

Resolution

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Theorem Proving

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Resolution

- An Alternative to Natural Deduction
- Natural deduction is (we can argue)
 - Good for human use, but maybe too complicated
 - Definitely complicated for automation
 - Not mechanical enough
 - Too many rules of inference

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Resolution: Motivation

- Designed to be automated
- Basis of logic programming
- Basis of Prolog

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Resolution in Propositional Logic in a Nutshell

- First convert the formulas to a very simple form.
- The form is called Conjunctive Normal Form (CNF).
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- CNF has just the connectives \wedge , \vee , \neg .
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- Every formula can be put in this form.

Example: The Election

Premise:

Rewrite in CNF

1. $E \rightarrow L \vee T$

$\neg E \vee L \vee T$

2. $\neg U \rightarrow \neg L$

$U \vee \neg L$

3. E

E

Conclusion:

$\neg T \rightarrow U$

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- Next Negate the conclusion to be derived and convert that to CNF.

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Example: The Election

Premise:

1. $E \rightarrow L \vee T$

2. $\neg U \rightarrow \neg L$

3. E

Conclusion:

$\neg T \rightarrow U$

Rewrite in CNF

$\neg E \vee L \vee T$

$U \vee \neg L$

E

Negation of conclusion

$\neg(\neg\neg T \vee U) \equiv \neg(T \vee U)$

In CNF $\neg T \wedge \neg U$

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The problem now becomes finding an inconsistency amongst

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$\neg E \vee L \vee T$

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$U \vee \neg L$

E WeChat: cstutorcs

$\neg T$

$\neg U$

Resolution Proof rule for Propositional Logic

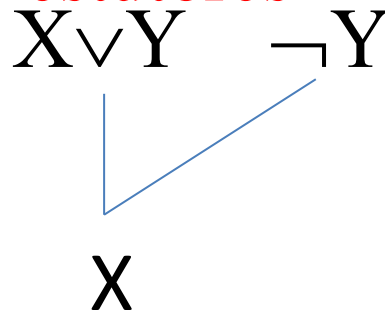
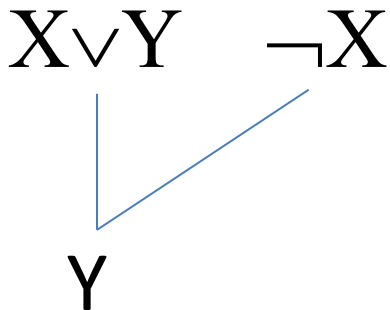
Unit resolution: \vee -elimination ($\vee E$)

$$\frac{X \vee Y, \neg X}{Y}$$

$$\frac{X \vee Y, \neg Y}{X}$$

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Take this a little further to **propositional binary resolution**:

$$\frac{X \vee Y, \neg X \vee Z}{Y \vee Z} \quad \frac{X \vee Y, Z \vee \neg Y}{X \vee Z}$$

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$$\begin{array}{c} X \vee Y \quad \neg X \vee Z \\ \diagdown \quad \diagup \\ Y \vee Z \end{array}$$

$$\begin{array}{c} X \vee Y \quad Z \vee \neg Y \\ \diagdown \quad \diagup \\ X \vee Z \end{array}$$

Justification of the propositional binary resolution

$$\begin{array}{l} X \vee Y \quad \neg X \vee Z \\ \swarrow \quad \searrow \\ Y \vee Z \end{array}$$

1. $X \vee Y$
2. $\neg X \vee Z$ given
3. $X \vee \neg X$ law of excluded middle
 4. X
 5. Z
 6. $Y \vee Z$
7. $X \rightarrow Y \vee Z$
 8. $\neg X$
 9. Y
 10. $Y \vee Z$
11. $\neg X \rightarrow Y \vee Z$
12. $Y \vee Z$

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assume

4, 2, $\vee E$

5, $\vee I$

4, 6, $\rightarrow I$

assume

8, 1, $\vee E$

9, $\vee I$

8, 10, $\rightarrow I$

3, 7, 11, dilemma

Back to the Election Example

We have

$\neg E \vee L \vee T$

$U \vee \neg L$

E

$\neg T$

$\neg U$

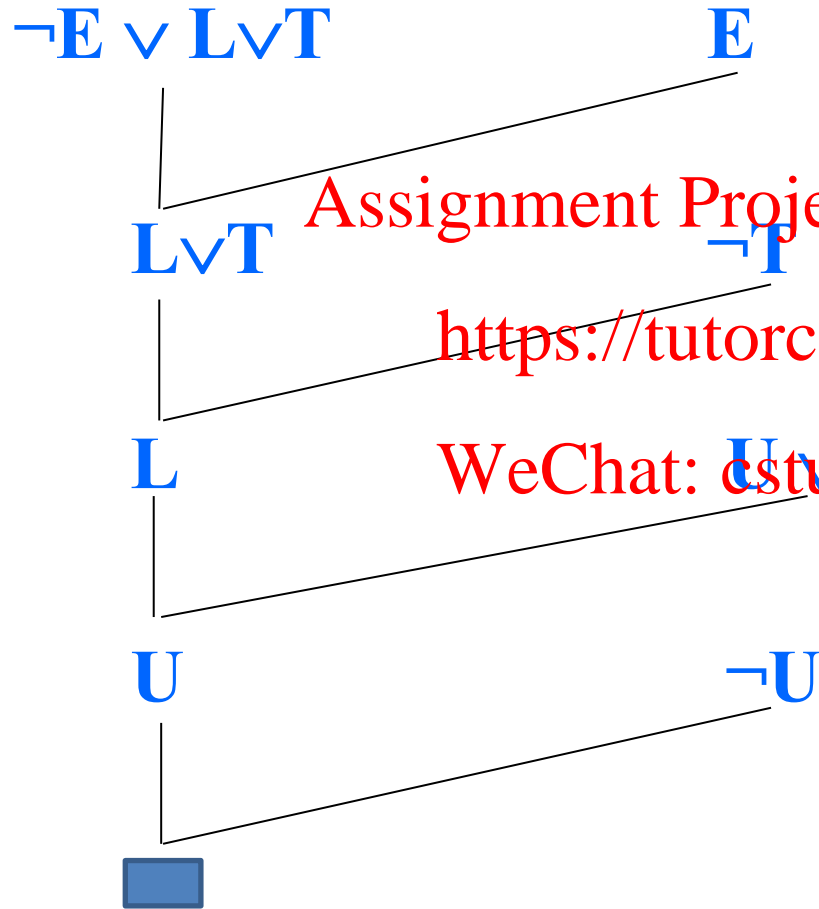
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and just one rule of inference, namely the resolution rule.

$\neg E \vee L \vee T, \quad U \vee \neg L, \quad E, \quad \neg T, \quad \neg U$



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This shows

Premise + \neg Conclusion is inconsistent.

So by proof by contradiction

Premise \vdash Conclusion.

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Now More Details

- What exactly is CNF
- How do we construct it
- Property of Resolution
- Extension to Predicate Logic
- Relationship to Prolog

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Conjunctive Normal Form (CNF)

A wff is in CNF if it is of the form:

$$W_1 \wedge W_2 \wedge \dots \wedge W_n, \quad n \geq 1$$

and each W_i is disjunction of literals.

Examples: the following are in CNF

P

$P \vee Q$

$P \vee \neg Q$

$(P \vee \neg Q) \wedge (R \vee \neg S \vee T)$

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Converting to CNF

Step 1: Eliminate \leftrightarrow and \rightarrow

Using $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

$P \rightarrow Q \equiv \neg P \vee Q$
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Step 2: Push negations in towards atoms

Using $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
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$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
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$\neg\neg P \equiv P$

Step 3: Use distributativity and commutativity

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

$(Q \wedge R) \vee P \equiv P \vee (Q \wedge R)$

Summary: How to do Resolution Proofs in Propositional Logic

Given a set of wffs S and a wff W , to show

$S \vdash W$

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by resolution, follow these steps:

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1. Convert all sentences in S to conjunctive normal form (CNF). WeChat: cstutorcs
2. Negate W (to get $\neg W$).
3. Convert $\neg W$ to CNF.
4. Apply resolution to $\text{CNF}(S)$ and $\text{CNF}(\neg W)$ until:

- a. Derive a falsity (contradiction). In this case W is proved.
- b. Can't apply any further resolution steps. In this case W is not proved.

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Property of Resolution

- Resolution is **refutation-complete** over first order logic (propositional and predicate logic).
- This means that if you write any set of sentences in first order logic which are **contradictory** or **unsatisfiable** (i.e., taken together they have no models), then the resolution method will eventually derive the Falsity, indicating that the sentences are contradictory.

Resolution for Predicate Logic

More general **first-order binary resolution**
inference rule:

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$$\frac{X \vee Y, \neg W \vee Z}{(Y \vee Z) \theta}$$

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if X and W unify with substitution θ .

Example:

$\text{athlete}(\text{arnie}) \vee \text{actor}(\text{arnie})$ $\neg \text{athlete}(P) \vee \text{healthy}(P)$

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$\text{actor}(\text{arnie}) \vee \text{healthy}(\text{arnie})$

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$\neg \text{athlete}(P) \vee \text{healthy}(P)$ is the CNF of
 $\forall P (\text{athlete}(P) \rightarrow \text{healthy}(P))$

Converting Predicate Logic Sentences to CNF

Step 1: as before

Step 2: as before but add moving negations inwards through quantifiers, using

$$\neg \exists X p(X) \equiv \forall X \neg p(X)$$

$$\neg \forall X p(X) \equiv \exists X \neg p(X)$$

Step 3: Standardize variables apart by renaming them: each quantifier should use a different variable.

Step 4: Skolemise. (don't worry about it)

Step 5: Drop universal quantifiers.

Step 6: Use distributativity and commutativity

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Relationship to Prolog

Prolog is based on unification and resolution.

Example:

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$p :- q.$

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$q :- r.$

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$r.$

Query: p

$p :- q.$

$q :- r.$

$r.$

Query: p

$\neg p$

$p \vee \neg q$

$\neg q$

$q \vee \neg r$

$\neg r$

r



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Example

sister_of(X,Y) :-
 siblings(X,Y),
 female(X).
siblings(X,Y) :-
 parent_of(Z,X),
 parent_of(Z,Y).
parent_of(tom, jill).
parent_of(tom, john).
female(jill).

sis(X,Y) \vee \neg sib(X,Y) \vee \neg f(X)
sib(X,Y) \vee \neg p(Z,X) \vee \neg p(Z,Y)
p(tom, jill)
p(tom, john)
f(jill)

Query: sister_of(X, Y) \neg sis(X,Y)

$\neg \text{sis}(X, Y)$

$\text{sis}(X,Y) \vee \neg \text{sib}(X,Y) \vee \neg f(X)$

$\neg \text{sib}(X,Y) \vee \neg f(X)$

$\text{sib}(X,Y) \vee \neg p(Z,X) \vee \neg p(Z,Y)$

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$\neg p(Z,X) \vee \neg p(Z,Y) \vee \neg f(X)$

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$p(\text{tom}, \text{jill})$

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$\neg p(\text{tom}, Y) \vee \neg f(\text{jill})$

$p(\text{tom}, \text{john})$

$Y = \text{john}$

$\neg f(\text{jill})$

$f(\text{jill})$

Answer is $X = \text{jill}, Y = \text{john}$

