P ntum Physics Assignment

h September 2023 11.59pm

Question 1

A beam of spin-1/2 particles are prepared in the following quantum state:

$$|\psi\rangle = |+\rangle_x + \sqrt{2}e^{i\pi/4}|-\rangle_x. \tag{1}$$

Answer the following questions.

1. Normalise this quantum state vector.
2. What are the possible results of a measurement of the spin component S_x, and with what probabilities do they occur?
3. Calculate the expectation value $\langle S_x \rangle$ and the uncertainty ΔS_x for this state. How does this quantity relate to your answer to Part 2 of this question? *Hint*: Make sure you write the vector in the correct basis!
4. What are the possible results of a measurement of the spin component S_z, and with what probabilities do they occur?
5. Calculate the expectation value $\langle S_z \rangle$ for this state.

Question 2

Consider a beam of spin-1/2 particles prepared in the quantum state:

$$|\psi\rangle_1 = \frac{\sqrt{3}}{2}|+\rangle - \frac{i}{2}|-\rangle. \tag{2}$$

Answer the following questions.

1. Show that the above state is normalised. Prove that the state

   $$|\psi\rangle_2 = e^{\frac{i\pi}{4}}\frac{\sqrt{3}}{2}|+\rangle - e^{\frac{i\pi}{4}}\frac{i}{2}|-\rangle. \tag{3}$$

   is also normalised.
2. Using Born's Rule, calculate the probability of measuring S_z and getting outcomes $+\hbar/2$ and $-\hbar/2$ for both $|\psi\rangle_1$ and $|\psi\rangle_2$.
3. Comment on your results in Part 2, in particular, on the impact of the overall phase on measurement outcomes.
4. Identify a spatial vector $\vec{n}$ and the associated spin operator $S_{\vec{n}}$ (written as a matrix) for which the state $|\psi\rangle_1$ is an eigenvector with eigenvalue $+\hbar/2$.

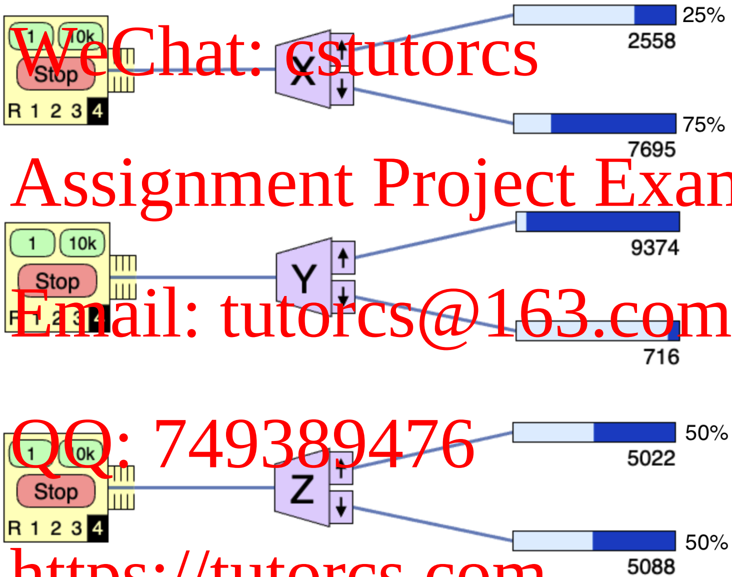
   Hint: Start by comparing the state $|\psi\rangle_1$ with the state $|+\rangle_n$ in your formula sheet.

5. What is the other eigenvector of this same spin operator, with eigenvalue $-\hbar/2$?

6. Calculate th n these two eigenvectors.

Question 3

Consider the follo h experiments. In this experiment the spins ejected from the source are *not* specific quantum state $|\psi\rangle$. Your job is to determine the state $|\psi\rangle$ from the the spin component S_x, S_y, or S_z on many copies of $|\psi\rangle$. The measurement elow. The statistics for S_y are intentionally left blank.



Answer the following questions.

1. Based on the measurement data above, determine the state vector $|\psi\rangle$ that describes the spin-1/2 particles exiting the source.

   Hint: This question involves using the Born rule in reverse. Consider which general state $|\psi\rangle = cos\frac{\theta}{2}\,|+\rangle + e^{i\phi} sin\frac{\theta}{2}\,|-\rangle$ on the Bloch sphere, or alternatively which general state $|\psi\rangle = a\,|+\rangle + b\,|-\rangle$, is consistent with the measured probabilities. If using the Bloch sphere, think about what the measured probabilities indicate about θ and ϕ of a general state $|\psi\rangle$ on the Bloch sphere.

2. Based on the state $|\psi\rangle$ that you have inferred, what are the possible results of a measurement of the spin component S_y, and with what the probabilities do they occur? Are they consistent with the measured data?

   Hint: Remember that there may be some statistical fluctuations in the data due to the sample size.

Question 4

An electron is placed in a controllable magnetic field $\vec{B}$. The initial spin state of the electron is $|\psi(t = 0)\rangle = |+\rangle$. Your goal is to make the spin precess to the state $|+\rangle_x$ by applying uniform magnetic fields. Answer the following questions.

(a) Consider the following experiment:

- First, you apply a magnetic field $\vec{B} = B_x \hat{x}$ in the x-direction for a time t_x, and then turn it off.
- Im[illegible]ou apply a magnetic field $\vec{B} = B_z \hat{z}$ in the z-direction for a ti[illegible]t off.

What t[illegible]you choose to ensure that the final state is $|+\rangle_x$?
Write y[illegible]the charge of the electron e, the mass of the electron m_e, and the[illegible]hs B_x and B_z.

(b) Now co[illegible]iment:

- Ins[illegible]gnetic fields sequentially, you apply them simultaneously with equal strengths. The resultant magnetic field is $\vec{B} = B_x \hat{x} + B_z \hat{z}$, where $B_x = B_z$. You apply this field for a time t, and then turn it off.

What time t should you choose to ensure that the final state is $|+\rangle_x$?
Write your answer in terms of the charge of the electron e, the mass of the electron m_e, and the magnetic field strengths $B_x = B_z$.

(c) For $B_x = B_z$ which of these experiments is quicker to perform?

Hint: Use the Bloch sphere picture to reason about this question. You may invoke the general solution for the precession of a spin-$\frac{1}{2}$ state about the direction of a uniform magnetic field derived in lectures.