Predicate Logic Tutorial 3

1.
    i)   Which of the following are correct formalisations of the following statement: "Bankers and estate agents are unpopular"?

         a. $\forall X\ (\text{banker}(X) \wedge \text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$
         b. $\forall X\ (\text{banker}(X)) \vee (\text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$
         c. $\forall X\ (\text{banker}(X) \vee \text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$
         d. $\forall X\ (\text{banker}(X) \rightarrow \text{unpopular}(X)) \wedge \forall X\ (\text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$

    ii)   Show $\vdash c \leftrightarrow d$, where c and d refer to the 3rd and 4th wffs above.

2.   Prove the following:

a. $\forall X\ (p(X) \rightarrow q(X) \wedge r(X)) \vdash \forall X\ (p(X) \rightarrow q(X)) \wedge \forall X\ (p(X) \rightarrow r(X))$
b. $\forall X\ (p(X) \rightarrow (q(X) \rightarrow r(X))) \vdash \forall X\ (p(X) \wedge q(X) \rightarrow r(X))$
c. $\forall X\ (p(X) \rightarrow \neg q(X)),\ p(a),\ \forall Y(q(Y) \vee s(Y)) \vdash s(a)$
d. $\forall X\ (p(X) \rightarrow m(X) \vee n(X)),\ \forall X\ (m(X) \rightarrow \exists Y q(X, Y)),\ \forall X\ (n(X) \rightarrow \exists Y q(X, Y)),$
       $p(a) \vdash \exists Y q(a, Y)$