Programming in Prolog

Unification and Search Strategy

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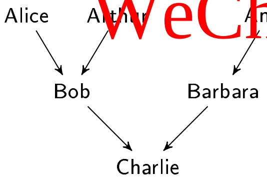
Dr Fariba Sadri
Claudia Schulz

How Prolog works? An informal example

Program

```
parent(alice, bob).
parent(arthur, bob).
parent(anna, barbara).
parent(bob, charlie).
parent(barbara, charlie).

grandparent(X, Z) :-
  parent(X, Y),
  parent(Y, Z).
```

Query

```
?- grandparent(X, charlie).
```

Who are the grand-parents of Charlie?

1. Find a rule that matches `granparent(X, charlie)`
2. Solve the query `?- parent(X,Y), parent(Y,charlie).`
3. Find a rule that matches `parent(X,Y)`
4. `parent(alice, bob)` works. Solve `?- parent(bob, charlie).`
5. `parent(bob, charlie)` is a fact: we have found a solution, `X = alice`.
6. Backtrack to point 4 to find other solutions (Arthur and Anna).

Prolog terms

A ***prolog term*** is one of the following:

- *Atom/Constant* [starts with lower case letter or anything between quotes]

  `john    computing_MSc    x123    'logic and ai'`

- *Number* [integer or float]

  `0    42    -1729    2.718    6.626E-34`

- *Variable* [starts with an upper case letter or an underscore]

  `My_variable    X    _Anonymous    _123    _`

- *Compound term* [$functor(t_1, \ldots, t_N)$:

  functor (*i.e.* a constant name) applied to N terms.

  N is called the arity of the term (*NB:* constants are 0-arity terms).]

```
dob(alice, 1970)
world_record('100m', 9.58, date(16, august, 2009))
'long function name 3'(X, cst, _)
```

A ***ground*** term is a term that contains no variable.

Substitutions

Definition

A substitution $\theta = \{X_1 \mapsto t_1, X_2 \mapsto t_2, \ldots, X_n \mapsto t_n\}$ is a mapping from variables to terms.

Applying a substitution to a term

A substitution $\theta = \{X_1 \mapsto t_1, X_2 \mapsto t_2, \ldots, X_n \mapsto t_n\}$ ***applied*** to a term t, written $t\theta$, is a new term s identical to t, where every occurrence of X_i has been replaced by t_i (for all i, simultaneously).
s is called an ***instance*** of t.

Examples

- `f(A,B)` $\{$`A`$\mapsto$`z`, `B` $\mapsto$ `y`$\}$ gives `f(z,y)`
- `g(X,f(c,X),Y)` $\{$`X`$\mapsto$`a`, `Y` $\mapsto$ `Z`$\}$ gives `g(a,f(c,a),Z)`
- `h(X,Y,Z)` $\{$`X`$\mapsto$`W`, `Y` $\mapsto$ `f(W)`, `Z` $\mapsto$ `f(b)`$\}$ gives `h(W,f(W),f(b))`

Unification

Definition

Two terms T_1 and T_2 unify if there exists a substitution θ such that $T_1\theta \equiv T_2\theta$.

Do the following terms unify?
If they do, give the instantiation of the variables.


```
 john & john        g(a, b, c) & h(a, b, c)
alice & Alice          p(X, 2) & p(f(g), Y)
    3 & 3.0             f(a,X) & f(X, b)
  518 & '518'       p(X, f(Y)) & p(a, X)
 _000 & Variable    p(X, f(Y)) & p(f(f(b)), X)
                    p(X, f(Y)) & p(Y, X)
```

Definition

Two terms T_1 and T_2 unify if there exists a substitution θ such that $T_1\theta \equiv T_2\theta$.

Do the following terms unify?
If they do, give the instantiation of the variables.

`john`	&	`john`	✓		`g(a, b, c)`	&	`h(a, b, c)`	✗
`alice`	&	`Alice`	✓		`p(X, 2)`	&	`p(f(g), Y)`	✓
`3`	&	`3.0`	✗		`f(a,X)`	&	`f(X, b)`	✗
`518`	&	`'518'`	✗		`p(X, f(Y))`	&	`p(a, X)`	✗
`_000`	&	`Variable`	✓		`p(X, f(Y))`	&	`p(f(f(b)), X)`	✓
					`p(X, f(Y))`	&	`p(Y, X)`	✗/✓

`=` vs. `==` vs. `is` vs. `=:=`

In the following, `S`, `T` and `X` are terms,
In the following, `Expr`, `Expr1` and `Expr2` are arithmetic expressions.

- `S = T` will succeed iff `S` and `T` unify
- `S == T` will succeed iff `S` and `T` are identical

- `X is Expr` will succeed iff
  the evaluation of `Expr` can be unified with `X`
- `Expr1 =:= Expr2` will succeed iff
  `Expr1` and `Expr2` evaluate to the same number

Opposite predicates:

`=`	`==`	`is`	`=:=`
`\=`	`\==`	undef.	`=\=`

`=` vs. `==` vs. `is` vs. `=:=`

Let's practice!

Term 1	Term 2	`=`	`==`	`is`	`=:=`
`james`	`james`				
`X`	`emily`				
`X`	`Y`				
`12`	`5+7`				
`X`	`3*4`				
`X-Y`	`15-2*5`				

= vs. == vs. is vs. =:=

Let's practice!

Term 1	Term 2	=	==	is	=:=
james	james	yes	yes	✗	✗
X	emily	X=emily	no	✗	✗
X	Y	X=_0, Y=_0	no	✗	✗
12	5+7	no	no	yes	yes
X	3*4	X=3*4	no	X=12	✗
X-Y	15-2*5	X=15, Y=2*5	no	no	✗

NB: `?- X=Y, X==Y.` will succeed
`?- X==Y, X=Y.` will not

Search Strategy: How Prolog answers queries

Prolog queries

A query is a conjunction of goals: `?- G1, G2, ..., Gn.`

An answer to a query is a substitution θ such that `G1`θ, `G2`θ, ..., `Gn`θ are logical consequences of the program.

But how does Prolog find this substitution
(or prove that it does not exists)?

Search Strategy: How Prolog answers queries

Prolog Search Startegy

1. To solve a query '`?- G1, G2, ..., Gn.`' start with solving the ***first*** goal `G1`.
2. To solve `G1`, find a fact/clause '`H :- B1, B2, ..., Bm`', whose head matches `G1` (i.e. $\exists\theta$ such that $\texttt{G1}\theta \equiv \texttt{H}\theta$).
   If more than one clause satisfy the above condition, we have reached a ***choice point***: in which case, select potential clauses ***from top to bottom***.

- 3.a If `G1` is the only goal in the query ($n = 1$) and the selected clause is a fact ('`H.`', $m = 0$) — ***succeed***

- 3.b If a such clause and substitution exist, solve query '`?-` $\texttt{B1}\theta$, $\texttt{B2}\theta$, `...`, $\texttt{Bm}\theta$, $\texttt{G2}\theta$, `...`, $\texttt{Gn}\theta$.' — (case 1)
- 3.c If no such clause and substitution, ***backtrack*** to the last choice point and pick the next satisfiable clause. — (case 2 for a previous goal)
- 3.d If there are no more choice points (i.e. all clauses for all choice points have been tried) — ***fail***

Search Strategy: How Prolog answers queries

Prolog Search Tree

- At each step, the applicable clauses represent alternative evaluations paths (i.e. different branches of the search tree)
- Prolog searches this tree, ***left to right***, ***depth-first***, to find a successful evaluation paths
- A path/branch of the search tree ***fails*** if the leaf query has no applicable clause
- A path/branch of the search tree ***succeeds*** if the leaf query is an empty conjunction

Search Strategy: Example 1

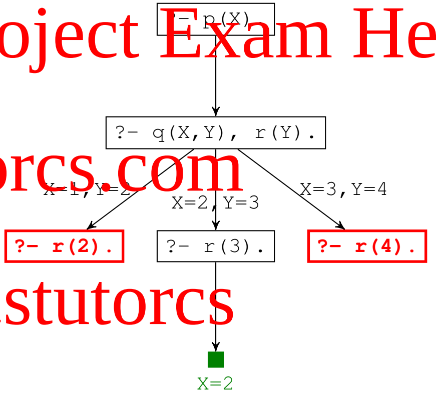
Program

```
p(X) :- q(X,Y), r(Y).
p(X) :- s(X).

q(1,2). q(2,3). q(3,4).
r(3).
s(1). s(2).
```

Query

```
?- p(X).
```



Search Strategy: Example 1

Program

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p(X) :- q(X,Y), r(Y).
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q(1,2). q(2,3). q(3,4).
r(3).
s(1). s(2).
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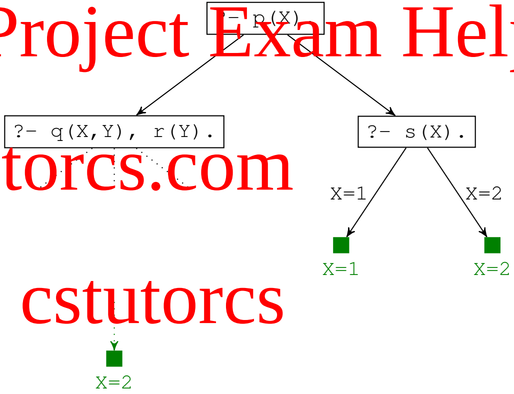
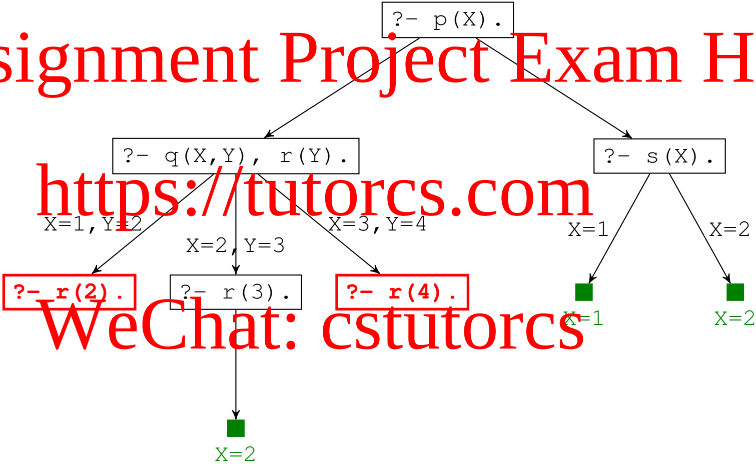
Query

```
?- p(X).
```



Search Strategy: Example 1 - Complete tree



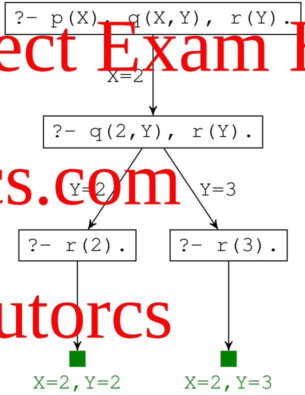
Search Strategy: Example 2

Program

```
p(2).
p(X) :- s(X).
p(X) :- t.

q(X,X).
q(1,5). q(2,3). q(3,1).

r(2). r(3). r(5).

s(1). s(4).
```

Query

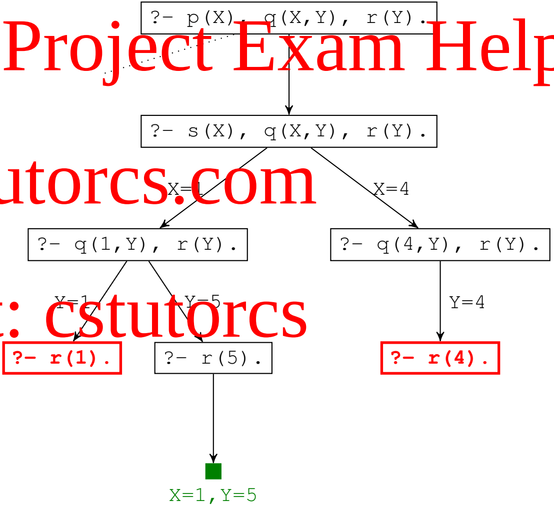
```
?- p(X), q(X,Y), r(Y).
```



Search Strategy: Example 2

Program

```
p(2).
p(X) :- s(X).
p(X) :- t.

q(X,X).
q(1,5). q(2,3). q(3,1).

r(2). r(3). r(5).

s(1). s(4).
```



Query

```
?- p(X), q(X,Y), r(Y).
```

Search Strategy: Example 2

Program

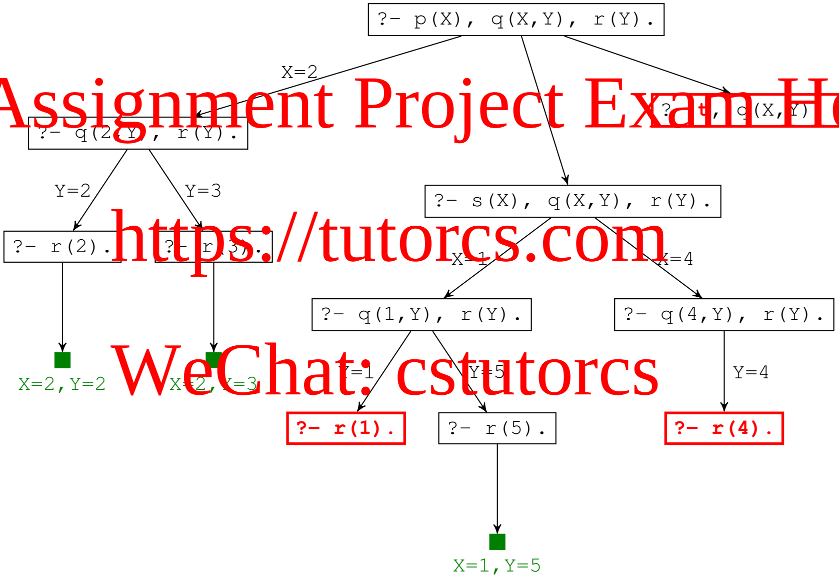
```
p(2).
p(X) :- s(X).
p(X) :- t.

q(X,X).
q(1,5). q(2,3). q(3,1).

r(2). r(3). r(5).

s(1). s(4).
```


Query

```
?- p(X), q(X,Y), r(Y).
```

```
?- p(X), q(X,Y), r(Y).
        |
        v
?- t, q(X,Y), r(Y).
```

Search Strategy: Example 2 - Complete tree



What have we learned today?

- What is unification and how it works
- What algorithm is used by Prolog to generate answers
- Why the order of clauses and the order of goals in clauses/queries matter.