Math 191 Sample Midterm 2A
Spring 2023, Wilkening

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Your Name: Solutions

1a. (5 points) Suppose A is a matrix with complex entries and $A^* = A$. Suppose V is a subspace of $\mathbb{C}^n$ such that $x \in V \Rightarrow Ax \in V$. Show that $A(V^\perp) \subset V^\perp$.

Let $y \in V^\perp$. Then for $x \in V$, $\langle z, x \rangle = \langle Ay, x \rangle$

$= \langle y, A^* x \rangle = \langle y, Ax \rangle = 0$ since $Ax \in V$

$\therefore z \in V^\perp$

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1b. (5 points) Suppose A is a real, symmetric $n \times n$ matrix. Show that e^A is positive definite.

$A = Q \Lambda Q^T$

$\lambda_j \in \mathbb{R} \Rightarrow e^{\lambda_j} > 0$

$e^A = Q e^\Lambda Q^T$ $\quad e^\Lambda = \begin{pmatrix} e^{\lambda_1} & & \\ & \ddots & \\ & & e^{\lambda_n} \end{pmatrix}$

$(e^A)^T = Q e^\Lambda Q^T = e^A \Rightarrow$ symmetric

and all eigenvalues of e^A are positive

2a. (2 points) Write down the overdetermined linear system $Ax = b$ whose least squares solution $x = \begin{pmatrix} C \\ D \end{pmatrix}$ gives the best-fit line $y(t) = C + Dt$ to the following points (t_i, b_i)

t_i	$-4/3$	1	4	5
b_i	$-13/4$	$-1/4$	$9/4$	$13/4$

$$\begin{pmatrix} 1 & -4/3 \\ 1 & 1 \\ 1 & 4 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} -13/4 \\ -1/4 \\ 9/4 \\ 13/4 \end{pmatrix}$$

in the sense that $\|r\|_2$ is minimized, where $r_i = b_i - y(t_i)$.

2b. (5 points) Find the Householder transformation $H_1 = I - \tau_1 v_1 v_1^T$ that reflects the first column of the matrix from part (a) onto the b_1 axis. (Find τ_1 and v_1, following the convention that $(v_1)_1 = 1$.)

$$a_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad \|a_1\| = \sqrt{4} = 2 \qquad w = a_1 + \|a_1\| e_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$v = \frac{w}{w_1} = \begin{pmatrix} 1 \\ 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} \qquad \tau = 1 + \frac{|a_1|}{\|a_1\|} = 1 + \frac{1}{2} = \frac{3}{2}$$

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2c. (3 points) After applying H_1 and computing and applying a second householder transformation, the above system becomes

$$\begin{pmatrix} -2 & -13/3 \\ 0 & -5 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ -0.3 \\ -0.4 \end{pmatrix}.$$

Compute C, D and the norm $\|r\|_2$ of the minimum residual.

$$-5D = -5 \Rightarrow \boxed{D = 1}$$

$$-2C - \frac{13}{3}D = -1$$

$$-2C = -1 + \frac{13}{3} = \frac{10}{3} \qquad \boxed{C = -5/3}$$

$$\|r\|_2 = \sqrt{(-.3)^2 + (-.4)^2}$$

$$= \sqrt{.09 + .16}$$

$$= \sqrt{.25}$$

$$= 0.5$$

3. (10 points) Let $A = \begin{pmatrix} 6 & 8 \\ 4 & -3 \end{pmatrix}$. Find all rank-1 matrices B such that $\|A - B\|_2$ is minimized.

Hint: if you can't figure out the SVD by inspection, AA^T is simpler than A^TA as a starting point to compute the SVD systematically.

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By inspection: $\begin{pmatrix} 6 & 8 \\ 4 & -3 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}}_{U} \underbrace{\begin{pmatrix} 10 & \\ & 5 \end{pmatrix}}_{S} \underbrace{\begin{pmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{pmatrix}}_{V^T}$

Or: $AA^T = \begin{pmatrix} 6 & 8 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} 6 & 4 \\ 8 & -3 \end{pmatrix} = \begin{pmatrix} 100 & 0 \\ 0 & 25 \end{pmatrix} = \begin{pmatrix} 10 & \\ & 5 \end{pmatrix}^2$

$= US^2U^T$ $\quad U = I, \; S = \begin{pmatrix} 10 & \\ & 5 \end{pmatrix}$

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$V = A^TUS^{-1} = \begin{pmatrix} 6 & 4 \\ 8 & -3 \end{pmatrix}\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}\begin{pmatrix} 1/10 & \\ & 1/5 \end{pmatrix}$

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$= \begin{pmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{pmatrix}$

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Once you have an SVD for A:

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$A_1 = U\begin{pmatrix} 10 & \\ & 0 \end{pmatrix}V^T \Rightarrow \|A - A_1\| = \left\|U\begin{pmatrix} 0 & \\ & 5 \end{pmatrix}V^T\right\|$

Eckart-Young: (best possible) $= \left\|\begin{pmatrix} 0 & \\ & 5 \end{pmatrix}\right\| = 5$

Any rank-1 B with $\|A - B\| = 5$ is also a minimizer.

$B = U\begin{pmatrix} \alpha & \\ & 0 \end{pmatrix}V^T, \quad A - B = U\begin{pmatrix} 10-\alpha & \\ & 5 \end{pmatrix}V^T$

need $-5 \le 10 - \alpha \le 5$

$5 \ge \alpha - 10 \ge -5 \rightarrow$

$B = \begin{pmatrix} \frac{3}{5}\alpha & \frac{4}{5}\alpha \\ 0 & 0 \end{pmatrix}$

$5 \le \alpha \le 15$

4. (10 points) Compute the pseudo-inverse of

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 2 \\ 2 & 1 \end{pmatrix}$$

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You can leave your answer as a product of 3 matrices if you wish, but compute each entry of each of those matrices.

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