QCC, Home Work 4

1. Draw the circuit for phase estimation with $t = 3$ lines for representing the value of φ.

2. Let $|u\rangle$ be the eigenvector of U with eigenvalue $U|u\rangle = \exp(2\pi i\varphi)|u\rangle$, $\varphi = 1/2 + 1/8$.

Let the initial state at the input of the phase estimation circuit be $|0\rangle|0\rangle|0\rangle|u\rangle$. Show the evolution of the state along the circuit up to the measurement blocks.

3. Let M_1, M_2, M_3 be the classical outputs of the three measurement blocks. Find the probabilities:

$$\Pr(M_1 = 0),\ \Pr(M_1 = 1),\ \Pr(M_2 = 0),\ \Pr(M_2 = 1),\ \Pr(M_3 = 0),\ \Pr(M_3 = 1).$$

4. Let now $\varphi = 1/2 + 1/8 + 1/64$.

Find numerically the quantum state at the end of the circuit BEFORE the measurement blocks (it is not necessary to present the evolution of the initial state, just the final state would be enough).

Find the probabilities of the eight possible classical outputs of the measurements:

$\Pr(M_1 = 0, M_2 = 0, M_3 = 0), \Pr(M_1 = 0, M_2 = 0, M_3 = 1), \ldots, \Pr(M_1 = 1, M_2 = 1, M_3 = 1).$

5. Let $x = 7$ and $N = 15$. Find the order r of x modulo N (classically, that is in the very usual way).

6. Draw the quantum circuit for finding the order of x modulo N assuming $t = 4$ (draw the inverse Fourier part explicitly with all needed gates). Indicate what is the total number of qubits, say r, at the input of the circuit.

7. Find explicitly the matrix U that should be used in the above circuit.