# Answers to Some Predicate Logic Formalisation Exercises from the Predicate Logic Notes

Predicates to be used:

lecTheatre/1            same/2

office/1                phd/1

contains/2             supervises/2

lecturer/1             happy/1

has/2                  completePhd/1

1.

311 is a lecture theatre and 447 is an office.

311 is a lecture theatre <span style="color:red">and</span> 447 is an office

∧

lecTheatre(311)    ∧    office(447)

2. Every lecture theatre contains a projector.

Every lecture theatre contains a projector.

$\forall x$ (           Assignment Project Exam Help

Every

Any            https://tutorcs.com

All

Everyone        WeChat: cstutorcs
Map to $\forall$

....

Every lecture theatre contains a projector.

$\forall X$ (lecTheatre(X) → )

$\forall X$ (lecTheatre(X) → contains(X, projector))

Alternatively:

$\forall X$ (lecTheatre(X) →

$\exists Y$ (projector(Y) $\wedge$ contains(X, Y)))

5

A universally quantified wff is usually like this:

∀**x** (　　　　→　　　　　)

Or like this:

∀**x** (　　　　↔　　　　　)

That is their principle connective is

→ or ↔.

3. Every office contains a telephone and either a desktop or a laptop computer.

Every office contains a telephone and either a desktop or a laptop computer.

$\forall X$ (office(X) ➔ .....)

... contains a telephone and either a desktop or a laptop computer.

$\forall X$ (office(X) ➔( .... $\wedge$ ...))

$\forall X$ (office(X) ➔(contains(X, telephone) $\wedge$ (...)))

... either a desktop or a laptop computer.

$\forall X$ (office(X) ➔(contains(X, telephone) $\wedge$ ( ... $\vee$ ...)))

$\forall X$ (office(X) ➔(contains(X, telephone) $\wedge$

(contains(X, desktop) $\vee$ contains(X, laptop))))

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4. Every lecturer has at least one office.

Every lecturer has at least one office.

$\forall$X (lecturer(X) $\rightarrow$         ...         )

... has at least one office.

$\forall$X (lecturer(X) $\rightarrow$ $\exists$Y (office(Y) $\wedge$ has(X, Y)))

At least one

Some

One

....

map to ∃

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An  existentially quantified wff is usually like this:

∃**x** (            ∧            )

Or like this:

∃**x** (            ∨            )

That is their principle connective is

∧ or ∨.

5. No lecturer has more than one office.

No lecturer has more than one office.

Not any lecturer has more than one office.

(There does not exist a lecturer who has more than one office.)

¬∃L (lecturer(L) ∧ *L has more than one office*)

¬∃L (lecturer(L) ∧ *L has at least 2 offices that are not the same*)

¬∃L (lecturer(L) ∧ there are *at least* *2 offices that L has and are not the same*)

¬∃L (lecturer(L) ∧

∃O1 ∃O2 (office(O1) ∧ office(O2) ∧

has(L, O1) ∧ has(L, O2) ∧

¬same(O1,O2)))

There are other ways of doing 5. For example:

$\forall$L (lecturer(L) → ¬ ($\exists$O1 $\exists$O2 (office(O1) $\wedge$ office(O2) $\wedge$ has(L, O1) $\wedge$ has(L, O2) $\wedge$ ¬same(O1,O2))))

$\forall$L $\forall$O1 $\forall$O2 (lecturer(L) $\wedge$ office(O1) $\wedge$ office(O2) $\wedge$ has(L, O1) $\wedge$ has(L, O2) → same(O1,O2))

6.  No lecturers share offices with anyone.

Try it .....

7.  Some lecturers supervise PhD students and some do not.

$(\exists L \ .....) \wedge (\exists L \ .....)$ or

$(\exists L1 \ .....) \wedge (\exists L2 \ .....)$

Some lecturers supervise PhD students and some do not.

(∃L (lecturer(L) ∧ *there is at least one PhD student that L supervises*)) ∧

(∃L (lecturer(L) ∧ *there is not at least one PhD student that L supervises*))

(∃L (lecturer(L) ∧ ∃S (phd(S)∧ supervises(L,S)))))∧

(∃L (lecturer(L) ∧ ¬∃S (phd(S) ∧ supervises(L,S))))

($\exists$L (lecturer(L) $\land$ $\exists$S (phd(S)$\land$ supervises(L,S))))$\land$

($\exists$L (lecturer(L) $\land$ ¬$\exists$S (phd(S) $\land$ supervises(L,S))))

Drop some unneccesary brackets:

$\exists$L (lecturer(L) $\land$ $\exists$S (phd(S)$\land$ supervises(L,S)))$\land$

$\exists$L (lecturer(L) $\land$ ¬$\exists$S (phd(S) $\land$ supervises(L,S)))

8. Each PhD student has an office, but all PhD students share their office with at least one other PhD student.

Try it ......

9. A lecturer is happy *if* the PhD students he/she supervises successfully complete their PhD.

∀L (lecturer(L) ∧ *his/her PhD students successfully complete* → happy(L))

∀L (lecturer(L) ∧ *all his/her PhD students successfully complete* → happy(L))

∀L (lecturer(L) ∧ ∀S *if S is L's PhD student then S successfully completes* → happy(L))

$\forall$L (lecturer(L) $\land$

$\forall$S *if S is L's PhD student then S successfully completes* $\rightarrow$ happy(L))

$\forall$L (lecturer(L) $\land$

$\forall$S (phd(S) $\land$ supervises(L,S) $\rightarrow$ completePhD(S))

$\rightarrow$ happy(L))

10.Not all PhD students complete their PhD.

Try it ....

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