

QCC 2022, Home Work 1

1. Two qubits are in the state

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{i}{4}|01\rangle - \frac{1}{2}|10\rangle + \frac{i\sqrt{3}}{4}|11\rangle.$$

We measure these 2 qubits. Find the classical outputs, quantum outputs, and the corresponding probabilities.

2. Four qubits are in the state

$$\frac{1}{\sqrt{2}}|0000\rangle + \frac{i}{4}|0001\rangle - \frac{1}{2}|1010\rangle + \frac{i\sqrt{3}}{4}|1011\rangle$$

We measure qubits 1 and 2 only. Find the classical outputs, quantum outputs, and the corresponding probabilities.

If we get classical outputs $i_1 = 1, i_2 = 0$, what is the quantum state of qubits 3 and 4 after the measurement?

3. Show how S gate changes the state $\alpha|0\rangle + \beta|1\rangle$.
4. Find the unitary matrix that corresponds to the Toffoli gate.
5. In quantum teleportation protocol we assume that qubits 1 and 2 are in the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

Let us now assume that we prepared 2 qubits in the state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ and sent qubit 1 to Alice and qubit 2 to Bob.

Draw a block diagram for quantum teleportation in these case, find states $|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle$, and classical and quantum measurement outputs and the corresponding probabilities. Show what Bob has to do upon receiving classical measurement results.