Lecture 6: **Studying the heart**

Learning objectives

✓ Develop the ability to theoretically derive models

✓ Analyse the effect that a change in one physical factor has on another

Scientific examples

✓ Blood flow through a blood vessel

✓ Hagen-Poiseuille equation from fluid dynamics

✓ Angioplasty

Maths skills

✓ Develop a plausible equation for a model

✓ Calculate the percentage increase / decrease of a variable

3.3 Modelling in action

fluids

- *Fluid dynamics* involves studying liquids and gases that are moving, which is important in many branches of science and engineering.

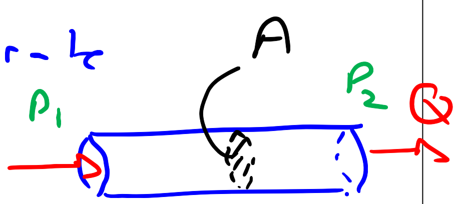
Case Study 2: **Let it flow**

volume

***Question** 3.3.1* $\Delta P = P_1 - P_2$

Develop a model for the flow rate (amount per unit time) of blood through a given blood vessel. (Hint: which factors are important; do they increase or decrease the rate?)

Use Q = volume flow rate



Depends on

Friction / viscosity μ $\mu \uparrow$ $Q \downarrow$

Area / diameter / radius $A = \pi R^2$ $A \uparrow$ $Q \uparrow$

Pressure difference ΔP $\Delta P \uparrow$ $Q \uparrow$

Length L $L \uparrow$ $Q \downarrow$

***Question* 3.3.1** *(continued)*

Plausible

$$Q = \frac{A \Delta P}{\mu L} = \frac{\pi R^2 \Delta P}{\mu L}$$

$$Q = A + \Delta P - \mu - L$$

$$Q = \pi R^2 \Delta P - \mu L$$

Dimensions?

"left-hand side" $\quad LHS = Q \Rightarrow \frac{mL}{s} \Rightarrow \frac{m^3}{s}$

$$RHS = \frac{\pi R^2 \Delta P}{\mu L} \Rightarrow \frac{m}{s}$$

Dimensions "[illegible]" correct $\Rightarrow$ not full story

The following formula (called the *Hagen-Poiseuille equation*) is often used to estimate such flows:

$$Q = \frac{\pi R^4 \Delta P}{8 \mu L}$$

Compare your formula with the Hagen-Poiseuille equation.

close – missing R^2 and $1/8$

- High levels of certain types of cholesterol in the blood can lead to blockages in coronary arteries, which can eventually lead to a heart attack.
- During a heart attack, a lack of blood supply causes heart muscle tissue to die and the dead tissue is replaced with scar tissue.

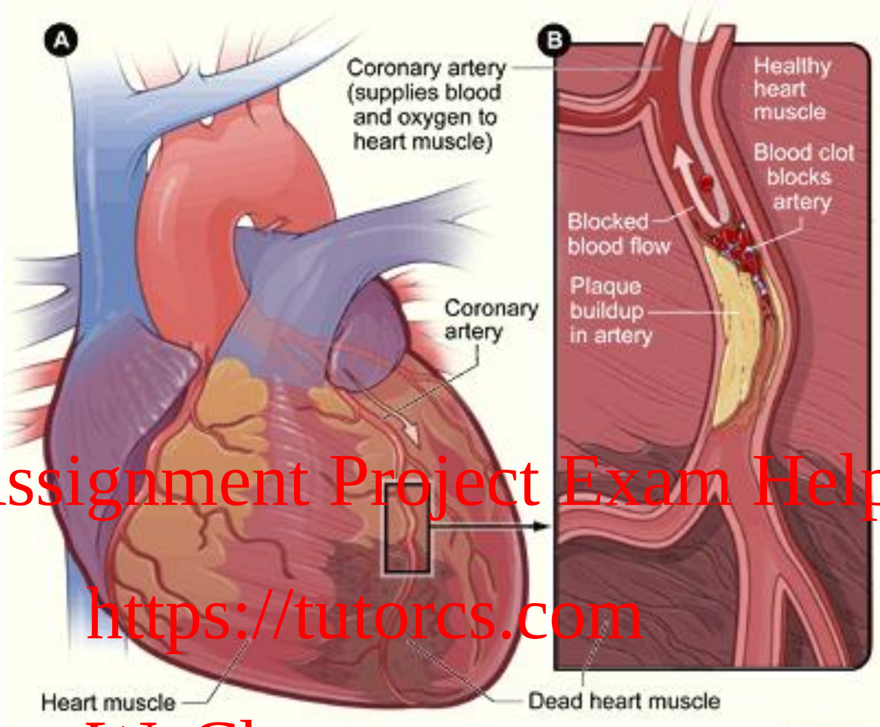


Figure 3.2: Left: heart and coronary artery showing dead heart muscle caused by a heart attack. Right: longitudinal section of a coronary artery with plaque build-up and a blood clot. (Source: www.nhlbi.nih.gov.)

- One surgical method of increasing blood flow through partially blocked arteries is an *angioplasty*.
- In a coronary angioplasty, a cardiologist inserts a balloon-tipped catheter under local anaesthetic, typically through the groin or arm.
- When the catheter is correctly positioned within the coronary artery, the doctor inflates the balloon to expand the blood vessel (and sometimes inserts a metallic stent to maintain the expansion).
- Angioplasties are much simpler and less invasive than coronary artery bypass surgery, but have a higher rate of recurrence of the original occlusion.

Question 3.3.2

Assume that a patient undergoing an angioplasty procedure shows a 30% increase in the diameter of a partially occluded artery. Use the Hagen-Poiseuille equation to calculate the resulting percentage increase in blood flow rate through that artery, and interpret your answer.

Given

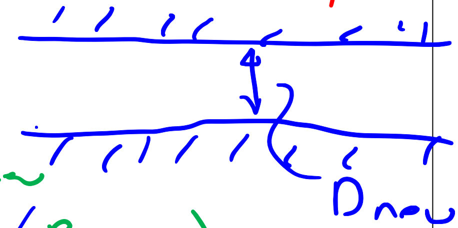
$$\boxed{D_{new} = D_{old} + 0.3\,D_{old}} \quad \text{"old"}$$

$$= 1.3\,D_{old}$$

$$Q = \frac{\pi R^4 \Delta P}{8\mu L}$$

assume $\Delta P, \mu, L$ same for old & new

"new"



$$R_{new} = D_{new}/2 \Rightarrow D_{new} = 2R_{new}$$

$$(\cancel{2}R_{new}) = (\cancel{2}R_{old}) + 0.3(\cancel{2}R_{old})$$

$$R_{new} = R_{old} + 0.3\,R_{old} = 1.3\,R_{old}$$

$$\frac{Q_{new}}{Q_{old}} = \frac{\cancel{\pi} R_{new}^4 \cancel{\Delta P} / \cancel{8\mu L}}{\cancel{\pi} R_{old}^4 \cancel{\Delta P} / \cancel{8\mu L}} = \left(\frac{R_{new}}{R_{old}}\right)^4$$

$$= \left(\frac{1.3\,\cancel{R_{old}}}{\cancel{R_{old}}}\right)^4 = 1.3^4 \simeq 2.9$$

$$Q_{new} = 2.9\,Q_{old}$$

Percentage increase in blood flow?

$$Q_{new} = Q_{old} + 1.9\,Q_{old}$$

new   old   change

The change is $1.9\,Q_{old}$ or 190% increase

Large change as $Q \propto R^4$

End of Case Study 2: Let it flow.