Solutions for Homework 2

Problem 1 Find the tensor product

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Solution:

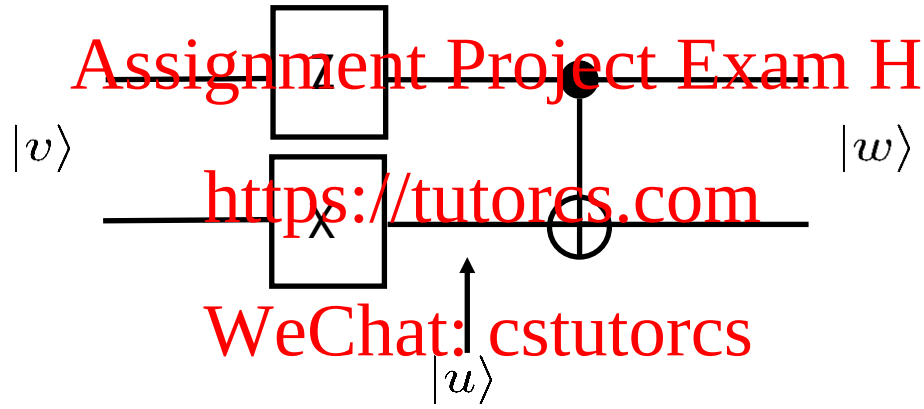
$$\begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$$

Home Work 2

Problem 2 For the input state

$$|v\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

find the state $|w\rangle$ at the output of the following circuit

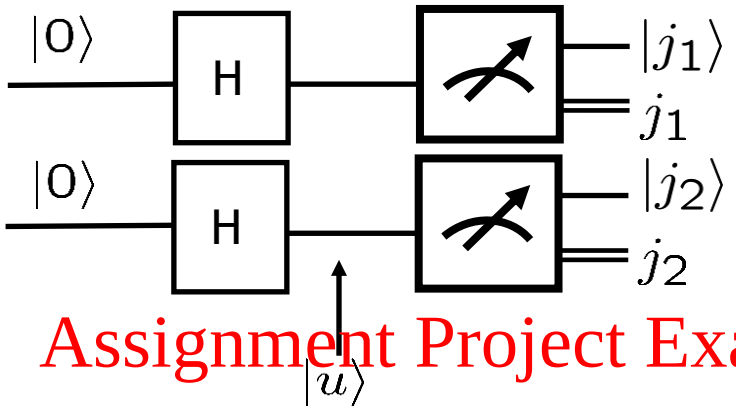


Solution

$$|u\rangle = \alpha_{00}|01\rangle + \alpha_{01}|00\rangle - \alpha_{10}|11\rangle - \alpha_{11}|10\rangle$$

$$|w\rangle = \alpha_{00}|01\rangle + \alpha_{01}|00\rangle - \alpha_{10}|10\rangle - \alpha_{11}|11\rangle$$

Problem 3



Find the classical and quantum outputs and the corresponding probabilities.

Solution

$$|u\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) = \frac{1}{2}(|00\rangle+|01\rangle+|10\rangle+|11\rangle)$$

Cl. Outputs	Q. Outputs	Probability
0 0	|0>|0>	¼
0 1	|0>|1>	¼
1 0	|1>|0>	¼
1 1	|1>|1>	¼

Problem 4 Let we have two qubits in the state

$$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

Let we act on the second qubit by the unitary rotation H. Find the new state of these two qubits.

Solution

$$\frac{1}{\sqrt{2}}|0\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) + \frac{1}{\sqrt{2}}|1\rangle\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$$