Home Work 4

1. Draw the circuite for phase estimation with $t = 3$ lines for representing the value of φ. At the end of the circuite for each of the 3 lines use one-qubit measurement block with

$$P_0 = |0\rangle\langle 0| \text{ and } P_1 = |1\rangle\langle 1|.$$

Hint. To get a circuite for the inverse DFT with $n = 3$ see the relation between Fig. 1 and Fig. 2 in HW 3.

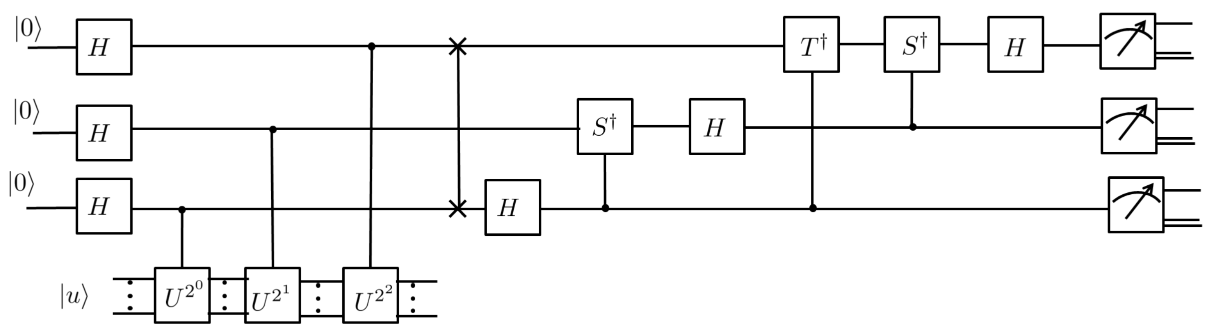

Figure 1: circuite for phase estimation with $t = 3$

2. Let U and $|u\rangle$ be so that $U|u\rangle = \exp(2\pi i\varphi)|u\rangle$, and let $\varphi = 1/2 + 1/8$.

The initial state at the input of the phase estimation circuite is $|0\rangle|0\rangle|0\rangle|u\rangle$. Show the evolution of the state along the circuite up to the measurement blocks.

The evolution (the state after each of the gates) of is :

$$
\begin{aligned}
H :& \frac{1}{2\sqrt{2}}(|0> + |1>)(|0> + |1>)(|0> + |1>)|u> \\
U^{2^0} :& \frac{1}{2\sqrt{2}}(|0> + |1>)(|0> + |1>)(|0> |u> + |1> U|u>) \\
=& \frac{1}{2\sqrt{2}}(|0> + |1>)(|0> + |1>)(|0> |u> + \exp(2\pi i \cdot \varphi)|1>) \\
U^{2^1} :& \frac{1}{2\sqrt{2}}(|0> + |1>)(|0> + \exp(2\pi i \cdot 2\varphi)|1>)(|0> + \exp(2\pi i \cdot \varphi)|1>)|u> \\
U^{2^2} :& \frac{1}{2\sqrt{2}}(|0> + \exp(2\pi i \cdot 4\varphi)|1>)(|0> + \exp(2\pi i \cdot 2\varphi)|1>)(|0> + \exp(2\pi i \cdot \varphi)|1>)|u> \\
=& \frac{1}{2\sqrt{2}}(|0> - |1>)(|0> + i|1>)(|0> + \exp(\pi i \cdot 5/4)|1>)|u> \\
:& \text{ further evolution is obtained similar to Problem 3} \\
\text{before meas. blocks :}& |1> |0> |1> |u>
\end{aligned}
$$

3. Let M_1, M_2, M_3 be the classical outputs of the three measurement blocks. Find the probabilities

$$\Pr(M_1 = 0),\ \Pr(M_1 = 1),\ \Pr(M_2 = 0),\ \Pr(M_2 = 1),\ \Pr(M_3 = 0),\ \Pr(M_3 = 1).$$

Answer:

$$\begin{aligned}
&\Pr(M_1 = 0) = 0, \ \ \Pr(M_1 = 1) = 1, \\
&\Pr(M_2 = 0) = 1, \ \ \Pr(M_2 = 1) = 0, \\
&\Pr(M_3 = 0) = 0, \ \ \Pr(M_3 = 1) = 1.
\end{aligned}$$

4. Let now $\varphi = 1/2 + 1/8 + 1/64$.

   Find numerically the quantum state at the end of the circuite BEFORE the the measurement blocks (it is not necessary to present the evolution of the initial state, just the final state would be enough).

   Answer:

$$\begin{aligned}
&\frac{1}{8}[(-0.0208 - 0.4228i)|000> + (0.1291 - 0.3607i)|001> + (0.2730 - 0.3012i)|010> \\
&+ (0.4669 - 0.2208i)|011> + (0.8940 - 0.0439i)|100> \\
&+ (7.3432 + 2.6274i)|101> + (-0.8417 - 0.7628i)|110> \\
&+ (-0.2436 - 0.5151i)|111>].
\end{aligned}$$

   Find the 9 probabilities:

   $\Pr(M_1 = 0, M_2 = 0, M_3 = 0),\ \Pr(M_1 = 0, M_2 = 0, M_3 = 1), \ldots, \Pr(M_1 = 1, M_2 = 1, M_3 = 1).$

   These probabilities are

$$0.0028,\ 0.0023,\ 0.0026,\ 0.0042,\ 0.0125,\ 0.9504,\ 0.0202,\ 0.0051.$$

5.

$$\begin{aligned}
7^2 \bmod 15 &= 4 \\
7^3 \bmod 15 &= 13 \\
7^4 \bmod 15 &= 1 \\
\to r &= 4.
\end{aligned}$$

6. See Fig. 2.

7. Nonzero elements of U are

$$u_{0,0} = 1,\ \ u_{7,1} = 1,\ \ u_{14,2} = 1,\ \ u_{6,3} = 1,\ \ u_{13,4} = 1,\ \ u_{5,5} = 1,\ \ u_{12,6} = 1,\ \ u_{4,7} = 1$$

$$u_{11,8} = 1,\ \ u_{3,9} = 1,\ \ u_{10,10} = 1,\ \ u_{2,11} = 1,\ \ u_{9,12} = 1,\ \ u_{1,13} = 1,\ \ u_{8,14} = 1$$

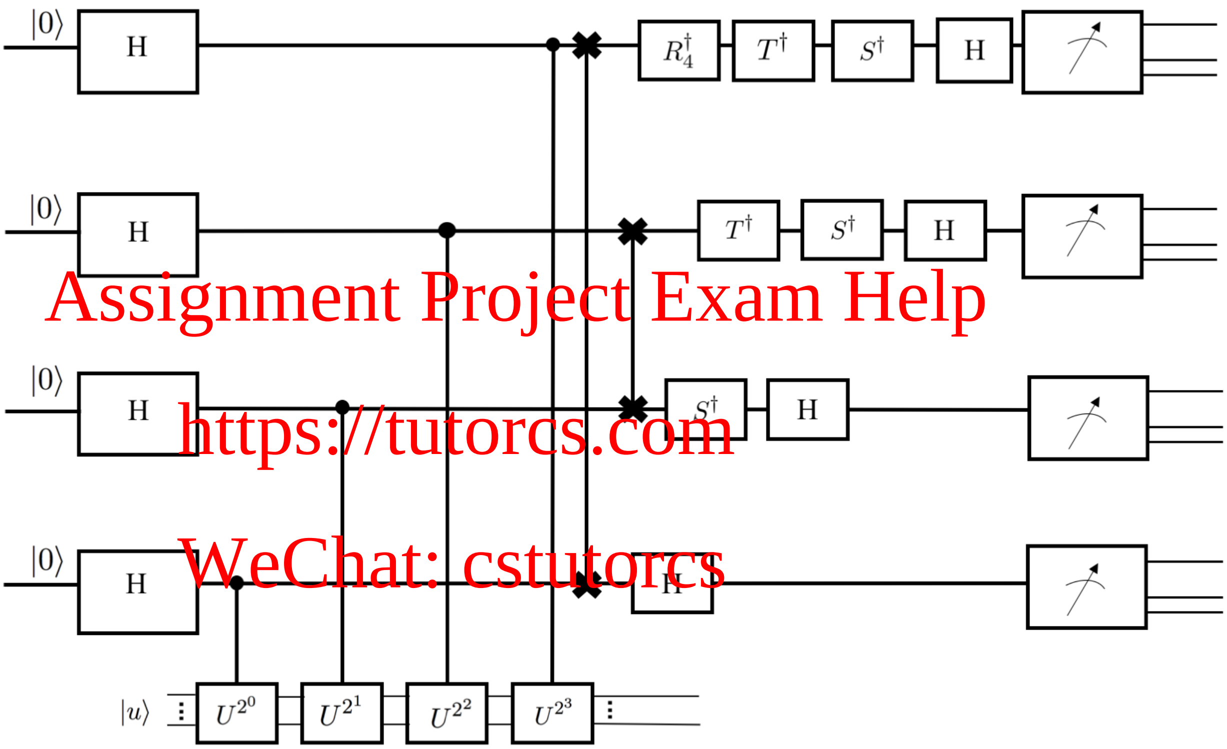

Figure 2: circuite for finding the order of x modulo N assuming $t = 4$