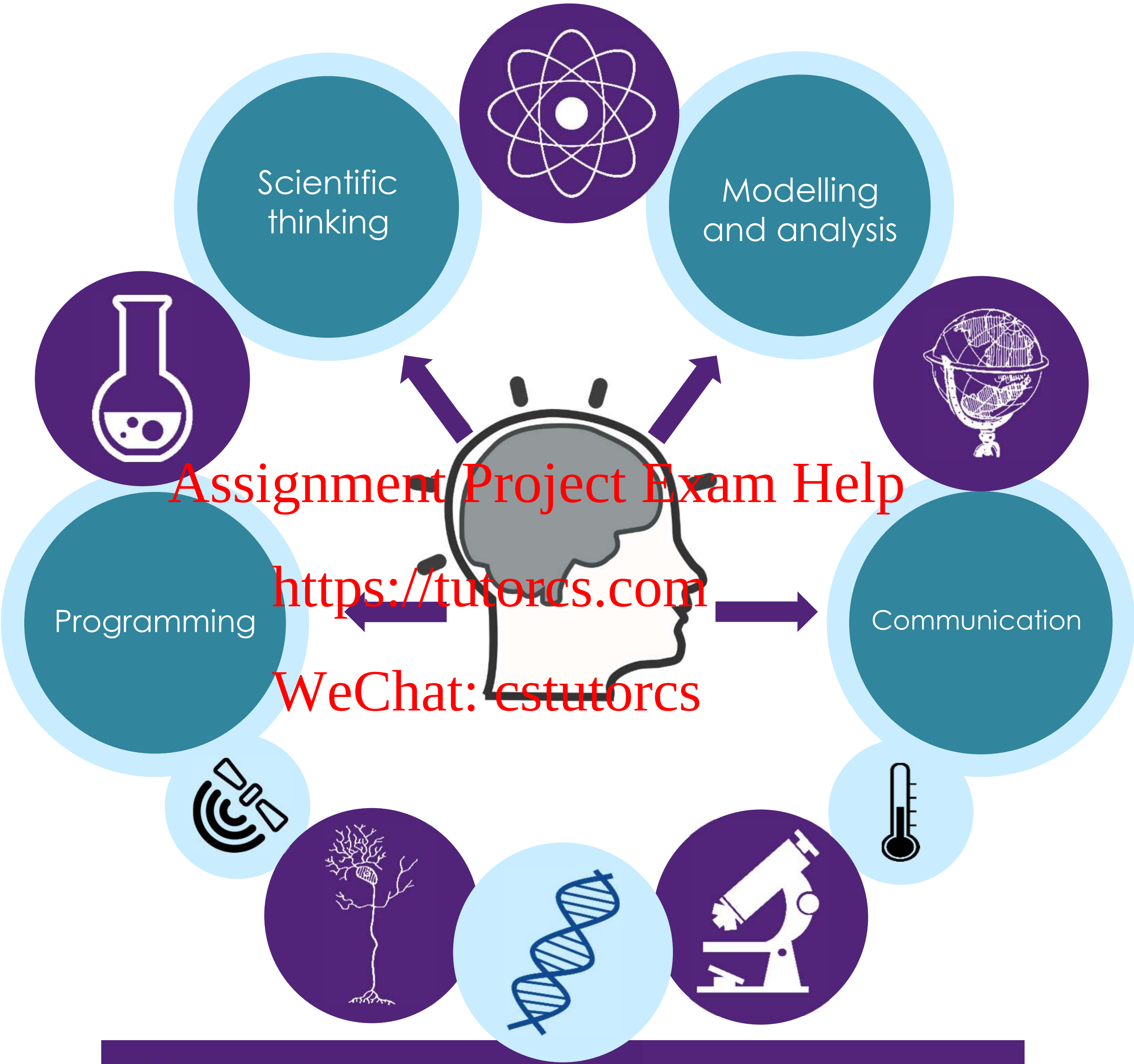
SCIE1000/1100
Theory and Practice in Science



THE UNIVERSITY OF QUEENSLAND
AUSTRALIA
CREATE CHANGE

Name: ______________________________

Contact details: ______________________________

Nineteenth edition, Semester One, 2021

SCIE1000/1100

The core content falls into four key areas: scientific thinking, modelling, programming and communication. The content is taught in a highly ``interleaved'' manner rather than in separate blocks, but the figure below may help you to appreciate the course goals and content.

Thinking

- scientific units
- dimensional analysis
- mechanistic modelling
- phenomeno-logical modelling
- hypotheses
- history of scientific thinking
- inductive reasoning
- Popperian science
- quantitative reasoning

Modelling

- linear equations
- quadratics
- power functions
- sine functions
- exponentials
- logarithms
- average rates of change
- meaning of derivatives
- Newton's method
- areas under curves
- differential equations
- Euler's method
- Systems of DEs
- Predator-prey models
- SIR models

Programming

- software design
- errors
- input and output
- variables
- calling functions
- conditionals
- loops
- arrays
- plotting
- writing functions

Communication

- units in communication
- communicating with graphs
- science in the media
- how to be precise, clear, and concise
- understand your purpose
- know your audience
- convey your key messages

Scientific Contexts

- blood alcohol
- breast cancer
- fluid flow
- heart disease
- atmospheric CO_2
- temperature
- species abundance
- climate change
- biodiversity
- UV light
- wind chill

- breathing
- daytimes/seasons
- radioactive decay
- pharmacokinetics
- recreational drugs
- contraception
- alcohol
- diabetes
- glycaemic index
- bioavailability

- populations
- unconstrained growth
- constrained growth
- resource management
- cancer
- lifecycle models
- turtles
- predator/prey models
- epidemics
- rubella
- catastrophes

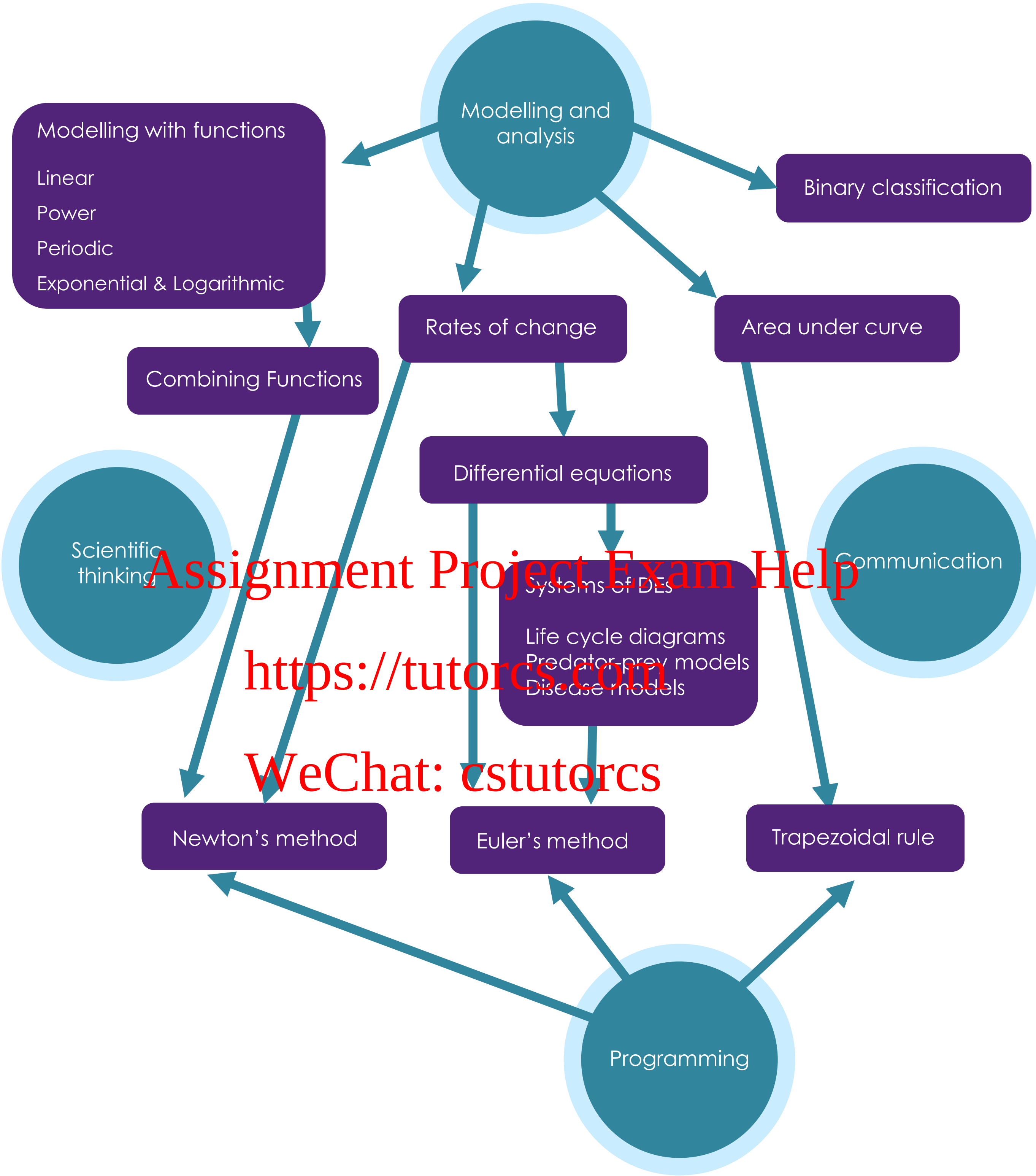

Modelling and analysis
Modelling with functions
Linear
Power
Periodic
Exponential & Logarithmic
Binary classification
Rates of change
Area under curve
Combining Functions
Differential equations
Scientific thinking
Communication
Systems of DEs
Life cycle diagrams
Predator-prey models
Disease models
Newton's method
Euler's method
Trapezoidal rule
Programming

Table of Contents

Lecture 1: **Welcome!**

Learning objectives

✓ Course rational

✓ Meet the lecturing staff

✓ What is a model in science?

Scientific examples

✓ Blood alcohol concentration

Maths skills

✓ Use the direction of effects that independent variables have on a dependent variable to find a plausible equation as a model

Course rationale

The purpose of science is to **understand**, **explain**, **predict** and **influence** phenomena. SCIE1000/1100 aims to help you to develop a range of relevant skills, integrating aspects of science, philosophy, mathematics and computing.

SCIE1000 and SCIE1100 use a unique delivery style at UQ, as they are taught in teams of two. Your lecturers are various discipline experts, including practising scientists, mathematicians and philosophers. Each brings something different to the discussion, and this is intentional. In class you will see that the study of any one topic is, quite naturally, interdisciplinary. A scientist's toolkit is never limited to "biology skills" or "chemistry skills." Consideration of a single topic may, at different times, requires us to think like a biologist, a chemist, a philosopher, a mathematician and a computer programmer, in order to achieve our aims.

The scientific contexts we examine are real and important. Almost every example and case study is based on a research paper, or is a reasonably accurate model of a real situation. For example, when we derive an equation to model blood alcohol content, the equation genuinely models *real* data. You can estimate *your own* blood alcohol content using the equation.

As compelling as the contexts are, memorising the specific details of any particular context is not your goal as a student in SCIE1000/1100. Instead, your focus will be on applying the underlying concepts to novel situations, understanding and practicing how to think critically, developing and applying models, interpreting your results, and communicating with clarity and precision.

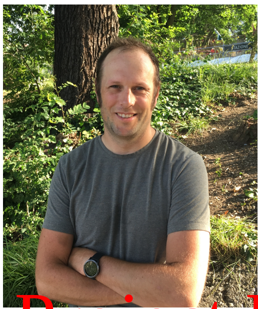
If this seems daunting, don't worry. Almost all of the mathematics we use will be familiar and you will be provided will resources to build your confidence; we will build your computer programming skills from scratch; and any scientific principles we need will be introduced as if you have never seen them before. You need only bring curiosity, diligence, and a determination to critically inquire.

Course Staff

Dr Sara Herke
Mathematics
SCIE1000 Course Coordinator

Prof Tim McIntyre
Physics

Dr Ava Greenwood
Mathematics



Dr Andrew Letten
Quantitative biology

A/Prof Michael Landsberg
Biochemistry

Prof Matthew Davis
Physics
SCIE1100 Course Coordinator

Dr Elizabeth Dun
Biology

Prof Anthony Richardson
Mathematical ecology

A/Prof Marcus Gallagher
Computer science

Dr Joel Katzav
Philosophy

Dr Peter Evans
Philosophy

Course assessment

You should **read the Electronic Course Profile (ECP)** in detail, where you will find the weighting of the assessment items as well as course policies.

Resources and getting help

Learning resources: There are many resources, which can all be accessed through the class Blackboard site. Course announcements are made on the Blackboard site as well, so please make a habit of checking the course Blackboard site regularly. Some specific resources you will want to make use of are:

- This course book, together with lecture annotations
- Lecture recordings
- Workshop tasks and solutions
- Specialised on-line learning modules (SOMSE)
- Selected past exams and solutions
- MyPyTutor for extra programming practice

Getting help:

- Online course discussion board.
- Talk to your tutors and peers during your scheduled workshops.
- Attend lecturer office hours.
- Check out "Course Help" on Blackboard for FAQs and support.

Chapter 1: A short discussion of nearly everything

Image 1.1: *The School of Athens* (1510 – 1511), Raphael (1483 – 1520), Stanze di Raffaello, Apostolic Palace, Vatican. (Source: en.wikipedia.org.)

The School of Athens (left) depicts some famous scientists, mathematicians and philosophers, including Plato, Aristotle, Euclid, Socrates and Pythagoras. Science and knowledge play fundamental roles in human history, culture and society.

In order to understand phenomena in the world around us, we use a variety of mathematical and scientific techniques, real-world data as well as "common-sense" in order to develop models. Whatever model we are considering, as scientists, we must compare its predictions to real-world data.

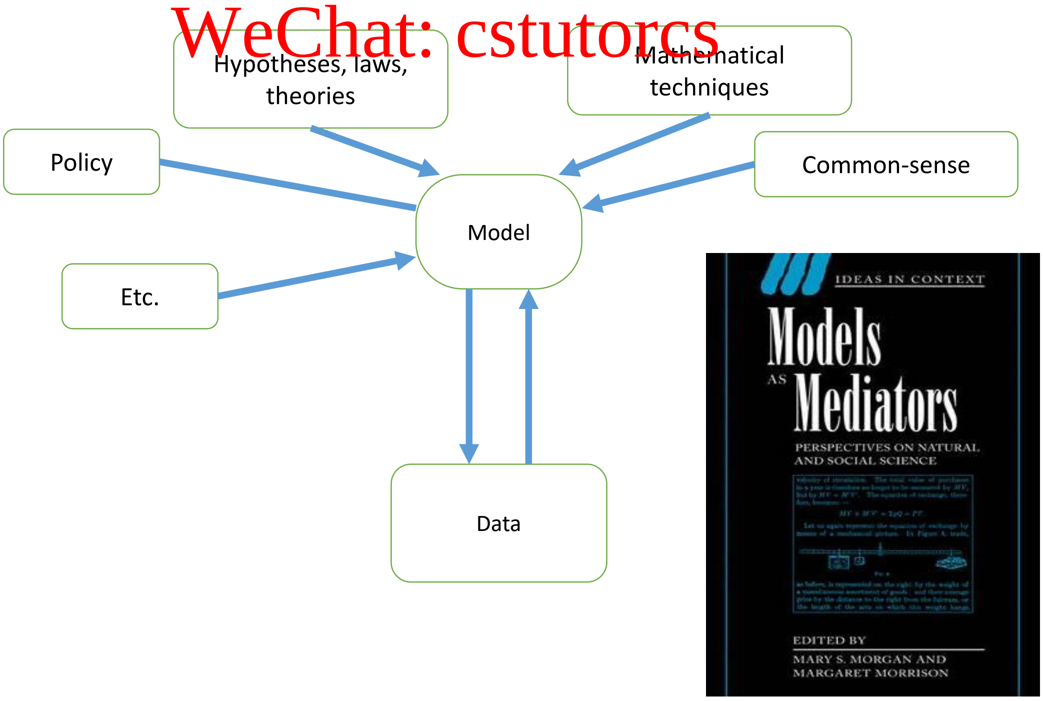


Image 1.2: *Philosophy of Models: Models as Mediators* [33]

1.1 A taste for SCIE1000: Bloody alcohol

***Question** 1.1.1*

(a) Suppose that you would like to understand how the blood alcohol level of a person who has consumed alcohol changes over time. What type of experiment would you set up? What would you measure? What would be your key question?

(continued over)

***Question* 1.1.1** *(continued)*

(b) Develop a model of the blood alcohol content of a person after they consume a quantity of alcohol.

Gender and sex

In SCIE1000, and often in science more generally, one thing we do is to aim to keep models as simple as possible. When modelling medical phenomena, it can be useful to have a variable that considers the influence of a person's sex. First, it is important to recognise the distinction between a person's sex and a person's gender.

The World Health Organisation [58] says:

> Gender interacts with but is different from sex, which refers to the different biological and physiological characteristics of females, males and intersex persons, such as chromosomes, hormones and reproductive organs. Gender and sex are related to but different from gender identity. Gender identity refers to a person's deeply felt, internal and individual experience of gender, which may or may not correspond to the person's physiology or designated sex at birth.

Often when we consider the influence of sex in scientific modelling, the relevant factors are related to *body fat percentage* and *hormones*. Under the hormonal effects of oestrogen, which is typically higher for females, comparatively larger amounts of fat tissue are deposited around the body. Within any category used to describe a person's sex, there still may also be quite a bit of natural variation around what are considered to be "standard" levels of fatty tissue and hormones. However, tracking these levels for individuals is quite a complex task. Thus, for the purpose of making simple models, we will sometimes use a binary choice for "sex"; for example, when modelling blood alcohol concentration. We hope that you recognise the limitations that come with making such an assumption in a model.

Learning objectives

✓ Review the quantitative language of science

✓ Introduce programming and some best-practices

✓ Understand the need for clear communication in science

✓ Develop critical evaluation skills

Scientific examples

✓ Mars Climate Orbiter

✓ Widmark formula

Maths skills

✓ Use dimensional analysis to check if an equation is plausible

1.2 Units

- A unit of measurement is an agreed upon quantity of something; any other quantity of the same kind can be described by giving the ratio between that quantity and the unit. For example, when we write that the minimum length of a long jump pit is 2.75 metres, we are saying that 2.75 is the ratio between the minimum length of the long jump pit and the length that we have agreed to call a metre.

- The same quantity can be described by comparison to different reference quantities (units). It is important to use the same units for measuring the same kinds of quantities in any one calculation, and to communicate the units used clearly. The consequences of not doing so can be severe.

Example *1.2.1*

The Mars Climate Orbiter was launched in 1998 as part of a $USD330 million project, but in September 1999 it crashed into Mars. Here is an extract from the report into the accident [34]:

(continued over)

Example 1.2.1 (continued)

"During the 9-month journey from Earth to Mars, propulsion maneuvers were periodically performed…coupled with the fact that the angular momentum (impulse) data was in English, rather than metric, units, resulted in small errors being introduced in the trajectory estimate over the course of the 9-month journey. At the time of Mars insertion, the spacecraft trajectory was approximately 170 km lower than planned…

…it was discovered that the small forces ΔVs reported by the spacecraft engineers for use in orbit determination solutions was low by a factor of 4.45 (1 pound force = 4.45 Newtons) because the impulse bit data contained in the AMD file was delivered in lb-sec instead of the specified and expected units of Newton-sec."

Photo 1.1: Mars Lander (proof test model) from the Viking program, launched 1975. (Source: PA.)

SI Units and prefixes

Australia adopted the **International System of units**, or **SI units**, in 1960 and this is the system still in use today. There are seven **SI base units**. The kinds of things they measure, their standard names and their symbols are shown in Figure 1.1.

(continued over)

SI Units and prefixes *(continued)*

Base quantity	SI base unit	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Figure 1.1: The seven SI base units.

Some lengths, like the charge radius of a proton, are much less than a metre; others, like the distance from the Earth to the centre of the Milky Way, are much more than a metre. To save ourselves writing numbers with many digits before or after the decimal place, we may write one of the SI prefixes, as shown in Figure 1.2, in front of a unit of measurement to indicate a fraction or multiple of the unit.

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^1	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

Figure 1.2: The 20 SI prefixes.

- One of the SI base units, the kilogram, has a prefix built into its name and its symbol. When writing about mass you should use prefixes in front of grams, or g, rather than the base unit.

Algebra for quantities and units

- If quantities have the same units then they can be added or subtracted and the result has the same units.
- If quantities are multiplied or divided then the corresponding units should be gathered on the right, then multiplied and divided (or cancelled) using the familiar algebra rules.
- We often write a dot, or leave a space, between units when they are to be multiplied, and use exponent notation to indicate "powers" of units. Negative exponents indicate quotients. So, for example, we write m^3 instead of $\mathrm{m} \cdot \mathrm{m} \cdot \mathrm{m}$, and $\mathrm{m} \cdot \mathrm{s}^{-1}$ or $\mathrm{m\ s}^{-1}$ instead of $\frac{\mathrm{m}}{\mathrm{s}}$.

Example 1.2.2

If $A = 12.3\ \mathrm{kg}$ and $B = 1.68\ \mathrm{m} \cdot \mathrm{s}^{-2}$ and $C = 4.62\ \mathrm{m}^2$, then

$$\begin{aligned}\frac{AB}{C} &= \frac{(12.3\ \mathrm{kg})\left(1.68\ \mathrm{m} \cdot \mathrm{s}^{-2}\right)}{4.62\ \mathrm{m}^2} = \frac{(12.3)(1.68)}{4.62}\ \frac{\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}}{\mathrm{m}^2} \\ &= 4.47\ \mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{-2}\end{aligned}$$

Derived units

Many natural and scientific quantities require more complex units than SI base units. These **can always be defined** in terms of the seven base units, and are called **SI derived units**.

Example 1.2.3

Some frequently-used SI derived units have been given special names and symbols. Figure 1.3 shows some well-known examples.

Quantity	Name	Symbol	In SI base units	In other SI Units
frequency	hertz	Hz	s^{-1}	-
force	newton	N	$m \cdot kg \cdot s^{-2}$	-
pressure, stress	pascal	Pa	$m^{-1} \cdot kg \cdot s^{-2}$	$N \cdot m^{-2}$
energy, work, quantity of heat	joule	J	$m^2 \cdot kg \cdot s^{-2}$	$N \cdot m$
power, radiant flux	watt	W	$m^2 \cdot kg \cdot s^{-3}$	$J \cdot s^{-1}$
electric potential difference, electromotive force	volt	V	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$	$W \cdot A^{-1}$
volume	litre	L or l	$10^{-3}\,m^3$	-
time	day	d	$86,400\,s$	24 h
time	hour	h	$3,600\,s$	60 min
time	minute	min	$60\,s$	-

Figure 1.3: Some well-known derived units and their SI base units.

Converting between different units for the same quantity

The algebra rules for quantities and units provide a neat way to convert between different units for the same base quantity. For example, an atmosphere, abbreviated to atm, is a unit for pressure. By definition, 1 atm is equal to 101.325 kPa. It follows that $\frac{1\text{ atm}}{101.325\text{ kPa}}$ is like 1, and we may introduce it as a multiplicative factor whenever we like.

For example, we can convert 233.05kPa to the units of atm as follows:

Question 1.2.4

The *cardiac output* CO of a heart is the volume of blood ejected by the heart during a particular time period, and equals the *stroke volume* SV multiplied by the *heart rate* HR. If each beat has a volume of 70 mL and the heart beats 1.5 times each second, calculate CO in $\mathrm{L} \cdot \mathrm{min}^{-1}$.

Dimensional analysis

Dimensional analysis is an important technique in science. It involves applying the following principle: an equation describing a physical situation can be true *only* if it is **dimensionally homogeneous**; that is, both sides of the equation have the same units. Dimensional analysis allows a quick check of whether a calculation is 'plausible'. If the dimensions do not match, then there **must** be an error.

About units and counting things

When a number represents a count of something, we should record what it counts. For example, after observing a friend's backyard I may write that I observed 17 chickens. However, this does not mean that "chickens" is a unit. For something to be a unit, or at least a scientific unit, it should be something we can describe using only powers of SI base units and constants.

1.3 Huh?

Question 1.3.1

In 2005 a psychologist, Dr Cliff Arnall, proposed a function which models how depressing each day of the year is. By finding the date on which the function takes its maximum value, Arnall identified "Blue Monday", the most depressing day of the year. The "Blue Monday" model still receives significant press attention. Arnall's model takes into account: weather (W), debt (D), monthly salary (d), time since Christmas (T), time since failing our new year's resolutions (Q), low motivational levels (M), and the feeling of the need to take action (N_a). His function has been variously reported as:

www.ft.com, en.wikipedia.org: $\dfrac{(W + (D - d)) \times T^Q}{M \times N_a}$

www.msnbc.msn.com: $\dfrac{[W + (D - d)] \times TQ}{M \times NA}$

www.scotsman.com: $([W + (D - d)] \times TQ)(M \times NA)$

www.peterboroughtoday.co.uk: $W + (D) \times T^Q / M \times Na$

news.bbc.co.uk, www.cbc.ca: $1/8W + (D - d)3/8 \times TQM \times NA$

Question 1.3.2

Write a short paragraph that explains to the general public in Australia why there is no underlying science behind "Blue Monday."

There are some excellent online articles that cover doubtful claims and errors in media reporting of quantitative science. The following links are very interesting, amusing, frightening and informative:

- "Behind the Headlines" [36]– provides a factual analysis of health-related claims, including the scientific background.
- "Bad Science" [21]– identifies and discusses mathematical and scientific errors in reported and published science.
- "Dodgy Boffins" [13]– discusses misuse of equations in the British media.
- "Helping Doctors and Patients Make Sense of Health Statistics" [20]– discusses the causes and impacts of errors in presenting and interpreting health statistics.

1.4 Programming

- Computation is important when formulating and applying models, particularly when dealing with complex phenomena. In SCIE1000 we will write programs to model some phenomena. Programming requires technical skill, experience and creativity.
- Refer to Appendix A for the Python programming instruction manual for this course.
- A *program* is a set of instructions that make a computer do something. Web browsers (for example, *Internet Explorer*), word processors (*Microsoft Word*) and "apps" for a mobile phone are all familiar examples of programs. Even the Python programming language itself is a program.
- The first step in programming is to understand exactly what problem is being solved, and hence specify **exactly** what the program should do; specifications should be precise, accurate and complete. Once the problem has been understood, the programmer needs to write a sequence of commands that together solve the problem.

- Even the best and most experienced computer programmers will sometimes (even often) write programs with errors in them. The consequences of software errors (*bugs*) can be very serious.
- Types of programming error include: incomplete (or incorrect) problem description, design faults in the software, unanticipated 'special cases', coding errors, logic errors and miscommunication between programmers. Section A.4 in Appendix A provides some advice on how to test and check for coding errors.

In Lecture 1, we developed a very rough model for the blood alcohol of a person who had consumed alcohol. The model we developed is quite similar to a model which is often used in courts to determine blood alcohol levels; this is called the *Widmark formula* and it was developed in 1932. The equation is:

$$B = \frac{A}{rM} \times 100\% - 0.015t,$$

where B is the blood alcohol concentration (as a %) at time t (where t is measured in hours since drinking commenced), A is the amount of alcohol consumed (in grams), M is the body mass (in grams) and r is an estimate of the proportion of body mass that is water.

Question 1.4.1

Consider the Widmark formula. What are the units of the constant 0.015? What does this constant represent physically?

Program 1.1: Blood Alcohol Concentration

```python
from pylab import *

M = float(input("What is the person's mass (in kg?) "))
s = int(input("Enter 1 for female, 0 for male: "))
n = int(input("Number standard drinks? "))
t = float(input("How many hours ago? "))

# 10g alcohol per standard drink
alcohol = 10*n

# mass of water (in grams) for the person
if s == 1:
    water = 1000*M*0.6
else:
    water = 1000*M*0.7

# metabolic rate of removal (% / hour)
rate = 0.015

BAC = (alcohol/water)*100 - rate*t
print("your blood alcohol concentration is ", round(BAC,4),"%")
```


***Question** 1.4.2*

Briefly summarise in words the purpose of each line of code in the above program. If you're not sure, have a guess. Note that you are not expected to be familiar with all of this programming syntax yet!

Chapter 2: Thinking and communicating

Lecture 3: Making sense in science

Learning objectives

✓ Understand the need for clear communication of scientific information

✓ Evaluate quantitative information presented by the media

Scientific examples

✓ Contraception and thrombosis

✓ Prostate cancer

✓ Tests for breast cancer

Maths skills

✓ Interpret quantitative information

✓ Evaluate percentages and ratios

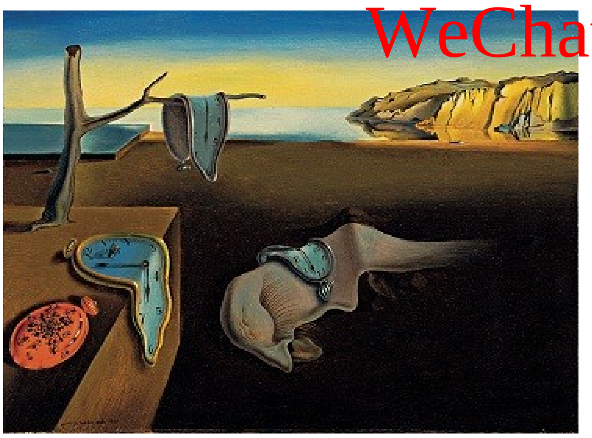


Image 2.1: *The Persistence of Memory* (1931), Salvador Dalí (Source: en.wikimedia.org)

In SCIE1000 you will develop your skills in scientific communication. Some key guiding principles of scientific communication are: be clear, know your purpose, know your audience, and identify your key messages. Refer to Appendix B "Communication in Science" for detailed advice and examples.

In this chapter, we explore the communication of scientific knowledge, particularly in the context of effectively communicating and interpreting quantitative data in a medical context. The goal is to motivate why clear scientific communication is so important.

2.1 Quantitative communication

- In SCIE1000, we will explore the fundamental skills and concepts that will help you with effective scientific analysis and communication.

- We are all producers and consumers of quantitative scientific information, in the form of scientific papers, assignments, media reports, the internet, and professional communications such as doctor/patient discussions.

- As a *producer* of such information, we should aspire to be accurate, honest, logical, unambiguous, concise, precise, not excessively technical, and always mindful of the intended audience. See Appendix B for detailed advice on how to communicate in science.

- As a *consumer*, we should aspire to be thoughtful, reflective, sceptical, logical and analytical, while at the same time open-minded and accepting of evidence that may differ from our preconceptions or opinions.

- The media and internet provide a continual bombardment of facts, reports, summaries, interpretations and opinions, often covering sophisticated concepts but written and read by non-experts. In many cases there are errors (or deliberate falsities) in such communications.

- You should form the habit of critically evaluating information, data and (claimed) conclusions.

- A useful approach (when checking your own work, or the work of others), is *rough estimation*, which is the process of calculating approximate values. It involves building rough, conceptual models, and then evaluating them 'for sense'.

- Estimating 'gives an idea' whether a particular value is plausible. Often, we aim to find an approximate value within an *order of magnitude* of the correct value (that is, within a factor of 10 of the correct value).

2.2 Losing patients with mathematics?

- Sometimes, particularly in a medical context, critically evaluating quantitative information is a matter of life and death. A paper from 2007 [20] presents the following key findings:

  - Many people (doctors, patients, journalists and politicians) do not understand health statistics.
  - Lack of understanding is due both to lack of knowledge, and intentional misrepresentation of information.

- The following paragraph is a quote from [20]:

  > "Statistical literacy is a necessary precondition for an educated citizenship in a technological democracy. Understanding risks and asking critical questions can also shape the emotional climate in a society so that hopes and anxieties are no longer as easily manipulated from outside . . ."

***Question** 2.2.1*

In [20], researchers asked 450 American adults (aged 35–70; 320 had attended college; 62 had a postgraduate degree) for answers to the following questions:

> "1. A person taking Drug A has a 1% chance of having an allergic reaction. If 1,000 people take Drug A, how many would you expect to have an allergic reaction?
>
> 2. A person taking Drug B has a 1 in 1,000 chance of an allergic reaction. What percent of people taking Drug B will have an allergic reaction?
>
> 3. Imagine that I flip a coin 1,000 times. What is your best guess about how many times the coin would come up heads in 1,000 flips?"

(continued over)

***Question** 2.2.1* *(continued)*

(a) What are the answers to the above three questions?

(b) What are the ramifications of getting these answers incorrect for doctors, journalists and politicians?

***Question** 2.2.2*

In 1995, an emergency announcement in the UK warned that third-generation oral contraceptive pills doubled the risk of potentially life-threatening blood clots (thrombosis). The announcement led to widespread concern and fear, and many women ceased using the contraceptives. Reports estimate that in the following year there were an additional 13,000 abortions and 13,000 births, with 800 additional pregnancies in girls under 16 years of age. The announcement omitted the following relevant information:

- young women have an absolute risk of spontaneous thrombosis of 1 in 10,000.

(continued over)

***Question** 2.2.2* *(continued)*

- the absolute risk of thrombosis when taking second-generation oral contraceptive pills is about 1 in 7000.
- the relative risk of thrombosis increases by a factor of 4 to 8 during a Caesarean birth.
- the relative risk of thrombosis during and after pregnancy increases by a factor of around 4.
- the absolute risk of dying from thrombosis during or after an abortion is around 1.1 in 10,000.

(a) Define the terms "absolute risk" and "relative risk".

(b) The Australian Medical Association (AMA) website [2] states that:

"... in order to support and enhance the collaborative nature of the doctor-patient relationship, patients must be able to make informed choices regarding their health care. An informed choice is dependent on receiving reliable, balanced health information, free from the influence of commercial considerations, that is communicated in a manner easily understood by patients."

Write a public health announcement that would better advise the public on the risks of the third generation contraceptive pill.

Case Study 1: **Cancer**

- Cancer is the name for a large group of diseases affecting many different parts of the body. It arises from the uncontrolled, rapid growth of abnormal cells that interfere with the usual bodily functions.
- Cancerous cells can *metastasise*, spreading to other parts of the body.
- Common cancers include cancers of the lung, prostate (males), breast (mostly females), colon, skin, bladder, kidney and blood (leukaemia).
- Smoking and excessive alcohol consumption are major risk factors.
- Cancer is a leading cause of human mortality. Figure 2.1 lists all leading causes of death for Australians.

Cause of death	2009	2018
Ischaemic heart disease	22587	17533
Dementia/Alzheimer disease	8280	13963
Cerebrovascular diseases	11216	9972
Lung/bronchus/trachea cancer	7786	8586
Chronic lower respiratory disease	5984	7889
Prostate cancer	3111	3264
Breast cancer	2799	3034
Blood/lymph cancer	3811	4612
Colon/rectal cancer	5244	5420
Diabetes	4176	4656
Diseases of the urinary system	3315	3384
Heart failure / ill-defined heart disease	3219	3192
Intentional self-harm	2337	3046
Pancreatic cancer	2204	3077
Accidental falls	1450	2952
Skin cancer	1837	2094

Figure 2.1: Leading causes of death in Australians. Population of 21.7 million in 2009; population of 24.9 million in 2018. (Source: Australian Bureau of Statistics.)

- Common cancer treatments include:
  - *Chemotherapy*, which involves the infusion of highly toxic chemicals into the body, killing rapidly dividing cells. (Recall that rapid division is a common characteristic of cancerous cells.)
  - *Radiation therapy*, which involves exposing cells to radiation and hence damaging their DNA, leading to cell death.
  - *Surgery*, which involves removing cancerous tissue from the individual.
  - *Stem cell transplants* (or *bone marrow transplants*), which involves infusing healthy stem cells into an individual with cancer.

- All of these treatments can have minor to major side effects, including fatigue, nausea, mouth ulcers, hair loss, cognitive problems, infection, anaemia, infertility, graft-versus-host disease, burns, cancer (!) or death.

- Determining the precise treatment regime and dosages involves a trade-off between the beneficial impact of reducing tumour size and the (often severe or life-threatening) side-effects resulting from the treatment.

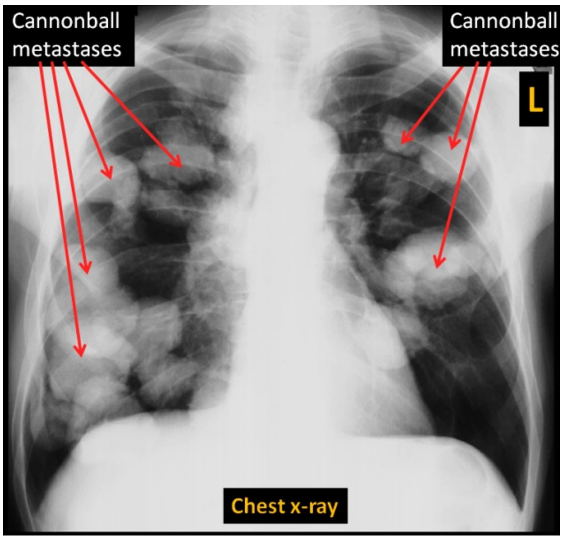


Photo 2.1: Chest X-ray displaying many classic "cannonball" metastases. (Source: Qld Health and DM.)

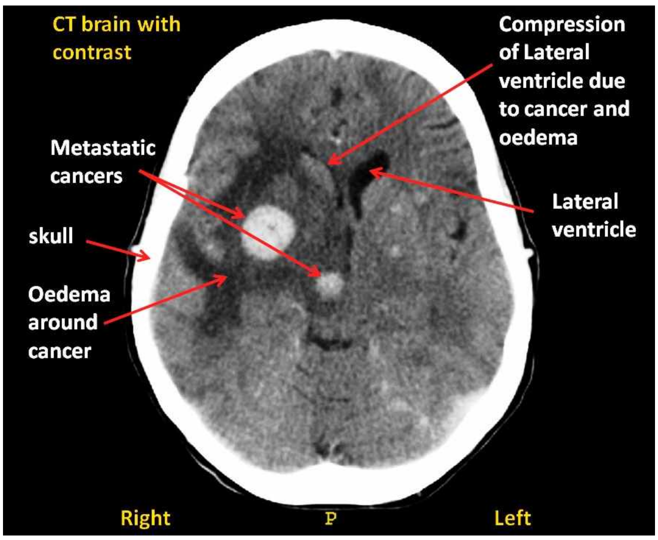


Photo 2.2: Axial CT image with contrast shows enhancing metastatic cancers with associated vasogenic oedema (swelling) in the brain. The metastases are due to breast cancer. (Source: Qld Health and DM.)

Question 2.2.3

Two commonly reported medical statistics are:

- the *5-year survival rate*, which is the percentage of people who are still alive five years after being diagnosed with a condition; and
- the *annual mortality rate*, which is the number of people dying from a given condition each year, often expressed as a rate per 100,000 people.

(a) The 5-year survival rate for prostate cancer in American men is 98%; for British men it is 71%.

(i) Assume that 1,000 British men and 1,000 American men receive a diagnosis of prostate cancer (at the same time). After 5 years how many men in each country are expected to have died?

(ii) Considering only the given data, which country has the 'better' health system, and why?

(b) The annual mortality rate for prostate cancer in American men is 26 deaths per 100,000; for British men it is 27 per 100,000. Considering only these data, which country has the 'better' health system, and why?

(continued over)

***Question** 2.2.3* *(continued)*

(c) The medical information given in Parts (a) and (b) is all correct. Explain how the (apparent) discrepancies could occur.

(d) Treatment for prostate cancer is invasive with many substantial side effects, including incontinence and impotence. Considering only prostate cancer, which country has the 'better' health system, and why?

- *Breast cancer* develops due to the uncontrolled growth of cells in breast tissue, which enlarge into one or more lumps within the breast. It is a comparatively common cancer, and is a leading cause of death in women; it also affects men, but at a much lower rate.

- Some risk factors for breast cancer identified in the paper [35] include:
  - sex traits: the risk for females is around 100 times that for males;
  - age: the risk of developing breast cancer rises rapidly with age;
  - affluence: breast cancer is more common in affluent societies;
  - pre-existing breast conditions (for example, increased breast density);
  - hormonal factors (such as age at menopause or oral contraceptive use);
  - high levels of alcohol consumption.
- Some factors that reduce the risk of breast cancer include having children (more offspring at an earlier age reduces risk), breastfeeding, and increased physical activity.
- Treatment options for breast cancer include chemotherapy, radiation therapy, hormonal methods and surgery, including total removal of the breast (mastectomy) and breast-conserving surgery (lumpectomy).

Question 2.2.4

In 2009, US authorities suggested that mammogram screening for breast cancer should start at 50 years of age rather than 40 years, and be conducted every second year rather than every year. Discuss reasons for and against this decision.

Lecture 4: **Risky business**

Learning objectives

✓ Analyse information about probabilities and risks

✓ Interpret binary classification test tables

✓ Become a critical consumer of information

Scientific examples

✓ Breast cancer

✓ HIV screening

Maths skills

✓ Calculate probabilities

Binary classification test

A *binary classification test* aims to classify objects, people or things into one of two groups. Examples include many medical tests, such as determining whether or not an individual has (or is likely to have) cancer. Most binary classification tests are imperfect: results can be *true positives*, *false positives*, *true negatives* or *false negatives*.

	Disease	
	Yes	**No**
Test +		
Test −		

Accuracy, sensitivity and specificity

The terms *accuracy*, *sensitivity* and *specificity* of a binary classification test are defined mathematically below. Explain what each term means and why it is important.

	Disease Yes	Disease No
Test +	A	B
Test −	C	D

$$N = A + B + C + D$$

$$\text{Accuracy} = \frac{A+D}{N}$$

$$\text{Sensitivity} = \frac{A}{A+C}$$

$$\text{Specificity} = \frac{D}{B+D}$$

Question 2.2.5

A paper [14] studied the effectiveness of combined mammography and ultrasound imaging to screen for breast cancer. A total of 203 patients returned "suspicious or malignant" test results, of whom 138 were later found to have cancer (via biopsy testing). A total of 2811 patients returned "normal or probably benign" test results, of whom 12 were later found to have cancer. Find the accuracy, sensitivity and specificity of the combined procedures.

Question 2.2.6

(a) What are some characteristics of a "good" binary classification test?

(b) Identify some negative impacts of false positive or false negative cancer test results.

(c) When might a test with a higher false positive test rate be 'better' than one with a lower rate. Give an example in which it would be worse.

(d) Are false positive results 'better' or 'worse' than false negative results?

Question 2.2.7

A paper [20] quotes an example in which 160 gynaecologists were asked:

"Assume you conduct breast cancer screening using mammography... You know the following information about the women in this region:

- *The probability that a woman has breast cancer is 1% (prevalence)*
- *If a woman has breast cancer, the probability that she tests positive is 90% (sensitivity)*
- *If a woman does not have breast cancer, the probability that she nevertheless tests positive is 9% (false-positive rate)*

A woman tests positive. She wants to know whether that means that she has breast cancer for sure, or what the chances are. What is the best answer?

A. The probability that she has breast cancer is about 81%.

B. Out of 10 women who test positive, about 9 have breast cancer.

C. Out of 10 women who test positive, about 1 has breast cancer.

D. The probability that she has breast cancer is about 1%."

(a) Without doing detailed calculations, what is **your** answer ?

(continued over)

Question 2.2.7 *(continued)*

(b) Investigate the answer to the question using a *probability flowchart* and a group of 1000 'typical' women.

(c) Repeat Part (b), instead using a binary classification table.

(d) What proportion of gynaecologists do you think could answer Part (a) correctly? What are the implications for you and/or your female relatives?

End of Case Study 1: Cancer.

Question 2.2.8

In the 1980s, blood screening in Florida found that 22 people who had donated blood tested positive for HIV. Once notified of the test results, seven of these donors committed suicide. (At that time, HIV was largely unheard of, and people were not regularly tested. Screening donors for the disease commenced after the discovery that transmission of HIV occurred through contact with infected blood.)

The HIV test has a very high *sensitivity* [percentage of infected individuals who correctly test positive] of about 99.9% and *specificity* [percentage of non-infected individuals who correctly test negative] of about 99.99%.

The *prevalence*, or rate of infection, for heterosexual men with low-risk behaviour, is around 1 in 10,000.

What is the (approximate) probability that someone who tests positive for HIV is infected?

Question 2.2.9

(From [20].) To investigate the quality of HIV counselling for heterosexual men with low-risk behaviour, an undercover client visited 20 public health centres in Germany, undergoing 20 HIV tests.

The client was explicit about belonging to a low risk group, as do the majority of people who take HIV tests. In the mandatory pre-test counselling session, the client asked: 'Could I possibly test positive if I do not have the virus? And if so, how often does this happen?'

The answers from the medical practitioners were:

No, certainly not	False positives never happen
Absolutely impossible	With absolute certainty, no
With absolute certainty, no	With absolute certainty, no
No, absolutely not	Definitely not ... extremely rare
Never	Absolutely not ... 99.7% specificity
Absolutely impossible	Absolutely not ... 99.9% specificity
Absolutely impossible	More than 99% specificity
With absolute certainty, no	More than 99.9% specificity
The test is absolutely certain	99.9% specificity
No, only in France, not here	Don't worry, trust me

(a) How would **you** answer the question?

(continued over)

***Question** 2.2.9* *(continued)*

(b) Recall that the Australian Medical Association (AMA) website [2] states:

"...in order to support and enhance the collaborative nature of the doctor-patient relationship, patients must be able to make informed choices regarding their health care. An informed choice is dependent on receiving reliable, balanced health information, free from the influence of commercial considerations, that is communicated in a manner easily understood by patients."

Comment on the responses from the German doctors, relating your answer to your answers to Question 2.2.8 and the AMA statement above.

Chapter 3: A career in modelling

Lecture 5: Learning how to model

Learning objectives

✓ Understand the framework for mathematical models in science

✓ Appreciate the uses and limitations of models

✓ Contrast data driven models with theoretically driven models

Scientific examples

✓ Blood flow through a bypass vein graft

Maths skills

✓ Understand and interpret data from a graph

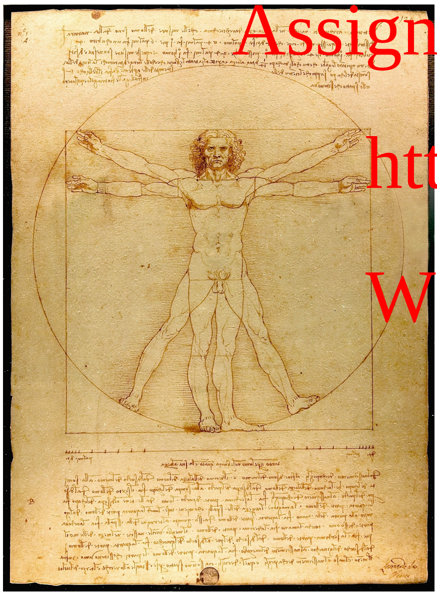


Image 3.1: *Vitruvian Man* (1490), Leonardo da Vinci. (Source: en.wikipedia.org)

In this chapter we discuss what mathematical modelling means in science, as well as different ways in which models are developed and understood. When developing models in science, there will always be a balance between theory and data driven models. We will cover a framework that discusses the differences between the two approaches, and note that they are often interlinked.

We will practice the skill of developing a theoretical mathematical model in the context of blood flow through a blood vessel, and what this means for angioplasty procedures. We will also consider the results of a long-term study which uses large amounts of data to model the risk of coronary heart disease.

3.1 Intro to philosophy of models in science

Science

Science aims to **understand**, **explain**, **predict** and **influence** phenomena. Understanding science, and thinking in a 'scientific manner', requires:

- *discipline knowledge and content* – the language, information, knowledge and skills specific to a discipline;
- *scientific thinking and logic* – the conceptual process of performing systematic investigations, hypothesising, thinking critically and defensibly, and making valid deductions and inferences;
- *communication and collaboration* – the process of working with others, sharing information and resources;
- *curiosity, creativity and persistence* – the relatively intangible characteristics that include the ability to ask and answer 'interesting' questions, and solve difficult problems in novel ways;
- *observation and data collection* – the processes and techniques used to collect useful data about particular phenomena;
- *modelling and analysis* – the process of developing conceptual representations of phenomena, then using approximation, mathematics, statistics and computation in order to allow predictions to be made.

- Most science is intrinsically quantitative, because quantifying phenomena allows us to measure, describe and compare them efficiently and precisely.
- Science often proceeds by:
  - observing and measuring values, such as the amount, frequency, magnitude, duration or rate of some phenomenon; and
  - Using these data and scientific thinking to answer predictive questions about that phenomenon, such as
    - "What will happen if . . . ?"
    - "How can . . . ?"
    - "What causes . . . ?"
    - "Why does . . . ?"

Models

Models are simplifications and approximations of reality, usually based on measured data, that allow us to understand phenomena, make predictions and evaluate possible impacts of interventions. All models need to strike an appropriate balance between accuracy and complexity.

- Models can be *physical* or *conceptual*. Examples of physical models include building scale models of bridges or dams, and subjecting the model to tests. In SCIE1000, we will focus on conceptual models.
- Ways of developing 'appropriate' models include:
  - using "common sense" and logical deduction;
  - using existing knowledge of similar phenomena; and
  - observing measured data and seeing what they "look like" (many phenomena change according to simple underlying patterns, such as at a constant rate or at a rate proportional to the current value).

- Some common ways of developing and presenting (conceptual) quantitative models are: words (descriptive text); values (for example, weight / height / age tables); pictures (such as graphs or flow diagrams); mathematics (using equations); and computer programs.
- Note that there is nothing "right" or "wrong" about each approach – each is suited to different uses and/or target audiences. See Appendix B for more information about communicating to a specific audience. Most models can be developed and presented in **all** of these ways.
- Scientists carefully examine the input and the output of a system in order to study and develop a deeper understanding of the system. If a system is well understood and there is a desired output, one may then control the input into the system in order to obtain that output (for example, in agriculture). If a system is understood and the input is known, then one may predict the output of that system; these predictions may cause some controversy, and the goal of a scientist is to refine and improve the understanding of systems in order to make better, more accurate predictions.

$$\text{Input} \quad \rightarrow \quad \text{System} \quad \rightarrow \quad \text{Output}$$

- We will refer to this concept as the ISO (Input, System, Output) view of models. Knowing any two of the three provides one with the opportunity to understand the third.
- There are also two types of ways in which models are derived. We will consider both of these in SCIE1000.

Mechanistic Models vs Phenomenological Models

A **mechanistic model** is a model that is derived theoretically by examining the individual physical components of a system.

A **phenomenological model** is a model that is derived empirically by fitting a hypothesized relationship to the observed data.

- When developing a mechanistic model, it is important to ask yourself what are the key factors that could influence the phenomenon in question.

- Phenomenological models are usually determined using statistical software. In SCIE1000, you will not be required to run such software; instead we will discuss the basic features of mathematical functions that can be used to fit various sets of observed data. In this course, *critical thinking* will be valued more than precision.

- Mathematics provides a range of logically valid techniques that allow us to deduce information that we cannot measure or obtain in other ways (due to physical, ethical or financial limitations).

***Question* 3.1.1**

For the following models, seen in earlier lectures, determine whether they were mechanistic or phenomenological models. Explain.

(a) The model for blood alcohol concentration we developed in Lecture 1.

(b) The Widmark formula for blood alcohol concentration we described in Lecture 2.

***Question* 3.1.2**

Suppose a model has been developed phenomenologically with no underlying mechanism to explain it. Should you use the model to make predictions by extrapolating it? Why or why not?

3.2 Mathematics and models

- Some people view mathematics as being an abstract process, unlike disciplines such as biology or chemistry that relate directly to the 'real' world. This view ignores the many links between mathematics and science:
  - Scientists use discipline knowledge and a special language to describe nature and the real world (for example, biologists use taxonomic categories, anatomical descriptions and medical terminology).
  - Mathematicians also use discipline knowledge and a special language to describe nature and the real world (for example, exponential and linear functions all describe relationships between natural phenomena).

Mathematics

Mathematics is a standardised formal language, way of thinking and body of knowledge that allows us to:

- develop models to represent reality;
- increase our understanding of phenomena when we can describe them mathematically;
- perform correct, logical deductions;
- communicate unambiguously; and
- draw conclusions and make predictions.

- Everyone needs to learn some mathematical language and thinking for their personal and professional lives, but to gain proficiency in science requires a higher level of mathematical knowledge and sophistication in its use.
- SCIE1000 aims to develop skills in using mathematics, but we do not study mathematics for its own sake, or develop new mathematical knowledge; if you wish to do that, enrol in discipline-based mathematics courses.
- Instead, we study mathematics **solely** to understand and model the 'real' world. For example, *mathematical functions* are the formal descriptions of patterns in data, *derivatives* describe the change in a value at a point in time, and *integrals* describe behaviour over a period of time.

Question 3.2.1

The following graph shows the measured blood flow rate in coronary artery bypass vein grafts during a single heart beat, after a patient underwent coronary bypass surgery (see [47]).

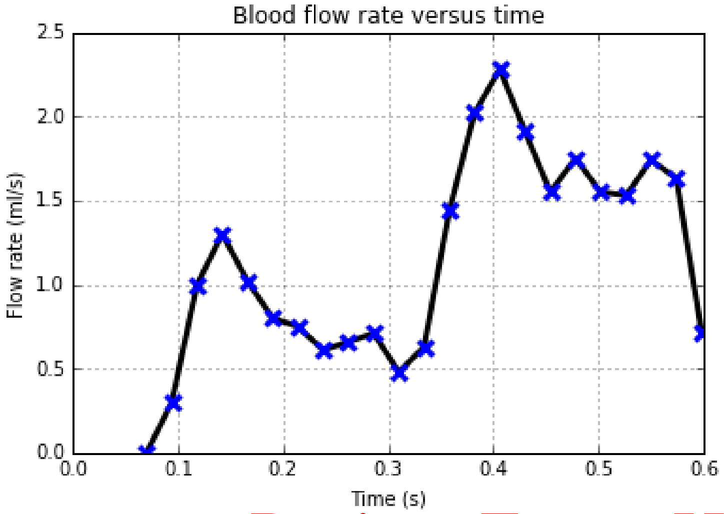


Figure 3.1: Blood flow rate in coronary artery bypass vein grafts, during a single heart beat.

Briefly discuss the physical meaning and significance of each of:

(a) The *height* or *value* of the graph at a given time.

(b) The *slope* or *rate of change* of the graph at a given time.

(c) The *area under the curve* of the graph over a time interval.

(d) Estimate the total blood flow through the vein graft in a heart beat.

Lecture 6: **Studying the heart**

Learning objectives

✓ Develop the ability to theoretically derive models

✓ Analyse the effect that a change in one physical factor has on another

Scientific examples

✓ Blood flow through a blood vessel

✓ Hagen-Poiseuille equation from fluid dynamics

✓ Angioplasty

Maths skills

✓ Develop a plausible equation for a model

✓ Calculate the percentage increase / decrease of a variable

3.3 Modelling in action

Case Study 2: **Let it flow**

- *Fluid dynamics* involves studying liquids and gases that are moving, which is important in many branches of science (particularly geology, physics, mathematics, environmental science and biomedical science) and engineering.

Question *3.3.1*

Develop a model for the flow rate of blood through a given blood vessel. (Hint: which factors are important; do they increase or decrease the rate?)

(continued over)

***Question** 3.3.1* *(continued)*

The following formula (called the *Hagen-Poiseuille equation*) is often used to estimate such flows:

Compare your formula with the Hagen-Poiseuille equation.

- High levels of certain types of cholesterol in the blood can lead to blockages in coronary arteries, which can eventually lead to a heart attack.
- During a heart attack, a lack of blood supply causes heart muscle tissue to die and the dead tissue is replaced with scar tissue.

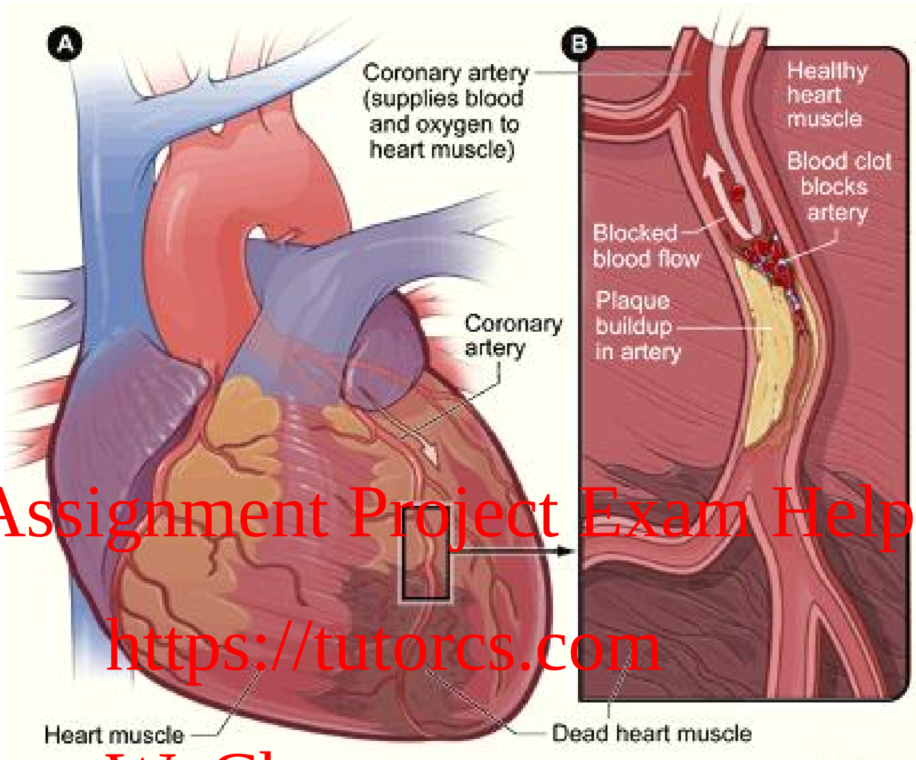


Figure 3.2: Left: heart and coronary artery showing dead heart muscle caused by a heart attack. Right: longitudinal section of a coronary artery with plaque buildup and a blood clot. (Source: www.nhlbi.nih.gov.)

- One surgical method of increasing blood flow through partially blocked arteries is an *angioplasty*.
- In a coronary angioplasty, a cardiologist inserts a balloon-tipped catheter under local anaesthetic, typically through the groin or arm.
- When the catheter is correctly positioned within the coronary artery, the doctor inflates the balloon to expand the blood vessel (and sometimes inserts a metallic stent to maintain the expansion).
- Angioplasties are much simpler and less invasive than coronary artery bypass surgery, but have a higher rate of recurrence of the original occlusion.

Question 3.3.2

Assume that a patient undergoing an angioplasty procedure shows a 30% increase in the diameter of a partially occluded artery. Use the Hagen-Poiseuille equation to calculate the resulting percentage increase in blood flow rate through that artery, and interpret your answer.

End of Case Study 2: Let it flow.

Lecture 7: **What the data says**

Learning objectives

✓ Interpret and communicate the results of a large-scale study

✓ Explain physical factors which may explain the trends seen in data

Scientific examples

✓ Coronary heart disease

✓ Carbon dioxide in the atmosphere (the Keeling curve)

Maths skills

✓ Describe mathematical trends in data

Case Study 3: **To the heart of the matter**

- Diseases of the circulatory system (including heart disease and stroke) are the leading cause of death in many western societies.
- Individuals, doctors and public health bodies all have an obvious interest in predicting the risk of suffering cardiovascular disease.
- In medicine and population health, risks are often specified as a probability of an identified event occurring in a given time period.
- Shortly we will encounter a famous, long-running study into cardiovascular health, called the *Framingham study*[1]. The study defines Coronary Heart Disease (CHD) as including:
  - *angina pectoris*, which is severe chest pain caused by a lack of blood to heart muscle;
  - *myocardial infarction*, commonly called a heart attack, arising from complete loss of blood supply to heart muscle; and
  - death due to cardiac arrest.

[1] All information from the Framingham study has been reproduced with permission from the National Heart, Lung, and Blood Institute as a part of the National Institutes of Health and the U.S. Department of Health and Human Services.

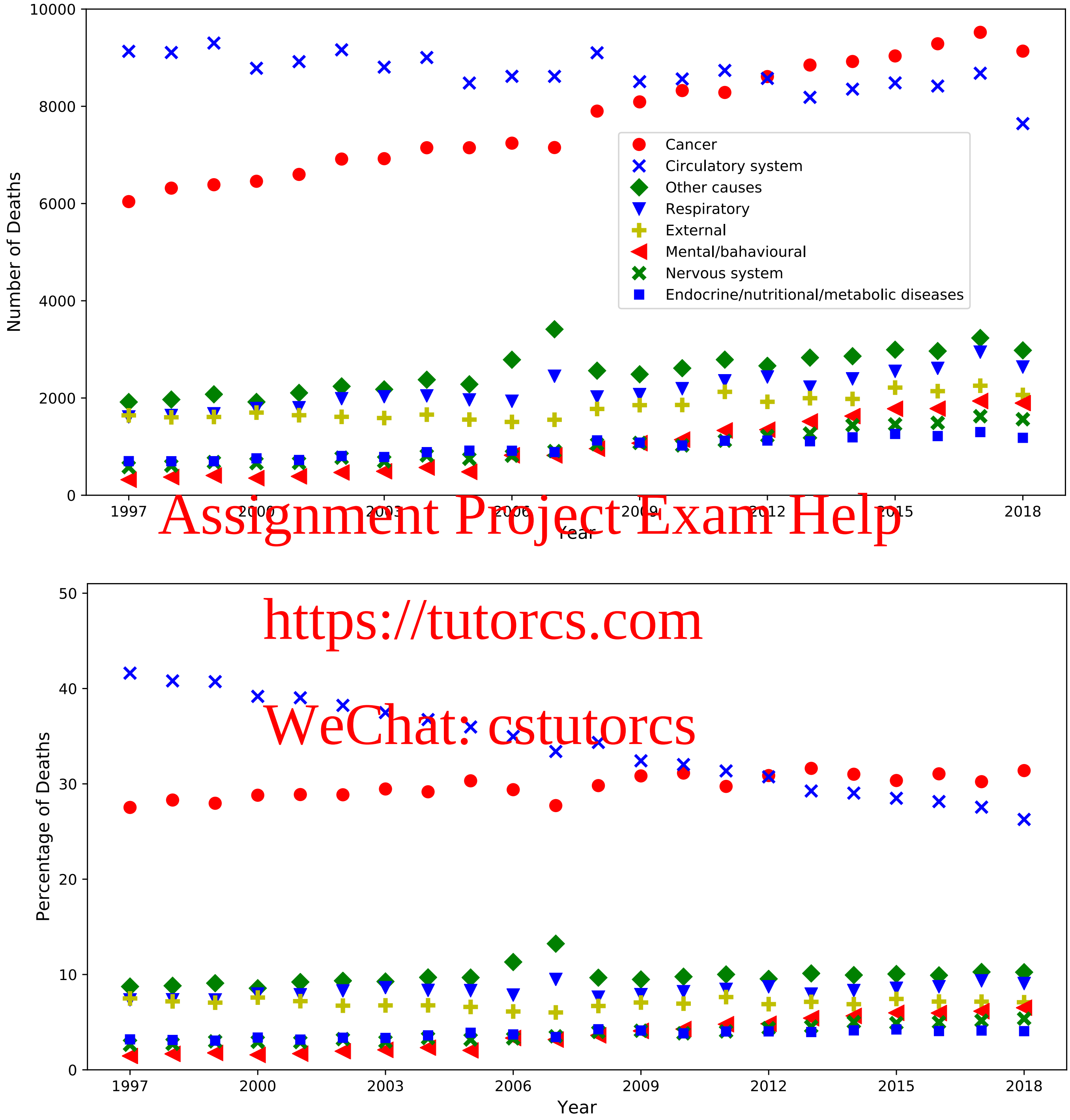


Figure 3.3: Leading causes of death in Queensland. The top graph shows total number of deaths attributed to a given cause, while the bottom graph shows the percentage of all deaths attributed to the given cause. (Data source: Australian Bureau of Statistics.)

- CHD is most often caused by *atherosclerosis*, which is a blockage of a coronary artery supplying blood to heart muscle tissue.

Question 3.3.3

Which factors or data are crucial when developing a model for estimating the likelihood that a person will suffer from CHD in the next 10 years? Does each factor increase or decrease the risk?

What is your "gut feeling" of the likelihood that your lecturer will suffer from CHD in the next 10 years?

- Until comparatively recently, little was known about the general causes of heart disease and stroke, although the rates of cardiovascular disease (CVD) in many societies had been rising for some time.
- In 1948, a study into heart disease commenced in Framingham, Massachusetts, which has become one of the best-known longitudinal health studies.
- The Framingham study (which continues today) has monitored the cardiovascular health of participants, identified a range of risk factors for CHD and included these factors in a mathematical risk model.

- One of the resources produced from the Framingham Study is a CHD Risk Prediction score sheet, used to predict the likelihood that a person will suffer CHD in the next ten years.

Step 1: Age

Age (Years)	Points Female	Points Male
30-34	-9	-1
35-39	-4	0
40-44	0	1
45-49	3	2
50-54	6	3
55-59	7	4
60-64	8	5
65-69	8	6
70-74	8	7

Step 2: LDL cholesterol

LDL (mmol/L)	Points Female	Points Male
≤ 2.59	-2	-3
2.60-3.36	0	0
3.37-4.14	0	0
4.19-4.91	2	1
≥ 4.92	2	2

Key	
Colour	Risk
green	very low
white	low
yellow	moderate
rose	high
red	very high

Step 3: HDL cholesterol

HDL (mmol/L)	Points Female	Points Male
≤ 0.9	5	2
0.91-1.16	2	1
1.17-1.29	1	0
1.30-1.55	0	0
≥ 1.56	-2	-1

Step 4: Blood pressure

(F: Female, M: Male)

Systolic (mm Hg)	Diastolic (mm Hg)				
	< 80	80-84	85-89	90-99	≥ 100
	Points	Points	Points	Points	Points
≤ 120	F: -3 M: 0				
120-129		0			
130-139			F: 0 M: 1		
140-159				2	
≥ 160					3

Note: When systolic and diastolic pressures provide different estimates for point scores, use the higher number.

Step 5: Diabetes

Diabetes	Points Female	Points Male
No	0	0
Yes	4	2

Step 6: Smoker

Smoker	Points Female	Points Male
No	0	0
Yes	2	2

Step 7: Sum points from Steps 1-6

Category	Points
Age	
LDL	
HDL	
Blood pressure	
Diabetes	
Smoker	
Point total	

Step 8: Determine risk from point total

Point total	10 Year CHD risk	
	Female	Male
≤ -3	1%	1%
-2	1%	2%
-1	2%	2%
0	2%	3%
1	2%	4%
2	3%	4%
3	3%	6%
4	4%	7%
5	5%	9%
6	6%	11%
7	7%	14%
8	8%	18%
9	9%	22%
10	11%	27%
11	13%	33%
12	15%	40%
13	17%	47%
14	20%	≥ 56%
15	24%	≥ 56%
16	27%	≥ 56%
≥ 17	≥ 32%	≥ 56%

Step 9: Compare to others of the same age

Age (Years)	Average 10 Yr risk	Low 10 Yr risk
30-34	F: <1% M: 3%	F: <1% M: 2%
35-39	F: 1% M: 5%	F: <1% M: 3%
40-44	F: 2% M: 7%	F: 2% M: 4%
45-49	F: 5% M: 11%	F: 3% M: 4%
50-54	F: 8% M: 14%	F: 5% M: 6%
55-59	F: 12% M: 16%	F: 7% M: 7%
60-64	F: 12% M: 21%	F: 8% M: 9%
65-69	F: 13% M: 25%	F: 8% M: 11%
70-74	F: 14% M: 30%	F: 8% M: 14%

Note: low risk was calculated for an individual of the same age, with normal blood pressure, LDL 2.60-3.36 mmol/L, HDL 1.45 mmol/L, non-smoker and no diabetes.

Figure 3.4: Framingham CHD risk assessment sheet for males and females

Question 3.3.4

Use the Framingham CHD risk assessment sheet in Figure 3.4 to estimate the probability that your lecturer will suffer CHD within 10 years. Compare this with your answers to Question 3.3.3.

Question 3.3.5

Briefly discuss some key points highlighted by the risk prediction sheet. (You may wish to mention such things as the comparative impact of different risk factors, some 'risk factors' commonly mentioned in the media that are not included, and some differences between males and females.)

End of Case Study 3: To the heart of the matter.

Chapter 4: A place with atmosphere

Image 4.1: *Starry Night* (1508 – 1512), Van Gogh (Source: commons.wikimedia.org)

The next three chapters introduce some tools for mathematical modelling, specifically mathematical functions. These include linear, quadratic, power, periodic, logarithmic and exponential functions, and combinations of these. You should have encountered these functions in previous study of mathematics. See Section C.1 in Appendix C for the pre-requisite mathematical tools we will use in this chapter. Use the online modules, available through the course website, for further support. Here our focus is on how these functions are applied in scientific contexts.

We begin the chapter with a motivating example: we consider data on atmospheric carbon dioxide. As a society, we would like to develop a mathematical model in order to get predictions or estimates of what to expect in our future. Remember that SCIE1000 is not a course on climate or climate change, so do not attempt to memorise any climate-related details. Instead, focus on the logical process of model development, and how this relates to other models in different contexts and in different areas of science.

4.1 Fully functional

Case Study 4: **Atmospheric CO_2**

- The broad scientific consensus is that:
  - Earth is undergoing a period of significant climate change;
  - global temperatures are likely to rise over coming years;
  - the warming is related to increasing concentrations of carbon dioxide (CO_2) in the atmosphere; and
  - the increase in atmospheric CO_2 concentration is a result of human activity.

- A famous, long-running study has monitored atmospheric CO_2 concentrations at the Mauna Loa observatory in Hawaii since 1958.
- When these data are plotted, the graph is called the *Keeling curve*, named after the initiator of the study, David Keeling.
- The Scripps Institution of Oceanography (which runs the study) describes the Keeling curve as "…almost certainly the best-known icon illustrating the impact of humanity on the planet as a whole…"
- Gases in the lower atmosphere mix fairly well, so scientists consider the Keeling curve data to be representative of the atmospheric CO_2 concentration world-wide.
- By July 2016, the level of atmospheric CO_2 was consistently above 400 parts per million (ppm). (When SCIE1000 was first offered in 2008, the figure was about 380 ppm.)
- Other data from ice-core samples show that CO_2 levels remained relatively constant at 280 ppm for thousands of years, but the level started increasing in the 19th century.
- Figure 4.1 is a plot of the Keeling curve data, taken from [28, 49].

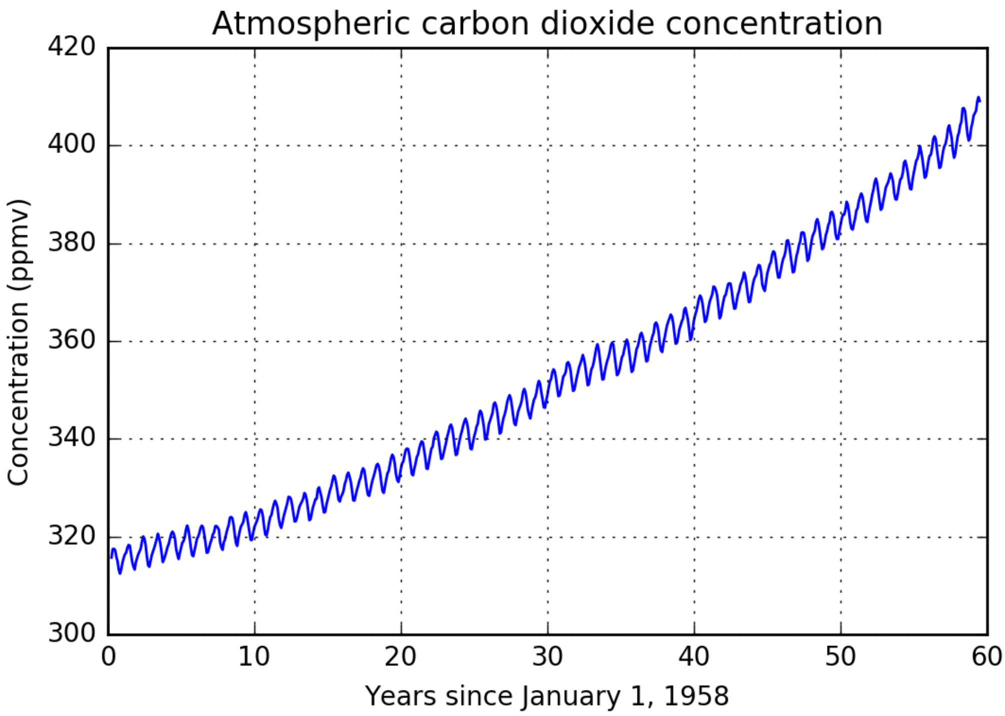


Figure 4.1: The Keeling curve.

Question 4.1.1

(a) Describe the main features of the Keeling curve graph.

(b) What physical factor(s) could cause these features?

(c) How could you mathematically model the Keeling curve?

End of Case Study 4: Atmospheric CO_2.

Lecture 8: **Hotter or colder?**

Learning objectives

✓ Understand what a mathematical function is and how they are used for modelling in science

✓ Develop linear models of real-world phenomenon

Scientific examples

✓ Temperature at different altitudes

Maths skills

✓ Use and interpret linear functions

✓ Find a linear function to fit given data

4.2 Temperature

Question 4.2.1

Although the SI units for measuring temperature are kelvin (K), when communicating to the public, the more commonly used units are degrees Celsius (°C) and degrees Fahrenheit (°F).

(a) What facts do you know about these two measures of temperature? For example, at what temperature (for each of these units of measure) does water freeze? What is a body temperature (for each of these units of measure) associated with a fever?

(continued over)

Question *4.2.1 (continued)*

(b) Let C represent temperature measured in degrees Celsius and let F represent temperature measured in degrees Fahrenheit. Using part (a), develop a linear equation for F as a function of C.

Case Study 5: **Higher than a kite**

Photo 4.1: Jetliner cruising at an altitude of about 10000 m. (Source: PA.)

- Scientists divide Earth's atmosphere into five primary regions: *troposphere*, *stratosphere*, *mesosphere*, *thermosphere* and *exosphere*.
- The *International Standard Atmosphere* (ISA) [27] is a model which further divides the atmosphere from the surface of Earth to the base of the thermosphere into eight layers. (Layer 0 is closest to the surface.)
- The ISA models various properties of each layer, including temperature, pressure and density.
- Layers in the ISA are defined as atmospheric regions in which *temperature is a linear function of altitude.*
- Figures 4.2 and 4.3 show various properties of the ISA temperature at different altitudes. (The *lapse rate* is the rate at which temperature changes as altitude increases.)

Layer	Name	Height at base (km)	Lapse rate (°C/km)	Temperature at base (°C)
0	Troposphere	0.0	−6.5	+15.0
1	Tropopause	11.0	+0	−56.5
2	Stratosphere	20.0	+1.0	−56.5
3	Stratosphere	32.0	+2.8	−44.5
4	Stratopause	47.0	+0	−2.5
5	Mesosphere	51.0	−2.8	−2.5
6	Mesosphere	71.0	−2.0	−58.5
7	Mesopause	84.852	NA	−86.2

Figure 4.2: Some properties of the layers within the International Standard Atmosphere.

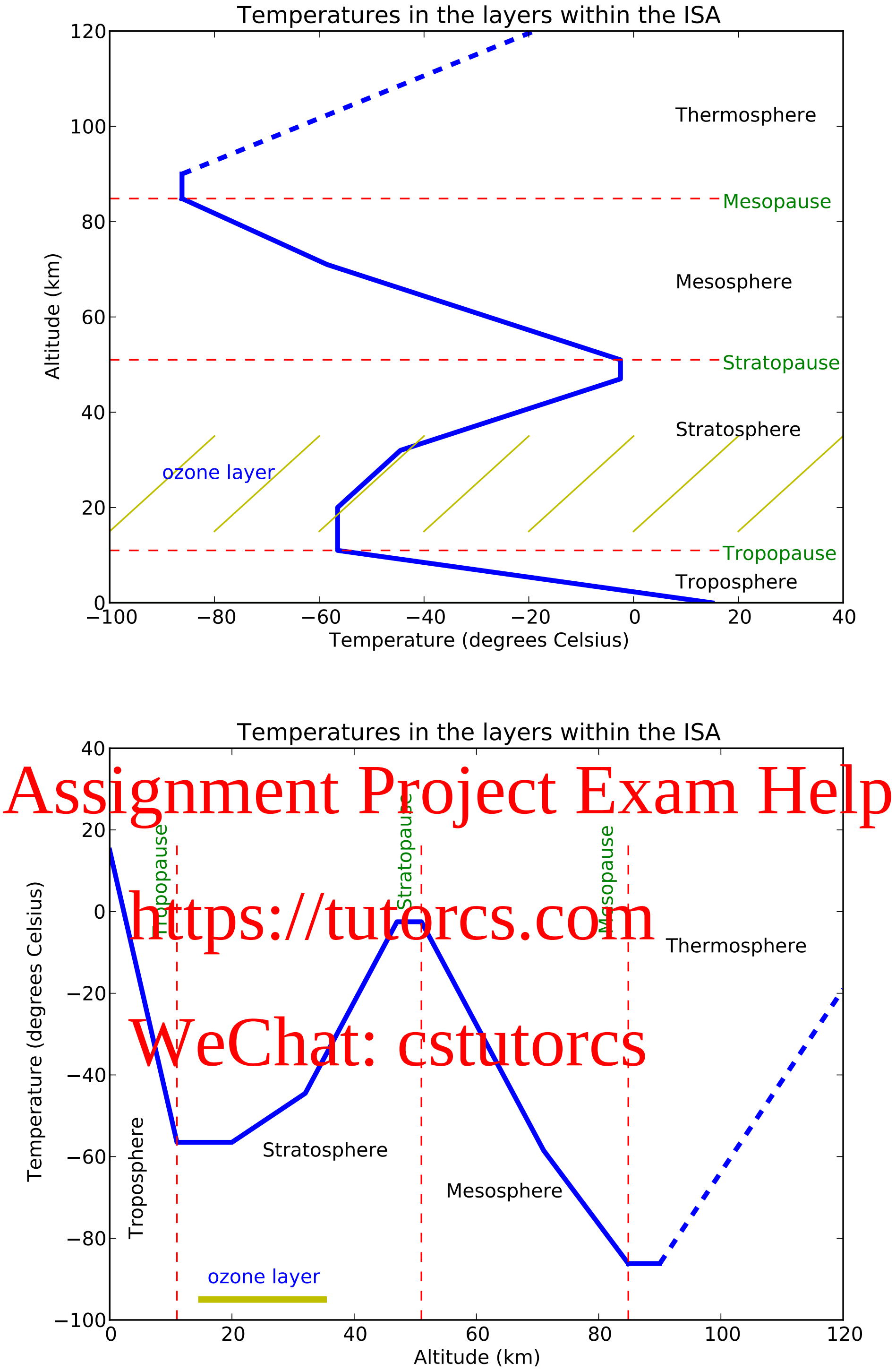


Figure 4.3: The relationships between temperature and altitude modelled by the ISA. The top graph shows altitude versus temperature, and the bottom graph shows temperature versus altitude. (The ISA does not model the thermosphere; temperature data displayed for the thermosphere are from other measurements.)

Question 4.2.2

Use the information about the ISA to answer the following.

(a) Write the troposphere temperature as a function of altitude.

(b) The Matterhorn is a mountain in the Swiss Alps, with a height of 4478 m above sea level. The summit air temperature can range from around 0 °C to −40 °C at different times of the year. Reconcile this with the temperature predicted by the ISA.

Photo 4.2: The Matterhorn – Italian side (Source: PA.)

(continued over)

***Question** 4.2.2* *(continued)*

(c) On an international flight, the following altitudes and external temperatures were reported on the in-flight information screen. The data are graphed in Figure 4.4.

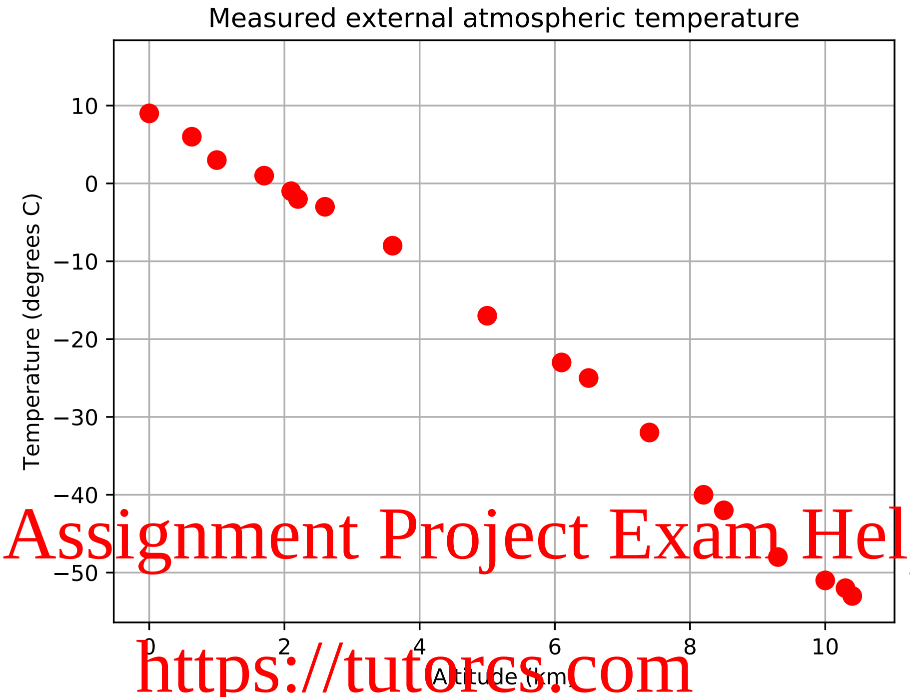


Figure 4.4: Measured external temperatures as a function of altitude.

Plot the function from Part (a) on the above graph and comment on the results.

(d) Write the temperature in ISA Layer 3 as a function of altitude.

End of Case Study 5: Higher than a kite.

Question 4.2.3

Keeling Model 1: Figure 4.5 shows a graph of the Keeling curve.

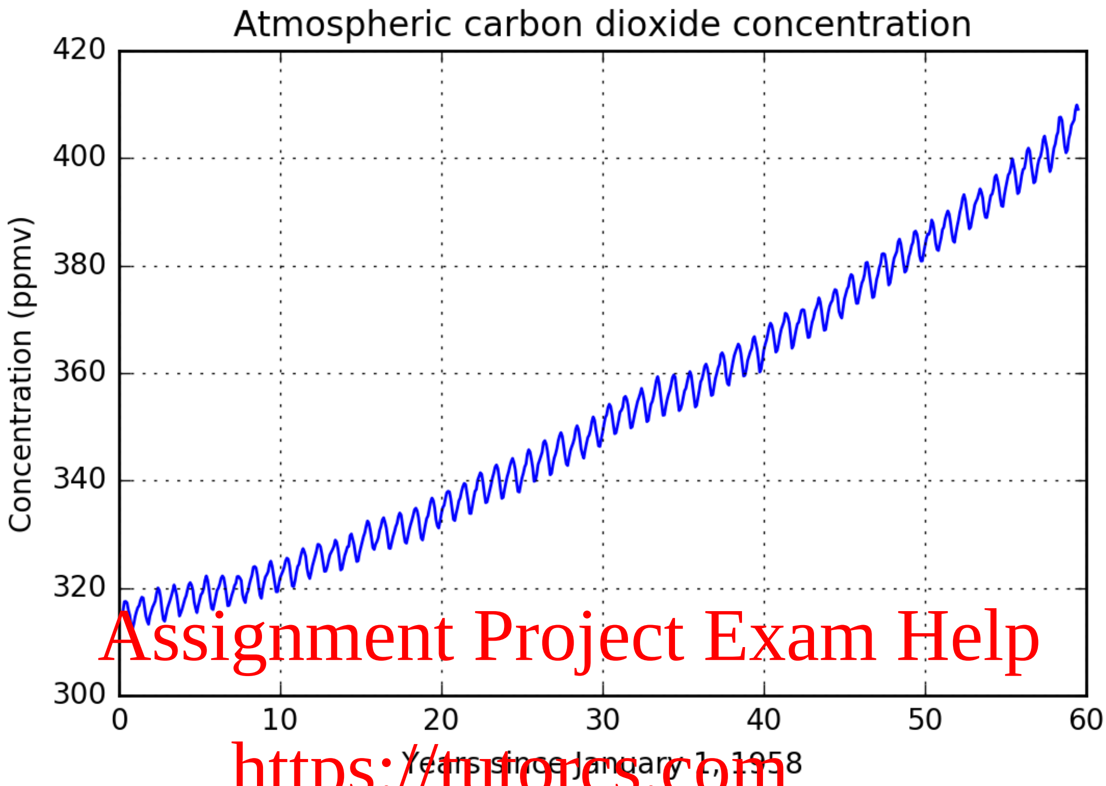


Figure 4.5: The Keeling curve.

(a) Find a rough linear model of the Keeling curve, and plot it on the graph.

(b) Discuss the limitations of your model.

Lecture 9: **A quadratic bird preference**

Learning objectives

✓ Interpret quadratic models of real-world phenomenon

✓ Critically analyse what information is being provided by a model

Scientific examples

✓ Climate change and Bicknell's thrush

Maths skills

✓ Use and interpret quadratic equations

4.3 Bend it!

- Many scientific phenomena relate in ways that are not straight lines.

Case Study 6: **Climate change and Bicknell's thrush**

Image 4.2: Bicknell's thrush, *Catharus bicknelli*. (Source: en.wikipedia.org.)

Image 4.3: Adirondack mountains, USA. (Source: PA.)

Example *4.3.1*

A paper [45] developed models for bird distributions using data from various altitudes, temperatures and locations in the north-eastern USA. The authors then used their models to predict the likely impact of rising temperatures on these distributions. Part of their study focused on Bicknell's thrush.

- The paper [45] built on earlier work that developed qualitative models of temperature and of Bicknell's thrush distributions.

(continued over)

Example 4.3.1 *(continued)*

- The authors combined these models to create a model for thrush distribution with respect to mean July temperatures across the breadth of their habitat.
- For the habitat in consideration, the study region was divided into cells, each 30 m by 30 m square.
- The study found that thrush habitats with July temperatures outside the range of 9.3 °C to 15.6 °C contained insignificant numbers of thrush.

Let T be a temperature within the range 9.3 °C – 15.6 °C. The proportion $p(T)$ of cells containing thrush is closely modelled by the quadratic function:

$$p(T) = -0.0747T^2 + 1.8693T - 10.918.$$

Question 4.3.2

The graph of $p(T) = -0.0747T^2 + 1.8693T - 10.918$ is shown in Figure 4.6.

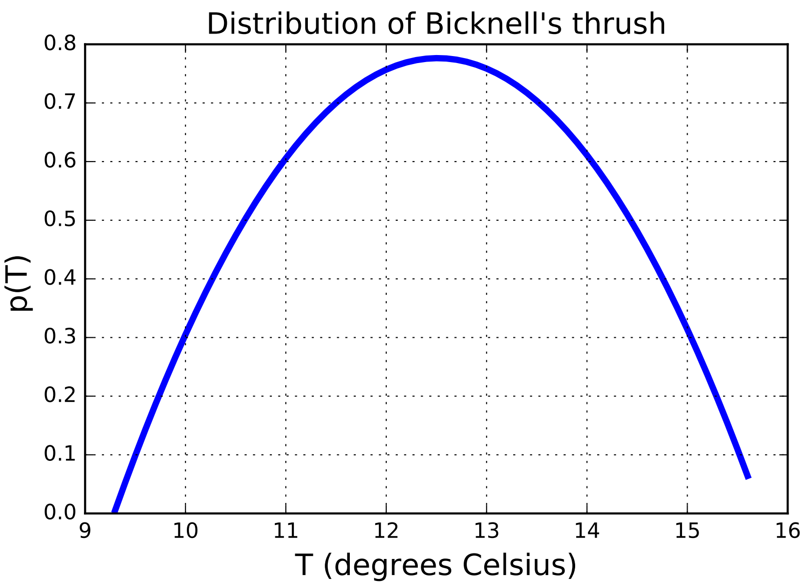


Figure 4.6: Distribution of Bicknell's thrush according to temperature.

(continued over)

Question 4.3.2 *(continued)*

(a) What is the probability that a thrush will be found in a sample area in which $T = 11$ °C?

(b) From the graph, at what (approximate) value of T is the thrush distribution most dense, and what is the (approximate) value of $p(T)$?

(c) There is no value of T for which $p(T) = 1$. Explain what this means in terms of the thrush distribution, and give reasons why it would happen.

(d) Average temperature rises in the region over the next century are predicted to range from 2.8 °C under a low greenhouse gas emission scenario, to 5.9 °C under a high emission scenario.

(i) How would the graph in Figure 4.6 change if the average temperature rose by 2.8 °C? What if it rose by 5.9 °C? Explain your answers.

(ii) Assuming a substantial rise in average July temperatures, what factors may be a concern to resident Bicknell's thrush?

(continued over)

Question 4.3.2 (continued)

Figure 4.7 shows the total area of existing thrush habitat, and the estimated amount of viable habitat available after predicted temperature increases under the low emission scenario and the high emission scenario.

Scenario (°C)	Habitat (hectares)
(current) +0°C	140000
+1°C	32000
+2°C	10000
+3°C	1000
+4°C	200
+5°C	75
+6°C	0

Figure 4.7: Total areas of viable habitat available to Bicknell's thrush under various climate change scenarios.

(e) What is the fraction of suitable habitat that would remain available for the thrush population if temperatures rise by 2.8 °C or 5.9 °C?

(f) If temperature increases occur at the higher end of predictions, what kind of survival strategies might the thrush utilise?

End of Case Study 6: Climate change and Bicknell's thrush.

Question 4.3.3

How can we be sure that the results of the study provide a good guide to the actual behaviour of the ecosystem? What idealisations are made in the model?

On page 538 of their paper, Rodenhouse et al. [45] say:

> *Although the prospect for retaining these species throughout most of the Northeast is not good under any of the projected scenarios, we also cannot predict with any precision when population declines and losses will occur. Predicting the pattern and timing of such losses would require knowing precisely how much temperature will change and where, the effects of site-specific factors such as slope and aspect, how disturbance regimes within spruce-fir forest may change, and last, the effects of climate change on montane food webs. We simply do not yet know enough about the ecology of montane biological communities to make more than qualitative projections of change for most species of this habitat type.*

They explain that:

- they don't know how fast new types of trees will replace old
- they don't know how different species will adapt to new habitats
- they can't factor in 'dramatic and unexpected' events

Does this mean that their model of the effects of climate change on birds in Northeastern USA is wrong?

Lecture 10: **Biodiversity**

Learning objectives

✓ Interpret power function models of real-world phenomenon

Scientific examples

✓ Biodiversity and species richness

Maths skills

✓ Understand the shape of power functions with powers between 0 and 1

✓ Solve power functions

4.4 (Super) powers

- Recall that linear and quadratic functions are examples of the more general group of *power functions*. Functions with different powers have graphs with different shapes, and hence can model different phenomena.

Question *4.4.1*

Lamington National Park in Queensland consists of about 20,000 hectares of rainforest. If you wanted to estimate the biodiversity in the park (the total number of different species found there), how might you do this?

Case Study 7: **Species-area curves and biodiversity**

Photo 4.3: Counting species in the field. (Source: DM.)

- Previously we discussed the abundance and distribution of a *single* species, *Bicknell's thrush*. Ecologists often study the *overall number of species* found in a region (sometimes called the *biodiversity* or *species richness*).

Species-area curves

In ecology, a *species-area curve* is a graph showing the number of distinct species observed, as a function of the size of the area surveyed.

Photo 4.4: Scribbly gum (*Eucalyptus racemosa*). Right: Scaly-breasted Lorikeet (*Trichoglossus chlorolepidotus*). (Source: PA.)

- Rather than performing a full count for an entire region, data from a smaller area can be extrapolated to estimate the regional species richness.

Example 4.4.2

Consider a 4 hectare property in eastern Brisbane. We wish to estimate the number of distinct, naturally occurring, native plant species (individuals greater than 2m in height), that occur on this land. Suppose that 30 cells (or *quadrats*), each 10m by 10m, are selected at random and for each cell we record the occurrence of new species not seen in previous cells. Figure 4.8 shows information on the previously unseen species, including the cumulative total C of species observed so far.

Cell(s)	New species observed	C
1	*Eucalyptus racemosa, Acacia fimbriata, Banksia integrifolia*	3
2	*Eucalyptus tereticornis, Alphitonia excelsa*	5
3	*Acacia disparrima*	6
4	*Acacia leiocalyx, Lophostemon suaveolens*	8
5	–	8
6	*Glochidion sumatranum*	9
7	–	9
8	–	9
9	*Eucalyptus crebra*	10
10	–	10
11 – 15	*Banksia robur, Melaleuca quinquinerva*	12
16 – 20	–	12
21 – 30	*Allocasuarina littoralis, Angophora leiocarpa*	14

Figure 4.8: Information on additional observed species.

Figure 4.9 is a species-area curve summarising the data in Figure 4.8:

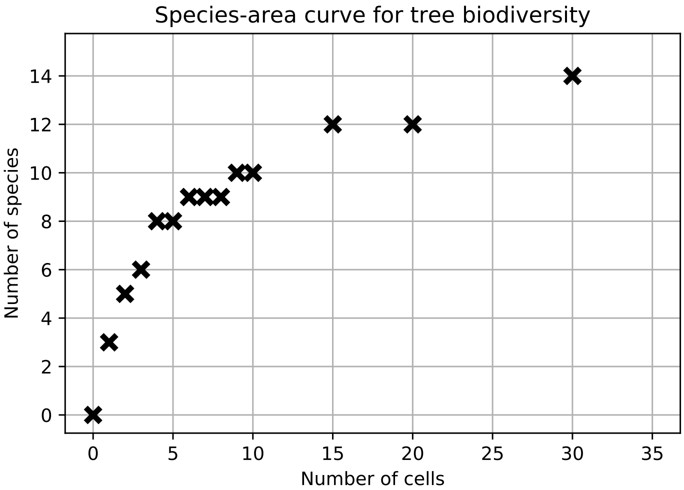


Figure 4.9: The number of distinct tree species recorded.

- The graph has a shape that is typical of many species-area curves: the number of distinct species initially rises rapidly as the area increases, but then rises less rapidly as the area becomes larger.

Equations for species-area curves

Species-area curves can be mathematically modelled using power functions, with power p between 0 and 1 (typically, p is between 0.2 and 0.5).

Their general form is $S(a) = Ma^p$, where S is the number of species occurring as a function of the area a, and M and p are constants depending on the geographical location, resource availability and similar factors.

***Question** 4.4.3*

With respect to a species-area curve $S = Ma^p$ (with p between 0 and 1):

(a) Discuss why species-area curves exhibit the general shape of this type of power function.

(b) How might a species-area curve model impact on field sampling techniques?

(continued over)

Question *4.4.3 (continued)*

(c) How do the values of M and p impact on the shape of the graph?

(d) What physical factors could affect the shape of the curve?

Example 4.4.4

Figure 4.10 shows the graph of $S(a) = 5a^{0.3}$ and the species data from Figure 4.8, where a is the number of 10m by 10m cells.

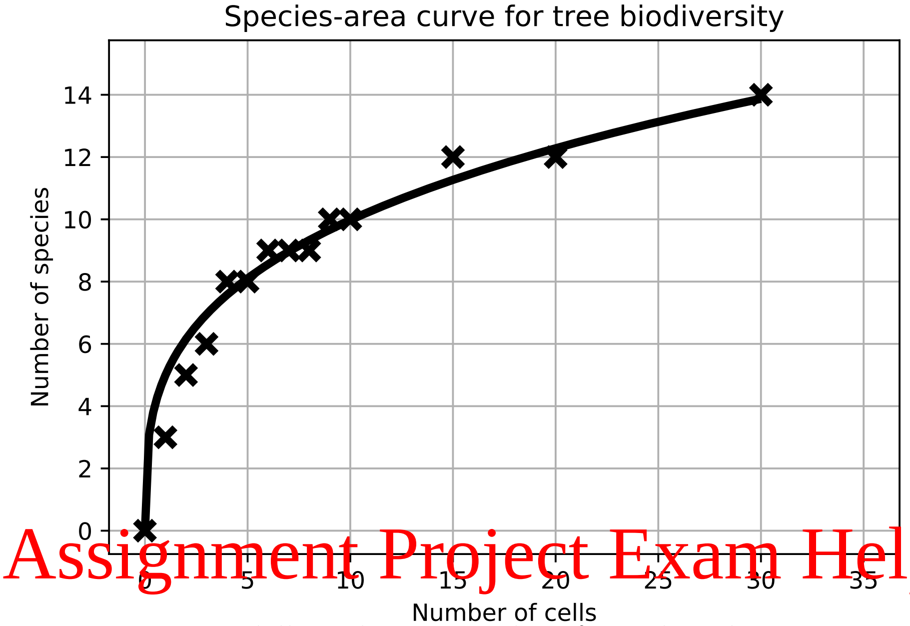


Figure 4.10: Modelling the species data from the 4 hectare property.

Question 4.4.5

Assume that this question refers to native, naturally occurring plants more than 2m high, growing on land ecologically similar to the 4 hectares of land in the previous example (that is, the model shown in Figure 4.10 is appropriate).

(a) Estimate the species richness (total number of species) on the 4 hectare (40000 m^2) property.

(continued over)

Question *4.4.5 (continued)*

(b) A typical conservation goal is to establish parks that preserve 10% of the representative land area. What fraction of species richness would be represented within such a park in the area near the 4 hectare property?

(c) Many people believe that the figure in Part (b) is too low. If the goal is to retain 75% of species, what proportion of land should be preserved?

Question 4.4.6

The paper [42] uses species-area curves to predict the reduction in species richness of vertebrate species in Mexican cloud forests. In the table, A_0 is the current area of cloud forest in two regions, A_1 is the predicted area in 2080 after climate change, and A_2 is the predicted area after climate change and forest clearing (here areas are measured in km^2). The corresponding numbers of endemic vertebrate species are S_0 (current), S_1 and S_2.

Region	S_0	A_0	S_1	A_1	S_2	A_2
Oaxaca	26	5160		2326	9	65
Chiapas	3	6037	2	797	1	45

(a) The species-area curve in Oaxaca is found to follow a power function with $p = 0.25$. Calculate the value of S_1 for Oaxaca.

(b) Which of Oaxaca or Chiapas would you suggest as the location for a new national park? Why? What other factors might influence your advice?

End of Case Study 7: Species-area curves and biodiversity.

Lecture 11: **Studying sunlight**

Learning objectives

✓ Interpret power function models of real-world phenomenon

✓ Evaluate the effectiveness of various models for the same data

Scientific examples

✓ UV light and SPF

✓ Carbon dioxide in the atmosphere

Maths skills

✓ Understand the properties and shapes of various types of power functions

✓ Understand transformations of power functions and their graphs

Case Study 8: **Ban the tan, man**

- Earlier, we saw that temperature in the lowest atmospheric layer (Troposphere) decreases as altitude increases, but temperature in the next layer (Stratosphere) increases from altitude 20 km to 50 km.
- The rise in temperature is due to interactions between the ozone layer and ultraviolet (UV) light.
- UV light is electromagnetic radiation with wavelengths shorter than visible light, and can be divided into: UVA (wavelength 315 – 400 nm); UVB (wavelength 280 – 315 nm); and UVC (wavelength 100 – 280 nm).
- The following sequence of chemical reactions occurs in the ozone layer:

$$\begin{array}{rcll}
O_2\ (g) & \xrightarrow{\text{UVC light}} & 2O\ (g) & \text{(exothermic: light energy} \longrightarrow \text{heat)} \\
O_2\ (g) + O\ (g) & \longrightarrow & O_3\ (g) & \\
O_3\ (g) & \xrightarrow{\text{UVB light}} & O_2\ (g) + O\ (g) & \text{(exothermic: light energy} \longrightarrow \text{heat)}
\end{array}$$

- Collectively, these reactions are called the *ozone-oxygen* cycle:
  - The net result of the first two reactions is that UVC light energy is converted to heat, and oxygen is converted into ozone, O_3.

- In the third reaction, UVB light is absorbed; this reaction is also exothermic, again converting light energy into heat.

- This cycle is extremely important to life on Earth:
  - UVC light is very damaging to life, but is completely absorbed.
  - Most UVB light is also absorbed; only around 1 part in 350 million reaches the surface of Earth. Exposure to this light causes sunburn, eye cataracts, visible ageing, genetic mutations in cells and skin cancer.
  - Almost all UVA light reaches the surface of Earth.

- The effectiveness of sunscreens at preventing UVB light from reaching the skin is measured by their *Sun Protection Factor*, SPF. When a product with SPF n is correctly applied to the skin, it blocks a fraction of $(n-1)/n$ of the usual amount of UVB light.

Question *4.4.7*

Write a function for the proportion of UVB light that is **not** blocked by sunscreen with SPF n, and draw a rough sketch of the graph of the function.

(continued over)

***Question** 4.4.7 (continued)*

What proportion of UVB light is not blocked by sunscreen with SPF 30? SPF 50? SPF 100?

Should people buy sunscreen with SPF 100?

- There has been a substantial depletion of total atmospheric ozone in recent decades, including formation of the *ozone hole* over Antarctica.
- It is well-accepted that this has anthropogenic causes, particularly the release of ozone depleting substances such as *chlorofluorocarbons* into the atmosphere. These had been used as refrigerants and aerosol propellants.
- The *Montreal Protocol*, adopted in 1989 and ratified by around 200 states, is an agreement on phasing out the use of CFCs. It represents one of the most significant international climate agreements ever.

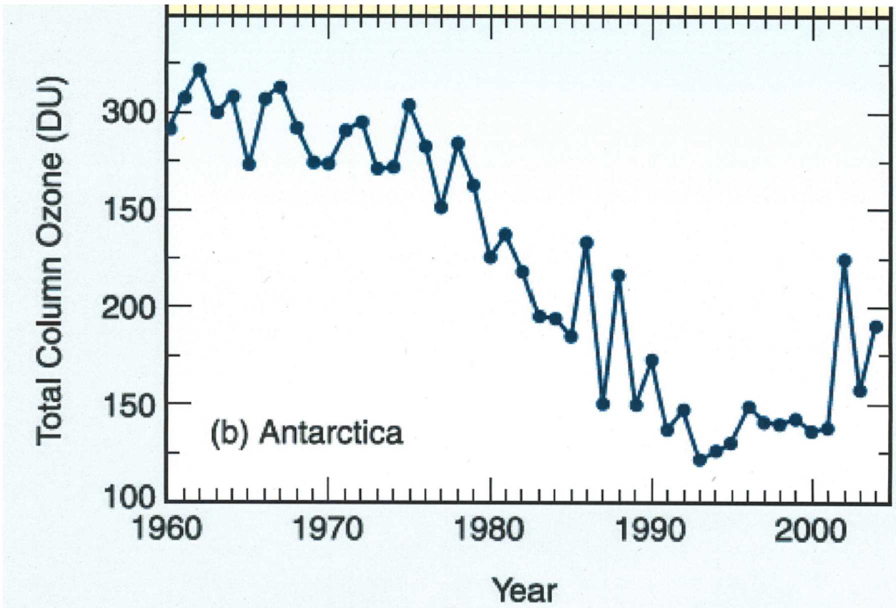


Figure 4.11: Ozone levels from 1960 to 2004 (source: IPCC (2005))

- There is evidence that the ozone layer has started repairing. Scientific modelling predictions made in 2005 suggested the ozone layer would return to its 1980 values by the year 2050 (see Figure 4.12).

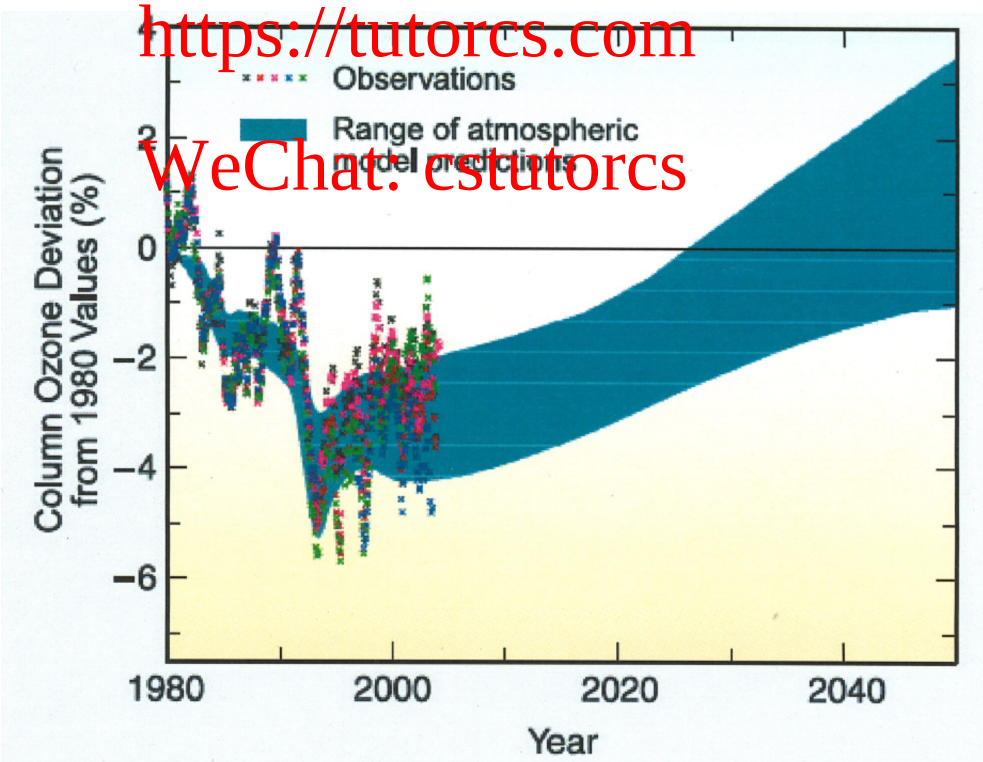


Figure 4.12: Ozone levels predictions (source: IPCC (2005))

- More recent evidence suggests the ozone layer will recover by the year 2075.

Question 4.4.8

In [38], fourteen authors asked the question *What would have happened to the ozone layer if chlorofluorocarbons (CFCs) had not been regulated?*. With some sophisticated modelling, they developed simulations to predict what would have happened if CFC's had not been regulated, and called this situation the "world avoided" scenario (see Figure 4.13). Note: The *Dobson unit* is often used to measure the amount of a gas in a vertical column of Earth's atmosphere, and 1 DU $= 0.4462$ mmol/m^2.

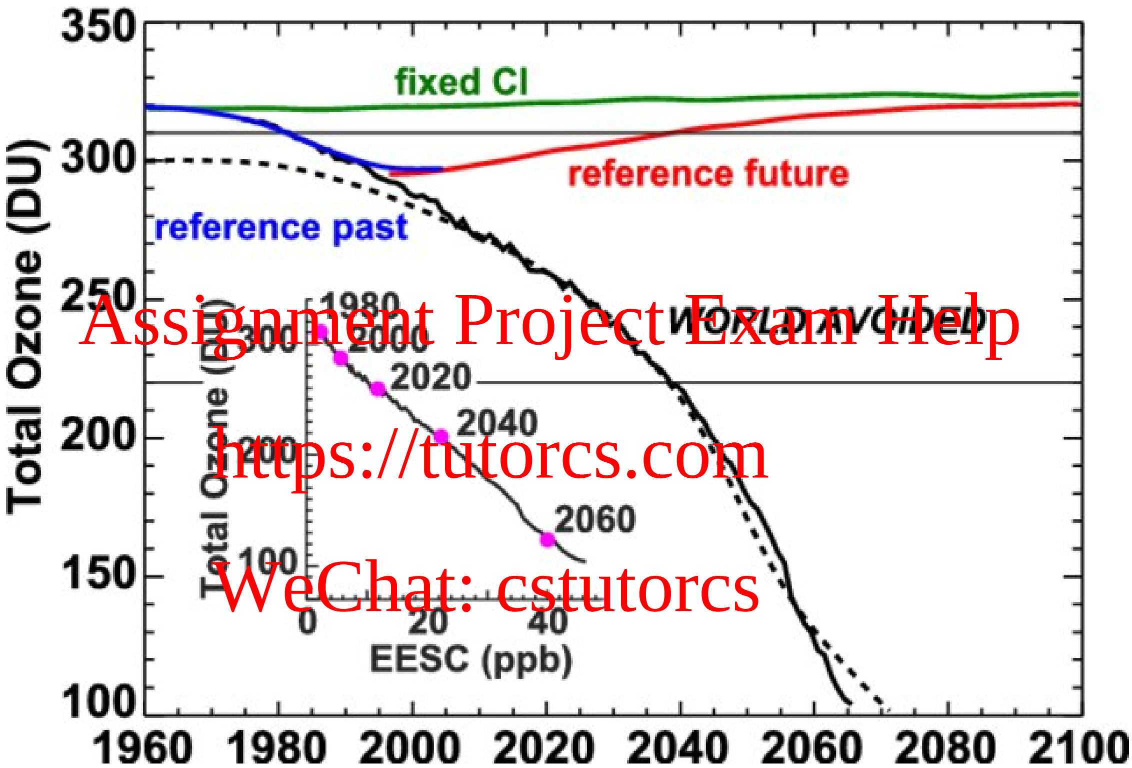


Figure 4.13: Annual average global ozone for the WORLD AVOIDED (solid black), reference future (red), fixed chlorine (green), and reference past (blue) simulations.

Consider the predictions for the year 2040 and comment on the relative percentage drop in ozone we would have faced if interventions had not been implemented.

End of Case Study 8: Ban the tan, man.

Question 4.4.9

Keeling Model 2: Figure 4.14 shows two plots: a graph of the function $y(t) = 1/3 \times t^{1.37} + 315$, and the Keeling curve.

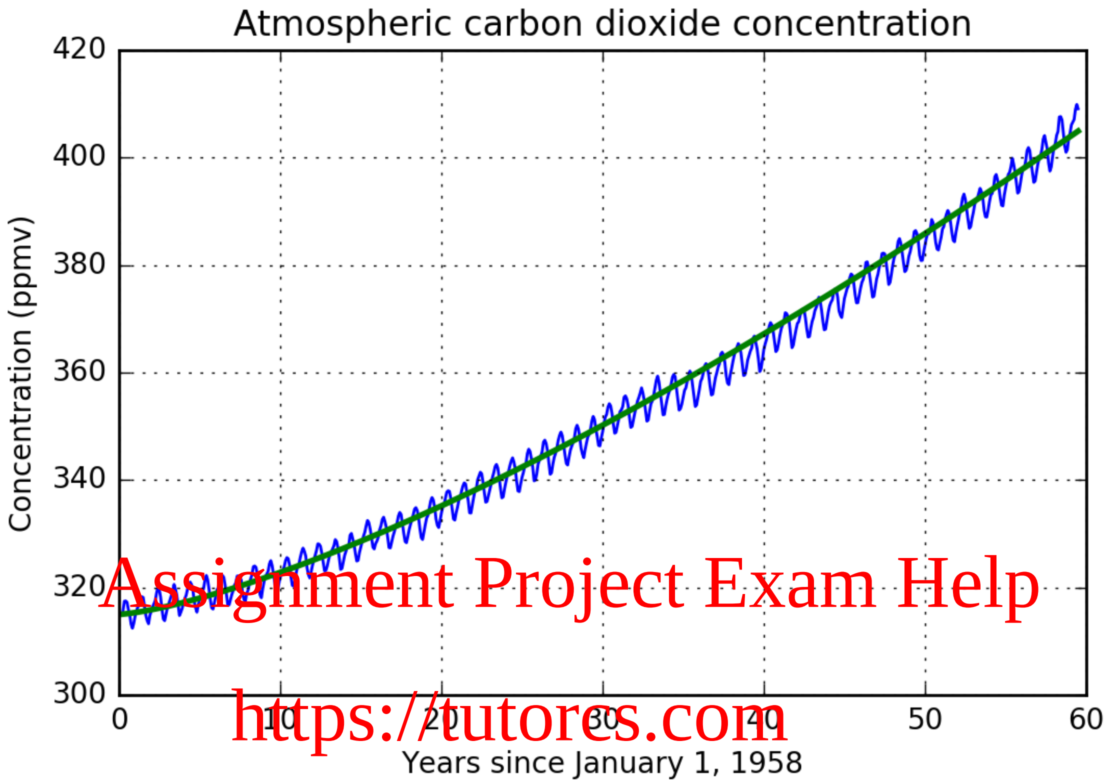


Figure 4.14: The Keeling curve and a power function model.

(a) Explain how each term in $y(t)$ impacts on the graph.

(b) Discuss the limitations of this model of the Keeling curve.

Chapter 5: Give us a wave!

Lecture 12: Breathing with sine

Learning objectives

✓ Interpret sine function models of real-world phenomenon

Scientific examples

✓ Breathing and lung capacity

Maths skills

✓ Understand sine functions and their transformations

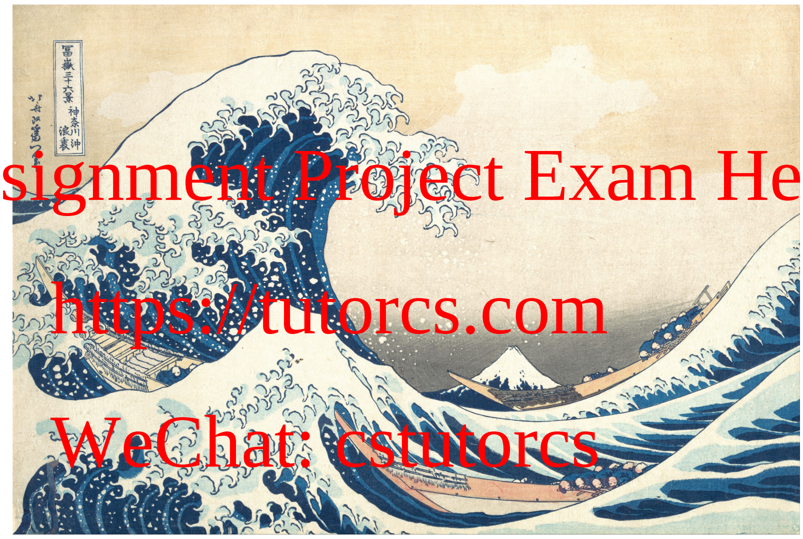


Image 5.1: *The Great Wave off Kanagawa* (1829–1833), Katsushika Hokusai. Print at the Metropolitan Museum of Art (Source: en.wikipedia.org)

Many phenomena in Science and nature *repeat* or *cycle*. These include: many aspects of weather and climate; ocean waves and tides; physiological processes, such as breathing and hormone levels; sound waves; and the voltages and currents in alternating current electricity.

In this chapter, we will discuss how to model cyclic or periodic phenomenon using a sine function. You should have encountered sine functions in previous study of mathematics. See Section C.2 in Appendix C for the pre-requisite mathematical tools we will use in this chapter. Use the online modules, available through the course website, for further support.

5.1 Waves, cycles and periodic functions

- Consider the four graphs in Figure 5.1, each of which shows climate-related data for Brisbane over a period of one year. If the graphs were extended over subsequent years, then an approximate cycling pattern would be observed.

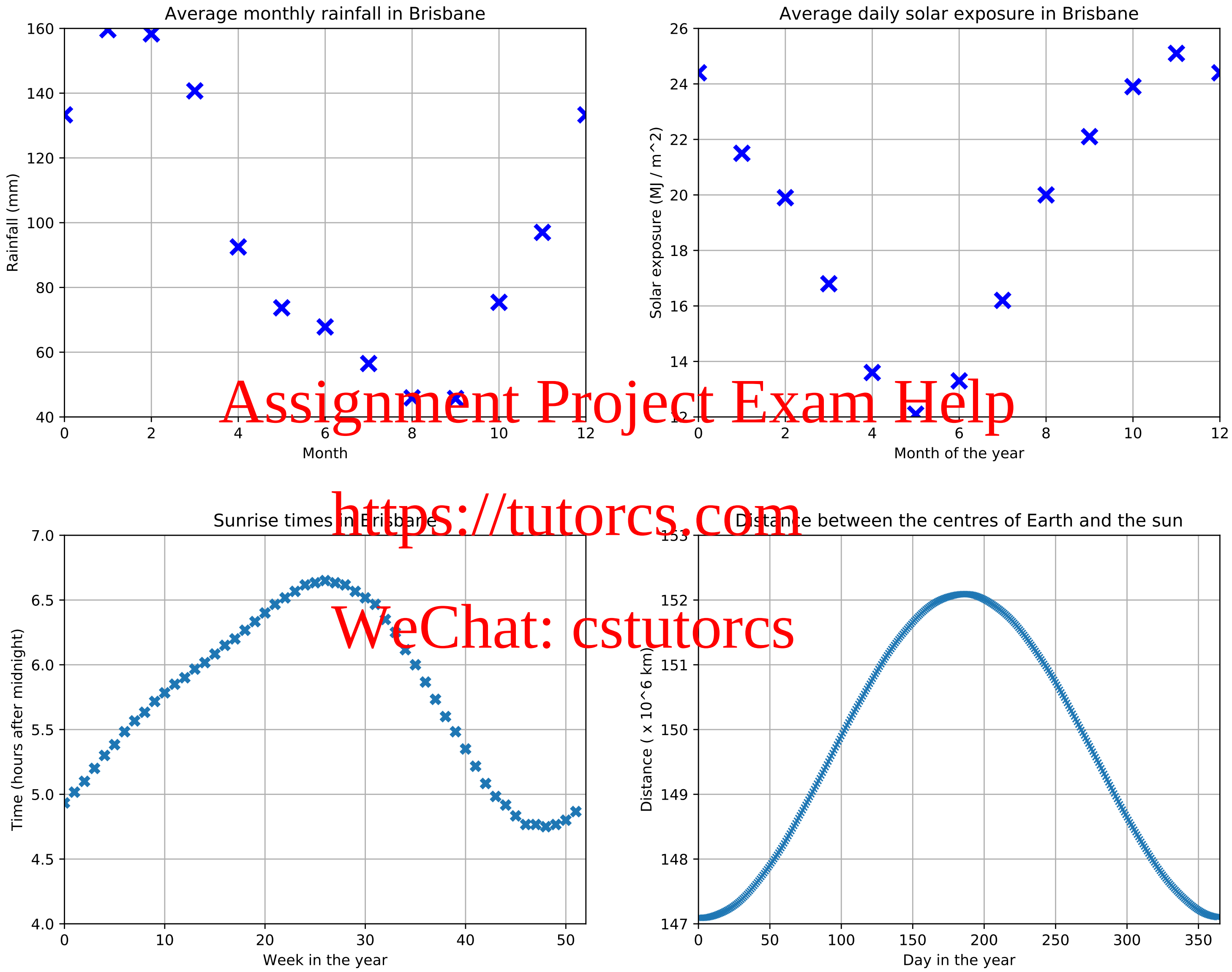
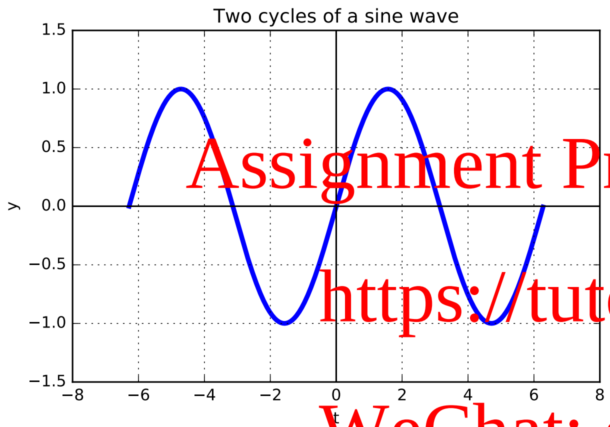


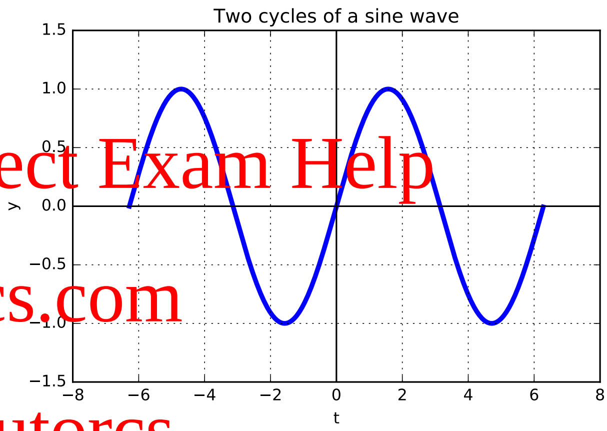
Figure 5.1: Four climate-related graphs. Top left: average monthly rainfall in Brisbane. Top right: average daily solar exposure in Brisbane. Bottom left: weekly sunrise times in Brisbane. Bottom right: daily distances between the centres of Earth and the sun.

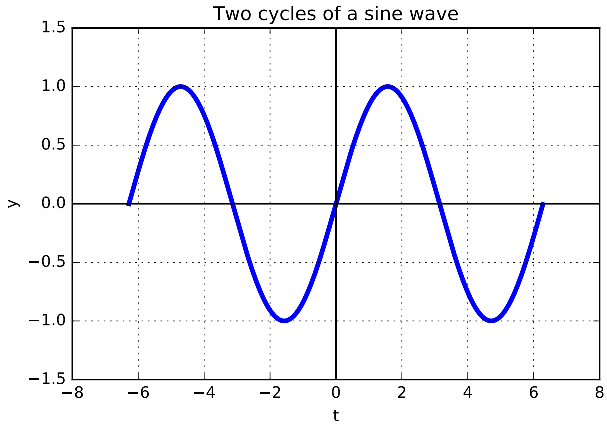
Question 5.1.1

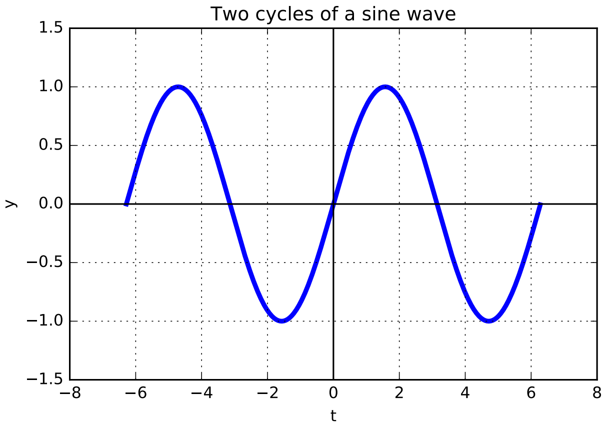
Consider the following four transformed sine functions. Draw a graph of each of these functions on the axes provided below. Each graph below already shows a graph of $y = \sin t$ for two full cycles (from $t = -2\pi$ to $t = 2\pi$).

(a) $y = \sin(t) + 0.5$	**(b)** $y = 1.5\sin(t)$
(c) $y = \sin(t - \frac{\pi}{2})$	**(d)** $y = \sin(\frac{2\pi}{5}t)$









It may be helpful to substitute some values in for t. If using a calculator, *make sure that your calculator is set to radians NOT degrees.*

Question 5.1.2

The general equation of a sine function is

$$y(t) = A \sin\left(\frac{2\pi}{P}(t - S)\right) + E.$$

Explain the *mathematical* meaning and impact of each of the constants A, P, S and E.

Case Study 9: **Heavy breathing**

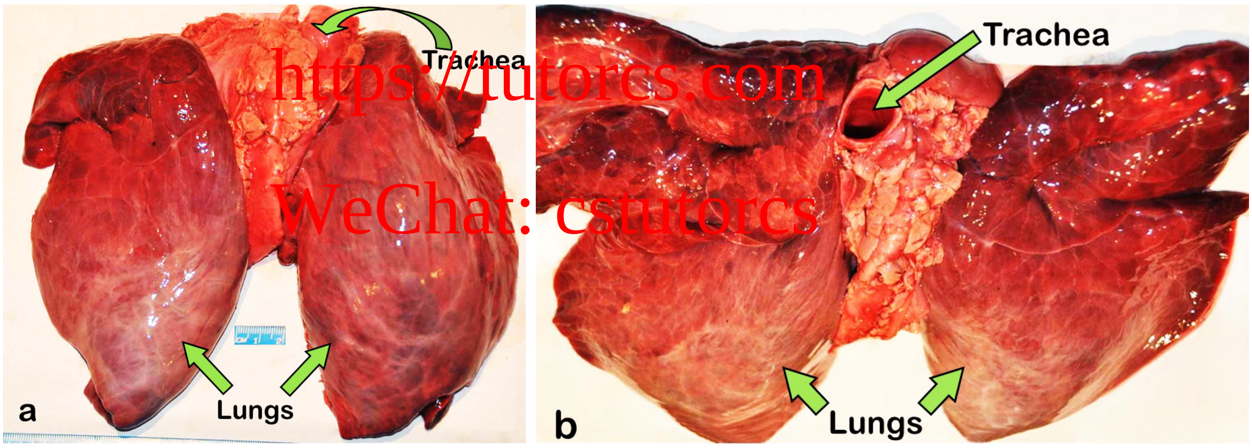


Photo 5.1: Calf lungs, (a) from the front, and (b) from the top. (Source: PA.)

- The volume and rate of air movement into and out of the lungs can be measured using a *spirometer* and graphed in a *spirogram*. (One common type of spirometer uses the Hagen-Poiseuille equation to measure air flow rates.) This information can be used to diagnose possible respiratory impairment.

- The maximum volume of air which can be expelled from the lungs in a single exhalation is called the **vital capacity**. After each exhalation the lung retains a volume of air, called the **functional residual capacity**.

- Normal breathing involves rhythmic inhalation and exhalation of air. The **tidal volume** is the difference in the volume of air in the lungs between a full inhale and a full exhale while breathing normally.

Question *5.1.3*

(a) Estimate the tidal volume, period between breaths, functional residual capacity and vital capacity for a resting adult human. Sketch a rough graph of the volume of air in the lung over time.

(b) Write a function using sine to model the volume of air in the lungs during normal breathing over time, based on Part (a).

(continued over)

***Question* 5.1.3** *(continued)*

(c) How would the function change after moderate physical activity?

(d) Suppose a patient is breathing very rapidly, with deep inhalations and exhalations. How would the original function change to model the breathing of this patient?

(e) Smoking and air pollution causes inflammation in the lungs, gradually destroying the lung tissue and leading to *emphysema*, a type of Chronic Obstructive Pulmonary Disease (COPD). The reduction of lung surface area decreases the ability to exchange carbon dioxide and oxygen.

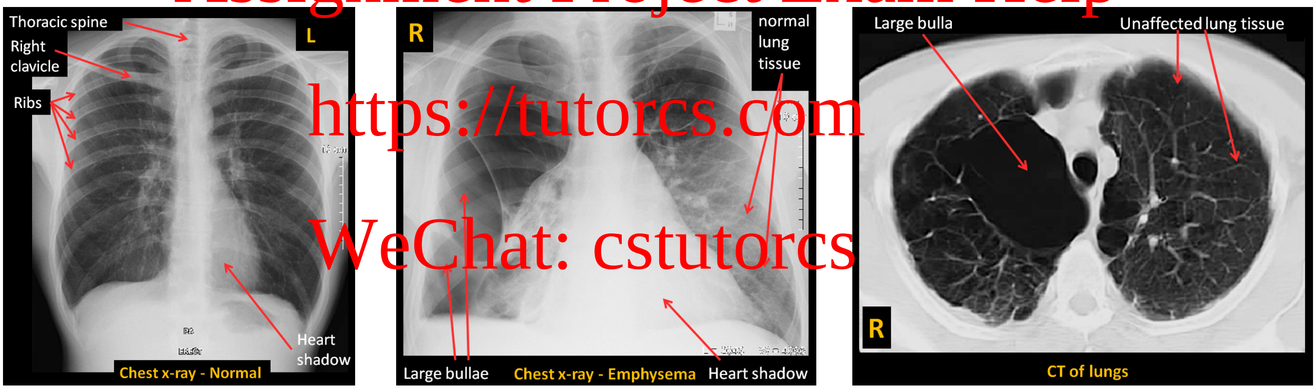


Photo 5.2: Left: x-ray of an adult male chest displaying normal lung tissue architecture and normal heart shadow. Middle: x-ray of a chest showing large emphysematous bullae within the right lung. Right: Axial CT showing one large, and multiple small, bullae of the alveolar air spaces in the right lung.(Source: Qld Health and DM.)

How would the function change for an individual with emphysema?

End of Case Study 9: Heavy breathing.

Lecture 13: **Sine and seasons**

Learning objectives

✓ Interpret sine function models of real-world phenomenon

Scientific examples

✓ Seasons on Earth

✓ Daylight hours

Maths skills

✓ Understand and interpret sine functions and their graphs

5.2 Days, seasons, cycles

Photo 5.3: Spring – Lotus flower, *Nelumbo nucifera* (Tokyo, Japan); Summer – Monument Valley (Utah, USA); Autumn – sugar maple, *Acer saccharum* (Vermont, USA); Winter – Pine tree (Canyonlands, Utah, USA). (Source: PA.)

- The amount of sunlight available at a location on Earth on a given day can be modelled using *daytime*, defined as the time between sunrise and sunset. (This is independent of clouds or weather events.)

- Daytime lengths vary through the year. The **summer solstice** and **winter solstice** are the days with the longest and shortest daytimes (respectively). The **vernal equinox** and **autumnal equinox**, are the days in spring and autumn (respectively) with daytimes of exactly 12 hours.

Question *5.2.1*

What causes seasons?

Question 5.2.2

Discuss the daytime lengths in midsummer and midwinter in each of:

(a) Brisbane;

<u>midsummer:</u>

<u>midwinter:</u>

(b) Singapore (which is very close to the equator); and

<u>midsummer:</u>

<u>midwinter:</u>

(c) Santa Claus village, Rovaniemi, Finland (north of the Arctic Circle).

<u>midsummer:</u>

<u>midwinter:</u>

Photo 5.4: Top left: road sign to Santa (Rovaniemi, Finland). Right: the official home of Santa (Santa Claus Village, Finland). Bottom left: Singapore. (Source: PA.)

- At large distances from the equator, summer daytimes are very long; on some occasions there is no sunrise or sunset for a period greater than one day. For simplicity, in such cases we say that the daytime is 24 hours.
- Similarly, in midwinter we say that the daytime is 0 hours.
- Figure 5.2 shows the daytimes in Brisbane at weekly intervals throughout a calendar year.[1] The graph of daytime lengths in every year will be very similar; clearly, the graph resembles a sine wave!

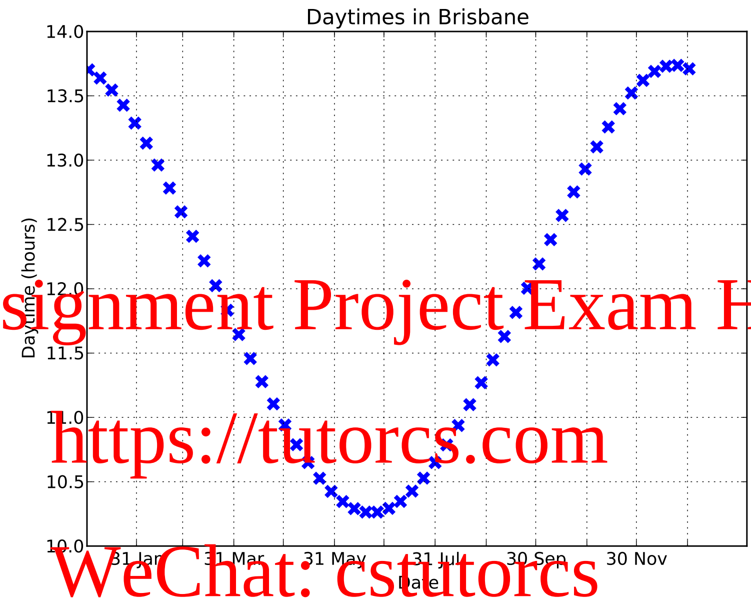


Figure 5.2: Daytimes in Brisbane during the year.

***Question** 5.2.3*

Use the graph in Figure 5.2 to answer the following questions.

(a) When are the solstices in Brisbane, and how long are the daytimes?

(b) When are the equinoxes in Brisbane?

[1] Daytimes were found by subtracting the sunrise time from the sunset time. Sunrise time is defined as the time at which any part of the sun is first visible on a clear, cloudless day. Sunset time is defined as the time at which any part of the sun is first **not** visible on a clear, cloudless day. This definition of sunset differs slightly from standard usage.

Question 5.2.4

On some international flights, in-flight maps show areas of night and day superimposed on the surface of Earth; see Figure 5.5 for two examples.

Photo 5.5: In-flight maps. (Source: PA.)

(a) Describe everything you can about the date and time of day in Brisbane when the first photograph was taken. Justify your answer.

(b) Describe everything you can about the date and time of day in Brisbane when the second photograph was taken. Justify your answer.

Case Study 10: **Modelling daytimes**

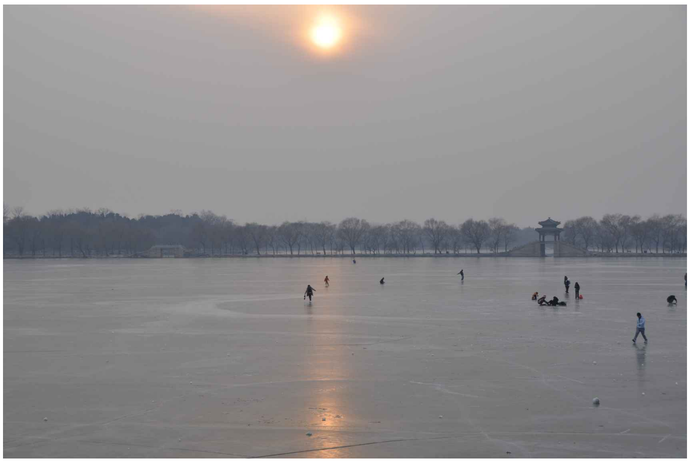


Photo 5.6: Sunrise over Kunming Lake in winter, Beijing, China. (Source: PA.)

Photo 5.7: Sunset over prison guard tower, near Krakow, Poland. (Source: PA.)

- Every location on Earth has a *latitude*, which is a measure of its distance from the equator. On any given day, **every location with the same latitude has the same daytime length.**

- At each location on Earth, the daytimes repeat in a yearly pattern. Therefore, they can be modelled using sin, as a function of the day of the year. (In reality, daytimes will vary slightly from these functions as days are discrete time steps whereas the Sun and Earth move continuously.)

Question *5.2.5*

If t is the day number in the year (starting from $t = 0$ on January 1st) then the length of the daytime in hours at any point in the southern hemisphere is given by the function

$$D(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right)$$

where K is a constant determined by the latitude of the point. At the equator $K \approx 0$ hours, and its value increases for more southerly locations.

(continued over)

Question 5.2.5 *(continued)*

$D(t)$ for Brisbane is plotted in Figure 5.3, where $K \approx 1.74$ hours.

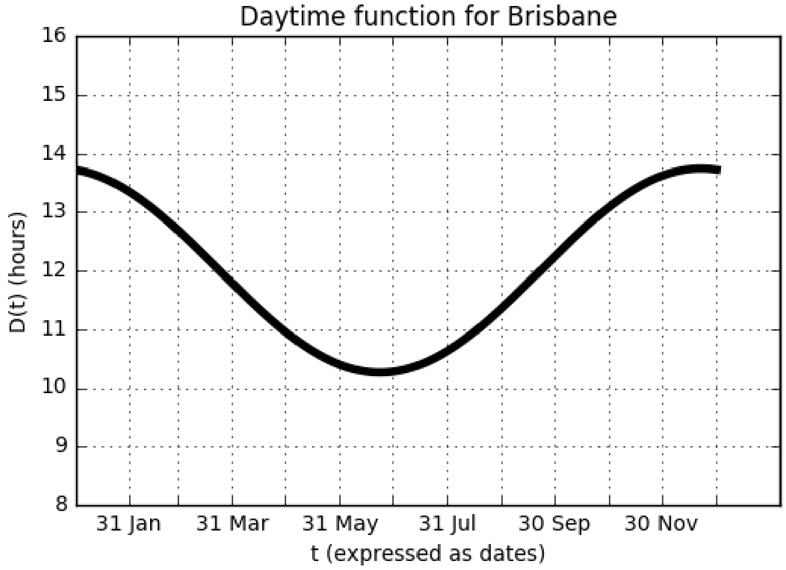


Figure 5.3: The daytime function for Brisbane.

Fill in the missing entries in the following table describing the physical and mathematical significance of each term in $D(t)$ for Brisbane, and indicate these on the graph.

value	mathematical meaning	physical meaning
12 hours	equilibrium of the sine wave	average number of daylight hours
1.74 hours		
365 days	period of the sine wave	number of days in a year, as the phenomenon follows a yearly cycle
264 days		

Question 5.2.6

Briefly explain how to mathematically find when the solstices occur in Brisbane, using the function $D(t) = 12 + 1.74 \sin\left(\frac{2\pi}{365}(t - 264)\right)$ (as opposed to reading off the graph).

Note that we modelled the number of daylight hours for the southern hemisphere. We will consider a model for the northern hemisphere in the next lecture.

End of Case Study 10: Modelling daytimes.

Lecture 14: **Track birds with sine**

Learning objectives

✓ Interpret sine function models of real-world phenomenon

Scientific examples

✓ Tracking migration of terns

Maths skills

✓ Understand and interpret sine functions and their graphs

Photo 5.8: Migrating Canada Geese, *Branta canadensis*, New York State, USA. (Source: PA.)

Many animals undertake *migration*, during which they move from one area to another, and then return. This often happens on an annual basis, according to seasons or weather patterns. Migratory behaviour occurs in all major animal groups (birds, reptiles, mammals, amphibians, fish, insects and crustaceans); see [9].

Examples of migration include: wildebeest and zebra on the Serengeti plains in Africa; geese “flying south for winter” in the northern hemisphere; humpback whales travelling north along the Queensland coast during winter; and sea turtles returning to beaches to lay eggs.

Question *5.2.7*

What are some of the reasons for, and benefits of, seasonal migration? How does this relate to daytimes?

Question 5.2.8

Recall that, if t is the day number in the year (starting from $t = 0$ on January 1st) then the length of the daytime in hours at any point in the southern hemisphere is given by the function

$$D(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right)$$

where K is a constant determined by the latitude of the point.
In Brisbane, $K \approx 1.74$ hours, whereas $K \approx 1$ hour for Townsville, and $K \approx 3.3$ hours for Hobart. The graph for Brisbane is shown in Figure 5.4.

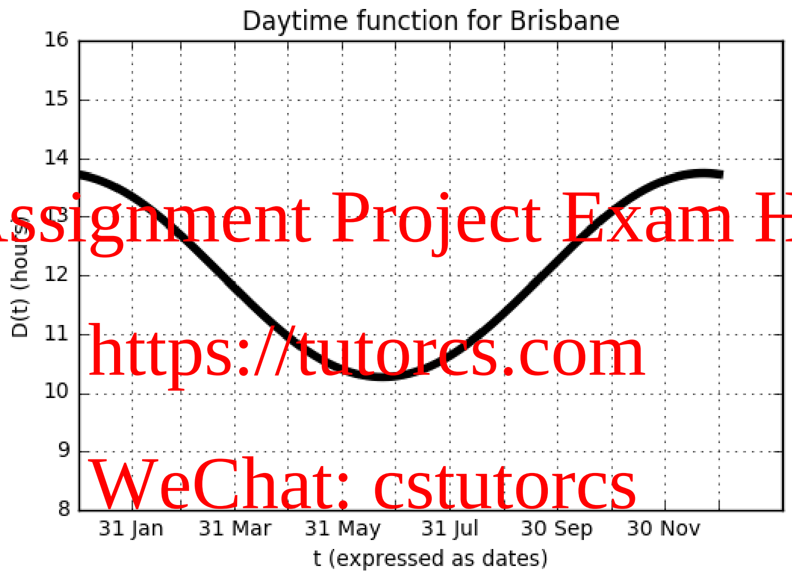


Figure 5.4: Daytimes in Brisbane over the year.

(a) Roughly sketch the graphs of $D(t)$ for Townsville and Hobart on the above graph.

(b) By how much is the daytime on the **summer** solstice in Hobart **longer** than in Townsville? By how much is the daytime on the **winter** solstice in Hobart **shorter** than in Townsville?

(continued over)

***Question** 5.2.8 (continued)*

(c) Consider a yellow wattlebird in Hobart and an eastern whipbird in Townsville. Assuming that the birds stay in their respective local habitats all year, what is the **total** number of daylight hours they each experience for the entire year? Use the models for daylight hours given for Hobart and Townsville on the previous page, and justify your answer.

(d) The eastern whipbird is sendentary and does not migrate. However, suppose that one brave whipbird flew to Hobart for the summertime, and then returned to Townsville. What effect would this have on the total number of daylight hours this bird would experience over that year?

(e) How might your answer to part (c) change if you were to consider the total number of "sunlight hours", as opposed to daylight hours?

Case Study 11: **To every thing, there is a season, tern, tern, tern.**

The Arctic tern, *Sterna paradisaea*, is a seabird that migrates annually from its breeding grounds in the Arctic to the Antarctic and back.

Image 5.2: Arctic tern in flight. (Source: en.wikipedia.org)

- Individual terns have been tracked travelling a distance of 400–700 km per day, and 80000 km in a year; this is the longest (known) migration of any animal.

- Figure 5.5 shows tracked migration routes of 11 Arctic terns (see [11]).

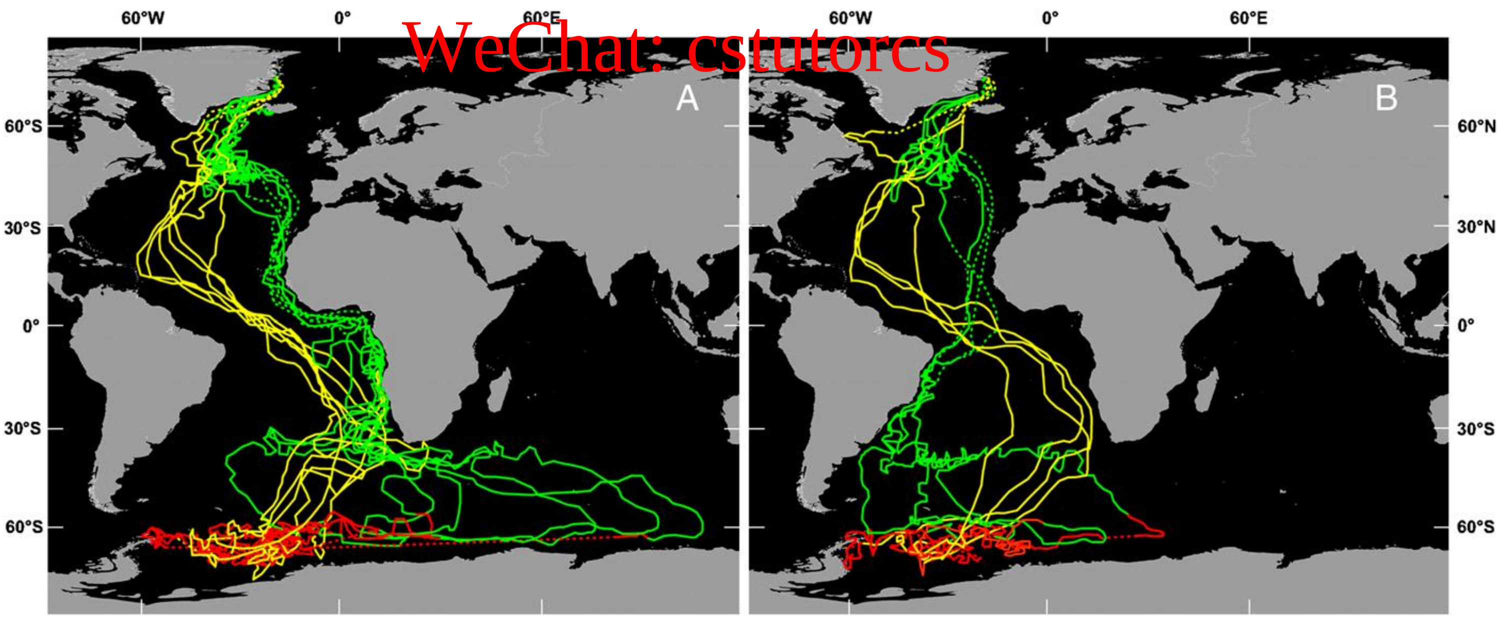


Figure 5.5: Interpolated geolocation tracks of 11 Arctic terns tracked from breeding colonies in Greenland ($n = 10$ birds) and Iceland ($n = 1$ bird). Green = autumn (postbreeding) migration (August/November), red = winter range (December/March), and yellow = spring (return) migration (April/May). Two southbound migration routes were adopted in the South Atlantic, either (A) West African coast (n = 7 birds) or (B) Brazilian coast. (This text is an extract from [11].)

Question 5.2.9

Researchers in [11] attached miniature light loggers to individual birds in Iceland and in Greenland, and retrieved the data one year later. The light loggers have an internal clock and they record when they are exposed to light. For this question, assume the birds are in the southern hemisphere.

(a) Describe how researchers could have used this data to determine the latitude of a tern on any given day.

(b) Describe how the researchers could have used the data from the light loggers to determine the longitude of a tern on any given day.

Question *5.2.10*

We have seen that the equation for the number of daylight hours in the southern hemisphere is $D_S(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right)$.

When the researchers track artic terns, they will need to use a different equation when the birds are in the northern hemisphere.

(a) Find a corresponding equation to model the number of daylight hours in the **northern** hemisphere, $D_N(t)$.

(b) A graph of $D_S(t)$ at the Antarctic Circle is shown below. Draw the corresponding graph for $D_N(t)$ at the Arctic Circle.

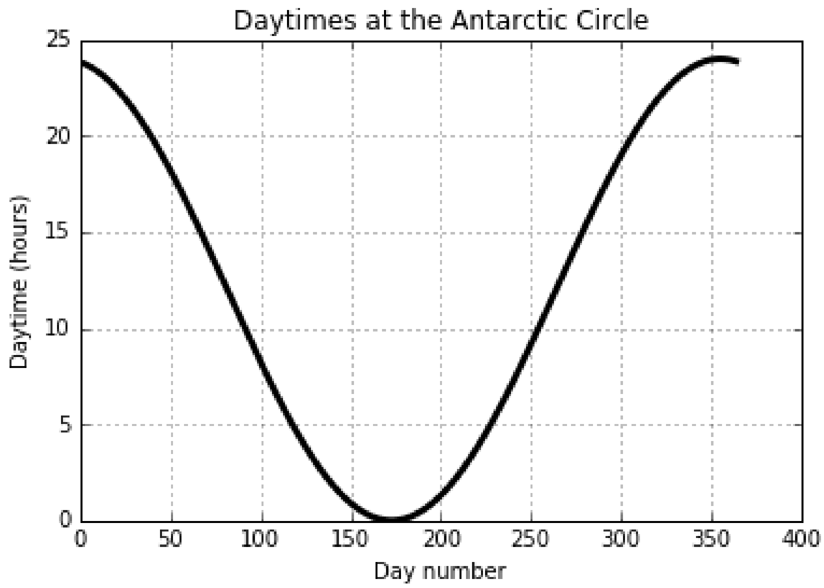


Figure 5.6: Daytimes at the Antarctic Circle.

- The paper [11] states that:

> “Locations were unavailable at periods of the year when birds were at very high latitudes and experiencing 24 h daylight. In addition, only longitudes were available around equinoxes, when day length is similar throughout the world. Overall, after omitting periods with light level interference and periods around equinoxes, the filtered data sets contained between 166 and 242 days of locations for each individual.”

***Question** 5.2.11*

Why did the researchers in [11] need to comment on high latitudes and the equinoxes?

End of Case Study 11: To every thing, there is a season, tern, tern, tern..

Chapter 6: **All about that base**

Lecture 15: **Isotopes and exponents**

Learning objectives

✓ Interpret exponential function models of real-world phenomenon

Scientific examples

✓ Radioactive decay

✓ Carbon dating

Maths skills

✓ Understand exponential functions and logarithms

✓ Doubling time and half-life

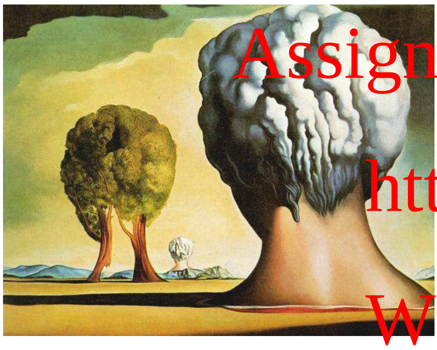


Image 6.1: *The Three Sphinxes of Bikini* (1947), Salvador Dalí (1904 – 1989), Morohashi Museum of Modern Art. (Source: Museum publication.)

Exponential functions are useful for modelling many natural phenomenon such as growing populations or radioactive decay of isotopes, as well as many "un-natural" phenomenon such compound interest.

Logarithms are closely related to exponential functions and you will have used them in previous mathematical study to solve exponential equations.

In this chapter we will review some of the properties of these important functions and discuss some of the scientific contexts in which they naturally arise. You should have encountered exponential and logarithmic functions in previous study of mathematics. See Section C.3 in Appendix C for the pre-requisite mathematical tools we will use in this chapter. Use the online modules, available through the course website, for further support.

We will also see how useful logarithms are for displaying and understanding data. Log plots and even log-log plots are incredibly useful for communication and understanding data in many scientific contexts.

6.1 Growth and decay

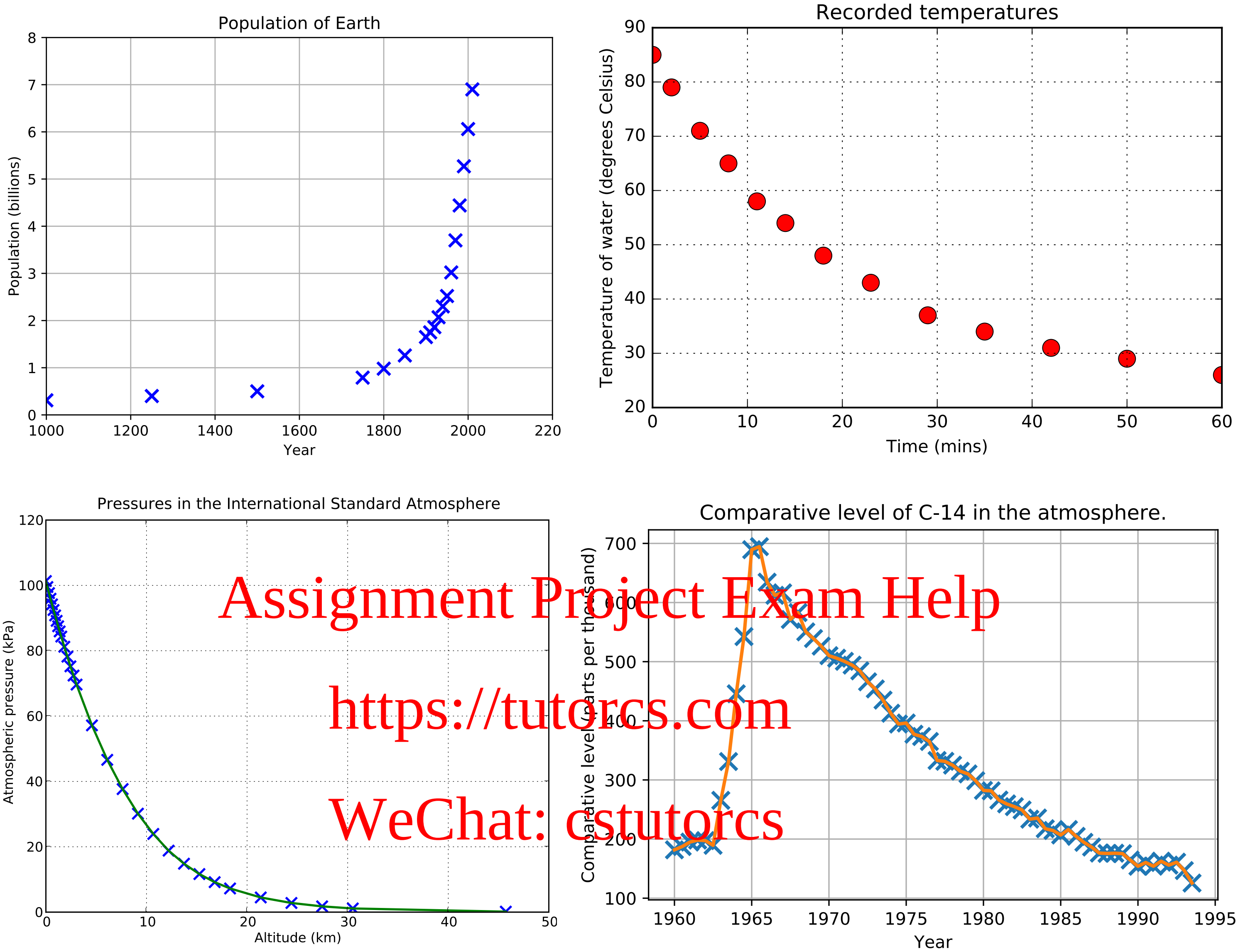


Figure 6.1: Some real, measured data. Top left: population of Earth over 1000 years. Top right: measured water temperatures in a simple experiment. Bottom left: atmospheric pressures in the international standard atmosphere. Bottom right: comparative level of atmospheric radioactive Carbon-14.

Question 6.1.1

Why did the comparative level of atmospheric radioactive Carbon-14 increase rapidly between 1960 and 1965, and why has it decreased since then?

- Science primarily studies phenomena that change. Often, the rate of change at any time is proportional to the amount that is currently there.
- This is typical of many populations. For example, each year the size of the global human population is increasing by around 1.5% of its current size.
- Any phenomenon that has a rate of change proportional to the current amount follows an *exponential* function. (We will see why later.)
- An exponential function is of the form $y(t) = Ca^{kt}$, where a is the *base* of the exponent. In many scientific contexts, Euler's number ($e \approx 2.718\ldots$) is used as the base, giving $y(t) = Ce^{kt}$.

Doubling time/Half-life

The **doubling time** for an exponentially growing quantity is the time it takes to increase to twice its current size.

The **halving time** or **half-life** for an exponentially decreasing quantity is the time it takes to decrease to half its current size.

Many exponential phenomena in science have relatively constant doubling times or half-lives over extended periods; knowing these values provides useful information about the phenomena.

***Example* 6.1.2**

Exponential functions occur frequently in models of nature and the social sciences. Some examples include unconstrained and constrained population growth, radioactive decay and carbon dating, modelling drug concentrations in blood, and modelling *habituation* to a stimulus.

- *Logarithms* (or *logs*) are very closely related to exponential functions.
- Logarithms are the *inverse* of exponentiation (in much the same way that division is the inverse of multiplication).

Logarithms and exponentials

The relationship between exponentials and logarithms is:

- If $y = 10^x$ then $x = \log_{10} y$ (and vice-versa).
- If $y = e^x$ then $x = \ln y$ (and vice-versa).

Question *6.1.3*

(a) Find $\log_{10} 1000$ and $\log_{10} 0.01$.

(b) If $y = e^{0.02t}$, find $\ln y$.

6.2 Exponentials in action

Case Study 12: **Radioactive decay**

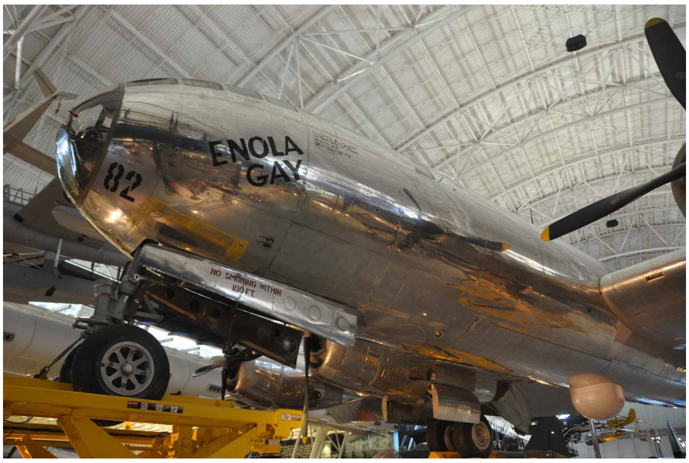


Photo 6.1: The B-29 Superfortress bomber "Enola Gay", National Air and Space Museum, Virginia, USA. (Source: PA.)

- Isotopes of an element behave the same way chemically but have different numbers of neutrons in the nucleus of the atom.
- One standard way of denoting isotopes is to write the name or chemical symbol of the element, hyphenated with its atomic mass. For example, Deuterium (an isotope of Hydrogen and the main ingredient in "Heavy water") is written as Hydrogen-2 or H-2.
- Not all atoms remain the same over time; some undergo *radioactive decay*, which involves rearrangement of the nucleus of the atom, sometimes changing it into a different element.
- Radioactive isotopes have useful applications in a range of sciences and industries, including chemistry, biology, medicine, physics and engineering. Therefore, it is important to understand how to model their decay.
- Radioactive decay is spontaneous, so there is no way of knowing *when* a *specific* individual atom is going to undergo decay.

- However, it *is* known that in any given time period a certain *proportion* of the total quantity in a sample will have decayed.
- Thus, radioactive material undergoes continuous decay at a rate **proportional** to the **quantity** of material, so the decay is an exponential process.

Decay constant

For a radioactive element, the *decay constant* k reflects the rate of decay of the element, and is a property of the chemical element. The half-life can be calculated from the value of k, and vice-versa.

***Example** 6.2.1*

Decay constants and half-lives vary greatly between radioactive elements. For example:

- Polonium-212 has a half-life of about 3×10^{-7} s.
- Uranium-238 has a half-life of about 4.5×10^{9} years.
- Carbon-14 has a half-life of about 5730 years.

***Example** 6.2.2*

Carbon-14 (C-14, also known as *radiocarbon*) is used to determine the age of organic-based artifacts (up to around 60,000 years).

Cosmic rays striking nitrogen in the upper atmosphere produce C-14. It then reacts chemically with oxygen to form radioactive carbon dioxide which permeates living creatures in a fixed proportion, either directly (by absorption from the atmosphere), or indirectly (via food chains).

When an organism dies, it ceases to accumulate C-14, and the remaining amount undergoes net decay over time. *Carbon dating* is the process of measuring the residual level of C-14 in organic artifacts, and thus deducing their age.

Question 6.2.3

The half-life of C-14 is 5730 years.

(a) Find the decay constant of C-14.

(continued over)

***Question** 6.2.3 (continued)*

(b) Consider the following extract from the paper [8].

> "The Shroud of Turin, which many people believe was used to wrap Christ's body, bears detailed front and back images of a man who appears to have suffered whipping and crucifixion. It was first displayed at Lirey in France in the 1350s . . . Very small samples from the Shroud of Turin have been dated by accelerator mass spectrometry in laboratories at Arizona, Oxford and Zurich. As Controls, three samples whose ages had been determined independently were also dated."

Researchers discovered that 91.9% of the expected original amount of C-14 was present (compared to that in new organic garments). Deduce the (approximate) age of the Shroud, and comment on your answer.

End of Case Study 12: Radioactive decay.

Lecture 16: **Chill out with logs**

Learning objectives

✓ Interpret exponential function models of real-world phenomenon

✓ Understand the form of log-plots

Scientific examples

✓ Newton's Law of Heating and Cooling

✓ Atmospheric pressure

Maths skills

✓ Understand and interpret exponential functions and their graphs

✓ Interpret log-plots

Case Study 13: **Hot stuff, cold stuff**

- Moving an object with one temperature to a location with a different (but constant) temperature leads to a gradual change in the temperature of the object to match that of the new location.

Question *6.2.4*

Explain why it is reasonable that an exponential function would model the temperature of an object moved to a location with a different temperature.

Question 6.2.5

In an experiment, the temperature of hot water in a container was recorded at various times over one hour; see Figure 6.2. The room temperature was 25 °C.

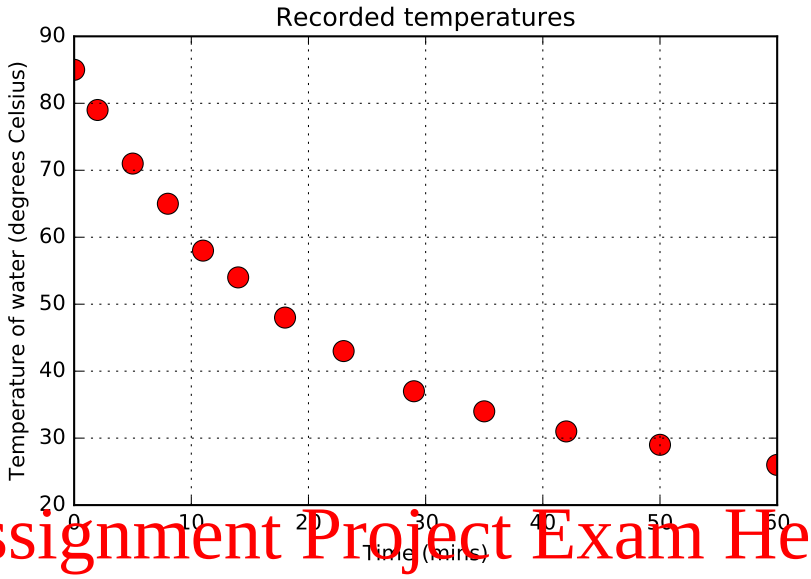


Figure 6.2: A graph of the measured temperatures.

Determine an equation for the water temperature at any time in minutes (note that the water approaches room temperature of 25 °C, not 0 °C.)

We can develop a computer program to model the temperature.

Program specifications: Write a program that plots the measured water temperatures and the function that models these temperatures.

Program 6.1: Temperatures

```
# Program to plot measured and modelled temperatures.
from pylab import *

# Initialise variables
times = array([0,2,5,8,11,14,18,23,29,35,42,50,60])
temps = array([85,79,71,65,58,54,48,43,37,34,31,29,26])
model = 60 * exp(-0.05 * times) + 25
# Draw graphs
plot(times, temps, 'ro', markersize=10, label="Data")
plot(times, model, 'k-', linewidth=3, label="Model")
text(30,40,"model")
text(10,50,"actual")
xlabel("Time (mins)")
ylabel("Temperature of water (degrees Celsius)")
title("Recorded temperatures")
grid(True)
legend()
show()
```

Output from the program is shown in Figure 6.3.

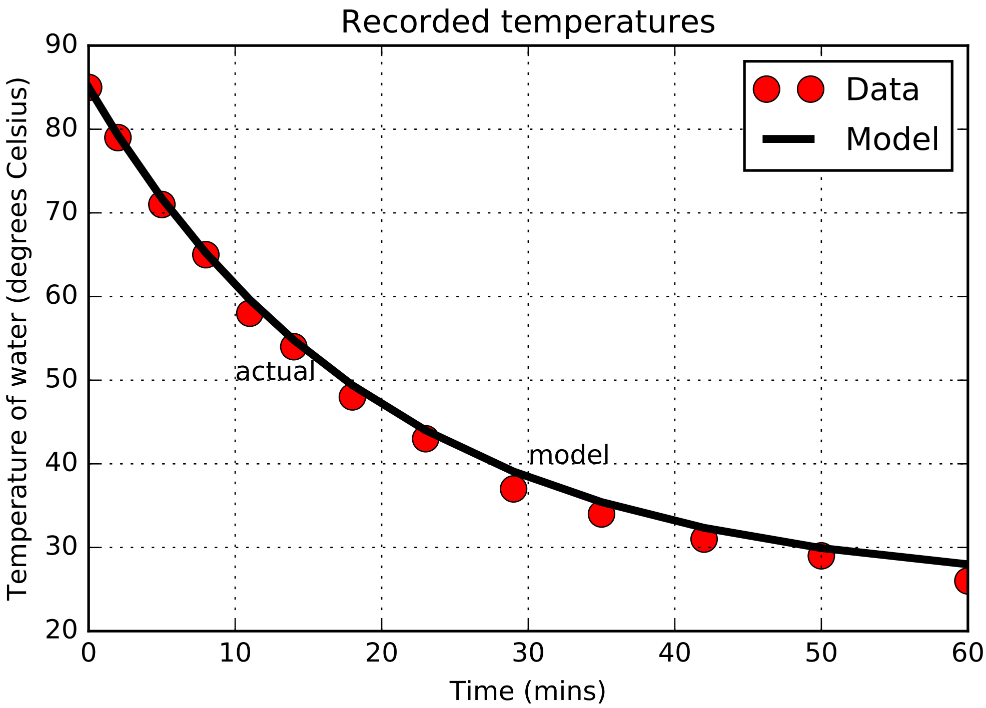


Figure 6.3: Modelled and actual water temperatures.

Question *6.2.6*

Do you think the model shown in Figure 6.3 is a good fit to the given data? If you were to use this model, justify your choice. If you were to modify the model, what change or changes would you suggest and why?

End of Case Study 13: Hot stuff, cold stuff.

6.3 Logarithms in action

- Logarithms provide a convenient mechanism for converting exponential data into a form that can make data analysis easier.

Question 6.3.1

Assume some data are modelled by the exponential function $D(t) = D_0 e^{kt}$. Demonstrate how a logarithmic transformation of the data values results in a linear model. Interpret the y-intercept and gradient of the linear model. (Hint: if x and y are positive then $\ln(xy) = \ln x + \ln y$.)

Question 6.3.2

Earlier we saw that the *International Standard Atmosphere* (ISA) [27] models various atmospheric properties, including temperature, pressure and density. Figure 6.4 shows atmospheric pressures in kilopascals (kPa) at various altitudes in the ISA, and Figure 6.5 shows a graph of these pressure data transformed using natural logarithm, ln.

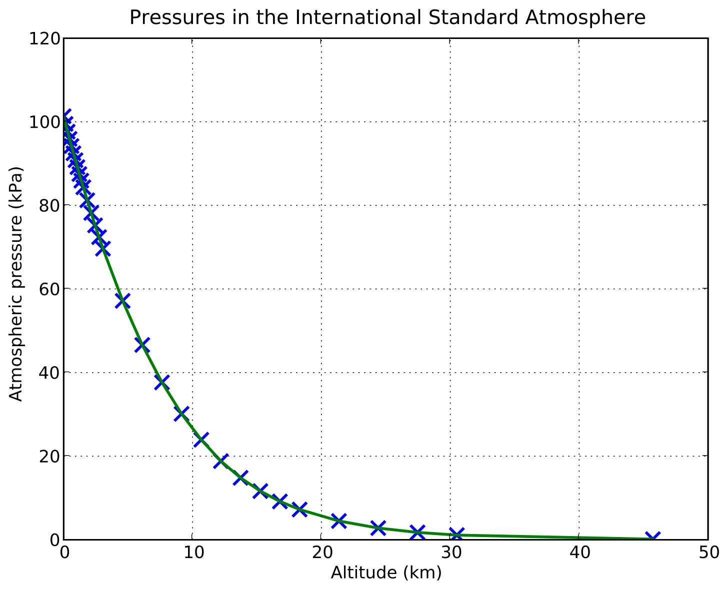


Figure 6.4: ISA pressures. *(continued over)*

Question 6.3.2 (*continued*)

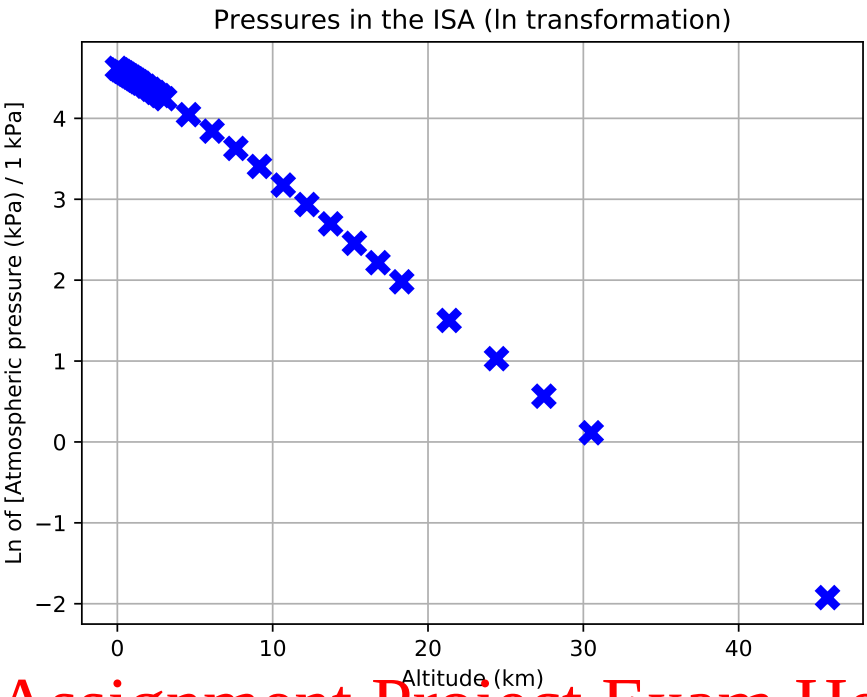


Figure 6.5: ISA pressures (transformed data).

(a) Use Figure 6.5 to estimate the pressure outside a jetliner cruising at 12000 m.

(b) Use Figure 6.5 and Question 6.3.1 to find an exponential model of pressures in the ISA.

(continued over)

Question 6.3.2 *(continued)*

When a jetliner is in flight, the pressure in the cabin is artifically raised to a higher level than the pressure outside. The *cabin altitude* is the altitude at which atmospheric pressure matches the pressure inside the cabin.

Modern planes typically cruise at an altitude of 12000 m, but maintain a cabin altitude of about 2000 m. Thus, the cabin pressure is around 75.6 kPa, while outside the cabin the atmospheric pressure is around 18.6 kPa. Note that on the ground, atmospheric pressure is around 100 kPa.

Photo 6.2: Bang? (Source: PA.)

Question 6.3.3

Keeling Model 3: Figure 6.6 shows two plots: a graph of the function $y(t) = 280 + 35e^{0.022t}$, and the Keeling curve.

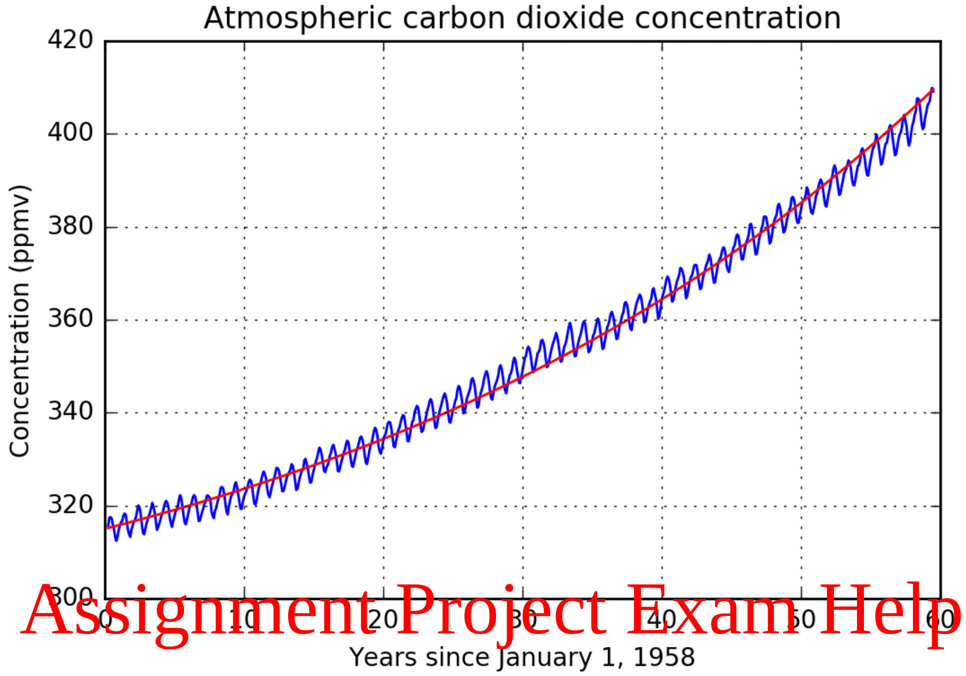


Figure 6.6: The Keeling curve and an exponential model.

(a) Explain mathematically how each term in $y(t)$ impacts on its graph.

(b) Data from ice-core samples show that long-term atmospheric CO_2 levels remained relatively constant at 280 ppmv. Explain the physical significance of the constants 280 ppmv and 35 ppmv.

(c) How effectively does $y(t)$ model the underlying Keeling curve trend?

Chapter 7: **How scientific reasoning works**

Image 7.1: Émilie Du Châtelet (1706-1749), mathematician, physicist and philosopher of the French Englightenment, portrait by Latour (Source: en.wikipedia.org)

Lecture 17: **First Philosophy Lecture**

Learning objectives

✓ Clarify the module question, ‘How is science rational?’

✓ Realise that some important questions are not able to be answered by experiment (e.g. how is science rational?)

✓ Develop clarity and precision in language (e.g. being clear about what we mean by the terms ‘theory’, ‘law’, ‘proof’ or ‘model’)

✓ Consider how models are created and used in science

✓ Examine how induction works

7.1 Introduction: science and the assumption of rationality

News Headlines:

"No Rational Person Can Deny Human-Induced Global Warming"

What does this mean? What, if anything, is scientific "rationality"?

Many people believe that science is rational and that this is because there is such a thing as *the scientific method* by means of which we are able to make reliable claims about the natural world. Results arrived at by means of the scientific method have a special status — *scientific knowledge is reliable knowledge*, unlike claims made on the basis of common sense. This is not to say that common sense never leads to truth or science always does, but rather that our grounds for accepting claims made in the name of science are stronger than those for accepting claims made on the basis of common sense — this by virtue of the method used for arriving at the claims made.

To illustrate the point, consider the conflict between the common sense view that the sun revolves around the earth (the earth feels stationary, the sun seems to move across the sky, etc.) and the scientific view that, in fact, the earth revolves around the sun. This conflict is typically resolved in favour of the scientific view. In general, where science and common sense conflict, common sense gives way.

So consider two public speakers A and B. A stands and proclaims that human-induced global warming is not occurring, B claims it is. When asked, A admits that he claims no global warming due to human activities is occurring because common sense suggests that any fluctuation is more likely to be part of a natural cycle of climate change. B, on the other hand, admits that she claims human-induced global warming is occurring because scientific evidence all points to the fact. Now, *regardless of who is in fact right, who do we have more reason to believe?* B would commonly be said to be the more credible of the two by virtue of the means employed for arriving at the claim — B's method is more reliable, more rational, than A's.

The general view underlying this resolution of the conflict seems to be, again, that scientific claims to knowledge have some kind of merit not shared by common sense claims. But if merit attaches to scientific knowledge then why? What makes science rational? The common answer, again, is that science employs a method which is rational and is the means by which scientific knowledge is arrived at. This is what I want to talk about for the next four lectures.

7.2 Getting Philosophical

Of course, at this point, in discussing the general nature of science, we are engaged in an activity other than science itself. No amount of scientific experimentation will tell us whether or how science is rational. Just think about it for a minute. Asking questions about what science is and how it works is not something that we can do in a lab. White coats, bunsen burners and experiments won't help. When we question our beliefs *about science* we step out of science itself to a more abstract level of discussion. We are engaged in *philosophical* debate and argument about the nature of science.

The following lectures are directed at introducing you to some of the philosophical issues that arise in attempting to explain the apparent rationality of science and scientific method.

7.3 Some Preliminaries

7.3.1 What is Science?

What is science? As we shall see, it is not a body of *facts*. Despite common views to the contrary, science is not in the business to putting forward proven facts. Scientific laws and theories are continually being overthrown in the face of problems, or anomalies, that the laws or theories fail to adequately account for – they are always *provisional* to some degree, as are the specific scientific claims that depend on them. Newton's Laws, for example, were never scientific facts. They were conjectures that were eventually overthrown by "better" laws, those of relativity theory.

Science is a way or method of thinking: thinking **critically** about **the empirical world** using **evidence** to try to justify **hypotheses**, **laws** and **theories** (collections of laws about some set of phenomena), put forward as **conjectures** that are subject to further critical testing against ever-increasing bodies of evidence.

Key Point: Science is not a body of facts. It is a way of thinking critically about the empirical world using evidence to make general conjectures.

Just think about the hypothesis of human-induced global warming. We appeal to evidence about humans increasing carbon dioxide levels in the atmosphere and their effect on the heat-retaining capacity of the atmosphere (the so-called "green house effect"), and go on to conjecture that *we* are thus the cause of a warmer climate. This conjecture is further tested by increasing bodies of evidence about past climate variation and its possible causes. And so it goes.

This conjecture is, of course, an hypothesis in the applied field of climate science. As applied science, it depends on a large number of even more fundamental physical and chemical laws, as well as associated mathematical principles that enable modelling of exponential growth of gas concentrations and summative effects of gas concentrations as a result of chemical reactions, etc. And these currently accepted laws and mathematical principles themselves are taken as (currently) justified. So how do we justify these? What justifies *these* fundamental laws and principles as acceptable?

For obvious reasons, we shall limit our discussion to *scientific* theory and set aside the many (interesting) issues surrounding the acceptability of mathematics used in science. The philosophy of mathematics is yet another area of philosophical enquiry with a heritage stretching back to the ancient Greek philosopher Pythagoras and beyond. The notion of mathematical truth, in particular, has been the subject of considerable study — most notably in the late 19th and early 20th centuries with key players like Bertrand Russell, David Hilbert and Kurt Gődel. Our focus, though, is squarely on scientific hypotheses, laws and theories.

7.3.2 Hypotheses, Laws, Theories and Models

Science is made up of many hypotheses and laws, and groups of them that work together to make up scientific theories — evolutionary theory, electromagnetic theory, and so on. Let's just stop for a minute to get clear on our terms here. As potential scientists you will come across the terms 'hypothesis', 'law' and 'theory' a lot, and the way they are used in science is sometimes different to how they get used "on the street", so some clarification may help here.

— **An Hypothesis:** a (scientifically testable) claim used to predict or explain some particular phenomenon or event.

  E.g. hypothesising that, since a ball began to move, it was acted on by some force. The hypothesis is that the ball was acted on by some force.

— **A Law:** a (scientifically testable) claim describing *a general regularity* in nature used to predict or explain some particular phenomenon or event.

  E.g. All bodies remain at rest unless acted upon by some force.

— **A Theory:** a set of interconnected laws and principles working together to form a *model* (typically involving significant idealisation) used to explain the general regularities themselves.

  E.g. Einstein's theory of general relativity (including $E = mc^2$, etc.) is a model that explains gravitational laws. Darwin's theory of evolution (including principles of natural selection, etc.) is a model that explains biological diversity.

Thus theories explain how the world works in general and why it works as it does by providing a model of the system in question. Hypotheses and laws are simply used to tell us why some particular thing happened but might themselves stand in need of explanation.

NB: Later in these lectures we'll pin down, more exactly, what we mean by "scientifically testable". For now, the above distinctions should be sufficiently clear.

Some more examples:

1. "Gravity is caused by undetectable particles exerting forces" may appear to be an hypothesis but is not (it is not testable — as we shall see later when discussing testability).

2. "The postman is sick" is an hypothesis (that one might offer to explain the lack of mail), but is not a law (it is not general).

3. "$F = ma$" is a law (describing a general regularity: when a force acts on an object it is caused to accelerate by an amount which when multiplied by the mass of the object, equals the force applied), but is not a theory. The relationship between the three quantity remains unexplained.

4. "Einstein's theory" explains general features of the observable world expressed by gravitational laws, etc.

Common Misuses and Abuses

Note that these key terms are not always used this way outside of science.

"Theory": People sometimes mean a mere *guess* or speculation lacking any support. (For example, "That is just a theory".) The former US President Ronald Reagan was famously reported as saying "Evolution is a theory, a scientific theory only . . . ". He meant it was just a guess, arguing that it was no more reasonable to believe than creationism (the view that the world was created by God rather than evolved).

"Law": People sometimes mean a general regularity in nature that has been *proven*. (For example, "But that is a law. It cannot be wrong.")

Scientists generally use the terms 'theory' and 'law' in a way that is neutral concerning whether they have no support, some support, or very strong support. More generally, the terms 'hypothesis', 'law' and 'theory' do not indicate a difference in how well established or proven a scientific claim is; they indicate the kind of claims in question — a specific claim that is general (law) or not, or an explanatory set of claims (theory). When we want to indicate that we are speaking of the *currently supported or accepted* view, we often speak of *the* theory of evolution, or the law of gravity. (Here we indicate that one from among many candidates is accepted.)

Don't forget about models!

— **Models:** idealised, i.e., simplified or distorted, representations of portions of the world. Models can be theory driven, that is, arrived at by the application of theory. Models can also be data driven, that is arrived at on the basis of data. Often, models are theory and data driven.

  E.g. The Keeling Curve is a data model of the accumulation of Carbon Dioxide in the Earth's atmosphere.

  It is partly because models are idealised that they are so useful. Data would be unmanageable if we did not simplify/distort it, e.g., by modelling it with a simple function. Theories would often be hard, if not impossible, to test or apply if they were not simplified in the process of their application. The idealised nature of models, therefore, enables them to connect our theories, hypotheses and laws – our 'ideas' – with data and thus enables testing our ideas.

7.3.3 The Task Ahead

Scientists typically believe that their way of thinking about the world involves some method, "the scientific method", and this rational method gives a way of justifying hypotheses, laws and theories (the results of scientific activity) as scientific knowledge. So what is this "method" and how does it produce scientific knowledge?

Notes:

7.4 Science and Inductive Reasoning

A popular view of scientific method is that science begins with particular observations of natural phenomena. From observation one logically arrives at general principles — scientific laws — by an inference known as *induction*.[1]

[1] "Induction" here is not to be confused with the mathematical inference known as "mathematical induction".

For example, imagine Boyle (1627-1691) studying the behaviour of a gas at constant temperature. He observes the following numerical measures of its volume at different pressures, in appropriate units:

Pressure	Volume
1	12
2	6
3	4
4	3

From an examination of these few measures he infers the general law that the product of the pressure and volume is constant (given constant temperature) — Boyle's Law.

Justification of scientific laws is thus by way of a special kind of generalising argument — using "inductive reasoning" to infer the law-like structure of the universe. Our belief in the laws of science is therefore rational since based on logical argument from evidence. ... Or so the story goes according to the inductivist.

In the same way, Avicenna (the 10-11th century Arabic philosopher, scientist, physician and mathematician) gave the following example of inductive inference, referring to the purgative effects of scammony[2] (not to be tried at home!):

It is observed that ingestion of scammony is followed by the discharge of red bile
This observation is repeated under circumstances in which other possible causes of the discharge of red bile have been excluded

---

Hence, all scammony according to its nature withdraws red bile.

We seem to use this kind of inference all the time in daily life. Why expect that the fire will burn me? Because it has numerous times in the past. Why think that food will nourish me? Because it has numerous times in the past.

The main principle underlying this use of "inductive inference" is the idea that *what occurs frequently does not do so by chance.*

In this way, from observations of lunar eclipses and other phenomena concerning light one inductively infers general principles or laws — e.g. that light travels in straight lines; that opaque bodies cast shadows; and that certain configurations of opaque and

[2]A twining plant having a stout taproot, *Convolvulus scammonia*, found in Syria, Asia Minor, Greece, etc. The dried milky juice was used as a medicine from ancient times.

luminous bodies place one opaque body in the shadow of another. (See the left side of Figure 7.1.)

These general principles or laws themselves can then serve as assumptions, along with some other facts concerning particular conditions etc., enabling one to *deduce* statements about phenomena. Continuing with the example above: the laws concerning opaque bodies and light, along with the fact that the earth and the moon are opaque bodies and the fact that on such-and-such a date the earth will pass across the line between the luminous sun and moon, can be used to deduce that there will be a lunar eclipse on that date. In this way we can make *predictions* about events not yet observed. (See the right side of Figure 7.1.)

Alternatively, by showing how an observation can be deduced from the laws one can progress from an already observed fact that the lunar surface has darkened to an understanding of *why* this took place. In this way, deducing the observation from laws about the nature of light and opaque bodies, along with particular facts, we can provide an *explanation* of events already observed.

Figure 7.1: Induction and deduction.

On this account, "induction" is the crucial process used to arrive at laws. It allows us to infer some general law-like property or relation from a number of particular, observed cases or events. Our experience of some novel scientific phenomenon (for example, the effect of some drug on the human nervous system) is usually very undiscriminating; we do not initially see the general principles at work — we begin with a confused mass. However, over time, with sufficiently many repeated occurrences of the phenomenon, we are able to infer the general principles underlying the phenomenon — we reason our way to a universal law-like feature of the universe. By examining many cases we can inductively infer a formal pattern.

Key Point: Inductivist models of scientific thinking claim scientific laws are justified by *induction* from observations of a particular phenomenon.

Notes:

7.5 The Renaissance: experimentation and mathematics

Notice the use made of an actively pursued set of experimental results in the second premise of Avicenna's inductive inference. Later, in the 16th and early 17th century, the Renaissance development of instruments like the telescope, microscope and accurate clocks meant that much more sophisticated experiments could be undertaken (e.g. in the microscopic realm and the astronomical realm). And more extensive use was made of increasing amounts of experimental evidence that was becoming available. The emphasis on active experimentation to acquire new and relevant data for theorising became increasingly significant.

Another significant Renaissance development was the increasing use of mathematics and mathematical modelling in scientific theorising.[3] In the Renaissance the idea of a "clockwork universe" developed — a conception of the universe as like a giant mechanical clock (typically with God the great Watchmaker who set it all in motion) governed by laws amenable to precise mathematical modelling, and there was a shift from qualitative analysis to quantitative analysis and measurement. Scientists began to theorise about motion with idealised mathematical models invoking frictionless planes, perfectly spherical bodies, etc. This reached its greatest expression in Newton's Laws of Motion, whose mathematical simplicity and wide applicability confirmed a view of the cosmos as essentially mathematical in nature.

A century before this great Newtonian triumph, Galileo had put the point clearly:

> Philosophy [i.e. science] is written in that vast book which stands forever open before our eyes, I mean the universe; but it cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in the mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.[4]

[3] It seems hard to imagine science without sophisticated mathematics involved but, remember, the calculus of infinitesimals — developed by Gottfried Leibnitz and Isaac Newton, and necessary for the modelling of motion, acceleration, population growth, etc. — and probability theory — developed by Blaise Pascal — were not developed until the 17th century.

[4] *Il Saggiatore (The Assayer).* Quoted in A.C. Crombie's *Grosseteste and Experimental Science*, Oxford (1953) p. 285.

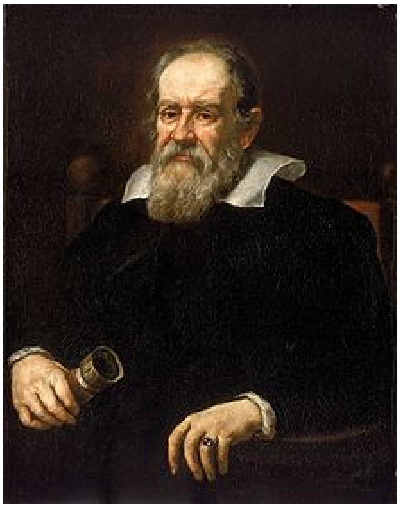


Image 7.2: Galileo, looking a touch haggard. (Source: en.wikipedia.org)

The book of nature is written in the language of mathematics, and that is why you need to get some mathematical skills under your belt before getting anywhere in science. If you don't like the maths, blame the Renaissance![5]

Key Points: In the Renaissance, technological innovation led to more *active experimentation* and an emphasis on the pursuit of observational evidence. Renaissance science also emphasised the *mathematical* structure of the scientific universe.

QUESTION: Could we do science purely *qualitatively* — i.e. using notions like *strong*, *weak*, *hot*, *cold*, *near*, *far*, *fast*, *slow*, etc. — instead of the Renaissance push to quantitative measurement? If not, why not?

Notes:

[5] For some examples of this increasingly mathematical approach to science see: H. Kearney, *Science and Change 1500-1700*, Weidenfeld & Nicolson (1971), pp. 66-7; A.C. Crombie, *Science, Optics and Music in Medieval and Early Modern Thought*, Hambledon Press (1990), p. 325.

Lecture 18: **Second Philosophy Lecture**

Learning objectives

✓ Recognise the proper role subjectivity has in science

✓ Consider challenges to the idea that induction is a rational process, including that of justifying induction; formulate a version of the principle of induction that addresses some of these challenges

✓ Understand the problem of induction

✓ Recognise what special challenges models pose for the inductivist view of science

7.6 A Common View of Science

Consistent with the account of scientific method described in the last lecture, a popular view of science is described by Alan Chalmers as follows:

> Scientific knowledge is proven knowledge. Scientific theories are derived in some rigorous way from the facts of experience acquired by observation and experiment. Science is based on what we can see and hear and touch, etc. Personal opinion or preferences and speculative imaginings have no place in science. Science is objective. Scientific knowledge is reliable knowledge because it is objectively proven knowledge.[6]

The notion of "proof" referred to here will, for the inductivist, be proof by inductive inference. On this "inductivist" interpretation of the popular view then, scientific reasoning is inductive and scientific method yields reliable knowledge through the application of inductive reasoning from the "facts of experience".

There are problems here though:

(i) The idea that scientific knowledge is *proven* knowledge can mislead us to think of such knowledge as certain. It is not.

(ii) The idea that science is objective and that "personal opinion or preferences and speculative imaginings have no place in science" is misguided.

[6] A. Chalmers, *What Is This Thing Called Science?*, (1976) p. 1.

(iii) The reliability of induction as a form of proof is difficult to justify.

Let us turn firstly to the problems concerning (ii).

7.6.1 Discovery versus Justification

With a little thought it seems obvious that there is a distinction to be made, first clearly drawn by John Herschel in the 1830s, between the means by which scientific theories are *discovered* — the context of discovery — and the means by which they are to be *justified* — the context of justification. How we discover a theory — i.e. the means by which we come to have the theory in our minds — is one thing, whereas establishing a theory as rationally acceptable — i.e. justifying it — is another thing.

The common view described above seems to wrongly characterise science by ruling as illegitimate that highly imaginative and creative aspect of science whereby practitioners "cook up" theories for consideration and testing. Who ever came up with the idea that there could be "dark matter" or "dark energy" and what in heaven's name were they on when they thought up the idea? Who cares? What is relevant is simply whether or not such an idea (indeed, whether any scientific law, hypothesis or theory) can be justified. For, irrespective of how someone came up with the idea, what matters from the point of view of understanding the universe is whether or not such an idea can be justified.

With this in mind, we do not need to give any account of the process whereby we *discover* scientific theories. We do not need to claim that the process is in any way rational or reliable, let alone suggest that it proceeds by way of induction. An account of scientific method as essentially inductive then need only be committed to the view that scientific laws are *justified* by induction. Personal opinion and preferences, and speculative imaginings play a key role in science. Science is, in this sense, clearly subjective. Imaginative theorising by individual subjects puts hypotheses on the table for consideration which might otherwise never have been considered (these hypotheses are not "objectively given" to us), and some of the greatest honours in science go to those who have used their subjective imagination in ways that are ingenious, and which have produced theories which have subsequently come to be seen as justified. But how are they justified? This, according to those advocating an inductive scientific method, is by way of inductive inference. And so to problem (i).

Key Point: The discovery of scientific hypotheses — a process entirely separate from justification — is a *creative* process that infuses science with subjectivity.

Notes:

7.6.2 Inductive Inference and Fallibilism

What *exactly* is the logic of inductive inference? Consistent with Avicenna's example, inductive inference, according to one of its most famous advocates, the philosopher J.S. Mill (1806-1873), consists in inferring from a finite number of observed instances of a phenomenon, that it occurs in *all* instances of a certain class that resemble the observed instances in certain ways.[7] For example, from the fact that ingestion of scammony is observed on a number of occasions to produce red bile we infer that its ingestion always produces red bile. Similarly, from the fact that John, Peter, etc. are all mortal we infer, by induction, that all humans are mortal. Let's look at this in more detail.

Observational (singular) statements

The simple inductivist account claims that science starts with observation. The scientist, with normal, unimpaired senses records what she sees without prejudice. Impartial reports as to how the world operates are justified by the use of the senses. Statements reporting these particular facts — often referred to as *observational statements* — serve as the basis for the derivation of scientific laws. Examples of some simple observation statements are:

> At midnight on Jan. 1 1975, Mars appeared at position x in the sky.
>
> Mrs Smith struck her husband.
>
> The water boiled at 100 degrees Celsius at sea level (at place p and time t).

Observing what is the case will establish such statements as true or false at a particular place and time. Such statements are *singular statements*; they describe a particular event or state of affairs at a particular place and time.

Universal (general) statements

Scientific statements however describe general patterns in nature. For example:

> Planets move in ellipses around their sun. (Astronomy)
>
> Animals in general have an inherent need for some kind of aggressive outlet. (Psychology)
>
> Water always boils at 100 degrees Celsius at sea level. (Physics)

These statements refer to *all* events of a particular kind at any place and time; they are not about particular events or states of affairs but suitably general. The laws and theories of science involve general statements of this kind; they are *universal statements*.

The problem then for those who think that science starts with particular observation statements is to explain how one can justifiably arrive at universal statements from

[7] J.S. Mill, *A System Of Logic*, Vol. I, p. 354.

particular ones. For example: just because water boiled when heated to 100^0C at sea level by person x_1 at place p_1 and time t_1, and by person x_2 at place p_2 and time t_2, and by person x_3 at place p_3 and time t_3, and ... and by person x_n at place p_n and time t_n, how can we thereby justifiably infer that the result holds in general, for all future times, places and persons? How can the general be justified on the basis of the particular? How can a set of observations about how the universe is now justify claims as to how the universe is in general?

Inductive reasoning

The inductivist reply is that under certain conditions we can generalise. So long as the following conditions are met, we may legitimately generalise to an appropriate universal statement:

1. The number of observation statements forming the basis of a generalisation is large.
2. The observations are repeated under a wide variety of conditions.
3. No accepted observation statement conflicts with the derived universal law.

Condition (1) helps rule out anomalous cases (e.g. a defective measuring instrument) and stops one jumping to conclusions prematurely. Condition (2) implies that it is not enough to increase our base of singular statements by simply repeating tests on the same subject under the same conditions. To rule out the possibility of the observed phenomenon being due to some hidden factor we ought to test for the phenomenon under as varied conditions as possible. For example, the claim 'All liquids contract when frozen' would seem a justified generalisation if water was not considered; testing liquids under a wide variety of conditions will include testing the liquid water which is unusual in that it expands when frozen. Now obviously if water is observed to expand when frozen then the universal law 'All liquids contract when frozen' is not justified. Hence condition (3) is necessary.

This kind of reasoning — from a finite list of singular statements to a universal statement, from some to all — is called *inductive reasoning* and the process of reasoning thus is called *induction*.[8] The simple Inductivist position can be summed up by saying: <u>science is based on inductive inference</u>.

> *Principle of Inductive Inference:*
> If a large number of As have been observed in the past, under a wide variety of conditions, to possess the property B without exception we can infer that all As have the property B.

[8] NB: there are other forms of inductive reasoning, like inferring from the fact that water has always boiled at 100^0C at sea level in the past that it will do so when I next boil it. The problems we go on to discuss apply equally to these other forms but for simplicity we shall concentrate on the simple form presented here.

Scientific knowledge is built up from and justified by a secure base of particular observation statements by induction.

Science then, on this account, is justified by its use of induction in inferring, from particular observations, general laws and theories; *scientific statements can be inductively justified by experience.* Scientific statements, based on observational and experimental evidence (i.e. the facts) are contrasted with statements of other kinds — those based on pure logic or mathematics, authority, tradition, prejudice, or any other foundation. Scientific statements are derived in a rigorous and objective manner from objective facts. Science is a body of such knowledge and scientific progress then is the piecemeal addition of laws and theories to that body of knowledge; the accumulation of facts, and new laws and theories arrived at via induction form the ever-growing observational base. This cumulative conception of scientific knowledge is sometimes called 'the bucket theory'.

Fallibilism

So the notion of proof that the inductivist relies on is inductive "proof". But it is important to realise that inductive proof falls short of certainty. Call it "proof" if you want, but it would be wrong to think inductive inference is anything like mathematical proof. When we prove Pythagoras's Theorem, we justify it as true, and because of the nature of mathematical proof we then take it to be established with *certainty*. There is no "probably" about it. This is not the case with inductive proof. Inductive inference cannot establish laws as certain. At best, it makes them *highly likely*.

For example, if water has been observed, on numerous occasions, to boil at 100^0C at sea level without exception, at best that only makes it *likely* or *probable* that all water boils at 100^0C at sea level. It is always left open to further contradictory evidence (evidence that we must recognise may be "out there", for all we know). We must recognise that our scientific investigations yield results that are clearly *fallible*. No-one in their right mind these days would claim that a currently "proven" scientific law is established as certain and so beyond revision. Since we cannot assume we have all the relevant data, scientific laws are always to be considered open to revision. In fact, we are constantly revising our scientific understanding of the world on the basis of new evidence and this involves admitting that we didn't have things "quite right" previously — a polite way of saying we were, in fact, *wrong* in what we previously thought!

If you think about it, everything we claimed to know about the scientific structure of the world in the past has been shown to be wrong. Scientists were once confident, for example, that Newton's laws of motion were absolutely certain, but new scientific evidence found in the early twentieth century led to developments in physics that resulted in their rejection in favour of more general relativity theory. Of course, you may want to say that Newton's laws weren't "wrong", they were just too general — they are a correct account of motion at low speeds. But that is to admit that, as general laws of motion (what they were put forward as describing), they *were* wrong. They are not true. More recently, scientists claim to have new evidence that suggests that they cannot account for some 80 percent of matter in the universe! They now speak of "dark matter" and "dark energy" — the stuff they can't yet detect but suppose is there. What revisions of our scientific laws will this lead to? We'll see. The point is, we can never discount the possibility of new evidence forcing revisions of our scientific understanding.

Science is an activity of constant testing of what we think we know — our *fallible* claims to knowledge — and constant searching for new information that might further confirm or refute what we think we know.

Bearing this in mind, the corresponding Principle of Inductive Inference should reflect this. It now says (and perhaps this what what many had in mind all along when it comes to induction):

> *The Weakened Principle of Inductive Inference:*
> If a large number of As have been observed in the past, under a wide variety of conditions, to possess the property B without exception we can infer that all As probably have the property B.

The common-sense account of science mentioned earlier that sees science as providing objectively *proven* knowledge is mistaken if 'proof' is read as 'proved with certainty'. Inductive proof can, at best, establish laws as reasonable-given-the-evidence.

Accepting this fallibilist shift, then, we may ask why we ought to accept the weakened principle? Can such a principle be justified? And so to problem (iii) identified earlier.

Key Points: Simple induction proceeds from the *singular* to the *general*. Legitimate inductive inference is subject to certain *conditions* (1 - 3 above). Unlike mathematical proof, scientific proof is *fallible* — induction only justifies scientific laws as *probable*.

Notes:

7.6.3 A Quick Puzzle about Models and Induction

Induction, as we have described it, involves drawing general conclusions from observations. But models, recall, simplify or distort reality. They are not accurate representations of what we observe in reality. Our data is, for example, never quite a straight line even when we model it as such. How then can our models be inductively inferred from what we observe? Does inductivism apply to models?

Notes:

7.6.4 The Problem of Induction

There are two ways we might try to justify the principle of induction: (a) we might try to justify it mathematically or logically — i.e. *a priori*; or (b) we might try to justify it scientifically, by experiment — i.e. *a posteriori*. It might not be obvious that these are the two kinds of defence one can offer, so let me elaborate.

A priori justification

Consider the inferential principle "If no As are Bs and all Cs are Bs then no As are Cs". This can be justified on the basis of purely logical considerations; it is self-evidently true. Just find yourself a nice peaceful darkened room to think about it and pure thinking alone, logic alone, should convince you of its truth. So too with "$2 + 2 = 4$". You don't need to know anything about how the world actually happens to be, no particular facts, to justify it. Such claims can be justified *a priori*.

Knowledge arrived at in this way, knowledge that can be established prior to knowledge of how the world happens to be, is called *a priori* knowledge. (It's stuff you can come to know just sitting in a dark cupboard with the door closed!) The most obvious examples of *a priori* knowledge include knowledge of mathematical principles or knowledge of logical principles. In this respect, mathematics (in particular) is quite distinct from science. Mathematics is an *a priori* discipline. Science is not *a priori* — you cannot develop scientific knowledge of the world isolated from any experience of how it happens to be, you must observe or otherwise actively engage with the world to see what, as a matter of fact, is the case.

A posteriori justification

Science is an *a posteriori* discipline. Pharmacology, for example, involves principles or claims that cannot be justified *a priori*. (If you are sitting in a dark cupboard, you'll need to open the door and have a look at the world outside to decide whether they are true!) Take a more ordinary example: "SCIE1000 students are the smartest students on campus". Pure reason, working alone prior to experience of how the world happens to be, cannot justify this claim. Sitting in a dark room with no idea of the outside world, one cannot prove it. We can only justify the claim given some knowledge of how the world is, after we have investigated the particular facts "on the ground", as it were. Such claims can only be justified *a posteriori*.

This kind of knowledge, knowledge that can only be established given some knowledge of how the world happens to be, is called *a posteriori* knowledge. Scientific knowledge is typically counted as the paradigm of *a posteriori* knowledge.

Here are some more examples of claims falling into the two categories of knowledge, justified in one of the two ways:

A PRIORI KNOWLEDGE	A POSTERIORI KNOWLEDGE
$2 + 2 = 4$	There are 2 houses of Parliament
All cats are cats	All cats land on their feet
Squares are rectangles	No celestial body has a square orbit
If it's not not 2016 then it's 2016	It is 2016

So what about the induction principle then? Does it fall under either heading?

The Principle of Induction? Provable *a priori* or *a posteriori*?

The Principle cannot be justified *a priori*. When we try to determine logically (*a priori*) how probable a specific law is given the number of observations we have made it seems always to be low — near zero. This is because no matter how many observations I make, compared with the general claim that things will *always* be as the law describes, my number of observations seems to pale into insignificance next to the potentially infinite number of possible situations I am making claims about. My evidential support for some law or theory always appears insignificant compared with the full strength of my general claim. Not only could I always be wrong (i.e. science is fallible, as discussed), but it seems that I can never even show a law to be probable on purely *a priori* grounds!

Consider an analogy. Suppose you are out on a lake in heavy fog. Everywhere you can see in your vicinity is water. This is how things are in your bit of the world at your particular time and you are utterly ignorant of everything else. You have only ever lived in a boat on water, with the rest of the world obscured by fog. How confident could you be in inferring, by induction, that the universe in general, throughout space (i.e. everywhere) and time (i.e. always), is water? Not very. How things seem here and now is not a very reliable guide to how they are everywhere and always. Yet, the (weakened) inductivist is essentially in this position, it seems. From a finite number of observations over (at best) a few hundred years, in our little local part of the cosmos, how confident can we be that we can know with a high probability how things are in the cosmos in general — as when we claim to have confidence in a scientific law? Not very. Our justification is very weak, as it involves judging the nature of the cosmos in its potential infinity from evidence of a small, finite part.

(A lot of effort has gone into trying to develop a notion of 'probability' and an inductive logic that will enable us to logically, *a priori*, calculate or estimate the degree of support theories attain given the body of evidence in their favour but that story continues and I leave it to those interested to read further ...[9])

Ok, so perhaps we can justify the induction principle *a posteriori*. That is, if we can find out enough about how the world is then perhaps we can discover (*a posteriori*) that this general principle is, as a matter of fact, true. The problem here is that this would mean trying to scientifically justify the principle — i.e. justify it experimentally (instead of mathematically or logically). But the very thing we are trying to justify is inductive scientific proof! We can't use a scientific proof to prove anything until we have a story about what scientific proof is. We can't, in particular, use a scientific proof to show that our method of scientific proof (the principle of induction) is justified.

So, we have a problem justifying the inductivist's account of what science is.

Key Points: Knowledge divides into the *a priori* and the *a posteriori*. Induction seems impossible to justify under either category.

Notes:

****

[9]See: W. Salmon, *The Foundations of Scientific Inference*, (1966).

So there is a problem trying to justify the principle of induction. This is known as "The Problem of Induction" or "Hume's Problem" since it was the Scottish philosopher David Hume who brought the problem into sharp relief in his *Treatise On Human Nature* (1739).[10] The inductivist who claims scientific knowledge based on observation seems unable to defend their use of the principle of induction.

Image 7.3: David Hume. (Source: en.wikipedia.org)

[10]You might like to look at Alan Musgrave's very readable account of the problem of induction and some responses in his *Common Sense, Science and Scepticism*, Cambridge University Press (1993), esp. Ch. 8 & 9.

Lecture 19: **Third Philosophy Lecture**

Learning objectives

✓ Understand the Popperian response to the problem of induction
✓ Understand the Popperian (falsificationist) hypothetico-deductive account of science
✓ Understand how falsifiability might be thought to be the hallmark of good science
✓ Recognise what special challenges models pose for falsificationism

7.7 Popperian Science

7.7.1 The Hypothetico-Deductivist Account of Science — Falsificationism

The philosopher of science Karl Popper certainly thought that induction (and the presumption of the uniformity of nature) was unjustifiable. He argued that we shouldn't pin our hopes on induction as an account of how science works because we'd then be relying on an unacceptable principle of induction. Popper claimed that science didn't, in fact, need induction at all. We can explain how science works and why it is rational without any need to rely on anything as suspect as induction. His account of science and scientific method has been widely accepted by the science community and has become known as "the hypothetico-deductive method", sometimes also called "falsificationism". And, properly understood, it contrasts in a number of respects with the "common-sense view" mentioned earlier.

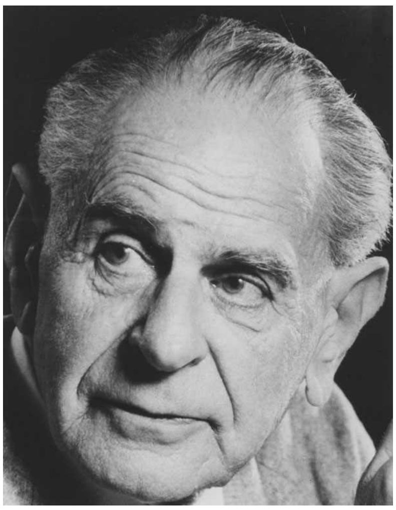


Image 7.4: Karl Popper. (Source: en.wikipedia.org)

According to Popper, science proceeds by conjectures (hypotheses) and refutations (deductive inferences that show the hypothesis to be false) — hypothesis and deduction (hence the name *hypothetico-deductivism*. Rather than trying to *prove* universal laws that constitute scientific knowledge by induction from observational evidence, as the inductivist would have us believe, science is in the business of proposing bold conjectures as laws describing what we see around us and then subjecting these conjectures to stringent tests to see if the law can withstand attempts to *falsify* it (hence the alternative name *falsificationism*). If it can withstand the tests then, though we are not in a position to claim it as true (when are we ever in such a position? — this is the force of the problem of induction) we may claim it as the best "law" currently available and so rational to believe, at least for the time being.

The popular notion that science is a body of established fact is entirely mistaken; conjectured scientific laws are, at any given time, those which have not yet been shown to be false and, on balance, are the best we currently can conceive of to account for the nature of the world around us.

Notes:

7.7.2 Inductive Proof vs. Falsification

Popper's proposed solution to the problem of induction derives its force from a logical difference between the (supposedly inductive) proof of a universal claim and the (clearly deductive) falsification or disproof of a universal. Consider the claim 'All metal expands when heated'. No number of instances of metals expanding when heated can be sufficient to prove the claim true yet a single instance of a metal not expanding when heated will be enough to prove the claim false — i.e. "falsify" it. The claim says that all As are B yet, if we can find a single A that is not B then the universal claim will be shown to be false. Unlike inductive proof, this is uncontroversial.

The falsity of a universal statement can be conclusively inferred from certain singular (particular) statements. Thus there *is* a clear and uncontroversial logical relation between singular and universal statements: singular statements, though they cannot inductively prove universal ones, can falsify them. It is this justifiable logical relation that Popper relies on to explain how it is that observation and scientific data relates to scientific laws and theories.

Laws are not justified by being proved by the data, they are justified by being not disproved by the data.

Falsifiability

Let us look more closely at just what we mean when we describe some statement as falsifiable. The following claims are all falsifiable in the Popperian sense:

1. ‘It never rains on Wednesdays.’
2. ‘All metals expand when heated.’
3. ‘Heavy objects fall straight downwards if not impeded.’
4. ‘When a ray of light is reflected from a plane mirror, the angle of incidence is equal to the angle of reflection.’

Claim (1) can be falsified if the world is observed to be such that it rains on some Wednesday. Claim (2) can be falsified if at some particular time a metal is observed not to expand under heating. Claim (3) can be falsified if a heavy object, which was not impeded in any way, was observed not to fall straight downwards. Claim (4) would be falsified if some ray of light at some time were observed to be reflected from a plane mirror in such a way that the angle of incidence was different from the angle of reflection.

The definition of “falsifiability” then is the following: *an hypothesis is falsifiable if there exists a logically possible observation statement or set of observation statements that are inconsistent with it — i.e., which if established as true would falsify the hypothesis.*

Scientific laws then (given the requirement that they be falsifiable) are testable in spite of their being unprovable; they can be tested by systematic attempts to falsify them.

Examples of claims that are not falsifiable are:

Logical Truths — e.g. ‘Either it is raining or it is not raining.’

Definitional Truths — e.g. ‘All bachelors are unmarried males.’

Mathematical Truths — e.g. ‘$2 + 2 = 4$’

Certain Modal Truths — e.g. ‘Luck is possible in sporting situations.’

This latter example is the stock-in-trade of many fortune-tellers and newspaper astrologists. Such claims can never be shown to be false because they are not capable of being falsified.

As further examples of seemingly unfalsifiable claims, consider the following:

1. 'The cosmos doubled in size overnight.'
2. 'God created the earth 6,000 years ago complete with fossil record.' (To test our faith perhaps — in this way, it is argued, we can consistently argue for creationism.)
3. 'The world came into existence only five minutes ago, complete with a "history".'

And what about:

4. 'Survival of the fittest'?

Falsifiability as the Demarcation of Scientific Statements

The falsificationist's view is that scientific hypotheses must provide information as to how the world is, and so therefore how it is not. In other words, scientific hypotheses must have some information content in the sense of ruling out certain possibilities. In fact, scientific statements, in general, whether hypotheses, laws or simply observation statements, must have information content in this sense. They must, in this sense, be "testable". Claims that are true or false regardless of how the world is tell us nothing about the world itself, are not falsifiable (i.e. not testable), and thus are not scientific (though they may appear to be scientific). Because scientific statements (including scientific hypotheses) make definite claims about the world they have informative content and so must be falsifiable.

This simple fact is used to test whether statements count as "scientific" or not. If I tell you that you may be lucky in sport today I might appear to be making a prediction about your future. In this sense it may appear that I am making a scientific claim about the future. Or, consider the claim that the electron may curve anti-clockwise in the cloud chamber and not clockwise. These claims are not falsifiable; they are not testable. (They only say that something *might* happen, and the mere *possibility* is not falsified by its *actually* not happening.) They rule nothing out. (Anything is *possible*!)

Key Point: In order to be scientific, an hypothesis must be *falsifiable*. We cannot inductively prove such statements but we can deductively *disprove* them.

Notes:

7.7.3 How Scientific Knowledge Advances

Scientific hypotheses then are by their very nature falsifiable. And the scientific method proceeds by putting forward such hypotheses, however arrived at. Then subjecting them to stringent testing to see if they are, in fact, false. Observations are compared to the consequences predicted by the hypothesis. If the observations conflict with the prediction then the (falsifiable) hypothesis is actually *falsified.* If not, if they pass stringent testing against our observations, then the hypothesis can be provisionally accepted (and if it is suitably *general* then it will be a provisionally accepted *law*) until and unless it is later falsified. If a law is falsified then we can establish this deductively, and we then go on to look for new bold hypotheses that will explain all that the old rejected law will explain and moreover which will explain the observation that led to the rejection of the old law.

There is no room for induction in this picture. Deriving predictions needs only deductive logic, and so too inferring that an hypothesis is false given our observations. Nothing on this picture "proves" hypotheses; they are simply useful conjectures that have been put forward and not yet been shown false.

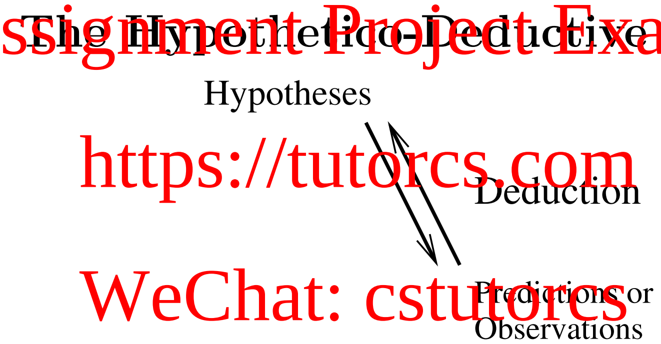


Figure 7.2: The Hypothetico-deductive account.

According to the hypothetico-deductive account then all our knowledge is, of its nature, provisional and will always be. At no stage are we in a position to be able to "prove" what we know to be true — it is always possible that it will turn out to be false. It is a simple fact of our intellectual history that nearly everything we have ever claimed at any time to know has later turned out to be false. A good example is Newton's Laws; they must have seemed so secure for two centuries until the Relativistic-turn early last century.

It is a mistake to try to prove a proposed law or theory to be true; to do so is to look for more assurance than will ever be available. What we can do is justify our preference for one candidate law (i.e. hypothesis) or theory (i.e. a collection of hypotheses) over another. (For example, the other was falsified, or was not scientific, or though scientific and not yet falsified was less informative so less useful than its preferred rival.) As theories are falsified we look for new bold conjectures that will explain all that the old rejected theory will explain and moreover which will explain the observation that led to the rejection of the old theory. *We learn from our mistakes; science progresses by trial and error.*

The popular notion that science is a body of established fact is entirely mistaken. Nothing in science is permanently established, it is changing all the time and not through the addition of new facts as the inductivist would have us believe. If we are rational then we will base our decisions on "the best available knowledge at the time", which is exactly what science provides us with.

Key Point: Popper's hypothetico-deductive account, falsificationism, describes science as a process of proposing hypotheses then deducing consequences from them for testing. Hypotheses are accepted as provisional unless they are falsified by observation.

Notes:

7.7.4 Hypothetico-Deductivism and the Problem of Induction

So, it is claimed, at no point does induction play a role in assessing the status of scientific knowledge; the problem of induction does not arise. To be sure, we may as a matter of psychological fact invoke inductive methods to think up a conjectured law or hypothesis just as we might have hit upon it in a moment of sublime inspiration, blind drunkenness, or a dream but the psychological means whereby the law or hypothesis was arrived at tells us nothing about the its status (as acceptable or unacceptable). We do not have to face the problem of justifying *how we came up with* our conjecture, whether it was by inductive inference or our dream-inspired method; the way we think up some law or hypothesis is not something requiring justification. What *does* require justification is why we might persist with such a claim and take it as something we can work with rather than abandoning it — this is done by seeing if it can pass those tests applied to it, not by any use of the problematic principle of induction.

Key Point: Since hypothetico-deductivism does not rely on induction to justify scientific laws, there is no problem of induction.

Notes:

7.7.5 A Quick Puzzle About Models and Falsificationism

Models, recall, simplify or distort reality. This means that we know, from the outset, that our models are not entirely accurate; they are, strictly speaking, false. But if they are false, it makes no sense to try and falsify them/to try to test their truth. What, then, are we doing when we are testing our models? Does Popper's falsificationism apply to models?

Notes:

Lecture 20: **Fourth Philosophy Lecture**

Learning objectives

✓ Recognise problems with the (falsificationist) hypothetico-deductive model of science

✓ Understand the Kuhnian criticism of falsificationism

✓ Understand the Kuhnian evolutionary account of science, including the notions of a paradigm and a paradigm shift

✓ Become familiar with the philosophy essay assignment

7.7.6 Problems

"Great" I hear you say. Problem solved. We can now describe scientific method as essentially hypothetico-deductive, thus establish our belief in scientific laws and theories as rational, and go on to explain how it might be that (returning to where we began) no rational person could deny human-induced global warming. Using (now) rationally justified scientific laws we can show how the hypothesis of human-induced global warming best explains the data we are confronted with, and so by inference to best explanation, rationally infer that such a phenomenon exists. All the evidence, along with rationally justified laws, points to the hypothesis and so we are rationally justified in accepting it.

But — and there is nearly always a 'but' in philosophical debate, as there often is in scientific debate — a hypothetico-deductive account does raise some interesting questions. Firstly, some have doubted that the hypothetico-deductive method actually avoids use of inductive justification of scientific laws. Recall the claim in §7.7.2: "laws are not justified by being proved by the data, they are justified by being not disproved by the data". Why should the mere fact that an hypothesis has not been disproved by the data give reason for believing it? Secondly, Popper presents cases where *one* theory is being tested against our experimental data, but hypotheses are tested in groups. When we "test" a theory, we are assuming a lot of other theories in the background. So if we find anomalous results should we reject the theory being "tested" or one of the other auxiliary hypotheses operating in the background? Lastly, even when anomalies *are* detected, we frequently do not go on to reject the theory thought to be at issue. Often we retain theories that have been successful and proved themselves powerful and look for ways to reconcile ourselves to the anomaly.

Notes:

7.8 Kuhn's View of Science and its Challenges

7.8.1 Kuhn on Scientific Revolutions

The observation that scientists sometimes protect theories from falsification has led to accounts of science, including elaborations of falsificationism, that see science as developing through stages. Thomas Kuhn's 1960s study of science and scientific method — *The Structure of Scientific Revolutions* — is a particularly famous work in this direction. Focussing heavily on how science actually functions, with a close study of the history of science, Kuhn introduced the notion of a "paradigm" into popular intellectual culture (for example, "Subvert the dominant paradigm"). His book was one of the most cited books of the twentieth century. According to Kuhn there are periods of "normal science" where people work to develop a particular scientific "paradigm" in a more-or-less Popperian way. Newtonian science formed one paradigm, for example, before being replaced by Einstein's revolution. Scientists commit to these paradigms and often persist with them even when the theories in the paradigm face contrary evidence. Eventually, the contrary evidence mounts and a rival paradigm appears. At this point there may be radical changes in our understanding of central scientific concepts (e.g. mass). So-called "revolutionary science" occurs and a new paradigm is adopted instead of the old one.

Crucially, according to Kuhn, the choice between theories from different paradigms is a place where scientific rationality is limited. We can think of a paradigm as a way of doing science within some domain. Each paradigm or way of doing science comes with its own methods of investigation, ways of explaining phenomena, important applications and characteristic types of argument. So, each paradigm comes with its own standards of what good science amounts to. Another way of putting this is by saying that paradigms are incommensurable; they have no common measure or standard that would allow their rational comparison. But then it seems that rationally choosing between theories from different paradigms will be a challenge. Proponents of each paradigm will have to use their own standards of good science in criticising their rivals' theories and thus to simply assume that the rival theories are problematic. In such a situation, according to Kuhn, there is no fully objective or rational, way of deciding which theory is correct.

For an example of incommensurability, consider the Cartesian and Newtonian paradigms. The Cartesian paradigm aimed to explain all motion by appealing to contact between material bodies (pushes and pulls). Thus, the motion of the planets was to be explained by appealing to vortices in an aether; the aether, on this paradigm, drags planets along. Now, from the Cartesian perspective, Newton's law of gravity should be rejected since it posits, or at least permits, action at a distance. For Newtonians, however, action at a distance was acceptable and did not provide a strong argument against Newtonian gravity.

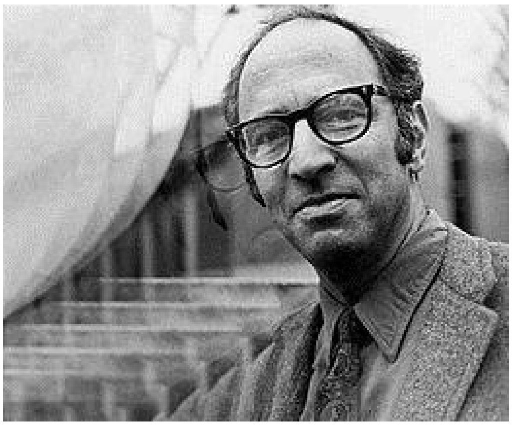


Image 7.5: Thomas Kuhn. (Source: en.wikipedia.org)

Notes:

7.8.2 Responding to Kuhn

There are, however, possible responses to Kuhn. First, if it really is true that we cannot rationally decide between rival paradigms, then it seems the rational thing to do is not to choose between them. In such a situation, we have to accept that we do not know which paradigm is correct. Further, scientists can continue to develop rival paradigms alongside each other. Indeed, we can encourage individual scientist to work in multiple paradigms.

Second, choosing between paradigms need not amount to begging the question by using the standards of one paradigm to judge the correctness of rival paradigms. There is always the possibility that those working within a paradigm will come to realise, using the standards of their own paradigm, that their paradigm is failing and needs to be abandoned.

Finally, there may at least be some standards for judging which paradigms are preferable that are, or should be, shared by different paradigms and that allow us objectively to decide between paradigms. This was the strategy preferred by Imre Lakatos in his 1970 text Falsification and the Methodology of Scientific Research Programmes. Lakatos agreed with Kuhn that when a paradigm confronts anomalies, it is sometimes rational to protect the paradigm from falsification. However, Lakatos also thought that we can compare the empirical successes and failures of rival paradigms over time and use such a comparison rationally to decide between the paradigms. If one paradigm has been making interesting new predictions over time and, further, these predictions have been successful while the new predictions of a rival paradigm invariably fail and give rise to anomalies, the paradigm making the new, successful predictions can rationally be preferred over its rival.

Notes:

7.8.3 Beyond Kuhn

Kuhn and Lakatos were by no means the only ones to develop an evolutionary account of science. Grace and Theodore de Laguna, for example, already develop an account that is in some ways more sophisticated than Kuhn's in their 1910 book *Dogmatism and Evolution: Essays in Modern Philosophy*, The MacMillan Company.

Image 7.6: Grace de Laguna and her daughter Frederica. (Source: fredericadelaguna-northernbooks.com)

There is also the later work of the radical Paul Feyerabend (see his entertaining 1975 essay "How to Defend Society Against Science", for example – http://www.galilean-library.org/manuscript.php?postid=43842) who argued that there is no such thing as *the* scientific method. He claims that close study of the history of science shows that there is *no* set of rules, *no method*, that describes "scientific progress". More recently, there is Deobarah Mayo's updated version of falsificationism, according to which some severe tests *can* reliably select true hypotheses (see her *Error and the Growth of Experimental Knowledge*, Chicago University Press (1996)).

Image 7.7: Deborah Mayo. (Source: larspsyll.wordpress.com)

We won't pursue this further. But there is a lot more to be said here. Those interested might look at Alan Chalmers' very readable book *What Is This Thing Called Science?*, available from the Library – much of the notes so far follow this text. (There are a number of editions but most after edition 1 are very good.) A very good introduction to some of the historical material is J. Losee's *A Historical Introduction to the Philosophy of Science*, 3rd ed. (1993), and D. Oldroyd's *The Arch of Knowledge*, University of New South Wales Press (1986). Further material is also mentioned in the notes and footnotes.

Notes:

7.9 Exercise

Create your own glossary by writing down definitions of the following terms:

(a) Induction

(b) Singular statement

(c) Universal statement

(d) Hypothesis

(e) Fallible

(f) Weak induction

(g) A priori

(h) A posteriori

(i) Hypothetico-deductive method

(j) Falsifiable

(k) Falsified

(k) Paradigm

(k) Scientific revolution

(k) Incommensurability

Chapter 8: **Combining curves**

Lecture 21: **Merging models**

Learning objectives

✓ Analyse more complex models of real-world phenomenon

✓ Compare models and critically evaluate predictions made by extrapolating

✓ Understand interactions between different factors in models

Scientific examples

✓ Atmospheric carbon dioxide

✓ Wind chill and apparent temperature

Maths skills

✓ Understand when and how to combine functions

✓ Interpret functions with multiple variables

Image 8.1: Right panel of *The Garden of Earthly Delights* (1503 – 1504), Hieronymus Bosch (c. 1450 – 1516), Museo del Prado, Madrid. (Source: en.wikipedia.org)

In this chapter we will explore some more complex functions, such as those with several variables and those which are combinations of functions we have previously seen. We will model the Keeling curve again, using combinations of the functions we saw in previous sections and we will model apparent temperature a function of several variables.

In this chapter, we introduce a type of function (which is the product of two functions we have already studied) called a **surge function**. These functions are most often used in pharmacokinetics, the study of what happens to a drug inside the body.

8.1 Keeling revisited

- Now we develop a more accurate mathematical model of the Keeling curve. Recall that the Keeling curve graphs the concentration of atmospheric carbon dioxide (in parts per million by volume) over time, since 1958.
- When we discussed power functions, we saw that the following function modelled the general trend of the Keeling curve quite well (increasing and bending upwards, that is increasing at an increasing rate)

$$y(t) = \frac{1}{3}t^{1.37} + 315.$$

- What we were missing from our model was a reflection of the cyclic variation that occurs in the Keeling curve data each year. We can now combine our function with a sine function in order to model the cyclic variation.

Question *8.1.1*

Give a rough sketch of the shape of the graph in each case:

(a) a power function added to a sine function;

(b) a power function multiplied by a sine function.

Which type of model is more appropriate to model the Keeling curve?

Question 8.1.2

Keeling Model 4: Figures 8.1, 8.2 and 8.3 plot the Keeling curve and the following function $y(t)$ over three different time periods.

$$y(t) = \frac{1}{3}t^{1.37} + 315 + 3.5\sin\left(\frac{2\pi}{1}(t-0.15)\right).$$

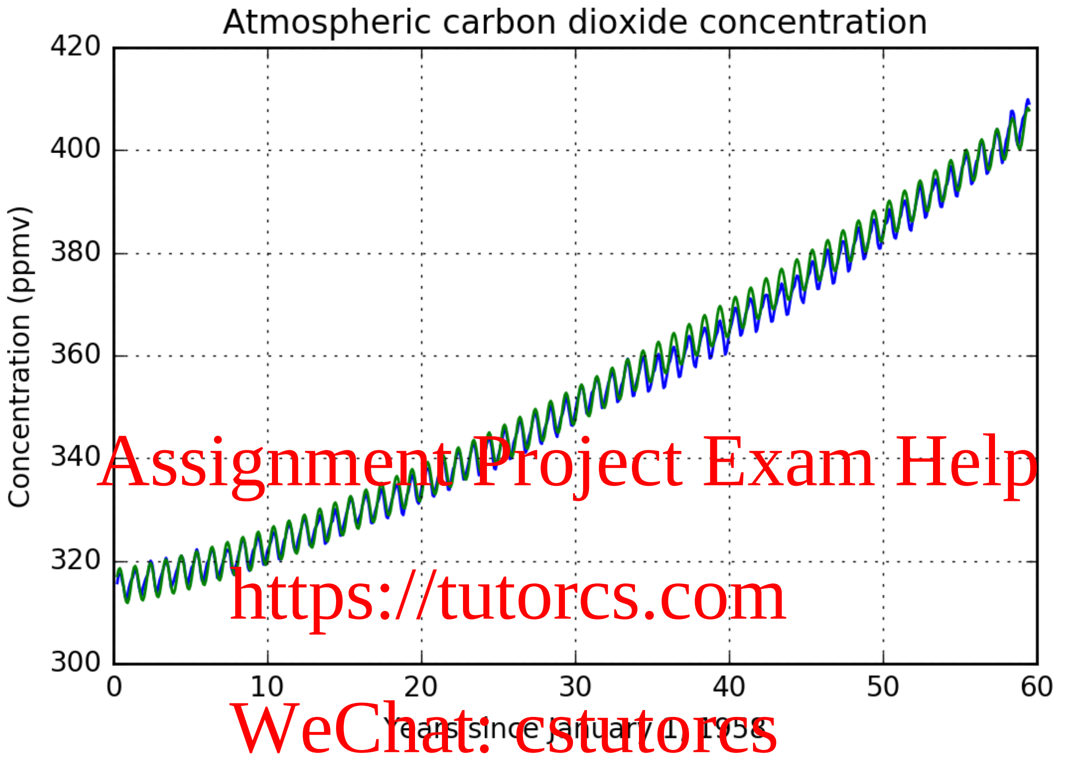


Figure 8.1: The Keeling curve and a model using sin and power functions.

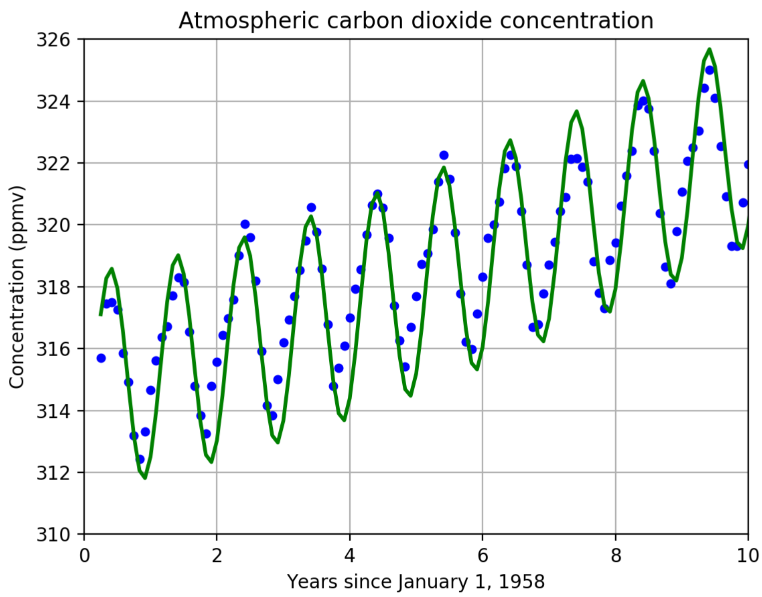


Figure 8.2: The Keeling curve and a model using sin and power functions (early years).

(continued over)

Question 8.1.2 (continued)

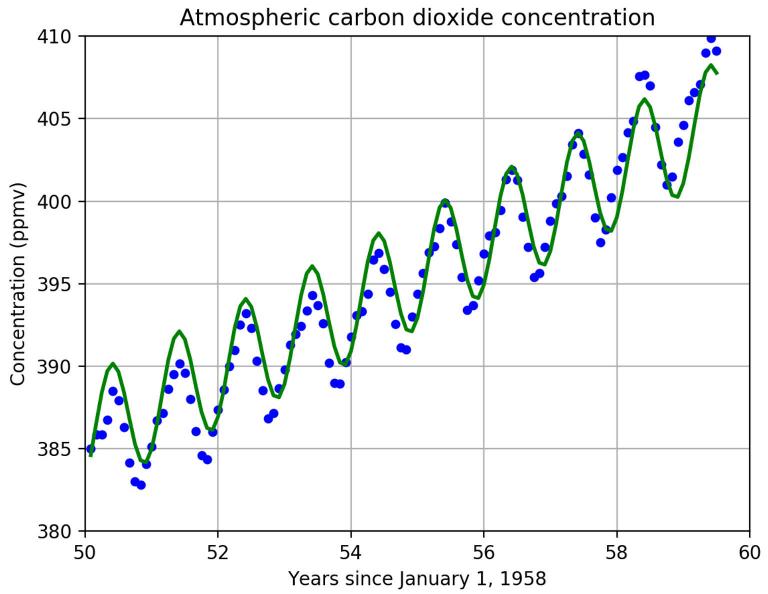


Figure 8.3: The Keeling curve and a model using sin and power functions (recent years).

(a) Explain how each term in $y(t)$ impacts on its graph. Recall that

$$y(t) = \frac{1}{3}t^{1.37} + 315 + 3.5\sin\left(\frac{2\pi}{1}(t - 0.15)\right).$$

(b) Given a physical justification for the term $(t - 0.15)$.

(c) How effectively does $y(t)$ model the Keeling curve?

Question 8.1.3

Consider the following three models of the Keeling curve.

- Model Q+S: $y(t) = 0.014t^2 + 0.7t + 315 + 3.5\sin(2\pi(t - 0.15))$.
- Model P+S: $y(t) = 1/3t^{1.37} + 315 + 3.5\sin(2\pi(t - 0.15))$.
- Model E+S: $y(t) = 280 + 35e^{0.022t} + 3.5\sin(2\pi(t - 0.15))$.

Figure 8.4 plots graphs of the Keeling curve and all three models.

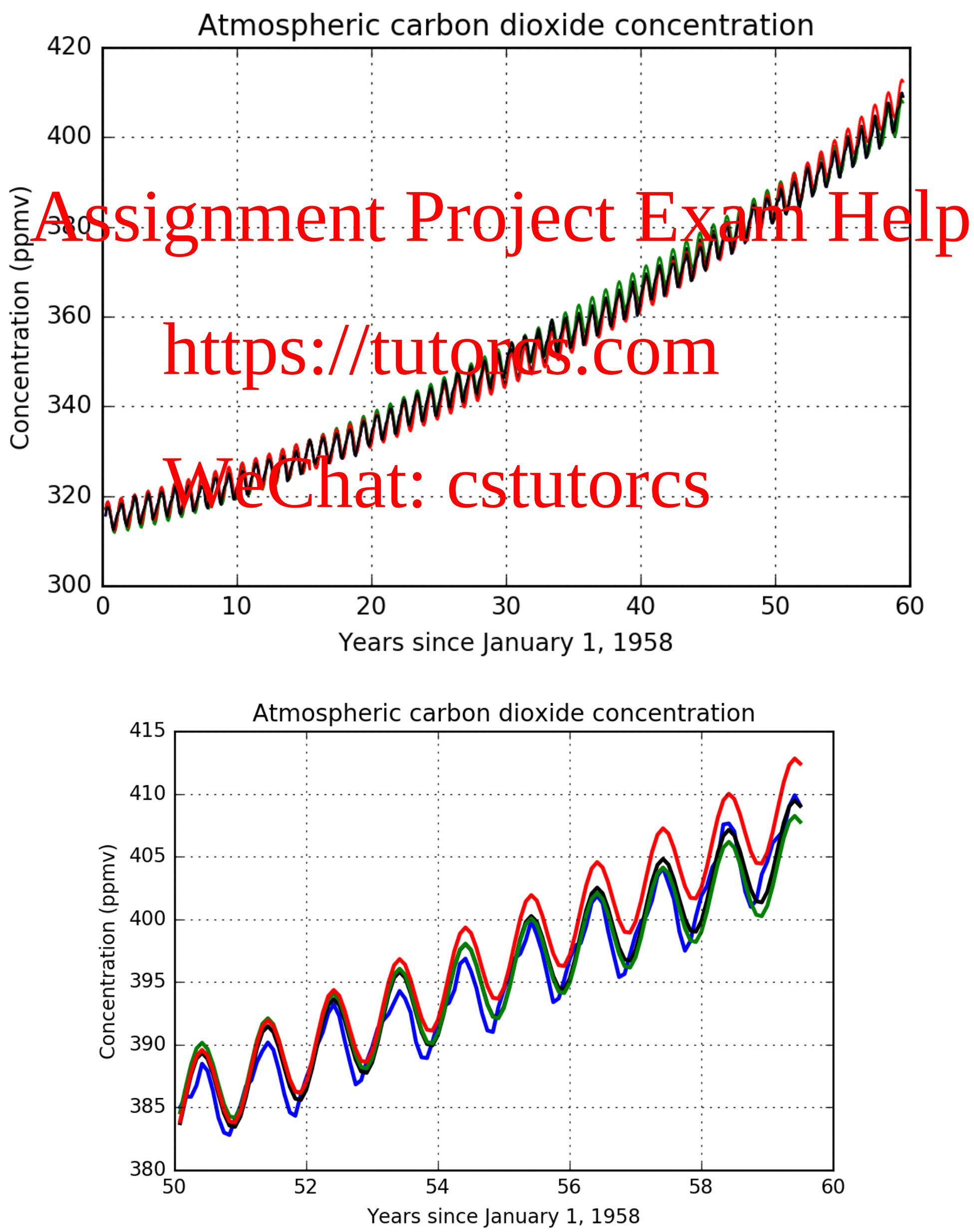


Figure 8.4: The Keeling curve and the three models for all years (top) and recent years (bottom).

(continued over)

Question 8.1.3 (continued)

(a) Which of the three models of the Keeling curve is correct? Why?

(b) Figure 8.5 extrapolates the models to the year 2058 (100 years after the Keeling study commenced). Which curve corresponds to each model?

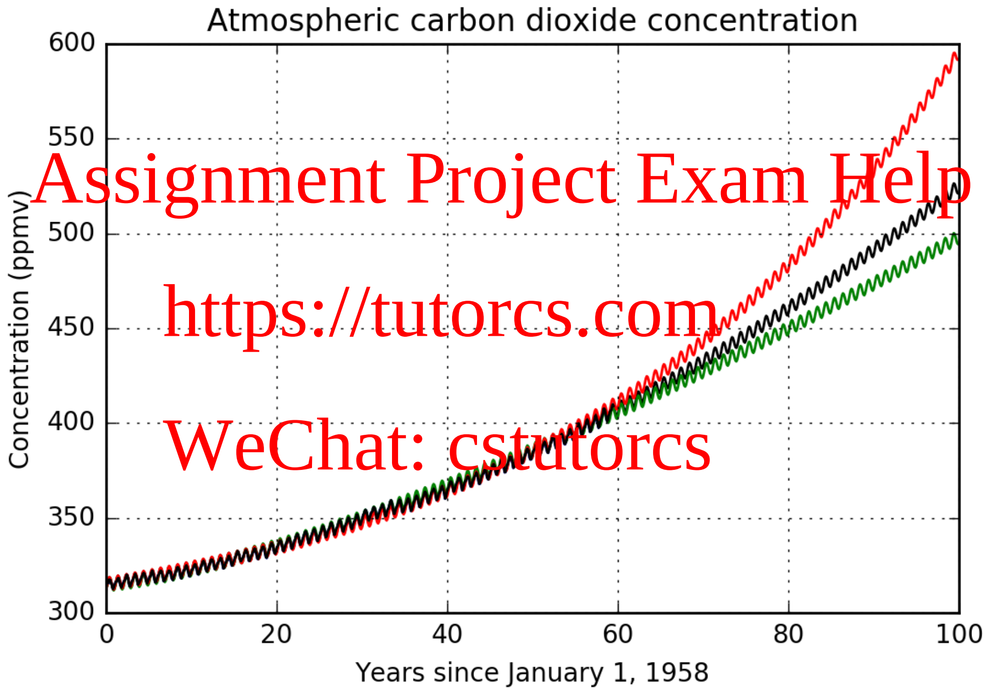


Figure 8.5: The three models of the Keeling curve, extrapolated to the year 2058.

(c) Briefly comment on the ramifications of the different predictions.

Case Study 14: **Apparent temperature for Aussies**

- Most weather apps now include a feature that explains what the "apparent temperature" is, or what the temperature "feels like".

Photo 8.1: Left: Snapshot of BOM mobile weather app (source: S.H.). Right: outback landscaper (source: https://pxhere.com/en/photo/949423, CC0).

Question 8.1.4

Derive an plausible equation that models apparent temperature. (Hint: start by deciding which factors are important, whether they increase or decrease the apparent temperature.)

Example 8.1.5

The model which is used by the Australian Bureau of Meteorology is based on the following function developed in [52]. Let T be the ambient air temperature in °C, H denote the relative humidity (%), and v be the wind speed in m/s. The perceived apparent temperature AT in °C can be modelled by:

$$\mathrm{AT} = T + 2.015\left(\frac{H}{100}\right)\exp\left(\frac{17.27T}{237.7+T}\right) - 0.7v - 4.00,$$

where we note that $\exp(x)$ is another way to write e^x.

Question 8.1.6

A simplified version of the model for apparent temperature in Australia is given by

$$\mathrm{AT} = T + 2\left(\frac{\mathrm{H}}{100}\right)e^{0.06T} - 0.7v - 4.$$

(a) For each of the three factors affecting apparent temperature, describe how you would expect an increase in that factor to affect the apparent temperature, giving a physical reason. Is this consistent with the model?

(continued over)

Question* 8.1.6 *(continued)

(b) Suppose that on one of the days during the 2009 heat wave in Mildura, at noon there was a relative humidity of 60%, wind speed of 3 m/s, and the apparent temperature AT was 4°C higher than the ambient air temperature T. What was the ambient air temperature T at that time?

End of Case Study 14: Apparent temperature for Aussies.

8.2 Apparent temperature and frostbite

Case Study 15: Wind chill

- In cold climates, the *apparent* temperature to the human body is often called the *wind chill* temperature.
- Because wind chill can cause major discomfort, and in cold climates can lead to serious injuries such as frostbite or even death, it is important to measure, model and predict the severity of wind chill.

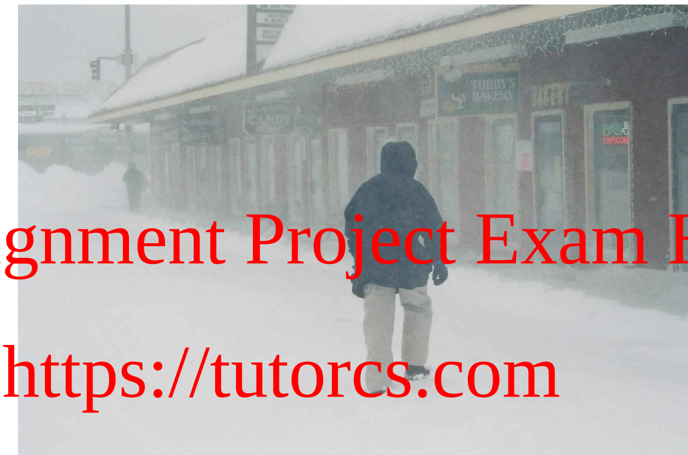


Photo 8.2: Blizzard, West Yellowstone, USA. (Source: PA.)

- It is possible to measure wind chill in a number of ways. In 2001, the US National Weather Service developed the most widely accepted model.
- Researchers exposed volunteers to various low temperatures and high wind speeds in a wind tunnel, recording their perceptions of temperatures, along with measurements of the physiological impact of wind chill on their faces.
- The researchers then formulated an equation that modelled the perceived wind chill temperature as a function of the ambient air temperature and the wind speed (for speeds of at least 5 km/h).
- Let T be the ambient air temperature in °C and v be the wind speed in km/h. The perceived wind chill temperature W in °C according to their model is:

$$W = 13.112 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16}$$

Program specifications: Write a program that inputs wind speed in km/h and air temperature in °C, then calculates the apparent wind chill temperature.

Program 8.1: Wind chill

```
# A program to calculate apparent wind chill temperatures.
from pylab import *

airT = float(input("Enter air temp. in degrees Celsius: "))
windS = float(input("Enter wind speed in km/h: "))
x = pow(windS,0.16)
windC = 13.112 + 0.6215 * airT - 11.37 * x + 0.3965 * airT * x
rt = round(windC,1)

print("An air temp. of ",airT," Celsius and wind speed of")
print( windS,"km/h gives a wind chill of",rt," Celsius.")
```

Here is the output from running the above program twice:


```
Enter air temp. in degrees Celsius: -19
Enter wind speed in km/h: 19
An air temp. of  -19  Celsius and wind speed of
19 km/h gives a wind chill of -29.0  Celsius.

Enter air temp. in degrees Celsius: -36
Enter wind speed in km/h: 135
An air temp. of  -36  Celsius and wind speed of
135 km/h gives a wind chill of -65.5  Celsius.
```

Photo 8.3: Mont Blanc. (Source: PA.)

- A common way to present information from the wind chill model is via a table of values, often with colour coding to show the risk of developing *frostbite*; see Figure 8.6.

		Air temperature (degrees Celsius)												
		10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50
Wind Speed (km/h)	5	10	4	-2	-7	-13	-19	-24	-30	-36	-41	-47	-53	-58
	10	9	3	-3	-9	-15	-21	-27	-33	-39	-45	-51	-57	-63
	15	8	2	-4	-11	-17	-23	-29	-35	-41	-48	-54	-60	-66
	20	7	1	-5	-12	-18	-24	-30	-37	-43	-49	-56	-62	-68
	25	7	1	-6	-12	-19	-25	-32	-38	-44	-51	-57	-64	-70
	30	7	0	-6	-13	-20	-26	-33	-39	-46	-52	-59	-65	-72
	35	6	0	-7	-14	-20	-27	-33	-40	-47	-53	-60	-66	-73
	40	6	-1	-7	-14	-21	-27	-34	-41	-48	-54	-61	-68	-74
	45	6	-1	-8	-15	-21	-28	-35	-42	-48	-55	-62	-69	-75
	50	5	-1	-8	-15	-22	-29	-35	-42	-49	-56	-63	-69	-76
	55	5	-2	-8	-15	-22	-29	-36	-43	-50	-57	-63	-70	-77
	60	5	-2	-9	-16	-23	-30	-36	-43	-50	-57	-64	-71	-78
	65	5	-2	-9	-16	-23	-30	-37	-44	-51	-58	-65	-72	-79
	70	5	-2	-9	-16	-23	-30	-37	-44	-51	-58	-65	-73	-80

Risk of developing frostbite:	
Low:	< 5% chance of developing frostbite
Increasing:	5% - 95% chance of developing frostbite in 10 to 30 mins.
High:	> 95% chance of developing frostbite in 5 to 10 mins.
Very high:	> 95% chance of developing frostbite in 2 to 5 mins.
Extreme:	> 95% chance of developing frostbite in 2 mins.

Figure 8.6: Wind chill temperatures at various ambient temperatures and wind speeds, colour-coded with frostbite risk factors.

- Frostbite is a medical condition in which intense cold causes tissues to freeze and die, most commonly in body extremities, particularly fingers and toes.
- Severe cases can lead to gangrene and the need for amputations.

End of Case Study 15: Wind chill.

Lecture 22: **Surging up and down**

Learning objectives

✓ Analyse the trend in data representing drug concentrations in the blood

Scientific examples

✓ Pharmacokinetics

Maths skills

✓ Understand the form of surge functions and their graphs

8.3 Drugs in the blood and surge functions

Question *8.3.1*

Suppose a patient consumes a drug. Sketch a graph of concentration of the drug in their blood over time. What are the key features of the graph and what are their physical meaning for the patient?

Some drug-related terminology

Broadly speaking, a *drug* is any externally derived chemical substance introduced into an organism that affects the function of that organism. Drugs may enhance physical or mental well-being, and include both medicinal and so-called recreational drugs.

Pharmacology studies the properties of drugs and their effects on living organisms.

Pharmacokinetics studies what happens to drugs inside the body, particularly the extent and rates of **absorption**, **distribution**, **metabolism** and **excretion**.

Drug concentrations

After the administration of a drug, key determinants of its impact on the body are the drug **concentration** in the bloodstream, which is commonly measured as mass per volume (such as mg/L), and the **time** over which that concentration occurs. Concentrations can be measured at various times after drug administration and plotted on a *drug concentration curve.*

- Mathematics and functions are particularly important when modelling the *change* in drug concentrations over time, as they help to predict the *impact* of the drug and the *timing* of subsequent interventions.

Case Study 16: **Zoloft and depression**

- Depression is one of the most common mental health problems.
- Unlike many health problems, depression (and other mental illnesses) can occur more frequently in young adults than in older adults; see Figure 8.7.

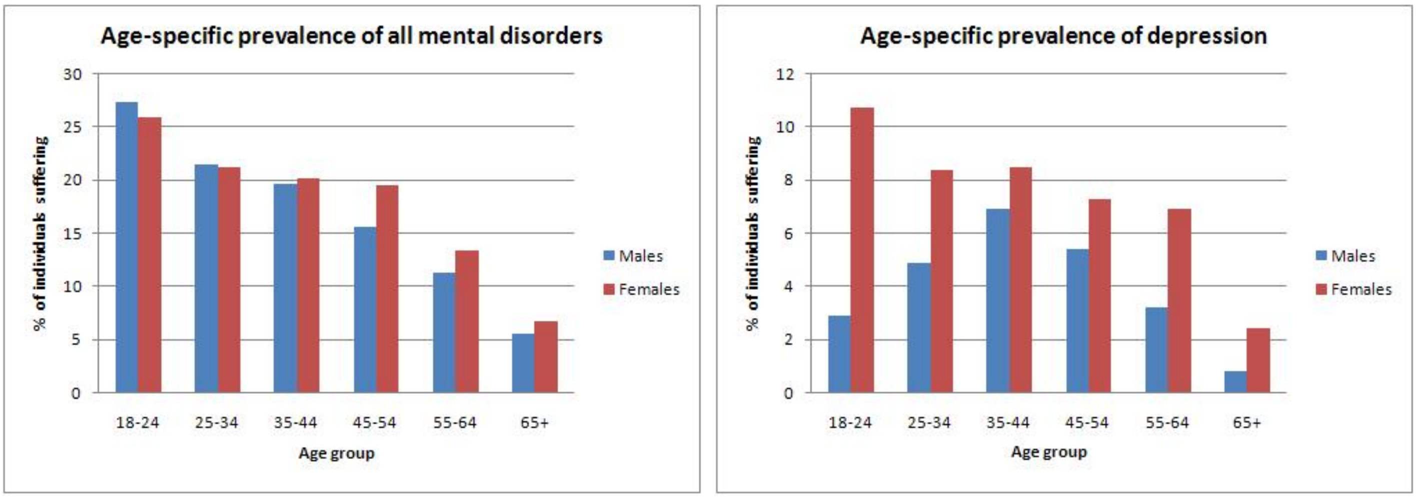


Figure 8.7: Age-specific prevalence of mental disorders and depression in Australian adults. (Source: National Survey of Mental Health and Wellbeing, 2007, Australian Bureau of Statistics.)

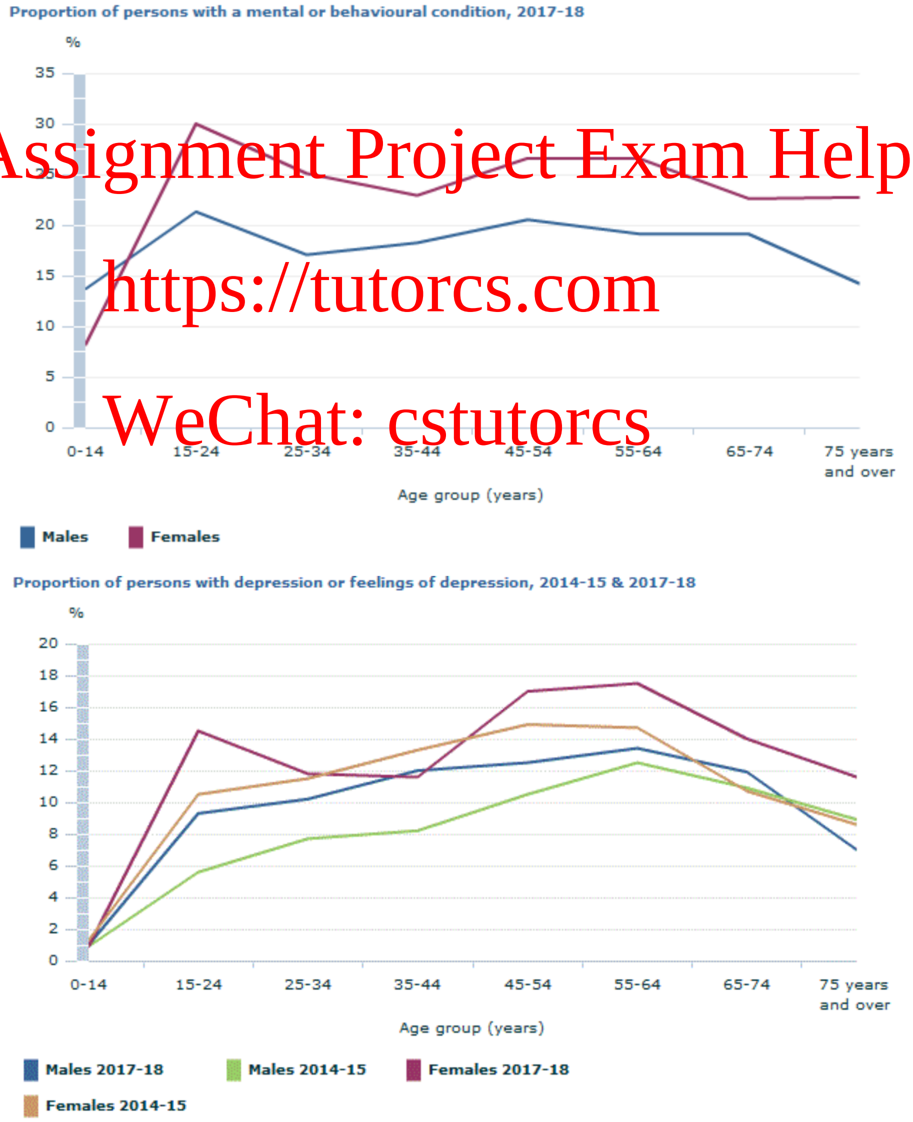


Figure 8.8: Age-specific prevalence of mental disorders and depression in Australian. (Source: National Survey of Mental Health and Wellbeing, 2018, Australian Bureau of Statistics.)

Question 8.3.2

Discuss the meaning and ramifications of the data represented in Figure 8.7 and Figure 8.8. Which figures are better for the communication of science? Why?

- There are multiple treatments available for depression, including a variety of therapy-based treatments, and pharmacological interventions.
- *Zoloft* (and a number of generically branded equivalents) is the brand name of the drug *sertraline hydrochloride*, which is an antidepressant of the SSRI class (Selective Serotonin Reuptake Inhibitor).
- The Consumer Medicine Information fact sheet explains that SSRIs "... are thought to work by blocking the uptake of a chemical called serotonin into nerve cells in the brain. Serotonin and other chemicals called amines are involved in controlling mood".
- Zoloft is the most commonly prescribed antidepressant in Australia, and one of the most prescribed drugs overall on the Australian Pharmaceutical Benefits Scheme.
- Zoloft is taken orally as a pill. The usual dosage ranges from 25 mg per day to 200 mg per day.
- Zoloft has a number of comparatively mild side effects (including insomnia, loss of appetite, and some sexual impairment), and is generally believed to be both effective and well tolerated.

Question *8.3.3*

Drug concentration curves (for sertraline or other drugs) allow pharmacologists to observe, measure and analyse factors including each of the following:
(a) the peak drug concentration C_{max};
(b) the time t_{max} at which C_{max} occurs;
(c) the *half-life* $t_{1/2}$ of the drug, which is the time taken for the concentration to fall to half of its previous value;
(d) the times at which the maximum rates of drug absorption/removal occur;
(e) the "total exposure" of the body to the drug.

Figure 8.9 shows the average blood sertraline concentrations for 11 young women involved in a study [46]. Participants received daily oral doses of sertraline over 30 days (to achieve 'steady state' concentrations), then a final dose was administered and blood concentrations monitored. Mark on the graph the values (or possible values) of each of (a) to (e) described above.

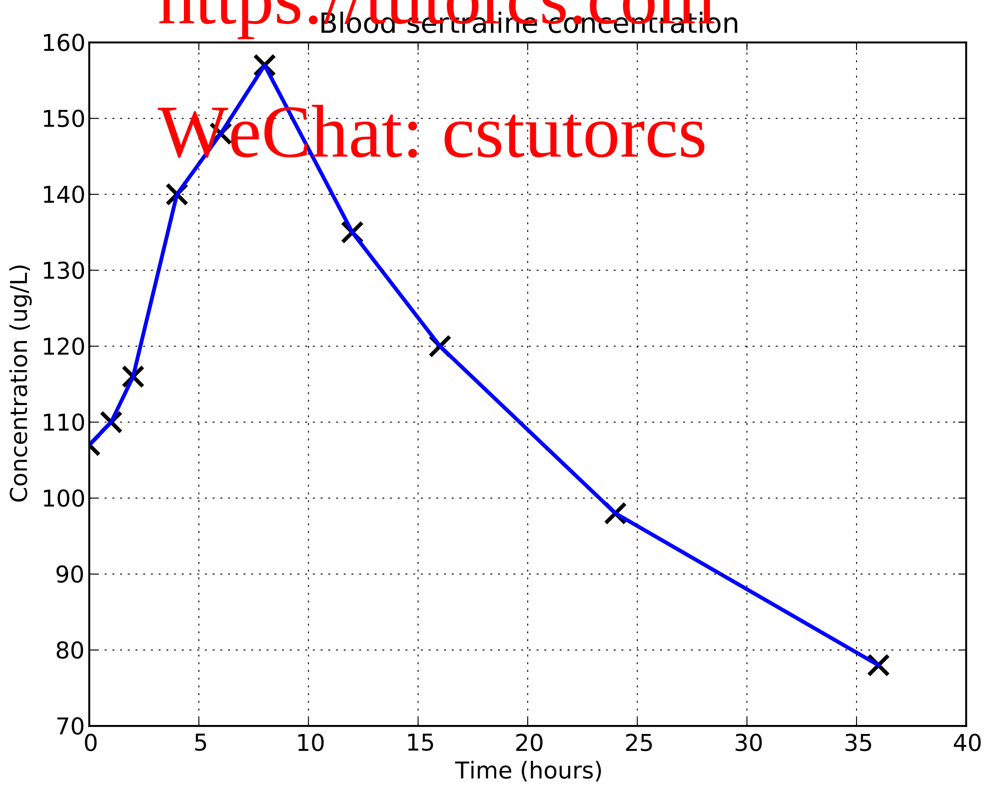


Figure 8.9: Blood sertraline concentrations in young women.

- Compare the information on Zoloft in the following example with some of the features/observations in Example 8.3.3. Also note the use of mathematical rates of change in the example.

Example 8.3.4

(The following is taken from the sertraline fact sheet at *www.pbs.gov.au*.)

“**Pharmacokinetics:** In humans, following oral once-daily dosing over the range of 50 to 200 mg for 14 days, mean peak plasma concentrations (C_{max}) of sertraline occurred between 4.5 to 8.4 hours post dosing. The average terminal elimination half-life of plasma sertraline is about 26 hours. Based on this pharmacokinetic parameter, steady-state sertraline plasma levels should be achieved after approximately one week of once-daily dosing. Linear dose-proportional pharmacokinetics were demonstrated in a single dose study in which the C_{max} and area under the plasma concentration time curve (AUC) of sertraline were proportional to dose over a range of 50 to 200 mg.

Dosage: Adults (18 years and older) The usual therapeutic dose for depression is 50 mg/day. ...patients not responding to a 50 mg/day dose may benefit from dose increases up to a maximum of 200 mg/day. Given the 24 hour elimination half-life of sertraline, dose changes should not occur at intervals of less than 1 week. The onset of therapeutic effect may be seen within 7 days ....

Use in Children and Adolescents aged less than 18 years: Sertraline should not be used in children and adolescents below the age of 18 years for the treatment of major depressive disorder. The efficacy and safety of sertraline has not been satisfactorily established for the treatment of major depressive disorder in this age group.

Overdosage: On the evidence available, sertraline has a wide margin of safety in overdose. Overdoses of sertraline alone of up to 13.5 g have been reported. Deaths have been reported involving overdoses of sertraline, primarily in combination with other drugs ....”

- The general shape of the blood sertraline concentration curve shown in Figure 8.9 is typical of many drug concentration curves. The corresponding functions are sometimes called *surge* functions.

Surge functions

In a **surge** function, the value initially rises rapidly before falling off exponentially over time. A general equation for a surge function is

$$f(t) = at^p e^{-bt}$$

where the values of a, p and b depend on the phenomenon $(0 < p < 1)$. Figure 8.10 shows the general shape of a surge function.

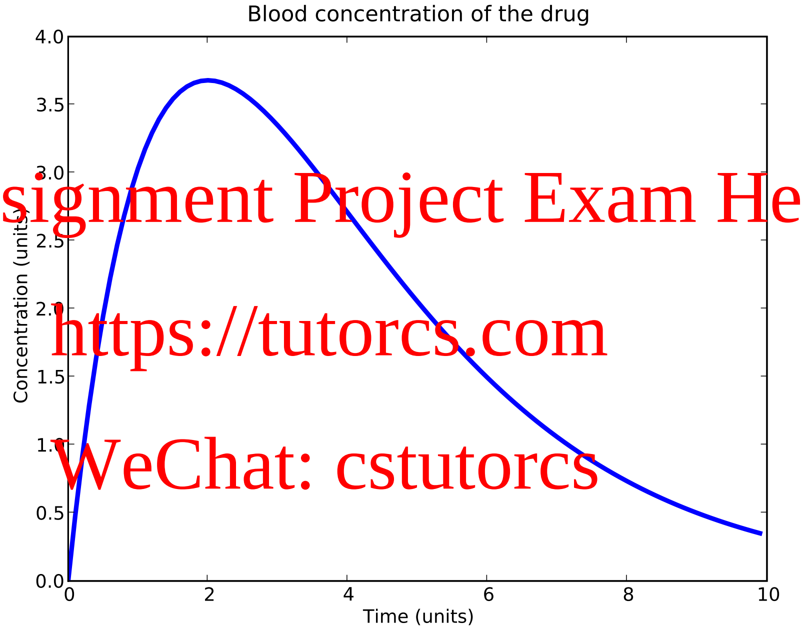


Figure 8.10: General shape of a surge function.

A surge function, as defined above, will reach a maximum when $t = \frac{p}{b}$.

Question 8.3.5

(a) Explain mathematically why functions of the form $f(t) = at^p e^{-bt}$ have a 'surge function shape'.

(b) Soon we will study some examples of surge functions, including blood concentrations of:

- paracetamol: $C_1(t) = 14t^{0.6}e^{-0.5t}$ μg/mL.
- a long-lasting contraceptive: $C_2(t) = 0.87t^{0.15}e^{-0.0008t}$ ng/mL.

Without drawing them, briefly discuss how the graphs of C_1 and C_2 would appear, including their similarities and differences. Time is measured in hours for both. Ignore the differences in concentration units.

End of Case Study 16: Zoloft and depression.

Chapter 9: **Sex, drugs and rates of change**

Lecture 23: **Average and instant change**

Learning objectives

✓ Understand the meaning and applications of average rates of change

✓ Understand the meaning and use of derivatives

Scientific examples

✓ Cyanide from smoking

✓ Metabolism of alcohol

Maths skills

✓ Calculate average rates of change

✓ Understand the meaning of derivatives

We are usually interested in studying phenomena whose values change over time. The primary mathematical tool that considers rates of change is the *derivative*. You should have encountered derivatives in previous study of mathematics. See Section C.4 in Appendix C for the pre-requisite mathematical tools we will use in this chapter. Use the online modules, available through the course website, for further support. In SCIE1000 we do not focus on how to *find* derivatives, but we instead focus on how to *use* and *interpret* them.

Image 9.1: *Skull with a burning cigarette* (c1885), Vincent van Gogh (1853 – 1890), Van Gogh Museum, Amsterdam. (Source: en.wikipedia.org)

Recall that in pharmacology, we saw that once a drug is administered, key determinants of the impact of the drug are its maximum blood concentration C_{max}, the time at which this occurs t_{max}. These are values which are calculated using derivatives.

- Pharmacokinetics is particularly concerned with the *rate* at which the drug concentration *changes*.
- The concept of one quantity changing as another quantity changes, and the rate at which the change occurs, is crucial to understanding and modelling many processes in science, engineering, social sciences and economics.
- We will cover two methods for analysing rates of change: *average* rates of change and *instantaneous* rates of change.

9.1 Rates of change

- The *average rate of change* measures the average change between two observed values of some phenomenon.
- In science, average rates of change are usually measured over time, such as 60 m s^{-1}.

***Example** 9.1.1*

The concentration of atmospheric CO_2 has risen by about 90 ppm over the last 60 years. Hence the average rate of change over this time is:

$$\frac{90 \text{ ppm}}{60 \text{ years}} = 1.5 \text{ ppm year}^{-1}.$$

- Rather than measure the average rate of change between two points, in many situations it is more useful to measure the *instantaneous* rate of change at a point.
- The mathematical term for an instantaneous rate of change is *derivative*.

Case Study 17: **Cigarettes**

- Nicotine is a highly addictive, poisonous alkaloid found in a number of plants, including tobacco. After inhaling tobacco smoke, nicotine typically enters your blood stream within five seconds, and reaches your brain after about 10 seconds.
- In addition to nicotine, tobacco products also contain a large number of other compounds (including heavy metals, poisons and radioactive materials), many of which are toxic or known carcinogens.

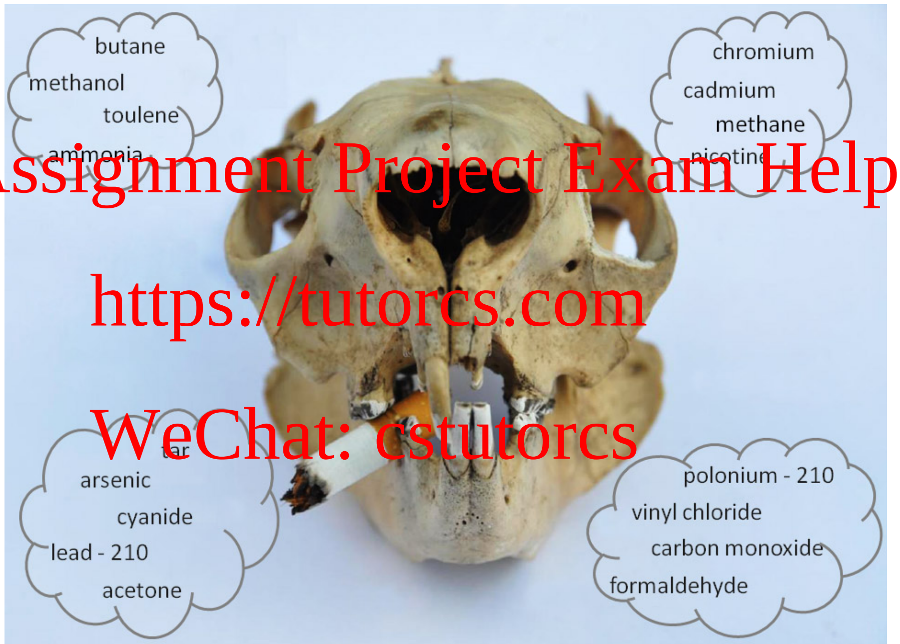


Photo 9.1: "Smoking kills": the joy of cigarette smoke. (Source: PA.)

Question 9.1.2

What types of parameters could be measured to investigate the physiological effects of smoking?

Question 9.1.3

While smoking tobacco, the body absorbs many chemical compounds in addition to nicotine, including cyanide (which is highly toxic to humans). Figure 9.1 shows blood cyanide concentrations after smoking a cigarette, starting at time $t = 0$ minutes; see [29].

t (min.)	0	5	10	15	20	25	35	65
conc. (μmol/L)	0.11	0.43	0.21	0.16	0.14	0.15	0.125	0.1

Figure 9.1: Measured cyanide concentrations in the blood of a person after smoking a cigarette.

The function $C(t) = 0.1 + 0.3t^{0.6}e^{-0.17t}$ μmol/L is a reasonable model of the measured blood cyanide concentrations. Figure 9.2 shows a plot of $C(t)$, along with the measured data values.

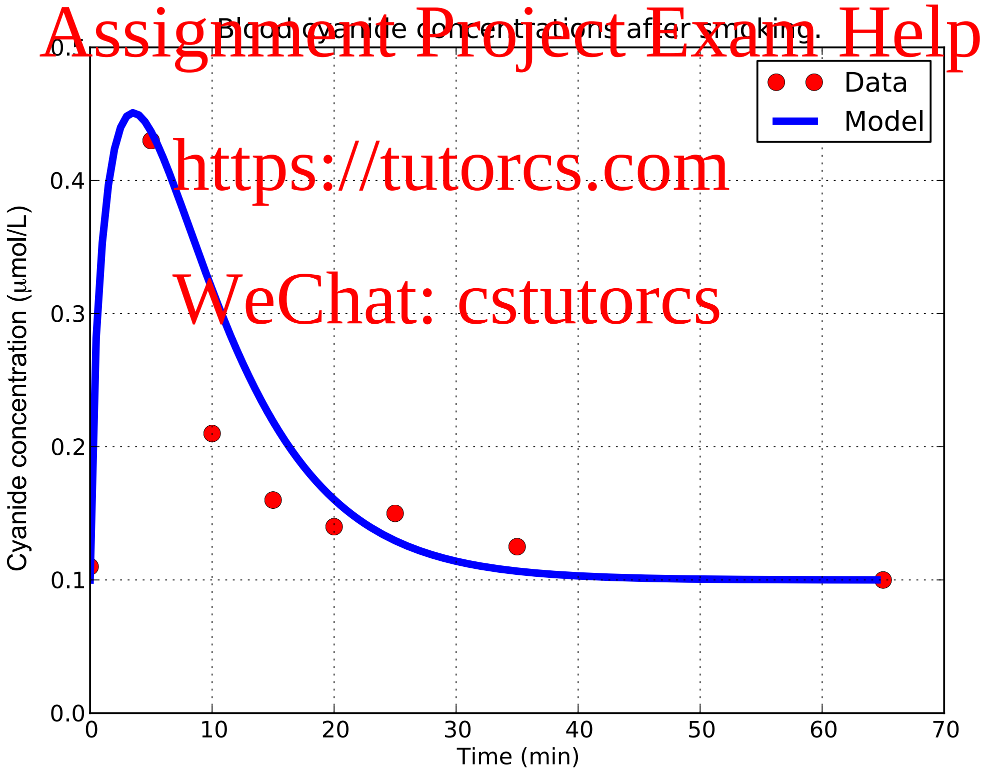


Figure 9.2: Measured and modelled blood cyanide concentrations after smoking a cigarette.

(continued over)

Question 9.1.3 *(continued)*

Using the graph of the model in Figure 9.2 we can estimate the **average** rate of change in concentration over different time intervals. For example: From $t = 0$ min to $t = 5$ min, the average rate of change is

$$\frac{\Delta c}{\Delta t} = \frac{0.43-0.1}{5-0} = 0.07\,\frac{\mu\text{mol}}{\text{L}\cdot\text{min}}.$$

From $t = 5$ min to $t = 65$ min, the average rate of change is

$$\frac{\Delta c}{\Delta t} = \frac{0.1-0.43}{65-5} = -0.006\,\frac{\mu\text{mol}}{\text{L}\cdot\text{min}}.$$

(a) What are some limitations in working with average rates of change in this context?

(b) The derivative of $f(t) = at^p e^{-bt}$ is $f'(t) = at^{p-1}e^{-bt}(p - bt)$. Thus, $C'(t) = 0.3t^{-0.4}e^{-0.17t}(0.6 - 0.17t)$.

(i) What is the physical meaning of C'?

(ii) For what value(s) of t is $C'(t) = 0$?

(iii) On the graph, identify all regions where C' is positive/negative.

(continued over)

Question *9.1.3 (continued)*

(c) What is happening physically when C' is:

(i) positive?

(ii) zero?

(iii) negative?

End of Case Study 17: Cigarettes.

9.2 Pleasures of the flesh and derivatives

Case Study 18: **Whisky (back to BAC)**

- A standard drink contains 10 g of alcohol.
- Usually, the measure of Blood Alcohol Concentration (BAC) is the percentage of total blood volume that is alcohol (or equivalently, grams of alcohol per litre of blood). In Australia the legal blood alcohol content for driving is 0.05%, or 0.5 g/L.
- Unlike many other drugs, the rate of alcohol metabolism is roughly constant (called a *zero-order* reaction in Chemistry).
- The rate of metabolism is usually independent of the BAC because typical levels of alcohol consumption saturate the metabolising capacity of enzymes within the liver.

Photo 9.2: Calf liver. (Source: PA.)

- The exact rate of metabolism varies between individuals, influenced by factors such as age, mass (weight) and sex.
- A graph of BAC from the time drinking commenced will show a rapid initial rise during the absorption phase, prior to a decline in concentration during the elimination phase.
- Because the rate of alcohol metabolism tends to be constant, a graph of BAC from the time of peak concentration shows a linear decline until metabolism is almost complete.

Question 9.2.1

Figure 9.3 shows some BAC measurements (see [56]). Let $B(t)$ represent the straight line modelling the BAC from $t = 1$ h to $t = 6$ h.

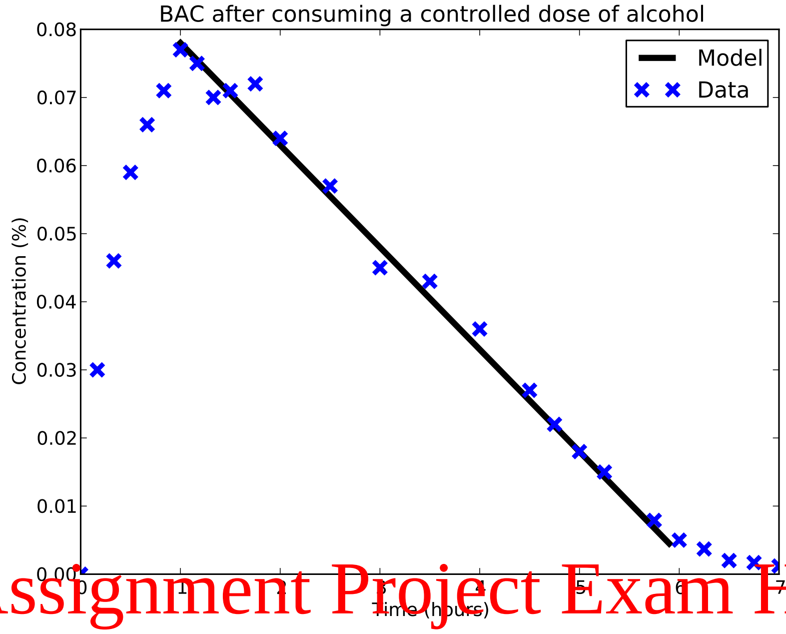


Figure 9.3: Measured blood alcohol concentrations.

(a) Find an equation for $B(t)$.

(b) Find $B'(t)$ (include units).

(c) Interpret, in words, what $B'(t)$ physically represents.

(continued over)

***Question* 9.2.1** *(continued)*

(d) Figure 9.4 shows measured BACs after researchers administered four different controlled doses of alcohol to study participants (see [56]). Discuss the value of B' for each of the graphs. What does this mean, and is it consistent with "real life"?

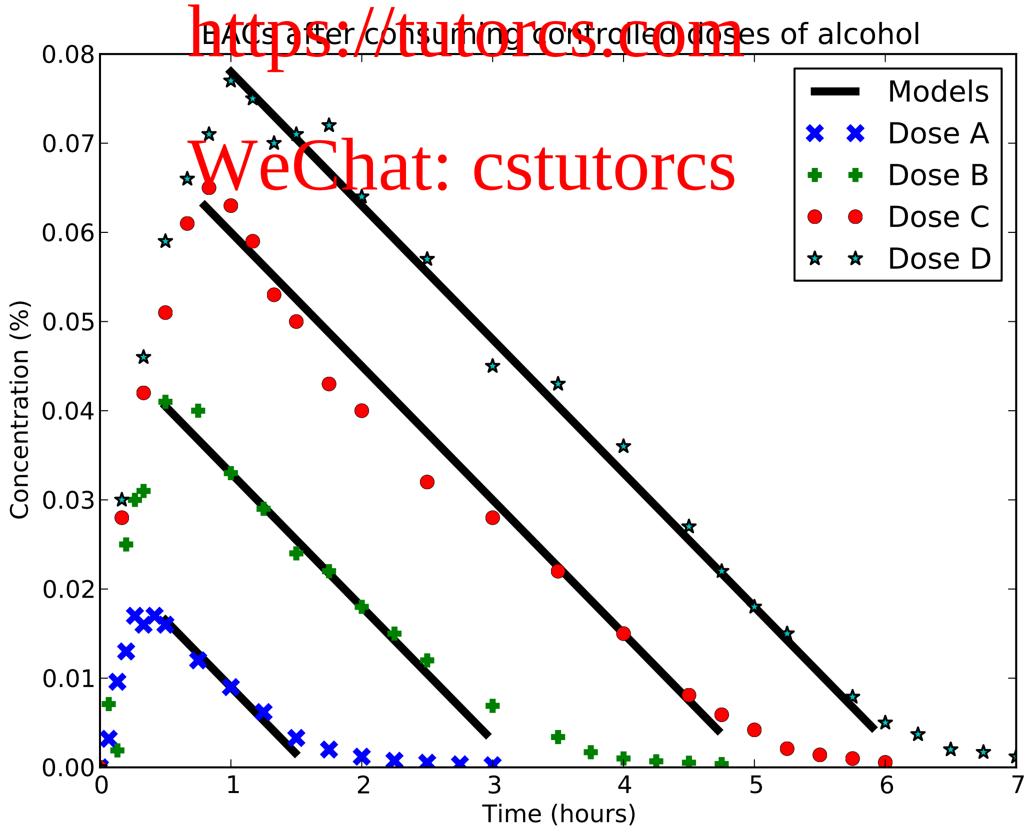


Figure 9.4: Measured BACs after administration of four different controlled doses of alcohol.

End of Case Study 18: Whisky (back to BAC).

Lecture 24: **If an equation cannot be solved...**

Learning objectives

✓ Understand how to find an approximate solution when an equation cannot be solved analytically

✓ Understand how to apply Newton's method

Scientific examples

✓ Depo-Provera for contraception

Maths skills

✓ Understand Newton's method and how to use it

9.3 Derivatives and Newton's method

Case Study 19: **Contraceptive calculations**

Photo 9.3: Various types of contraceptive including: oral contraceptive, condoms, injected contraceptives and traditional herbal methods. (Source: PA.)

- Each of the many different methods of contraception has advantages and disadvantages.
- Figure 9.5 compares the effectiveness of various methods of contraception, based on data given in [10].

Method	Typical use	Ideal use	1-year
Chance	85	85	
Spermicides	26	6	40
Periodic Abstinence	25	1 – 9	63
Cap			
Parous Women	40	26	42
Nulliparous Women	20	9	56
Sponge			
Parous Women	40	20	42
Nulliparous Women	20	9	56
Diaphragm	20	6	56
Withdrawal	19	4	
Condom	14	3	61
Oral pill	5	0.1	71
IUD	0.1 – 2.0	0.1 – 1.5	80
Depo-Provera IM 150 mg	0.3	0.3	70
Female Sterilisation	0.5	0.5	100
Male Sterilisation	0.15	0.10	100

Figure 9.5: The expected percentage of women who will experience an unintended pregnancy when using various methods of contraception for a year, through either typical use or ideal (very careful) use. Also shown is the average percentage of women continuing to use that method of contraception after one year.

- Depo-subQ Provera 104 is a long-term female contraceptive administered as an injection every $12 - 13$ weeks.

- The active ingredient in a standard 0.65 mL dose is 104 mg of the artificial female hormone medroxyprogesterone acetate (MPA), which is similar to progesterone.

- It is 99.7% effective, which is very high when compared to many other forms of contraception.

- Commonly quoted benefits are convenience and reliability.

- Studies have identified some side effects, including breakthrough bleeding, reduced libido, weight gain and potentially, reduced bone density.

***Example** 9.3.1*

Figure 9.6 shows some pharmacokinetic parameters of MPA after a single subcutaneous injection of Depo-SubQ Provera 104 in healthy women. The data are based on results in [10], with a sample size of $n = 42$ women.

	C_{max} (ng/mL)	t_{max} (day)	C_{91} (ng/mL)	AUC_{0-91} (ng day/mL)	$AUC_{0-\infty}$ (ng day/mL)	$t_{1/2}$ (day)
Mean	1.56	8.8	0.402	66.98	92.84	43
Min	0.53	2.0	0.133	20.63	31.36	16
Max	3.08	80.0	0.733	139.79	162.29	114

Figure 9.6: Pharmacokinetic parameters of MPA.

In Figure 9.6:

- C_{max} = peak blood concentration;
- t_{max} = the time at which C_{max} occurs;
- C_{91} = blood concentration at 91 days;
- AUC_{0-91} = the area under the concentration-time curve over 91 days;
- $AUC_{0-\infty}$ = the area under the concentration-time curve over an indefinite time period, and
- $t_{1/2}$ = half-life removal of MPA from the body.

Question 9.3.2

The blood concentration of an injected long-lasting female contraceptive (medroxyprogesterone acetate or MPA) in ng/mL can be modelled by the function $C(t) = 1.4t^{0.15}e^{-0.02t}$. The graph of $C(t)$ is:

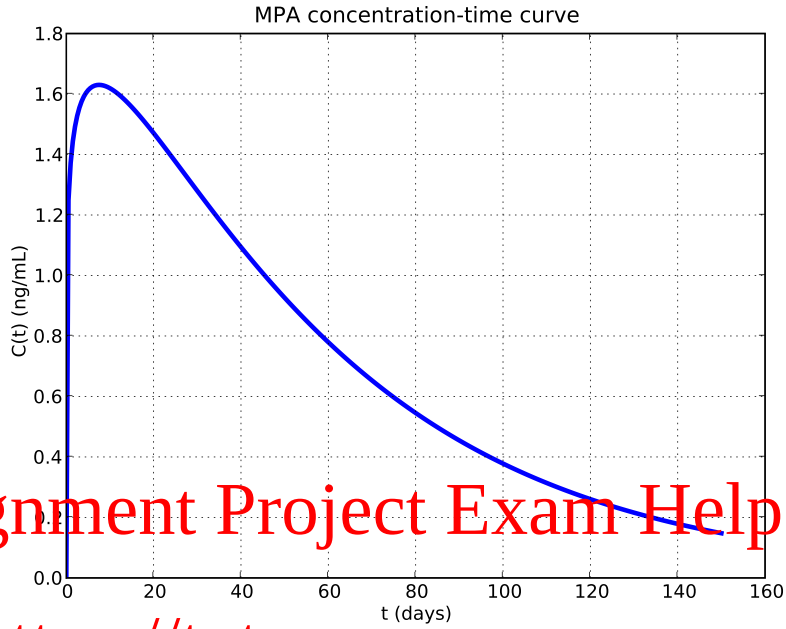


Figure 9.7: Modelled blood concentration after an injection of MPA.

(a) The minimum blood concentration for reliable contraception is 0.3 ng/mL. Estimate the time at which reliable contraception ceases.

(b) Write an equation that should be solved to accurately find this time.

(c) Do you think you could solve the equation in Part (b)?

- Some equations are difficult or impossible to solve **exactly**. An alternative is to find an **approximate** solution, using *solution-finding* algorithms, which involve repeatedly applying similar mathematical steps or *iterations*.

- Usually, a *numerical error* is associated with the approximate solutions. These errors can often be reduced by performing more iterations.

- *Newton's method* is an iterative solution-finding algorithm which uses an *initial estimate* of a solution and a derivative to find a solution. Newton's method does not always *converge* to a solution, but will usually converge if the initial estimate is 'good enough'.

- Note that Newton's method only solves equations of the form $f(x) = 0$. Before applying Newton's method, the equation may need rearranging. For example, to use Newton's method to solve the equation in Part (b) of Question 9.3.2, we instead solve $C(t) - 0.3 = 0$.

Newton's method

Informal description: To solve $f(x) = 0$:

1. Choose an initial estimate of the solution.

2. Calculate a new estimate using the old estimate and the derivative. (The new estimate is hopefully better than the old.)

3. Stop if the new estimate is sufficiently accurate or if too many steps have been taken. Otherwise, return to Step 2.

Formal description: To solve $f(x) = 0$:

1. Let x_0 be an initial estimate of a solution of f that is 'sufficiently close' to an actual solution of f. At the ith iteration ($i = 0, 1, 2, \ldots$), x_i is the current approximation of the actual solution.

2. Calculate the next estimate: $$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

3.(a) If the value of x_{i+1} is sufficiently accurate then stop; x_{i+1} is the estimated solution.

(continued over)

Newton's method *(continued)*

(b) If x_{i+1} is not sufficiently accurate after a certain number of steps then stop, because the method is probably not converging to a solution. Choose a 'better' value for x_0 and start again.

(c) Otherwise, return to Step 2.

- Newton's method is based on equations of straight lines!
- Let the initial estimate of an unknown solution of $f(x)$ be x_0. Newton's method calculates the next estimate x_1 by extending a line from the point $(x_0, f(x_0))$ to the x-axis, with the slope of the line equal to the value of the derivative f' at the point x_0; see Figure 9.8.

Question 9.3.3

Annotate the following figure to describe how Newton's method repeatedly uses a guess and the tangent line to find the next guess.

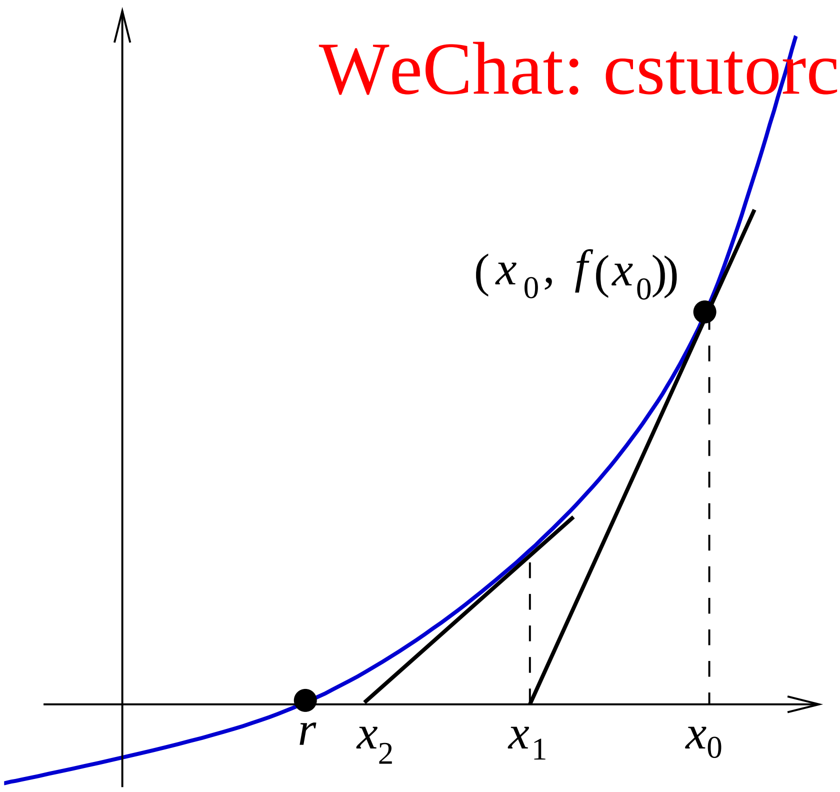


Figure 9.8: Two steps of Newton's method.

- If x_0 is sufficiently close to the solution, then the new approximation x_1 will be better than x_0.

- These steps continue until either Newton's method finds a sufficiently accurate approximation to the solution, or the process has taken too many steps without converging to a solution.

Example *9.3.4*

Use Newton's method to estimate $x = \sqrt{12}$.

Newton's method is used to solve equations of the form $f(x) = 0$. We want to solve $x = \sqrt{12}$, which does not have the right form. However, we know that $x = \sqrt{12}$ will be a solution to $x^2 - 12 = 0$, and this equation has the form for Newton's method. Hence, let $f(x) = x^2 - 12$; we aim to solve $f(x) = 0$.

To apply Newton's method, we first need to find the derivative and then choose an initial estimate of the solution:

- Because $f(x) = x^2 - 12$, we have $f'(x) = 2x$.
- We know that $\sqrt{12}$ is between 3 and 4, so we will use $x_0 = 3$ as the initial estimate of the solution. (We could choose other estimates but $x_0 = 3$ is likely to be "close" to the solution.)

Now we have everything we need to use Newton's method. Recall that the equation for finding the next estimate of the solution is:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$

Performing three steps of Newton's method gives the results shown in Figure 9.9, with the last column showing the sequence of approximations to the solution.

(continued over)

Example 9.3.4 (continued)

i	x_i	$f(x_i) = x_i^2 - 12$	$f'(x_i) = 2x_i$	x_{i+1}
0	3	-3	6	3.5
1	3.5	0.25	7	3.4642857
2	3.4642857	0.001275	6.92857	3.4641016

Figure 9.9: Using three iterations of Newton's method to find $\sqrt{12}$.

After three steps, the estimate of $\sqrt{12}$ is $x_3 = 3.4641016$. Note that:

- The estimated solution barely changed between x_2 and x_3.
- The estimate of the solution is quite accurate; in fact, x_3 is correct to seven decimal places.

Example 9.3.5

A patient receives an injection of Depo-subQ Provera 104. After the dose, the concentration $C(t)$ of MPA in her blood (in ng/mL at time t in days) is modelled by $C(t) = 1.4t^{0.15}e^{-0.02t}$. Figure 9.10 shows the graph of $C(t)$.

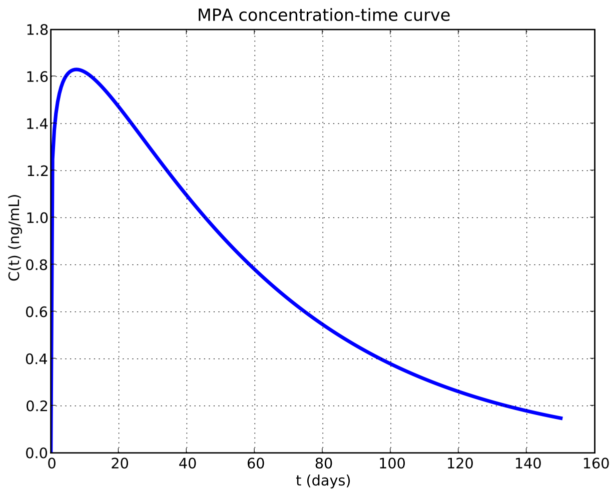


Figure 9.10: A model of the blood concentration of MPA.

- Now we will demonstrate how to use derivatives and Newton's method to determine the timing of a follow-up injection. This process is ideally suited to being performed by a computer.
- Reliable contraception ceases when $C(t) = 0.3$ ng/mL, so $C(t) - 0.3 = 0$.
- Hence the function $f(t)$ for Newton's method is:

$$f(t) = 1.4t^{0.15}e^{-0.02t} - 0.3.$$

- The derivative is:

$$f'(t) = 1.4e^{-0.02t}\left(0.15t^{-0.85} - 0.02t^{0.15}\right).$$

- We can use $t_0 = 50$ days as the initial estimate for the solution. (Remember that you usually have a choice of many different initial values.)

Question 9.3.6

Use one step of Newton's method to find a "better" approximate solution to $f(t) = 0$ for this example.

- The following program was run, and on the fourth step gave: $t_4 \approx 112.440$ days. Thus, the blood concentration of MPA decreased to 0.3 ng/mL at around 112 days, or about 16 weeks.
- For reference, the time recommended by the manufacturer for follow-up injections is 12–13 weeks, which provides a reasonable safety margin.

Program 9.1: Using Newton's method for contraception

```
# Use Newton's method to find the follow-up time for a
# contraceptive injection.
from pylab import *

# Define the function and its derivative.
def func(t):
    return 1.4 * pow(t,0.15) * exp(-0.02*t) - 0.3

def funcDash(t):
    val1 = 1.4 * exp(-0.02*t)
    val2 = 0.15 * pow(t,-0.85) - 0.02*pow(t,0.15)
    return(val1 * val2)

# Initialise variables
val = float(input("What is the initial estimate? "))

# Loop through steps of Newton's method.
i=0
while abs (func(val)) > 0.0001:
    val = val - func(val) / funcDash(val)
    i = i+1
    print("Step ",i,":",round(val,3))

print("Estimated time is:",round(val,3),"days")
```


Here is the output from running the above program:

```
What is the initial estimate? 50
Step  1 : 89.769
Step  2 : 108.467
Step  3 : 112.302
Step  4 : 112.44
Estimated time is: 112.44 days
```

End of Case Study 19: Contraceptive calculations.

Lecture 25: **Eat before you drink**

Learning objectives

✓ Analyse the shapes of graphs and the effect of different constants

✓ Use the meaning of derivatives and to understand and interpret shapes of graphs

Scientific examples

✓ Alcohol exposure on a full vs empty stomach

Maths skills

✓ Graph functions

✓ Using derivatives to find maxima or minima

Case Study 20: **Drink deriving**

- In practice (particularly in legal cases), models of BAC use the *Widmark formula*, developed in 1932. The equation is:

$$B = \frac{A}{rM} \times 100\% - Vt$$

  where B is the BAC at time t since commencing drinking, A is the amount of alcohol consumed in grams, V is the rate at which the body eliminates alcohol measured in % per time period, M is the body mass in grams and the *Widmark factor* r estimates the proportion of body mass that is water.

- The precise value of r depends on factors such as sex, age and percentage body fat. Reasonable estimates are $r \approx 0.7$ for adult males and $r \approx 0.6$ for adult females. The typical value for V is 0.015 % hr^{-1}.

Question 9.3.7 *(continued)*

(a) What is the physical meaning of the term rM in the Widmark formula?

(b) The Widmark formula is: $B = \dfrac{A}{rM} \times 100\% - Vt$. Sketch rough graphs of B against time for a "typical" male and a "typical" female who each consume the same amount of alcohol (that is, assume A is constant).

(continued over)

Question *9.3.7* *(continued)*

(c) Use the Widmark formula to justify Australian government guidelines that to remain below the legal driving BAC limit, within the first hour "men should drink at most two drinks and women at most one".

Question *9.3.8*

In the Widmark formula, the absorption term assumes that the body absorbs alcohol **immediately** after consumption. The following variation, from [43], takes into account absorption time.

(continued over)

Question *9.3.8 (continued)*

$$B = \frac{A}{rM} \times \left(1 - e^{-kt}\right) \times 100\% - Vt$$

where k is the rate at which the body absorbs alcohol.

(a) Expand this variation of the Widmark formula and compare it with the "standard" Widmark formula (which is $B = \frac{A}{rM} \times 100\% - Vt$.) Sketch rough graphs of each.

(continued over)

***Question** 9.3.8* *(continued)*

(b) Recall that $B = \dfrac{A}{rM} \times \left(1 - e^{-kt}\right) \times 100\% - Vt$.

If t is measured in hours, what are the units of k?

(c) What factors could influence the value of k?

(d) Let t_{max} be the time at which BAC reaches its maximum value B_{max}. For larger values of k, will t_{max} be larger, smaller or unchanged? Why? Will B_{max} be larger, smaller or unchanged? Why?

(e) Explain briefly how to find the values of t_{max} and B_{max}.

(continued over)

***Question** 9.3.8 (continued)*

(f) When consuming alcohol with food in the stomach, $k \approx 2.3$/hr, but when the stomach contains no food, $k \approx 6$/hr. When a "typical" man of mass 80 kg consumes 4 standard drinks with food in his stomach, we have

$$B(t) \approx 0.0714(1 - e^{-2.3t}) - 0.015t$$

$$B'(t) \approx 0.164e^{-2.3t} - 0.015.$$

Find t_{max} and B_{max} for this man.

(g) If the same man consumes the same amount of alcohol, but on an empty stomach, we have $t_{max} \approx 0.56$ hours and $B_{max} \approx 0.0605$ %. Compare this with your answer to Part (f), and relate this to Part (d).

Now we can develop a computer model of food consumption and BACs.

Program specifications: Write a program that plots BAC curves up to 5 hours after alcohol consumption on both a full and empty stomach, for men or women of varying masses and for various levels of alcohol consumption.

Program 9.2: BACs and food consumption

```
# Program to compare BACs for men and women of varying masses
# and levels of alcohol consumption, on full and empty stomachs.
from pylab import *

alcohol = float(input("How many g of pure alcohol consumed? "))
weight = float(input("Person's mass (in kg)? "))
sex = float(input("Type 1 if male, 2 if female: "))
if sex == 1:
    mult = 100 * alcohol / (0.7*weight*1000)
else:
    mult = 100 * alcohol / (0.6*weight*1000)
times = arange(0,5.1,0.1)

# Calculate the BAC at each time.
BACfull = mult * (1 - exp(-times * 2.3)) - 0.015 * times
BACempt = mult * (1 - exp(-times *   6)) - 0.015 * times
i = 0

# BAC cannot be negative.
while i<size(times):
    BACfull[i] = max(BACfull[i], 0)
    BACempt[i] = max(BACempt[i], 0)
    i = i+1

plot(times,BACfull,"k--",linewidth=4, label="Full stomach")
plot(times,BACempt,"b-",linewidth=4, label="Empty stomach")
grid(True)
xlabel("Time (hours)")
ylabel("BAC (%)")
title("BAC curve for a full stomach versus an empty stomach")
legend()
show()
```

Figure 9.11 shows the output from running the above program for an 80 kg male consuming four standard drinks:

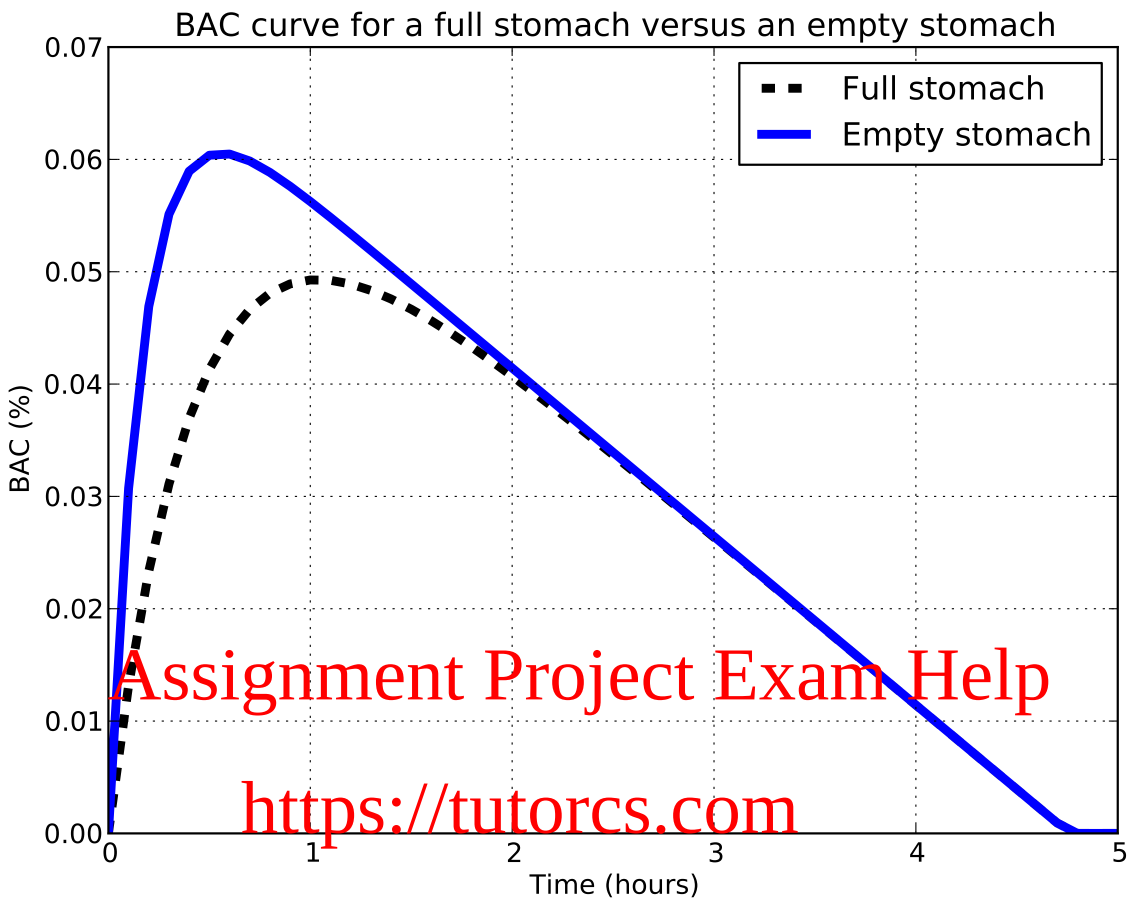


Figure 9.11: Predicted BACs when consuming alcohol on a full stomach compared to an empty stomach.

Question *9.3.9*

Briefly compare the graph in Figure 9.11 with your answers to Question 9.3.8. What are some of the physical implications of the graph?

End of Case Study 20: Drink deriving.

Chapter 10: **You, me and AUC**

Lecture 26: **To binge or not to binge...**

Learning objectives

✓ Interpret the meaning of the area under a curve in various scientific contexts

✓ Understand the meaning of the area under a curve

Scientific examples

✓ Exposure to alcohol with different drinking patterns

Maths skills

✓ Estimate the area under a curve

Image 10.1: *The Drunks* (1629), Diego Velazquez (1599 – 1660), Museo del Prado, Madrid. (Source: en.wikipedia.org)

We have noted that in pharmacology, the area under a drug concentration curve has an important physical meaning. Specifically, a key determinant of the impact of a drug once it has entered the bloodstream is the total exposure of the body to the drug, which is the area under the curve (AUC). Other related phenomena include the *bioavailability* of drugs administered by different routes, the and even such well-known concepts as the *Glycaemic Index* (GI) of foods.

In this chapter we study areas under curves (AUCs). The primary mathematical tool for analysing AUCs is the *integral*. While SCIE1000 does not cover integration techniques, we will discuss several methods of finding or estimating AUCs, and we will mainly use the *trapezoid rule*. More importantly, you will need to know how to use and interpret the results.

10.1 Areas under curves

- Given a graph, the *area under the curve* or AUC of that graph is the area bounded by that curve, the x-axis and two points on the x-axis.
- The AUC often has a useful physical meaning, which depends on what is being graphed.

***Question** 10.1.1*

What is meant by the AUC in each of the following.

(a) A graph of velocity versus time.

(b) A graph of electricity consumption in a household versus time.

(c) A graph of chlorine concentration in water versus time.

Case Study 21: **Dying for a drink**

Photo 10.1: Left: mellow and yellow. Right: better red than dead. (Source: DM.)

Question 10.1.2

Figure 10.1 shows a graph with a line fitted to some measured blood alcohol concentrations (see [56]).

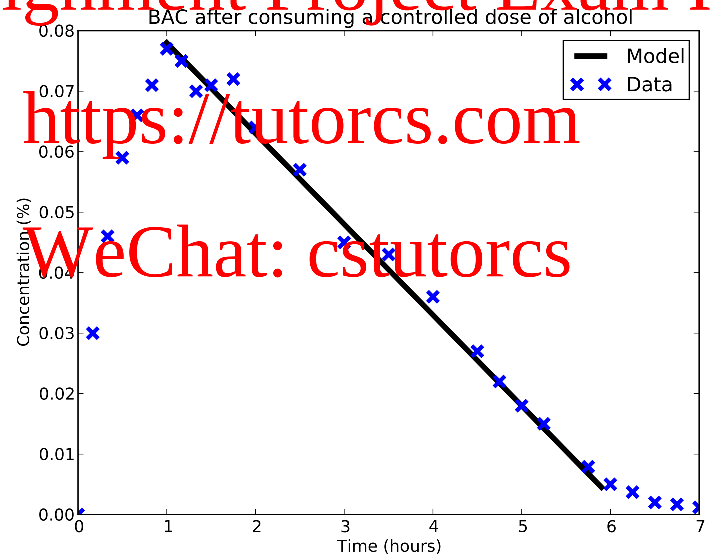


Figure 10.1: A graph of measured BACs.

(a) What are the units of the AUC in the graph?

(b) What does the AUC represent and why is it significant?

- In addition to the immediate risks associated with alcohol consumption (such as accidents), the risk of many negative long-term health effects is increased by both the frequency and volume of consumption.
- Thus, long-term health risks are affected by the *exposure* to alcohol, (that is, the area under the alcohol concentration curve).
- Recall that the "standard" *Widmark formula* is:

$$B = \frac{A}{rM} \times 100\% - Vt.$$

- The "standard" Widmark formula, does not account for absorption, but it can be used to estimate the area under the alcohol concentration curve.
- Since each standard drink contains 10 grams of alcohol, and alcohol is removed from the blood at a rate of $0.015\%/h$, we have

$$B = \frac{10n}{rM} \times 100\% - 0.015t,$$

  where B is measured in %, n is the number of standard drinks, M is the person's mass in grams, and t is measured in hours.
- Recall that r is the proportion of the person's mass that is water. Typically, $r \approx 0.7$ for males (on average) and $r \approx 0.6$ for females (on average).

***Question** 10.1.3*

The Widmark formula is used to estimate blood alcohol content (BAC); see Question 9.3.7. For a 'typical' 70 kg man drinking n standard drinks, his estimated % BAC at time t in hours since commencing drinking is

$$B = \frac{10n}{490} - 0.015t.$$

(a) Define the *total exposure to alcohol* E as the AUC of B from $t = 0$ until the BAC reaches 0 again. Find an expression for E for this man. [Hint: You will need to find an expression for the time at which his BAC returns to 0.]

(continued over)

***Question** 10.1.3 (continued)*

(b) Assume that long-term damage to internal organs from consumption of alcohol is proportional to the total exposure to alcohol E (which is simplistic, but not unreasonable). Discuss the impact on E of "one extra drink for the road".

(continued over)

Question *10.1.3 (continued)*

(c) A 'typical' man with mass 70 kg consumes two standard drinks every day. A second 'typical' man with the same mass consumes 14 standard drinks once a week, but does not drink at any other time. Estimate the weekly value of E for each man. What are some of the physical ramifications of your answer in relation to binge drinking?

(d) For a 'typical' woman of mass 60 kg, $E = 0.0257n^2$. Find the ratio of the values of E for the 'typical' woman and 'typical' man. What does this mean?

Now we can develop a computer model for total exposure to alcohol.

Program specifications: Write a Python program that uses the Widmark formula to graph the total exposure to alcohol for a person who consumes from zero to 14 standard drinks.

Program 10.1: Wilful exposure (to alcohol)

```
# Calculate exposure to alcohol.
from pylab import *

mass = float(input("Enter the person's mass in kg: "))
sex = float(input("Enter 1 if male, 2 if female: "))
drinks = arange(0,15)
if sex == 1:
    water = 1000 * mass * 0.7
else:
    water = 1000 * mass * 0.6
# Estimate E for each number of drinks and plot graph
peakBAC = 1000 * drinks / water
tBAC0 = peakBAC / 0.015
AUC = tBAC0 * peakBAC / 2.0
plot(drinks, AUC, "bo", markersize=8)
grid(True)
xlabel("Number of drinks")
ylabel("Total exposure (% hours)")
title("Total exposure to alcohol")
show()
```

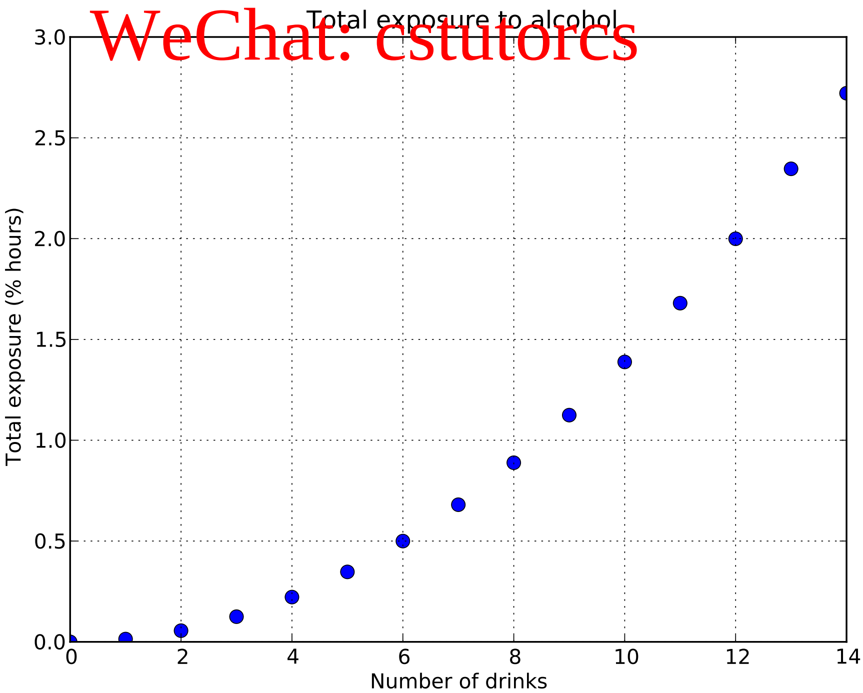



Figure 10.2: Program output showing total exposure to alcohol according to the number of drinks consumed.

End of Case Study 21: Dying for a drink.

Lecture 27: **Exposure and trapezoids**

Learning objectives

✓ Calculate and interpret the meaning of the area under a curve

Scientific examples

✓ Exposure to nicotine

✓ Exposure to glucose

Maths skills

✓ Estimate the area under a curve using rectangles

✓ Estimate the area under a curve using the trapezoid rule

- When a function is known, it may be possible to calculate the area under the curve using definite integrals. A brief reminder of this is included in Section C.5 of Appendix C.
- More often, AUCs are used in practical applications in which the only available information is a collection of measured data values, and the function $f(x)$ is **not** known. In such cases, AUCs are estimated approximately, by summing the areas of geometric shapes of "narrow" width, such as rectangles (called *Riemann sums*), or *trapezoids* (called the *trapezoid rule*).

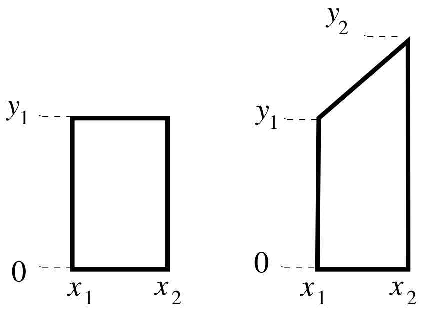
***Question** 10.1.4*

Give some advantages and disadvantages of each way of finding AUCs.

	Integration	Summing geometric shapes
Pros		
Cons		

Question 10.1.5

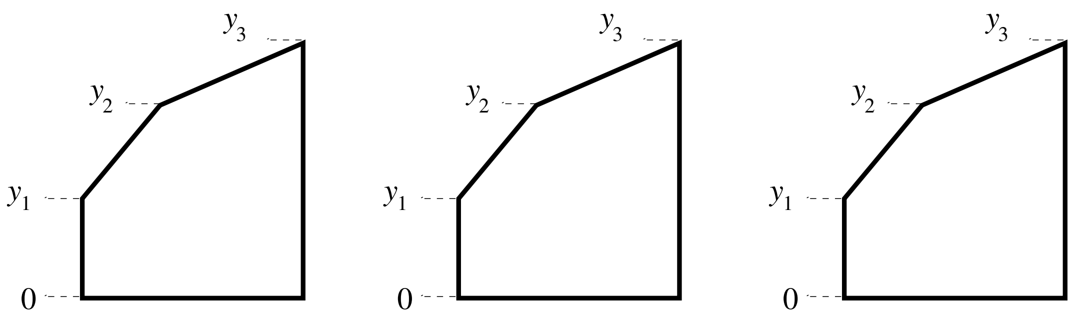
(a) Derive expressions for the areas of the rectangle and the trapezoid.



(continued over)

Question *10.1.5 (continued)*

(b) Show how to find the approximate area of the following shape in three ways, using rectangles in two different ways, and using trapezoids.



(c) Comment on the three approaches you used in Part (b). Which is likely to be most accurate? How do the two methods using rectangles relate to the method using trapezoids?

Question 10.1.6

Let $N(t)$ be an unknown function representing the blood nicotine concentration of a person after smoking a cigarette. Figure 10.3 shows some concentrations, measured experimentally (see [3]).

t (min.)	0	6	20	35	65	95
Value (ng/mL)	5	15.4	11.3	9.8	8	7

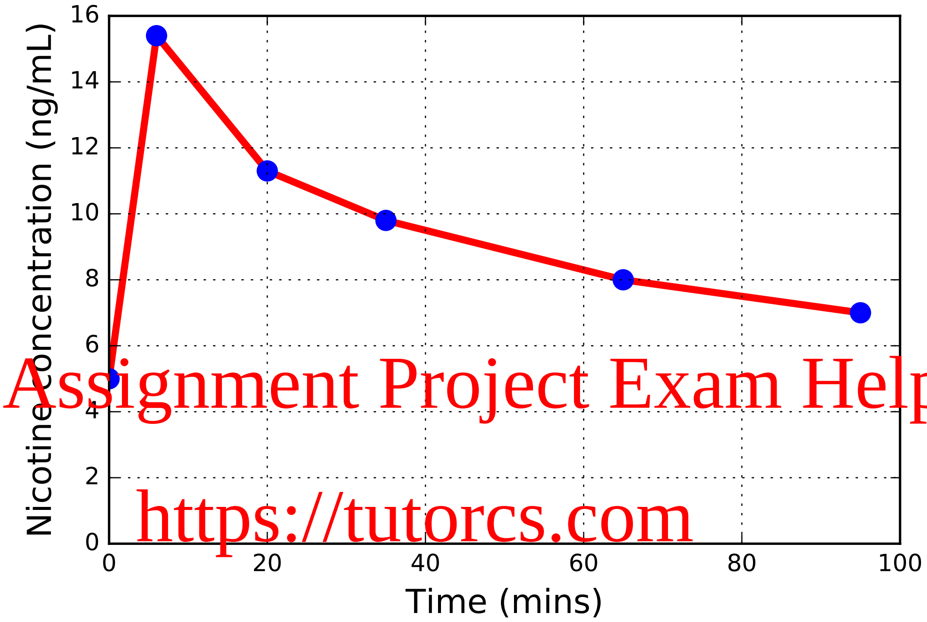


Figure 10.3: Measured blood nicotine concentrations after smoking.

The following calculations use areas of rectangles to estimate partially the AUC of the nicotine concentration curve. Link the calculations to rectangles on the graph and then complete the calculation for the remaining points.

$$\begin{array}{rcrcl}(6-0) & \times & 5 & = & 30 \\ (20-6) & \times & 15.4 & = & 215.6 \\ (35-20) & \times & 11.3 & = & 169.5\end{array}$$

Question 10.1.7

Repeat Question 10.1.6 but instead use the trapezoid rule.

t (min.)	0	6	20	35	65	95
Value (ng/mL)	5	15.4	11.3	9.8	8	7

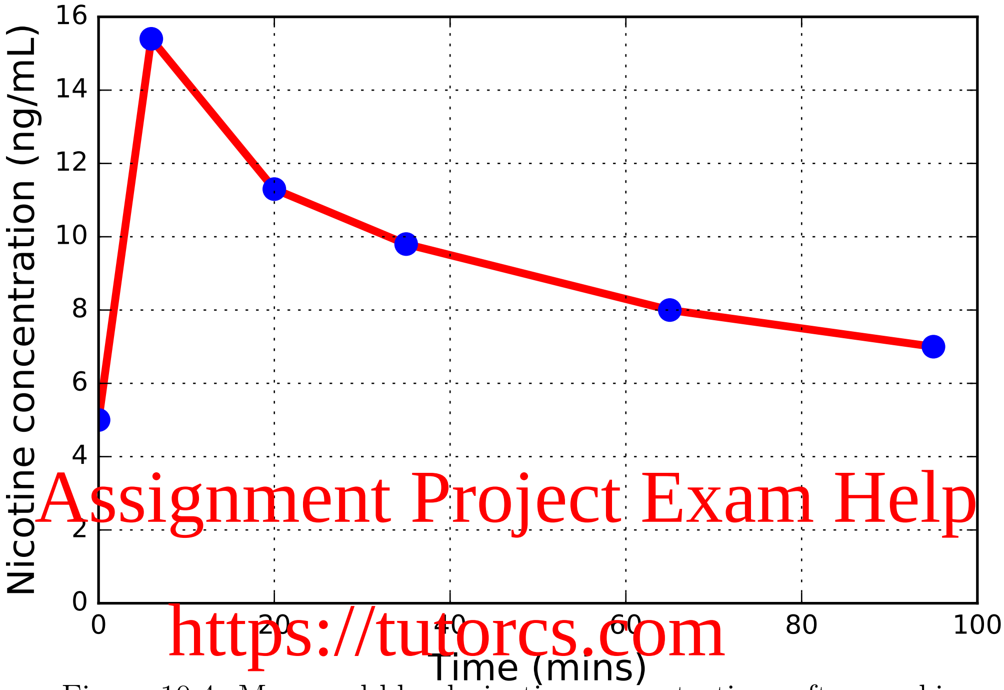


Figure 10.4: Measured blood nicotine concentrations after smoking.

$$\begin{aligned}
(6-0) &\times (5+15.4)/2 &= 61.2\\
(20-6) &\times (15.4+11.3)/2 &= 186.9\\
(35-20) &\times (11.3+9.8)/2 &= 158.25
\end{aligned}$$

We can perform the previous calculations more efficiently using a program.

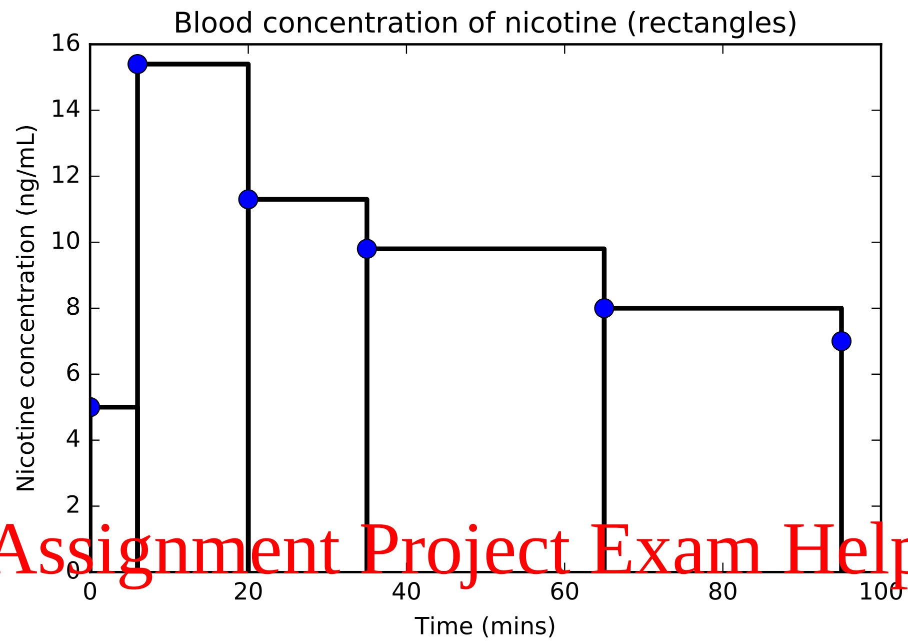
Program specifications: Write a Python program that estimates the AUC for $N(t)$ using rectangles or the trapezoid rule. The program must output the total AUC and draw a graph showing the shapes used in the sums.

Program 10.2: AUC for nicotine

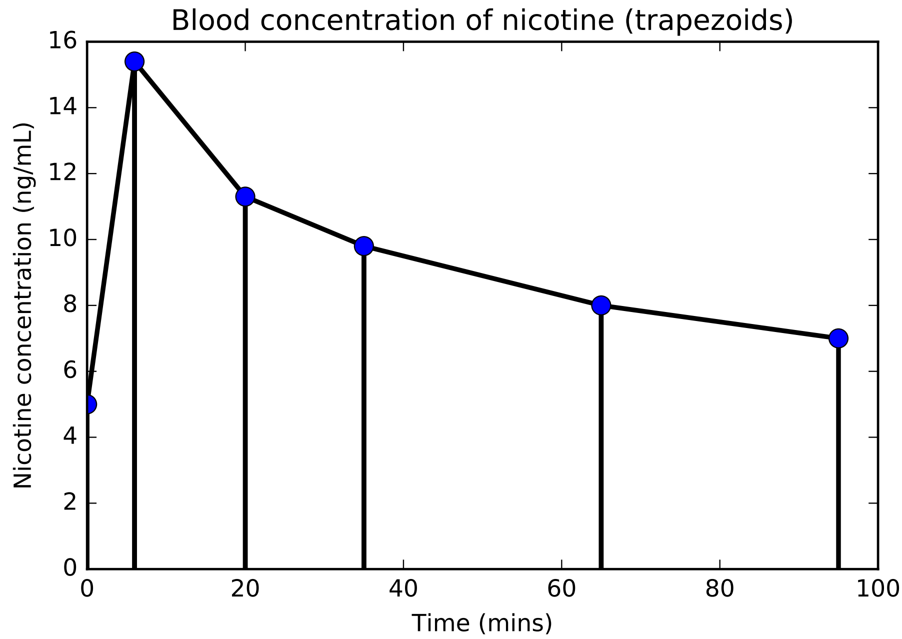
```
# Use rectangles or trapezoids to estimate AUC for nicotine.
from pylab import *

# Initialise variables
type = float(input("Type 1 for rectangles, 2 for trapezoids: ")
  )
t = array([0, 6, 20, 35, 65, 95])
concs = array([5, 15.4, 11.3, 9.8, 8, 7])
area = 0

# Sum the areas of each shape
i = 1
while i < size(t):
    width = t[i] - t[i-1]
    if type == 1:
        height = concs[i-1]
        shapeX = array([t[i-1], t[i-1], t[i], t[i]])
        shapeY = array([0, height, height, 0])
    else:
        height = (concs[i-1] + concs[i])/2
        shapeX = array([t[i-1], t[i-1], t[i], t[i]])
        shapeY = array([0, concs[i-1], concs[i], 0])
    area = area + height * width
# Plot each shape
    plot(shapeX, shapeY, "k-",linewidth=2)
    i = i + 1

# Give the output.
print("The estimated AUC is",area,"ng min / mL")
plot(t, concs, "bo", markersize=8)
xlabel("Time (mins)")
ylabel("Nicotine concentration (ng/mL)")
if type == 1:
    title("Blood concentration of nicotine (rectangles)")
else:
    title("Blood concentration of nicotine (trapezoids)")
show()
```

Here is the output from running the above program twice:

```
Type 1 for rectangles, 2 for trapezoids: 1
The estimated AUC is 949.1 ng min / mL
```




```
Type 1 for rectangles, 2 for trapezoids: 2
The estimated AUC is 898.35 ng min / mL
```

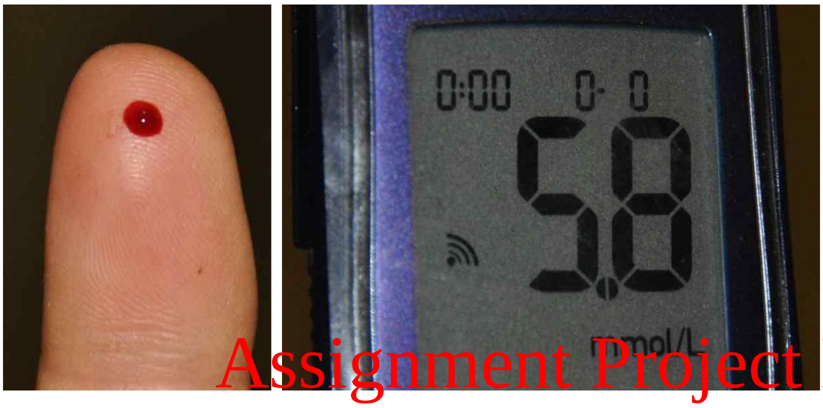
10.2 All in the blood

Case Study 22: Sweet P's

- The hormone *insulin* regulates conversion of glucose into usable energy in the body. *Diabetes mellitus* is a group of chronic diseases in which insufficient insulin is produced, or insensitivity to insulin develops. This leads to high levels of blood glucose.
- There are three main types of diabetes: *type* 1 (insulin-dependent diabetes, typically present at birth, representing about 10–15% of all cases), *type* 2 (non-insulin-dependent diabetes, which accounts for 85–90% of all cases) and *gestational diabetes* (developed in 3–8% of pregnancies).
- Typical signs of diabetes include:
  - *polyuria* (excessive urination, often with a sweet taste)
  - *polydipsia* (excessive thirst)
  - *polyphagia* (excessive hunger).

- Once type 1 or type 2 diabetes becomes established, it is usually permanent.
- Diabetes, particularly type 2, is becoming increasingly common in societies with a "western lifestyle". Around 1.7 million Australians have diabetes, although only about half of them are aware of it.
- The Framingham heart study has shown that diabetes significantly reduces life expectancy (by 7.5 years for men aged over 50, and 8.2 years for women).
- Untreated diabetes can cause blindness, kidney failure and cardiovascular disease including blockages in small arteries. Some patients require amputations after blocked peripheral circulation causes the death of soft tissue.

- An *Oral Glucose Tolerance Test* (OGTT) is a common test for diabetes.
- Prior to taking the test, the patient fasts for around 12 hours. During the test, the patient is administered a measured oral dose of glucose, with blood samples taken immediately prior to ingestion of the glucose and at various intervals for 2 hours afterwards.
- The graph in Figure 10.5 compares the measured blood glucose levels for a non-diabetic person with those from a hypothetical diabetic person.



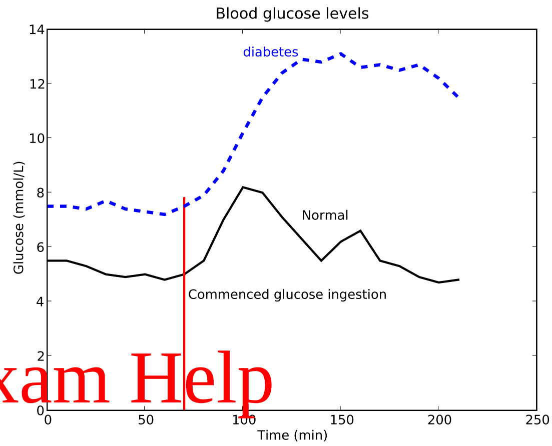


Photo 10.2: Left: bloody finger. Right: measured blood glucose concentration. (Source: PA.)

Figure 10.5: Normal blood glucose levels and levels indicative of diabetes.

- Figure 10.6 shows the World Health Organisation guidelines for blood glucose levels indicating various stages of health or disease.

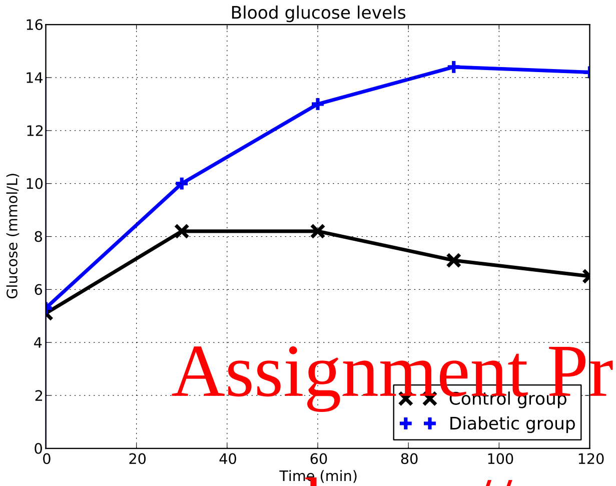
	Blood glucose level (mmol/L)			
time (hr)	Normal	IFG	IGT	DM
$t = 0$	< 6.1	$\geq 6.1, < 7.0$	< 7.0	≥ 7.0
$t = 2$	< 7.8	< 7.8	≥ 7.8	≥ 11.1

Figure 10.6: World Health Organisation guidelines for blood glucose levels as indicators of: Impaired Fasting Glycaemia (IFG); Impaired Glucose Tolerance (IGT or *pre-diabetes*); and Diabetes Mellitus (DM).

- It is possible that an individual blood glucose measurement might be within the normal range at some instant in time, but outside that range over a longer time period. It is often very useful to analyse AUCs as well.

Question 10.2.1

Figure 10.7 shows measured blood glucose levels for a diabetic group and a control group, adapted from [24].



time (mins)	diabetic (mmol/L)	control (mmol/L)
0	5.3	5.1
30	10	8.2
60	13	8.2
90	14.4	7.1
120	14.2	6.5

Figure 10.7: Mean blood glucose levels for diabetic and non-diabetic (control) groups.

(a) Roughly estimate the AUC for the control group.

(b) Use the trapezoid rule to calculate the AUC for the control group.

(continued over)

***Question* 10.2.1** *(continued)*

(c) The paper [24] found that the "glucose AUC" for the control group is around 265 mmol/L/min. Comment on the units and compare the given AUC for the control group with your values in Parts (a) and (b).

(d) How does the concept of measuring the AUC for a blood glucose curve relate to the concept of *total exposure to alcohol* discussed earlier.

End of Case Study 22: Sweet P's.

Lecture 28: **Areas in action**

Learning objectives

✓ Interpret different applications of areas under curves and ratios of these areas

Scientific examples

✓ Glycaemic Index

✓ Bioavailability of a drug

Maths skills

✓ Estimate areas under curves

✓ Interpret graphs

Case Study 23: **Hi GI!**

- The *Glycaemic Index* or GI of foods is often mentioned in marketing campaigns and in association with dietary health claims.

- GIs range between 0 and 100, and indicate the relative extent by which blood glucose levels rise after the consumption of a food. Hence, GI scores are only valid for foods containing carbohydrates.

- Researchers classify foods into the following GI categories.

  - *Low GI* – when GI is ≤ 55, digestion of carbohydrates is slow, with a slow rise and lower peak in blood sugar level. Examples of Low GI foods include cherries, skim milk, apples, chick peas, oranges and carrots.
  - *Medium GI* – when the GI is between 56 and 69, the digestion of carbohydrates occurs at a moderate rate. Examples of Medium GI foods include include boiled potato, honey, ice cream and sultanas.
  - *High GI* – when the GI is 70 or higher, the digestion of carbohydrates is fast, leading to a rapid rise and high peak in blood sugar level. Examples of High GI foods include mashed potato, white bread, cornflakes, watermelon and steamed white rice.

- The many claimed health benefits of Low GI diets include weight loss and improved weight control, improved management of diabetes, reduced risk of cardiovascular disease and increased physical stamina.

- Criticisms of focusing on GIs as a dietary tool include:
  - GIs can vary greatly for a given food, depending on how ripe it is, and how it is processed, stored and cooked; moreover, the GI of a food may be less important than the *quantity* consumed;
  - Measured GIs may not be very exact or reliable; the GI of a given food is measured at different times of the day then the results can differ quite substantially.
- Researchers calculate the GI of a food in the following way:
  - Ten healthy people fast overnight. In the morning, each person consumes a controlled dose of the test food, with known total carbohydrate content (typically 50 g). Over a 2 hour period, researchers measure the increase in the blood glucose level above baseline for each participant, produce graphs, and calculate the AUCs for the test food using the trapezoid rule.
  - On a separate day, participants undergo the same procedure, but consume a glucose solution which contains the same amount of total carbohydrate. Researchers calculate the AUC for glucose (the reference food) for each individual.
  - The **definition** of the GI is: divide the AUC above base level of the test food by the AUC above base level for glucose, and multiply by 100%. An average of the individual GI scores represents the overall GI for the test food.

***Question** 10.2.2*

Use a diagram to illustrate calculation of GI for a test food.

Question 10.2.3

A study [57] recorded the following blood glucose levels when an individual consumed a controlled quantity of bread in one test, then lentils in another test. The increase in the AUCs above baseline glucose levels is $29\text{mmol/L} \cdot \text{min}$ for lentils, and $66\text{mmol/L} \cdot \text{min}$ for bread. Given that the GI of bread is around 70, find the GI of lentils.

Time	Glucose (mmol/L)	
(mins)	Bread	Lentils
0	4.3	4.3
15	4.8	4.8
30	6.2	4.8
45	5.5	4.3
60	5.0	4.4
90	3.9	4.5
120	4.1	4.7

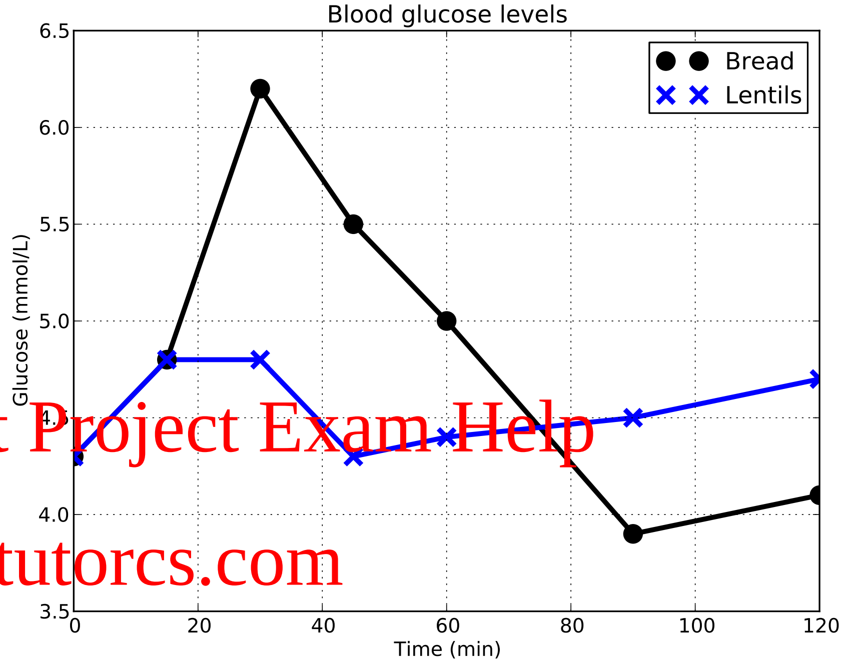


Figure 10.8: Mean blood glucose levels.

End of Case Study 23: Hi GI!.

Case Study 24: **Bioavailability of drugs**

- Drugs can be administered via many routes, including:
  - orally, such as the contraceptive pill;
  - as a gas, such as nicotine from a cigarette;
  - through the skin, such as a nicotine patch;
  - nasally, such as "snorted" cocaine;
  - intravenously, such as chemotherapy drugs for treating cancer;
  - sublingually, such as nitroglycerin used to treat angina; and
  - rectally, such as a paracetamol suppository.

- Different routes of drug delivery are required depending on the drug type, duration and frequency of treatment, and the condition of the patient. Oral administration is common, but other routes may be more convenient for drugs that cause nausea or vomiting, or for patients who cannot swallow.

- After administration of a drug, it typically needs to pass through a number of stages before it enters general circulation and has a chance to act. This can have a substantial impact on the proportion of the dose available to achieve the desired pharmacological impact. For example, the following *first pass effects* reduce the availability of orally-administered drugs:
  - how readily and rapidly the drug dissolves in the digestive tract;
  - whether the drug is damaged by acidic stomach contents;
  - whether the drug is partially metabolised by bacteria in the gut;
  - how much of the drug is absorbed across the intestinal wall;
  - the digestive health of the individual (for example, vomiting or diarrhoea may cause mechanical expulsion of the drug); and
  - how much of the drug is metabolised in the liver prior to entering general circulation (because blood travels from the small intestine to the liver and then to the rest of the body).

- After administering a drug by a given route, its relative **bioavailability** F is the fraction of the dose that enters general circulation compared to a dose administered via a more direct route, usually *intravenously* (IV).

Definition of bioavailability

If $R(t)$ is the blood concentration of a drug after giving a dose by some route and $I(t)$ is the concentration after an IV dose of **the same size**, then the bioavailability F of the drug administered via this route is

$$F = \frac{\displaystyle\int_0^\infty R(t)\,dt}{\displaystyle\int_0^\infty I(t)\,dt}.$$

Question 10.2.4

(a) Explain the meaning of the expression for F.

(b) How is the method of calculating the bioavailability of a drug similar to the method of calculating the GI of a food? How is it different?

Question 10.2.5

In [44] and [32], on separate occasions, test subjects were each administered 1000 mg doses of paracetamol. In [44], doses were intravenous and in oral tablet form. In [32], an aqueous dose was administered as a rectal suppository. Figure 10.9 plots the blood concentrations of paracetamol obtained for these subjects over the following six hours.

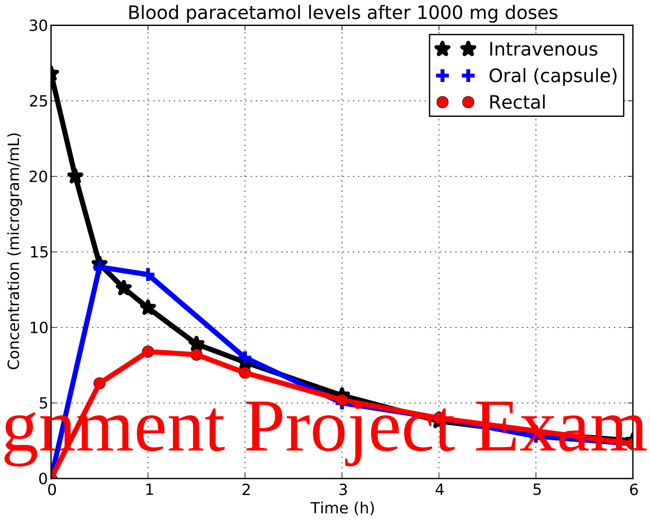


Figure 10.9: Blood concentration curves for paracetamol administered in various ways.

(a) Discuss the shapes of the three curves in Figure 10.9.

(b) **Estimate** the bioavailabilities of the oral and rectal doses.

(continued over)

Question 10.2.5 *(continued)*

(c) For the IV dose (see [44]), the blood paracetamol concentration in μg/mL at time t in hours after dosing is modelled using the equation

$$I(t) = 13.8e^{-2.55t} + 13e^{-0.28t}.$$

Figure 10.10 plots the measured values and $I(t)$. Using integration, the AUC for this curve is 51.84 μg hr/mL. In [44], the AUC for the oral tablet dose is around 44 μg hr/mL. Calculate the bioavailability of the oral tablet dose.

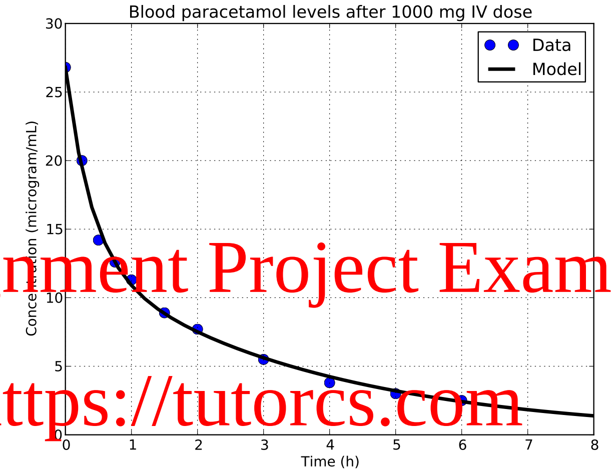


Figure 10.10: The graph of $I(t)$, following an intravenous dose of 1000 mg of paracetamol.

(d) In [32], the AUC given for the rectal dose is around 2290 μg/mL/min. Comment on the AUC units, then calculate the dose bioavailability.

(e) Compare your answers to Parts (c) and (d) with those to Part (b).

End of Case Study 24: Bioavailability of drugs.

Chapter 11: **Differential equations and populations**

Lecture 29: **What's a DE?**

Learning objectives

✓ Understand what a differential equation (DE) is and how they arise naturally in scientific contexts

Scientific examples

✓ Human population growth

✓ E. coli

Maths skills

✓ Understand what a DE is

✓ Know how to check if a function is a solution to a DE

✓ Know the exponential DE and its solution

Image 11.1: *Lascaux Cave*, Depiction of aurochs, horses and deer, UNESCO World Heritage Site, Part of prehistoric sites and decorated caves of the Vésère Valley (Source: en.wikimedia.org.)

Differential equations are very useful, "natural" tools for modelling a huge range of phenomena in science and other areas. For example, if you know that a population is changing in size at a rate proportional to its current value, then you can write and solve a simple DE to represent what is happening. Many natural phenomena display this kind of relationship, so knowing how to write and solve the DE allows the phenomena to be studied.

Here we will investigate changing phenomena in the context of life cycles and populations, and explore how to model these phenomena. The mathematical content that underpins this section is the *differential equation* or DE. Differential equations describe how the value(s) of one or more quantities are changing, and typically involve a function and its derivative(s). We will begin with an introduction to differential equations (DEs), and then study the exponential and logistic DEs, and use them to model changes in the sizes of simple populations. We will also learn how to use *Euler's method* to solve DEs numerically.

11.1 Introduction to differential equations

- Typically, developing a mathematical model of a phenomenon involves deriving one or more equations that predict the *value* of the phenomenon.

- Sometimes, this is difficult or impossible. Instead, it may be possible to write an equation for how the value is *changing*, then use mathematical techniques to deduce information about the *value*.

- An equation for the rate at which the value of a phenomenon is changing is called a *differential equation*.

- To understand differential equations, it is essential to be clear about the distinction between the *value* of a phenomenon, and the *rate at which that value is changing*.

Differential equations

If y is an unknown function of t, then a **differential equation** is an equation that involves a combination of t, y and/or the derivatives of y.

In all of the examples we will study, the DE will be of the form $y' = \ldots$. (That is, the DE will be an equation for the rate of change of y.)

A particular function y is called a **solution** to a DE if the DE is true when y and its derivative(s) are substituted into the DE.

Thus, a solution to a DE is **another function** which, when **substituted** into the DE, makes the DE **true**.

- Just as with any mathematical model, there are two steps to modelling with DEs: *writing* the equations, and then *solving* them.

- DEs are often very "natural" ways of representing phenomena. That is, it often makes "more sense" to write an equation for a rate of change of some value than to write an equation for the value.

Question 11.1.1

Write a DE to model each of the following.

(a) The straight line distance $D(t)$ travelled by a car increases by 10 m/s.

(b) The human population P of Earth is increasing at about 1.1% per annum. Assume this growth rate continues.

(c) According to the Australian Bureau of Statistics, during 2013 Australia had: birth rate 1.34%; death rate 0.64%; 504000 people inward migration; 268000 people outward emigration. If these changes continue indefinitely, write a DE for the Australian population $P(t)$ in year t.

(d) The *von Bertalanffy growth model* states that the rate of increase of the length $L(t)$ of a shark of age t years is proportional to the difference between a fixed maximum length M and its current length $L(t)$. The constant of proportionality is an intrinsic positive growth rate r.

(e) *Newton's Law of Cooling* states that the rate of change of the temperature $T(t)$ of an object placed in an environment with fixed temperature F is proportional to the relative difference in the temperatures.

- Once a DE has been written for the *rate of change* of a phenomenon, that DE can (sometimes) be *solved* to give the *value* of the phenomenon.
- This (usually) requires an additional piece of measured information, such as the value of the phenomenon at some time, often $t = 0$.

***Example** 11.1.2*

Refer to Question 11.1.1. The following additional information applies in each case.

(a) Initially the car has travelled 0 metres.

(b) The human population of Earth in July 2014 was about 7.24 billion.

(c) The human population of Australia at the start of 2013 was 22.9 million.

(d) In [22], it is shown that:

- the maximum length of a female *sand tiger* or *grey nurse* shark is $M = 295.8$ cm;

- the intrinsic growth rate is $r = 0.11$ yr^{-1}; and
- the length of a typical shark at birth is 110 cm.

(e) An item with temperature 85 °C is placed in a room with constant temperature 25 °C.

- We can solve some DEs *analytically*, using integration and algebra.
- We will next look at two very well-studied types of DEs and their solutions.

11.2 The exponential DE

- Earlier we studied exponential growth and decay. On Page 114 we state "Any phenomenon that has a rate of change proportional to the current amount follows an exponential function".
- This occurs precisely because such phenomena satisfy simple DEs whose solutions are exponential functions.

DE for exponential growth and decay

Any quantity $N(t)$ whose rate of change at any time is proportional to the value of N, with rate of change equal to a constant r per time period, follows the DE

$$N' = rN.$$

The solution to this DE is the exponential function

$$N(t) = N_0 e^{rt},$$

where N_0 is the value of N at $t = 0$.

Example *11.2.1*

It can be shown that $N(t) = N_0 e^{rt}$ is the solution to $N' = rN$.

The derivative of $N(t)$ is: $N'(t) = r \times N_0 e^{rt}$.

We can see that this is equivalent to $N'(t) = rN(t)$.

- Many populations can be modelled effectively using exponential functions, for some periods of time.

Case Study 25: **Poo**

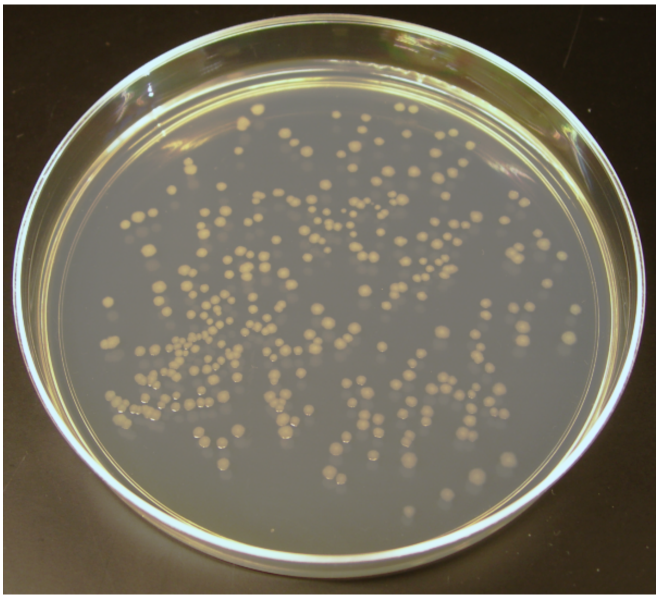


Photo 11.1: E. coli colonies on a plate (Source: commons.wikimedia.org.)

- *Escherichia coli* (usually shortened to *E. coli*) are bacteria commonly found in the lower intestine of warm-blooded animals, including humans.

- Most strains of *E. coli* are harmless in the digestive system, or even beneficial to the host. However, some strains produce toxins, and can cause food poisoning, gastrointestinal infections and urinary tract infections.

- One such strain is O104:H4, which caused outbreaks of illness in Europe in 2011. Around 50 people died and more than 4000 became ill. Contamination was traced to a farm that grew organic sprouted vegetables.

- Because *E. coli* can survive outside the body for some time, tests for *E. coli* are often used to identify faecal contamination in environmental samples or foods during hygiene checks.

- Under simplifying assumptions (such as relatively unlimited resources) the rate of increase of a population of *E. coli* at any time is proportional to the population size at that time.

- Hence the population follows an exponential function, and it makes sense to discuss the *doubling time* of the population.

- Under favourable conditions, the doubling time for a population of *E. coli* may be an hour, or even shorter. The rapid growth rate is one reason why good hygiene standards are important in food preparation.

- When studying populations of bacteria, microbiologists commonly count *colony-forming units* (CFU), which is the number of *live* bacterial cells. (Direct counts of individuals include both dead and living cells.)

***Question** 11.2.2*

A population of *E. coli* has a growth constant of 1% per minute. Let $N(t)$ be number of CFU's of E. coli at time t minutes, and assume at time $t = 0$ there are 1000 CFU's. Write and solve a DE to model this population.

***Question** 11.2.3*

A study [51] investigates *E. coli* contamination of pre-cooked meat. Researchers contaminated some ham with 10^7 CFU of *E. coli* and then sliced the ham. The same blade was then used to slice clean ham. The number of CFU were counted on each of 100 slices of the second ham, showing that Slice 1 contained around 580 CFU and Slice 100 contained 9 CFU.
To answer the following questions, assume the number of *E. coli* on the ham can be modelled by $N(t) = N_0 e^{0.01t}$ where N_0 is the initial number of CFU and t is in minutes.

(continued over)

Question 11.2.3 (continued)

(a) How many CFU will be on Slice 1 after 24 hours, assuming it is stored under ideal growing conditions for *E. coli*?

(b) At what time does Slice 100 contain 580 CFU?

(c) Human faeces can contain 10^9 CFU per gram. Given the results of this study, what does this mean for hygiene practices in food preparation?

(continued over)

***Question** 11.2.3 (continued)*

(d) Consider the E.coli on Slice 1. The mass of 1 CFU is approximately 1×10^{-12}g and the mass of the Earth is approximately 6×10^{24}kg. According to the exponential model, how long it would take for the mass of the E.coli on Slice 1 to reach the mass of the Earth?

End of Case Study 25: Poo.

Lecture 30: **A limit to the madness**

Learning objectives

✓ Analyse the form of population growth under limited resource constraints

✓ Understand the meaning and use of carrying capacity

Scientific examples

✓ Fish populations

✓ Human population

Maths skills

✓ Know the logistic DE and its solution

✓ Interpret graphs

11.3 Limited scope for growth

- Exponential growth models are *unconstrained*, so the growth continues indefinitely with a constant proportional rate of increase, say r.

- This can be accurate over limited time periods, but in reality populations cannot continue to show unconstrained growth forever. Many *constrained* growth models assume that there is a maximum population size.

Carrying capacity of an ecosystem

The **carrying capacity** K of an ecosystem is the maximum population size of a particular organism that is supported by resources within the ecosystem. Resources may include food, water, shelter and sunlight. The carrying capacity for a particular organism often changes over time; for simplicity, we will assume it remains constant.

A population size below the carrying capacity will typically increase towards the carrying capacity, whereas a population size above the carrying capacity

(continued over)

Carrying capacity of an ecosystem *(continued)*

(which may occur when, for example, a lake is overstocked with fish) will typically decrease to the carrying capacity.

***Question** 11.3.1*

Let $N(t)$ be the size of a fish population in a certain lake at any time t in months, and let the carrying capacity of the lake be K fish. Draw a rough sketch of $N(t)$ versus time.

Photo 11.2: Kiss me, red. (Source: PA.)

- We have modelled unconstrained growth with the exponential DE. This assumes a constant growth rate, r.

$$N' = rN$$

- One way to model *constrained growth* is to assume that the growth rate varies with the population size N, rather than remaining constant. That is,

$$N' = g(N)N$$

where $g(N)$ is an *effective growth rate* that changes as N changes.

Question 11.3.2

Assume that a population has an unconstrained growth rate of r. As the population N increases, the effective growth rate g reduces linearly from r, until the population reaches the carrying capacity K at which point the effective growth rate is 0. Derive an expression for g as a function of N.

Logistic DE

Any function $N(t)$ that changes at a rate proportional to its value (with unconstrained growth rate r), **and also** in reverse proportion to how close the value is to a carrying capacity K, is modelled by the logistic DE

$$N' = r\left(1 - \frac{N}{K}\right) N.$$

- In the logistic model, for $N < K$, the change in the population N' will:
  - **increase** as the population size gets larger and there are more individuals who can reproduce; and
  - **decrease** as the population size gets larger as individuals compete for scarce resources.

- The power of the logistic model is the interaction between two opposing factors, growth and competition.

Question 11.3.3

What does the logistic DE predict for N' in the following extreme cases:

(a) N is approximately equal to K?

(b) N is much less than K?

Solution to the logistic DE

The logistic DE can be solved exactly. If N_0 is the value of N at time 0, the exact solution is

$$N(t) = \frac{K\,N_0}{N_0 + (K - N_0)e^{-rt}}$$

Example 11.3.4

A fish species with initial population $N_0 = 30$ and unconstrained growth rate of 10% per month lives in a reef with a carrying capacity of K=1000 fish. The function $N(t)$ gives the number of fish at time t months, and Figure 11.1 graphs $N(t)$ for 80 months, showing the typical "S"-shaped logistic curve.

$$N(t) = \frac{30000}{30 + 970e^{-0.1t}}$$

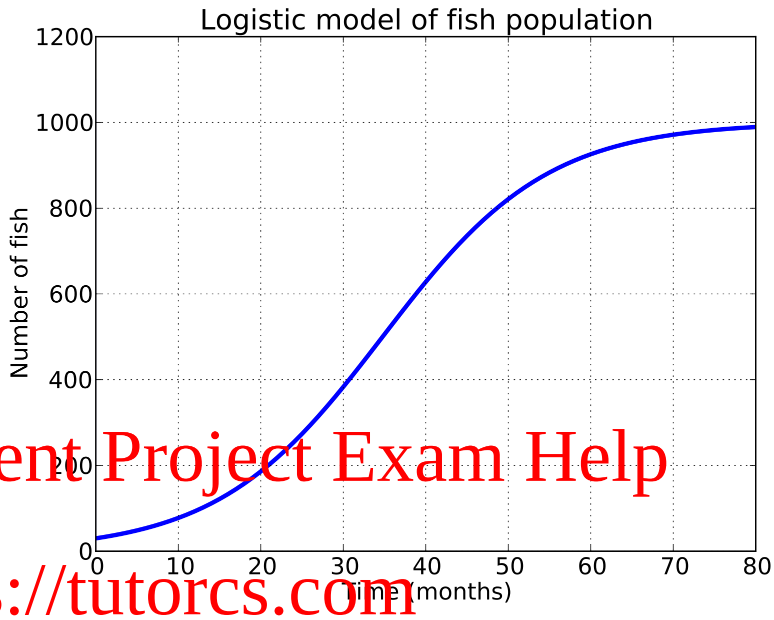


Figure 11.1: Logistic model with an initial population of 30 fish.

Question 11.3.5

With reference to Example 11.3.4, at what population size is the value of N' largest? Explain your answer briefly.

Case Study 26: **Overpopulation annoys us all**

- The human population of Earth is rising very rapidly. In 1950 the global population was about 2.5 billion, and in 2012 it first exceeded 7 billion.
- The doubling time has reduced in recent centuries: it took about 300 years for the population to double to one billion, then 120 years to double again, then 47 years to double again.

Question *11.3.6*

(a) The following graphs show the human population of Earth over 1000 years (left) and transformed using ln (right). Is the population shown in Figure 11.2 growing exponentially?

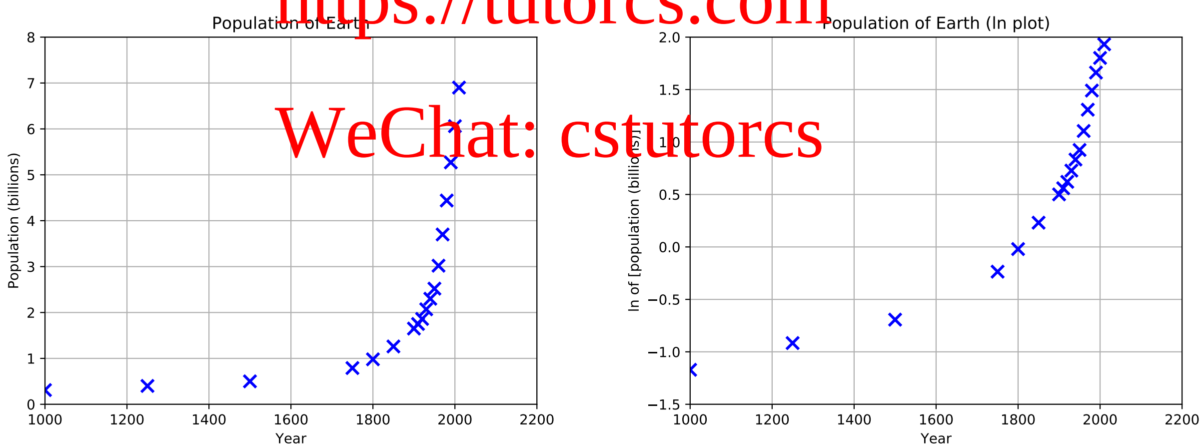


Figure 11.2: Human population of Earth over 1000 years (left), and transformed using ln (right).

(continued over)

***Question** 11.3.6 (continued)*

(b) If the population were growing exponentially, then the shape of the graph showing the annual change in global population would also be exponential. This is because $P(t) = P_0e^{rt}$ has $P'(t) = rP_0e^{rt}$.

Figure 11.3 shows the annual change in global population since 1950. Interpret this graph. What does it mean for global population since 1950, and what might it mean into the future?

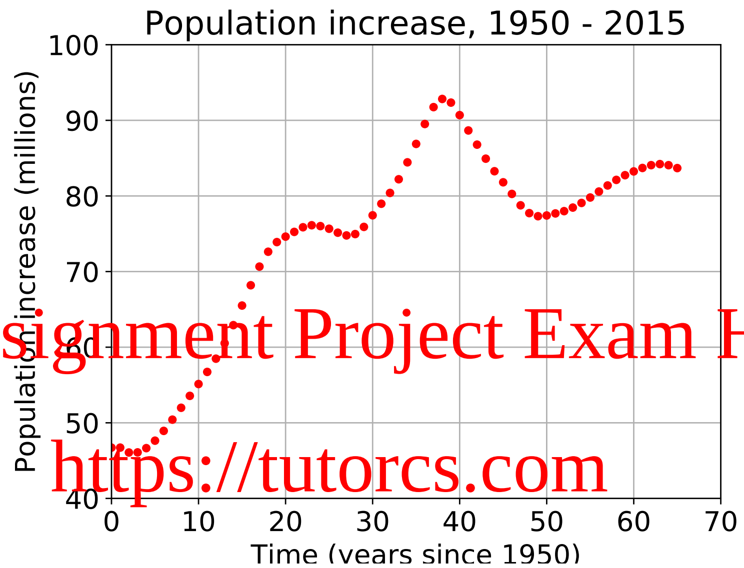


Figure 11.3: Annual change in global population, 1950–2015.

- The United Nations has estimated the populations of all countries, and also globally, for each year until 2100; see [55]. Their models take into account predicted shifts in demographic patterns in each country.
- Figure 11.4 shows the projected global population based on different levels of fertility.

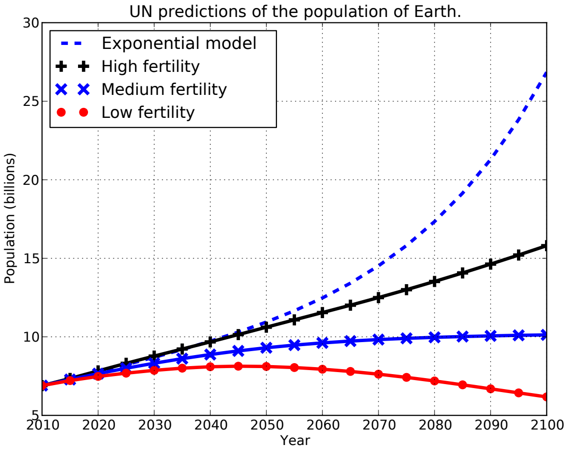


Figure 11.4: UN predictions of global population, 2010–2100.

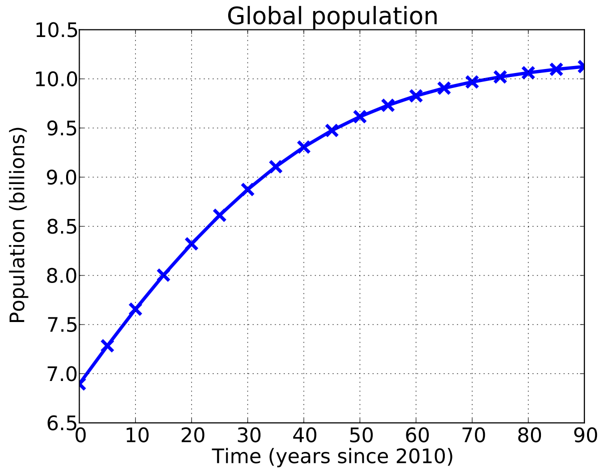


Figure 11.5: Predicted global population with medium fertility.

Question 11.3.7

Figure 11.5 shows the UN projected global population $P(t)$ at five year intervals from 2010 ($t = 0$) until 2100 ($t = 90$), assuming 'medium fertility'.

(a) Is it reasonable to model $P(t)$ using a logistic DE, $P' = rP\left(1 - \dfrac{P}{K}\right)$?

(b) Estimate the value of K in the model. Discuss the statement: 'K is the carrying capacity of Earth for humans'.

End of Case Study 26: Overpopulation annoys us all.

Lecture 31: **Oysters in Chesapeake Bay**

Learning objectives

✓ Use a logistic DE to study a population and make predictions

✓ Make recommendations to government or invested parties based on modelling predictions

Scientific examples

✓ Over-harvesting oysters

Maths skills

✓ Understand the logistic DE and its solution

Case Study 27: **Overfishing annoys an oyster**

Photo 11.3: Oysters. (Source: MG.)

- Chesapeake Bay is a large estuary on the east coast of the United States, near the states of Virginia and Maryland. The bay has a surface area of more than 11000 km^2, with a shoreline length of more than 18000 km.

- In the past, the bay supported a diverse range of flora and fauna, including an abundant shellfish population, most notably oysters. However, it has experienced serious environmental degradation due to over-use, overfishing, and polluted runoff from agriculture, urban areas and industry.

- Substantial *marine dead zones*, which are areas of water so low in oxygen that they are unable to support life, are often found within the bay.

- Harvesting oysters is a long-term commercial industry in Chesapeake Bay, however, the size of the population (and hence the harvest) has drastically reduced, due to over harvesting and environmental damage; see Figure 11.6.

- Between 1982 and 2008, the value of the oyster harvest declined by 88%.
- Considerable research, money and education are being devoted to developing and implementing a sustainable, comprehensive management strategy.
- Figure 11.6 shows the historical annual official oyster harvest data for the Maryland part of the bay.

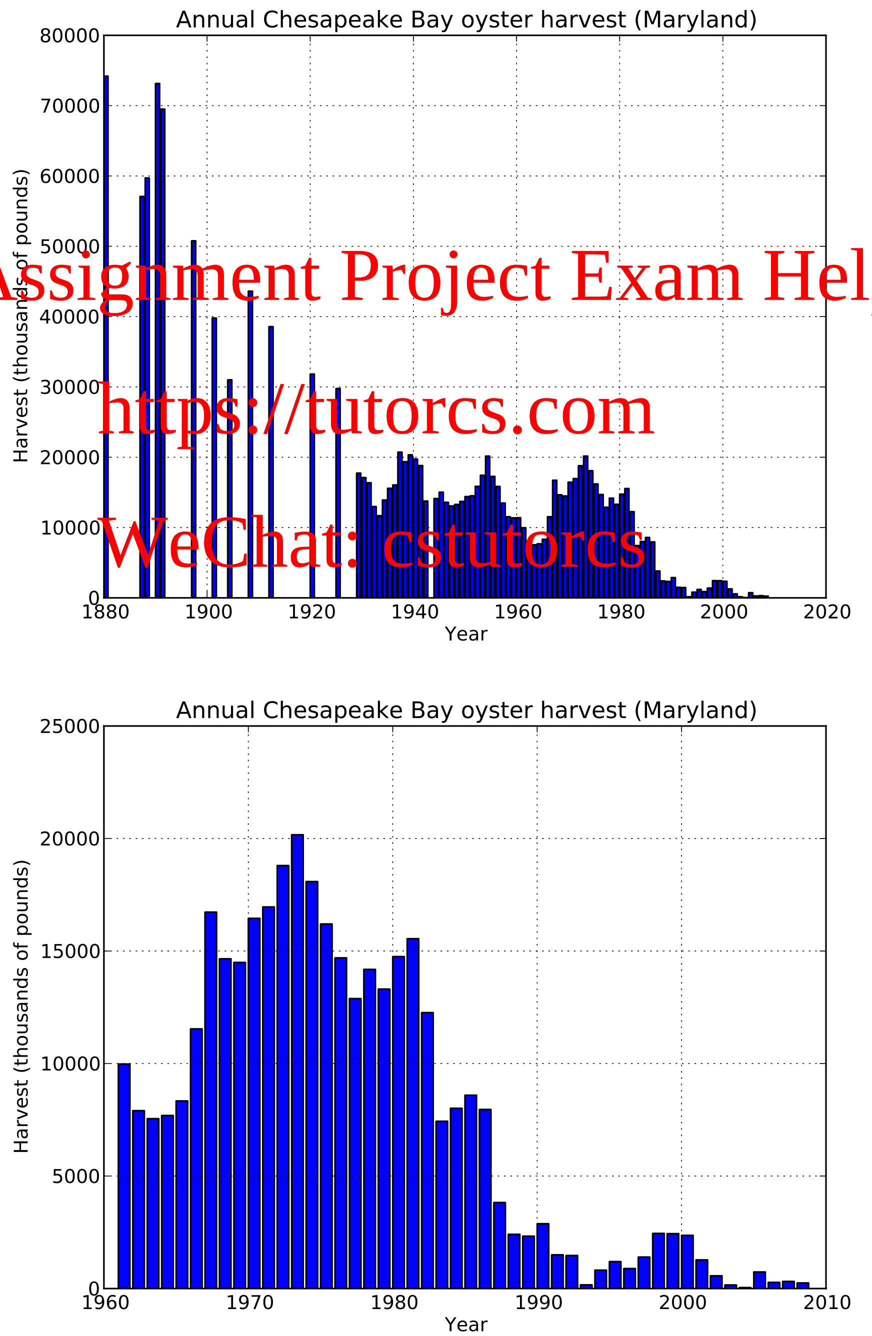


Figure 11.6: Annual Chesapeake Bay oyster harvests. Top: 1880 – 2008. Bottom: 1960 – 2008.

- The paper [54] studied the population of market-sized oysters in the Maryland part of the Chesapeake Bay.
- Using data from 1994 – 2007, researchers found that the effective unconstrained growth rate of market-sized oysters is around $r = 0.133$ per year.
- The estimated carrying capacity of the Maryland part of the bay is around 5×10^9 market-sized oysters.

***Question** 11.3.8*

(a) Write a logistic DE for the population $N(t)$ of market-sized oysters, assuming no harvesting.

(b) In 2007, the population of market-sized oysters in Chesapeake Bay was 81×10^6. Find the annual increase in the population size at that time.

(c) The 2007 harvest was 50×10^6 oysters. Is this sustainable? Why?

(d) Find the largest number of oysters that could be harvested *from the 2007 population* each year without reducing the total population.

(continued over)

***Question** 11.3.8 (continued)*

(e) If there was no oyster harvesting conducted for a few years, how would your answer to Part (d) change? Why?

(f) In resource management, especially fisheries management, the *Maximum Sustainable Yield* (MSY) of a population is defined to be the *largest possible harvest size that could be maintained indefinitely.* Explain how the MSY relates to population growth rates. What is the **population size** at which the MSY can be sustainably harvested?

(continued over)

***Question** 11.3.8 (continued)*

(g) Find the MSY of oysters in Chesapeake Bay. Noting that there are around 5 oysters per pound, comment on the historical harvest rates in Figure 11.6.

(h) A logistic growth model of a population with unconstrained growth rate $r = 0.133$ per year, current population $N_0 = 81 \times 10^6$, carrying capacity $K = 5 \times 10^9$ and no harvesting predicts the following population:

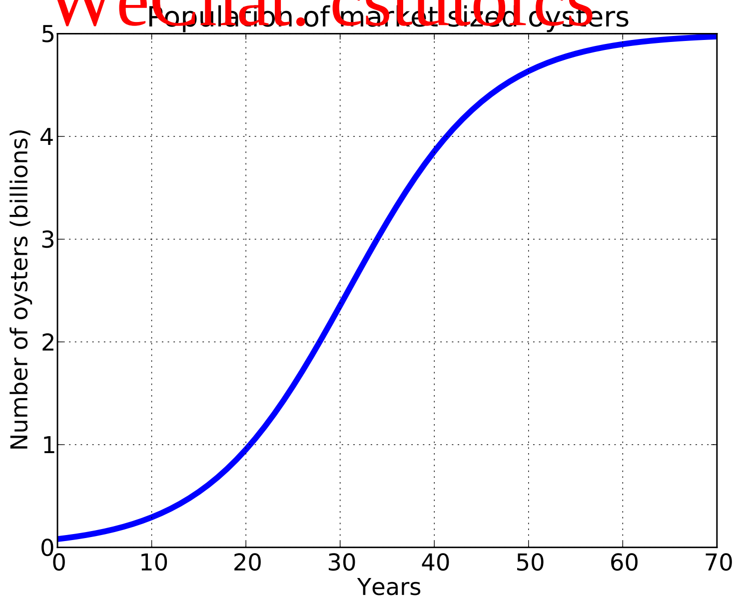


Figure 11.7: Logistic growth of the Chesapeake Bay oyster population with no harvesting.

(continued over)

***Question** 11.3.8 (continued)*

Make some brief recommendations to assist the government with long-term oyster stock management in Chesapeake Bay.

Photo 11.4: Over-exploited? (Source: MG.)

(continued over)

***Question** 11.3.8* *(continued)*

The 2008 paper [54] considers the economic value ("net present value") of the oyster industry, which also considers expected net returns of future harvests. They explain that unrestricted harvesting is unsustainable. They recommend that decreasing harvesting rates will increase the net present value, and shutting down the fishery for a number of years will allow stock to recover and significantly increase the net present value. They noted:

– As a result of habitat degradation, they may have overestimated the carrying capacity, so to test the sensitivity of the results to this parameter, they re-ran the models with a reduced carrying capacity and achieved a near identical optimal harvest rate as the original model.

– To test the sensitivity of their results to the intrinsic growth rate, they tested different growth rates. For example, even when cutting the growth rate in half, the net present value for the optimisation model's recommendation was still six times greater than the net present value for the unrestricted harvesting policy.

Given what you know from the philosophy of science component of the course about models being strictly false, how does the authors' clarification help give confidence in their recommendations?

End of Case Study 27: Overfishing annoys an oyster.

Lecture 32: **Approximating solutions to DEs**

Learning objectives

✓ Use Euler's method to find approximate solutions to differential equations

Scientific examples

✓ Population growth

Maths skills

✓ Understand how and why Euler's method works

11.4 Euler's method

- We have seen some examples of DEs and their solutions, but for more complex DEs finding an exact solution is often not possible analytically.

- Instead, we can often find approximate solutions using numerical algorithms.

- One numerical algorithm for finding an approximate solution to a DE is called *Euler's method*.

- Euler's method is a useful approach, that you have possibly used before without knowing, based on a simple observation: if the value of some quantity is changing at a certain amount per time period, then it is possible to estimate the future value as follows:

  (future value) = (current value) +
  (estimated change per time period) × (number of time periods)

Question 11.4.1

(See Question 11.1.1 and Example 11.1.2.) The human population of Earth in July 2014 was 7.295 billion, and was expected to grow by about 80.245 million over the next year (note that this is a growth of 1.1%).

(a) Assuming the population increases by the same number each year, predict the population in July 2017.

(b) Instead of growing by a fixed number each year, the global population has a growth rate of about 1.1% per annum. Estimate the population in July 2015, July 2016 and July 2017.

(continued over)

Question *11.4.1 (continued)*

(c) Compare your answers to Parts (a) and (b), and explain the difference.

- Euler's method proceeds by approximating the unknown function as a series of short **straight lines**, starting from the initial point, each with:
  - **width** equal to a chosen step size h;
  - **slope** equal to the slope calculated using the DE; and
  - **height** equal to the width multiplied by the slope.
- The following is a more formal description of Euler's method.

Euler's method

Given an unknown quantity y, a DE of the form $y' = \ldots$, and a value of y (say y_0) at a given time (say t_0):

1. Choose a small *step size* h, and start at the initial point (t_0, y_0).

2. Use the DE to find the slope y'_i at the current time, by substituting the current values of t_i and y_i into the DE.

3. Advance the current point to the end point of a short straight line, by setting $t_{i+1} = t_i + h$ and $y_{i+1} = y_i + h \times y'_i$.

The new point (t_{i+1}, y_{i+1}) is the next approximate function value.

4. If t has reached the desired end-point then stop, else return to Step 2.

Question *11.4.2*

Draw a diagram illustrating Euler's method.

Example 11.4.3

Consider a population of algae growing at a rate of 10% per day, with an initial population of algal cells of 100 per mL. Thus, this population can be modelled by the DE

$$y' = 0.1y \qquad \text{where} \qquad y_0 = 100.$$

Use Euler's method to estimate the population after 5 days, using a step size of $h = 1$ day.

Answer:

We use (t_0, y_0) and the DE to approximate the next point (t_1, y_1). We compute:
$t_1 = t_0 + h = 0 + 1 = 1$ days
$y_1 = y_0 + hy_0' = y_0 + h(0.1y_0) = 100 + 1 \times 0.1 \times 100 = 110 \text{ mL}^{-1}$
Hence the new point is $(t_1, y_1) = (1, 110)$.
Our approximation for the concentration of algal cells after 1 day is $y(1) \approx 110 \text{ mL}^{-1}$.

We use (t_1, y_1) and the DE to approximate the next point (t_2, y_2). We compute:
$t_2 = t_1 + h = 1 + 1 = 2$ days
$y_2 = y_1 + hy_1' = y_1 + h(0.1y_1) = 110 + 1 \times 0.1 \times 110 = 121 \text{ mL}^{-1}$.
Hence the new point is $(t_2, y_2) = (2, 121)$.
Our approximation for the concentration of algal cells after 2 days is $y(2) \approx 121 \text{ mL}^{-1}$.

We use (t_2, y_2) and the DE to approximate the next point (t_3, y_3). We compute:
$t_3 = t_2 + h = 2 + 1 = 3$ days
$y_3 = y_2 + hy_2' = y_2 + h(0.1y_2) = 121 + 1 \times 0.1 \times 121 = 133.1 \text{ mL}^{-1}$.
Hence the new point is $(t_3, y_3) = (3, 133.1)$.
Our approximation for the concentration of algal cells after 3 days is $y(3) \approx 133.1 \text{ mL}^{-1}$.

We use (t_3, y_3) and the DE to approximate the next point (t_4, y_4). We compute:
$t_4 = t_3 + h = 3 + 1 = 4$ days
$y_4 = y_3 + hy_3' = y_3 + h(0.1y_3) = 133.1 + 1 \times 0.1 \times 133.1 = 146.41 \approx 146.4 \text{ mL}^{-1}$.
Hence the new point is $(t_4, y_4) = (4, 146.4)$.
Our approximation for the concentration of algal cells after 4 days is $y(4) \approx 146.4 \approx 146.4 \text{ mL}^{-1}$.

We use (t_4, y_4) and the DE to approximate the next point (t_5, y_5). We compute:

$t_5 = t_4 + h = 4 + 1 = 5$ days

$y_5 = y_4 + hy_4' = y_4 + h(0.1y_4) = 146.4 + 1 \times 0.1 \times 146.4 = 161.04 \approx 161.0 \text{ mL}^{-1}$.

Hence the new point is $(t_5, y_5) = (5, 161.0)$.

Our approximation for the concentration of algal cells after 5 days is $y(5) \approx 161.0 \text{ mL}^{-1}$. We have our final answer. After 5 days, there the concentration of algal cells is approximately 161 mL^{-1}.

(continued over)

***Example** 11.4.3 (continued)*

The following table summarises the above calculations:

i	t_i (days)	y_i (mL^{-1})	$t_{i+1} = t_i + h$ (days)	y_{i+1} (mL^{-1}) $= y_i + h \times y_i'$ $= y_i + h \times 0.1 \times y_i$
0	0	100	1	110
1	1	110	2	121
2	2	121	3	133.1
3	3	133.1	4	146.4
4	4	146.41	5	161.0

Figure 11.8: Five steps of Euler's method.

Figure 11.9 plots the five points calculated above, marked as asterisks, with straight lines approximating the function between these points. This graph shows an approximate solution to the differential equation over these 5 days.

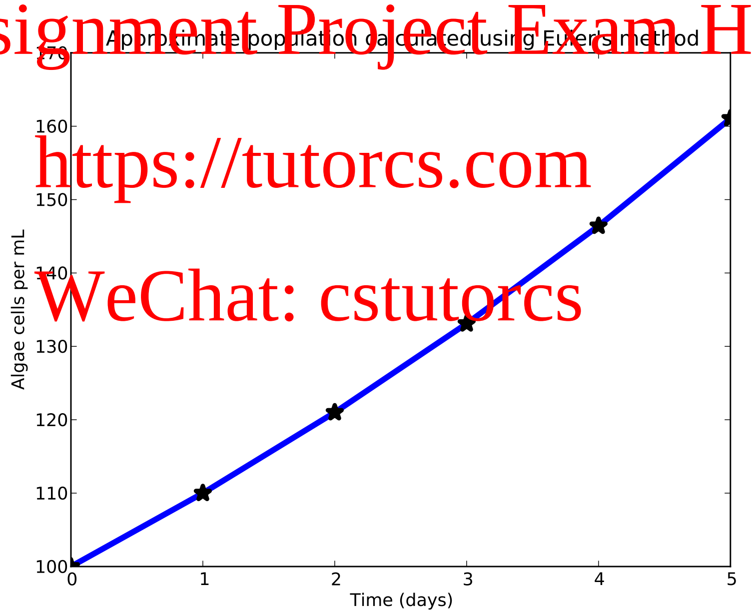


Figure 11.9: Approximate solution to the DE $y' = 0.1y$ with $y(0) = 100$ per mL.

Question *11.4.4*

How will our answer differ if we repeat the problem in the previous example, but using $h = 2.5$ days instead of $h = 1$ day?

Repeat the problem in the previous example, but using $h = 2.5$ days.

There are some important things to know about Euler's method.

- It gives an **approximate** solution, not an exact one. There will be numerical inaccuracies in the answer, particularly over a large range of t values.
- The choice of step size is very important: smaller values will give a more accurate answer, but take longer to calculate.
- Despite these limitations, the method can give very good approximate solutions to quite difficult problems.

Example *11.4.5*

In Example 11.4.3 we used a step size of $h = 1$ to solve $y' = 0.1y$.

Figure 11.10 shows approximate solutions with a step size of $h = 2.5$ (bottom curve), $h = 1$ (middle curve) and the exact, true solution (top curve).

As h becomes smaller, the solution becomes more accurate (that is, it moves closer to the true solution).

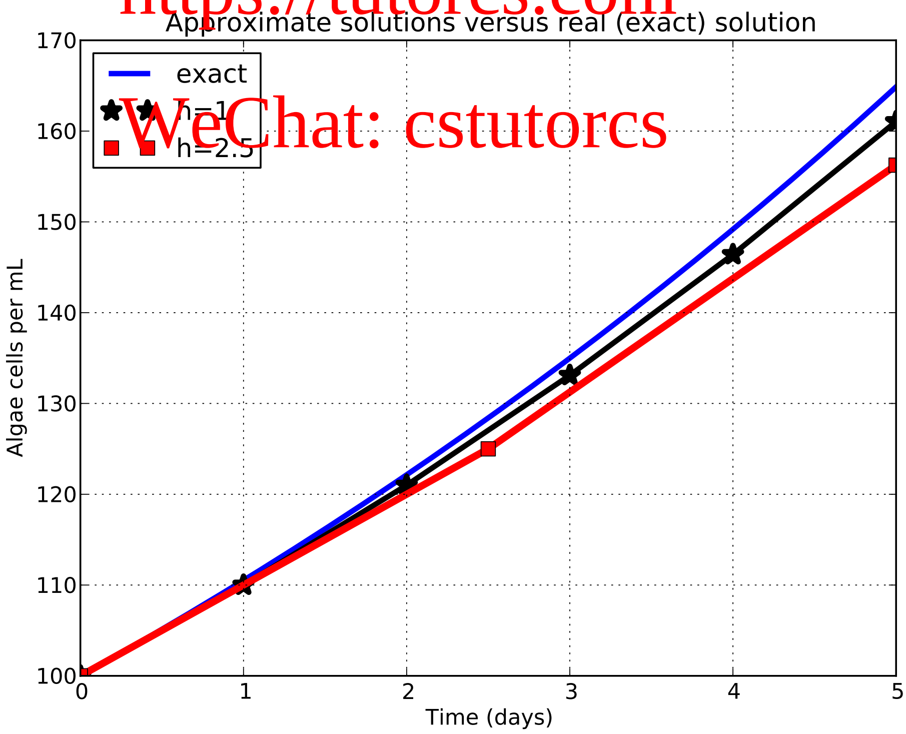


Figure 11.10: Approximate and exact solutions to the DE $y' = 0.1y$ with $y(0) = 100$ cells per mL.

Chapter 12: **Systems of DEs**

Image 12.1: *The wild hunt: Åsgårdsreien* (1872), Peter Nicolai Arbo (1831 – 1892), Nasjonalgalleriet, Oslo. (Source: en.wikipedia.org)

Here we introduce some simple systems of DEs that allow us to model organisms with multiple life stages, and situations in which multiple populations interact, such as predator/prey relationships.

We will use systems of DEs to model the spread of infectious diseases, and investigate the potential impact of pandemics. We approximate solutions to systems of DEs using Euler's method.

12.1 Introduction to systems of differential equations

- The DE models we have studied so far have all modelled a single, distinct phenomenon.
- Often, multiple factors interact, requiring more sophisticated models.
- For example:
  - in predator-prey relationships, changes in population sizes of *two* species are interrelated;
  - in species with multiple distinct life stages, changes in the population sizes within each stage depend on the numbers in other stages; and
  - the rates at which epidemics spread through populations are influenced by the number of infected individuals **and also** by the number of susceptible individuals.

- Typically, models for these more complex situations use a *system* of DEs (that is, more than one DE).
- Just as with single DEs, analytical solutions exist for some systems of DEs, but other systems require approximate solutions.
- Euler's method can be used to solve a system of DEs approximately, by applying a single iteration to each equation in turn, and then repeating.

Photo 12.1: Predators: Siberian tigers, *Panthera tigris altaica*. (Source: PA.)

***Question** 12.1.1*

If we were to study all humans on Earth, what would be some of the important life-stages to consider, and what would be some important transitions to keep track of?

Lecture 33: **Stages of life**

Learning objectives

✓ Draw and interpret life cycle diagrams

✓ Understand how to get a system of DE's from a life cycle diagram

✓ Solve and interpret systems of DE's

✓ Analyse different proposals for saving an endangered species

Scientific examples

✓ Endangered turtles

Maths skills

✓ Understand and be able to use Euler's method for systems of DE's

12.2 Going through a difficult stage

- We previously modelled populations using exponential and logistic DEs. In each case we assumed that populations were *homogeneous*; that is, every individual in the population had an identical impact on population growth.
- Many organisms have different life stages, each with substantial differences in typical survival rates and reproduction rates.
- For example, in many species, small juveniles have a low survival rate and do not reproduce, whereas mature individuals have a high survival rate and typically do reproduce.
- Hence, simple models based on single DEs are inaccurate for more advanced organisms, particularly those with long life spans. In such cases, systems of DEs give rise to better models.
- In one type of model, populations are classified into groups based on their *life stages*, such as *juvenile* or *breeding adult*.

- Rather than applying a constant growth rate to every individual in the population, a system of DEs includes:
  - the *distribution* of the population within the distinct groups;
  - differing rates of *reproduction* and *death* within groups; and
  - the *transitions* of individuals between groups.
- *Life-cycle diagrams* are useful aids to writing the equations in a system of DEs. These diagrams show the rates of *transition* between stages.

Life-cycle diagram

Life-cycle diagrams represent all possible transitions between stages in the life-cycle of an organism. Each stage is represented as a circle in the diagram, with a directed arrow joining Stage A to Stage B whenever it is possible for an individual to transition from Stage A to Stage B. Each arrow has an associated number, which is the *rate* of transition.

- In a life cycle diagram, not all pairs of stages will have an arrow between them, as some particular transitions may not be possible.
- In order to draw the life-cycle diagram for an organism, we need to know the number of stages, all possible transitions to and from each stage, including reproduction, transitions due to the passage of time, and deaths, as well as the number or probability associated with each possible transition.
- We used a dashed arrow to represent reproduction rates to distinguish it from transitions between stages (an adult does not *become* the offspring).
- Once we have drawn a life-cycle diagram, it is usually easy to write a system of DEs for the number of individuals in each stage.

Question 12.2.1

Consider an idealised fish species with two distinct life stages: juvenile and adult. Each month, on average:

- Juveniles do not breed, have a 50% probability of surviving to adulthood, and a 50% probability of dying.
- Adults produce 5 offspring (juveniles), and then die.

Photo 12.2: Bighead Gurnard Perch (*Neosebastes pandus*). (Source: DM.)

(a) Draw a life-cycle diagram for this fish, with juvenile and adult stages.

(continued over)

Question *12.2.1* *(continued)*

(b) Let the populations of juveniles and adults at any time be $J(t)$ and $A(t)$ respectively. Write a system of DEs for these populations.

(c) Assume that a specific population comprises 20 juveniles and 3 adults at time $t = 0$ months. Use Euler's method and a step size of 1 to estimate the number of fish in each stage at time $t = 2$ months.

Case Study 28: **Total turtle turmoil**

Image 12.2: Loggerhead sea turtle. (Source: en.wikipedia.org.)

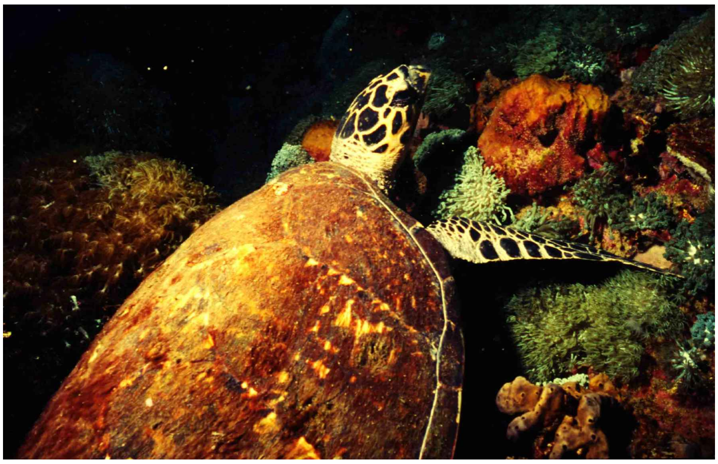


Photo 12.3: Sea turtle species. (Source: DM.)

- The loggerhead sea turtle (*Caretta caretta*) is a large marine turtle, reaching a length of around 1 m and a mass of more than 100 kg.
- The species is distributed throughout temperate, subtropical and tropical regions, and nests in a number of countries, including Australia.
- Individuals often live for more than 50 years.
- The species is listed as threatened, largely due to human activity, so is likely to become endangered within the foreseeable future.
- Ecologists have studied these turtles in detail, in order to better understand how populations change over time, to investigate possible management strategies and predict the impacts of further environmental change and human activity.
- Researchers in [4] and [6] found that these turtles move through seven distinct stages during their life cycle, and developed a population model based on these stages.
- (For interest, the researchers used a matrix model rather than a system of DEs. However, such models are equivalent to using a system of DEs and Euler's method with a step size of 1.)

- We will study a simplified version of their model, with the seven stages collapsed into three for ease of calculation.
- Figure 12.1 shows the life stages used for the simplified model, along with the estimated proportion of the total turtle population, and the global number of individuals, in each stage.

Stage	Description	Age (years)	Proportion	Global population
A	hatchlings	< 1	0.20651	1445570
B	youth	$1 - 23$	0.79097	5536790
C	breeding adult	$24 - 54$	0.00252	17640

Figure 12.1: Loggerhead sea turtles classified into three life stages.

- Each year, turtles transition with the following (rounded) probabilities:
  - Hatchlings become youths with probability $p = 0.675$ or die with probability $p = 0.325$.
  - Youths become breeding adults with probability $p = 0.000434$ and youths die with probability $p = 0.230$.
  - Breeding adults produce new hatchlings (77.4 per adult), and die with probability $p = 0.191$.
- The estimated global population across all life stages was 7 million.

***Question** 12.2.2*

(a) Draw a life-cycle diagram for the three life stages of this turtle.

(continued over)

Question *12.2.2 (continued)*

(b) Write a system of DEs for the turtle population.

(c) Use Euler's method with a step size of 1 year to estimate the number of hatchlings after one year.

(d) Using Euler's method, after one year there are 5236685 turtles in the youth stage and 16673 breeding adults. What does this indicate about future turtle populations?

This population can be modelled using a computer program.

Program 12.1: Turtles

```python
# Uses Euler's method to model the turtle population.
from pylab import *

# Initialise variables.
maxt = 30
Apops = zeros(int(maxt+1))
Bpops = zeros(int(maxt+1))
Cpops = zeros(int(maxt+1))
Apops[0] = 1445570
Bpops[0] = 5536790
Cpops[0] = 17640
stepsize = 1

# Step through Euler's method for 30 years.
i = 1
while i < (maxt+1):
    dA = -Apops[i-1] + 77.4 * Cpops[i-1]
    dB = 0.675 * Apops[i-1] - 0.230434 * Bpops[i-1]
    dC = 0.000434 * Bpops[i-1] - 0.191 * Cpops[i-1]
    Apops[i] = Apops[i-1] + stepsize * dA
    Bpops[i] = Bpops[i-1] + stepsize * dB
    Cpops[i] = Cpops[i-1] + stepsize * dC
    i = i + 1

# Output the graph.
times = arange(0, maxt+1)
plot(times, Apops, "bx", markersize=8,mew=2,label='Stage A')
plot(times, Bpops, "r+", markersize=8,mew=2,label='Stage B')
plot(times, Cpops, "gs", markersize=6,mew=2,label='Stage C')
plot(times, Apops+Bpops+Cpops, "ko", markersize=6,mew=2,label='
    Total')
xlabel("Time (years)")
ylabel("Number of turtles")
title("Turtle population")
grid(True)
legend()
# savefig('turtlepop.pdf',format='pdf')
show()
```

Figure 12.2 shows the output from running the program.

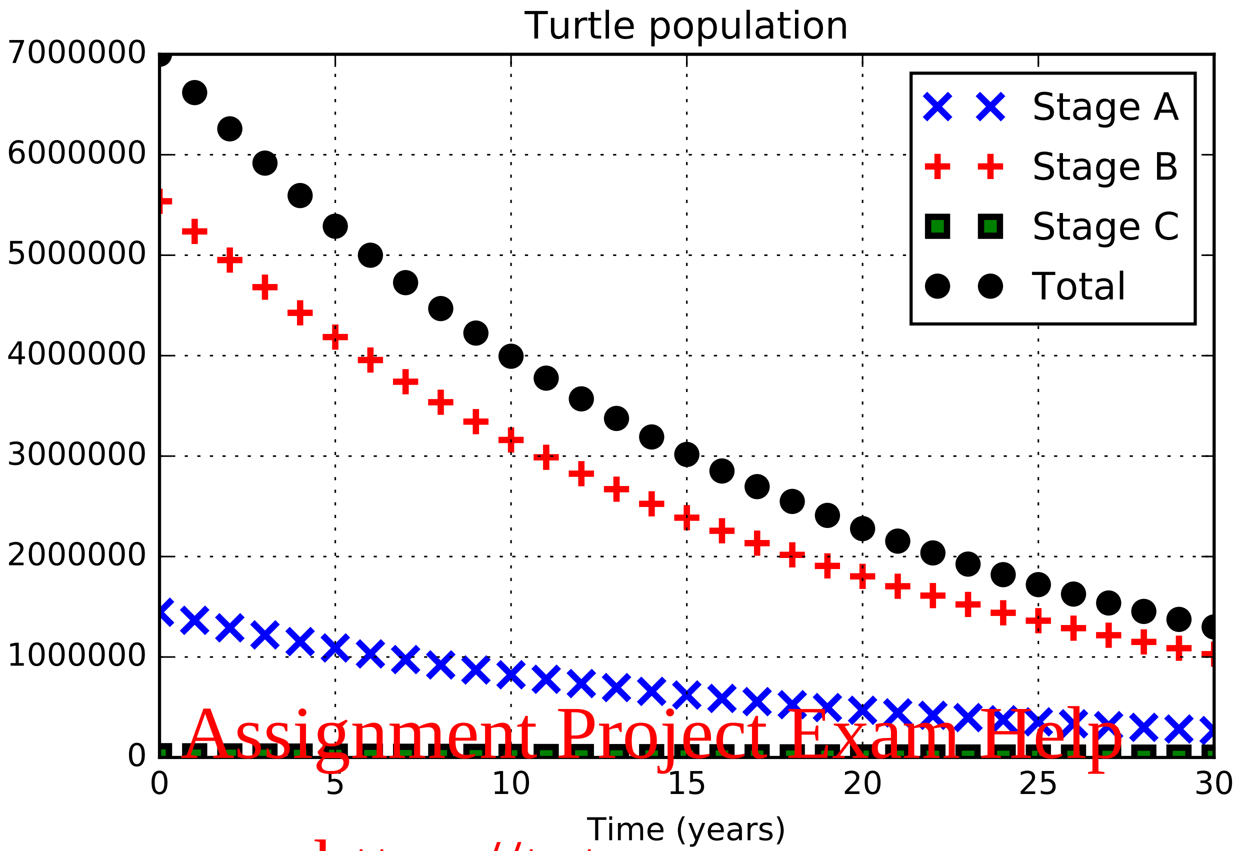


Figure 12.2: Turtle population modelled using Euler's method.

Question 12.2.3

Researchers and authorities have proposed various conservation strategies for the sea turtle; see [7]. Briefly discuss some possible strategies, and explain how the population model would change to reflect them.

End of Case Study 28: Total turtle turmoil.

Lecture 34: **Predator-prey systems**

Learning objectives

✓ Analyse the interaction between populations in which one species relies on another as their food source

✓ Critically evaluate real-world data compared with model predictions

Scientific examples

✓ Canadian lynx and snowshoe hare

Maths skills

✓ Interpret the meaning of terms in a system of DE's

12.3 Eat or be eaten

- In addition to modelling individual organisms with multiple life stages, systems of DEs can also model interactions between multiple species.
- For example, the classical *predator/prey* problem in ecology considers what happens to the populations of two species when one preys on the other.
- In laboratory situations there is control over these interactions. In nature, inter-species interactions are highly complex. We will first investigate a controlled example, then model a real interaction. The controlled example is very simple, but is not completely unreasonable.

Photo 12.4: Left: skeleton of *Tyrannosaurus rex*. Right: skeleton of *Triceratops horridus*. (Source: PA.)

Case Study 29: **It's just not cricket**

- One method of predicting what may happen in a real-world situation is to simulate it in a laboratory.
- Unpredictable phenomena complicate and impact predator/prey interactions in nature. However, controlled laboratory simulations can give valuable insight into real situations.
- Consider a controlled, time-compressed laboratory experiment simulating the effects of immigration, emigration, births and deaths on populations of frogs (predators) and crickets (prey).
- Initially the experiment comprises 60 frogs and 400 crickets. Each day:
  - 15 crickets are introduced into the experiment (modelling immigration and birth of crickets);
  - 25% of the frogs each eat a cricket (death of crickets);
  - 12 frogs are removed (modelling emigration and death of frogs); and
  - for each 25 crickets present, one new frog is introduced (modelling birth and immigration of frogs based on available food resources).

Photo 12.5: Left: Striped burrowing frog, *Litoria alboguttata*. Right: cricket. (Source: DM.)

Question 12.3.1

Let $F(t)$ and $C(t)$ be the populations of frogs and crickets at time t in days.

(a) Write DEs involving the rate of change of **each** of the populations.

(b) Using differentiation it can be shown that solutions to the DEs are:

$$F(t) = 40\sin 0.1t + 60 \qquad C(t) = 100\cos 0.1t + 300.$$

(continued over)

***Question** 12.3.1 (continued)*

Figure 12.3 shows $C(t)$. On the graph, sketch $F(t) = 40\sin 0.1t + 60$.

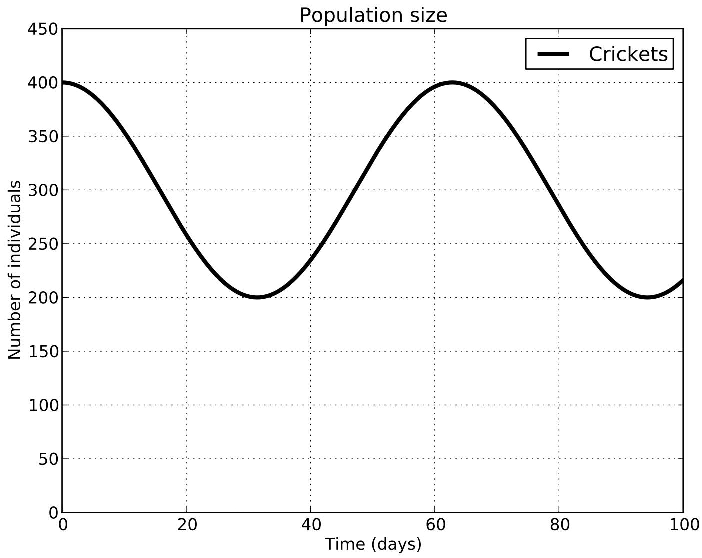


Figure 12.3: Population of crickets.

(c) Interpret the population dynamics. One population is “leading” and the other “lagging”. Identify which is which, and explain your answer.

Photo 12.6: Left: genuine KTF. Right: fossilised cricket. (Source: PA.)

End of Case Study 29: It’s just not cricket.

- Now we will develop a more realistic predator/prey model. In general, such models are based on the assumptions that:
  - the prey has no other predators, and the predator no other prey;
  - there is no significant change to the environment or species' genetics;
  - the prey species is not resource limited, so breeds rapidly and individuals **do not** compete with each other;
  - the predator species is resource limited, so breeds more slowly and individuals **do** compete with each other.

Question *12.3.2*

Let W be a population of wolves (predators) and E be a population of elk (prey). How are the values of the *rates of change* of each of W and E influenced by the values of W and E, using the assumptions above? In each case, identify whether the impact is positive or negative.

(a) E' is: positively/negatively dependent on the value of E.

(b) E' is: positively/negatively dependent on the value of W.

(c) W' is: positively/negatively dependent on the value of E.

(d) W' is: positively/negatively dependent on the value of W.

Photo 12.7: Left: gray wolf (*Canis lupus*). Right: elk (*Cervus canadensis*). (Source: PA.)

- The best-known predator/prey model is the **Lotka-Volterra** model.

Lotka-Volterra model

Let $P(t)$ and $Q(t)$ be the population sizes of a predator (for example, wolf) and prey (for example, elk) species respectively, at any time t. The following system of DEs forms the *Lotka-Volterra model*:

$$\begin{aligned} Q' &= aQ - bPQ \\ P' &= -cP + dPQ \end{aligned}$$

where a, b, c and d are positive constants whose values depend on various characteristics of the species and their physical interactions.

Question *12.3.3*

Carefully explain the meaning of each term in the Lotka-Volterra equations. In particular, explain the physical relevance of the terms involving PQ.

Case Study 30: **Snowshoe hares and Canadian lynx**

Image 12.3: Canadian lynx chasing a snowshoe hare. (Source: www.animalspedia.com.)

- The Canadian lynx, *Lynx canadensis*, is a member of the feline family distributed predominantly in Canada and Alaska. Lynx are carnivorous, with individuals weighing 8 to 15 kg, and living for up to 15 years.
- The primary food source (up to 95%) of the Canadian lynx is the snowshoe hare, *Lepus americanus*. The hare has large hind feet (for moving on snow) and turns white in winter.
- People have hunted these lynx and hares for their fur for many years. Harvest records dating from the 1730s allow long-term population estimates.
- Figure 12.4 (from [30]) graphs these data over 90 years, and shows a series of reasonably regular fluctuations in the sizes of both populations. Note the similarity to the periodic population movements in the laboratory-controlled predator/prey relationship between frogs and crickets.

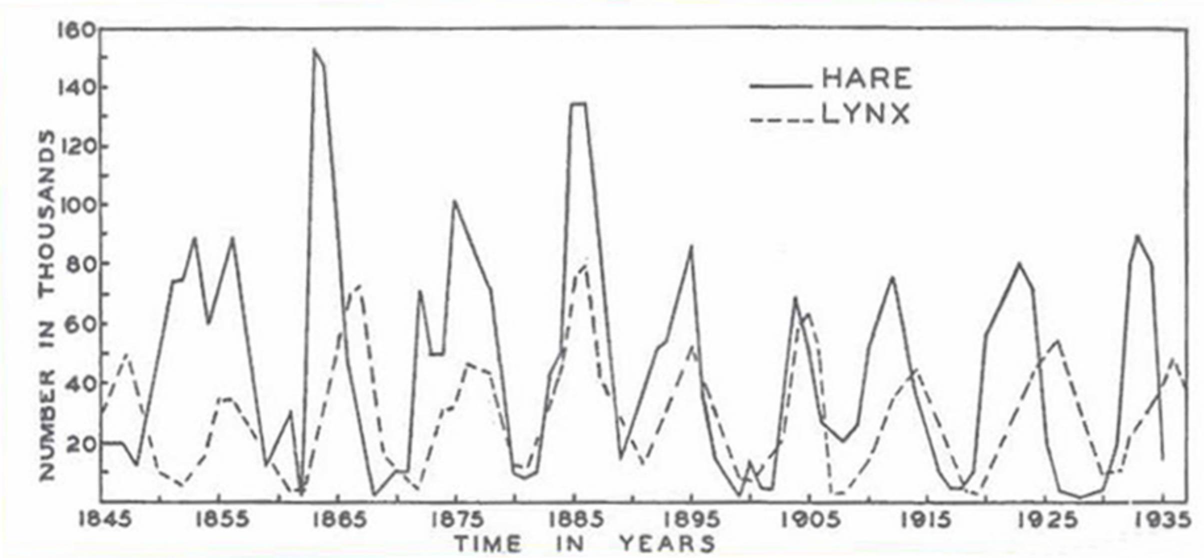


Figure 12.4: Numbers of Canadian lynx and snowshoe hares. (Source: [30].)

Question 12.3.4

Let $L(t)$ and $H(t)$ be the populations of lynx (predators) and hares (prey) respectively, **in thousands**. The Lotka-Volterra equations are:

$$H' = aH - bHL \qquad\qquad L' = -cL + dHL$$

(a) If either population suddenly became extinct, what does the model predict will happen to the other population? Simplify the differential equations and then sketch rough graphs of the populations.

(b) What would you expect to happen in reality?

Example 12.3.5

Figures 12.5 and 12.6 show data from the Canadian Government and the Hudson's Bay Company, estimating the populations of hare and lynx in part of their range from 1900 to 1920. (All populations are in thousands.)

(continued over)

***Example* 12.3.5** *(continued)*

Year	Hares	Lynx	Year	Hares	Lynx	Year	Hares	Lynx	Year	Hares	Lynx
1900	30	4	1905	20.6	41.7	1910	27.1	7.4	1915	19.5	51.1
1901	47.2	6.1	1906	18.1	19	1911	40.3	8	1916	11.2	29.7
1902	70.2	9.8	1907	21.4	13	1912	57	12.3	1917	7.6	15.8
1903	77.4	35.2	1908	22	8.3	1913	76.6	19.5	1918	14.6	9.7
1904	36.3	59.4	1909	25.4	9.1	1914	52.3	45.7	1919	16.2	10.1
									1920	24.7	8.6

Figure 12.5: Populations of lynx and hares (in thousands).

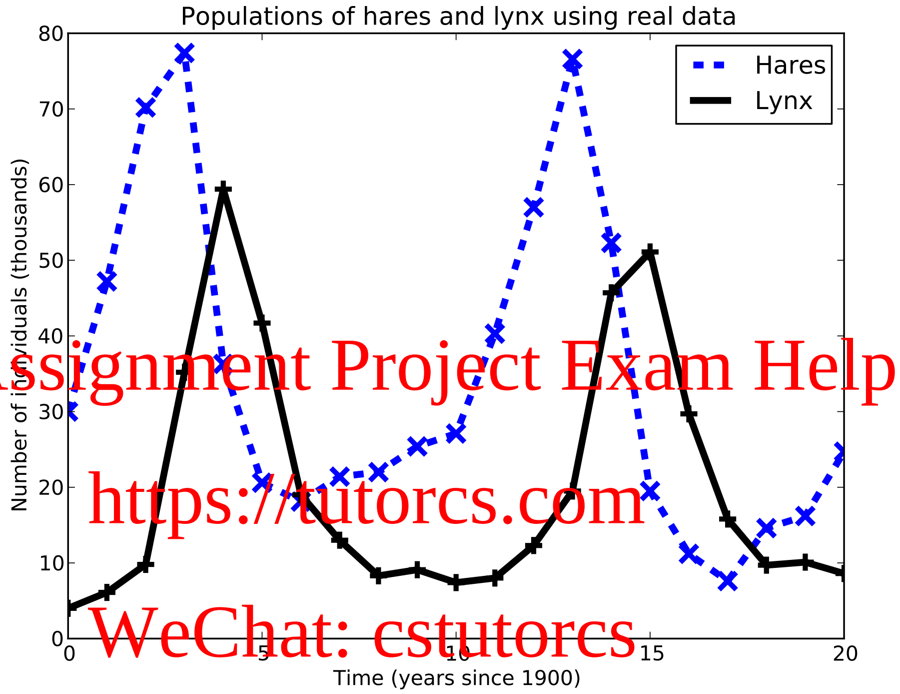


Figure 12.6: Graph of the populations of lynx and hares (in thousands).

- Experimentation and analysis show that for this time period, reasonable values for the constants a, b, c and d in the Lotka-Volterra equations are: $a = 0.484$, $b = 0.028$, $c = 1$ and $d = 0.032$ (in appropriate units).

Now we can use Euler's method to model the population sizes.

Program specifications: Develop a Python program that uses Euler's method with step size of 0.1 year to model the populations of lynx and hares.

Program 12.2: Lotka-Volterra model of hares and lynx.

```
# Uses Euler's method and Lotka-Volterra equations to model
# populations of lynx and hare from 1900 to 1920.
from pylab import *

# Initialise variables for Euler's method.
ss=0.01

time=arange(0,40.1,ss)
a = 0.484
b = 0.028
c = 1
d = 0.032

H = zeros(int(size(time)))
L = zeros(int(size(time)))
H[0] = 30.0
L[0] = 4.0

nn=size(time)

# Step through Euler's method with stepsize ss.
# Repeatedly calculate derivatives then update the 'next' values.
i = 0
while i < nn-1:
    dH =  a*H[i] - b*H[i]*L[i]
    dL = -c*L[i] + d*H[i]*L[i]

    H[i+1] = H[i] + ss*dH
    L[i+1] = L[i] + ss*dL
    i = i+1

# Output graphs.
xlabel("Time (years since 1900)")
ylabel("Number of individuals (thousands)")
title("Modelled populations of hares and lynx")
plot(time, H, "b-", linewidth=3, label='H(t)')
plot(time, L, "k-", linewidth=3, label='L(t)')
legend()
show()
```

Example 12.3.6

Below is the output of the above program.

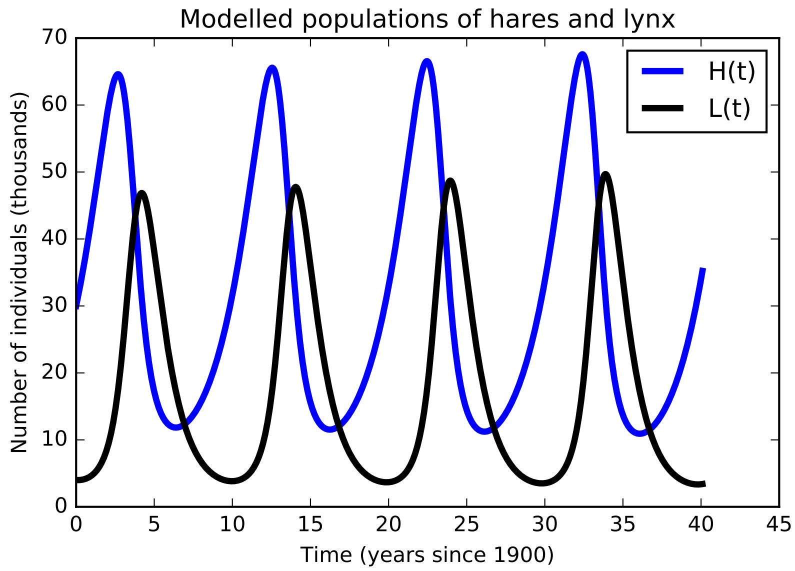


Figure 12.7: Modelled lynx and hare populations.

- At time $t = 0$ years (corresponding to year 1900), data show that there were 30 (thousand) hares and 4 (thousand) lynx in the monitored region.
- Figure 12.8 compares the modelled population sizes over 20 years with the real (measured) data for each population.

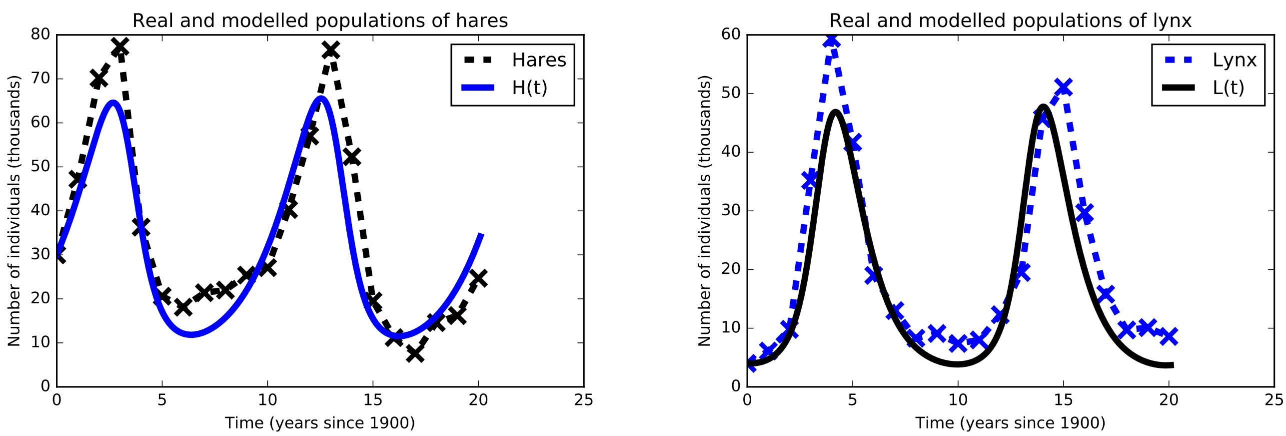


Figure 12.8: Real and modelled populations for hare (left) and lynx (right).

Question 12.3.7

(a) Comment on the results in Example 12.3.6.

(b) Critically evaluate the following possible media statement:

> *A survey has shown that the populations of lynx and snowshoe hares are both in decline. We need to act promptly or else one or both species will become extinct.*

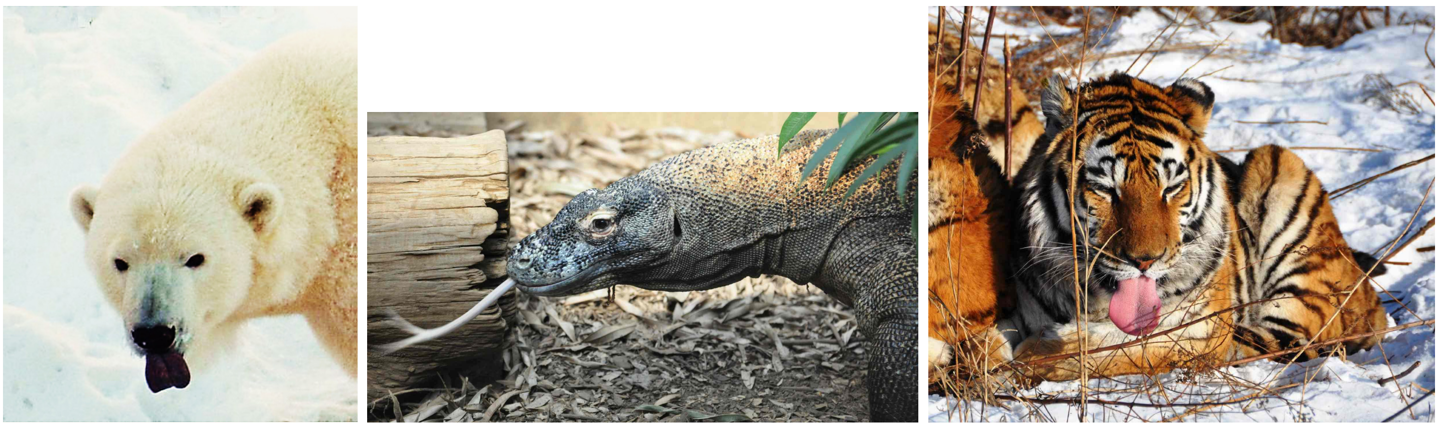


Photo 12.8: Three top predators. Left: polar bear, *Ursus maritimus*. Centre: Komodo dragon, *Varanus komodoensis*. Right: Siberian tiger, *Panthera tigris altaica*. (Source: PA.)

End of Case Study 30: Snowshoe hares and Canadian lynx.

Learning objectives

✓ Understand the mechanisms for modelling epidemics

Scientific examples

✓ Disease models

✓ Rubella

Maths skills

✓ Interpret the meaning of terms in a system of DE's

12.4 Epidemics and SIR models

- In this section, we will use systems of DEs to model the large-scale spread of communicable disease through a population over time.

Epidemic

A large-scale occurrence of disease in a human population is called an **epidemic** if new cases of the disease arise at a rate that "substantially exceeds what is expected" in a given time period. Localised occurrences are called **outbreaks**, and global occurrences are often called **pandemics**.

- Modelling diseases is important to understanding how they spread, and how their impact may be mitigated through approaches such as quarantine, vaccination and public health campaigns.

Photo 12.9: Images commemorating the bubonic plague in Eyam, the "Plague Village", UK. Left: stained glass 'Plague Window'. Centre: first page of the list of names of villagers who died from plague in 1665–6. Right: tombstone. (Source: PA.)

- Modelling disease spread often begins with estimates of the number of secondary infections that typically arise from an individual with the disease, and the rate at which individuals recover from the disease.

Basic reproduction number and infectious period

The **infectious period** of a disease is the average length of time during which an infective individual can infect a susceptible individual. Many diseases are infectious before symptoms become apparent.

The **basic reproduction number** R_0 of a disease is the average number of secondary infections caused by a single infective individual in a completely susceptible population, in the absence of any preventive interventions.

The value of R_0 is determined by factors including how infectious the disease is, how it is spread and the duration of the infectious period.

- Figure 12.9 gives information for some well-known communicable diseases.

Disease	Transmission method	R_0	Infectious period
Rubella	Airborne droplet	≈ 5	2 weeks
Measles	Airborne droplet	$12 - 18$	10 days
Whooping cough	Airborne droplet	$12 - 17$	3 weeks
Mumps	Airborne droplet	$4 - 7$	14 days
Swine flu	Airborne droplet	$1.4 - 1.6$	6 days
Seasonal influenza	Airborne droplet	$2 - 3$	6 days
COVID-19	Airborne droplet	$1 - 3$	$7 - 14$ days
Polio	Faecal/oral	$5 - 7$	$6 - 20$ days
HIV/AIDS	Sexual contact	$2 - 5$	unlimited
Syphilis	Sexual contact	≈ 1.5	up to 2 years
Human papillomavirus	Sexual contact	$1 - 3$	very variable
Pneumonic plague	Airborne droplet	≈ 1.3	2 days (100% death rate)

Figure 12.9: Transmission methods, infectious periods and values of R_0 for some communicable diseases.

Infection rate and recovery rate

The **infection rate** a is the rate at which secondary infections arise from a single infective individual, and is defined to equal the basic reproduction number divided by the infectious period (IP). Thus, $a = \dfrac{R_0}{IP}$.

The **recovery rate** b is the rate at which an infective individual recovers, and is defined to equal 1 divided by the infectious period. Thus, $b = \dfrac{1}{IP}$.

Question 12.4.1

Calculate the infection rate and the recovery rate for rubella.

- The SIR (**S**usceptible, **I**nfective, **R**emoved) epidemic model is used to model many diseases, including rubella, measles, cholera and bubonic plague.

SIR model of epidemics

The *SIR* epidemic model classifies a population into three distinct **compartments** or groups, and uses a system of DEs to predict the changes in the number of people in each group. At any time t:

(1) The *susceptible* compartment $S(t)$ is the group of people who are susceptible to the disease.

(2) The *infective* compartment $I(t)$ is the group of people who have the disease and can infect susceptible people.

(3) The *removed* compartment $R(t)$ is the group of people who cannot catch the disease, either because they have permanently recovered, are naturally immune, or have already died from the disease.

(continued over)

SIR model of epidemics *(continued)*

The only possible transitions in the SIR model are that: a susceptible person can become infective; and an infective person can become removed.

The model assumes that there are no births, no deaths from any other causes, and that the population mixes homogeneously.

Question *12.4.2*

Draw a "life-cycle diagram" for the SIR model, with infection rate a and recovery rate b.

Effective reproduction number and infection rate

Often, not everyone in a population is susceptible to a disease. The **effective reproduction number**, R_e, is an estimate of the *average* number of secondary infections arising from an infective individual. If a population of size N contains S susceptible individuals then

$$R_e(t) = R_0 \times \frac{S(t)}{N}.$$

Similarly, if a is the infection rate in a fully susceptible population, then the **effective infection rate** in a population that is not fully susceptible is

$$a_e(t) = a \times \frac{S(t)}{N}.$$

The equations for the SIR model

If a population of N people at time t is divided into three compartments, susceptible $S(t)$, infective $I(t)$ and removed $R(t)$, then the SIR model is:

$$\begin{aligned} S' &= -a \times \frac{S}{N} \times I \\ I' &= a \times \frac{S}{N} \times I - bI \\ R' &= b\, I \end{aligned}$$

where a is the infection rate and b is the recovery rate. Note that the population size N remains constant because the numbers of people moving between compartments always balance.

Question 12.4.3

Explain what each of the terms in each of the SIR equations represents.

Case Study 31: **Rubella**

- **Rubella** (or **German measles**) was (and in some countries, still is) a common disease, particularly in childhood.
- In most cases, symptoms are very mild, and may even pass unnoticed. However, if a woman is infected during the first 20 weeks of pregnancy then spontaneous abortion can occur (in about 20% of cases), or the child may be born with congenital rubella syndrome (CRS), which is a range of incurable conditions including deafness, blindness and intellectual impairment.
- The risk of developing CRS in an unborn child is as high as 90% if the mother is infected during the first 10 weeks of pregnancy.
- There was a rubella epidemic in the USA between 1962 and 1965. Data from [40] show that during 1964–65 there were
  - 12.5 million rubella cases
  - 11000 abortions (spontaneous and surgical)
  - 20000 infants born with CRS (12000 deaf, 3580 blind, 1800 with intellectual impairment)

- During that epidemic, 1% of all children born in New York were affected.
- A vaccine was introduced in 1969 and is routinely administered in many countries. In Queensland, the Department of Health recommends all children have combined MMR (measles, mumps and rubella) vaccines at the ages of 12 months and 4 years.
- Vaccination campaigns have greatly reduced the incidence of rubella and the frequency of outbreaks. The Centers for Disease Control and Prevention announced that rubella was eliminated from the USA in 2004.
- In January 2008, at least four babies in Sydney became infected with rubella. All were less than 12 months old, so were under the age for vaccination with the MMR vaccine.

***Example** 12.4.4*

Assume that a population of 10000 people contains 10 people infective with rubella, and that everyone else is susceptible. Using the values of a and b from above, the SIR equations for rubella are:

$$\begin{aligned} S' &= -2.5 \times \frac{S}{10000} \times I \\ I' &= 2.5 \times \frac{S}{10000} \times I - 0.5I \\ R' &= 0.5I \end{aligned}$$

where $I(0) = 10$, $S(0) = 9990$ and $R(0) = 0$.

***Question** 12.4.5*

Use Euler's method and a stepsize of one week to estimate the number of people in each category after one week.

Now we can develop a computer program to model a rubella epidemic.

Program 12.3: SIR model of rubella.

```python
# This program uses Euler's method and the SIR equations to
# model the spread of rubella in a population with a proportion
# of between 0 and 1 of the population being vaccinated.
from pylab import *

# Input parameters for the model.
maxt = float(input("Over how many weeks should the model run? "))

# Initialise variables for rubella; values of a and b are per week.
N = 10000
a = 2.5
b = 0.5

# Initialise variables for Euler's method. The stepsize is 0.1 week.
ss = 0.1
time=arange(0, maxt+0.1, ss)
nn=size(time)
SA = zeros(int(nn))
IA = zeros(int(nn))
RA = zeros(int(nn))

# Set the initial number of people in each category.
IA[0] = 10
SA[0] = N - IA[0]
RA[0] = N - SA[0] - IA[0]

# Step through Euler's method with stepsize ss.
i = 0
while i < nn-1:
    dS = -a * SA[i] * IA[i]/N
    dI = a * SA[i] * IA[i]/N - b * IA[i]
    dR = b * IA[i]
    SA[i+1] = SA[i] + ss*dS
    IA[i+1] = IA[i] + ss*dI
    RA[i+1] = RA[i] + ss*dR
    i = i+1
# Output
xlabel("Time (weeks)")
ylabel("Number of people")
title("SIR model of rubella")
plot(time, SA, "b--", linewidth=5,label="Susceptible")
plot(time, IA, "r-", linewidth=5,label="Infective",)
plot(time, RA, "k-.", linewidth=5,label="Removed")
legend(loc="center right")
grid(True)
savefig('rubella1.pdf',format='pdf')
show()
```

Example 12.4.6

Figure 12.10 shows the program output for a period of 12 weeks.

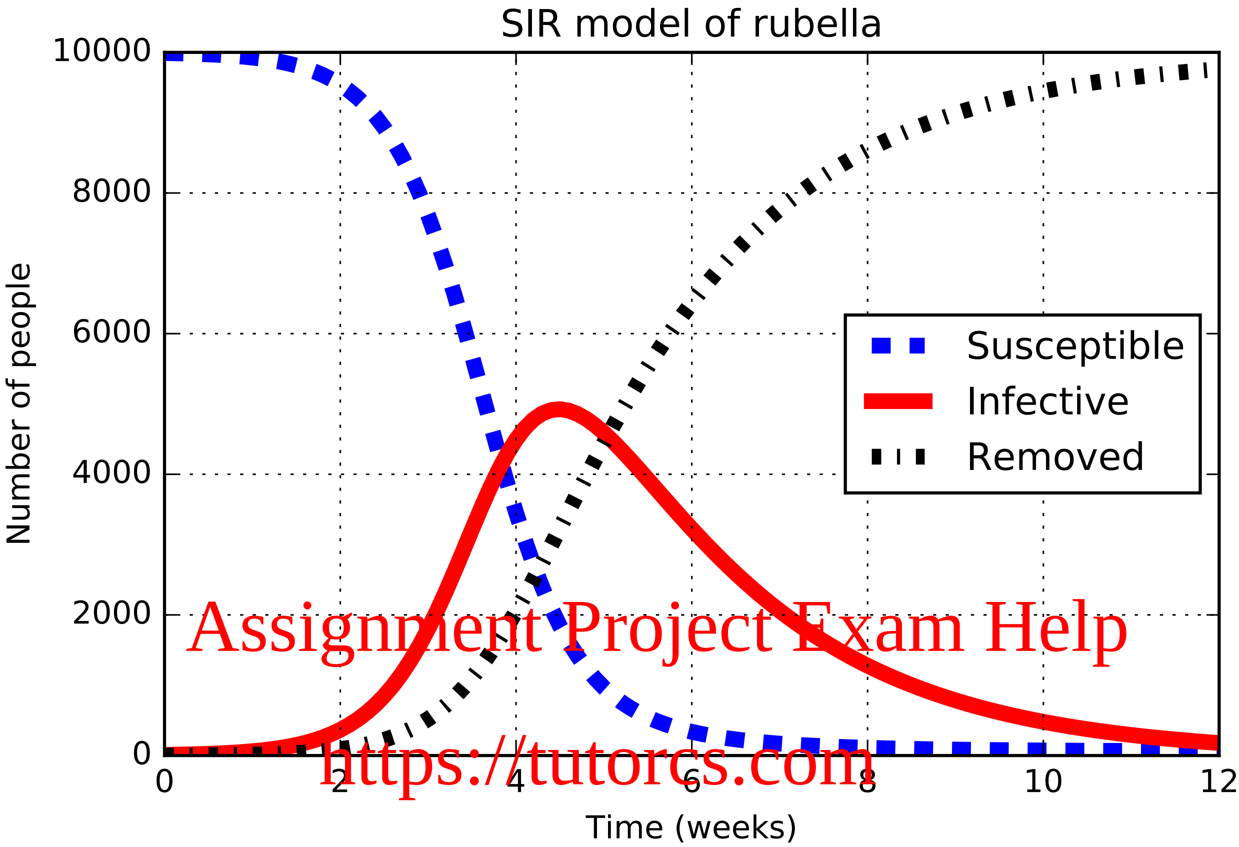


Figure 12.10: A rubella epidemic modelled using Euler's method, showing the numbers of people who are susceptible $S(t)$, infective $I(t)$ and removed $R(t)$.

Figure 12.10 shows that:

- An epidemic occurs. (This is expected, because the population mostly comprises susceptible individuals.)
- The epidemic lasts for about 12 weeks.
- The peak number of infectives is 4925, which occurs about 4.4 weeks after infectives first entered the population.

End of Case Study 31: Rubella.

Lecture 36: **Modelling for public health**

Learning objectives

✓ Analyse predictions of outbreaks and pandemics

✓ Understand and interpret the threshold vaccination levels to avoid epidemics

Scientific examples

✓ Vaccination targets

✓ Catastrophe planning

Maths skills

✓ Interpret differential equations

Case Study 32: **Vaccinations**

Question 12.4.7

Find an expression for R_0 (the basic reproduction number of a disease) in terms of the infection rate a and the recovery rate b. What does this expression indicate about the relative values of a and b in order for an epidemic to begin?

- An epidemic occurs if introducing a group of infected people into a population causes an increase in the number of infectives in the population (that is, $I' > 0$).

Question 12.4.8

If a is the infection rate and b is the recovery rate then the DE for I' is:

$$I' = a \times \frac{S}{N} \times I - bI.$$

(a) Show that for an epidemic to occur, the proportion of susceptibles in the population must be more than $1/R_0$ (according to the SIR model).

(continued over)

***Question** 12.4.8 (continued)*

(b) With reference to the effective reproduction number, R_e, explain why an epidemic will occur if a fraction of more than $1/R_0$ of a population is susceptible.

(c) If the susceptible proportion of a population is smaller than $1/R_0$, then the population is said to have *herd immunity* to the disease. Why is herd immunity to, say, Rubella, desirable?

(d) Mass public vaccination aims to vaccinate a certain proportion of people. What level of coverage do authorities typically aim for? Why?

Question *12.4.9*

Explain why the target vaccination rate for measles is (at least) 95%. What is the figure for rubella?

Question *12.4.10*

Should vaccinations be compulsorily enforced? Why or why not?

Example *12.4.11*

In 1998, a paper published in the Lancet (with lead author Dr Andrew Wakefield) claimed to identify a link between the MMR (Measles, Mumps and Rubella) vaccine and autism in children. The findings had a large impact on the public perception of the vaccination. As a result, more than 3 million young people in the UK were not fully vaccinated.

In recent years, the study linking MMR vaccines with autism has been completely discredited. Wakefield was found to have committed scientific fraud by falsifying data, to have acted dishonestly and irresponsibly, and to have a conflict of interest. A number of his research papers were retracted by the journals that had previously published them, and in 2010 he was struck off the UK medical register.

Question 12.4.12

In 1995/6, around 91% of 2 year old children in the UK had received the MMR vaccine. The figure then dropped steadily to 79.9% in 2003/4, and then rose to 92.3% in 2012/13. Figure 12.11 shows the number of laboratory confirmed cases of measles and rubella in England over recent years.

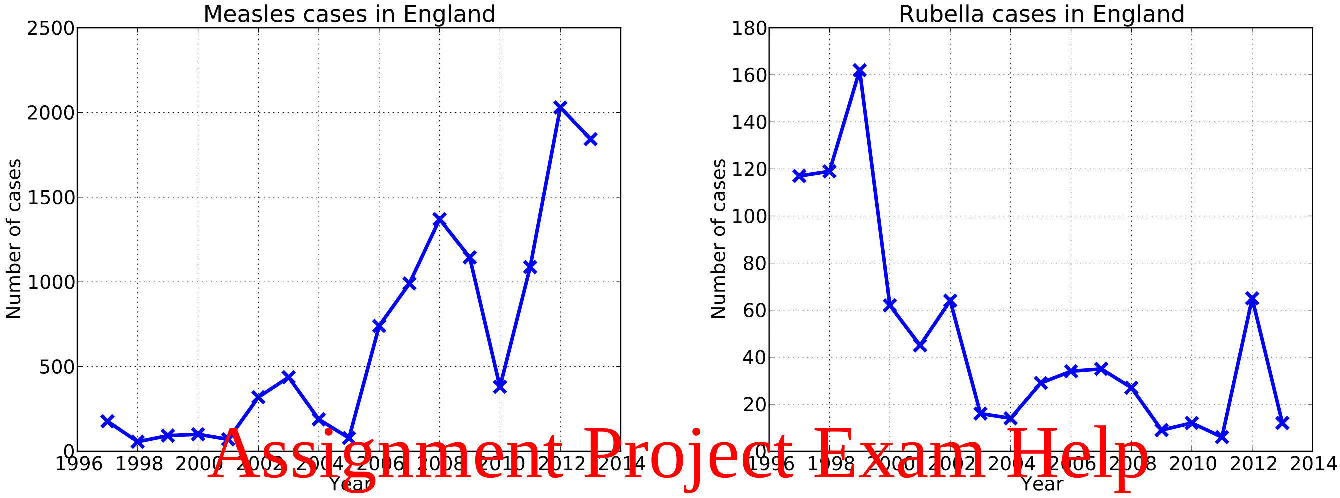


Figure 12.11: Number of laboratory confirmed cases of measles (left) and rubella (right) in England.

(a) Why might the measles graph in Figure 12.11 have that shape?

(b) Why might the rubella and measles graphs have different shapes?

Question 12.4.13 *(continued)*

How could the SIR model the effects of different vaccination rates?

Question 12.4.14

Earlier we modelled a rubella epidemic in a city with 9990 susceptible people and 10 infective people. Figure 12.12 shows the predicted numbers of infective people $I(t)$ under five scenarios, with vaccination rates of 0%, 20%, 40%, 60% and 80%.

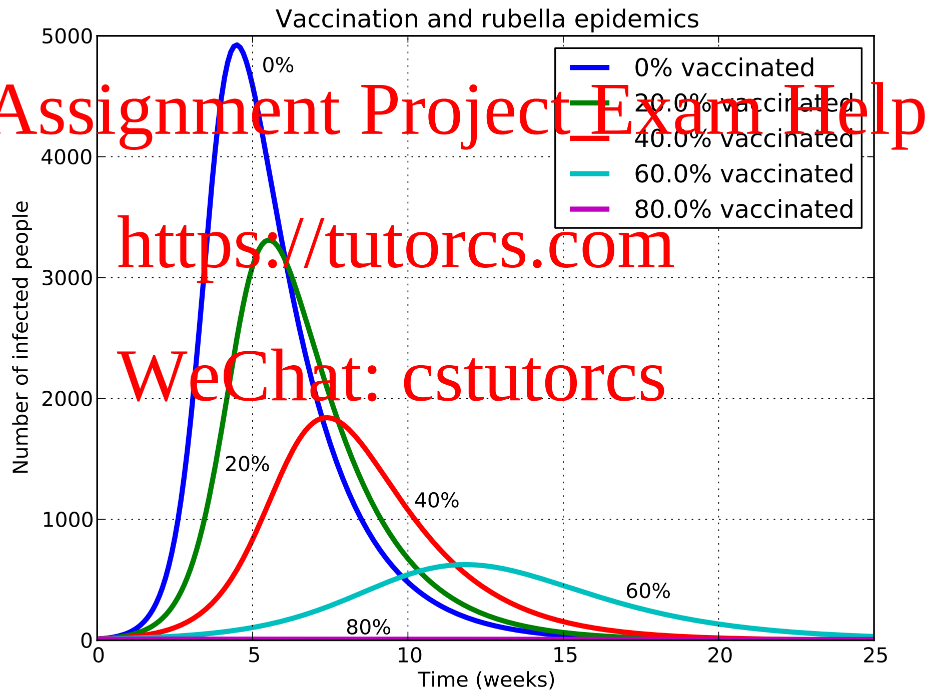


Figure 12.12: The effect of different vaccination rates on a rubella epidemic.

Interpret and explain the graphs. What are the benefits of increased vaccination rates?

End of Case Study 32: Vaccinations.

12.5 Catastrophe planning

- Many governments conduct *catastrophe planning*, modelling the potential impacts of disastrous events, such as nuclear explosions, terrorist strikes, tsunamis, earthquakes and pandemics. Much of this work is highly secret, partly for security reasons, but also because some of the predicted outcomes are too frightening for public release.

***Example* 12.5.1**

In terms of numbers of fatalities, five of the worst catastrophes in (European) Australian history are:

- Spanish influenza in 1918–19, causing more than 12000 deaths;
- a polio epidemic in 1946–55, causing more than 1000 deaths;
- the COVID-19 pandemic starting in 2020, causing more than 900 deaths;
- a naval battle in the Second World War, causing 727 deaths; and
- a bubonic plague epidemic in 1900–1910, causing 550 deaths.

In addition, thousands of indigenous Australians died from communicable diseases introduced by European settlement.

***Example* 12.5.2**

In the 1300s, the bubonic plague or *Black Death* killed around 20 million Europeans in six years, which was about one third of the population. In the worst affected urban areas, around half of the population died. The plague returned regularly, with around 100 epidemics occurring in the next 400 years.

(continued over)

***Example* 12.5.2** *(continued)*

Photo 12.10: Plague monuments. Left: Brno, Czech Republic. Centre: Vienna, Austria. Right: Olomouc, Czech Republic. (Source: PA.)

***Example* 12.5.3**

A Spanish influenza pandemic occurred in 1918–1919. Within six months the global death toll was 25 million (more than the number who died from combat in the First World War). The flu was so virulent and deadly that it 'burnt itself out', disappearing completely within 18 months.

Case Study 33: **Avian influenza**

- From 2008–2020, the SCIE1000/1100 course book contained the following warning from the The World Health Organisation (WHO):
  - the risk of an influenza pandemic is high;
  - H5N1 (avian) influenza is endemic in many bird populations;
  - bird-to-human transmission has already caused fatalities; and
  - there is a serious risk that the virus could mutate and become human-to-human transmissible, leading to a "nightmare scenario".

Photo 12.11: Left: Marabou Stork, *Leptoptilos crumeniferus*. Right: Painted Desert, USA. (Source: PA.)

- We will use the **SIRD** model to investigate the potential impact of a catastrophe caused by human-transmissible avian influenza.
- The SIRD model divides the population into **four** distinct compartments: Susceptible, $S(t)$; Infective, $I(t)$; Recovered, $R(t)$; and Dead, $D(t)$.
- The only possible movements of people *between* compartments are: susceptible people can become infective; infective people can recover or die.

Question *12.5.4*

Draw a "life-cycle diagram" for the SIRD model, with infection rate a, recovery rate b and death rate c.

- There has never been a verified case of human-to-human transmission of avian influenza, but by choosing reasonable values for all parameters, we can model a hypothetical avian flu epidemic.
- We can also modify this SIRD model with reasonable parameters for COVID-19, and compare the model predictions against real data.

- Researchers estimate the following values for the Spanish Flu pandemic in 1918–1919. We will use these values in our SIRD model of a hypothetical avian flu outbreak.

$$
\begin{aligned}
a &= \text{infection rate} & b &= \text{recovery rate} & c &= \text{mortality rate}\\
&= 1.9 \text{ week}^{-1}; & &= 1.4 \text{ week}^{-1}; & &= 0.065 \text{ week}^{-1}.
\end{aligned}
$$

Question *12.5.5*

Using the values for the infection rate and recovery rate of the Spanish Flu, calculate the values of R_0 and the infectious period (IP). Are these values reasonable for a “flu”?

- Now we can use Euler’s method and the SIRD model to investigate various scenarios in a city such as Brisbane with $N = 2 \times 10^6$.

Example *12.5.6*

There is one infective person in a city in which $N = 2 \times 10^6$ and everyone else is susceptible. Figure 12.13 shows the results. The model predicts that the disease outbreak will last for about 45 weeks, around 870000 people will become ill, the largest number of infective people at any time will be about 59500, and that approximately 38500 people will die.

(continued over)

Example 12.5.6 *(continued)*

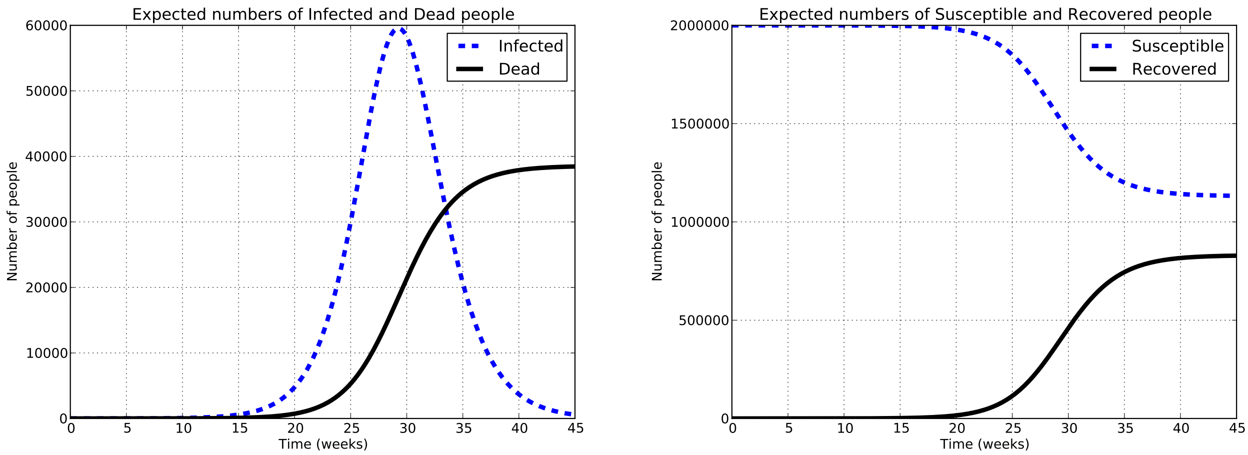


Figure 12.13: The impact of a possible human-transmissible avian influenza epidemic on a city of two million people, as modelled by Euler's method.

- Is this speculative catastrophe model realistic? Figure 12.14 shows the number of deaths per week in the Spanish flu historical records for some cities compared with the number of deaths per week predicted by our SIRD model.

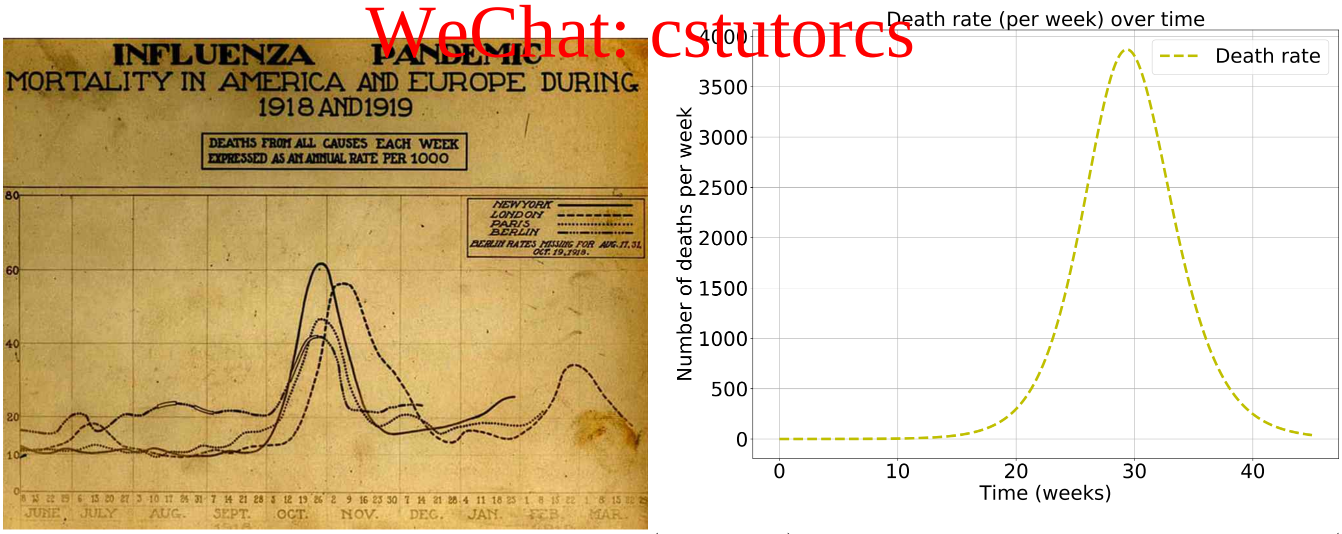


Figure 12.14: Left: Spanish flu death rates (per week) in several cities, from 1918–1919. (Source: en.wikipedia.org). Right: Number of deaths (per week) predicted from the SIRD model in the previous example.

- This suggests that our catastrophe model is (at least) plausible.

End of Case Study 33: Avian influenza.

Appendix A: **Programming manual**

A.1 Getting started

- A *computer program* a list of commands which directs a computer to perform specific calculations and operations.
- If you want to write a list of instructions for a person to follow, you must write in a language (comprising a vocabulary and rules of grammar and so on) that the reader understands. Similarly, if you want to write a list of commands for a computer to follow, you must write in a language that the machine understands. An established vocabulary and grammar for commanding a computer is called a *programming language*.
- There are many different programming languages, each suited to various uses. In SCIE1000 we will use the language *Python*.[1]
- The standard vocabulary of Python includes *reserved words*, which are words with precise meanings in the language, and *constants* such as numbers typed into the file. Commands are built out of reserved words, constants, and the following elements:
  - **Variables** allow data values to be stored and manipulated. Python allows variables that store single data values, and variables called *arrays* that store multiple data values.
  - **Boolean expressions** are logical expressions that evaluate as being true or false. In Python, boolean expressions are built using variables, numbers, the values `True` and `False`, mathematical symbols such as ">", and logical words such as `and`, `or`.
  - **Function calls** direct the computer to complete a computation described in a pre-written piece of code called a *function*. Functions can be written by the programmer themselves, or located in *modules*, which are libraries of useful code.

[1] Python was named after *Monty Python's Flying Circus*. We use Python because it is modern, freely available, fairly easy to learn, used in real science applications, and illustrates many important general computing concepts. Python users include Youtube, Google, Yahoo!, CERN and NASA.

- Commands vary between computer languages, however the following command types are typical:
  - **Comments** are ignored by the computer, but make programs easier to understand for people reading them. In Python programs, lines commencing with `#` are comments.
  - **Assignment commands** set the value of a variable. In Python, the equals sign "=" is the assignment operator. A typical assignment command has the form `variable = calculation`.
  - **Input commands** allow data to be entered into the program from the keyboard or a file. In Python, the command `input` is used to obtain data from the keyboard.
  - **Output commands** display data on the screen or write it to a file. In Python, the command `print` displays text, while `plot` and `show` together display a graph.
  - **Calculations** permit the computer to perform a range of mathematical calculations Python supports all standard calculations. The results of a calculation are often stored in a variable for later use.
  - **Conditional execution** allows the computer to execute certain commands if, and only if, a boolean expression is true. Python supports a number of conditional commands, including `if-elif-else`.
  - **Loops** execute commands multiple times, continuing provided that a boolean expression is true. Python supports a number of loops, including `while` loops.

A.2 Basic use of Python

We will write and run Python programs throughout semester. We start with a brief description of what a program is, and how to write, save and run programs. Then we cover some useful Python commands to use in your programs.

The usual approach to Python programming is to write a sequence of Python commands, save them to a file, and then run them. Saving programs in files means that

- programs can be run multiple times, on different computers.
- programs can be debugged more easily. For example, once a section of a program is thoroughly tested, it does not need any further testing.
- teams of people can design and write different parts of the program.
- problems of much greater complexity can be solved.

A.2.1 Creating Python programs

The specific methods for creating, saving and running python programs depend on the application in which you write and run your code. You are strongly encouraged to download Anaconda, a free and open-source distribution of the Python and R programming languages, for your own computer (instructions are available in the Learning Resources section of Blackboard). The Anaconda distribution contains Jupyter Notebook and Spyder, two applications in which you can write and run Python code. They are designed for different purposes:

- Jupyter Notebook makes it easy to produce interactive documents, where you can write and execute “snippets of code” in between pieces of written text. Your tutorial worksheets are prepared in Jupyter notebook.

- Spyder is designed for writing and running code only, and has enhanced features for debugging your code. You may prefer Spyder when writing a stand-alone program.

A.2.2 Comments in Python programs

The language Python is designed to be as “readable” as possible by English speakers. The vocabulary borrows from the English language, and the grammar rules are somewhat similar to English as well. Even so, the technical nature of the task means that it is sometimes difficult to read Python code and understand what it is doing.

One way to help the reader (who may be you, several weeks or months after you wrote the program!) quickly understand what your code is doing is to include *comments*. Any line starting with `#` in a Python program is a comment. The computer does not execute a comment, it just skips over it looking for the next command.

Just as annotations or margin notes can help the reader understand a dense and highly technical passage in a textbook, comments help the reader understand a piece of code. Always use comments where appropriate.

A.2.3 Importing modules

The vocabulary for Python provides a basic level of functionality. A more sophisticated vocabulary of instructions, including many mathematical functions and other useful operations, can be accessed by directing the computer to read through, or “import,” one or more *modules* before it processes your program. Each module, or part thereof, you import adds to the vocabulary that the computer understands. We will use the module called `pylab`. With this in mind, all of your programs should have the following first line:


```
from pylab import *
```

This line instructs the computer to read through the module `pylab` and remember all of the new vocabulary it finds.

A.2.4 Printing to the screen

The *print* command outputs text and the results of calculations to the screen.

Printing text

To print text to the computer screen, use the **`print`** command in the following ways:

`print()`	prints a blank line.
`print("message")`	prints the message.
`print(expression)`	prints the **value** of the mathematical expression.

When printing multiple items (which may include a mixture of expressions and messages), items must be separated by commas.

Anything within quotation marks is printed *as it is*, whereas anything not inside quotation marks is evaluated as an expression and the *answer* is printed.

The following program demonstrates use of the **`print`** command. Note that line numbers are added for ease of reference and are not part of the program. (If you wish to see how it works, type this program into a Python edit window, save it and then run it. Take care to type everything correctly.)

Program A.1: Printing things

```
1 from pylab import *
2
3 # Print some messages.
4 print("This is a message")
5 print("This is first", "and this is second")
6
7 # Print the results of some calculations.
8 print(3+4)
9 print("3*4 =",3*4)
```

Note that Lines 3 and 7 of the program contain comments. Also, you can use blank lines (such as Lines 2 and 6) to make your program more readable. Here is the output from running the program:

```
This is a message
This is first and this is second
7
3*4 = 12
```

A.2.5 Numerical calculations

As suggested by Lines 8 and 9 of the previous program, Python can use standard mathematical operations. The following table shows how to do this. (In each case, the letters a and b represent numbers.)

Mathematics	Python	Mathematics	Python
$a+b$	`a+b`	$a-b$	`a-b`
$a \times b$	`a*b`	$a \div b$	`a/b`
a^b	`a**b`	$(...)$	`(...)`

Important note

You may have seen `a^b` used to represent a^b on your calculator. In Python `a^b` means something completely different so be careful to use `a**b`.

The following program demonstrates some mathematical operations.

Program A.2: Simple calculations

```python
from pylab import *

# Simple calculations:
print(3+2, 3-2, 3*2, 3/2)
print("3 squared =",3**2)

# Python correctly applies order of operations:
print(2+3*4," and ", (2+3)*4)
```

Here is the output from running the program:

```
5 1 6 1.5
3 squared = 9
14 and 20
```

Use spaces within expressions (almost) anywhere, to make the program easier to read and understand. When using spaces, remember that your main goal is *communication*; use your judgement about what works. The following example demonstrates one approach.

Program A.3: Spacing inside Python programs

```python
from pylab import *

# Adding one space between numbers and symbols is reasonable.
print(6 + 4)

# You normally do not use space between brackets and numbers.
print((2 + 3) * (6 - 4))

# Sometimes spaces are used to show order of operations.
print(2 + 3*4 + 5*6)
```


Note that in Python, the symbol `e` represents scientific notation. For example, 6.02×10^{23} is displayed as `6.02e+023`, and 3×10^{-4} as `3e-04`.

A.3 Variables and functions

A.3.1 Variables

A program can "remember" values by *assigning values* to *variables*. The programmer can create and use as many variables as needed in the program.

Each variable has a name, chosen by the programmer, which is used to access it. When Python encounters a word that it does not recognise, it assumes that the word is the name of a new variable. To introduce a variable, you should use it on the left-hand side of an assignment command. For example, to assign a value to a new variable called `level`, use the command

```
level = expression
```

where `expression` is either a value (such as `3` or `-2.25`), or an expression (such as `2 + 4`). Python calculates the result from the expression on the right hand side of the equals sign, and assigns that value to the variable `level`.

After you have introduced a variable and assigned it a value, Python will recognise the name as referring to whatever value the variable has been assigned. You can use that variable name in subsequent calculations. You can use another assignment statement to assign a new value to the same variable. The following program gives some examples.

Program A.4: Variables

```
1  from pylab import *
2
3  width = 20
4  height = 45
5  print("For a rectangle of size", width,"by", height)
6  print("the area is ",width * height)
7  perimeter = 2 * width + 2 * height
8  print("and the perimeter is", perimeter)
9
10 width = width + 5
11 height = height + 5
12 print("For a rectangle of size", width,"by", height)
13 print("the area is ",width * height)
14 perimeter = 2 * width + 2 * height
15 print("and the perimeter is", perimeter)
```


In line 3, a new variable called `width` is introduced. Python knows this is a new variable because “width’ is not a reserved word, and this is the first time it encounters it in the program. The variable is assigned the value 20. In lines 5, 6 and 7 the variable name `width` appears, in each case the value 20 is used in the place indicated. In line 10, `width` is assigned a new value, equal to the value it had (20) plus 5; so the new value is 25.

Here is the output from running the program:

```
For a rectangle of size 20 by 45
the area is  900
and the perimeter is 130
For a rectangle of size 25 by 50
the area is  1250
and the perimeter is 150
```

Choosing variable names

Always choose *meaningful* names for your variables, to make the program easier to understand. Examples of names are `x`, `height`, `NumFish` and `x7`. Do not use spaces or other "special" characters in variable names. Also, note that variable names are case sensitive, so `numfish` and `NumFish` are different.

A.3.2 Python functions

Just as a program can remember values by storing them as variables, it can also remember pieces of code in the form of functions. A function is a piece of code that acts on one or more values and produces some output. When you instruct the computer to run a function, you are said to be "calling" the function.

A function is not executed at the point in the program where it is defined, rather, it is executed at the place(s) in which it is called. To call a Python function that has already been defined, type the name of the function followed by parentheses enclosing the value, or list of values, to use. Here is a list of some mathematical functions in Python and a desciption of the value they return:

`sqrt(value)`	square root of the value
`sin(value)`	sine of the value (the value is in radians)
`exp(value)`	e raised to the given power
`log(value)`	ln(value)
`log10(value)`	$\log_{10}$(value)
`round(value, numOfDigits)`	returns the value rounded to the specified number of digits after the decimal.

The following example demonstrates how to use mathematical functions.

Program A.5: Functions

```
from pylab import *

val = 9
print("The square root of 9 equals", sqrt(val))
print("e^1 = ", exp(1))
print("When rounded to two decimal places, e^1 = ", round(exp
    (1), 2))
print("When rounded to the nearest integer, e^1 = ", round(exp
    (1), 0))
print("log to base 10 of 1000 equals",log10(1000))

# Evaluate sin of 90 degrees. First, convert to radians.
angleDeg = 90
angleRad = angleDeg * pi/180
print(angleDeg,"degrees =", angleRad,"radians.")
print(sin(angleRad))
```


Here is the output from running the program.


```
The square root of 9 equals 3.0
e^1 =  2.71828182846
When rounded to two decimal places, e^1 =  2.72
When rounded to the nearest integer, e^1 =  3.0
log to base 10 of 1000 equals 3.0
90 degrees = 1.5707963267948966 radians.
1.0
```

A function must be defined before it can be called. Many useful functions are defined in the module `pylab`. Provided that your program begins with the line

`from pylab import *`

you can use any of the functions that have been defined there. These functions are efficient and extremely well-tested.

Python also includes the constant `pi`, which (approximately) equals π.

A.3.3 The input function

It is often useful or necessary to ask the user to enter some input from the keyboard. The `input(...)` function prints a message on the screen, waits for the user to enter data at the keyboard followed by the `Enter` key, and returns the data they entered so that it can be stored as a variable. For example, the command

```
userAge = input("Please enter your age:")
```

will display the message "Please enter your age:", and cause the program execution to wait for the user to enter data at the keyboard. When the user finally hits the `Enter` key, whatever they typed will be stored in the variable `userAge`.

Users sometimes enter strange things. If the user responds to the prompt above by typing "a%6J@", and the program goes on to use `userAge` as if it is a number, things might get weird. It is good programming practice to make sure that the data entered is of the type expected. In this course we will only ask the user to input numbers. We can make sure that what the user enters is a number by calling the input function inside the function `float`[2]. The `float` function tries its best to interpret what the user entered as a number, with digits after the demical point allowed; if it cannot, the program execution will cease with an error message.

The following example program shows how to use the `input` function inside the `float` function.

[2] Float is short for "floating point decimal", a phrase which refers to the format in which the computer stores the value in its memory.

Program A.6: Input

```
from pylab import *

# Input two values, then multiply and divide them.
a = float(input("Tell me a number: "))
b = float(input("Tell me another number: "))

prod = a * b
quot = a / b

print(a, "*", b, "=", prod)
print(a, "/", b, "=", quot)
```

Here is the output from running the program.

```
Tell me a number: 8

Tell me another number: 2
8.0 * 2.0 = 16.0
8.0 / 2.0 = 4.0
```


A.4 Software errors and bugs

All computer programmers make errors. An error in a program is called a *bug*. A key skill in programming is minimising the number of errors, and then identifying and fixing any that occur. There are many different types of error, including incomplete problem description, design faults in the software, unanticipated 'special cases', coding errors and logic errors.

In real life, the consequences arising from programming errors can be very serious: for example, they have caused plane crashes, rocket explosions and failure of entire transport systems. In SCIE1000 we will not give you control of aeroplanes, rockets or even small transport systems. The impact of any programming errors will be minor. You may feel a bit frustrated and need to ask for help, but no lasting damage will occur. Rather, you should learn from the process of finding and fixing your errors.

Avoiding errors

When writing programs, make sure that you:

- Understand the question **before** you start programming;
- Think about the best and most logical way to solve the problem;
- Consider planning your program on paper first;
- Put comments in your program so you know what you are trying to do;
- Test your programs on a range of data;
- Check some output carefully to make sure it is correct; and
- Pay attention to any error messages!
- Try writing a few lines at a time, then running your program to make sure those lines are doing what you think they are doing.

Fear not!

Do not be afraid of error messages! Never let the fear of error messages stop you from playing around with Python and trying different commands. Getting an error message **does not** mean that you will fail the course. If it helps you to figure out what you did wrong, then you have learned something! Equally important, do not ignore error messages. They give you useful advice about what is going wrong.

The following Python program illustrates a number of errors.

Program A.7: Multiple errors

```
1  from pylab import *
2  
3  # Input two values, then multiply and divide them.
4  a = float(input("Tell me a number: "))
5  b = float(input("Tell me another number: "))
6  
7  prod = a * b
8  quot = a / bb
9  
10 print(a, "*", b, "=", prod)
11 print(a, "/", b, "=", prod)
```

Here is the output from running the program:

```
1  Tell me a number: 12
2  Tell me another number: 4
3  
4  Traceback (most recent call last):
5    File "inputerror.py", line 8, in <module>
6      quot = a / bb
7  NameError: name 'bb' is not defined
```


To help you to identify the error:

1. The **last line** of the error message indicates **what** error occurred. In this case, it says `NameError: name 'bb' is not defined`
2. The third last line (Line 5) indicates **where** the error was detected. In this case, it says: `File "inputerror.py", line 8, in <module>`. This gives the name of the file and the line number where the error occurred.

The information in the error message allows the error to be located and then identified. In Line 8 of the program in the example above, the programmer has accidentally typed '`bb`' instead of '`b`'. Because the variable `bb` does not already have a value, the program cannot divide `a` by `bb`.

If a program contains multiple errors, Python will display the message for the first one it encounters. After you find and fix the error, Python may give a different error message. This is usually a good sign, indicating that the first error is fixed and you can move on to the next one.

Here is some output from running the previous program with `bb` changed to `b`.

```
Tell me a number: 12

Tell me another number: 4
12.0 * 4.0 = 48.0
12.0 / 4.0 = 48.0
```

Look carefully at the output — although there was no error message, there is still an error. The output says that `12.0 / 4.0 = 48.0`, which is incorrect. Hence you should **always check your output**, as it may be incorrect even if there is no error message. In this case, Line 11 should print the variable `quot`.

Finally, another type of error can arise. If you enter "0" for the second input number, then an error message will arise, saying that you cannot divide by zero! This is a type of "special case" error, which only arises for certain values. To avoid such errors, you need to test your program on a range of "special cases".

For reference, three common error messages are:

Error	**Explanation and possible causes**
`SyntaxError`	The command is not understood by Python. Perhaps: • You used incorrect bracket types (e.g. `( )` instead of `[ ]`) • You have forgotten a bracket • Your indentation is incorrect (wrong number of spaces at the start of a line)
`NameError`	There is no variable with the given name. Perhaps: • You have mistyped the name of a variable. • You have forgotten to set a starting value for a variable.
`IndexError`	You have used an invalid index to an array or sequence.

A.5 Conditionals

A.5.1 Introduction to conditionals

Programs often require the computer to do different things depending on whether a certain condition is true or false. For example, you might want to print various messages depending on the answer to the condition “are you aged over 18 and hence legally allowed to vote in Australia?” Python supports this by means of the *conditional command*, demonstrated below.

Program A.8: The basic `if`

```
from pylab import *

age = float(input("What is your age? "))
if age >= 18:
    print("You can vote.")
print("Finished!")
```


Here is the output from running the program twice.


```
What is your age? 24
You can vote.
Finished!

What is your age? 17
Finished!
```

Here are some things to note about the conditional command.

- In the first line, the word `if` and the colon `:` are essential.
- The text between `if` and `:` must be a boolean expression (a logical expression that is either true or false).
- After the first line, any lines that are indented by four spaces will run if, and only if, the condition is true.
- Lines of the program that occur after any indented lines are outside the scope of the conditional command and therefore will run whether the condition is true or false.

A.5.2 Boolean Expressions

Boolean expressions are logical statements which are either true or false. In Python they often take the form of a comparison between two variables, or a variable and a value. Python supports a number of operations for comparing variables and values:

Operation	**Mathematics**	**Python**
Greater than	$a > b$	`a > b`
Less than	$a < b$	`a < b`
Greater than or equal	$a \geq b$	`a >= b`
Less than or equal	$a \leq b$	`a <= b`
Equal to	$a = b$	`a == b`
Not equal to	$a \neq b$	`a != b`

Notice that the operator for checking whether two things are equal in Python is **==** and not just a single **=** sign.

Multiple conditions can be combined using the Python commands **and**, **or**, **not**, matching standard English usage of the words.

A.5.3 The `else` statement

In the previous example, the program prints a message if it is legal to vote, but gives no output if voting is not legal. When programming, we often have **two** possible situations – run some commands if a condition is true, and run other commands if the condition is false. This is done in the following way.

Program A.9: An `if-else`

```
from pylab import *

# Is it legal to vote?
age = float(input("What is your age? "))
if age >= 18:
    print("You can vote.")
else:
    print("You cannot vote.")
print("Finished!")
```

Here is the output from running the program twice.

```
What is your age? 24
You can vote.
Finished!

What is your age? 17
You cannot vote.
Finished!
```

If there are more than two conditions to check then the command `elif` is used; it means "else if". Once again, we can use this to extend our example.

Program A.10: An `if-elif-else`

```python
from pylab import *

# Is it legal to vote?
age = float(input("What is your age? "))
if age > 18:
    print("You can vote.")
elif age == 18:
    print("You can vote for the first time.")
else:
    print("You cannot vote.")
print("Finished!")
```


Here is the output from running the program three times.

```
What is your age? 24
You can vote.
Finished!

What is your age? 17
You cannot vote.
Finished!

What is your age? 18
You can vote for the first time.
Finished!
```

A.6 Loops

A.6.1 Introduction to loops

Programs often require some commands to run multiple times. For example, to model the growth of a population over 50 years, rather than writing 50 identical sections of code it is more convenient to write a single section, and run it 50 times. The programming concept which allows lines of code to execute multiple times is called a *loop*; here is some Python code demonstrating a loop.

Program A.11: Loops, 1

```
from pylab import *

# Print squares and cubes of numbers from 1 to 4.
i = 1
while i<5:
    print(i, i*i, i*i*i)
    i = i + 1
print("Finished!")
```


Here is the output from running the program.

```
1 1 1
2 4 8
3 9 27
4 16 64
Finished!
```

Here are some things to note about loops.

- In the first line, the word `while` and the colon `:` are essential, and the text between `while` and `:` must be a boolean expression.
- After the first line, any lines that are indented by four spaces will run while the condition is true.
- Lines of the program that occur after any indented lines will run once the condition is false.

Make sure you understand what is happening in the example loop above. A *loop control* variable, called `i`, is initially set to a value of 1. Each time the loop runs, the value of `i` is increased by 1, ensuring that the loop runs the required number of times, and stops when the condition `i<5` is false.

A.6.2 Loops and conditionals

There are some similarities between loops and conditionals. The *body* of each is indented by four spaces, and will only run if the initial condition is true. In a conditional, the body only runs once; in a loop, the body is run while the condition remains true. In both cases, the first line *after* the indentation will run after the conditional/loop has finished.

Multiple loops and conditionals can be *nested* within each other; indent by an extra four spaces each time.

A.6.3 Loop forever...

A *while* loop continues to run commands in the loop body the condition is false. You must take care to choose a condition that will stop the loop at some stage. Consider the following loop:

Program A.12: Infinite loop


```
from pylab import *

i = 1
while i < 5:
    print("forever...")
```

Notice that nothing within the body of the loop changes the value of `i`, so the condition `i < 5` is always true, and the loop will never terminate. This is called an *infinite loop*.

Stopping infinite loops

If you run a Python program and it seems to be taking a long time, it **may** contain an infinite loop. If you suspect that a running program contains an infinite loop, you can terminate it by pressing the "stop" button in the Juypter tool bar (if you are working in Spyder, press either the "stop" button on the top right of the console or `Ctrl+C`).

A.7 Arrays

A.7.1 Introduction to arrays

Apart from when we were plotting graphs, we have only used Python to store individual data values in variables. A Python *array* is a different type of variable, one which allows *multiple* items of data to be stored in the **same** variable. The following program creates and prints an array called `primes`.

Program A.13: Our first array


```
1 from pylab import *
2
3 # Create an array containing the first 5 prime numbers.
4 primes = array([2, 3, 5, 7, 11])
5 print("Primes are:", primes)
6 print("Primes is an array with", size(primes), "elements")
```


Here is the output from running the program.

```
Primes are: [ 2  3  5  7 11]
Primes is an array with 5 elements
```

Here are some things to note about arrays.

- Arrays are variables, so must have meaningful names.
- Python uses square brackets [and] to distinguish arrays from other variables. These are shown in Line 4 of the above program, and in the output.
- The `array(...)` function provides one way to create an array. See below for a discussion of other ways.

- Line 5 uses the `print` command to display the entire contents of an array.
- The `size` function returns the number of elements in an array. The above array holds five values, so its `size` is 5.

A.7.2 Three functions for creating arrays

We shall discuss three different functions for creating an array:

- The function `array(...)` was used in the program above. Use this function when your array is small and you know exactly which entries will go into the array at the time you are making it.

- The function `zeros(...)` creates an array filled with zeros. It takes one value, an integer which tells it how many entries the array should have. To make sure that the value you pass to the function is understood to be an integer, you may need to enclose the value in the function `int(...)` as shown in the example below.

- Sometimes, particularly when plotting graphs, you want to create an array filled with many equally spaced values. The Python function `arange` accomplishes this. The function takes three values: the first value to be placed in the array; a value which indicates when to stop; and a value which indicates how far apart values in the array should be. For example, the Python function

  ```
  X = arange(a,b,s)
  ```

  creates an array `X` of values starting at `a`, increasing by an equally spaced step of `s` each time, and stopping at the **last value less than** b.

  The following example programs demonstrates the use of each function.

Program A.14: Creating arrays

```
from pylab import *

# Create three arrays to demonstrate the functions
# array(...), zeros(...) and arange(...)

X = array([2, -1.2, 13, 4.578])
print("Array X is ", X)

Y = zeros(int(5))
print("Array Y is ", Y)

Z = arange(1, 2.5, 0.25)
print("Array Z is ", Z)
```

Here is the output from running the program.


```
Array X is  [ 2.    -1.2   13.    4.578]
Array Y is  [0. 0. 0. 0. 0.]
Array Z is  [ 1.   1.25  1.5   1.75  2.   2.25]
```


A.7.3 Operations on arrays

Another way to create an array is by applying Python commands to already existing arrays. Most Python commands we have already seen also act element-by-element on entire arrays at once, producing new arrays as the result. Pay particular attention to Lines 6 and 7 of the following example program.

Program A.15: Creating new arrays from old

```
1 from pylab import *
2 
3 # Create arrays of primes and powers of 10
4 primes = array([2,3,5,7,11])
5 pows = array([0.01, 0.1, 1, 10, 100, 1000])
6 primeSq = primes * primes
7 pows = log10(pows)
8 print("Squares:  ", primeSq)
9 print("log(pows):", pows)
```

Here is the output from running the program.

```
Squares:    [  4   9  25  49 121]
log(pows): [-2. -1.  0.  1.  2.  3.]
```

A.7.4 Accessing individual array entries

In addition to dealing with an entire array, it is often useful to access individual entries in the array. The **index** of an entry refers to the *position* of that entry in the array (somewhat similar to the room numbers in the corridor of a building). To access individual entries, type the name of the array, immediately followed by the index surrounded by square brackets. For example, `A[i]` refers to the value at position `i` in the array `A`.

In Python, the **first** entry in an array has index `0`. This is important to remember! If `A` is an array of size n (so it contains n entries), then valid values of the index are from 0 to $n-1$ (inclusive). This is illustrated in Figure A.1.

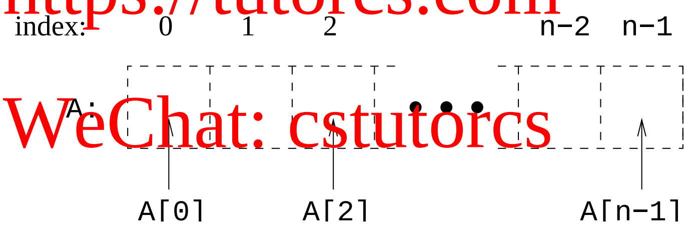


Figure A.1: The index of each entry in an array A with n entries

The following program demonstrates how to access individual array entries.

Program A.16: Accessing individual array elements

```python
from pylab import *

# Create an array of primes and access various entries
primes = array([2,3,5,7,11])
print(primes)
print(primes[0], primes[4])
primes[0] = 13
primes[1] = 2 * primes[1] + 1
print(primes)
```

Here is the output from running the program.

```
[ 2  3  5  7 11]
2 11
[13  7  5  7 11]
```

A.7.5 Arrays and loops

The following program uses the function `zeros(int(n))` to create an array with `n` cells each equal to zero[3] then uses a loop to place values in the array.

Program A.17: Arrays and loops

```
from pylab import *

# Create an empty array then put values in it
X = zeros(int(5))
i=0
while i<5:
    X[i] = i*i
    i = i+1
print(X)
```


Here is the output from running the program.

```
[  0.   1.   4.   9.  16.]
```

[3]Using `int` ensures that Python treats the number `n` as an integer. There are various data types for numbers, which is a topic that goes beyond the scope of this course.

A.8 Writing functions

A.8.1 Why write new functions?

Earlier we saw that Python includes mathematical functions such as `sqrt` and `sin`. Creating your own, new functions can be very useful. Once written, you can reuse them in multiple places or in different programs, without rewriting the commands each time.

Working with functions involves two related but distinct activities:

- *creating*, or *defining*, the function. You *define* a function by giving it a name, giving a name to the values that will be passed to the function, and specifying the Python commands that actually do what the function requires.

- *using*, or *calling*, the function. You *call* the function by typing its name in one of your Python commands, followed by parentheses enclosing the values you are passing to the function.

A.8.2 Writing a new function

In an earlier example, before using trigonometric `sin` we needed to convert the angle from degrees to radians. Here is some Python code demonstrating this.

Program A.18: Degrees to radians, 1

```
angleDeg = 90
angleRad = angleDeg * pi/180
print(sin(angleRad))
```

Converting from degrees to radians is a common calculation, so it may be useful to create a new function called (say) `toRadians` to do the conversion. Once the new function is written, the Python code could become

Program A.19: Degrees to radians, 2

```
angleDeg = 90
angleRad = toRadians(angleDeg)
print(sin(angleRad))
```

The second line has changed — now the new function performs the conversion to radians. Of course, before using the new function, you need to write it. Here is a Python program showing the new function.

Program A.20: Converting to radians

```
def toRadians(deg):
# This function converts degrees to radians.
    rad = deg * pi/180
    return (rad)
```

Pay careful attention to this example, as it demonstrates a number of important aspects of writing new functions. The first line and the last line are particularly important. Take the time to understand what is happening.

First line:

- The *name* of this function is `toRadians`. You should always choose **meaningful names** for your functions, to help you remember what the functions do.

- The word `def`, the brackets `(...)` and the colon `:` are essential!

- The word `deg` inside the brackets saya that one value will be passed to the function and it will be referred to as `deg`. In this function (in degrees) to be converted to radians. The common terminology is that this value is *passed into the function*. The values passed to a function are sometimes called *arguments* to the function. You can choose any name for these arguments.

Last line:

- The last line 'returns' the value calculated by the function; in this case, it is the angle converted to radians.

Remaining lines:

- Except for the first line (or any comments), **every line** in the function **must** be indented; that is how Python can tell where the function ends. In SCIE1000, we will always use **four spaces** for indentation.

- These lines must perform the calculations required by the function, and create the value to be returned.

Here is an example showing how to use the new function.

Program A.21: Using a new function

```
from pylab import *

def toRadians(deg):
# This function converts degrees to radians.
    rad = deg * pi/180
    return (rad)

angleDeg = float(input("Enter the angle in degrees: "))
angleRad = toRadians(angleDeg)
print(angleDeg,"degrees equals",angleRad,"radians.")
print("and sin() of this equals", sin(angleRad))
```


Here is the output from running the program:


```
Enter the angle in degrees: 90
90 degrees equals 1.57079632679 radians.
and sin() of this equals 1.0
```


A.8.3 Some notes on functions

Note that:

- Variables that are given values **inside** a function are **not accessible** outside the function, even if you use the same name in both places.
- You can call other functions from within a function.
- You can define multiple functions inside the same file. Remember that indenting shows where the body of each function starts and ends.
- The lines inside a function are **only** used when you call the function.
- A Python function is not restricted to only performing mathematical calculations. It can do anything that a program does.

- If the 'return' statement is omitted, then the function will not return a value. In this case, the function is called by typing its name with whatever arguments it needs in parentheses.

Functions and good programming practice

When defining your own functions, the following are recognised as good programming practice:

- New functions should be defined at the top of the file, just under the import statements.
- A function should not use variables that are not passed into the function. That is, every piece of data your function needs to use in order to compute its output should be provided in the form of a value passed into the function.

A.9 Graphs

A.9.1 Plotting graphs

Drawing graphs is important in computer modelling. In Python, the `plot(...)` function organises a graph, and the `show(...)` function displays the graph.

To make the most basic graph, call the `plot(...)` function and then the `show(...)` function. To the `plot(...)` function you must pass two arrays of the same size: the first contains the x-coordinates of the points to be plotted; the second contains the corresponding y-coordinates. The `show(...)` function does not require anything to be passed.

You may pass more information to the `plot(...)` function if you would like to control more features of the plot, like the colour of the line or the way in which points are marked.

You may make more than one `plot(...)` call before calling `show(...)`. In this case your graphs will be plotted on the same set of axes.

The following example plots three graphs on the same set of axes. Make sure you remember to use the `show()` function at the end of your program; if you forget, the graph will not be displayed.

Program A.22: Using plot(...) and show(...)

```
# This program demonstrates multiple plotting styles.
from pylab import *

# make some arrays of the same size
A = arange(0,10,1)
Exponential = A**2
Linear1 = 10*A
Linear2 = 3*A+20

# Plot the exponential function using a solid line.
plot(A, Exponential)
# Plot the first linear function using discrete points marked
   by 'x'.
plot(A, Linear1, 'x')
# Plot the second linear function using a wide solid line.
plot(A, Linear2, linewidth=3)
show()
```


Here is the output from running the program:

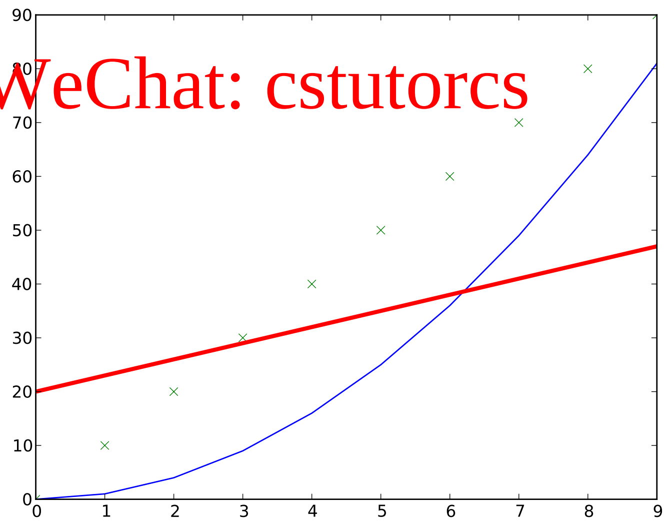


Figure A.2: Three graphs.

A.9.2 Graphing smooth functions

Computers cannot draw perfectly smooth curves: instead, they approximate smooth curves by drawing very short straight lines between points that are very close together. The more data points there are, the smoother the curve appears. The number of points needed to produce a smooth curve varies between different graphs. It is common to choose points whose x-coordinates are *equally spaced*, with a "small" spacing. Use the `arange(...)` function to do this.

Program A.23: Plotting graphs, 2

```python
# This program shows how to use arange()
from pylab import *

# Plot sin(x) with x-coordinates separated by 0.5.
X1 = arange(0.0, 4.1, 0.5)
plot(X1, sin(X1), linewidth=3)

# Plot cos(x) with x-coordinates separated by 0.1.
X2 = arange(0.0, 4.1, 0.1)
plot(X2, cos(X2), linewidth=3)
show()
```

Figure A.3 shows the output from running the program.

A.9.3 Customising your graphs.

Python provides a number of commands to customise your graphs. The commands in the following program should be self-explanatory.

Program A.24: Some customised plots


```python
# This program shows how to customise graphs.
from pylab import *

# Create equally spaced points and plot sin and cos.
X = arange(0.0, 4.1, 0.1)
plot(X, sin(X), linewidth=3, label="sin(x)")
plot(X, cos(X), "--", linewidth=3, label="cos(x)")

# Create title and label axes.
title("Graphs of sin(x) and cos(x)")
xlabel("x")
ylabel("y")

# Draw a grid, create a legend and display the graph.
grid(True)
legend()
show()
```

Figure A.4 shows the output from running this program.

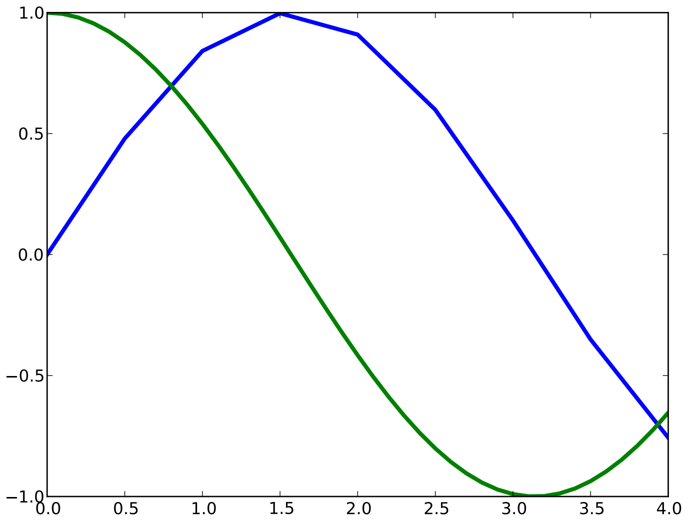


Figure A.3: Two graphs, with spacings of 0.5 and 0.1 between points.

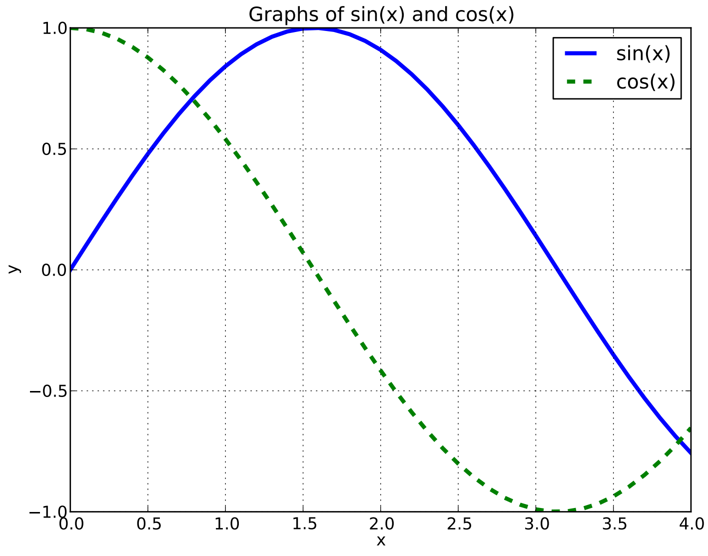


Figure A.4: Customising graphs.

A.10 Python summary

- All programs must commence with the line:

```
from pylab import *
```

- All lines commencing with '#' are comments.

```
# Use lots of comments to explain what your program does.
```

- The `print(...)` command displays text on the screen. Text inside quotation marks is displayed. Text outside quotation marks is treated as calculations or variables, and the values are printed.

- Values can be assigned to variables using '='.

- Use the operation `**` to find a power.


```
print("x cubed =", x**3)
```


- Useful inbuilt functions include `sqrt(value)`, `sin(value)`, `exp(value)`, `log(value)` and `log10(value)`.


```
print("The square root of x =", sqrt(x))
```

- The command `float(input(message))` reads input from the keyboard.

```
numFish = float(input("How many fish are there?"))
```

- The bodies of conditionals, loops and functions must be indented.

- The conditional commands `if`, `else` and `elif` control which commands are run, when a particular condition is true or false.

```
if x==y:
    print("x equals y")
elif x<y:
    print("x is less than y")
else:
    print("x is greater than y")
```

- The `while` loop allows commands to be run multiple times. Make sure that your loops will stop sometime!

```
i = 0
while i<5:
    print(i, "squared =",i**2)
    i = i+1
```

- To create a new function, *define* it at the top of your file (just under the `import` statements), then *call* it wherever you need it.

```
def toRadians(deg):
# Define a new function to convert degrees to radians.
    rad = deg * pi/180
    return (rad)

# Call the function:
print("Converted to radians, 90 degrees is", toRadians(90))
```


- Arrays allow multiple values to be stored in a "table", using a single variable name. The *index* of an element is its location in the array.

```
A = array([1,10,100,1000])
print("The last entry is",A[3])
B = log10(A)
C = zeros(int(10))
```

- Graphs are drawn using `plot()`, followed by `show()`. Commands such as `title()`, `xlabel()` and `ylabel()` allow you to customise your graphs.

```
x = arange(0,10.1,0.1)
y = x**2
plot(x, y)
show()
```

Appendix B: **Communication in Science**

Image B.1: *Communication*, by Joan M. Mas (Source: https://www.flickr.com/photos/dailypic/1459055735, CC BY-NC 2.0)

In SCIE1000 we will focus on four principles essential for effective communication of science which will be of lifelong use to you in this course, in your degree, and in your personal and professional life. You will apply one or more of these principles in a task each week in your SCIE1000 tutorials. We have designed these tasks to build upon one another so that you develop good habits for communicating science.

Science by its nature aims to be:

- Precise
- Clear, and
- Concise

Consequently, **communication by scientists about science** aims to be:

- Precise
- Clear, and
- Concise

...and in that order. Communication that is vague, is not scientific communication. Communication that is unclear or ambiguous is not scientific communication. And communication that is long-winded and rambles from one topic to another is not scientific communication. But do not sacrifice being **precise** and **clear** only to be concise! In other words, if you need to use more words, numbers or visuals to make something crystal **clear** to as many readers or listeners as possible, then do so.

Not all communication is **precise**, **clear** and **concise**, and nor should it be. These are characteristics are typical of the way scientists communicate with other scientists. There are situations and contexts where being precise, clear and concise are not the most effective way to communicate. For example, some communication aims to entertain, persuade, delight in playing with words, or create confusion or complex ideas. But in learning to become a scientist, honing your skills at being precise, clear and concise when you communicate is important.

As developing scientists we expect you to become **precise**, **clear** and **concise** in your writing and speaking about science. This is not a skill that comes naturally to most people, but your participation and efforts in this course and throughout your degree will have you communicating science precisely, clearly and concisely sooner than you think.

The four principles we focus on in SCIE1000 are:

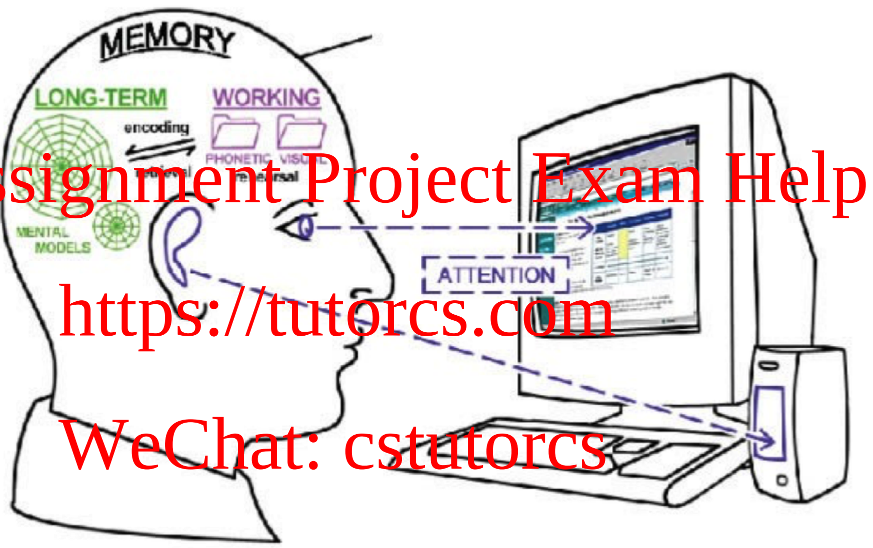
1. Being clear
2. Knowing your purpose
3. Knowing your audience
4. Identifying key messages

Applying these four principles will get you well on the way to being precise, clear and concise when **you** communicate science. Here is why and how to do it...

B.1 Principle 1: Being clear

Why be clear?

Being clear is important because of the way the human brain works. The working memory part of the brain processes all incoming communication and turns it into understanding and long term memory. The working memory is only able to process a limited amount of information at any one time, and absorb only small amounts of new information at any one time. The brain then has to integrate the new information with its own mental models for it to be remembered in the long term.



Much of the information and ideas in science is new to many people. So an effective way to communicate science is to combine information the person already knows with small chunks of information that is new to them. Doing so makes it easier for the brain to process, or understand, and more likely to be remembered in the long term.

Useful tactics for being clear

Whether you use words, computer code, numbers, symbols and/or pictures, being clear in your communication is important. Some useful tactics to achieve clarity in your communication are:

1. **Identify what is FAMILIAR and NEW to your intended reader or listener.**
   People learn and remember information by adding and integrating it with what they already know. By identifying what is familiar and new, you can make decisions about how to present your information which will help another person learn and remember it (see below).

2. **Provide MORE explanation for new information than for the familiar.**
   The human brain requires more effort to understand and remember new information than the familiar. Providing extra explanation means less effort for the brain, and increased likelihood that it will be understood and remembered.

3. **Organise new information in SMALL CHUNKS AFTER familiar information.**
   The working memory of the brain can process only a limited amount of new information at any one time. By organising new information into small chunks you help ensure the working memory is able to process it. When you place new information after familiar you make it easier for the brain to combine the two and pass them together to long term memory.

4. **Present your work NEATLY and LOGICALLY.**
   Lay out your communication in a way that another person can easily see each step in your thinking; make it easy for them to follow the process you used to get from A to B.

5. **Provide DEFINITIONS and LABELS**
   Provide definitions and labels for all data, spreadsheet columns, variables, symbols, drawings, pictures, technical terms and jargon. It reduces effort required by the working memory in processing fiddly details and allows it to concentrate on the important stuff – remember there is only a limited amount the working memory can do at any one time.

6. **REVIEW your communication from a different perspective**
   Learning to see your work from the perspective of another person, including

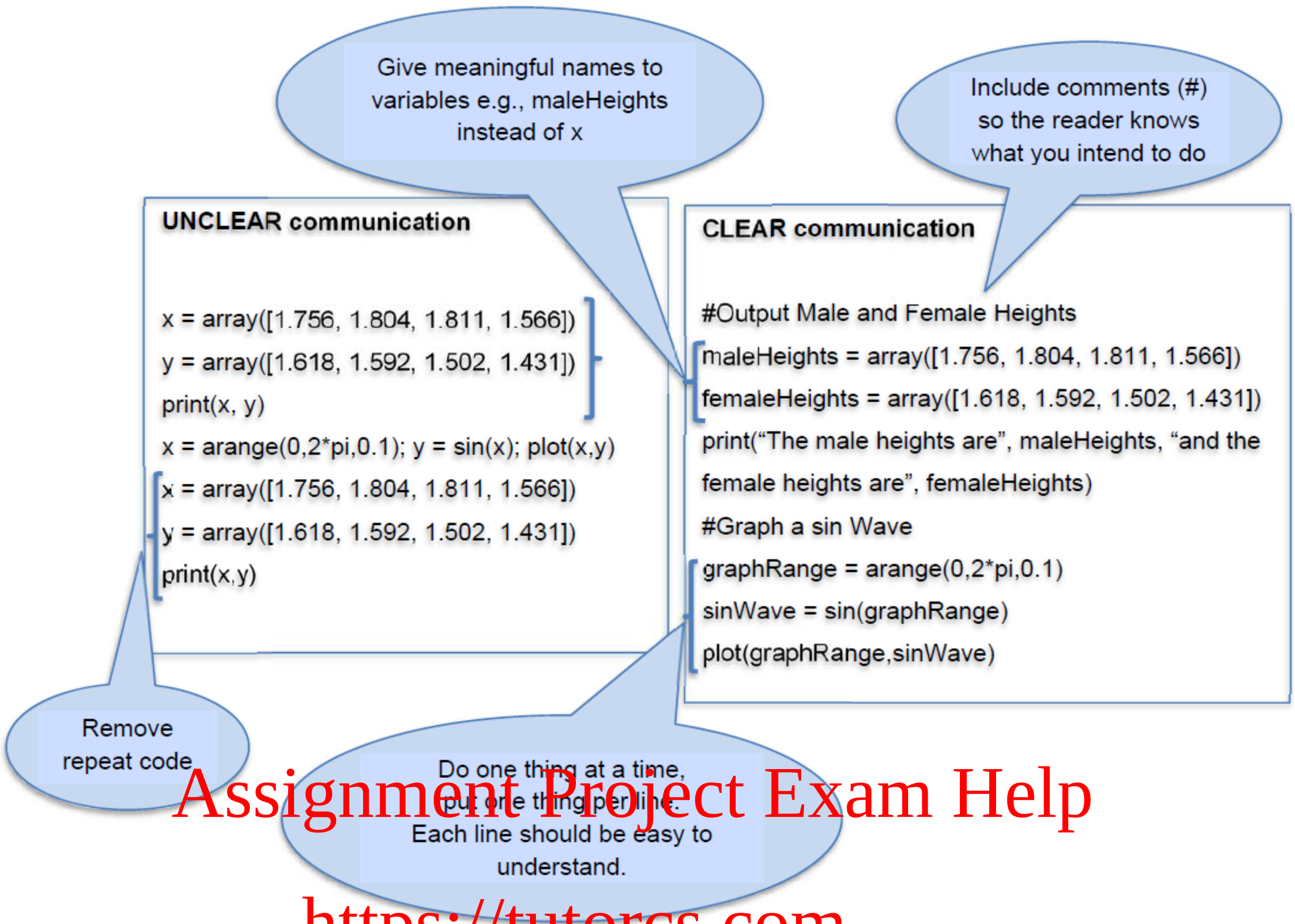
your future self, is a valuable skill. When you look back at your notes will you remember what it is you meant? Ask another person to look over your work and tell you what is unclear for them. It is not always possible to see your own work from another person's perspective because we all make assumptions about what other people know that may not be correct. In SCIE1000 we encourage and expect you to ask other students for feedback on how to make your work clearer.

Examples of being clear

Here are examples of the tactics to be clear being put to use in a variety of contexts.

COMPUTER CODE should be written for people, not computers. By being clear in your code, you make it easy for people to maintain, change and repair your code. In the example code below:

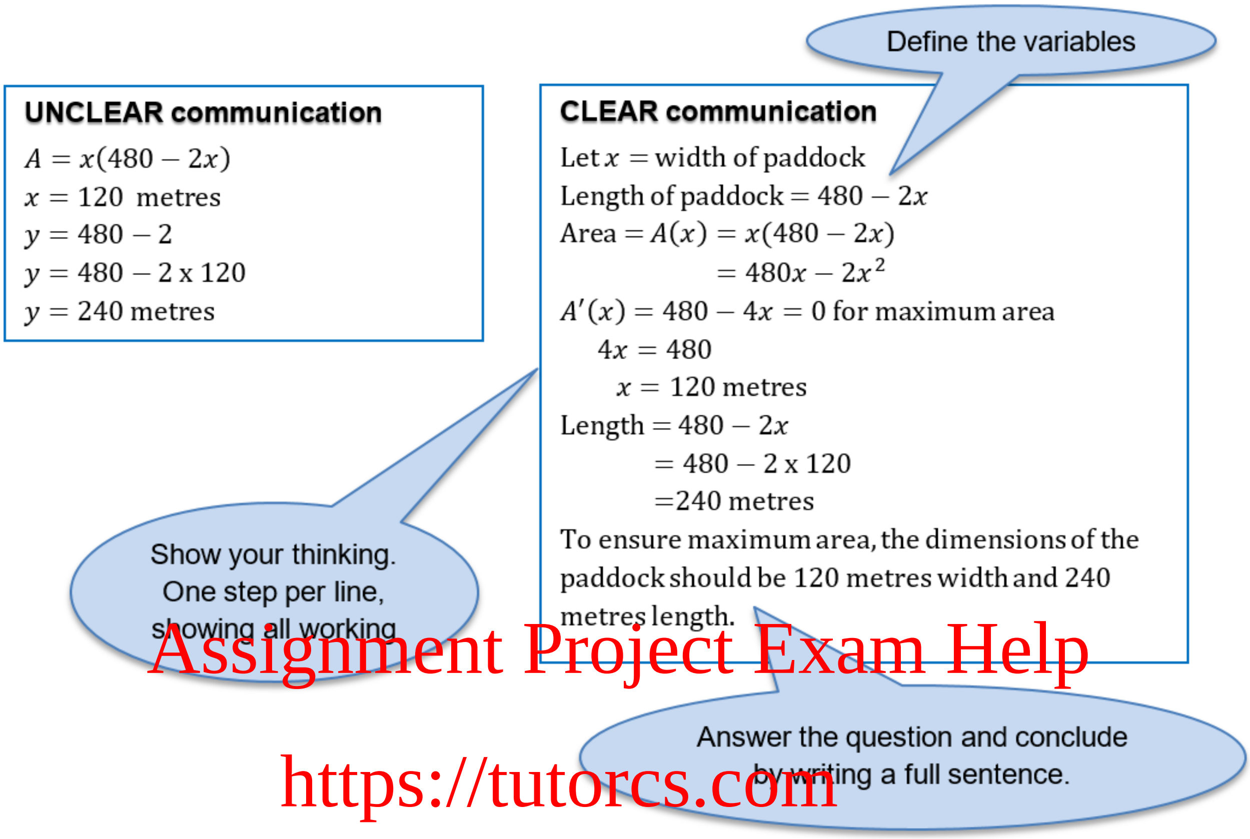
1. the variables are the NEW information. The FAMILIAR is the functions e.g., plot, sin.
2. the meaningful variable names provide MORE explanation about the new and act as LABELS.
3. by removing repeated code there is only a SMALL CHUNK of new information in this example.
4. by placing one step per line the code is NEAT and LOGICAL.
5. Adding comments that tell the reader the intentions of your code serves as a LABEL to enable the working memory to more easily process the code that follows. In Python comments are denotes with a # at the start.



MATHEMATICS should be written with a mix of words, sentences and numbers so that mathematicians and non-math specialists alike can follow your logic. One reason for this that most users of mathematical outcomes are not mathematicians. In the example calculations below:

1. the variables and conclusion are the NEW information. The FAMILIAR is the symbols e.g., $=$, $-$, $'$.
2. the DEFINITIONS of the variables and the full sentence explaining the conclusion provide MORE explanation about the new information
3. by placing one step per line the calculation is NEAT and LOGICAL

Problem: A farmer plans to use a river as one boundary of a rectangular paddock. If the farmer has 480m of fencing to be used to fence the other 3 sides, what dimensions should the paddock be to ensure maximum area?

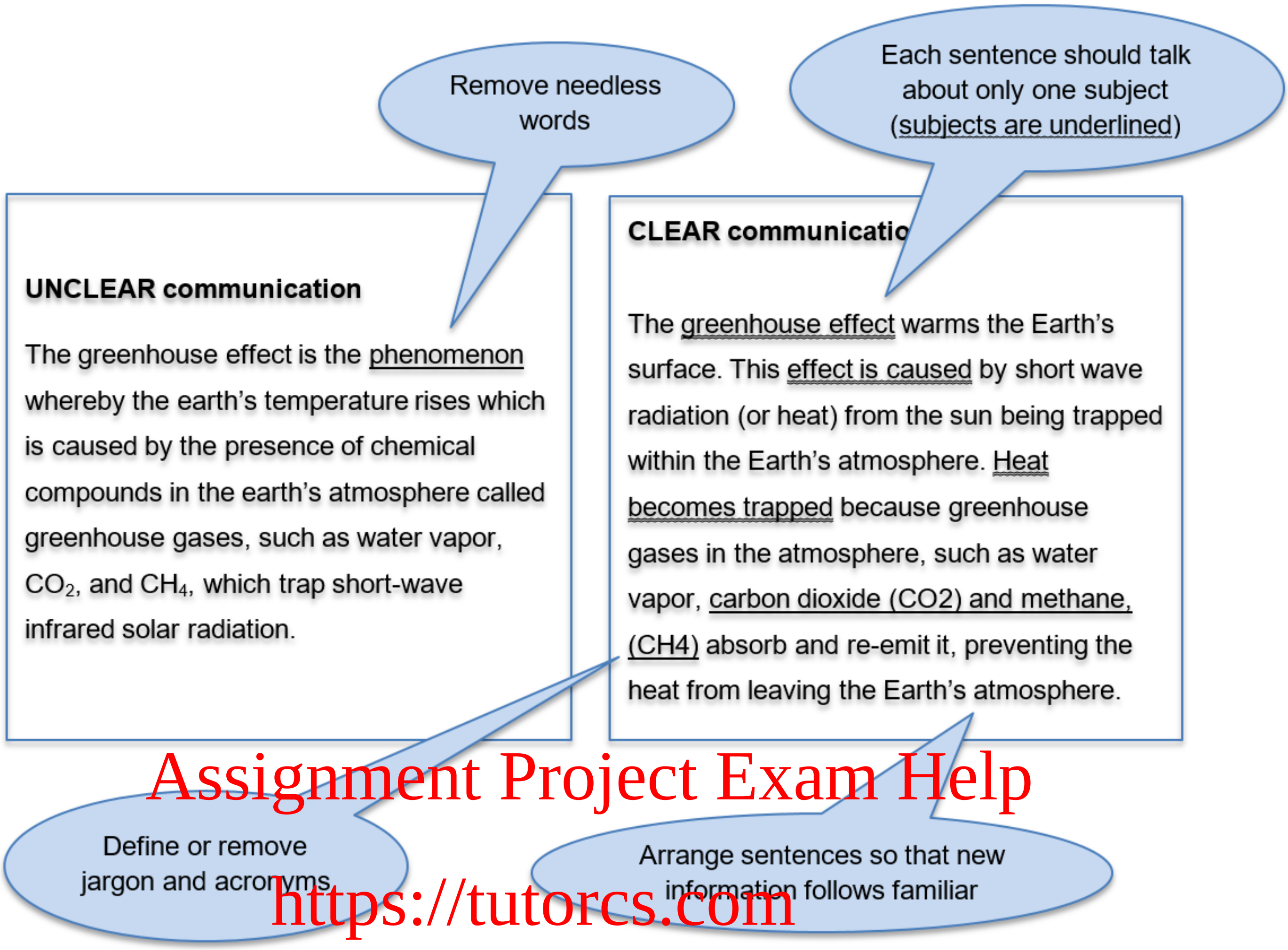


SENTENCES and **PARAGRAPHS** Whether speaking or writing in words, there are simple tactics to help you be clear. Here are some shown in the example below:

1. Each sentence begins with FAMILIAR information with NEW information AFTER. For example, each sentence (except the first) begins with information that was introduced in the previous sentence and ends with new information.

2. Shorter sentences break new information into SMALL CHUNKS

3. Shorter sentences and the patterns of new information after familiar help organise the information LOGICALLY so the reader can see each step of the writer's thinking.



B.2 Principle 2: Knowing your purpose

Why know your purpose?

Why am I communicating with you? That is a question you should always ask when you begin any form of communication. Knowing your specific purpose helps you to make decisions about how to tailor your communication to meet specific needs. If you don't know your purpose, your communication may not have the effect you want.

By knowing your purpose you can decide what information to include or exclude, what to emphasise or de-emphasise, what needs labelling or explaining, and how to structure and approach your communication for best effect.

How to know your purpose

Since communication usually occurs between two or more people, the best way to identify your purpose is to consider it from two perspectives: what is it that you want to do and what is it that you want your audience to do? By audience, we mean the person or people who will read, hear or see your communication.

It is quite normal for there to be more than one purpose for your communication. For example you might want to both inform and entertain your audience. You might also want your audience to understand and give you feedback. Recognizing which purpose(s) are most important to you will help you prioritize decisions when creating your communication.

Some useful tactics to know your purpose include:

1. Be SPECIFIC. The more specific you are about your purpose, the easier it will be for you to make decisions about how to best communicate your information.

2. THINK and WRITE what you want to ACHIEVE. Each time you communicate there is a reason you want other people to know about what you have to say, and outcome you want to see happen. Try to articulate it and write it down in as much detail as possible. Time you spend doing this will save you time and make your job easier when you create your communication.

3. THINK and WRITE how you want others to RESPOND. The outcome you hope to achieve will require the listener or reader to respond in some way. What do you want them to do? How do you want them to think, feel or act? What do you want them to comprehend? Write down your answers so you can refer back to them to help you make decisions when creating your communication.

4. STATE your purpose(s) near the start when you communicate. Doing so allows the reader or listener to be aware of their role, what will be useful to them, and think about how they want to respond.

5. ANALYSE the task. In the case of an assessment task, to identify the reason you are being asked to communicate, analyse what you are being asked to do and why. The 'why'is your purpose. A useful technique to

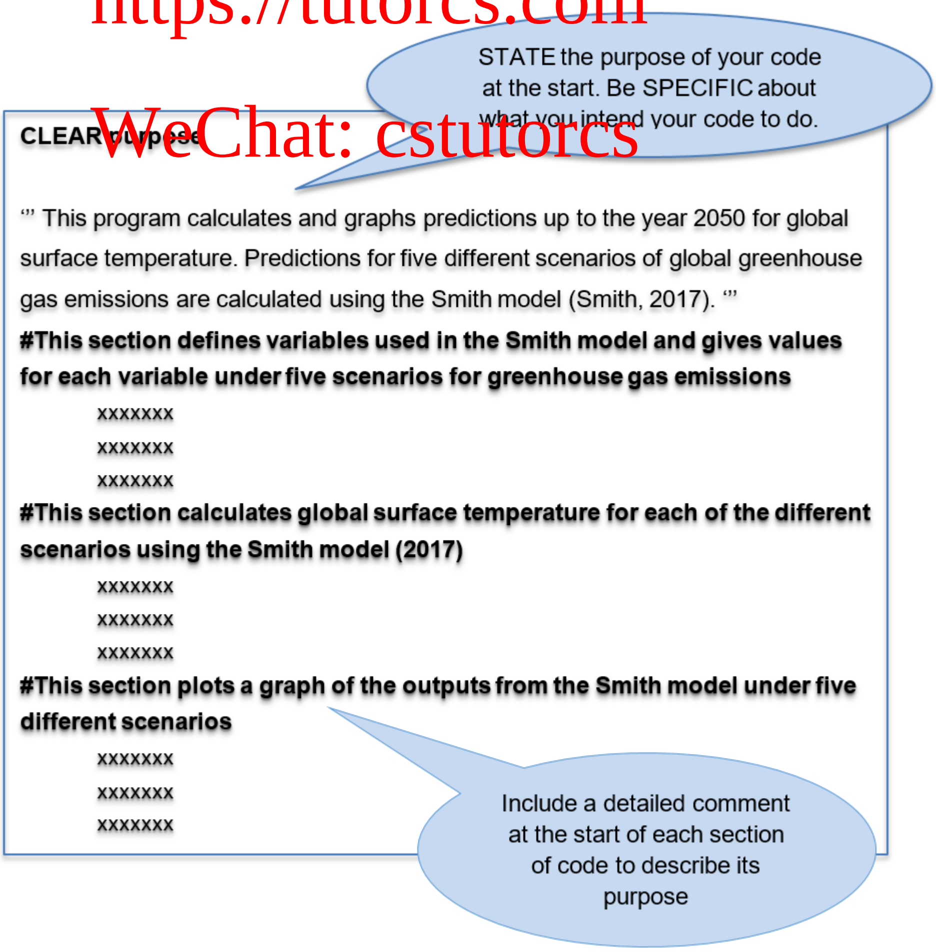
analyse assessment questions is to underline words in the question you think are important.

Examples

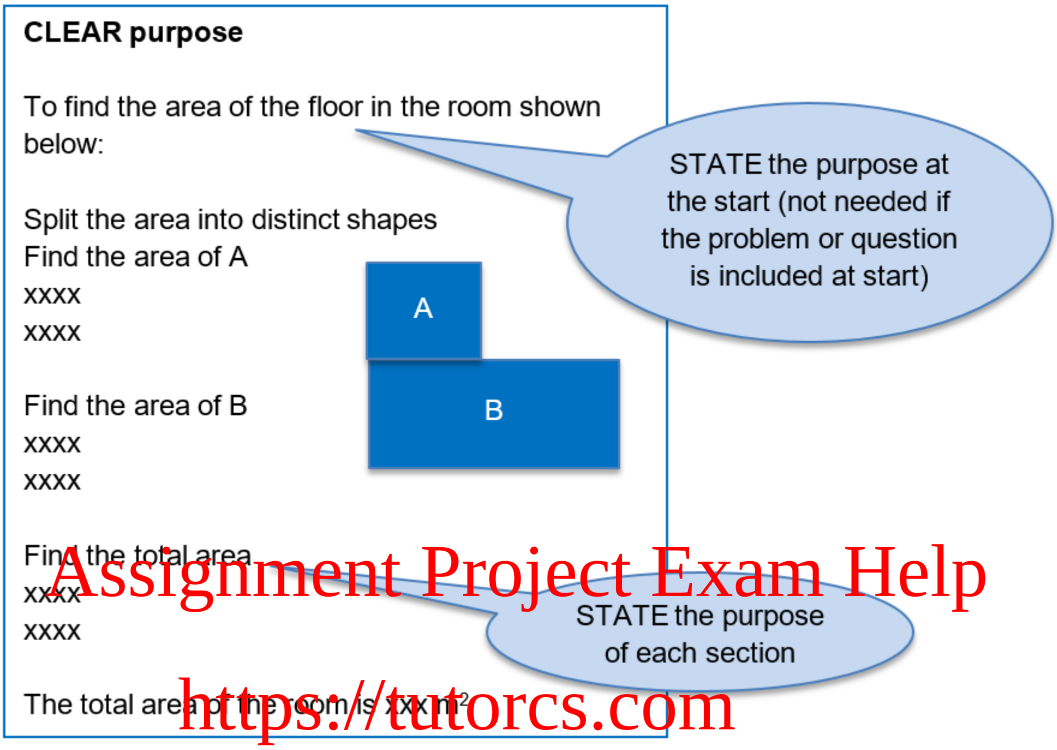
COMPUTER CODE should be written for people, not computers.

1. By STATING the purpose for writing the code at the start, it will be easier and faster for the reader (often the person who will maintain, repair or change the code) to follow your logic. In Python, purposes are denoted by '" at the start and end of a paragraph, collection of sentences or descriptive block.

2. By STATING the purpose or intent at the start of each section of code it prepares the working memory for what to expect and makes easier the process of understanding the code that follows. In Python comments are denoted by a # at the start of the sentence.



MATHEMATICS should be written with a clear purpose near the start. This may be as simple as including the question or rational for the calculations, or instead you may choose to state the purpose as shown in the example below.



SENTENCES AND PARAGRAPHS When writing or speaking about the purpose of your communication in words, you can:

1. Either state the purpose explicitly or you can infer it. For example you can say 'In this essay I will. . . ', or include a title such as 'A report on the evaluation of. . . .', or you could pose a question, as seen in the example below, or choose a format or forum that has a commonly understood purpose, such as a court hearing or executive summary.
2. Place the purpose in the title or in the first few paragraphs, or both.

A title can infer the purpose

No obvious purpose

The greenhouse effect is the phenomenon whereby the earth's temperature rises which is caused by the presence of chemical compounds in the earth's atmosphere called greenhouse gases, such as water vapor, CO_2, and CH_4, which trap short-wave infrared solar radiation.

CLEAR purpose

What is the greenhouse effect?

The greenhouse effect warms the Earth's surface. This effect is caused by short wave radiation (or heat) from the sun being trapped within the Earth's atmosphere. Heat becomes trapped because greenhouse gases in the atmosphere, such as water vapor, carbon dioxide (CO2) and methane, (CH4) absorb and re-emit it, preventing the heat from leaving the Earth's atmosphere.

B.3 Principle 3: Knowing your audience

Why know your audience?

Communication usually involves more than one person. So if you are the communicator, the person to whom you are communicating is your audience. To communicate effectively, you need to adjust what you say or write to suit your audience. You are most likely already aware of this as you probably speak about what you did on your big night out in a different way when you speak with your best friend, your grandparents or your five year old cousin.

The more details you know about your audience the easier it is for you to make decisions about what information to include or exclude, what to emphasise or de-emphasise, what needs labelling or explaining, what types of words to use, and how to approach your communication for best effect.

Tactics to know your audience

There are many sources of information you can use to get to know your audience. Whenever possile it is best to avoid making assumptions. If you have the opportunity, ask your audience questions directly. If that is not possible, do your research; ask people who know them and use the internet. Some useful tactics to get to know your audience include:

1. **Find out the DEMOGRAPHICS of your intended audience.** Demographics include factors such as age, gender, geographic location, ethnicity, language, the sector in which they work, etc.

2. **Determine what they ALREADY KNOW about the topic.** Having a good idea of what types of information are familiar and new to your audience will help you create clear communication. Their interests, education in science and age can provide clues to what aspects of science are likely to be familiar to them.

3. **Find out what they BELIEVE about the topic.** Beliefs have a strong influence on how people remember, interpret and use information. The cultural, religious and political background of your audience can provide clues.

4. **Determine their INTERESTS and PRIORITIES.** People are more likely to pay attention to, and remember, things that they find interesting, things that they need, or things which are important to them. Find out what it is that your audience values. Knowing the occupation, time availability, hobbies, studies, and/or company profile of your audience can provide clues.

5. **Ascertain what types of communication ENGAGE their attention.** Information is more likely to be processed by the working memory of the human brain if the person is paying attention, so finding out where the attention of your audience is focused is an advantage. For example, is your audience more likely to encounter and pay attention to a story, phone conversation, written report or video? Knowing this will help you decide what format to use in your communication.

Examples

Example Audience	Characteristics of Audience
A classmate who missed last week's lecture	*Demographic*: similar age & language to you *Knowledge*: similar scientific knowledge to you *Beliefs*: values science, values university study *Interests & priorities:* similar interests to yours (studying science), priority is to do well in studies for this class *Type of Comm*: In person conversation and/or shared study notes
A person who submits a question to the "Dr Science" Q&A Facebook page	*Demographic*: visit sites like https://sproutsocial.com/insights/new-social-media-demographics/ for detailed information *Knowledge*: you can gain clues about this from the question being asked *Beliefs*: values science, you can also gain clues from the question being asked *Interests & priorities:* interested in details of science, more clues can be gained from the question they ask *Type of Comm*: Facebook
The Australian association of farmers for sustainable futures need to provide a recommendation to its stakeholders about Y	*Demographic*: farmers, mostly male… look up members list for the association for more clues *Knowledge*: some scientific knowledge, plenty of on-ground observational knowledge of ecosystems, business knowledge *Beliefs*: value science, value practical solutions *Interests & priorities:* business, sustainability, land management *Type of Comm*: in person, social networks, local community groups

B.4 Principle 4: Identifying key messages

Why identify a key message?

Have you ever been to a lecture, or read an article and thought, wow that was fascinating! But when you try to summarise what it was about to another person, you can't quite remember? Chances are that the speaker or writer did not make their key message obvious, or maybe they didn't know what it was! Identifying key messages in communication from other people is a skill, but one that is made much easier when the person communicating has a clear and precise idea of the main message they want to convey.

Knowing your key message helps you make decisions about what information to include or exclude, what to emphasise or de-emphasise, what needs labelling or explaining, what types of words to use, and how to approach your communication for best effect.

Tactics to identify a key message

Your key message is the one thing you want your readers to know, consider, do or think about. It is your take home message; you will be satisfied if your audience remember nothing else. Ideally a key message should be easy to recall and repeat in conversation. Here are some tactics you can use to identify your key message:

1. **SUMMARISE in one sentence the specific purpose of communicating to your specific audience.**
   …you now have a rough draft for your key message.

2. **It is the IDEA not the words that is most important.**
   When developing your message, knowing what idea you want to get across is more important than worrying about the exact wording. Once you have crystalized the idea, you can find the best wording to make that idea stick, or elaborate that idea by using examples.

3. **REVISIT your key message as you create your piece of communication.**
   You will find that your message changes and improves as you clarify what it is you want to say. Often this only happens while you are creating the whole piece of communication. Revisiting your key message during the process also helps reminds you of your goals and assess if you are on the right track.

4. **TEST your message.**
   Have someone read/listen to your communication and tell you what they think the key message is, or what it is that they understood from your communication. If what they tell you is not the same as you intended, you need to revisit and

5. **CHANGE your message for different audiences and purposes.**
   What works in one context, may not work in another. The wording and sometimes the idea in your message may need to change when you communicate with different audiences or for different purposes.

6. **BEGIN and END with your key message.**
   You know you have a successful key message when your audience can recall and repeat what it is. To help this happen, place your message near the start when the attention of the audience is at its highest, and near then end because when it is repeated it is more likely to stay in people's memory.

Examples

MATHEMATICS Mathematical calculations and visual displays of data all have key messages.

1. Usually the key message is the outcome from the calculation or the conclusion from analysis of the data.

2. The key message should always be stated in words, either as a conclusion at the end of a calculation as shown in the example above for being clear, or as a caption or annotation on a table, figure or graph as shown below.

3. The key message in mathematical work directs direct the reader to what you consider to be the most important message from the data or calculation.

Use the figure or table caption to tell the reader the key message

No obvious key message

Change in Median House Prices 2014-2017				
City	Jun-14	Jun-15	Jun-16	Jun-17
Sydney	$812,929	$1,000,616	$1,046,068	$1,178,417
Melbourne	$608,863	$668,030	$752,083	$865,712
Brisbane	$479,989	$490,855	$529,438	$546,043
Adelaide	$464,029	$479,285	$492,252	$524,968
Perth	$614,297	$605,089	$577,778	$555,788
Canberra	$583,473	$616,313	$664,133	$723,299
Darwin	$648,584	$325,972	$351,187	$404,522
Hobart	$326,460	$654,270	$594,144	$666,686
National	$628,561	$701,827	$743,264	$818,416

Remove needless columns and data. If appropriate and ethical, replace with summary data to illustrate your key message

CLEAR key message

Table 1: Median House Prices increased in most Australian Capital Cities between 2014 and 2017.

City	**Jun-14**	**Jun-17**	**% change over 3 years**
Sydney	$812,929	$1,178,417	45.0
Melbourne	$608,863	$865,712	42.2
Brisbane	$479,989	$546,043	13.8
Adelaide	$464,029	$524,968	13.1
Perth	$614,297	$555,788	-9.5
Canberra	$583,473	$723,299	24.0
Darwin	$648,584	$404,522	-37.6
Hobart	$326,460	$666,686	104.2
National	**$628,561**	**$818,416**	**30.2**

Use formatting to highlight the data that supports your key message

COMPUTER CODE Tactics for making your key message obvious to your reader when writing code is pretty much the same as identifying your purpose, the only difference being that the comments you add for every section of the code need to support the stated purpose. In the example shown above for purpose, the comment for each section refers to one or more parts of the description of purpose at the start of the code. This approach also helps make the code clear to the reader by laying out each step in your thinking.

SENTENCES and PARAGRAPHS Useful tactics for making your key message obvious in writing include those described above for being clear, but for different reasons as described below.

1. The paragraph in the clear example below BEGINS and ENDS with the same message, albeit stated in different words i.e., 'the greenhouse effect warms...' and 'prevents the heat from leaving...'

2. The first sentence (topic sentence) orients the reader's mind to what type of information will follow. The last sentence (concluding sentence) reiterates the key message so the reader can check they understood the content of the paragraph and to remind the reader of the key message.

3. Each sentence in the paragraph supports the key message stated in the first sentence. Doing so helps to reinforce the key message and makes it easier for the working memory part of the brain to process and move to long term memory.

No obvious key message

The greenhouse effect is the phenomenon whereby the earth's temperature rises which is caused by the presence of chemical compounds in the earth's atmosphere called greenhouse gases, such as water vapor, CO_2, and CH_4, which trap short-wave infrared solar radiation.

The key message in this example is fuzzy. Is it that temperatures rise? Or that chemical compounds are present? or that infra-red

CLEAR key message

What is the greenhouse effect?

The greenhouse effect warms the Earth's surface. This effect is caused by short wave radiation (or heat) from the sun being trapped within the Earth's atmosphere. Heat becomes trapped because greenhouse gases in the atmosphere, such as water vapor, carbon dioxide (CO2) and methane, (CH4) absorb and re-emit it, preventing the heat from leaving the Earth's atmosphere.

The paragraph BEGINS and ENDS with the key message i.e., that the greenhouse effect warms the Earth

CLIPS www.clips.edu.au

An online UQ resource to help improve your communication in science

COMMUNICATING WITH NUMBERS | DISPLAYING DATA | WRITING | PRESENTATIONS

Appendix C: Prerequisite maths review

C.1 Linear, quadratic, and power functions

- The basic mathematical tool used to describe quantitative relationships and patterns in models is the mathematical *function*.

Mathematical functions

A *function* is a rule that converts input value(s) to output value(s). There is exactly one output value for each input value. Often, the input values are called x or t (for time). On a graph, they are represented on the *horizontal* axis. If f is the name of a function, then $f(t)$ denotes the output that arises from applying f to the input value t. On a graph, these values are represented on the *vertical* axis.

- People study a range of functions, including linear, quadratic, power, periodic, exponential, logarithmic, and combinations of these. These functions are interesting **precisely** because they model natural phenomena.

Linear functions

Linear functions have equations $y(x) = mx + c$, where m is the *slope* and c is the *y-intercept* of the line.

If (x_1, y_1) and (x_2, y_2) are two points on the line then

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

- Given a straight line, we can calculate the equation $y = mx+c$ that models the line by finding two points on the line and using them to calculate the slope m, and then using a point on the line to plug into the equation to find the value c.

- Note that sometimes it is easy to read the value of c straight from the graph. When doing this, be careful to read the labels of the axes very carefully.

Quadratics and modelling

Quadratic functions have a power of x (or t, or ...) equal to 2, with equations of the form $y(x) = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$. The graphs of quadratics are parabolas.

- Quadratics are important in practical modelling, particularly when modelling over short time periods. They are the simplest functions with *local optimal* values, that is, local *maximum* or *minimum* values.
- Linear equations and quadratic equations are two special cases of a more general class of functions, called *power functions*.

General form of power functions

Power functions have equations

$$y(x) = Mx^p + c$$

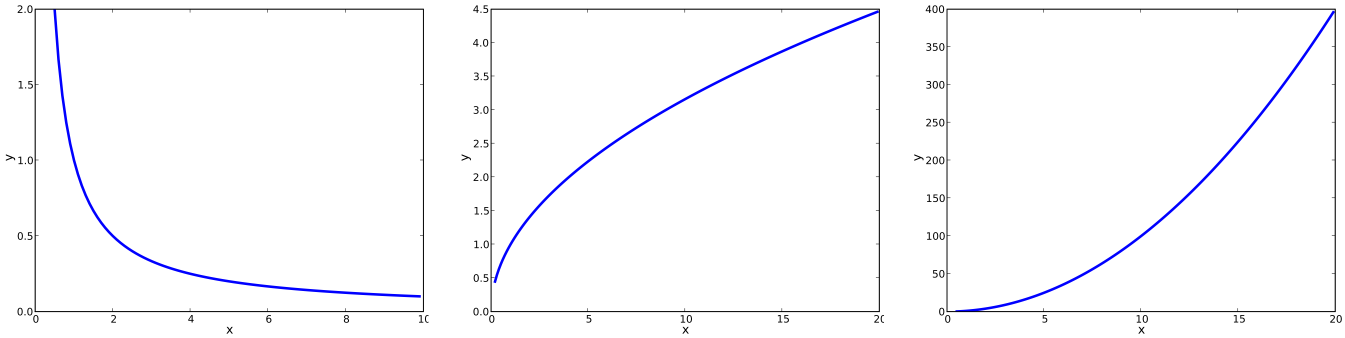
where M, p and c are constants. Changing the value of these constants generates graphs with different shapes, which makes power functions useful for modelling a range of phenomena. For example, changing the value of:

- the power p creates graphs that increase or decrease, at different rates;
- the constant M *scales* the vertical height of the graph at each point; and
- the constant c *shifts* the graph up or down.

Figure C.1 illustrates how the value of the power p affects the general shape of the corresponding graph, **for positive values of** C **and** x. Figure C.2 illustrates this, showing some equations and their graphs.

Power, p	General shape of the graph
< 0	curve, **decreasing less rapidly** as x increases
0	horizontal line
> 0 and < 1	curve, **increasing less rapidly** as x increases
1	straight line
> 1	curve, **increasing more rapidly** as x increases

Figure C.1: Different powers and the general shapes of the corresponding graphs.



$y = x^{-1}$ $\qquad$ $y = x^{1/2} = \sqrt{x}$ $\qquad$ $y = x^2$

Figure C.2: Graphs showing the shapes of some power functions.

C.2 Periodic functions

- To represent cyclic phenomena, or data that give rise to waves, the most common choices are the *trigonometric functions* sin and cos.
- These functions are defined in the context of geometry and angles. However, you **do not** need to think of them in a geometric context when modelling.
- In SCIE1000 we will always use the function sin (we could have used cos, noting that a cosine function can be considered as a shift of a sine function).

Sine functions

The function $\sin(t)$ has period 2π, amplitude 1, equilibrium (centre value) 0, and equals 0 when $t = 0$. Figure C.3 shows a simple sine wave $y(t) = \sin(t)$.

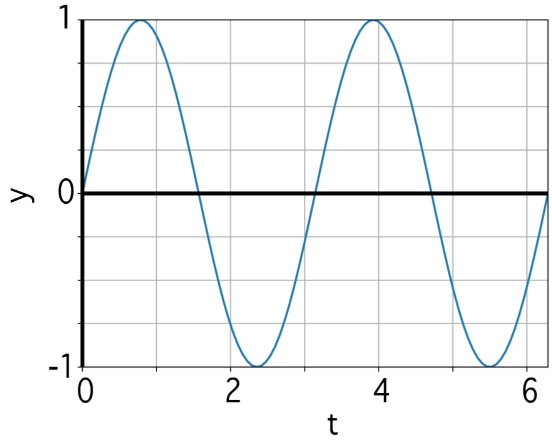


Figure C.3: Graph of $y(t) = \sin(t)$.

The general equation of a sine function is

$$y(t) = A \sin\left(\frac{2\pi}{P}(t - S)\right) + E.$$

The constants in this equation are

A: the amplitude of the sine wave

P: the period of the wave

S: the shift right of the wave

E: the equilibrium value

- Varying the values of the constants A, P, S and E within a general sin function alters the cyclic model, allowing us to model a range of cyclic phenomena.

C.3 Exponential and logarithmic functions

Exponential functions

Exponential functions have equations

$$f(x) = Ca^{kx},$$

where C, a and k are constants. The constant C is a scaling factor. The constant a is called the **base** of the exponential. The two most common values used for the base a are

- the number 10; and
- *Euler's number*, denoted e, where $e \approx 2.71828\ldots$.

The constant k is the **growth rate** or **decay rate**. If C is positive then:

- If k is *positive*, the function displays exponential *growth*.
- If k is *negative*, the function displays exponential *decay*.

Note that when $x = 0$ the function value equals C.

- Two useful rules for manipulating exponentials are:

$$a^x a^y = a^{x+y}$$

$$(a^x)^y = a^{xy}$$

Logarithmic functions

Suppose we have an exponential function of the form

$$y(x) = a^x.$$

The inverse function of the exponential is called the *logarithm* (or *log* for short), defined as

$$x(y) = \log_a(y).$$

- Note that there are a number of conventions when referring to the base. On your calculator you will notice a button labelled "ln" which means log to the base e (also called a natural logarithm), The button labelled "log" represents log to the base 10.
- However, mathematically, "log" (without a subscript to show the base) may sometimes be used to represent log to the base e, whereas in other cases "ln" will be used. Other bases are shown by writing the base as a subscript immediately after "log", eg log2. In this course we mostly work with log to the base e, and write this as ln, but always be careful to use the context to decide if this is true.

- Two useful rules for manipulating logarithmic functions (independent of base) are:

$$\log(x^n) = n\log(x)$$

$$\log(x \cdot y) = \log(x) + \log(y)$$

C.4 Rates of change

Average rate of change

Let (x_1, y_1) and (x_2, y_2) be two points. The **average rate of change of y with respect to x** between these points is the *slope of the straight line joining the points*. As we saw earlier, the slope equals the change in y values divided by the change in x values, so:

$$\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

(Note that Δ is the Greek capital letter "Delta", and usually means "the change in the value of".)

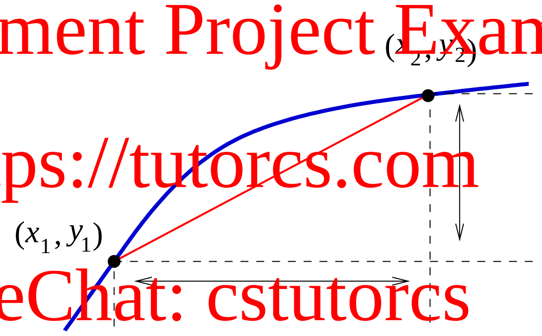


Figure C.4: Average rate of change.

- Rather than measure the average rate of change between two points, in many situations it is more useful to measure the *instantaneous* rate of change at a point. The mathematical term for an instantaneous rate of change is *derivative*. In SCIE1000, you will not be finding derivatives (in general), but will need to interpret and use them.

Derivatives

If $y = f(x)$ is a function, then the derivative y' is a new function that gives the instantaneous rate at which y is changing with respect to x.

The **value** of the **derivative** at any point describes the behaviour of the **function** at that point. At any point:

- if y' is **positive** then the function y is **increasing**;
- if y' is **negative** then the function y is **decreasing**; and
- if the function y has a **local maximum** (peak) or **local minimum** (trough) at a point, then y' **equals zero** at that point.

The *derivative of the derivative*, or *second derivative*, is denoted f''.

- A constant function has no slope and thus the derivative of a constant is 0. That is, if $y(x) = c$ for some constant c, then $y'(x) = 0$.
- The derivative of a linear function is equal to the slope of that function. That is, if $y(x) = mx + c$ for some constants m and c, then $y'(x) = m$.
- If $f(x)$ and $g(x)$ are functions whose derivatives are defined, then the derivative of $f(x) + g(x)$ is $f'(x) + g'(x)$ and the derivative of $f(x) - g(x)$ is $f'(x) - g'(x)$.

C.5 Area under the curve

- Given a graph, the *area under the curve* or *AUC* of that graph is the area bounded by that curve, the x-axis and two points on the x-axis.

AUC and Definite integrals

Given a function $f(x)$, the AUC from the point $x = a$ to the point $x = b$ is called the *definite integral of $f(x)$ from a to b*, written as

$$\int_a^b f(x)\,dx.$$

- There are two common ways of calculating AUCs.
- First, if the function $f(x)$ is known, then the *Fundamental Theorem of Calculus* gives an "easy" mathematical approach for finding the AUC between two points a and b on the x-axis:
  - Find an *antiderivative* or *integral* of $f(x)$, say $F(x)$.
  - Substitute the value b into $F(x)$.
  - Substitute the value a into $F(x)$.
  - Subtract the second value from the first one.
  - The answer gives the required AUC.

- More often, AUCs are used in practical applications in which the only available information is a collection of measured data values, and the function $f(x)$ is **not** known.
- In such cases, AUCs are estimated approximately, by summing the areas of geometric shapes of "narrow" width, such as rectangles (called *Riemann sums*), or *trapezoids* (called the *trapezoid rule*).

List of Figures

List of Images

List of Original Photographs

These notes contain a large number of original photographs. Qld Health images are copyright to the State of Queensland (Queensland Health) 2010 and are used with permission. In other cases, the photographer or photograph owner is identified, as follows:

AR: Anthony Richardson	DH: Dominic Hyde	DM: Delphia Manietta	WG: William Grey
MG: Melanie Giandzi	MGa: Marcus Gallagher	PA: Peter Adams	POD: Peter O'Donoghue
SH: Sara Herke			

List of Programs

Bibliography

[1] Australian Institute of Health and Welfare (AIHW) and Australasian Association of Cancer Registries (AACR), *Cancer in Australia 2001*. AIHW cat. no. CAN 23. Canberra: AIHW (Cancer Series no. 28, 2004).

[2] Australian Medical Association, accessed 27 December 2010, `<ama.com.au/node/2931>`

[3] Benowitz *et al.*, *Nicotine intake and dose response when smoking reduced-nicotine content cigarettes*, Clinical Pharmacology and Therapeutics **80** (2006) 703–714.

[4] Caswell, *Matrix population models: Construction, analysis and interpretation*, 2nd Edition, 2001. Sinauer Associates, Sunderland, Massachusetts. ISBN 0-87893-096-5.

[5] Centers for Disease Control and Prevention, accessed 27 December 2010, `www.cdc.gov`

[6] Crouse *et al.*, *A Stage-Based Population Model for Loggerhead Sea Turtles and Implications for Conservation*, Ecological Society of America **68** (1987) 1–13.

[7] Crowder *et al.*, *Predicting the impact of turtle excluder devices on loggerhead sea turtle populations*, Ecological Applications **4(3)** (1994) 437–445.

[8] Damon *et al.*, *Radiocarbon Dating of the Shroud of Turin*, Nature **337: 6208** (1989) 611–615.

[9] Dingle *et al.*, *What Is Migration?*, BioScience **57(2)** (2007) 113–121.

[10] Drugs.com, accessed 11 September 2010, `<www.drugs.com>`

[11] Egevang *et al.*, *Tracking of Arctic terns* Sterna paradisaea *reveals longest animal migration*, PNAS **107** (2010) 2078–2081.

[12] Engber, *Vodka + movies = pseudoscience*, Chronicle of Higher Education, October 29, 2004.

[13] Engber, "Dodgy Boffins", accessed 17th August 2012,

`<www.slate.com/articles/health_and_science/science/2012/05/science_in_the_telegraph_and_the_daily_mail_what_s_wrong_with_british_journalism_.single.html#pagebreak_anchor_2>`

[14] Duijm *et al.*, *Sensitivity, specificity and predictive values of breast imaging in the detection of cancer*, British Journal of Cancer **76(3)** (1997) 377–381.

[15] Explorers Web 2004, accessed 6 September 2010,

`<www.explorersweb.com/adventureweather/charts>`

[16] Framingham Heart Study 2010, accessed 31 August 2010,

`<www.framinghamheartstudy.org/participants/index.html>`

[17] Framingham Heart Study 2010, accessed 31 August 2010,

`<www.framinghamheartstudy.org/about/index.html>`

[18] Framingham Heart Study 2010, accessed 31 August 2010,

`<www.framinghamheartstudy.org/about/milestones.html>`

[19] Galileo, *Dialogue Concerning the Two Chief World Systems*, trans. Stillman Drake, Berkeley: Univ. California Press (1953) 207.

[20] Gigerenzer *et al.*, *Helping Doctors and Patients Make Sense of Health Statistics*, Psych. Science in the Public Interest **8 (2)** (2007) 53–96.

[21] Goldacre, "Bad Science", accessed 19th September 2012, `<www.badscience.net>`

[22] Goldman *et al.*, *A re-examination of the age and growth of sand tiger sharks, Carcharias taurus, in the western North Atlantic: the importance of ageing protocols and use of multiple back-calculation techniques*, Environ Biol Fish **77:3-4** (2006) 241–252.

[23] Grau *et al.*, *Breast cancer in Rubens paintings*, Breast Cancer Research and Treatment **68** (2001), 89–93.

[24] Greenbaum *et al.*, *Impaired β-cell function, incretin effect, and glucagon suppression in patients with type 1 diabetes who have normal fasting glucose*, Diabetes **51** (2002) 943 – 950.

[25] Haub, "How many people have ever lived on Earth?", accessed 5th July 2013, `<www.prb.org>`

[26] Hoegh-Guldberg *et al.*, *Coral Reefs Under Rapid Climate Change and Ocean Acidification*, Science **318:5857** (2007) 1737 – 1742.

[27] International Organization for Standardization, ISO 2533:1975.

[28] Keeling *et al.*, *Exchanges of atmospheric CO_2 and $^{13}CO_2$ with the terrestrial biosphere and oceans from 1978 to 2000*, I. Global aspects, SIO Reference Series, No. 01-06, Scripps Institution of Oceanography, San Diego, (2001), 88 pages.

[29] Lundquist *et al.*, *Cyanide concentrations in blood after cigarette smoking, as determined by a sensitive fluorimetric method*, Clinical Chemistry, **33:7** (1987), 1228 – 1230.

[30] MacLulich, *Fluctuations in the numbers of the varying hare (Lepus americanus)*, University of Toronto Studies, Biological series **43** (1937).

[31] Maher, *Growth and development of the Yellow-bellied Sunbird* Nectarinia jugularis *in North Queensland*, Emu **91** (1991) 58–61.

[32] Moolenaar *et al.*, *Absorption rate and bioavailability of paracetamol from rectal aqueous suspensions*, Pharmacy World and Science, **1:1** (1979), 201–206.

[33] Morgan, Mary S., and Margaret Morrison (eds.) (1999), *Models as Mediators: Perspectives on Natural and Social Science*, Cambridge: Cambridge University Press.

[34] Stephenson *et al.*, *Mars Climate Orbiter Mishap Investigation Board, Phase I Report*,

`<ftp.hq.nasa.gov/pub/pao/reports/1999/MCO_report.pdf>`

[35] National Breast and Ovarian Cancer Centre, *Breast cancer risk factors: a review of the evidence*, National Breast and Ovarian Cancer Centre, Surry Hills, NSW, 2009.

[36] National Health Service, accessed 19 September 2012, `<www.nhs.uk/News/Pages/NewsIndex.aspx>`

[37] National Heart, Lung and Blood Institute 2008, accessed 31 August 2010,

`<www.nhlbi.nih.gov/health/dci/Diseases/Hbc/HBC_WhatIs.html>`

[38] Newman em et al., What would have happened to the ozone layer if chlorofluorocarbons (CFCs) had not been regulated?, *Atmos. Chem. Phys.*, 9 (2009) 2113–2128.

[39] Niyomnaitham *et al.*, *Bioequivalence study of 50 mg sertraline tablets in healthy Thai volunteers*, J Med Assoc Thailand **92:9** (2009), 1229–33.

[40] Pan American Health Organization, *EPI Newsletter Volume XX, Number 4*, August 1998.

[41] Popper, *The Logic of Scientific Discovery*, London: Hutchinson (1934).

[42] Ponce-Reyes *et al.*, *Vulnerability of cloud forest reserves in Mexico to climate change*, Nature Climate Change **2** (2012), 448–52.

[43] Posey and Mozayani, *The estimation of BAC, Widmark revisited*, Forensic Science, Medicine and Pathology, **3** (2007) 33–39.

[44] Rawlins *et al.*, *Pharmacokinetics of paracetamol (Acetaminophen) after intravenous and oral administration*, European Journal of Clinical Pharmacology **11** (1977), 283 – 286.

[45] Rodenhouse *et al.*, *Potential effects of climate change on birds of the northeast*, Mitigation and Adaptation Strategies for Global Change, **13** (2008) 517–540.

[46] Ronfeld *et al.*, *Pharmacokinetics of sertraline and its N-demethyl metabolite in elderly and young male and female volunteers*, Clinical Pharmacokinetics, **32** Suppl 1 (1997) 22–30.

[47] Salm *et al.*, *Blood flow in coronary artery bypass vein grafts: volume versus velocity at cardiovascular MR imaging*, Radiology, **232** 915–920.

[48] Schwartz *et al.*, *The role of numeracy in understanding the benefit of screening mammography*, Annals of Internal Medicine, **127** (1997) 966–972.

[49] Scripps Institution of Oceanography 2010, La Jolla, CA, accessed 24 September 2013,
`<scrippsco2.ucsd.edu/data/atmospheric_co2.html>`

[50] Seidl *et al.*, *The calculation of blood ethanol concentrations in males and females*, International Journal of Legal Medicine, **114** (2000) 71–77.

[51] Sheen and Hwang, *Mathematical modelling the cross-contamination of E. coli* O157:H7 *on the surface of ready-to-eat meat product while slicing*, Food Microbiology **27** (2010) 37–43.

[52] Robert G. Steadman, Norms of apparent temperature in Australia, *Aust. Met. Mag.*, **Vol 43**, 1994, 1–16.

[53] Stenseth *et al.*, *Population regulation in snowshoe hare and Canadian lynx*, PNAS **94** (1997), 5147–5152.

[54] Wieland and Kasperski, *Estimating Net Present Value in the Northern Chesapeake Bay Oyster Fishery*, Prepared for: NOAA Chesapeake Bay Office, Non-native Oyster Research Program, Grant: NA05NMF4571231, (2008).

[55] United Nations, Department of Economic and Social Affairs, Population Division, *World Population Prospects: The 2010 Revision*, CD-ROM Edition (2011).

[56] Wilkinson *et al.*, *Pharmacokinetics of ethanol after oral administration in the fasting state*, Journal of Pharmacokinetics and Biopharmaceutics, **5:3** (1977) 207–224.

[57] Wolever *et al.*, *The glycemic index: methodology and clinical implications*, American Journal of Clinical Nutrition, **54** (1991) 846–854.

[58] World Health Organisation, *Gender*, accessed 5 November 2020, `https://www.who.int/health-topics/gender`

[59] World Health Organisation, *Global burden of disease*, accessed 10 October 2012, `www.who.int/healthinfo/global_burden_disease/en`

Index