq 300 \quad \text{Month 1}$$
$$0.05x_{12} + 0.07x_{22} \leq 300 \quad \text{Month 2}$$
$$0.05x_{13} + 0.07x_{23} \leq 300 \quad \text{Month 3}$$

Storage Capacity

$$2s_{11} + 3s_{21} \leq 10{,}000 \quad \text{Month 1}$$
$$2s_{12} + 3s_{22} \leq 10{,}000 \quad \text{Month 2}$$
$$2s_{13} + 3s_{23} \leq 10{,}000 \quad \text{Month 3}$$

One final set of constraints must be added to guarantee that I_m and D_m will reflect the increase or decrease in the total production level for month m. Suppose that the production levels for March, the month before the start of the current production scheduling period, had been 1500 units of component 322A and 1000 units of component 802B for a total production level of 1500 + 1000 = 2500 units. We can find the amount of the change in production for April from the relationship

$$\text{April production} - \text{March production} = \text{Change}$$

Using the April production variables, x_{11} and x_{21}, and the March production of 2500 units, we have

$$[illegible] x_{21}) - 2500 = \text{Change}$$

Note t[illegible]sitive or negative. A positive change reflects an increase in the total [illegible] negative change reflects a decrease in the total production level. [illegible] in production for April, I_1, and the decrease in production for Ap[illegible]nstraint for the change in total production for the month of April:

$$(x_{11} + x_{21}) - 2500 = I_1 - D_1$$

Of course, we cannot have an increase in production and a decrease in production during the same one-month period; thus, either, I_1 or D_1 will be zero. If April requires 3000 units of production, $I_1 = 500$ and $D_1 = 0$. If April requires 2200 units of production, $I_1 = 0$ and $D_1 = 300$. This approach of denoting the change in production level as the difference between two nonnegative variables, I_1 and D_1, permits both positive and negative changes in the total production level. If a single variable (say, c_m) had been used to represent the change in production level, only positive changes would be possible because of the nonnegativity requirement.

Using the same approach in May and June (always subtracting the previous month's total production from the current month's total production), we obtain the constraints for the second and third months of the production scheduling period:

$$(x_{12} + x_{22}) - (x_{11} + x_{21}) = I_2 - D_2$$
$$(x_{13} + x_{23}) - (x_{12} + x_{22}) = I_3 - D_3$$

Linear programming models for production scheduling are often very large. Thousands of decision variables and constraints are necessary when the problem involves numerous products, machines, and time periods. Data collection for large-scale models can be more time-consuming than either the formulation of the model or the development of the computer solution.

The initially rather small, two-product, three-month scheduling problem has now developed into an 18-variable, 20-constraint linear programming problem. Note that in this problem we were concerned only with one type of machine process, one type of labor, and one type of storage area. Actual production scheduling problems usually involve several machine types, several labor grades, and/or several storage areas, requiring large-scale linear programs. For instance, a problem involving 100 products over a 12-month period could have more than 1000 variables and constraints.

Figure 4.6 shows the optimal solution to the Bollinger Electronics production scheduling problem. Table 4.10 contains a portion of the managerial report based on the optimal solution.

Consider the monthly variation in the production and inventory schedule shown in Table 4.10. Recall that the inventory cost for component 802B is one-half the inventory cost for component 322A. Therefore, as might be expected, component 802B is produced heavily in the first month (April) and then held in inventory for the demand that will occur in future months. Component 322A tends to be produced when needed, and only small amounts are carried in inventory.

FIGURE 4.6 THE SOLUTION FOR THE BOLLINGER ELECTRONICS PROBLEM

WEB file
Bollinger

Optimal Objective Value = 225[illegible]

Variable	Value	Reduced Cost
[illegible]	500.00000	0.00000
[illegible]	3200.00000	0.00000
[illegible]	5200.00000	0.00000
[illegible]	0.00000	0.17222
[illegible]	200.00000	0.00000
[illegible]	400.00000	0.00000
[illegible]	2500.00000	0.00000
[illegible]	2000.00000	0.00000
X23	0.00000	0.12778
S21	1700.00000	0.00000
S22	3200.00000	0.00000
S2[illegible]	[illegible]	0.00000
I1	500.00000	0.00000
I2	2200.00000	0.00000
I3	0.00000	0.07222
D[illegible]	[illegible]	[illegible]
D2	0.00000	0.70000
D3	0.00000	0.62778

Constraint	Slack/Surplus	Dual Value
1	0.00000	20.00000
2	0.00000	10.00000
3	0.00000	20.12778
4	0.00000	10.15000
5	0.00000	20.42778
6	0.00000	10.30000
7	0.00000	20.72778
8	0.00000	10.45000
9	150.00000	0.00000
10	20.00000	0.00000
11	80.00000	0.00000
12	100.00000	0.00000
13	0.00000	-1.11111
14	40.00000	0.00000
15	4900.00000	0.00000
16	0.00000	0.00000
17	8600.00000	0.00000
18	0.00000	-0.50000
19	0.00000	-0.50000
20	0.00000	-0.42778

TABLE 4.10 MINIMUM COST PRODUCTION SCHEDULE INFORMATION FOR THE BOLLINGER ELECTRONICS PROBLEM

Activity	April	May	June
Production			
Component 322A	500	3200	5200
C[illegible]	2500	2000	0
	3000	5200	5200
Endi[illegible]			
C[illegible]	0	200	400
C[illegible]	1700	3200	200
Mac[illegible]			
Sc[illegible]	250	480	520
Slack capacity hours	150	20	80
Labor usage			
Scheduled hours	200	300	260
Slack capacity hours	100	0	40
Storage usage			
Scheduled storage	5100	10,000	1400
Slack capacity	4900	0	8600
Total production, inventory, and production-smoothing cost = $225,295			

The costs of increasing and decreasing the total production volume tend to smooth the monthly variations. In fact, the minimum-cost schedule calls for a 500-unit increase in total production in April and a 2200-unit increase in total production in May. The May production level of 5200 units is then maintained during June.

The machine usage section of the report shows ample machine capacity in all three months. However, labor capacity is at full utilization (slack = 0 for constraint 13 in Figure 4.6) in the month of May. The dual value shows that an additional hour of labor capacity in May will decrease total cost by approximately $1.11.

A linear programming model of a two-product, three-month production system can provide valuable information in terms of identifying a minimum-cost production schedule. In larger production systems, where the number of variables and constraints is too large to track manually, linear programming models can provide a significant advantage in developing cost-saving production schedules. The Management Science in Action, Optimizing Production, Inventory, and Distribution at the Kellogg Company, illustrates the use of a large-scale multiperiod linear program for production planning and distribution.

Workforce Assignment

Workforce assignment problems frequently occur when production managers must make decisions involving staffing requirements for a given planning period. Workforce assignments often have some flexibility, and at least some personnel can be assigned to more than one department or work center. Such is the case when employees have been cross-trained on two or more jobs or, for instance, when sales personnel can be transferred between stores. In the following application, we show how linear programming

MANAGEMENT SCIENCE IN ACTION

OPTIMIZING PRODUCTION, INVENTORY, AND DISTRIBUTION AT THE KELLOGG COMPANY*

The Kellogg Company is the largest cereal producer in the w[illegible]er of convenience food[illegible]Tarts and Nutri-Grain ce[illegible]ces more than 40 differe[illegible] countries, on six contine[illegible]s its products in more th[illegible]loys more than 15,600 pe[illegible]anization. In the cereal b[illegible]oordinates the production [illegible] a total of approximately 90 production lines and 180 packaging lines.

Kellogg has a long history of using linear programming for production planning and distribution. The Kellogg Planning System (KPS) is a large-scale, multiperiod linear program. The operational version of KPS makes production, packaging, inventory, and distribution decisions on a weekly basis. The primary objective of the system is to minimize the total cost of meeting estimated demand; constraints involve processing line capacities, packaging line capacities, and satisfying safety stock requirements.

A tactical version of KPS helps to establish plant budgets and make capacity-expansion and consolidation decisions on a monthly basis. The tactical version was recently used to guide a consolidation of production capacity that resulted in projected savings of $35 to $40 million per year. Because of the success Kellogg has had using KPS in their North American operations, the company is now introducing KPS into Latin America, and is studying the development of a global KPS model.

*Based on G. Brown, J. Keegan, B. Vigus, and K. Wood, "The Kellogg Company Optimizes Production, Inventory, and Distribution," *Interfaces* (November/December 2001): 1–15.

can be used to determine not only an optimal product mix, but also an optimal workforce assignment.

McCormick Manufacturing Company produces two products with contributions to profit per unit of $10 and $9, respectively. The labor requirements per unit produced and the total hours of labor available from personnel assigned to each of four departments are shown in Table 4.11. Assuming that the number of hours available in each department is fixed, we can formulate McCormick's problem as a standard product-mix linear program with the following decision variables:

$$P_1 = \text{units of product 1}$$
$$P_2 = \text{units of product 2}$$

TABLE 4.11 DEPARTMENTAL LABOR-HOURS PER UNIT AND TOTAL HOURS AVAILABLE FOR THE McCORMICK MANUFACTURING COMPANY

	Labor-Hours per Unit		
Department	**Product 1**	**Product 2**	**Total Hours Available**
1	0.65	0.95	6500
2	0.45	0.85	6000
3	1.00	0.70	7000
4	0.15	0.30	1400

The linear program is

$$
\begin{aligned}
\text{Max} \quad & 10P_1 + 9P_2 \\
\text{s.t.} \quad & \\
& 0.65P_1 + 0.95P_2 \le 6500 \\
& 0.45P_1 + 0.85P_2 \le 6000 \\
& 1.00P_1 + 0.70P_2 \le 7000 \\
& 0.15P_1 + 0.30P_2 \le 1400 \\
& P_1, P_2 \ge 0
\end{aligned}
$$

The op[illegible]ear programming model is shown in Figure 4.7. After rounding, i[illegible] product 1, 1795 units of product 2, and a total profit of \$73,590. W[illegible], departments 3 and 4 are operating at capacity, and departments [illegible] approximately 1062 and 1890 hours, respectively. We would anticipate that the product mix would change and that the total profit would increase if the workforce assignment could be revised so that the slack, or unused hours, in departments 1 and 2 could be transferred to the departments currently working at capacity. However, the production manager may be uncertain as to how the workforce should be reallocated among the four departments. Let us expand the linear programming model to include decision variables that will help determine the optimal workforce assignment in addition to the profit-maximizing product mix.

Suppose that McCormick has a cross-training program that enables some employees to be transferred between departments. By taking advantage of the cross-training skills, a limited number of employees and labor-hours may be transferred from one department to another. For example, suppose that the cross-training permits transfers as shown in Table 4.12. Row 1 of this table shows that some employees assigned to department 1 have cross-training skills that permit them to be transferred to department 2 or 3. The right-hand column shows that, for the current production planning period, a maximum of 400 hours can be transferred from department 1. Similar cross-training transfer capabilities and capacities are shown for departments 2, 3, and 4.

FIGURE 4.7 THE SOLUTION FOR THE McCORMICK MANUFACTURING COMPANY PROBLEM WITH NO WORKFORCE TRANSFERS PERMITTED

WEB file McCormick

Optimal Objective Value = 73589.74359

Variable	Value	Reduced Cost
1	5743.58974	0.00000
2	1794.87179	0.00000

Constraint	Slack/Surplus	Dual Value
1	1061.53846	0.00000
2	1889.74359	0.00000
3	0.00000	8.46154
4	0.00000	10.25641

TABLE 4.12 CROSS-TRAINING ABILITY AND CAPACITY INFORMATION

From Department	Cross-Training Transfers Permitted to Department				Maximum Hours Transferable
	1	2	3	4	
1	[illegible]	yes	yes	—	400
2	[illegible]	—	yes	yes	800
3	[illegible]	—	—	yes	100
4	[illegible]	yes	—	—	200

...ments are flexible, we do not automatically know how many hours of labor should be assigned to or be transferred from each department. We need to add decision variables to the linear programming model to account for such changes.

b_i = the labor-hours allocated to department i for $i = 1, 2, 3$, and 4
t_{ij} = the labor-hours transferred from department i to department j

The right-hand sides are now treated as decision variables.

With the addition of decision variables b_1, b_2, b_3, and b_4, we write the capacity restrictions for the four departments as follows:

$$0.65P_1 + 0.95P_2 \le b_1$$
$$0.45P_1 + 0.85P_2 \le b_2$$
$$1.00P_1 + 0.70P_2 \le b_3$$
$$0.15P_1 + 0.30P_2 \le b_4$$

The labor-hours ultimately allocated to each department must be determined by a series of labor balance equations, or constraints, that include the number of hours initially assigned to each department plus the number of hours transferred into the department minus the number of hours transferred out of the department. Using department 1 as an example, we determine the workforce allocation as follows:

$$b_1 = \begin{pmatrix}\text{Hours}\\ \text{initially in}\\ \text{department 1}\end{pmatrix} + \begin{pmatrix}\text{Hours}\\ \text{transferred into}\\ \text{department 1}\end{pmatrix} - \begin{pmatrix}\text{Hours}\\ \text{transferred out of}\\ \text{department 1}\end{pmatrix}$$

Table 4.11 shows 6500 hours initially assigned to department 1. We use the transfer decision variables t_{i1} to denote transfers into department 1 and t_{1j} to denote transfers from department 1. Table 4.12 shows that the cross-training capabilities involving department 1 are restricted to transfers from department 4 (variable t_{41}) and transfers to either department 2 or department 3 (variables t_{12} and t_{13}). Thus, we can express the total workforce allocation for department 1 as

$$b_1 = 6500 + t_{41} - t_{12} - t_{13}$$

Moving the decision variables for the workforce transfers to the left-hand side, we have the labor balance equation or constraint

$$b_1 - t_{41} + t_{12} + t_{13} = 6500$$

This form of constraint will be needed for each of the four departments. Thus, the following labor balance constraints for departments 2, 3, and 4 would be added to the model.

$$b_2 - t_{12} - t_{42} + t_{23} + t_{24} = 6000$$
$$b_3 - t_{13} - t_{23} + t_{34} \qquad = 7000$$
$$b_4 - t_{24} - t_{34} + t_{41} + t_{42} = 1400$$

Finally, umber of hours that may be transferred from each department is transfer capacity constraint must be added for each of the four de constraints are

$$t_{12} + t_{13} \leq 400$$
$$t_{23} + t_{24} \leq 800$$
$$t_{34} \qquad \leq 100$$
$$t_{41} + t_{42} \leq 200$$

The complete linear programming model has two product decision variables (P_1 and P_2), four department workforce assignment variables (b_1, b_2, b_3, and b_4), seven transfer variables (t_{12}, t_{13}, t_{23}, t_{24}, t_{34}, t_{41}, and t_{42}), and 12 constraints. Figure 4.8 shows the optimal solution to this linear program.

Variations in the workforce assignment model could be used in situations such as allocating raw material resources to products, allocating machine time to products, and allocating salesforce time to stores or sales territories.

McCormick's profit can be increased by \$84,011 − \$73,590 = \$10,421 by taking advantage of cross-training and workforce transfers. The optimal product mix of 6825 units of product 1 and 1751 units of product 2 can be achieved if $t_{13} = 400$ hours are transferred from department 1 to department 3; $t_{23} = 651$ hours are transferred from department 2 to department 3; and $t_{24} = 149$ hours are transferred from department 2 to department 4. The resulting workforce assignments for departments 1 through 4 would provide 6100, 5200, 8051, and 1549 hours, respectively.

If a manager has the flexibility to assign personnel to different departments, reduced workforce idle time, improved workforce utilization, and improved profit should result. The linear programming model in this section automatically assigns employees and labor-hours to the departments in the most profitable manner.

Blending Problems

Blending problems arise whenever a manager must decide how to blend two or more resources to produce one or more products. In these situations, the resources contain one or more essential ingredients that must be blended into final products that will contain specific percentages of each. In most of these applications, then, management must decide how much of each resource to purchase to satisfy product specifications and product demands at minimum cost.

Blending problems occur frequently in the petroleum industry (e.g., blending crude oil to produce different octane gasolines), chemical industry (e.g., blending chemicals to produce fertilizers and weed killers), and food industry (e.g., blending ingredients to produce soft drinks and soups). In this section we illustrate how to apply linear programming to a blending problem in the petroleum industry.

The Grand Strand Oil Company produces regular and premium gasoline for independent service stations in the southeastern United States. The Grand Strand refinery manufactures the gasoline products by blending three petroleum components. The gasolines are sold at different prices, and the petroleum components have different costs. The firm wants

FIGURE 4.8 THE SOLUTION FOR THE McCORMICK MANUFACTURING COMPANY PROBLEM

WEB file
McCormick

```
Optimal Objective Value =          84011.29945
```

Variable	Value	Reduced Cost
[illegible]	6824.85900	0.00000
[illegible]	1751.41200	0.00000
[illegible]	6100.00000	0.00000
[illegible]	5200.00000	0.00000
[illegible]	8050.84700	0.00000
[illegible]	1549.15300	0.00000
[illegible]	0.00000	7.45763
T12	0.00000	8.24859
T13	400.00000	0.00000
T42	0.00000	8.24859
T23	550.84750	0.00000
T24	149.15250	0.00000
T34	0.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	0.79096
2	640.11300	0.00000
3	0.00000	8.24859
4	0.00000	8.24859
5	0.00000	0.79096
6	0.00000	0.00000
7	0.00000	8.24859
8	0.00000	8.24859
9	0.00000	7.45763
10	0.00000	8.24859
11	100.00000	0.00000
12	200.00000	0.00000

to determine how to mix or blend the three components into the two gasoline products and maximize profits.

Data available show that regular gasoline can be sold for $2.90 per gallon and premium gasoline for $3.00 per gallon. For the current production planning period, Grand Strand can obtain the three petroleum components at the cost per gallon and in the quantities shown in Table 4.13.

Product specifications for the regular and premium gasolines restrict the amounts of each component that can be used in each gasoline product. Table 4.14 lists the product specifications. Current commitments to distributors require Grand Strand to produce at least 10,000 gallons of regular gasoline.

The Grand Strand blending problem is to determine how many gallons of each component should be used in the regular gasoline blend and how many should be used in the

TABLE 4.13 PETROLEUM COST AND SUPPLY FOR THE GRAND STRAND BLENDING PROBLEM

Petroleum Component	Cost/Gallon	Maximum Available
1	\$2.50	5,000 gallons
2	\$2.60	10,000 gallons
[illegible]	\$2.84	10,000 gallons

TABLE 4.14 PR[illegible]NS FOR THE GRAND STRAND BL[illegible]

[illegible]	Specifications
Regular gasoline	At most 30% component 1 At least 40% component 2 At most 20% component 3
Premium gasoline	At least 25% component 1 At most 45% component 2 At least 30% component 3

premium gasoline blend. The optimal blending solution should maximize the firm's profit, subject to the constraints on the available petroleum supplies shown in Table 4.13, the product specifications shown in Table 4.14, and the required 10,000 gallons of regular gasoline.

We define the decision variables as

$$x_{ij} = \text{gallons of component } i \text{ used in gasoline } j, \text{ where } i = 1, 2, \text{ or } 3 \text{ for components } 1, 2, \text{ or } 3, \text{ and } j = r \text{ if regular or } j = p \text{ if premium}$$

The six decision variables are

x_{1r} = gallons of component 1 in regular gasoline
x_{2r} = gallons of component 2 in regular gasoline
x_{3r} = gallons of component 3 in regular gasoline
x_{1p} = gallons of component 1 in premium gasoline
x_{2p} = gallons of component 2 in premium gasoline
x_{3p} = gallons of component 3 in premium gasoline

The total number of gallons of each type of gasoline produced is the sum of the number of gallons produced using each of the three petroleum components.

Total Gallons Produced

$$\text{Regular gasoline} = x_{1r} + x_{2r} + x_{3r}$$
$$\text{Premium gasoline} = x_{1p} + x_{2p} + x_{3p}$$

The total gallons of each petroleum component are computed in a similar fashion.

Total Petroleum Component Use

$$\text{Component 1} = x_{1r} + x_{1p}$$
$$\text{Component 2} = x_{2r} + x_{2p}$$
$$\text{Component 3} = x_{3r} + x_{3p}$$

[illegible] function of maximizing the profit contribution by identifying the [illegible]tal revenue from both gasolines and the total cost of the three petr[illegible] multiplying the \$2.90 per gallon price by the total gallons of regu[illegible]er gallon price by the total gallons of premium gasoline, and the [illegible]n figures in Table 4.13 by the total gallons of each component used [illegible] function:

$$\begin{aligned} \text{Max} \quad & 2.90(x_{1r} + x_{2r} + x_{3r}) + 3.00(x_{1p} + x_{2p} + x_{3p}) \\ & - 2.50(x_{1r} + x_{1p}) - 2.60(x_{2r} + x_{2p}) - 2.84(x_{3r} + x_{3p}) \end{aligned}$$

When we combine terms, the objective function becomes

$$\text{Max} \quad 0.40x_{1r} + 0.30x_{2r} + 0.06x_{3r} + 0.50x_{1p} + 0.40x_{2p} + 0.16x_{3p}$$

The limitations on the availability of the three petroleum components are

$$\begin{aligned} x_{1r} + x_{1p} &\le 5{,}000 \quad \text{Component 1} \\ x_{2r} + x_{2p} &\le 10{,}000 \quad \text{Component 2} \\ x_{3r} + x_{3p} &\le 10{,}000 \quad \text{Component 3} \end{aligned}$$

Six constraints are now required to meet the product specifications stated in Table 4.14. The first specification states that component 1 can account for no more than 30% of the total gallons of regular gasoline produced. That is,

$$x_{1r} \le 0.30(x_{1r} + x_{2r} + x_{3r})$$

The second product specification listed in Table 4.14 becomes

$$x_{2r} \ge 0.40(x_{1r} + x_{2r} + x_{3r})$$

Similarly, we write the four remaining blending specifications listed in Table 4.14 as

$$\begin{aligned} x_{3r} &\le 0.20(x_{1r} + x_{2r} + x_{3r}) \\ x_{1p} &\ge 0.25(x_{1p} + x_{2p} + x_{3p}) \\ x_{2p} &\le 0.45(x_{1p} + x_{2p} + x_{3p}) \\ x_{3p} &\ge 0.30(x_{1p} + x_{2p} + x_{3p}) \end{aligned}$$

The constraint for at least 10,000 gallons of regular gasoline is

$$x_{1r} + x_{2r} + x_{3r} \ge 10{,}000$$

The complete linear programming model with six decision variables and 10 constraints is

$$
\begin{aligned}
\text{Max}\quad & 0.40x_{1r} + 0.30x_{2r} + 0.06x_{3r} + 0.50x_{1p} + 0.40x_{2p} + 0.16x_{3p} \\
\text{s.t.}\quad & \\
& x_{1r} + x_{1p} \le 5{,}000 \\
& x_{2r} + x_{2p} \le 10{,}000 \\
& x_{3r} + x_{3p} \le 10{,}000 \\
& x_{1r} \le 0.30(x_{1r} + x_{2r} + x_{3r}) \\
& x_{2r} \ge 0.40(x_{1r} + x_{2r} + x_{3r}) \\
& x_{3r} \le 0.20(x_{1r} + x_{2r} + x_{3r}) \\
& x_{1p} \ge 0.25(x_{1p} + x_{2p} + x_{3p}) \\
& x_{2p} \le 0.45(x_{1p} + x_{2p} + x_{3p}) \\
& x_{3p} \ge 0.30(x_{1p} + x_{2p} + x_{3p}) \\
& x_{1r} + x_{2r} + x_{3r} \ge 10{,}000 \\
& x_{1r}, x_{2r}, x_{3r}, x_{1p}, x_{2p}, x_{3p} \ge 0
\end{aligned}
$$

Try Problem 15 as another example of a blending model.

The optimal solution to the Grand Strand blending problem is shown in Figure 4.9. The optimal solution, which provides a profit of $7100, is summarized in Table 4.15. The optimal blending strategy shows that 10,000 gallons of regular gasoline should be produced. The regular gasoline will be manufactured as a blend of 1250 gallons of component 1, 6750 gallons of component 2, and 2000 gallons of component 3. The 15,000 gallons of premium gasoline will be manufactured as a blend of 3750 gallons of component 1, 3250 gallons of component 2, and 8000 gallons of component 3.

FIGURE 4.9 THE SOLUTION FOR THE GRAND STRAND BLENDING PROBLEM

WEB file Grand

Optimal Objective Value = 7100.00000

Variable	Value	Reduced Cost
X1R	1250.00000	0.00000
X2R	6750.00000	0.00000
X3R	2000.00000	0.00000
X1P	3750.00000	0.00000
X2P	3250.00000	0.00000
X3P	8000.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	0.50000
2	0.00000	0.40000
3	0.00000	0.16000
4	1750.00000	0.00000
5	2750.00000	0.00000
6	0.00000	0.00000
7	0.00000	0.00000
8	3500.00000	0.00000
9	3500.00000	0.00000
10	0.00000	-0.10000

TABLE 4.15 GRAND STRAND GASOLINE BLENDING SOLUTION

	Gallons of Component (percentage)			
Gasoline	**Component 1**	**Component 2**	**Component 3**	**Total**
Regular	1250 (12.5%)	6750 (67.5%)	2000 (20%)	10,000
Premium	3750 (25%)	3250 (21⅔%)	8000 (53⅓%)	15,000

[illegible] slack and surplus variables associated with the product specification [illegible]s 4–9) in Figure 4.9 needs some clarification. If the constraint is a [illegible]f the slack variable can be interpreted as the gallons of component [illegible]n amount of the component use specified by the constraint. For exa[illegible]00 for constraint 4 shows that component 1 use is 1750 gallons below the maximum amount of component 1 that could have been used in the production of 10,000 gallons of regular gasoline. If the product specification constraint is a ≥ constraint, a surplus variable shows the gallons of component use above the minimum amount of component use specified by the blending constraint. For example, the surplus of 2750.000 for constraint 5 shows that component 2 use is 2750 gallons above the minimum amount of component 2 that must be used in the production of 10,000 gallons of regular gasoline.

NOTES AND COMMENTS

A convenient way to define the decision variables in a blending problem is to use a matrix in which the rows correspond to the raw materials and the columns correspond to the final products. For example, in the Grand Strand blending problem, we define the decision variables as follows:

This approach has two advantages: (1) it provides a systematic way to define the decision variables for any blending problem; and (2) it provides a visual image of the decision variables in terms of how they are related to the raw materials, products, and each other.

		Final Products	
		Regular Gasoline	**Premium Gasoline**
Raw Materials	**Component 1**	x_{1r}	x_{1p}
	Component 2	x_{2r}	x_{2p}
	Component 3	x_{3r}	x_{3p}

SUMMARY

In this chapter we presented a broad range of applications that demonstrate how to use linear programming to assist in the decision-making process. We formulated and solved problems from marketing, finance, and operations management, and interpreted the computer output.

Many of the illustrations presented in this chapter are scaled-down versions of actual situations in which linear programming has been applied. In real-world applications, the problem may not be so concisely stated, the data for the problem may not be as readily available, and the problem most likely will involve numerous decision variables and/or constraints. However, a thorough study of the applications in this chapter is a good place to begin in applying linear programming to real problems.

PROBLEMS

Note: The following problems have been designed to give you an understanding and appreciation of the broad range of problems that can be formulated as linear programs. You should be able to formulate a linear programming model for each of the problems. However, you will need access to a linear programming computer package to develop the solutions and make the requested inte[illegible]

SELF test

1. The W[illegible] Commerce periodically sponsors public service seminars and pr[illegible]tional plans are under way for this year's program. Advertising [illegible]sion, radio, and newspaper. Audience estimates, costs, and maxin[illegible]ons are as shown.

Constra[illegible]	Television	Radio	Newspaper
Audience per advertisement	100,000	18,000	40,000
Cost per advertisement	$2000	$300	$600
Maximum media usage	10	20	10

To ensure a balanced use of advertising media, radio advertisements must not exceed 50% of the total number of advertisements authorized. In addition, television should account for at least 10% of the total number of advertisements authorized.

a. If the promotional budget is limited to $18,200, how many commercial messages should be run on each medium to maximize total audience contact? What is the allocation of the budget among the three media, and what is the total audience reached?

b. By how much would audience contact increase if an extra $100 were allocated to the promotional budget?

2. The management of Hartman Company is trying to determine the amount of each of two products to produce over the coming planning period. The following information concerns labor availability, labor utilization, and product profitability.

Department	Product (hours/unit) 1	2	Labor-Hours Available
A	1.00	0.35	100
B	0.30	0.20	36
C	0.20	0.50	50
Profit contribution/unit	$30.00	$15.00	

a. Develop a linear programming model of the Hartman Company problem. Solve the model to determine the optimal production quantities of products 1 and 2.

b. In computing the profit contribution per unit, management doesn't deduct labor costs because they are considered fixed for the upcoming planning period. However, suppose that overtime can be scheduled in some of the departments. Which departments would you recommend scheduling for overtime? How much would you be willing to pay per hour of overtime in each department?

c. Suppose that 10, 6, and 8 hours of overtime may be scheduled in departments A, B, and C, respectively. The cost per hour of overtime is $18 in department A, $22.50 in department B, and $12 in department C. Formulate a linear programming model that can be used to determine the optimal production quantities if overtime is made available. What are the optimal production quantities, and what is the revised total contribution to profit? How much overtime do you recommend using in each department? What is the increase in the total contribution to profit if overtime is used?

3. The employee credit union at State University is planning the allocation of funds for the coming year. The credit union makes four types of loans to its members. In addition, the [illegible] risk-free securities to stabilize income. The various revenue-[illegible]ogether with annual rates of return are as follows:

[illegible]nvestment	Annual Rate of Return (%)
[illegible]ns	8
[illegible]	10
[illegible]oans	11
[illegible]	12
Risk-free securities	9

The credit union will have $2 million available for investment during the coming year. State laws and credit union policies impose the following restrictions on the composition of the loans and investments.

- Risk-free securities may not exceed 30% of the total funds available for investment.
- Signature loans may not exceed 10% of the funds invested in all loans (automobile, furniture, other secured, and signature loans).
- Furniture loans plus other secured loans may not exceed the automobile loans.
- Other secured loans plus signature loans may not exceed the funds invested in risk-free securities.

How should the $2 million be allocated to each of the loan/investment alternatives to maximize total annual return? What is the projected total annual return?

4. Hilltop Coffee manufactures a coffee product by blending three types of coffee beans. The cost per pound and the available pounds of each bean are as follows:

Bean	Cost per Pound	Available Pounds
1	$0.50	500
2	$0.70	600
3	$0.45	400

Consumer tests with coffee products were used to provide ratings on a scale of 0–100, with higher ratings indicating higher quality. Product quality standards for the blended coffee require a consumer rating for aroma to be at least 75 and a consumer rating for taste to be at least 80. The individual ratings of the aroma and taste for coffee made from 100% of each bean are as follows.

Bean	Aroma Rating	Taste Rating
1	75	86
2	85	88
3	60	75

Assume that the aroma and taste attributes of the coffee blend will be a weighted average of the attributes of the beans used in the blend.

a. What is the minimum-cost blend that will meet the quality standards and provide 1000 pounds of the blended coffee product?

b. What is the cost per pound for the coffee blend?

c. Determine the aroma and taste ratings for the coffee blend.

d. If additional coffee were to be produced, what would be the expected cost per pound?

5. Ajax [illegible] a new additive for airplane fuels. The additive is a mixture of th[illegible] C. For proper performance, the total amount of additive (amo[illegible] amount of C) must be at least 10 ounces per gallon of fuel. Howe[illegible]sons, the amount of additive must not exceed 15 ounces per gallo[illegible]d of the three ingredients is critical. At least 1 ounce of ingredi[illegible]ry ounce of ingredient B. The amount of ingredient C must be at [illegible] of ingredient A. If the costs per ounce for ingredients A, B, and C [illegible].09, respectively, find the minimum-cost mixture of A, B, and C for each gallon of airplane fuel.

6. G. Kunz and Sons, Inc., manufactures two products used in the heavy equipment industry. Both products require manufacturing operations in two departments. The following are the production time (in hours) and profit contribution figures for the two products.

		Labor-Hours	
Product	**Profit per Unit**	**Dept. A**	**Dept. B**
1	$25	6	12
2	$20	8	10

For the coming production period, Kunz has available a total of 900 hours of labor that can be allocated to either of the two departments. Find the production plan and labor allocation (hours assigned in each department) that will maximize the total contribution to profit.

7. As part of the settlement for a class action lawsuit, Hoxworth Corporation must provide sufficient cash to make the following annual payments (in thousands of dollars).

Year	1	2	3	4	5	6
Payment	190	215	240	285	315	460

The annual payments must be made at the beginning of each year. The judge will approve an amount that, along with earnings on its investment, will cover the annual payments. Investment of the funds will be limited to savings (at 4% annually) and government securities, at prices and rates currently quoted in *The Wall Street Journal*.

Hoxworth wants to develop a plan for making the annual payments by investing in the following securities (par value = $1000). Funds not invested in these securities will be placed in savings.

Security	Current Price	Rate (%)	Years to Maturity
1	$1055	6.750	3
2	$1000	5.125	4

Assume that interest is paid annually. The plan will be submitted to the judge and, if approved, Hoxworth will be required to pay a trustee the amount that will be required to fund the plan.

a. Use linear programming to find the minimum cash settlement necessary to fund the annual payments.
b. Use the dual value to determine how much more Hoxworth should be willing to pay now to reduce the payment at the beginning of year 6 to $400,000.
c. [illegible] determine how much more Hoxworth should be willing to pay [illegible] payment to $150,000.
d. [illegible] ual payments are to be made at the end of each year. Reformulate [illegible] modate this change. How much would Hoxworth save if this [illegible] otiated?

8. [illegible] ff's Department schedules police officers for 8-hour shifts. The [illegible] shifts are 8:00 A.M., noon, 4:00 P.M., 8:00 P.M., midnight, and 4:00 A.M. An officer beginning a shift at one of these times works for the next 8 hours. During normal weekday operations, the number of officers needed varies depending on the time of day. The department staffing guidelines require the following minimum number of officers on duty:

Time of Day	Minimum Officers on Duty
8:00 A.M.–Noon	5
Noon–4:00 P.M.	6
4:00 P.M.–8:00 P.M.	10
8:00 P.M.–Midnight	7
Midnight–4:00 A.M.	4
4:00 A.M.–8:00 A.M.	6

Determine the number of police officers that should be scheduled to begin the 8-hour shifts at each of the six times (8:00 A.M., noon, 4:00 P.M., 8:00 P.M., midnight, and 4:00 A.M.) to minimize the total number of officers required. (*Hint:* Let x_1 = the number of officers beginning work at 8:00 A.M., x_2 = the number of officers beginning work at noon, and so on.)

SELF test

9. Reconsider the Welte Mutual Funds problem from Section 4.2. Define your decision variables as the fraction of funds invested in each security. Also, modify the constraints limiting investments in the oil and steel industries as follows: No more than 50% of the total funds invested in stock (oil and steel) may be invested in the oil industry, and no more than 50% of the funds invested in stock (oil and steel) may be invested in the steel industry.
   a. Solve the revised linear programming model. What fraction of the portfolio should be invested in each type of security?
   b. How much should be invested in each type of security?
   c. What are the total earnings for the portfolio?
   d. What is the marginal rate of return on the portfolio? That is, how much more could be earned by investing one more dollar in the portfolio?

10. An investment advisor at Shore Financial Services wants to develop a model that can be used to allocate investment funds among four alternatives: stocks, bonds, mutual funds, and cash. For the coming investment period, the company developed estimates of the annual rate of return and the associated risk for each alternative. Risk is measured using an index between 0 and 1, with higher risk values denoting more volatility and thus more uncertainty.

Investment	Annual Rate of Return (%)	Risk
Stocks	10	0.8
Bonds	3	0.2
Mutual funds	4	0.3
Cash	1	0.0

Bec[illegible] ey market fund, the annual return is lower, but it carries essenti[illegible] e is to determine the portion of funds allocated to each investi[illegible] o maximize the total annual return for the portfolio subject to th[illegible] illing to tolerate.

[illegible] risk for all investment alternatives. For instance, if 40% of a clien[illegible] ocks, 30% in bonds, 20% in mutual funds, and 10% in cash, the t[illegible] would be 0.40(0.8) + 0.30(0.2) + 0.20(0.3) + 0.10(0.0) = 0.44. An investment advisor will meet with each client to discuss the client's investment objectives and to determine a maximum total risk value for the client. A maximum total risk value of less than 0.3 would be assigned to a conservative investor; a maximum total risk value of between 0.3 and 0.5 would be assigned to a moderate tolerance to risk; and a maximum total risk value greater than 0.5 would be assigned to a more aggressive investor.

Shore Financial Services specified additional guidelines that must be applied to all clients. The guidelines are as follows:

- No more than 75% of the total investment may be in stocks.
- The amount invested in mutual funds must be at least as much as invested in bonds.
- The amount of cash must be at least 10%, but no more than 30% of the total investment funds.

a. Suppose the maximum risk value for a particular client is 0.4. What is the optimal allocation of investment funds among stocks, bonds, mutual funds, and cash? What is the annual rate of return and the total risk for the optimal portfolio?

b. Suppose the maximum risk value for a more conservative client is 0.18. What is the optimal allocation of investment funds for this client? What is the annual rate of return and the total risk for the optimal portfolio?

c. Another more aggressive client has a maximum risk value of 0.7. What is the optimal allocation of investment funds for this client? What is the annual rate of return and the total risk for the optimal portfolio?

d. Refer to the solution for the more aggressive client in part (c). Would this client be interested in having the investment advisor increase the maximum percentage allowed in stocks or decrease the requirement that the amount of cash must be at least 10% of the funds invested? Explain.

e. What is the advantage of defining the decision variables as is done in this model rather than stating the amount to be invested and expressing the decision variables directly in dollar amounts?

11. Edwards Manufacturing Company purchases two component parts from three different suppliers. The suppliers have limited capacity, and no one supplier can meet all the company's needs. In addition, the suppliers charge different prices for the components. Component price data (in price per unit) are as follows:

	Supplier		
Component	1	2	3
1	$12	$13	$14
2	$10	$11	$10

Each supplier has a limited capacity in terms of the total number of components it can supply. However, as long as Edwards provides sufficient advance orders, each supplier can devote its capacity to component 1, component 2, or any combination of the two components, if the total number of units ordered is within its capacity. Supplier capacities are as follows:

Supplier	1	2	3
Capacity	600	1000	800

[illegible] n plan for the next period includes 1000 units of component 1 and [illegible] 2, what purchases do you recommend? That is, how many units of [illegible] be ordered from each supplier? What is the total purchase cost for [illegible]

12. [illegible] ompany (ASC) is a buyer and distributor of seafood products that [illegible] nd specialty seafood outlets throughout the Northeast. ASC has a frozen storage facility in New York City that serves as the primary distribution point for all products. One of the ASC products is frozen large black tiger shrimp, which are sized at 16–20 pieces per pound. Each Saturday ASC can purchase more tiger shrimp or sell the tiger shrimp at the existing New York City warehouse market price. The ASC goal is to buy tiger shrimp at a low weekly price and sell it later at a higher price. ASC currently has 20,000 pounds of tiger shrimp in storage. Space is available to store a maximum of 100,000 pounds of tiger shrimp each week. In addition, ASC developed the following estimates of tiger shrimp prices for the next four weeks:

Week	Price/lb.
1	$6.00
2	$6.20
3	$6.65
4	$5.55

ASC would like to determine the optimal buying-storing-selling strategy for the next four weeks. The cost to store a pound of shrimp for one week is $0.15, and to account for unforeseen changes in supply or demand, management also indicated that 25,000 pounds of tiger shrimp must be in storage at the end of week 4. Determine the optimal buying-storing-selling strategy for ASC. What is the projected four-week profit?

13. Romans Food Market, located in Saratoga, New York, carries a variety of specialty foods from around the world. Two of the store's leading products use the Romans Food Market name: Romans Regular Coffee and Romans DeCaf Coffee. These coffees are blends of Brazilian Natural and Colombian Mild coffee beans, which are purchased from a distributor located in New York City. Because Romans purchases large quantities, the coffee beans may be purchased on an as-needed basis for a price 10% higher than the market price the distributor pays for the beans. The current market price is $0.47 per pound for Brazilian Natural and $0.62 per pound for Colombian Mild. The compositions of each coffee blend are as follows:

	Blend	
Bean	Regular	DeCaf
Brazilian Natural	75%	40%
Colombian Mild	25%	60%

Romans sells the Regular blend for $3.60 per pound and the DeCaf blend for $4.40 per pound. Romans would like to place an order for the Brazilian and Colombian coffee beans that will enable the production of 1000 pounds of Romans Regular coffee and 500 pounds of Romans DeCaf coffee. The production cost is $0.80 per pound for the Regular blend. Because of the extra steps required to produce DeCaf, the production cost for the DeCaf blend is $1.05 per pound. Packaging costs for both products are $0.25 per pound. Formulate a [illegible]el that can be used to determine the pounds of Brazilian Natur[illegible]hat will maximize the total contribution to profit. What is the op[illegible] the contribution to profit?

14. The p[illegible] Classic Boat Corporation must determine how many units of the [illegible]uce over the next four quarters. The company has a beginning i[illegible]1 boats, and demand for the four quarters is 2000 units in quarte[illegible]2, 3000 units in quarter 3, and 1500 units in quarter 4. The firm h[illegible]acity in each quarter. That is, up to 4000 units can be produced [illegible] quarter 2, 2000 units in quarter 3, and 4000 units in quarter 4. Each boat held in inventory in quarters 1 and 2 incurs an inventory holding cost of $250 per unit; the holding cost for quarters 3 and 4 is $300 per unit. The production costs for the first quarter are $10,000 per unit; these costs are expected to increase by 10% each quarter because of increases in labor and material costs. Management specified that the ending inventory for quarter 4 must be at least 500 boats.

a. Formulate a linear programming model that can be used to determine the production schedule that will minimize the total cost of meeting demand in each quarter subject to the production capacities in each quarter and also to the required ending inventory in quarter 4.

b. Solve the linear program formulated in part (a). Then develop a table that will show for each quarter the number of units to manufacture, the ending inventory, and the costs incurred.

c. Interpret each of the dual values corresponding to the constraints developed to meet demand in each quarter. Based on these dual values, what advice would you give the production manager?

d. Interpret each of the dual values corresponding to the production capacity in each quarter. Based on each of these dual values, what advice would you give the production manager?

SELF test

15. Seastrand Oil Company produces two grades of gasoline: regular and high octane. Both gasolines are produced by blending two types of crude oil. Although both types of crude oil contain the two important ingredients required to produce both gasolines, the percentage of important ingredients in each type of crude oil differs, as does the cost per gallon. The percentage of ingredients A and B in each type of crude oil and the cost per gallon are shown.

Crude Oil	Cost	Ingredient A	Ingredient B
1	$0.10	20%	60%
2	$0.15	50%	30%

Crude oil 1 is 60% ingredient B

Each gallon of regular gasoline must contain at least 40% of ingredient A, whereas each gallon of high octane can contain at most 50% of ingredient B. Daily demand for regular and high-octane gasoline is 800,000 and 500,000 gallons, respectively. How many gallons of each type of crude oil should be used in the two gasolines to satisfy daily demand at a minimum cost?

16. The Ferguson Paper Company produces rolls of paper for use in adding machines, desk calculators, and cash registers. The rolls, which are 200 feet long, are produced in widths of 1½, 2½, and 3½ inches. The production process provides 200-foot rolls in 10-inch widths only. The firm must therefore cut the rolls to the desired final product sizes. The seven cutting alternatives and the amount of waste generated by each are as follows:

Alternative	Number of Rolls 1½ in.	2½ in.	3½ in.	Waste (inches)
[illegible]	6	0	0	1
[illegible]	0	4	0	0
[illegible]	2	0	2	0
[illegible]	0	1	2	½
[illegible]	1	3	0	1
6	1	2	1	0
7	4	0	1	½

The minimum requirements for the three products are

Roll Width (inches)	1½	2½	3½
Units	1000	2000	4000

a. If the company wants to minimize the number of 10-inch rolls that must be manufactured, how many 10-inch rolls will be processed on each cutting alternative? How many rolls are required, and what is the total waste (inches)?

b. If the company wants to minimize the waste generated, how many 10-inch rolls will be processed on each cutting alternative? How many rolls are required, and what is the total waste (inches)?

c. What are the differences in parts (a) and (b) to this problem? In this case, which objective do you prefer? Explain. What types of situations would make the other objective more desirable?

17. Frandec Company manufactures, assembles, and rebuilds material handling equipment used in warehouses and distribution centers. One product, called a Liftmaster, is assembled from four components: a frame, a motor, two supports, and a metal strap. Frandec's production schedule calls for 5000 Liftmasters to be made next month. Frandec purchases the motors from an outside supplier, but the frames, supports, and straps may be either manufactured by the company or purchased from an outside supplier. Manufacturing and purchase costs per unit are shown.

Component	Manufacturing Cost	Purchase Cost
Frame	$38.00	$51.00
Support	$11.50	$15.00
Strap	$ 6.50	$ 7.50

Three departments are involved in the production of these components. The time (in minutes per unit) required to process each component in each department and the available capacity (in hours) for the three departments are as follows:

Component	Department		
	Cutting	Milling	Shaping
Frame	3.5	2.2	3.1
Support	1.3	1.7	2.6
Strap	0.8	—	1.7
Capacity	350	420	680

a. F[illegible]ar programming model for this make-or-buy application. H[illegible]nent should be manufactured and how many should be p[illegible]
b. W[illegible] manufacturing and purchasing plan?
c. How many hours of production time are used in each department?
d. How much should Frandec be willing to pay for an additional hour of time in the shaping department?
e. Another manufacturer has offered to sell frames to Frandec for $45 each. Could Frandec improve its position by pursuing this opportunity? Why or why not?

18. The Two-Rivers Oil Company near Pittsburgh transports gasoline to its distributors by truck. The company recently contracted to supply gasoline distributors in southern Ohio, and it has $600,000 available to spend on the necessary expansion of its fleet of gasoline tank trucks. Three models of gasoline tank trucks are available.

Truck Model	Capacity (gallons)	Purchase Cost	Monthly Operating Cost, Including Depreciation
Super Tanker	5000	$67,000	$550
Regular Line	2500	$55,000	$425
Econo-Tanker	1000	$46,000	$350

The company estimates that the monthly demand for the region will be 550,000 gallons of gasoline. Because of the size and speed differences of the trucks, the number of deliveries or round trips possible per month for each truck model will vary. Trip capacities are estimated at 15 trips per month for the Super Tanker, 20 trips per month for the Regular Line, and 25 trips per month for the Econo-Tanker. Based on maintenance and driver availability, the firm does not want to add more than 15 new vehicles to its fleet. In addition, the company has decided to purchase at least three of the new Econo-Tankers for use on short-run, low-demand routes. As a final constraint, the company does not want more than half the new models to be Super Tankers.

a. If the company wishes to satisfy the gasoline demand with a minimum monthly operating expense, how many models of each truck should be purchased?
b. If the company did not require at least three Econo-Tankers and did not limit the number of Super Tankers to at most half the new models, how many models of each truck should be purchased?

SELF test

19. The Silver Star Bicycle Company will be manufacturing both men's and women's models for its Easy-Pedal 10-speed bicycles during the next two months. Management wants to develop a production schedule indicating how many bicycles of each model should be produced in each month. Current demand forecasts call for 150 men's and 125 women's models to be shipped during the first month and 200 men's and 150 women's models to be shipped during the second month. Additional data are shown:

Model	Production Costs	Labor Requirements (hours) Manufacturing	Assembly	Current Inventory
M[illegible]	[illegible]	2.0	1.5	20
W[illegible]	[illegible]	1.6	1.0	30

[illegible]y used a total of 1000 hours of labor. The company's labor rela-[illegible]w the combined total hours of labor (manufacturing plus assem-[illegible]ase by more than 100 hours from month to month. In addition, the [illegible]ly inventory at the rate of 2% of the production cost based on the inventory levels at the end of the month. The company would like to have at least 25 units of each model in inventory at the end of the two months.

a. Establish a production schedule that minimizes production and inventory costs and satisfies the labor-smoothing, demand, and inventory requirements. What inventories will be maintained and what are the monthly labor requirements?

b. If the company changed the constraints so that monthly labor increases and decreases could not exceed 50 hours, what would happen to the production schedule? How much will the cost increase? What would you recommend?

20. Filtron Corporation produces filtration containers used in water treatment systems. Although business has been growing, the demand each month varies considerably. As a result, the company utilizes a mix of part-time and full-time employees to meet production demands. Although this approach provides Filtron with great flexibility, it has resulted in increased costs and morale problems among employees. For instance, if Filtron needs to increase production from one month to the next, additional part-time employees have to be hired and trained, and costs go up. If Filtron has to decrease production, the workforce has to be reduced and Filtron incurs additional costs in terms of unemployment benefits and decreased morale. Best estimates are that increasing the number of units produced from one month to the next will increase production costs by \$1.25 per unit, and that decreasing the number of units produced will increase production costs by \$1.00 per unit. In February Filtron produced 10,000 filtration containers but only sold 7500 units; 2500 units are currently in inventory. The sales forecasts for March, April, and May are for 12,000 units, 8000 units, and 15,000 units, respectively. In addition, Filtron has the capacity to store up to 3000 filtration containers at the end of any month. Management would like to determine the number of units to be produced in March, April, and May that will minimize the total cost of the monthly production increases and decreases.

21. Greenville Cabinets received a contract to produce speaker cabinets for a major speaker manufacturer. The contract calls for the production of 3300 bookshelf speakers and 4100 floor speakers over the next two months, with the following delivery schedule:

Model	Month 1	Month 2
Bookshelf	2100	1200
Floor	1500	2600

Greenville estimates that the production time for each bookshelf model is 0.7 hour and the production time for each floor model is 1 hour. The raw material costs are \$10 for each bookshelf model and $12 for each floor model. Labor costs are $22 per hour using regular production time and $33 using overtime. Greenville has up to 2400 hours of regular production time available each month and up to 1000 additional hours of overtime available each month. If production for either cabinet exceeds demand in month 1, the cabinets can be stored at a cost of $5 per cabinet. For each product, determine the number of units that should be manufactured each month on regular time and on overtime to minimize total produ [illegible]

22. TriCi [illegible] makes Styrofoam cups, plates, and sandwich and meal conta [illegible] le calls for the production of 80,000 small sandwich containers [illegible] containers, and 65,000 meal containers. To make these conta [illegible] re melted and formed into final products using three machines [illegible] hine M1 can process Styrofoam sheets with a maximum width [illegible] apacity of machine M2 is 16 inches, and the width capacity of [illegible] The small sandwich containers require 10-inch-wide Styrofoam sheets; thus, these containers can be produced on each of the three machines. The large sandwich containers require 12-inch-wide sheets; thus, these containers can also be produced on each of the three machines. However, the meal containers require 16-inch-wide Styrofoam sheets, so the meal containers cannot be produced on machine M1. Waste is incurred in the production of all three containers because Styrofoam is lost in the heating and forming process as well as in the final trimming of the product. The amount of waste generated varies depending upon the container produced and the machine used. The following table shows the waste in square inches for each machine and product combination. The waste material is recycled for future use.

Machine	Small Sandwich	Large Sandwich	Meal
M1	20	15	—
M2	24	28	18
M3	32	35	36

Production rates also depend upon the container produced and the machine used. The following table shows the production rates in units per minute for each machine and product combination. Machine capacities are limited for the next week. Time available is 35 hours for machine M1, 35 hours for machine M2, and 40 hours for machine M3.

Machine	Small Sandwich	Large Sandwich	Meal
M1	30	25	—
M2	45	40	30
M3	60	52	44

a. Costs associated with reprocessing the waste material have been increasing. Thus, TCM would like to minimize the amount of waste generated in meeting next week's production schedule. Formulate a linear programming model that can be used to determine the best production schedule.

b. Solve the linear program formulated in part (a) to determine the production schedule. How much waste is generated? Which machines, if any, have idle capacity?

23. EZ-Windows, Inc., manufactures replacement windows for the home remodeling business. In January, the company produced 15,000 windows and ended the month with 9000 windows in inventory. EZ-Windows' management team would like to develop a production schedule for the next three months. A smooth production schedule is obviously desirable because it maintains the current workforce and provides a similar month-to-month operation. However, given the sales forecasts, the production capacities, and the storage [illegible] management team does not think a smooth production schedule [illegible] quantity each month possible.

	February	March	April
[illegible]	15,000	16,500	20,000
[illegible]	14,000	14,000	18,000
[illegible]	6,000	6,000	6,000

The company's cost accounting department estimates that increasing production by one window from one month to the next will increase total costs by $1.00 for each unit increase in the production level. In addition, decreasing production by one unit from one month to the next will increase total costs by $0.65 for each unit decrease in the production level. Ignoring production and inventory carrying costs, formulate and solve a linear programming model that will minimize the cost of changing production levels while still satisfying the monthly sales forecasts.

24. Morton Financial must decide on the percentage of available funds to commit to each of two investments, referred to as A and B, over the next four periods. The following table shows the amount of new funds available for each of the four periods, as well as the cash expenditure required for each investment (negative values) or the cash income from the investment (positive values). The data shown (in thousands of dollars) reflect the amount of expenditure or income if 100% of the funds available in any period are invested in either A or B. For example, if Morton decides to invest 100% of the funds available in any period in investment A, it will incur cash expenditures of $1000 in period 1, $800 in period 2, $200 in period 3, and income of $200 in period 4. Note, however, if Morton made the decision to invest 80% in investment A, the cash expenditures or income would be 80% of the values shown.

Period	New Investment Funds Available	Investment A	Investment B
1	1500	−1000	−800
2	400	−800	−500
3	500	−200	−300
4	100	200	300

The amount of funds available in any period is the sum of the new investment funds for the period, the new loan funds, the savings from the previous period, the cash income from investment A, and the cash income from investment B. The funds available in any period can be used to pay the loan and interest from the previous period, placed in savings, used to pay the cash expenditures for investment A, or used to pay the cash expenditures for investment B.

Assume an interest rate of 10% per period for savings and an interest rate of 18% per period on borrowed funds. Let

$$S(t) = \text{the savings for period } t$$
$$L(t) = \text{the new loan funds for period } t$$

Then, in any period t, the savings income from the previous period is $1.1S(t-1)$, and the loan a[illegible]om the previous period is $1.18L(t-1)$.

A[illegible]tment A is expected to have a cash value of $3200 (assuming a [illegible]d investment B is expected to have a cash value of $2500 (assum[illegible] B). Additional income and expenses at the end of period 4 will [illegible] in period 4 less the repayment of the period 4 loan plus interes[illegible]

S[illegible]ariables are defined as

$$x_1 = \text{[illegible]portion of investment A undertaken}$$
$$x_2 = \text{the proportion of investment B undertaken}$$

For example, if $x_1 = 0.5$, $500 would be invested in investment A during the first period, and all remaining cash flows and ending investment A values would be multiplied by 0.5. The same holds for investment B. The model must include constraints $x_1 \leq 1$ and $x_2 \leq 1$ to make sure that no more than 100% of the investments can be undertaken.

If no more than $200 can be borrowed in any period, determine the proportions of investments A and B and the amount of savings and borrowing in each period that will maximize the cash value for the firm at the end of the four periods.

25. Western Family Steakhouse offers a variety of low-cost meals and quick service. Other than management, the steakhouse operates with two full-time employees who work 8 hours per day. The rest of the employees are part-time employees who are scheduled for 4-hour shifts during peak meal times. On Saturdays the steakhouse is open from 11:00 A.M. to 10:00 P.M. Management wants to develop a schedule for part-time employees that will minimize labor costs and still provide excellent customer service. The average wage rate for the part-time employees is $7.60 per hour. The total number of full-time and part-time employees needed varies with the time of day as shown.

Time	Total Number of Employees Needed
11:00 A.M.–Noon	9
Noon–1:00 P.M.	9
1:00 P.M.–2:00 P.M.	9
2:00 P.M.–3:00 P.M.	3
3:00 P.M.–4:00 P.M.	3
4:00 P.M.–5:00 P.M.	3
5:00 P.M.–6:00 P.M.	6
6:00 P.M.–7:00 P.M.	12
7:00 P.M.–8:00 P.M.	12
8:00 P.M.–9:00 P.M.	7
9:00 P.M.–10:00 P.M.	7

One full-time employee comes on duty at 11:00 A.M., works 4 hours, takes an hour off, and returns for another 4 hours. The other full-time employee comes to work at 1:00 P.M. and works the same 4-hours-on, 1-hour-off, 4-hours-on pattern.

a. Develop a minimum-cost schedule for part-time employees.

b. What is the total payroll for the part-time employees? How many part-time shifts are needed? Use the surplus variables to comment on the desirability of scheduling at least some of the part-time employees for 3-hour shifts.

c. Assume that part-time employees can be assigned either a 3-hour or a 4-hour shift. Develop a minimum-cost schedule for the part-time employees. How many part-time [illegible]nd what is the cost savings compared to the previous schedule?

Case Prob[illegible] AN ADVERTISING CAMPAIGN

The [illegible]cale restaurant located in St. Petersburg, Florida. To help plan an a[illegible] the coming season, Flamingo's management team hired the adve[illegible] Johnson (HJ). The management team requested HJ's recommen[illegible]he advertising budget should be distributed across television, radio, and newspaper advertisements. The budget has been set at $279,000.

In a meeting with Flamingo's management team, HJ consultants provided the following information about the industry exposure effectiveness rating per ad, their estimate of the number of potential new customers reached per ad, and the cost for each ad.

Advertising Media	Exposure Rating per Ad	New Customers per Ad	Cost per Ad
Television	90	4000	$10,000
Radio	25	2000	$ 3,000
Newspaper	10	1000	$ 1,000

The exposure rating is viewed as a measure of the value of the ad to both existing customers and potential new customers. It is a function of such things as image, message recall, visual and audio appeal, and so on. As expected, the more expensive television advertisement has the highest exposure effectiveness rating along with the greatest potential for reaching new customers.

At this point, the HJ consultants pointed out that the data concerning exposure and reach were only applicable to the first few ads in each medium. For television, HJ stated that the exposure rating of 90 and the 4000 new customers reached per ad were reliable for the first 10 television ads. After 10 ads, the benefit is expected to decline. For planning purposes, HJ recommended reducing the exposure rating to 55 and the estimate of the potential new customers reached to 1500 for any television ads beyond 10. For radio ads, the preceding data are reliable up to a maximum of 15 ads. Beyond 15 ads, the exposure rating declines to 20 and the number of new customers reached declines to 1200 per ad. Similarly, for newspaper ads, the preceding data are reliable up to a maximum of 20; the exposure rating declines to 5 and the potential number of new customers reached declines to 800 for additional ads.

Flamingo's management team accepted maximizing the total exposure rating, across all media, as the objective of the advertising campaign. Because of management's concern with attracting new customers, management stated that the advertising campaign must reach at least 100,000 new customers. To balance the advertising campaign and make use of all advertising media, Flamingo's management team also adopted the following guidelines.

- Use at least twice as many radio advertisements as television advertisements.
- Use no more than 20 television advertisements.

- The television budget should be at least $140,000.
- The radio advertising budget is restricted to a maximum of $99,000.
- The newspaper budget is to be at least $30,000.

HJ agreed to work with these guidelines and provide a recommendation as to how the $279,000 advertising budget should be allocated among television, radio, and newspaper advertising [illegible]

Manage[illegible]

Develop a [illegible] to determine the advertising budget allocation for the Flamingo [illegible]n of the following in your report.

1. A s[illegible]ommended number of television, radio, and newspaper adv[illegible]get allocation for each medium. Show the total exposure and indicate the total number of potential new customers reached.
2. How would the total exposure change if an additional $10,000 were added to the advertising budget?
3. A discussion of the ranges for the objective function coefficients. What do the ranges indicate about how sensitive the recommended solution is to HJ's exposure rating coefficients?
4. After reviewing HJ's recommendation, the Flamingo's management team asked how the recommendation would change if the objective of the advertising campaign was to maximize the number of potential new customers reached. Develop the media schedule under this objective.
5. Compare the recommendations from parts 1 and 4. What is your recommendation for the Flamingo Grill's advertising campaign?

Case Problem 2 PHOENIX COMPUTER

Phoenix Computer manufactures and sells personal computers directly to customers. Orders are accepted by phone and through the company's website. Phoenix will be introducing several new laptop models over the next few months and management recognizes a need to develop technical support personnel to specialize in the new laptop systems. One option being considered is to hire new employees and put them through a three-month training program. Another option is to put current customer service specialists through a two-month training program on the new laptop models. Phoenix estimates that the need for laptop specialists will grow from 0 to 100 during the months of May through September as follows: May—20; June—30; July—85; August—85; and September—100. After September, Phoenix expects that maintaining a staff of 100 laptop specialists will be sufficient.

The annual salary for a new employee is estimated to be $27,000 whether the person is hired to enter the training program or to replace a current employee who is entering the training program. The annual salary for the current Phoenix employees who are being considered for the training program is approximately $36,000. The cost of the three-month training program is $1500 per person, and the cost of the two-month training program is $1000 per person. Note that the length of the training program means that a lag will occur between the time when a new person is hired and the time a new laptop specialist is available. The number of current employees who will be available for training is limited. Phoenix estimates that the following numbers can be made available in the coming months: March—15; April—20; May—0; June—5; and July—10. The training center has the capacity to start new three-month and two-month training classes each month; however, the total number of students (new and current employees) that begin training each month cannot exceed 25.

Phoenix needs to determine the number of new hires that should begin the three-month training program each month and the number of current employees that should begin the two-month training program each month. The objective is to satisfy staffing needs during May [illegible] the lowest possible total cost; that is, minimize the incremental sala[illegible] ing cost.

[illegible] nd Phoenix Computer would like to develop a plan for hiring new [illegible] ing the mix of new hires and current employees to place in the trai[illegible]

Ma[illegible]

Perf[illegible] Phoenix Computer problem and prepare a report that summarizes your findings. Be sure to include information on and analysis of the following items:

1. The incremental salary and training cost associated with hiring a new employee and training him/her to be a laptop specialist.
2. The incremental salary and training cost associated with putting a current employee through the training program. (Don't forget that a replacement must be hired when the current employee enters the program.)
3. Recommendations regarding the hiring and training plan that will minimize the salary and training costs over the February through August period as well as answers to these questions: What is the total cost of providing technical support for the new laptop models? How much higher will monthly payroll costs be in September than in January?

Case Problem 3 TEXTILE MILL SCHEDULING

The Scottsville Textile Mill* produces five different fabrics. Each fabric can be woven on one or more of the mill's 38 looms. The sales department's forecast of demand for the next month is shown in Table 4.16, along with data on the selling price per yard, variable cost per yard, and purchase price per yard. The mill operates 24 hours a day and is scheduled for 30 days during the coming month.

TABLE 4.16 MONTHLY DEMAND, SELLING PRICE, VARIABLE COST, AND PURCHASE PRICE DATA FOR SCOTTSVILLE TEXTILE MILL FABRICS

Fabric	Demand (yards)	Selling Price ($/yard)	Variable Cost ($/yard)	Purchase Price ($/yard)
1	16,500	0.99	0.66	0.80
2	22,000	0.86	0.55	0.70
3	62,000	1.10	0.49	0.60
4	7,500	1.24	0.51	0.70
5	62,000	0.70	0.50	0.70

*This case is based on the Calhoun Textile Mill Case by Jeffrey D. Camm, P. M. Dearing, and Suresh K. Tadisnia, 1987.

TABLE 4.17 LOOM PRODUCTION RATES FOR THE SCOTTSVILLE TEXTILE MILL

	Loom Rate (yards/hour)	
Fabric	Dobbie	Regular
1	4.63	—
[illegible]	4.63	—
[illegible]	5.23	5.23
[illegible]	5.23	5.23
[illegible]	4.17	4.17

[illegible] 2 can be manufactured only on the [illegible]

The mill has two types of looms: dobbie and regular. The dobbie looms are more versatile and can be used for all five fabrics. The regular looms can produce only three of the fabrics. The mill has a total of 38 looms: 8 are dobbie and 30 are regular. The rate of production for each fabric on each type of loom is given in Table 4.17. The time required to change over from producing one fabric to another is negligible and does not have to be considered.

The Scottsville Textile Mill satisfies all demand with either its own fabric or fabric purchased from another mill. Fabrics that cannot be woven at the Scottsville Mill because of limited loom capacity will be purchased from another mill. The purchase price of each fabric is also shown in Table 4.16.

Managerial Report

Develop a model that can be used to schedule production for the Scottsville Textile Mill, and at the same time, determine how many yards of each fabric must be purchased from another mill. Include a discussion and analysis of the following items in your report:

1. The final production schedule and loom assignments for each fabric.
2. The projected total contribution to profit.
3. A discussion of the value of additional loom time. (The mill is considering purchasing a ninth dobbie loom. What is your estimate of the monthly profit contribution of this additional loom?)
4. A discussion of the objective coefficients' ranges.
5. A discussion of how the objective of minimizing total costs would provide a different model than the objective of maximizing total profit contribution. (How would the interpretation of the objective coefficients' ranges differ for these two models?)

Case Problem 4 WORKFORCE SCHEDULING

Davis Instruments has two manufacturing plants located in Atlanta, Georgia. Product demand varies considerably from month to month, causing Davis extreme difficulty in workforce scheduling. Recently Davis started hiring temporary workers supplied by WorkForce Unlimited, a company that specializes in providing temporary employees for firms in the greater Atlanta area. WorkForce Unlimited offered to provide temporary employees under three contract options that differ in terms of the length of employment and the cost. The three options are summarized:

Option	Length of Employment	Cost
1	One month	$2000
[illegible]	Two months	$4800
[illegible]	Three months	$7500

The l[illegible]e more expensive because WorkForce Unlimited experiences great[illegible]porary workers who are willing to commit to longer work assig[illegible]

Over the next six months, Davis projects the following needs for additional employees:

Month	January	February	March	April	May	June
Employees Needed	10	23	19	26	20	14

Each month, Davis can hire as many temporary employees as needed under each of the three options. For instance, if Davis hires five employees in January under Option 2, WorkForce Unlimited will supply Davis with five temporary workers who will work two months: January and February. For these workers, Davis will have to pay 5($4800) = $24,000. Because of some merger negotiations under way, Davis does not want to commit to any contractual obligations for temporary employees that extend beyond June.

Davis's quality control program requires each temporary employee to receive training at the time of hire. The training program is required even if the person worked for Davis Instruments in the past. Davis estimates that the cost of training is $875 each time a temporary employee is hired. Thus, if a temporary employee is hired for one month, Davis will incur a training cost of $875, but will incur no additional training cost if the employee is on a two- or three-month contract.

Managerial Report

Develop a model that can be used to determine the number of temporary employees Davis should hire each month under each contract plan in order to meet the projected needs at a minimum total cost. Include the following items in your report:

1. A schedule that shows the number of temporary employees that Davis should hire each month for each contract option.
2. A summary table that shows the number of temporary employees that Davis should hire under each contract option, the associated contract cost for each option, and the associated training cost for each option. Provide summary totals showing the total number of temporary employees hired, total contract costs, and total training costs.
3. If the cost to train each temporary employee could be reduced to $700 per month, what effect would this change have on the hiring plan? Explain. Discuss the implications that this effect on the hiring plan has for identifying methods for reducing training costs. How much of a reduction in training costs would be required to change the hiring plan based on a training cost of $875 per temporary employee?

4. Suppose that Davis hired 10 full-time employees at the beginning of January in order to satisfy part of the labor requirements over the next six months. If Davis can hire full-time employees for $16.50 per hour, including fringe benefits, what effect would it have on total labor and training costs over the six-month period as compared to hiring only temporary employees? Assume that full-time and temporary employees both work approximately 160 hours per month. Provide a recommendatio[illegible] to hire additional full-time employees.

Case Problem 5 DUKE ENERGY COAL ALLOCATION*

Duke Ene[illegible]istributes electricity to customers in the United States and Latin [illegible] purchased Cinergy Corporation, which has generating facilities [illegible] Indiana, Kentucky, and Ohio. For these customers Cinergy has been spending $725 to $750 million each year for the fuel needed to operate its coal-fired and gas-fired power plants; 92% to 95% of the fuel used is coal. In this region, Duke Energy uses 10 coal-burning generating plants: five located inland and five located on the Ohio River. Some plants have more than one generating unit. Duke Energy uses 28–29 million tons of coal per year at a cost of approximately $2 million every day in this region.

The company purchases coal using fixed-tonnage or variable-tonnage contracts from mines in Indiana (49%), West Virginia (20%), Ohio (12%), Kentucky (11%), Illinois (5%), and Pennsylvania (3%). The company must purchase all of the coal contracted for on fixed-tonnage contracts, but on variable-tonnage contracts it can purchase varying amounts up to the limit specified in the contract. The coal is shipped from the mines to Duke Energy's generating facilities in Ohio, Kentucky, and Indiana. The cost of coal varies from $19 to $35 per ton and transportation/delivery charges range from $1.50 to $5.00 per ton.

A model is used to determine the megawatt-hours (mWh) of electricity that each generating unit is expected to produce and to provide a measure of each generating unit's efficiency, referred to as the heat rate. The heat rate is the total BTUs required to produce 1 kilowatt-hour (kWh) of electrical power.

Coal Allocation Model

Duke Energy uses a linear programming model, called the coal allocation model, to allocate coal to its generating facilities. The objective of the coal allocation model is to determine the lowest-cost method for purchasing and distributing coal to the generating units. The supply/availability of the coal is determined by the contracts with the various mines, and the demand for coal at the generating units is determined indirectly by the megawatt-hours of electricity each unit must produce.

The cost to process coal, called the add-on cost, depends upon the characteristics of the coal (moisture content, ash content, BTU content, sulfur content, and grindability) and the efficiency of the generating unit. The add-on cost plus the transportation cost are added to the purchase cost of the coal to determine the total cost to purchase and use the coal.

*The authors are indebted to Thomas Mason and David Bossee of Duke Energy Corporation, formerly Cinergy Corp., for their contribution to this case problem.

Current Problem

Duke Energy signed three fixed-tonnage contracts and four variable-tonnage contracts. The company would like to determine the least-cost way to allocate the coal available through these contracts to five generating units. The relevant data for the three fixed-tonnage contracts are as follows:

S[illegible]	Number of Tons Contracted For	Cost ($/ton)	BTUs/lb
R[illegible]	350,000	22	13,000
P[illegible]	300,000	26	13,300
A[illegible]	275,000	22	12,600

For example, the contract signed with RAG requires Duke Energy to purchase 350,000 tons of coal at a price of $22 per ton; each pound of this particular coal provides 13,000 BTUs.

The data for the four variable-tonnage contracts follow:

Supplier	Number of Tons Available	Cost ($/ton)	BTUs/lb
Consol, Inc.	200,000	32	12,250
Cyprus Amax	175,000	35	12,000
Addington Mining	200,000	31	12,000
Waterloo	180,000	33	11,300

For example, the contract with Consol, Inc., enables Duke Energy to purchase up to 200,000 tons of coal at a cost of $32 per ton; each pound of this coal provides 12,250 BTUs.

The number of megawatt-hours of electricity that each generating unit must produce and the heat rate provided are as follows:

Generating Unit	Electricity Produced (mWh)	Heat Rate (BTUs per kWh)
Miami Fort Unit 5	550,000	10,500
Miami Fort Unit 7	500,000	10,200
Beckjord Unit 1	650,000	10,100
East Bend Unit 2	750,000	10,000
Zimmer Unit 1	1,100,000	10,000

For example, Miami Fort Unit 5 must produce 550,000 megawatt-hours of electricity, and 10,500 BTUs are needed to produce each kilowatt-hour.

The transportation cost and the add-on cost in dollars per ton are shown here:

Transportation Cost ($/ton)

Supplier	Miami Fort Unit 5	Miami Fort Unit 7	Beckjord Unit 1	East Bend Unit 2	Zimmer Unit 1
RAG	[illegible]	5.00	4.75	5.00	4.75
Peabod[illegible]	[illegible]	3.75	3.50	3.75	3.50
Americ[illegible]	[illegible]	3.00	2.75	3.00	2.75
Consol	[illegible]	3.25	2.85	3.25	2.85
Cyprus	[illegible]	5.00	4.75	5.00	4.75
Adding[illegible]	[illegible]	2.25	2.00	2.25	2.00
Waterl[illegible]	[illegible]	2.00	1.60	2.00	1.60

Add-On Cost ($/ton)

Supplier	Miami Fort Unit 5	Miami Fort Unit 7	Beckjord Unit 1	East Bend Unit 2	Zimmer Unit 1
RAG	10.00	10.00	10.00	5.00	6.00
Peabody	10.00	10.00	11.00	6.00	7.00
American	13.00	13.00	15.00	9.00	9.00
Consol	10.00	10.00	11.00	7.00	7.00
Cyprus	10.00	10.00	10.00	5.00	6.00
Addington	5.00	5.00	6.00	4.00	4.00
Waterloo	11.00	11.00	11.00	7.00	9.00

Managerial Report

Prepare a report that summarizes your recommendations regarding Duke Energy's coal allocation problem. Be sure to include information and analysis for the following issues:

1. Determine how much coal to purchase from each of the mining companies and how it should be allocated to the generating units. What is the cost to purchase, deliver, and process the coal?
2. Compute the average cost of coal in cents per million BTUs for each generating unit (a measure of the cost of fuel for the generating units).
3. Compute the average number of BTUs per pound of coal received at each generating unit (a measure of the energy efficiency of the coal received at each unit).
4. Suppose that Duke Energy can purchase an additional 80,000 tons of coal from American Coal Sales as an "all or nothing deal" for $30 per ton. Should Duke Energy purchase the additional 80,000 tons of coal?
5. Suppose that Duke Energy learns that the energy content of the coal from Cyprus Amax is actually 13,000 BTUs per pound. Should Duke Energy revise its procurement plan?
6. Duke Energy has learned from its trading group that Duke Energy can sell 50,000 megawatt-hours of electricity over the grid (to other electricity suppliers) at a price of $30 per megawatt-hour. Should Duke Energy sell the electricity? If so, which generating units should produce the additional electricity?

Appendix 4.1 EXCEL SOLUTION OF HEWLITT CORPORATION FINANCIAL PLANNING PROBLEM

In Appendix 2.2 we showed how Excel could be used to solve the Par, Inc., linear programming problem. To illustrate the use of Excel in solving a more complex linear programming problem, we show the solution to the Hewlitt Corporation financial planning problem presented [illegible]

[illegible] ation and solution of the Hewlitt Corporation problem are show[illegible] cribed in Appendix 2.2, our practice is to put the data required for t[illegible] rt of the worksheet and build the model in the bottom part of the [illegible] nsists of a set of cells for the decision variables, a cell for the obje[illegible] ells for the left-hand-side functions, and a set of cells for the righ[illegible] straints. The cells for each of these model components are scre[illegible] cision variables are also enclosed by a boldface line. Descriptive labels are used to make the spreadsheet easy to read.

Formulation

The data and descriptive labels are contained in cells A1:G12. The screened cells in the bottom portion of the spreadsheet contain the key elements of the model required by the Excel Solver.

FIGURE 4.10 EXCEL SOLUTION FOR THE HEWLITT CORPORATION PROBLEM

WEB file
Hewlitt

	A	B	C	D	E	F	G	H	I	J	K	L
1	Hewlitt Corporation Cash Requirements											
2												
3		Cash										
4	Year	Rqmt.				Bond						
5	1	430			[illegible]	2	3					
6	2	210		Price ($1000)	1.15	1	1.35					
7	3	222		Rate	0.08875	0.055	0.1175					
8	4	231		Years to Maturity	5	6	7					
9	5	240										
10	6	195		Annual Savings Multiple		1.0[illegible]						
11	7	225										
12	8	255										
13												
14	Model											
15												
16	F	B1	B2	B3	S1	S2	S3	S4	S5	S6	S7	S8
17	1728.794	144.988	187.856	228.188	636.148	501.606	349.682	182.681	0	0	0	0
18												
19					Cash Flow		Net Cash		Cash			
20	Min Funds	1728.7939		Constraints	In	Out	Flow		Rqmt.			
21				Year 1	1728.794	1298.794	430	=	430			
22				Year 2	711.6057	501.6057	210	=	210			
23				Year 3	571.6818	349.6818	222	=	222			
24				Year 4	413.6809	182.6809	231	=	231			
25				Year 5	240	0	240	=	240			
26				Year 6	195	0	195	=	195			
27				Year 7	225	0	225	=	225			
28				Year 8	255	0	255	=	255			

Decision Variables Cells A17:L17 are reserved for the decision variables. The optimal values (rounded to three places), are shown to be $F = 1728.794$, $B_1 =$ [illegible]4.988, $B_2 =$ [illegible]187.856, $B_3 =$ 22[illegible]88, [illegible] = [illegible]6.148, $S_2 = 50$[illegible]$.606$, $S_3 = 349.682$, $S_4 = 1$[illegible]2.681, and $S_5 = S_6 = S_7 = S_8 = 0$.

Objectiv[illegible] The formula =A17 has been placed into cell B20 to reflect the [illegible]ds required. It is simply the value of the decision variable, [illegible]tal funds required by the optimal solution is shown to be [illegible]94.

Left-Har[illegible] [illegible]hand sides for the eight constraints represent the annual [illegible]flow. They are placed into cells G21:G28.
[illegible]21 = E21 – F21 (Copy to G22:G28)

For this problem, some of the left-hand-side cells reference other cells that contain formulas. These referenced cells provide Hewlitt's cash flow in and cash flow out for each of the eight years.* The cells and their formulas are as follows:

Cell E21 = A17

Cell E22 = SUMPRODUCT($E$7:$G$7,$B$17:$D$17)+$F$10*E17

Cell E23 = SUMPRODUCT($E$7:$G$7,$B$17:$D$17)+$F$10*F17

Cell E24 = SUMPRODUCT($E$7:$G$7,$B$17:$D$17)+$F$10*G17

Cell E25 = SUMPRODUCT($E$7:$G$7,$B$17:$D$17)+$F$10*H17

Cell E26 = (1+E7)*B17+F7*C17+G7*D17+F10*I17

Cell E27 = (1+F7)*C17+G7*D17+F10*J17

Cell E28 = (1+G7)*D17+F10*K17

Cell F21 = SUMPRODUCT(E6:G6,B17:D17)+E17

Cell F22 = F17

Cell F23 = G17

Cell F24 = H17

Cell F25 = I17

Cell F26 = J17

Cell F27 = K17

Cell F28 = L17

Right-Hand Sides The right-hand sides for the eight constraints represent the annual cash requirements. They are placed into cells I21:I28.
Cell I21 = B5 (Copy to I22:I28)

Excel Solution

We are now ready to use the information in the worksheet to determine the optimal solution to the Hewlitt Corporation problem. The following steps describe how to use Excel to obtain the optimal solution.

*The cash flow in is the sum of the positive terms in each constraint equation in the mathematical model, and the cash flow out is the sum of the negative terms in each constraint equation.

Step 1. Select the **Data** tab

Step 2. Select **Solver** from the **Analysis** group

St... When the **Solver Parameters** dialog box appears (see Figure 4.11):

Enter B20 in the **Set Target Cell** box

Select the **Equal to: Min** option

Enter A17:L17 in the **By Changing Cells** box

S... **onstraint** dialog box appears:

... the left-hand box of the **Cell Reference** area

... middle drop-down button

... he **Constraint** area

S... **Parameters** dialog box reappears (see Figure 4.11):

Step 6. When the **Solver Options** dialog box appears:

Select **Assume Linear Model**

Select **Assume Non-Negative**

Click **OK**

Step 7. When the **Solver Parameters** dialog box reappears:

Choose **Solve**

Step 8. When the **Solver Results** dialog box appears:

Select **Keep Solver Solution**

Select **Sensitivity** in the **Reports** box

Click **OK**

The Solver Parameters dialog box is shown in Figure 4.11. The optimal solution is shown in Figure 4.10; the accompanying sensitivity report is shown in Figure 4.12.

FIGURE 4.11 SOLVER PARAMETERS DIALOG BOX FOR THE HEWLITT CORPORATION PROBLEM

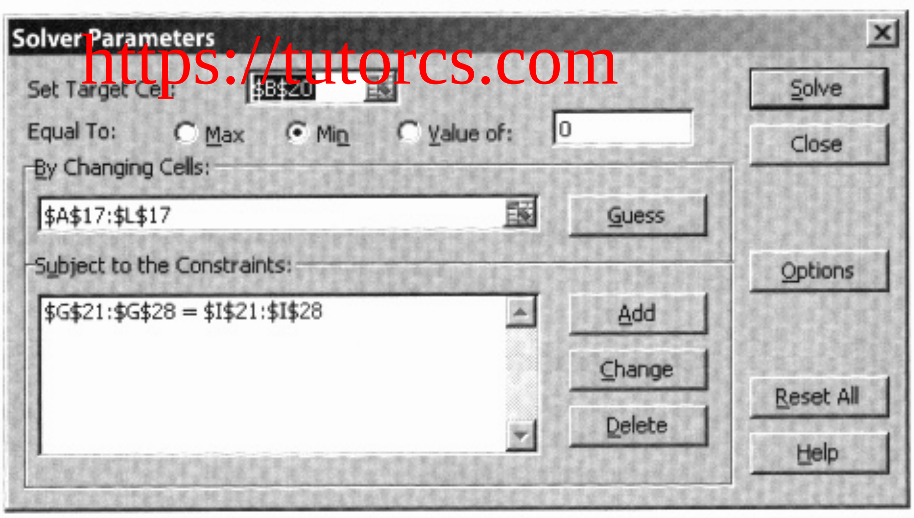

FIGURE 4.12 EXCEL'S SENSITIVITY REPORT FOR THE HEWLITT CORPORATION PROBLEM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$A$17	F	[illegible]	0	1	1E + 30	1
$B$17	B1	[illegible]	0	0	0.067026339	0.013026775
$C$17	B2	[illegible]	0	0	0.012795531	0.020273774
$D$17	B3	[illegible]	0	0	0.022906851	0.749663022
$E$17	S1	[illegible]	0	0	0.109559907	0.05507386
$F$17	S2	[illegible]	0	0	0.143307365	0.056948823
$G$17	S3	[illegible]	0	0	0.210854199	0.059039182
$H$17	S4	[illegible]	0	0	0.413598622	0.061382404
$I$17	S5	0	0.064025159	0	1E + 30	0.064025159
$J$17	S6	0	0.012613604	0	1E + 30	0.012613604
$K$17	S7	0	0.021318233	0	1E + 30	0.021318233
$L$17	S8	0	0.670839393	0	1E + 30	0.670839393

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$G$21	Year 1 Flow	430	1	430	1E + 30	1728.793855
$G$22	Year 2 Flow	210	0.961538462	210	1E + 30	661.5938616
$G$23	Year 3 Flow	222	0.924556213	222	1E + 30	521.6699405
$G$24	Year 4 Flow	231	0.888996359	231	1E + 30	363.6690626
$G$25	Year 5 Flow	240	0.854804191	240	1E + 30	189.9881496
$G$26	Year 6 Flow	195	0.760364454	195	2149.927647	157.8558478
$G$27	Year 7 Flow	225	0.718991202	225	3027.962172	198.1879195
$G$28	Year 8 Flow	255	0.670839393	255	1583.881915	255

Discussion

Figures 4.10 and 4.12 contain essentially the same information as that provided in Figure 4.4. Recall that the Excel sensitivity report uses the term *shadow price* to describe the *change* in value of the solution per unit increase in the right-hand side of a constraint. This is the same as the Dual Value in Figure 4.4.