Logic Tutorial 2

1. An Exercise taken from Year 1 Logic – with my thanks to Prof. Ian Hodkinson

Consider a set of objects labelled A, B, C, … placed on a 3*3 grid, and the following atomic formulas talking about the objects:

[x next-to y] means (that is, it is true if) x and y are adjacent (horizontally or
        vertically, but not diagonally);

[x sees y] means x and y are in the same row or the same column;

[x left-of y] means x is in a column to the left of the column of y;

[x above y] means x is in a row above the row of y.

(a) For the placements shown in figure below, which of the following evaluate to true, and why?

   i.   [A sees B] ↔ [B sees C]
   ii.   [B next-to D] ∨ [B next-to E]
   iii.   ¬ ([A left-of F] ∧ [F above A])
   iv.   ¬ ([E left-of D] → ¬[D next-to C]) → ¬[A sees E]
   v.   ([E sees D] ∨ [F sees E]) → ¬([B above E] ↔ [B next-to C])

A		D
C	F	B
	E	

(b) Place the 6 objects A, … , F on the grid so that all the formulas above are true.

2. For each of the following determine if it is a tautology, inconsistency or contingency by drawing the truth table.

   a.   P ∧ (P ∨ Q)   d. (P ∧ (Q ∨ P)) ↔ P

   b.   (P ∨ Q) ∧ (P → Q)   e. (P → Q) → (¬ P ∨ Q)

   c.   Q ∧ ¬ P ∧ (P ∨ (Q → P))   f. ((P → Q) ∧ (R → S) ∧ (P ∨ R)) → (Q ∨ S)