Lecture 25: Eat before you drink

Learning objectives

✓ Analyse the shapes of graphs and the effect of different constants

✓ Use the meaning of derivatives and to understand and interpret shapes of graphs

Scientific examples

✓ Alcohol exposure on a full vs empty stomach

Maths skills

✓ Graph functions

✓ Using derivatives to find maxima or minima

Case Study 20: Drink deriving

- In practice (particularly in legal cases), models of BAC use the *Widmark formula*, developed in 1932. The equation is:

$$B = \frac{A}{rM} \times 100\% - Vt$$

where B is the BAC at time t since commencing drinking, A is the amount of alcohol consumed in grams, V is the rate at which the body eliminates alcohol measured in % per time period, M is the body mass in grams and the *Widmark factor* r estimates the proportion of body mass that is water.

- The precise value of r depends on factors such as sex, age and percentage body fat. Reasonable estimates are $r \approx 0.7$ for adult males and $r \approx 0.6$ for adult females. The typical value for V is 0.015 % hr^{-1}.

Question 9.3.6

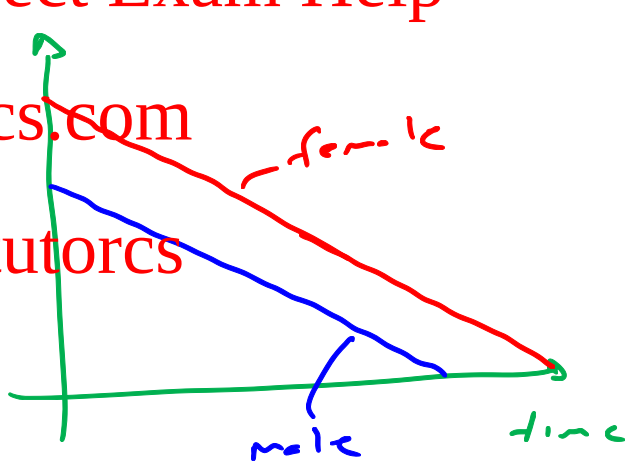
(a) What is the physical meaning of the term rM in the Widmark formula?

r - proportion of the body that is water
M - mass of the person

↳ rM is the mass of water in the body (in grams)

(b) The Widmark formula is: $B = \frac{A}{rM} \times 100\% - Vt$. Sketch rough graphs of B against time for a "typical" male and a "typical" female who each consume the same amount of alcohol (that is, assume A is constant).

Similar assumptions:
A - same amount of alcohol
V - same rate (starve)



Different male/female
r: $r_{male} > r_{female}$
M: $M_{male} > M_{female}$
$(rM)_{male} > (rM)_{female}$

Question 9.3.6 (continued)

(c) Use the Widmark formula to justify Australian government guidelines that, to remain below the legal driving BAC limit, within the first hour "men should drink at most two drinks and women at most one".

0.05%.

(standard drink)

$B = \frac{A}{rM} \times 100\% - Vt$

Interested in B_{max} $\rightarrow$ $t = 0$

$A = 10n$

$B_{max} = \frac{10n}{rM} \times 100\%$ $M = 80000\,g$

Male - $M \sim 80kg$, $r = 0.7$

Find B_{max} for $n = 1, 2, 3$

$(B_{max})_{n=1} = \frac{10 \times 1}{0.7 \times 80000} \times 100 \simeq 0.018\%$

$(B_{max})_{n=2} = \frac{10 \times 2}{0.7 \times 80000} \times 100 \simeq 0.036\%$

$(B_{max})_{n=3} = \frac{10 \times 3}{0.7 \times 80000} \times 100 \simeq 0.054\%$

$\Rightarrow$ "safe" with $n = 2$, not 3

(cut-off $n = 2.8$)

Female - $M = 60kg = 60000g$, $r = 0.6$

$(B_{max})_{n=1} = \frac{10 \times 1}{0.6 \times 60000} \times 100 \simeq 0.028\%$

$(B_{max})_{n=2} = \frac{10 \times 2}{0.6 \times 60000} \times 100 \simeq 0.056\%$

$\Rightarrow$ "safe" with $n = 1$, not 2

(cut-off $n = 1.8$)

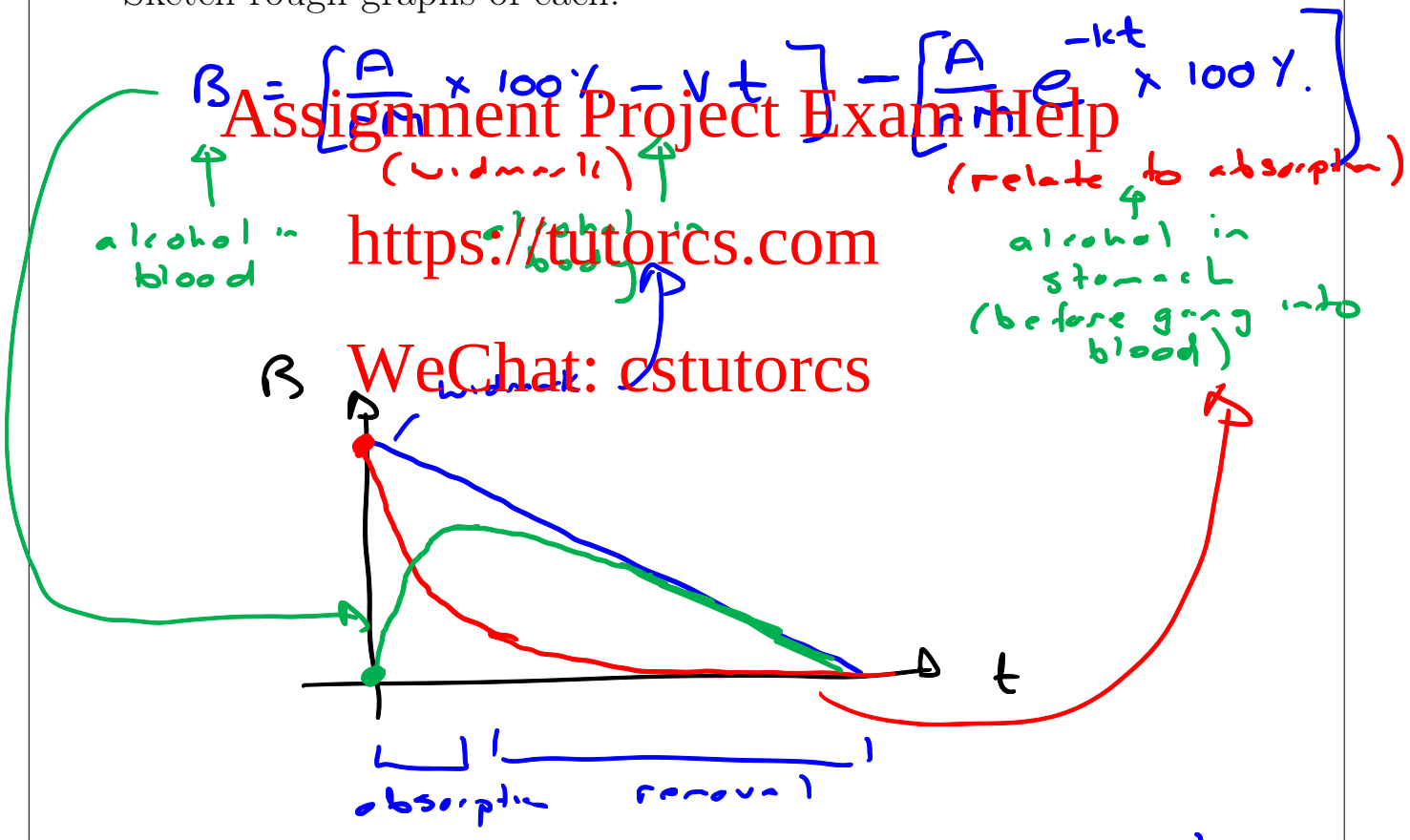
Question 9.3.7

In the Widmark formula, the absorption term assumes that the body absorbs alcohol **immediately** after consumption. The following variation, from [43], takes into account absorption time.

$$B = \frac{A}{rM} \times \left(1 - e^{-kt}\right) \times 100\% - Vt$$

where k is the rate at which the body absorbs alcohol.

(a) Expand this variation of the Widmark formula and compare it with the "standard" Widmark formula (which is $B = \frac{A}{rM} \times 100\% - Vt$.) Sketch rough graphs of each.



we previously assumed alcohol went straight into blood, now accounting for absorption (after drinking)

Question 9.3.7 (continued)

(b) Recall that $B = \dfrac{A}{rM} \times \left(1 - e^{-kt}\right) \times 100\% - Vt$.

If t is measured in hours, what are the units of k?

we know kt is unitless

t is in hours

k is in $\frac{1}{\text{hour}}$, per hour, hour^{-1}

(c) What factors could influence the value of k?

Food in stomach

size of stomach

very person-to-person

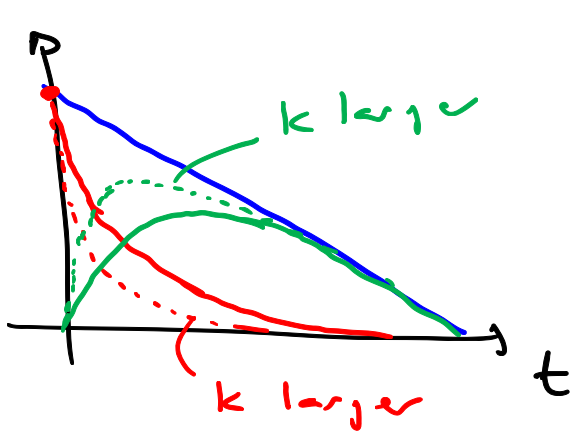
(d) Let t_{max} be the time at which BAC reaches its maximum value B_{max}. For larger values of k, will t_{max} be larger, smaller or unchanged? Why? Will B_{max} be larger, smaller or unchanged? Why?

Physical: k is rate of absorption into blood (after drinking)

$k \uparrow$ $e^{-kt} \downarrow$ $(1 - e^{-kt}) \uparrow$

B_{max} is larger

(show t_{max} earlier)



(e) Explain briefly how to find the values of t_{max} and B_{max}.

Find maximum, set $B' = 0$, solve for t_{max}

$B_{max} = B(t_{max})$

Question 9.3.7 (continued)

(f) When consuming alcohol with food in the stomach, $k \approx 2.3$/hr, but when the stomach contains no food, $k \approx 6$/hr. When a "typical" man of mass 80 kg consumes 4 standard drinks with food in his stomach, we have

$$B(t) \approx 0.0714(1 - e^{-2.3t}) - 0.015t$$

$$B'(t) \approx 0.164e^{-2.3t} - 0.015.$$

Find t_{max} and B_{max} for this man.

$B'(t) = 0 \Rightarrow 0.164e^{-2.3t} - 0.015 = 0$

$e^{-2.3t} = \frac{0.015}{0.164}$

$-2.3t = \ln\left(\frac{0.015}{0.164}\right)$

$t_{max} = \frac{-1}{2.3}\ln\left(\frac{0.015}{0.164}\right)$

≈ 1.0 hours

$B_{max} = B(t_{max})$

$= 0.0714(1 - e^{-2.3(1)}) - 0.015(1)$

$\approx 0.048\%$

(g) If the same man consumes the same amount of alcohol, but on an empty stomach, we have $t_{max} \approx 0.56$ hours and $B_{max} \approx 0.0605$ %. Compare this with your answer to Part (f), and relate this to Part (d).

$k = 6$/hr

	k (/hr)	t_{max} (hr)	B_{max} (%)
full	2.3	1.0	0.048
empty	6	0.56	0.0605

As $k \uparrow$, $t_{max} \downarrow$, $B_{max} \uparrow$ (as expected)

Program specifications: Write a program that plots BAC curves up to five hours after alcohol consumption on both a full and empty stomach, for men or women of varying masses and for various levels of alcohol consumption.

Program 9.2: BACs and food consumption

```
# Program to compare BACs for men and women of varying masses
# and levels of alcohol consumption, on full and empty stomachs.
from pylab import *

alcohol_mass = float(input("How many grams of pure alcohol consumed? "))
mass_person = float(input("Person's mass (in kg)? "))
sex_person = float(input("Type 1 if male, 2 if female: "))
if sex_person == 1:
    mult = 100 * alcohol_mass / (0.7 * mass_person * 1000)
else:
    mult = 100 * alcohol_mass / (0.6 * mass_person * 1000)
times = arange(0,5.1,0.1)

# Calculate the BAC at each time.
bac_full = mult * (1 - exp(-times * 2.3)) - 0.015 * times
bac_empt = mult * (1 - exp(-times *   6)) - 0.015 * times
i = 0

# BAC cannot be negative.
while i<size(times):
    bac_full[i] = max(bac_full[i], 0)
    bac_empt[i] = max(bac_empt[i], 0)
    i = i+1

# Plot graph.
plot(times,bac_full,"k—",linewidth=2, label="Full stomach")
plot(times,bac_empt,"b–",linewidth=2, label="Empty stomach")
grid(True)
xlabel("Time (hours)")
ylabel("BAC (%)")
xlim(0,5)
ylim(0,0.07)
legend()
show()
```

Figure 9.6 shows the output from running the above program for an 80 kg male consuming four standard drinks (40 g alcohol):

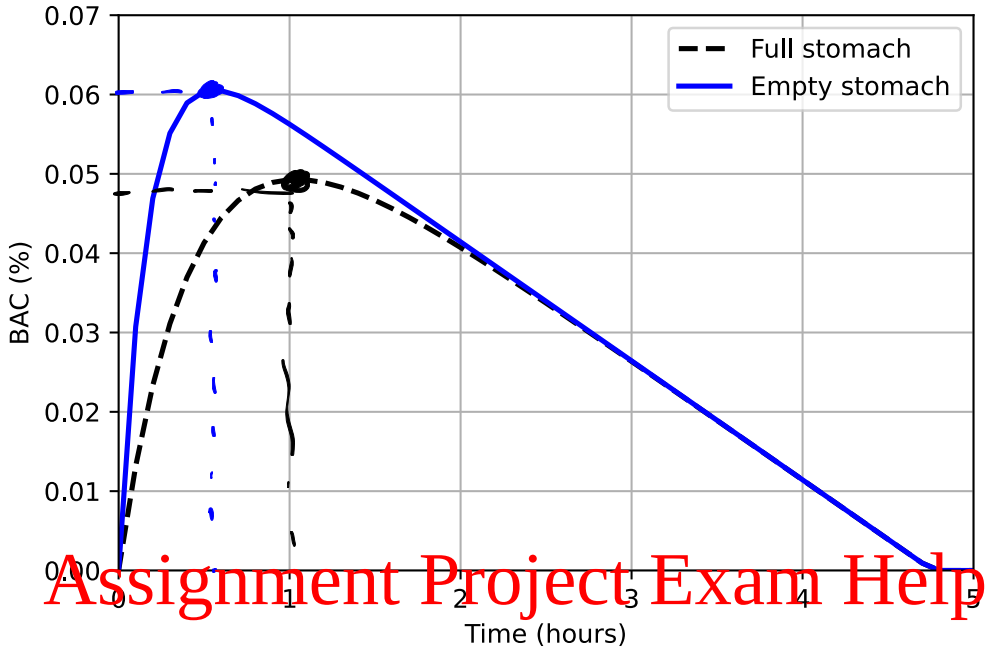


Figure 9.6: Predicted BACs when consuming alcohol on a full stomach compared to an empty stomach.

Question 9.3.8

Briefly compare the graph in Figure 9.6 with your answers to Question 9.3.7. What are some of the physical implications of the graph?

Values are consistent with calculations

Empty stomach – higher level (higher maximum BAC)

– higher exposure (area under curve)

– not a huge difference – especially at later times

End of Case Study 20: Drink deriving.