HW1 Solutions

Problem 1

Classical outputs	Quantum outputs	Probability
00	$\|00\rangle$	1/2
01	$\|01\rangle$	1/16
10	$\|10\rangle$	1/4
11	$\|11\rangle$	3/16

Problem 2

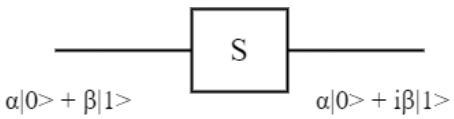
Classical outputs	Quantum outputs	Probability
00	$\gamma\|00\rangle\left(\frac{1}{\sqrt{2}}\|00\rangle + \frac{i}{4}\|01\rangle\right)$, $\gamma^2 = \frac{1}{\frac{1}{2}+\frac{1}{16}} = \frac{16}{9}$ $\implies \|00\rangle\left(\frac{2\sqrt{2}}{3}\|00\rangle + \frac{i}{3}\|01\rangle\right)$	9/16
10	$\|10\rangle\left(-\frac{2}{\sqrt{7}}\|10\rangle + \frac{\sqrt{3}i}{\sqrt{7}}\|11\rangle\right)$	7/16
01	—	0
11	—	0

If $j_1 = 1$ and $j_2 = 0$ then the state of qubits 3 and 4 is:

$$-\frac{2}{\sqrt{7}}|10\rangle + \frac{\sqrt{3}i}{\sqrt{7}}|11\rangle$$

Problem 3

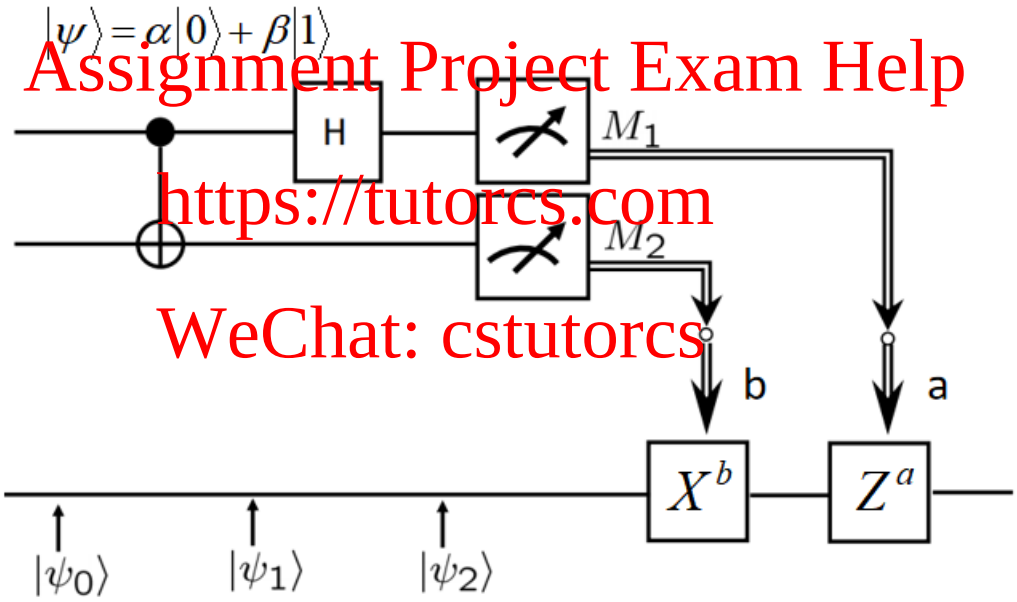
$$U_S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \implies$$



Problem 4

$$U_{\text{Toffoli}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Problem 5



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\begin{aligned}|\psi_0\rangle &= \frac{1}{\sqrt{2}}\alpha|0\rangle\left(|01\rangle - |10\rangle\right) + \frac{1}{\sqrt{2}}\beta|1\rangle\left(|01\rangle - |10\rangle\right) \\ &= \frac{1}{\sqrt{2}}\left(\alpha|001\rangle - \alpha|010\rangle + \beta|101\rangle - \beta|110\rangle\right)\end{aligned}$$

$$\begin{aligned}|\psi_1\rangle &= \frac{1}{\sqrt{2}}\left(\alpha|001\rangle - \alpha|010\rangle + \beta|111\rangle - \beta|100\rangle\right) \\ &= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle\left(|01\rangle - |10\rangle\right) + \beta|1\rangle\left(|11\rangle - |00\rangle\right)\right]\end{aligned}$$

$$\begin{aligned}|\psi_2\rangle &= \frac{1}{2}\left[\alpha\left(|0\rangle + |1\rangle\right)\left(|01\rangle - |10\rangle\right) + \beta\left(|0\rangle - |1\rangle\right)\left(|11\rangle - |00\rangle\right)\right] \\ &= \frac{1}{2}\left[|00\rangle\left(\alpha|1\rangle - \beta|0\rangle\right) + |01\rangle\left(-\alpha|0\rangle + \beta|1\rangle\right) + |10\rangle\left(\alpha|1\rangle + \beta|0\rangle\right) + |11\rangle\left(-\alpha|0\rangle - \beta|1\rangle\right)\right]\end{aligned}$$

Classical outputs	Quantum outputs	Probability
00	$\|00\rangle\left(\alpha\|1\rangle - \beta\|0\rangle\right)$	1/4
01	$\|01\rangle\left(-\alpha\|0\rangle + \beta\|1\rangle\right)$	1/4
10	$\|10\rangle\left(\alpha\|1\rangle + \beta\|0\rangle\right)$	1/4
11	$\|11\rangle\left(-\alpha\|0\rangle - \beta\|1\rangle\right)$	1/4

Value of M_1M_2	Bob's Unitary Operator
00	ZX
01	Z
10	X
11	I_4