THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2017

SCIE1000 Theory and Practice in Science

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

One A4 sheet of handwritten or typed notes double sided is permitted

Materials To Be Supplied To Students:

None

Instructions To Students:

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

Total of 120 marks on paper; questions carry the indicated number of marks. Write all answers on the examination paper. The final page contains important information, which you will use when answering the questions. You may detach that page from the examination paper if you wish. Do not write any answers on that page.

For Examiner Use Only

Page	Mark
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	

Total ________

To answer each question you will need to use the information on Page 12.

1. (a) Show that 1 mm of rain is equivalent to 1000 m^3 of rainwater per square kilometre. (Show all working, with units.) (3 marks)

(b) This question refers to the map of rainfall in Adelaide, on Page 12. If all of the rainwater that fell in the area of that map on that day could be collected and added to Adelaide's water supply, estimate for how long it would supply Adelaide's water needs? Show all working, include units in your answer, and justify any assumptions you make. (Hint: recall that the distance from Elizabeth to Echunga is 60 km. The answer to Part (a) might be useful.)

(11 marks)

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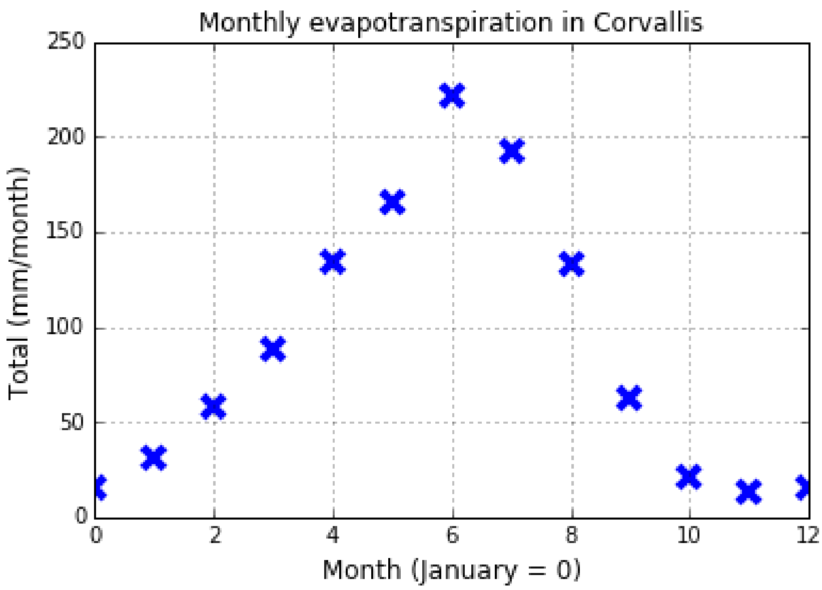
2. The graph shows average evapotranspiration in mm/month, in the town of Corvallis, in Oregon in the USA. In all parts of this question, let $V(t)$ be the evapotranspiration in month t, where t ranges from 0 (January) to 12 (the next January).



(a) Find a linear function to model $V(t)$ from Month 0 (January) to Month 6 (July). Show all working. How effective is your model during those months?

(5 marks)

(b) Find a sine model for $V(t)$ **for the whole year**, in the following form, showing working.

$$V(t) = A\sin\left(\frac{2\pi}{P}(t - S)\right) + E.$$

(6 marks)

(continued over)

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(c) Recall that your sine function from Part (b) is of the form $V(t) = A\sin\left(\frac{2\pi}{P}(t-S)\right) + E$.

Consider a sine model for evapotranspiration in another location, called Newtown, compared to the model for Corvallis. Parts (i) and (ii) below give comparative information for Corvallis and Newtown; apart from the given information, assume that each location has identical conditions. In each case, describe which of the constants A, P, S and/or E would have their value(s) impacted by the information, and whether the value(s) would be larger or smaller in the Newtown model than in Corvallis. Justify your answers, and treat each part separately.

(i) Corvallis is in the northern hemisphere, with cold, cloudy winters and warm, sunny summers; Newtown is in the southern hemisphere, with cool, sunny winters, and hot, sunny summers. (5 marks)

(ii) Winter days in Corvallis are about an hour shorter than winter days in Newtown. Summer days in Corvallis are about an hour longer than summer days in Newtown. (5 marks)

(d) The following graph and table show information about the average evapotranspiration $E(t)$ and rainfall $R(t)$ in month t, both in mm/month, in the town of Corvallis. The graph shows values of E and R over a whole year, and the table shows values from Month 3 (April) until Month 8 (September). Rainfall amounts are not known in Months 4, 5 and 7.

(continued over)

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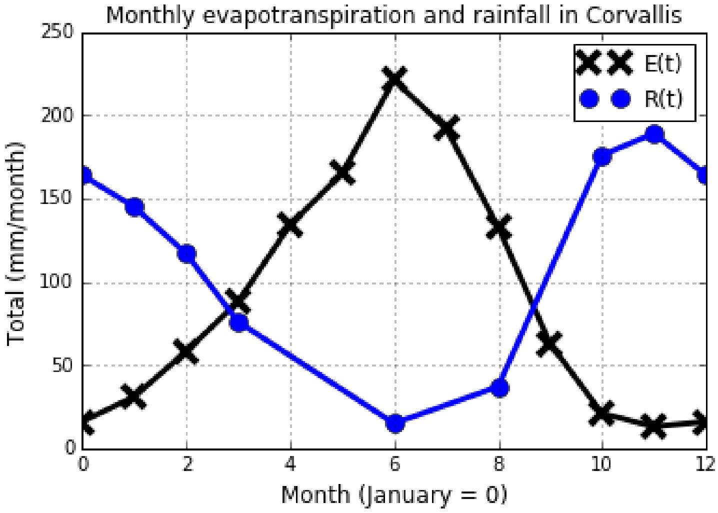

Month	$E(t)$	$R(t)$
3	88	76
4	134	unknown
5	165	unknown
6	221	15
7	192	unknown
8	133	37

Find the **exact** area **between** the graphs of $E(t)$ and $R(t)$ from Month 3 until Month 8 (inclusive). (Hint: first find the area under the curve (AUC) for each of $E(t)$ and $R(t)$.) (10 marks)

(e) Find the units of the area in Part (d), and briefly explain the physical meaning of the area. (2 marks)

Do not write here

3. Let $M_a(t)$ be the mass of pollutants in grams per square metre (g/m^2) that have accumulated on the surface of a carpark on any day t since the most recent rainy day. The following differential equation models M_a on non-rainy days:

$$M_a' = k_d - k_b M_a$$

where the positive constant k_d is the rate of pollution deposition in the car park and the positive constant k_b is the rate of removal due to wind and traffic.

(a) Find the units of M_a', k_d and k_b. (3 marks)

(b) The mass of accumulated pollutants is said to be at *equilibrium* if it does not change over time. For a given carpark, $k_d = 1.2$ and $k_b = 0.2$, both measured in the units from Part (a), and $M_a = 6$ g/m^2. Show that the mass of accumulated pollutants is at equilibrium.

(3 marks)

(c) The solution to the differential equation given above is

$$M_a = \frac{k_d}{k_b}(1 - e^{-k_b t}).$$

Describe briefly what the model predicts will happen to the mass of accumulated pollutants as t increases, and sketch a rough graph.

(5 marks)

(continued over)

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(d) (i) For a particular carpark in which $k_d = 1.2$ and $k_b = 0.2$, the solution to the differential equation given in Part (c) is

$$M_a = 6(1 - e^{-0.2t}).$$

Find the day number on which the mass of accumulated pollutants is 5 g/m^2.

(5 marks)

(ii) Using $t_0 = 0$, apply one step of Newton's method to estimate the value of t for which

$$6\left(1 - e^{-0.2t}\right) - 5 = 0.$$

(Hint: if $y(t) = 6(1 - e^{-0.2t}) - 5$ then $y'(t) = 1.2e^{-0.2t}$.)

(5 marks)

(iii) After multiple steps of Newton's method, what (approximate) value would you expect the current estimate t_i to have? Explain your answer briefly.

(3 marks)

(continued over)

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(e) **In the box**, write all of the output produced by the following partial program when the value 2 is entered from the keyboard. (Round all values to three decimal places.)

(7 marks)

```
def Val(x):
    print("here",x)
    y = 6 - 6*exp(-0.2*x)
    return(y)

t = array([0, 5, 10, 15])
M = zeros(4)
d = eval(input("Value?"))
i = 0
while i<3:
    M[i] = Val(t[i])
    i = i + d
print(M)
```


(f) **In the box**, write all of the output produced by the following partial program. (8 marks)

```
t = array([0, 5, 10, 15])
i = 1
while i<=3:
    if t[i] > t[i-1]:
        a = t[i]
        t[i] = t[i-1]
        t[i-1] = a
        print(t[i-1], t[i])
    i = i + 1
print(t)
```


Do not write here

4. Peter installs a new, empty tank to catch rainwater from his roof, which has a total area of 200 m^2. He installs a diverter, which directs the first 400 L of rainfall water onto his garden, and directs the rest of the water into his tank. When rain starts falling, Peter's roof contains 5 g/m^2 of pollutants. Assume that each mm of rain will wash off 20% of the remaining pollutants.

Let r be the amount of rain that falls in a rain storm, measured in mm. Then let $P(r)$ be the total mass of pollutants on Peter's roof after rainfall r, $G(r)$ be the total mass of pollutants diverted onto his garden, and $T(r)$ be the total mass of pollutants directed into his tank, all measured in grams, g.

(a) Explain briefly why P, G and T are functions of rainfall r, rather than time. (2 marks)

(b) Demonstrate why $P(0) = 1000$, $G(0) = 0$ and $T(0) = 0$. (2 marks)

(c) Demonstrate clearly why the following system of differential equations approximately models the values of $P(r)$, $G(r)$ and $T(r)$ when it commences raining.

$$P' = -0.2P \qquad G' = 0.2P \qquad T' = 0.$$

(3 marks)

(continued over)

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(d) Recall that the system of equations from Part (c) is:

$$P' = -0.2P \qquad G' = 0.2P \qquad T' = 0$$

with $P(0) = 1000$, $G(0) = 0$ and $T(0) = 0$. Use **two steps** of Euler's method, with a stepsize of 1 mm, to demonstrate that after 2 mm of rain has fallen, there will be 640 g of pollutants remaining on Peter's roof, 360 g diverted to his garden, and none in his tank.

(5 marks)

(e) Explain clearly why the system of differential equations in Part (d) does not apply for any rainfall after the first 2 mm. (Hint: 1 mm of rain is equivalent to 1 L/m^2.)

(2 marks)

(f) Write a system of differential equations for pollutants similar to the equations in Part (c), but instead for rainfall amounts larger than 2 mm. What are the values of $P(2)$, $G(2)$ and $T(2)$?

(5 marks)

(continued over)

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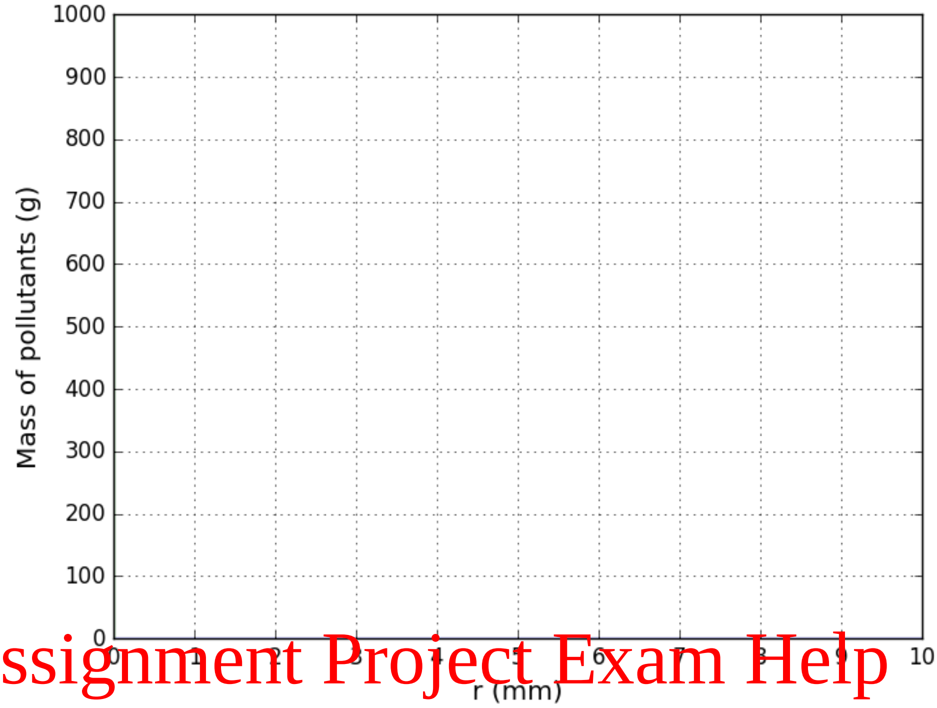
(g) Sketch rough graphs of $P(r)$, $G(r)$ and $T(r)$ on the following axes, for rainfall amounts between $r = 0$ mm and $r = 10$ mm. Take care to identify each graph clearly. (Hint: exact values are not important for $r > 2$; instead show the shapes of the graphs.)

(6 marks)



(h) Describe briefly how the graphs in Part (g) would change in each of the following cases. Treat each case as being separate.

(i) Rather than diverting the first 400 L onto the garden, instead the first 600 L is diverted. (4 marks)

(ii) Heavy rainfall two days earlier meant that a lower mass of pollutants had accumulated. (4 marks)

(i) Research suggests that keeping pet animals can reduce levels of stress. Draw a picture of a happy, friendly puppy. (1 mark)

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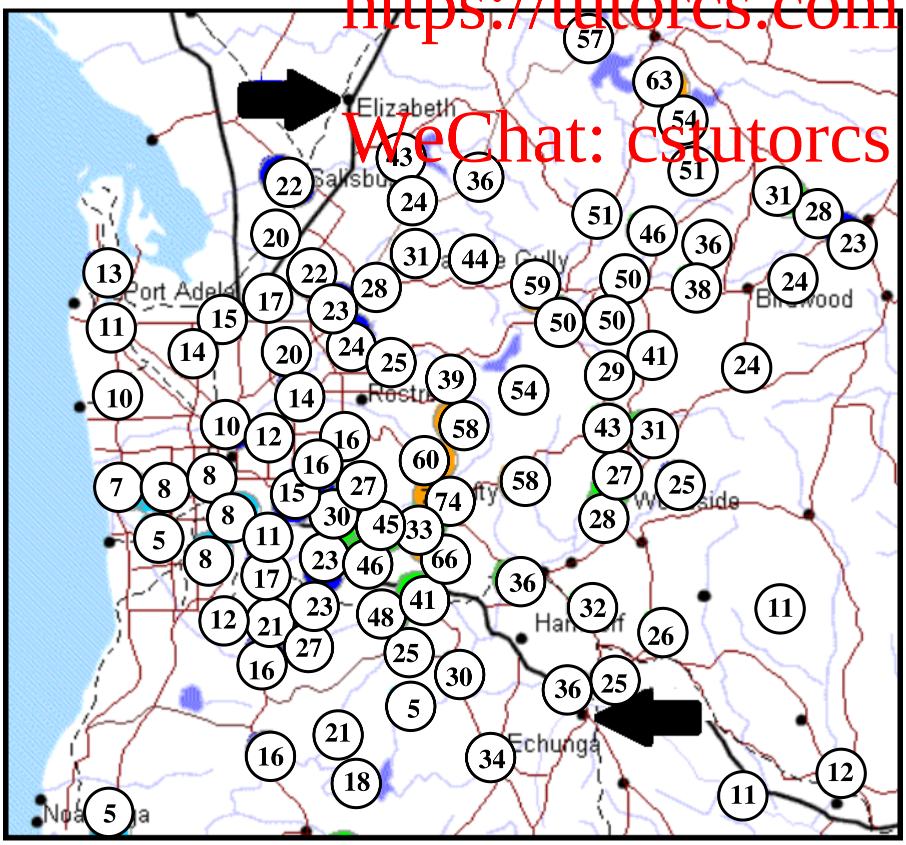
Some data: The city of Adelaide has a population of about 1.25 million. On average, each person in Adelaide uses 0.2 m^3 of water per day, which is 200 L per day.

Measuring rainfall amounts: Rainfall in Australia is often measured in millimetres (mm), representing the depth to which that amount of rainfall would cover an area if none of the fallen water evaporated, soaked in or ran off as stormwater. For example, a rainfall of 10 mm means that an open-topped container with vertical sides would accumulate rainfall water to a depth of 10 mm.

Stormwater runoff models: When rain falls on a hard surface, such as a concrete car park or a building roof, stormwater runoff can carry a significant load of pollutants into surrounding waterways. These pollutants can include hydrocarbons, nitrogen and suspended solids (such as fine dirt particles). *Stormwater runoff models* can be used to predict and hence better manage the quantity of pollutants that are involved. Such models typically involve two components: *accumulation* of pollutants on the hard surface between rainfall events; and *wash-off* of the pollutants during rainfall.

Household rainwater tanks: When rainwater is collected into a water tank from a house roof, the collected water will include pollutants washed off the roof. When rain starts falling, a *diverter* can be used to direct the first runoff water onto a garden, and only direct subsequent runoff water into the tank.

Adelaide rainfall event: In the 24 hours to 9am on September 30th 2016, heavy rain fell on the city of Adelaide. The following map shows locations of weather stations around Adelaide (marked as circles) and the rainfall recorded at that weather station in mm (the number inside each circle). Where there are no weather stations, any rainfall amount is unknown, but can be estimated from amounts at nearby stations.



Two locations are highlighted by arrows on the map: Elizabeth (near the top left) and Echunga (near the bottom right). The distance from Elizabeth to Echunga in a straight line is 60 km. You may assume that the map is drawn to scale.

Evapotranspiration: *Evapotranspiration* is the name given to the transfer of water from the surface of Earth to the atmosphere. It includes both *evaporation* of water from soil and waterbodies, and *transpiration* of water as water vapour from plants.

END OF EXAMINATION

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