Chapter 9: Sex, drugs and rates of change

Lecture 23: Average and instant change

Learning objectives

✓ Understand the meaning and applications of average rates of change

✓ Understand the meaning and use of derivatives

Scientific examples

✓ Cyanide from smoking

✓ Metabolism of alcohol

Maths skills

✓ Calculate average rates of change

✓ Understand the meaning of derivatives

We are usually interested in studying phenomena whose values change over time. The primary mathematical tool that considers rates of change is the *derivative*. You should have encountered derivatives in previous study of mathematics. See Section C.4 in Appendix C for the pre-requisite mathematical tools we will use in this chapter. SOMS6 Use the online modules, available through the course website, for further support. In SCIE1000 we do not focus on how to *find* derivatives, but we instead focus on how to *use* and *interpret* them.

Image 9.1: *Skull with a burning cigarette* (c1885), Vincent van Gogh (1853 – 1890), Van Gogh Museum, Amsterdam. (Source: en.wikipedia.org)

Recall that in pharmacology, we saw that once a drug is administered, two key determinants of the impact of the drug are its maximum blood concentration $\boxed{C_{max}}$, and the time at which this occurs $\boxed{t_{max}}$. These are values which are calculated using derivatives.

- Pharmacokinetics is particularly concerned with the *rate* at which the drug concentration *changes*.
- The concept of one quantity changing as another quantity changes, and the rate at which the change occurs, is crucial to understanding and modelling many processes in science, engineering, social sciences and economics.
- We will cover two methods for analysing rates of change: *average* rates of change and *instantaneous* rates of change.

9.1 Rates of change

- The *average rate of change* measures the average change between two observed values of some phenomenon.

- In science, average rates of change are usually measured over time, such as 60 m s^{-1}.

On average travel 60m in every second

***Example** 9.1.1*

The concentration of atmospheric CO_2 has risen by about 90 ppm over the last 60 years. Hence the average rate of change over this time is:

$$\frac{90 \text{ ppm}}{60 \text{ years}} = 1.5 \text{ ppm year}^{-1}.$$

- Rather than measure the average rate of change between two points, in many situations it is more useful to measure the *instantaneous* rate of change at a point.
- The mathematical term for an instantaneous rate of change is *derivative*.

Case Study 17: **Cigarettes**

- Nicotine is a highly addictive, poisonous alkaloid found in a number of plants, including tobacco. After inhaling tobacco smoke, nicotine typically enters your blood stream within five seconds, and reaches your brain after about 10 seconds.
- In addition to nicotine, tobacco products also contain a large number of other compounds (including heavy metals, poisons and radioactive materials), many of which are toxic or known carcinogens.

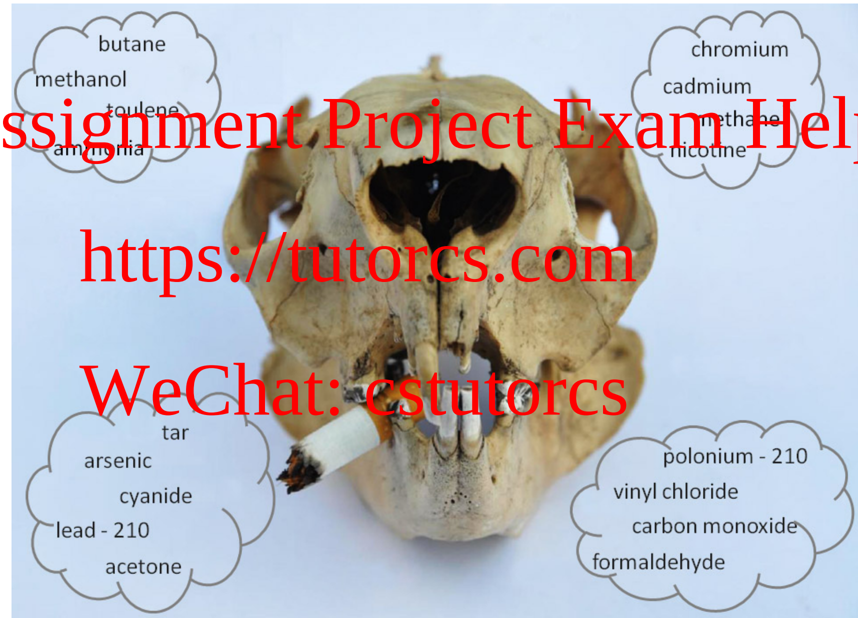


Photo 9.1: "Smoking kills": the joy of cigarette smoke. (Source: PA.)

Question 9.1.2

What types of parameters could be measured to investigate the physiological effects of smoking?

- Lung capacity - impacts on respiration
- Heart rate
- concentration of chemicals
- Blood pressure
- O_2 in blood

Question 9.1.3

While smoking tobacco, the body absorbs many chemical compounds in addition to nicotine, including cyanide (which is highly toxic to humans). Table 9.1 shows blood cyanide concentrations after smoking a cigarette, starting at time $t = 0$ minutes; see [29]. - 1987

t (min.)	0	5	10	15	20	25	35	65
conc. (μmol/L)	0.11	0.43	0.21	0.16	0.14	0.15	0.125	0.1

Table 9.1: Measured cyanide concentrations in the blood of a person after smoking a cigarette.

surge

The function $C(t) = 0.1 + 0.3t^{0.6}e^{-0.17t}$ μmol/L is a reasonable model of the measured blood cyanide concentrations. Figure 9.1 shows a plot of $C(t)$, along with the measured data values.

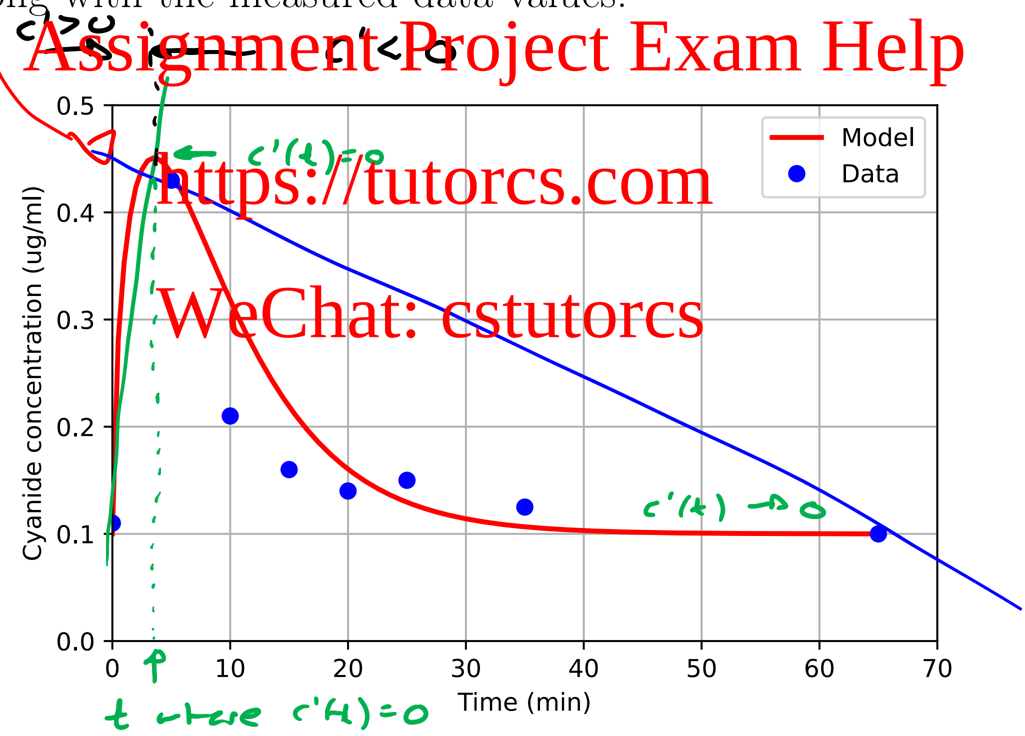


Figure 9.1: Measured and modelled blood cyanide concentrations after smoking a cigarette.

Rapidly absorbed

Background level in the body (present is some food)

Exponential (fast) decay

***Question** 9.1.3 (continued)*

Using the graph of the model in Figure 9.1 we can estimate the **average** rate of change in concentration over different time intervals. For example: From $t = 0$ min to $t = 5$ min, the average rate of change is

$$\frac{\Delta c}{\Delta t} = \frac{0.43-0.1}{5-0} = 0.07\,\frac{\mu\text{mol}}{\text{L}\cdot\text{min}}.$$

increasing

From $t = 5$ min to $t = 65$ min, the average rate of change is

$$\frac{\Delta c}{\Delta t} = \frac{0.1-0.43}{65-5} = -0.006\,\frac{\mu\text{mol}}{\text{L}\cdot\text{min}}.$$

decreasing (slower)

(a) What are some limitations in working with average rates of change in this context?

might miss changes that occur over a short time frame.

might not correctly represent change over shorter time periods

(b) The derivative of $f(t) = at^p e^{-bt}$ is $f'(t) = at^{p-1}e^{-bt}(p - bt)$.

Thus, $C'(t) = 0.3t^{-0.4}e^{-0.17t}(0.6 - 0.17t)$

(i) What is the physical meaning of C'?

Instantaneous rate of change of the blood cyanide concentration at a particular time

(ii) For what value(s) of t is $C'(t) = 0$?

At $t \simeq 4$ min, $c'(t) = 0$ (from graph)

Use function $c'(t) = 0$, solve for t

$$0.3t^{-0.4}e^{-0.17t}(\underbrace{0.6 - 0.17t}_{=0}) = 0$$

$0.6 = 0.17t$ or $t = \frac{0.6}{0.17} \simeq 3.5$ min

(iii) On the graph, identify all regions where C' is positive/negative.

***Question* 9.1.3** *(continued)*

(c) What is happening physically when C' is:

(i) positive?

concentration increasing

absorption / distribution dominating

(ii) zero?

concentration is constant

absorption / distribution $\simeq$ metabolism / excretion

"equilibrium"

(iii) negative?

concentration is decreasing

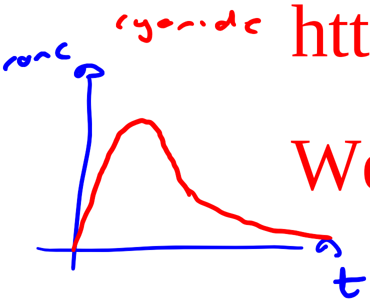
metabolism / excretion is dominating

End of Case Study 17: Cigarettes.

9.2 Pleasures of the flesh and derivatives

Case Study 18: **Whisky (back to BAC)**

- A standard drink contains 10 g of alcohol.
- Usually, the measure of Blood Alcohol Concentration (BAC) is the percentage of total blood volume that is alcohol (or equivalently, grams of alcohol per litre of blood). In Australia the legal blood alcohol content for driving is 0.05%, or 0.5 g/L.
- Unlike many other drugs, the rate of alcohol metabolism is roughly constant (called a *zero-order* reaction in Chemistry).
- The rate of metabolism is usually independent of the BAC because typical levels of alcohol consumption saturate the metabolising capacity of enzymes within the liver.



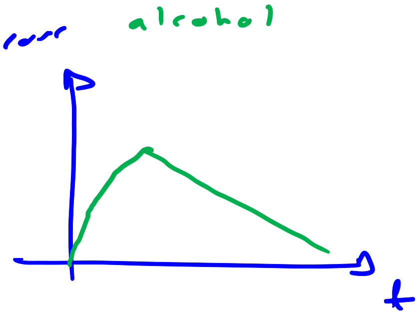


Photo 9.2: Calf liver. (Source: PA.)

- The exact rate of metabolism varies between individuals, influenced by factors such as age, mass (weight) and sex.
- A graph of BAC from the time drinking commenced will show a rapid initial rise during the absorption phase, prior to a decline in concentration during the elimination phase.
- Because the rate of alcohol metabolism tends to be constant, a graph of BAC from the time of peak concentration shows a linear decline until metabolism is almost complete.

Question 9.2.1

Figure 9.2 shows some BAC measurements (see [56]). Let $B(t)$ represent the straight line modelling the BAC from $t = 1$ hour to $t = 6$ hours.

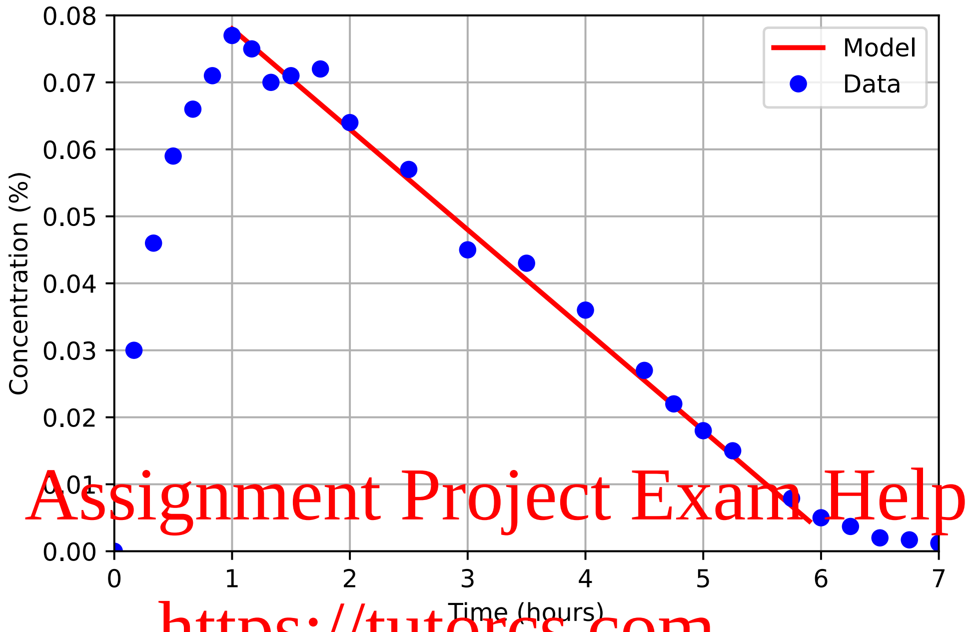


Figure 9.2: Measured blood alcohol concentrations.

Fitting a linear model to match the line drawn above, we find

$$\boxed{B(t) = -\underbrace{0.015}_{m}t + 0.093,}$$

where B is in % and t is in hours.

(a) Find $B'(t)$ (include units).

$B'(t) = -0.015$ %/hour

(b) Interpret, in words, what $B'(t)$ physically represents.

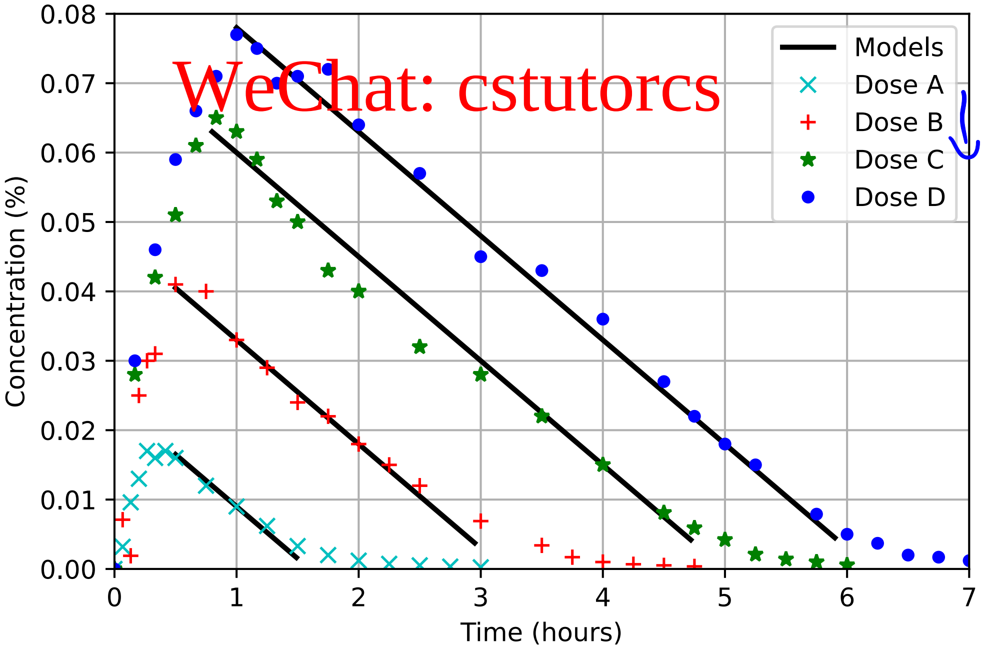
Instantaneous rate of removal of alcohol from the blood.

Question *9.2.1 (continued)*

(d) Figure 9.3 shows measured BACs after researchers administered four different controlled doses of alcohol to study participants (see [56]). Discuss the value of B' for each of the graphs. What does this mean, and is it consistent with "real life"?

Same slope (independent of dose)

Rate of removal of alcohol from the blood is constant, independent of dose



more standard drinks

Figure 9.3: Measured BACs after administration of four different controlled doses of alcohol.

End of Case Study 18: Whisky (back to BAC).