

## Solutions to Predicate Logic Tutorial 3

Q1.

- i) c and d.
- ii) You have to show  $\vdash c \rightarrow d$  and  $\vdash d \rightarrow c$ . I will show the first.

showing  $\vdash c \rightarrow d$ :

- |  |                      |
|--|----------------------|
| 1. $\forall X (\text{banker}(X) \vee \text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$ | assume               |
| 2. $\text{banker}(a)$  | assume               |
| 3. $\text{banker}(a) \vee \text{estate\_agent}(a)$   | 2, $\vee I$          |
| 4. $\text{banker}(a) \vee \text{estate\_agent}(a) \rightarrow \text{unpopular}(a)$             | 1, $\forall E$       |
| 5. $\text{unpopular}(a)$   | 3,4, $\rightarrow E$ |
| 6. $\text{banker}(a) \rightarrow \text{unpopular}(a)$  | 2,5, $\rightarrow I$ |
| 7. $\forall X (\text{banker}(X) \rightarrow \text{unpopular}(X))$                              | 6, $\forall I$       |

In an almost identical way you can show

$\forall X (\text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$

Then use  $\wedge I$  to derive

$\forall X (\text{banker}(X) \rightarrow \text{unpopular}(X)) \wedge \forall X (\text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$

Then by  $\rightarrow I$  you get  $c \rightarrow d$ , discharging 1.

Showing  $\vdash d \rightarrow c$ :

- |  |                         |
|--|-------------------------|
| 1. $\forall X (\text{banker}(X) \rightarrow \text{unpopular}(X)) \wedge \forall X (\text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$ | assume                  |
| 2. $\forall X (\text{banker}(X) \rightarrow \text{unpopular}(X))$  | 1, $\wedge E$           |
| 3. $\forall X (\text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$   | 1, $\wedge E$           |
| 4. $\text{banker}(a) \vee \text{estate\_agent}(a)$   | assume                  |
| 5. $\text{banker}(a) \rightarrow \text{unpopular}(a)$  | 2, $\forall E$          |
| 6. $\text{estate\_agent}(a) \rightarrow \text{unpopular}(a)$   | 3, $\forall E$          |
| 7. $\text{unpopular}(a)$   | Proof by cases, 4, 5, 6 |
| 8. $\text{banker}(a) \vee \text{estate\_agent}(a) \rightarrow \text{unpopular}(a)$   | $\rightarrow I$ , 4, 7  |
| 9. $\forall X (\text{banker}(X) \vee \text{estate\_agent}(X) \rightarrow \text{unpopular}(X))$   | $\forall I$ , 8         |

Then by  $\rightarrow I$  you get  $d \rightarrow c$ , discharging 1.

Q2.

a.

- |  |                      |
|--|----------------------|
| 1. $\forall X (p(X) \rightarrow q(X) \wedge r(X))$ | given                |
| 2. $p(a)$  | assume               |
| 3. $p(a) \rightarrow q(a) \wedge r(a)$             | 1, $\forall E$       |
| 4. $q(a) \wedge r(a)$                              | 3, $\rightarrow E$   |
| 5. $q(a)$  | 4, $\wedge E$        |
| 6. $p(a) \rightarrow q(a)$                         | 2,5, $\rightarrow I$ |
| 7. $\forall X (p(X) \rightarrow q(X))$             | 6, $\forall I$       |

Similarly we prove

$\forall X (p(X) \rightarrow r(X))$

And then apply  $\wedge I$  to get:

$$\forall X (p(X) \rightarrow q(X)) \wedge \forall X (p(X) \rightarrow r(X))$$

b.

1.  $\forall X (p(X) \rightarrow (q(X) \rightarrow r(X)))$  given
2.  $p(a) \wedge q(a)$  assume
3.  $p(a)$  2,  $\wedge E$
4.  $q(a) \rightarrow r(a)$  1, 3,  $\forall \rightarrow E$
5.  $q(a)$  2,  $\wedge E$
6.  $r(a)$  4, 5,  $\rightarrow E$
7.  $p(a) \wedge q(a) \rightarrow r(a)$  2, 6,  $\rightarrow I$
8.  $\forall X (p(X) \wedge q(X) \rightarrow r(X))$  7,  $\forall I$

c.

1.  $\forall X (p(X) \rightarrow \neg q(X))$  given
2.  $p(a)$  given
3.  $\forall Y (q(Y) \vee s(Y))$  given
4.  $\neg q(a)$  1, 2,  $\forall \rightarrow E$
5.  $q(a) \vee s(a)$  3,  $\forall E$
6.  $s(a)$  4, 5,  $\vee E$

d. Hint: Think of using proof by cases. Then it's easy.

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