Chapter 5: Give us a wave!

Lecture 12: Breathing in, breathing out

Learning objectives

✓ Interpret sine function models of real-world phenomenon

Scientific examples

✓ Breathing and lung capacity

Maths skills

✓ Understand sine functions and their transformations

Functions
- power
- oscillations

Image 5.1: *The Great Wave off Kanagawa* (1829–1833), Katsushika Hokusai. Print at the Metropolitan Museum of Art (Source: en.wikipedia.org)

Many phenomena in Science and nature *repeat* or *cycle*. These include: many aspects of weather and climate; ocean waves and tides; physiological processes, such as breathing and hormone levels; sound waves; and the voltages and currents in alternating current electricity.

In this chapter, we will discuss how to model cyclic or periodic phenomenon using a sine function. You should have encountered sine functions in previous study of mathematics. See Section C.2 in Appendix C for the pre-requisite mathematical tools we will use in this chapter. Use the online modules, available through the course website, for further support.

SOMSE

5.1 Waves, cycles and periodic functions

- Consider the four graphs in Figure 5.1, each of which shows climate-related data for Brisbane over a period of one year. If the graphs were extended over subsequent years, then an approximate cycling pattern would be observed.

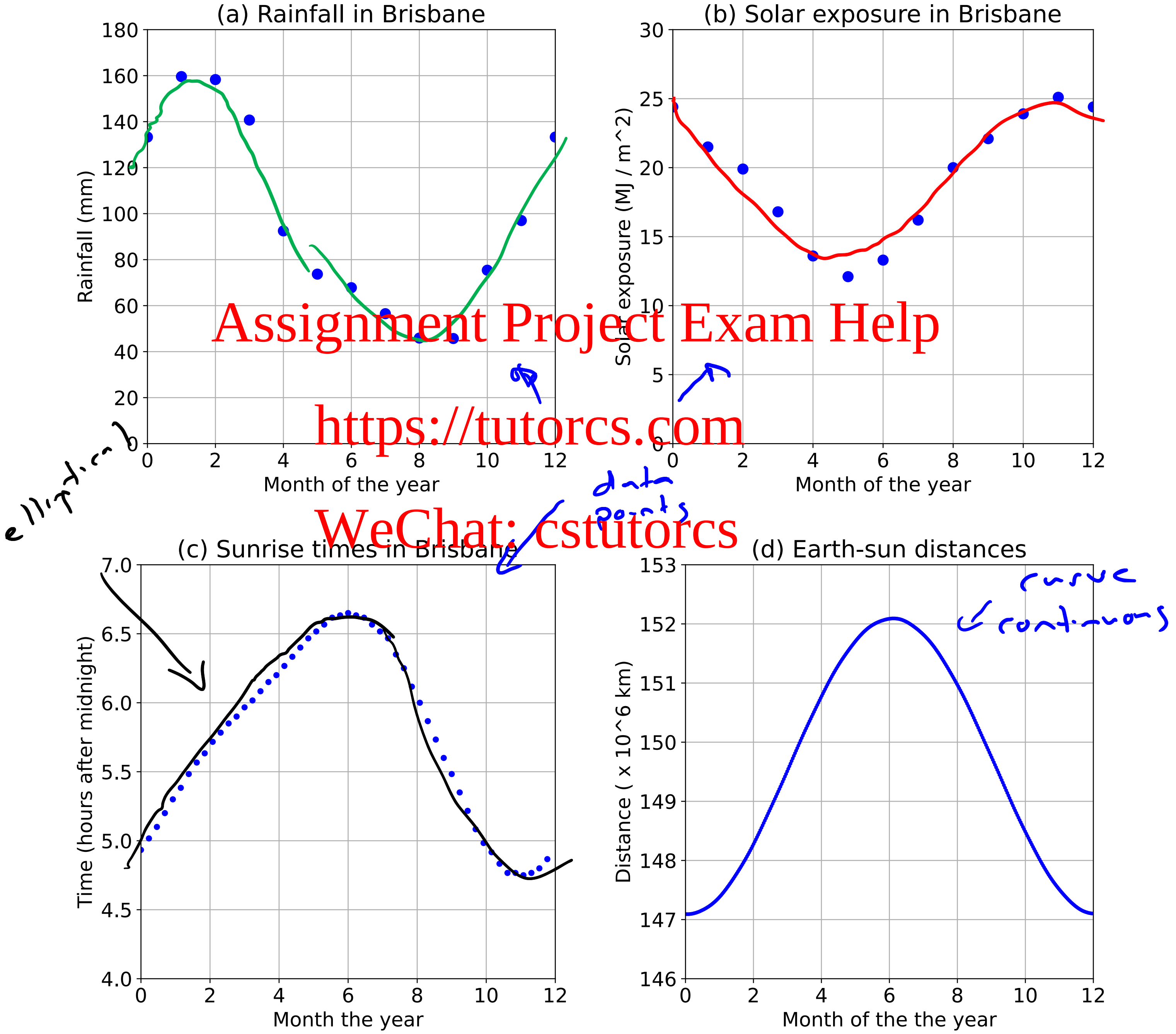


Figure 5.1: Four climate-related graphs. (a) Average monthly rainfall in Brisbane. (b) Average monthly solar exposure in Brisbane. (c) Weekly sunrise times in Brisbane. (d) Daily distances between the centres of Earth and the sun.

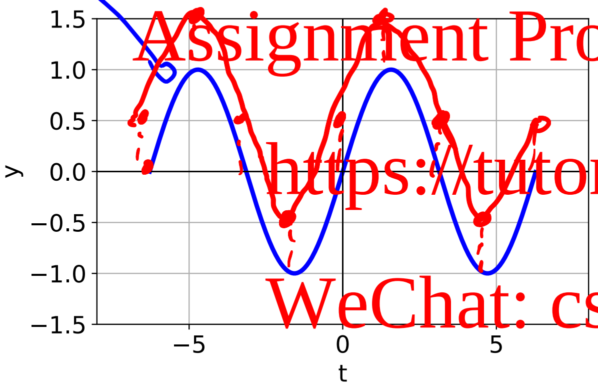
$y = \sin(t)$ – cycle in 2π

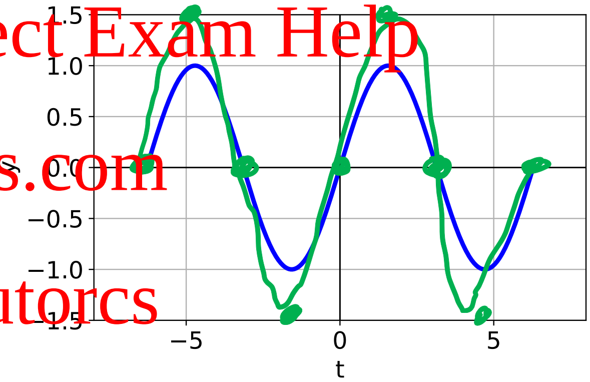
Question 5.1.1

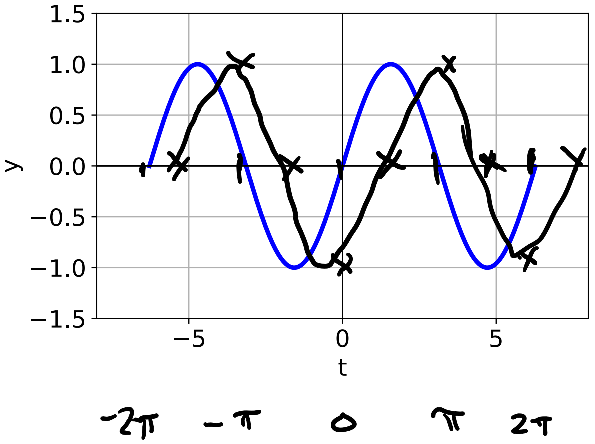
Consider the following four transformed sine functions. Draw a graph of each of these functions on the axes provided below. Each graph below already shows a graph of $y = \sin t$ for two full cycles (from $t = -2\pi$ to $t = 2\pi$).

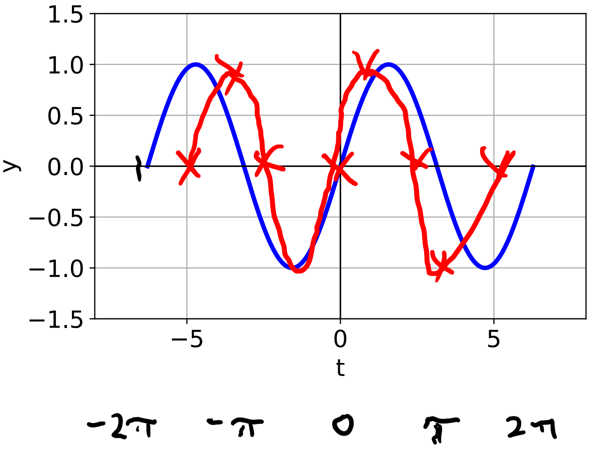
(a) $\boxed{y = \sin(t) + 0.5}$ vertical shift (by 0.5 units upwards)	**(b)** $\boxed{y = 1.5\sin(t)}$ vertical scale (×1.5 - expanded)
(c) $\boxed{y = \sin(t - \frac{\pi}{2})}$ horizontal shift (here to right)	**(d)** $\boxed{y = \sin(\frac{2\pi}{5}t)}$ horizontal scale (< 2π - contracts)

y = sin(t)





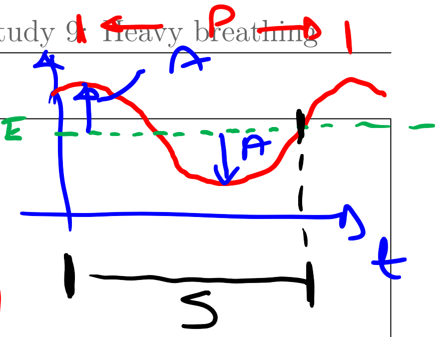




It may be helpful to substitute some values in for t. If using a calculator, *make sure that your calculator is set to* <u>*radians*</u> *NOT degrees.*

$t = 0 \Rightarrow \sin(0) = 0$

$t = 5 \Rightarrow \sin(2\pi) = 0$



Question 5.1.2

The general equation of a sine function is

$$y(t) = \boxed{A}\sin\left(\frac{2\pi}{\boxed{P}}(t - \boxed{S})\right) + \boxed{E}.$$

Explain the *mathematical* meaning and impact of each of the constants A, P, S and E.

E - average y-value - vertical shift (equilibrium)

A - amplitude - vertical scale
difference between equilibrium & max (or min)

P - period, horizontal scale
time between successive peaks
(one complete cycle)

S - shift - horizontal shift
time at which y = equilibrium
and increasing

Case Study 9: Heavy breathing

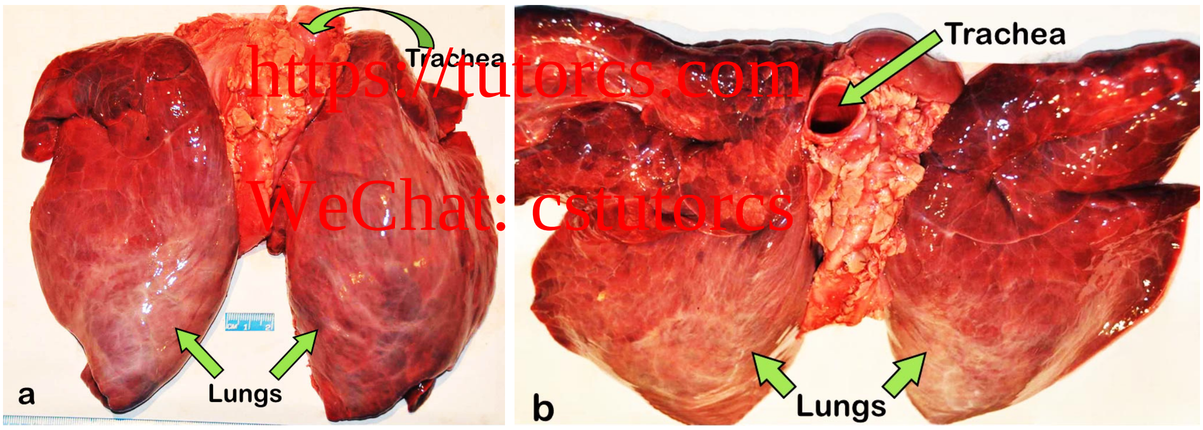


Photo 5.1: Calf lungs, (a) from the front, and (b) from the top. (Source: PA.)

- The volume and rate of air movement into and out of the lungs can be measured using a *spirometer* and graphed in a *spirogram*. (One common type of spirometer uses the Hagen-Poiseuille equation to measure air flow rates.) This information can be used to diagnose possible respiratory impairment.
- The maximum volume of air which can be expelled from the lungs in a single exhalation is called the **vital capacity**. After each exhalation the lung retains a volume of air, called the **functional residual capacity**.

- Normal breathing involves rhythmic inhalation and exhalation of air. The **tidal volume** is the difference in the volume of air in the lungs between a full inhale and a full exhale while breathing normally.

$V = A\sin\left(\frac{2\pi}{P}(t-s)\right) + E$ − Find A, P, E

s is not important

Question 5.1.3

(a) Estimate the tidal volume, period between breaths, functional residual capacity and vital capacity for a resting adult human. Sketch a rough graph of the volume of air in the lung over time.

− tidal volume ~ 500 mL = 0.5 L
relates to amplitude $2A = 0.5\text{ L} \Rightarrow A = 0.25\text{ L}$

− period between breaths
in 30 s, 6 full breaths $\Rightarrow P \approx \frac{30\text{s}}{6} = 5\text{s}$

− vital capacity $\Rightarrow$ balloon, larger bag!
about 5 L

− functional residual capacity
(total volume lungs − vital capacity)
full volume = 6 L $\Rightarrow$ frc = 1 L

(b) Write a function using sine to model the volume of air in the lungs during normal breathing over time, based on Part (a).

$$V = 0.25\sin\left(\frac{2\pi}{5}t\right) + 3.5$$

V is in Litres

t is in seconds

Question 5.1.3 *(continued)*

(c) How would the function change after moderate physical activity?

A ↑ , P ↓

(d) Suppose a patient is breathing very rapidly, with deep inhalations and exhalations. How would the original function change to model the breathing of this patient?

A ↑ , P ↓↓

(e) Smoking and air pollution cause inflammation in the lungs, gradually destroying the lung tissue and leading to *emphysema*, a type of Chronic Obstructive Pulmonary Disease. The reduction of lung surface area decreases the ability to exchange carbon dioxide and oxygen.

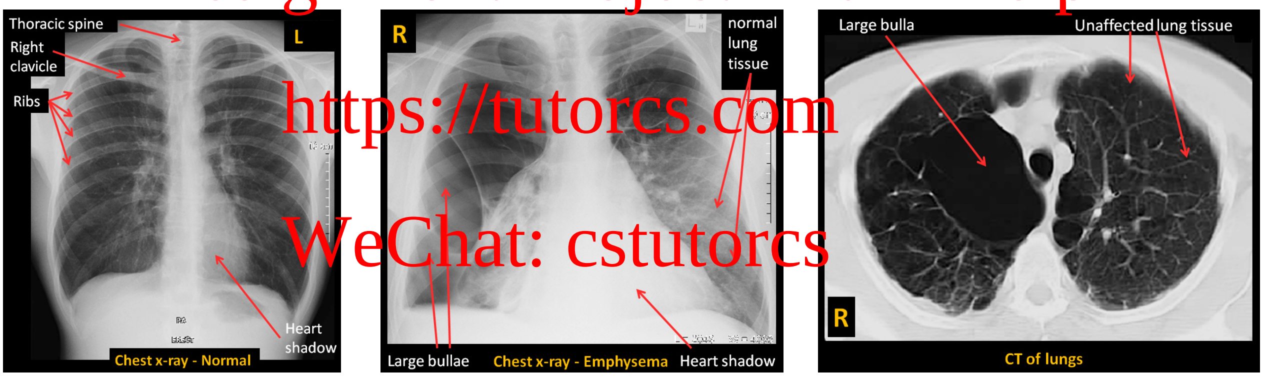


Photo 5.2: Left: x-ray of an adult male chest displaying normal lung tissue architecture and normal heart shadow. Middle: x-ray of a chest showing large emphysematous bullae within the right lung. Right: Axial CT showing one large, and multiple small, bullae of the alveolar air spaces in the right lung.(Source: Qld Health and DM.)

How would the function change for an individual with emphysema?

short shallow breaths

A ↓ P ↓ , E ↑

End of Case Study 9: Heavy breathing.