Math 191 Sample Midterm 2B
Spring 2023, Wilkening

Your Name:

1a. (4 points) Let $A = $ D of an arbitrary $m \times n$ real matrix A. Show that the singular values $\sigma_i = S_{ii}$ hich orthogonal matrices U and V are used.

1b. (4 points) Suppose A is an $m \times n$ matrix with real entries. Prove that $\|A\|_2 = \sigma_1$, where σ_1 is the largest singular value of A and we recall the definition

$$\|A\|_2 = \max_{x \neq 0} \frac{\|Ax\|_2}{\|x\|_2}$$

2. (8 points) Compute the LDL^T factorization of $A = \begin{pmatrix} 3 & 3 & 1 \\ 3 & a & 2 \\ 1 & 2 & b \end{pmatrix}$, assuming it exists. Which values of a and b cause (i) A to be singular? (ii) A to be positive definite?

3. A real 3×3 matrix A, not necessarily symmetric, has eigenvalues 5, 3 and 0 with corresponding eigenvectors u, v and w, all unit vectors. Suppose z is a unit vector orthogonal to u and to v.

(a) (3 points) Find any solution of $Ax = b$ if $b = 2u + 3v$.

(b) (5 points) Find the minimum norm least squares solution of $Ax = b$ if $b = 2u + 3v + 4z$. (The answer will contain ww^T somewhere in the formula. Justify your answer.)

4. (8 points) Compute the polar decompositon $A = Q|A|$ of

$$A = \begin{pmatrix} 9 & 6 & 6 \\ 6 & 0 & 12 \\ 6 & 6 & 0 \end{pmatrix} = \begin{pmatrix} 2/3 & 1/3 & \\ 2/3 & -2/3 & \\ 1/3 & 2/3 & \end{pmatrix} \begin{pmatrix} 18 & & \\ & 9 & \\ & & \end{pmatrix} \begin{pmatrix} 2/3 & 1/3 & 2/3 \\ 1/3 & & \\ & & \end{pmatrix}$$

i.e., compute the matrix entries of the partial isometry Q with the same nullspace as A and the symmetric, positive sem|.

5. (8 points) Draw the tilted ellipse $(5/2)x^2+2xy+y^2=1$ and find the half-lengths of its major and minor axes from the eigenvalues of the corresponding matrix S for which $(5/2)x_1^2+2x_1x_2+x_2^2=x^TSx$.