To answer each question you will need to use the information on Page 14.

1. Consider the model of standard weight of a fish species given on Page 14.

   (a) For a particular species of fish, when the length L is measured in mm, the standard weight in grams g is modelled by

   $$W(L) = aL^3.$$

   However, the United States of America makes use of imperial units, where the length of fish is measured in inches, and the weight is measured in pounds.

   (i) If a fish weighs $W = 2530$ g, how much does it weigh in pounds? (3 marks)

   (ii) If a particular species of fish has a standard weight of $W = 2530$ g for an individual of length $L = 216$ mm, what is the numerical value of the constant a? Show your working, and include units in your answer. (3 marks)

(continued over)

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(iii) Find the equivalent model for this species for the standard weight in pounds, using length measured in inches. Simplify your answer fully, show all working, and include units.

(4 marks)

(continued over)

Do not write here

(b) The general model of standard weight of a fish species given on Page 14 is

$$W(L) = aL^b.$$

Briefly justify why the value of b for many species of fish is close to 3. (Hint: It may help to develop a rough model for the volume of a fish, and consider what happens as a fish grows and its length increases.)

(5 marks)

(c) Consider two species of fish, with values of a and b as shown in the table.

Species	Value of a	Value of b
Species 1	0.0004	3
Species 2	0.0002	3

Briefly describe one physical factor that could impact the value of a for a fish species. Explain how your factor would differ between Species 1 and Species 2. (5 marks)

(continued over)

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(d) Consider two species of fish, Species 1 with standard weight modelled as $W_1(L)$, and Species 2 with weight modelled as $W_2(L)$, where:

$$W_1(L) = aL^{3.2} \qquad \text{and} \qquad W_2(L) = 2aL^{2.8}.$$

L is measured in inches, and $W_1(L)$ and $W_2(L)$ are measured in pounds. For what length is $W_1(L) = W_2(L)$? For what lengths is $W_1(L) > W_2(L)$?

(5 marks)

(e) Draw a single set of axes below, and sketch rough curves for $W_1(L)$ and $W_2(L)$ from Part (d) on the same graph. Clearly identify each curve, and the point of intersection. (5 marks)

(continued over)

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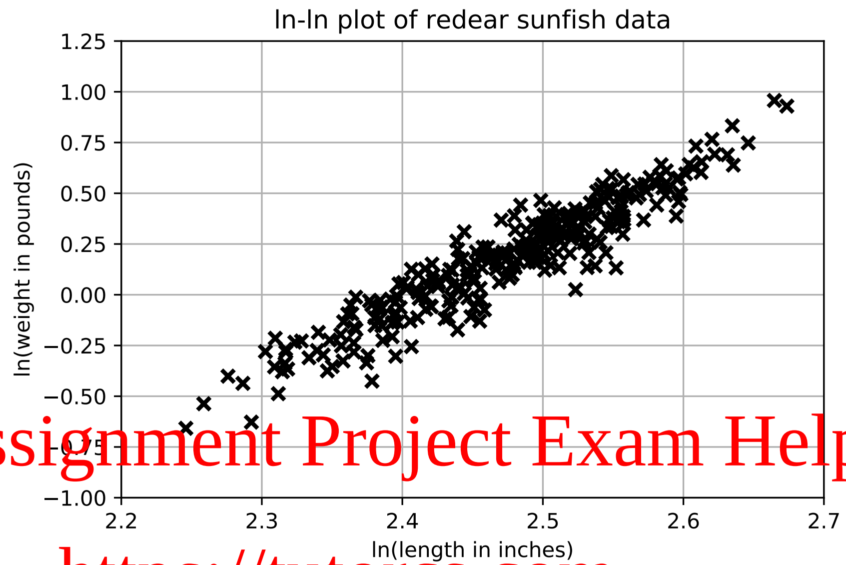
(f) A sample of 300 redear sunfish was taken from a pond, and the fish lengths and weights were recorded. The graph shows these data with natural logarithm (ln) of weight on the vertical axis and natural logarithm of length on the horizontal axis. Using data in the graph, find a model in the following form for the standard weights of redear sunfish. Show all working.

$$W(L) = aL^b.$$

[Hint: $\ln(aL^b) = \ln a + b \ln L$.] (10 marks)



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2. For each of Parts (a) to (c), **in the box**, write all of the output produced by the print statements in the Python program when the given values are entered from the keyboard until either the next input is asked for, or the program ends. Assume that the program is run from the beginning each time. **Do not write the input lines**. Round all values to two decimal places.

```
from pylab import *

# Calculate standard weight
def getStW(L):
    a = 0.004;
    b = 3;
    W = a*(L**b);
    return(W);

# Main program:

# Classify individual redear sunfish
run = 1

while (run == 1):
    fishL = eval(input("Length (inches)? "))
    fishW = eval(input("Weight (pounds)? "))
    if (fishL < 6) or (fishL > 15):
        print("Error: outside model range")
    else:
        StW = getStW(fishL)
        print("StW = ",StW," pounds")
        relW = 100 * fishW / StW
        print("relW = ",relW)
        if relW < 90:
            print("Weight: light")
        elif relW > 110:
            print("Weight: heavy")
        else:
            print("Weight: OK")
    run = eval(input("Enter 1 to run again: "))
print("So long and thanks for all the fish!")
```

(a) 7 and 1.47 (5 marks)

(b) 5 and 1.04 (5 marks)

(c) 9 and 2.43 (5 marks)

(d) Explain the use of the variable `run` in the program. (5 marks)

Do not write here

3. Consider a population of largemouth bass. Use information from Page 14 and Euler's method with an appropriate stepsize to estimate the missing values A, B, and C in the following table. Show your working.

(10 marks)

Fish age (years)	Length (mm)	Standard weight (g)
2	250	A
3	B	C

Do not write here

4. Consider the predator-prey model for largemouth bass and gizzard shad given on Page 14. Find the (non-zero) values of G and L for each of the following situations:

(a) Both G and L remain unchanged over time.

(b) G is increasing but L is decreasing.

(10 marks)

Do not write here

5. This question refers to the tests for the distribution of smallmouth bass given on Page 14. The tests were applied to 13719 distinct locations, of which 567 contained smallmouth bass.

(a) Construct the binary classification table for Test A, and hence compute the overall correct classification (accuracy).

(10 marks)

(continued over)

Do not write here

(b) In each of the following scenarios, which of Test A, Test B, or Test C would you recommend as being most useful before carrying out the suggested intervention? Justify your answers briefly.

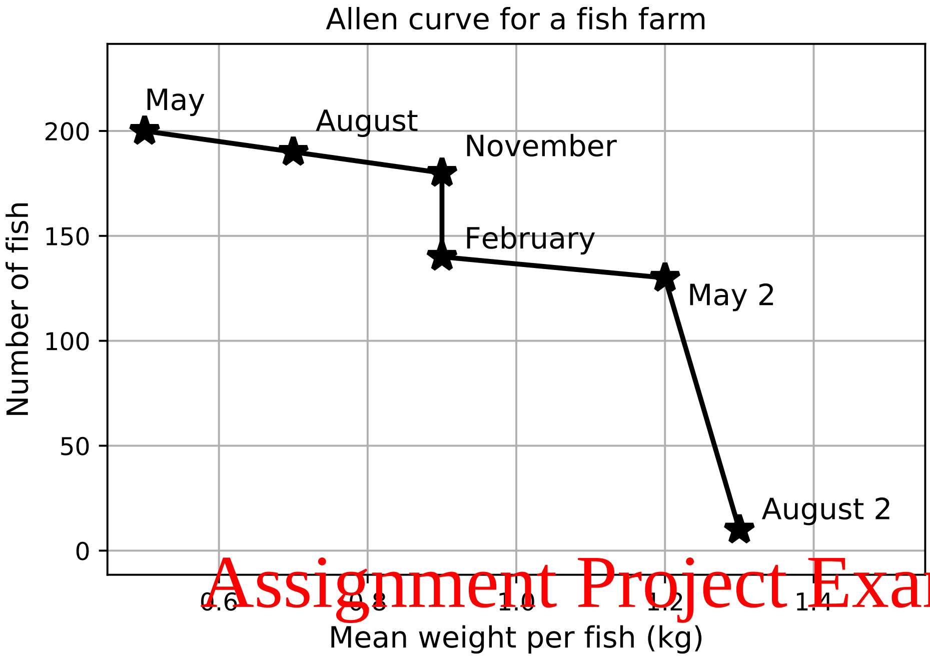
(i) Smallmouth bass are a bad environmental pest, and government wants to stop their spread. There is an effective intervention that can eradicate smallmouth bass from identified sites. (3 marks)

(ii) The government wants to release some young fish of a different and rare species in a number of locations and wants to maximise the chances of survival. Unfortunately, smallmouth bass will prey on the released fish. (3 marks)

(c) In Part (b) you recommended which test(s) were most suitable in the given scenarios. In practice, what other information might be considered when choosing which test(s) to apply? (4 marks)

Do not write here

6. Felicity operates the *Family Fun Fishing Farm* near Philadelphia in the USA. (Her motto is “We hatch them, you catch them. Summer fun for everyone!”) The following Allen curve and table give data for one of her ponds, with observations made at 3 month intervals from May in one year to August in the following year. (The second May and August observations include the label “2”.)



Month	Number of fish	Mean weight (kg)
May	200	0.5
August	190	0.7
November	180	0.9
February	140	0.9
May 2	130	1.2
August 2	10	1.3

(a) Find the total production for the given time period using the trapezoidal method. Show all working, and include units in your answer.

(10 marks)

(continued over)

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(b) Briefly describe what happened between May 2 and August 2. What might have caused this? (4 marks)

(c) Briefly describe what happened between November and February. What might have caused this? (3 marks)

(d) Felicity asks you for some strategies that could increase the total production of this fish pond. Make two suggestions to her, and describe how each suggestion would impact on the Allen curve if the suggestion were successful. (3 marks)

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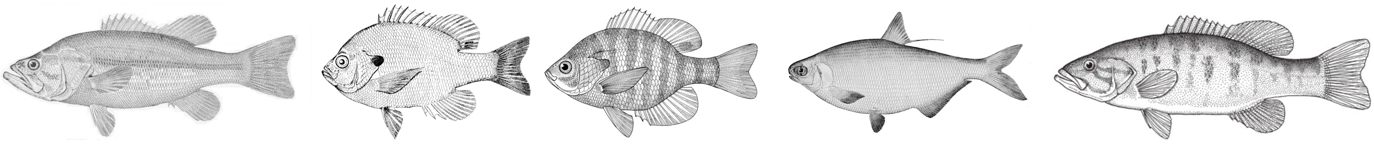
Units: There are 2.54 cm in an inch, 10 mm in a cm and 2.2 pounds in a kg. To be consistent with industry terminology, this exam paper uses the term 'weight' instead of the more correct 'mass'.

Modelling fish growth: Below are illustrations of five North American freshwater fish species: largemouth bass (*Micropterus salmoides*); redear sunfish (*Lepomis microlophus*); bluegill (*Lepomis macrochirus*); gizzard shad (*Dorosoma cepedianum*); smallmouth bass (*Micropterus dolomieu*).



Monitoring fish growth is important in aquaculture and environmental management. Two important measures are weight and length.

The *standard weight* for a fish species is the expected weight $W(L)$ of a healthy individual fish as a function of its length. Standard weights are often modelled as follows, where a and b are constant for a given species; for many fish species, the value of b is around 3:

$$W(L) = aL^b.$$

The *relative weight* of an individual fish of a given species is the weight of that individual expressed as a percentage of the standard weight of a fish of the same length.

Fish length can be modelled as a function of age using the *von Bertalanffy growth model*. The model states that the rate of increase of the length $L(t)$ of a fish of age t is proportional to an intrinsic positive growth rate r and the difference between a fixed maximum length M and its current length $L(t)$. This can be represented in a differential equation as $L' = r(M - L)$.

Information for largemouth bass: Sampling a population of largemouth bass showed the maximum length to be 600 mm, the unconstrained length growth rate to be $r = 0.2$ per year, and the average length at age 2 to be 250 mm. The standard weight $W(L)$ of largemouth bass can be modelled as follows, where W is measured in g and length L measured in mm:

$$W(L) = 4.48 \times 10^{-6} \times L^{3.2}.$$

Predator-Prey model: Researchers developed a model of predator (largemouth bass) and prey (gizzard shad) in the upper Mississippi river. Let $L(t)$ be the **percentage** of largemouth bass in the **entire fish population** at time t years, and $G(t)$ be the percentage that is gizzard shad. Then:

$$G' = 2.1G - 6GL \qquad L' = -20.5L + 1.1GL \qquad G(0) = 22 \qquad L(0) = 0.9.$$

Distribution of smallmouth bass: Researchers developed a range of tests to investigate the distribution of smallmouth bass in North America, using climate data and other factors. The question they asked was: given a region, do smallmouth bass occur in this region? Test A had a sensitivity of 4.1% and a specificity of 99.9%, Test B had a sensitivity of 91.9% and a specificity of 90.7%, and Test C had a sensitivity of 81.3% and a specificity of 93.5%. The *overall correct classification* of a test is the percentage of sites **correctly classified** either as containing smallmouth bass (true positives) or as not containing smallmouth bass (true negatives).

Allen curve and total production: For a given region, an *Allen curve* for a fish species and time period can be plotted as follows. At various dates during the time period, measure the number of fish and the mean weight of those fish. Plot numbers of fish on the vertical axis and mean weights on the horizontal axis, connecting points from consecutive observations. The *total production* of the region is defined to equal the area under the Allen curve (between the curve and the horizontal axis) for observations during the time period.

END OF EXAMINATION