THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Student Number |__|__|__|__|__|__|__|__|

Family Name ____________________

First Name ____________________

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School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2020

SCIE1000 Theory and Practice in Science

This paper is for all students.

Examination Duration: You have a 24 hour window in which you must complete your exam. You can access and submit your exam at any time within the 24 hour window. Even though you have the entire 24 hours to complete and submit your exam, the expectation is that it will take most students around 2 hours to complete. The exam will remain open **only** for the duration shown.

Materials Permitted While Completing The Exam:

- Calculators – any non-graphical calculator is permitted.
- The SCIE1000 lecture book, workshop activities & solutions, and your personal notes from the course are permitted.
- One unmarked bilingual dictionary is permitted.

You may not make use of any other material. This includes web-sites, books, or any other material or software. (No electronic aids are permitted e.g. laptops, phones except for annotating this exam paper.)

Instructions To Students: Answer all questions on the exam paper in the spaces provided or on your own paper. You can print the exam and write on the exam paper, or write your answers on blank paper, or write electronically on a suitable device. Scan or photograph your work if necessary and upload your answers to Blackboard as a single pdf file.

Who to Contact: Since students may not all undertake the online exam at the same time, or in the same time zone, and that some questions may be randomised, responding to student queries and/or relaying corrections to exam content during the exam will not be feasible. Course coordinators will not be able to respond to academic queries during the exam.

If you have any concerns or queries about a particular question, or need to make any assumptions to answer the question, state these at the start of your solution to that question. You may also include queries you may have made with respect to a particular question, should you have been able to 'raise your hand' in an examination room. If you experience any technical difficulties during the exam, contact the Library AskUs service via the Live Chat or Phone for advice (open 7:00am – 10:00pm AEST every day during the final exam period). You should ask the library staff for an email documenting the advice provided so you can provide this to the course coordinator.

Certification (must be signed before submission):
I certify that my submitted answers are entirely my own work and that I have neither given nor received any unauthorised assistance on this assessment item.

Signed:________________________________Date:______________________

To answer each question you will need to use the information on Page 18. Your solutions will be marked on the correctness and clarity of your explanation and communication. Include units in your answer wherever relevant.

1. Exposure to high seawater temperatures causes coral bleaching; that is, the expulsion of *zooxanthellae* from the coral structure. In [8], the authors found that most zooxanthellae exhibited normal characteristics at temperatures between 28 °C and 30 °C, but after 12 hours at 30 °C, there was an indication of stress and potential cell death. For this question, we will therefore assume that 30 °C is the threshold beyond which zooxanthellae cannot survive.

   (a) Calculate the total exposure to temperatures above the survivable threshold experienced by these zooxanthellar cells in the following quote from [8]:

   > "*Within 6 hrs at 32° C, zooxanthellar cells showed symptoms of late-stage apoptosis... including organelle fragmentation... and cell disintegration.*"

   (2 marks)

   (b) Consider an 18 hour experiment in which a collection of zooxanthellar cells experienced varying water temperatures. The following measurements show the temperature at various times, where time is measured in hours since the experiment began.

time (h)	0	3	7	10	15	18
temperature (°C)	30.0	30.0	32.5	33.0	33.5	31.5

(question continued over)

(i) Graph the data provided in the table on the previous page; ensure that your graph is clearly communicated. Your graph should clearly demonstrate temperatures experienced by the zooxanthellar cells during this 18 hour experiment and the survivable threshold should also be clearly indicated. Assume that the temperature *changed linearly* between recorded measurements.

[You may use the axes below, or hand draw your own axes (if working on a separate paper)].

(2 marks)

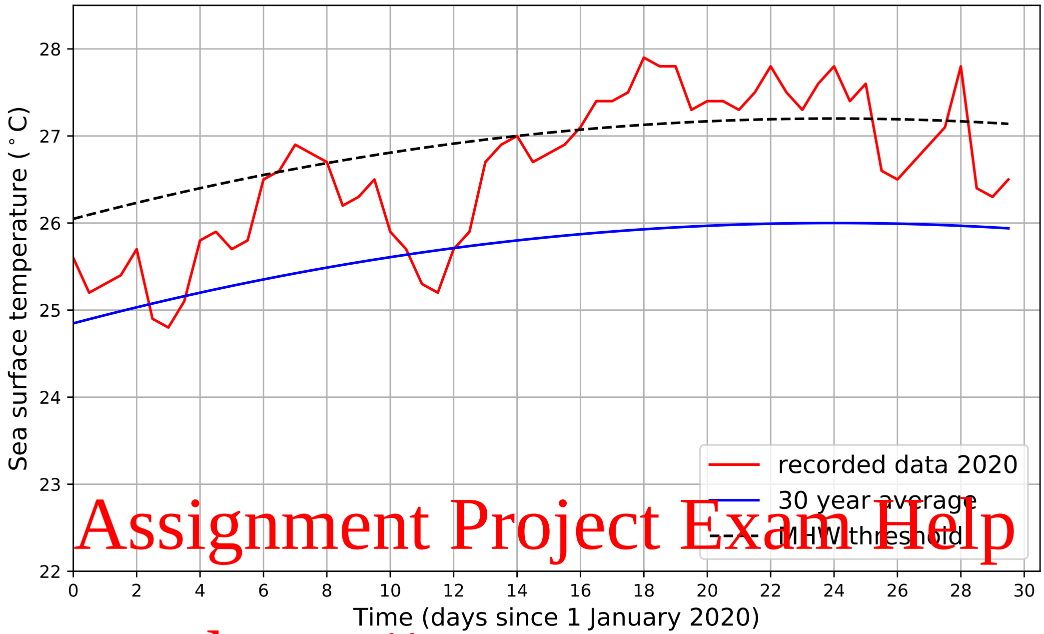
(ii) Calculate the total exposure to temperatures above the survivable threshold experienced by the zooxanthellar cells in this 18 hour experiment. Show all of your working.

(3 marks)

Do not write here

2. Marine heat waves (MHWs) have been studied by many researchers using several different definitions. The authors of [3] set out a unifying definition of a marine heat wave (see Page 18).

   Suppose that the following data represents ocean temperatures in a hypothetical coral reef, called Reef X, measured at 12 noon each day in January 2020.



   (a) Note that in the above figure, $t = 0$ days represents January 1st. According to the figure, what is the 30 year average sea surface temperature for the date of January 9th?

   (1 mark)

   (b) Suppose that you wanted to model the 30-year average sea surface temperature for Reef X using a mathematical function. What type of function would you use? Why?

   (2 marks)

(question continued over)

(c) How many marine heat waves occurred in Reef X during January 2020? How long did each one last for?

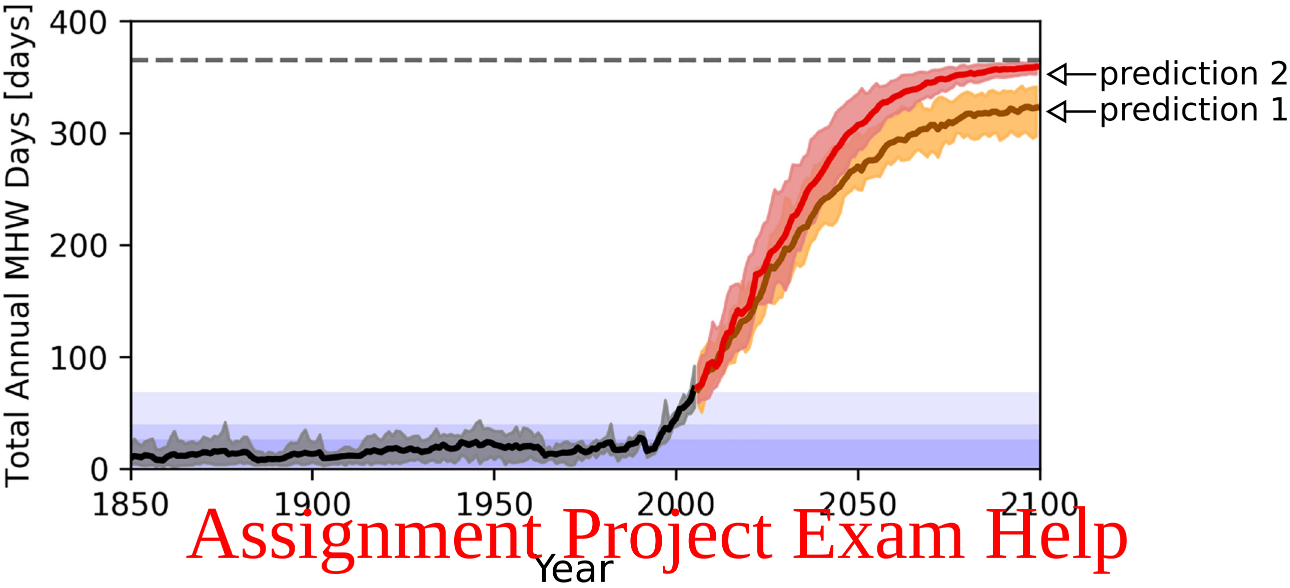
(2 marks)

(d) Describe what a marine heat wave is in a manner that would be well-understood by the general public in Australia.

(2 marks)

Do not write here

3. In [6] the authors predict the total number of days of the year in which there is at least one part of the world experiencing a MHW. The graph below comes from [6] and contains historical data on MHWs until the year 2005, followed by two predictions (prediction 1 and prediction 2). The shadings near the lines in the graph show the uncertainties in the measurements and modelled predictions, and the blue shading shows the "expected range of natural variability" [6].



(a) Suppose that another researcher, trying to construct a simpler model that recreates prediction 1, defines a function $T(t)$, where T is the total annual MHW days and t is the time measured in *years since 1950*. Their function is a reasonably good fit to the existing data (starting at the year 1950) as well as the total annual MHW days from prediction 1 (for the years 2005 to 2100).

(i) Estimate a reasonable value for $T(0)$ based on the graph and explain the physical significance of the value in this context.

(1 mark)

(ii) The researcher determines that the rate of change for their function will have the form

$$T'(t) = 0.04\,T\left(1 - \frac{T}{L}\right).$$

Estimate the value for L and explain its physical significance in this context.

(2 marks)

(question continued over)

(iii) A student working with the researcher suggests that a slightly better fit could be achieved by adjusting some of the parameters. They demonstrate that either one of the following changes does indeed result in a slightly better fit:

- changing constant 0.04/year to 0.045/year OR
- setting $L = 375$ days.

Which one of these changes would be better to implement? Why? Relate your answer to the notion of phenomenological and mechanistic modelling.

(2 marks)

(b) Suppose that in the future, the total number of days of the year in which there are MHWs observed for the year 2050 is approximately 200, which is less than the predicted value for 2050. Would this mean that the model in part (a) is falsified? Why or why not? Use the concepts covered in the philosophy of science component of SCIE1000 to justify your answer.

(2 marks)

Do not write here

4. The concentrations of hydrogen ions and carbonate ions can each be modelled in terms of the aqueous carbon dioxide of seawater using a power function. That is, if H is the concentration of hydrogen ions, C is the concentration of carbonate ions and X is the concentration of aqueous carbon dioxide, where all concentrations are measured in $\text{mol} \cdot \text{kg}^{-1}$ of sea water, then

$$H = aX^b \quad \text{and} \quad C = mX^p$$

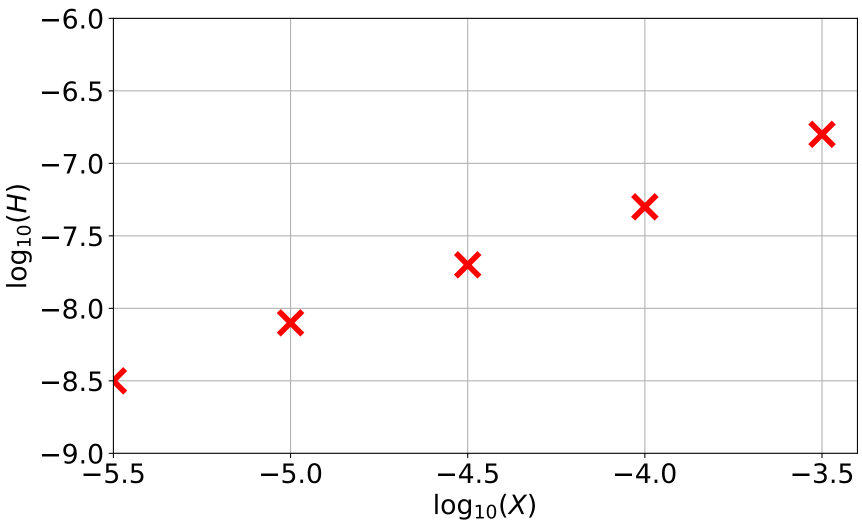
where a, b, m and p are constants. In this question, we will construct one of these models; namely the model for hydrogen ion concentration as a function of aqueous carbon dioxide concentration.

(a) Using the definition of ocean acidification from Page 18, for each of the constants in the above models, determine whether they must by positive or negative. Justify your answers. [If you are writing your answers on a separate paper, hand draw this table to organise your answers.]

(2 marks)

constant	positive or negative	justification
a		
b		
m		
p		

(b) The following figure is adapted from [4] and shows a log-log plot for concentration of hydrogen ions against concentration of aqueous carbon dioxide in seawater (that is, $\log_{10} H$ against $\log_{10} X$).



(question continued over)

(i) Find a linear equation that models the data in the plot on the previous page. Show all of your working. [You do not need to include units here.]

(2 marks)

(ii) Use the linear model you obtained in part (i) to calculate values for the constants a and b (for the model $H = aX^b$). Show all of your working and reasoning. [You do not need to include units here.]

(3 marks)

Do not write here

5. *Fungia scutaria* is a common species of Hawaiian reef coral. The effects of increased ocean temperatures on this species were studied in [5]. Using data from that paper, a scientist has implemented a model in the form of the following Python program. The program is shown on pages 10 and 11. The output from one run of the program is shown at the top of page 12.

   Recall that, in Python, a backslash (\ ) is a continuation character. It is placed at the end of a line to indicate that the current command continues on the next line.

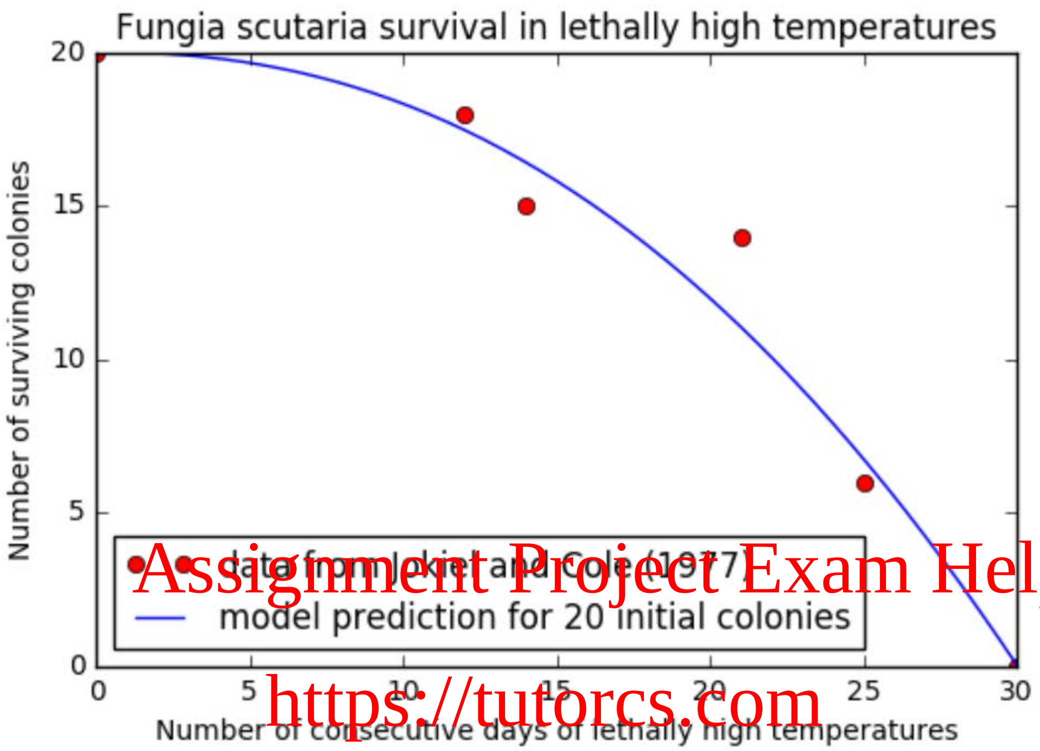
```
from pylab import *

# a model predicting the proportion of Fungia scutaria colonies
# surviving after t days of lethally high temperatures
def p(t):
    return 1 - (t/30)**2.25

# the derivative of the model above
def p_dash(t):
    return -2.25/30*(t/30)**1.25

# COMMENT TO GO HERE
def compute_number_days(surviving_proportion):
    t = 10
    i = 0
    while i < 2:
        t = t - (p(t) - surviving_proportion)/p_dash(t)
        i = i+1
    return t


# data from Jokiel and Cole (1977)
times_data = array([0, 12, 14, 21, 25, 30])
colonies_data = array([20, 18, 15, 14, 6, 0])

# make some arrays to store the computed data
ss = 0.1
times_model = arange(0, 30+ss, ss)
N = size(times_model)
colonies_model = zeros(N)

# use the model to populate the colonies_model array
initial_colonies = colonies_data[0]
i = 0
while i < N:
    colonies_model[i] = initial_colonies*p(times_model[i])
    i = i + 1

# Provide information about how model was made, including a plot of the data
# and the corresponding model prediction to show the fit
print("Jokiel and Cole (1977) reported data from an experiment in which 20\
 colonies")
print("of Fungia scutaria were exposed to lethally high temperatures. Using\
```

```
 that ")
print("data, we have developed a model to predict the proportion of initial\
 colonies")
print("surviving after t days of lethal temperatures.")

plot(times_data, colonies_data, "ro", label="data from Jokiel and Cole (1977)")
plot(times_model, colonies_model, "b-", label="model prediction for 20 initial\
 colonies")
title("Fungia scutaria survival in lethally high temperatures")
xlabel("Number of consecutive days of lethally high temperatures")
ylabel("Number of surviving colonies")
legend(loc="lower left")
show()

# Instruct user, get data and make prediction
print("If you know the number of Fungia scutaria colonies at a site both\
 before and")
print("after a suspected MHW event, you can use our model to estimate the\
 number of")
print("consecutive days that the site was exposed to lethally high\
 temperatures.\n")

initial_value = float(input("Enter the number of colonies at the site before\
 the suspected MHW event: "))

observed_value = float(input("Enter the number of colonies at the site after\
 the suspected MHW event: "))

surviving_proportion = observed_value/initial_value
prediction = compute_number_days(surviving_proportion)

print("\nOur model predicts that the site was exposed to",\
 round(prediction, 1), "consecutive days of")
print("lethally high temperatures.")
```


(question continued over)

The output from one run of the program is shown in the box below.

```
Jokiel and Cole (1977) reported data from an experiment in which 20 colonies
of Fungia scutaria were exposed to lethally high temperatures. Using that
data, we have developed a model to predict the proportion of initial colonies
surviving after t days of lethal temperatures.
```




```
If you know the number of Fungia scutaria colonies at a site both before and
after a suspected MHW event, you can use our model to estimate the number of
consecutive days that the site was exposed to lethally high temperatures.

Enter the number of colonies at the site before the suspected MHW event: 383
Enter the number of colonies at the site after the suspected MHW event: 234

Our model predicts that the site was exposed to 20.7 consecutive days of
lethally high temperatures.
```

(a) The commenting of the code is incomplete. Use the space below to write a comment that could go in place of the line

```
# COMMENT TO GO HERE
```

(2 marks)

Do not write here

(b) Let p denote the proportion of *Fungia scutaria* colonies that will survive t days of lethal temperatures. Write down, in the usual mathematical notation, the model used in the program to predict p.

(1 mark)

(c) Use your answer to (b) to determine the number of days at lethal temperatures that would result in a proportion of 234/383 *Fungia scutaria* colonies surviving.

(2 marks)

(d) Compare the output of the program with your answer to (c). Comment on the comparison, and make any suggestions to the programmer that you think may be appropriate.

(2 marks)

Do not write here

6. In [7], the authors "...report on a simple population dynamic model that indicates that fish predators could in theory help regulate COTS population levels..." They consider a system of differential equations that models the interactions between coral, crown-of-thorns starfish (COTS), and some fish that are predators of the COTS. Here is a simplified version of the model:

$$C' = s_1 C \left(1 - \frac{C}{s_2}\right) - s_3 C A \tag{1}$$

$$A' = s_4 A \left(1 - \frac{A}{s_5 C}\right) - s_6 A F \tag{2}$$

$$F' = s_7 F \left(1 - \frac{F}{s_8 A}\right) \tag{3}$$

In the model: t represents time in years; C, A and F represent the normalised population density of corals, COTS and fish that prey on COTS, respectively; and $s_1, \ldots, s_8$ are constants to be determined from data. The population densities can be thought of as a proportion of a theoretical maximum, so C, A, F are unitless and must be between 0 and 1. The constants s_2, s_5 and s_8 are unitless, and the constants s_1, s_3, s_4, s_6, and s_7 have units year^{-1}.

(a) A reader of the paper interprets Equation (3) of the model as follows:

> The model predicts logistic growth for the population density of predator fish, but with a carrying capacity that is proportional to the population density of COTS.

Give a similar interpretation of Equation (2) of the model.

(2 marks)

(b) Use two iterations of Euler's method, with a step size of 0.1 years, applied to the model to predict the population densities of coral and COTS when $t = 0.2$ years. You should use the parameter values

$$s_1 = 12\,\text{year}^{-1}, \quad s_2 = 1, \quad s_3 = 37\,\text{year}^{-1}, \quad s_4 = 2\,\text{year}^{-1},$$
$$s_5 = 1, \quad s_6 = 25\,\text{year}^{-1}, \quad s_7 = 0.3\,\text{year}^{-1}, \quad s_8 = 1.$$

and initial conditions

$$C(0) = 0.60, \qquad A(0) = 0.10, \qquad F(0) = 0.00.$$

(Note that the condition $F(0) = 0.00$ is equivalent to assuming that there are no predator fish).

(3 marks)

(question continued over)

(Use this page for your working for question 6 (b)).

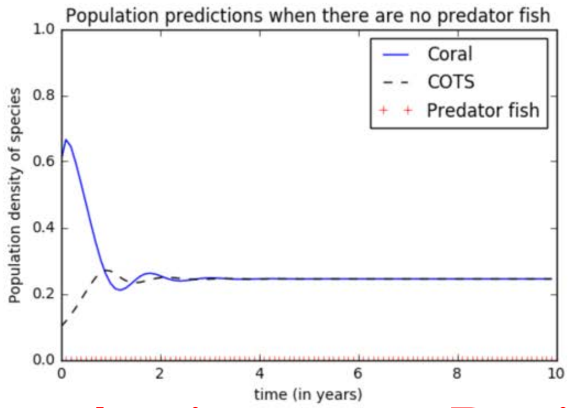
(question continued over)

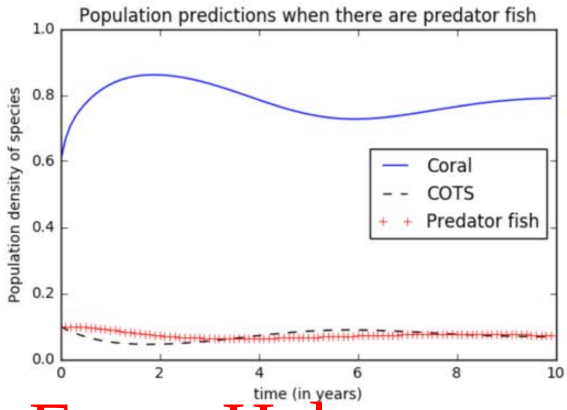
(c) The following graphs were produced by a Python program that approximates the functions $C(t), A(t)$ and $F(t)$ described in the model. The program uses Euler's method with a step size of 0.1 years, and the parameter values described in (b). The first graph is produced using the initial conditions

$$C(0) = 0.60, \quad A(0) = 0.10, \quad F(0) = 0.00,$$

and the second using the initial conditions

$$C(0) = 0.60, \quad A(0) = 0.10, \quad F(0) = 0.10.$$





Based on these graphs, and any other information available in this problem, briefly describe the scientific work that is done in [7]. Your audience is a UQ student who is about to begin SCIE1000.

(2 marks)

END OF EXAMINATION

Do not write here

This page can be used if you need extra space to answer any of the questions. Clearly indicate the question(s) that you are answering. Ensure that you also put a note within the body of the relevant question(s). Only scan or photograph this page if you write answers on it.

You do not need to scan or photograph any of the following pages - they will not be graded.

(information and formula sheets over)

Information on corals

Coral reefs: Corals are marine invertebrates which typically grow in large underwater ecosystems called coral reefs. Most corals require sunlight to grow and are found at shallow depths of the ocean.

Coral bleaching: Coral bleaching occurs when corals expel an algae, known as *zooxanthellae*. The zooxanthellae are normally hosted by the coral in a symbiotic relationship, and they are what contribute the colour of corals. The loss of zooxanthellae means a loss in pigmentation, and the resulting coral appears to be white or "bleached". Corals derive much of their energy from the symbiotic relationship with zooxanthellae, and coral bleaching can often lead to the death of the coral structure itself. Coral bleaching is largely caused by rising temperatures of seawater; corals themselves appear to be more resilient to increases in seawater temperature, but the zooxanthellae are not [8].

Ocean acidification: Rising carbon dioxide levels in the atmosphere have resulted in rising levels of aqueous carbon dioxide and hydrogen ion concentrations in seawater, as well as a decrease in carbonate ion concentrations [4]; this effect is called *ocean acidification*. Corals skeletons are formed of calcium carbonate and are affected by ocean acidification. The authors of [4] remark "*Ocean acidification is potentially one of the greatest threats to marine ecosystems and global carbon cycling.*"

Marine heat waves: A marine heat wave (MHW) is defined as an anomalously warm event in the sea surface temperature "lasting five days or more, with temperatures warmer than the 90th percentile based on a 30-year historical baseline period" [3]. Thus the 90th percentile for the 30-year historical baseline measurements is the *MHW threshold*. Historical data is based on the particular location and the time of the year. A value from a data set which is above the 90th percentile is one which is greater than 90% of the values in the data set.

Crown-of-thorn starfish: "Crown-of-thorns starfish (also known as COTS) are marine invertebrates that feed on coral. They occur naturally on reefs throughout the Indo-Pacific region, and when conditions are right, they can reach plague proportions and devastate hard coral communities" [1].

References

[1] Crown-of-Thorns Starfish, Australian Institute of Marine Science, Australian Government, `https://www.aims.gov.au/docs/research/biodiversity-ecology/threats/cots.html`, accessed 16 April 2020.

[2] Coral reproduction, Great Barrier Reef Marine Park Authority, Australian Government, `http://www.gbrmpa.gov.au/the-reef/corals/coral-reproduction`, accessed 16 April 2020.

[3] Hobday *et al.* (2016) A hierarchical approach to defining marine heatwaves, *Progress in Oceanography.* **141** 227–238.

[4] Irie T., Bessho K., Findlay H.S., and Calosi P. (2010) Increasing costs due to ocean acidification drives phytoplankton to be more heavily calcified: optimal growth strategy of coccolithophores. *PLoS ONE* **5**: e13436.

[5] Jokiel P., and Cole S. (1977). Effects of temperature on the mortality and growth of Hawaiian reef corals. *Marine Biology* **43** pp. 201-208.

[6] Oliver *et al.* (2019) Projected Marine Heatwaves in the 21st Century and the Potential for Ecological Impact, *Front. Mar. Sci.* **6**, 734.

[7] Ormond R., Bradbury R., Bainbridge S., Fabricius K., Keesing J., Devantier L., Medlay P. and Steven A. (1990) Test of a Model of Regulation of Crown-of-Thorns Starfish by Fish Predators *Lecture Notes in Biomathematics* **88 (01)** 189-207.

[8] Sammarco P.W. and Strychar K.B. (2013) Responses to high seawater temperatures in zooxanthellae Octocorals, *PLoS ONE* **8 (2)** e54989.

(formula sheet over)

SCIE1000 / SCIE1100 Formula Sheet

Base quantity	SI unit name	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^1	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

Quantity	Name	Symbol	SI units	SI base units
frequency	hertz	Hz	-	s^{-1}
force	newton	N	-	$m \cdot kg \cdot s^{-2}$
pressure, stress	pascal	Pa	$N \cdot m^{-2}$	$m^{-1} \cdot kg \cdot s^{-2}$
energy, work, quantity of heat	joule	J	$N \cdot m$	$m^2 \cdot kg \cdot s^{-2}$
power, radiant flux	watt	W	$J \cdot s^{-1}$	$m^2 \cdot kg \cdot s^{-3}$
electric potential difference, electromotive force	volt	V	$W \cdot A^{-1}$	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
Celsius temperature	degree Celsius	°C	-	K

function type	general form
linear	$y = mx + c$
quadratic	$y = ax^2 + bx + c$
power	$y = ax^p$
periodic	$y = A\sin(\frac{2\pi}{P}(t - S)) + E$
exponential	$y = Ce^{kt}$
surge	$y = at^p e^{-bt}$

Newton's method $\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$

Trapezoid rule $\boxed{A_{trap} = (x_2 - x_1)(\frac{y_1+y_2}{2})}$

Euler's method $\boxed{t_{i+1} = t_i + h \qquad y_{i+1} = y_i + hy_i'}$

True Status

	Yes	No
Test Positive	A	B
Test Negative	C	D

$$N = A + B + C + D$$

$$\text{accuracy} = \frac{A + D}{N}$$

$$\text{sensitivity} = \frac{A}{A + C}$$

$$\text{specificity} = \frac{D}{B + D}$$

Lotka-Volterra model

$$Q' = aQ - bPQ$$
$$P' = -cP + dPQ$$

SIR model

$$S' = -a\frac{S}{N}I$$
$$I' = a\frac{S}{N}I - bI$$
$$R' = bI$$