Predicate Logic
Some Exercises in Semantics

With my thanks to everyone who has taught logic in the Department

Assume the Universe of integers ≥ 1

For each of the following decide if it is true or false.

1. $\exists X$ (X is the product of two odd integers)

   $\exists X, Y, Z$ ($X=Y*Z \wedge odd(Y) \wedge odd(Z)$)

2. $\forall X$ (X is the product of two odd integers)

   $\forall X \exists Y, Z$ ($X=Y*Z \wedge odd(Y) \wedge odd(Z)$)

3. $\forall Y \exists X$ (X is an even factor of Y)

   $\forall Y \exists X, Z$ ($Y=X*Z \wedge even(X)$)

4. $\forall X \ \exists Y\ (Y>X)$
5. $\forall X\ \forall Y\ (X*Y \geq X)$
6. $\exists Y\ \forall X\ (Y \leq X)$
7. $\exists Y\ \forall X\ (X \leq Y)$
8. $\forall X\ (\exists Y(X= Y^2) \rightarrow \text{even}(X))$
9. $\forall X\ (\forall Y\ \neg(X= Y^2) \rightarrow \text{odd}(X))$
10. $\forall X\ (\neg\forall Y\ (X \text{ divides } Y) \rightarrow \neg(X=1)\ )$

Answers

1. True. Take 15=3*5.
2. False. Take 20.
3. False. Take 15.
4. True. For every X take X+1.
5. True.
6. True. Y=1.
7. False.

8. False. Take X= 25.

$25=5^2$, but 25 is not even.

9. False. Take 24.

10.

$\forall X\ (\neg\forall Y\ (X \text{ divides } Y) \rightarrow \neg(X=1)\ ) \equiv$

$\forall X\ (X=1 \rightarrow \forall Y\ (X \text{ divides } Y)\ )$

True.