Chapter 10: You, me and AUC

Lecture 26: To binge or not to binge...

Learning objectives

✓ Interpret the meaning of the area under a curve in various scientific contexts

✓ Understand the meaning of the area under a curve

Scientific examples

✓ Exposure to alcohol with different drinking patterns

Maths skills

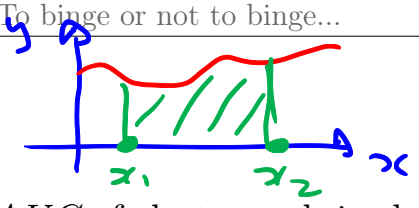
✓ Estimate the area under a curve

Image 10.1: *The Drunks* (1629), Diego Velazquez (1599 – 1660), Museo del Prado, Madrid. (Source: en.wikipedia.org)

We have noted that in pharmacology, the area under a drug concentration curve has an important physical meaning. Specifically, a key determinant of the impact of a drug once it has entered the bloodstream is the total exposure of the body to the drug, which is the area under the curve (AUC). Other related phenomena include the *bioavailability* of drugs administered by different routes and the *Glycaemic Index* (GI) of foods.

In this chapter we study areas under curves (AUCs). The primary mathematical tool for analysing AUCs is the *integral*. While SCIE1000 does not cover integration techniques, we will discuss several methods of finding or estimating AUCs, and we will mainly use the *trapezoid rule*. More importantly, you will need to know how to use and interpret the results.

10.1 Areas under curves

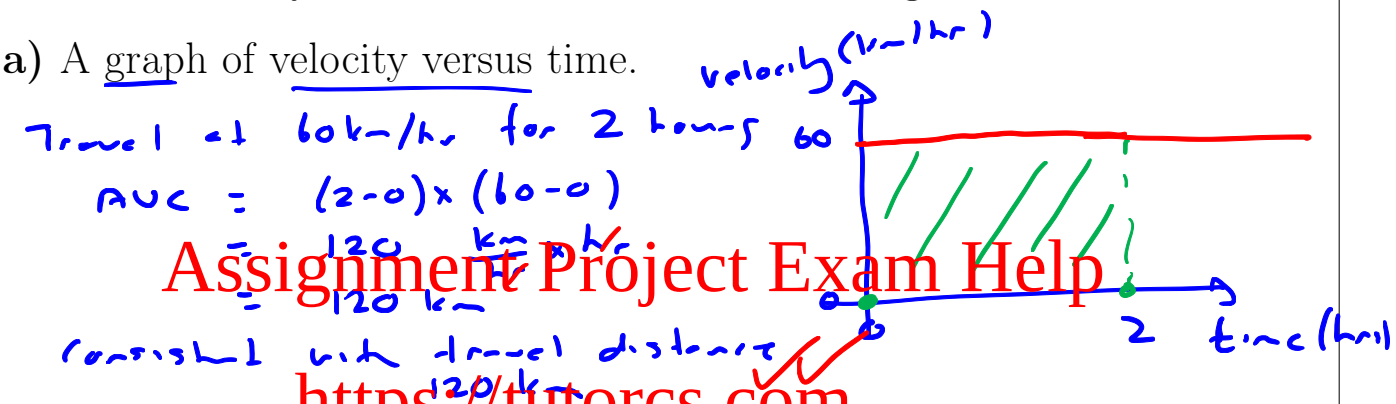


- Given a graph, the *area under the curve* or *AUC* of that graph is the area bounded by that curve, the x-axis and two points on the x-axis.
- The AUC often has a useful physical meaning, which depends on what is being graphed.

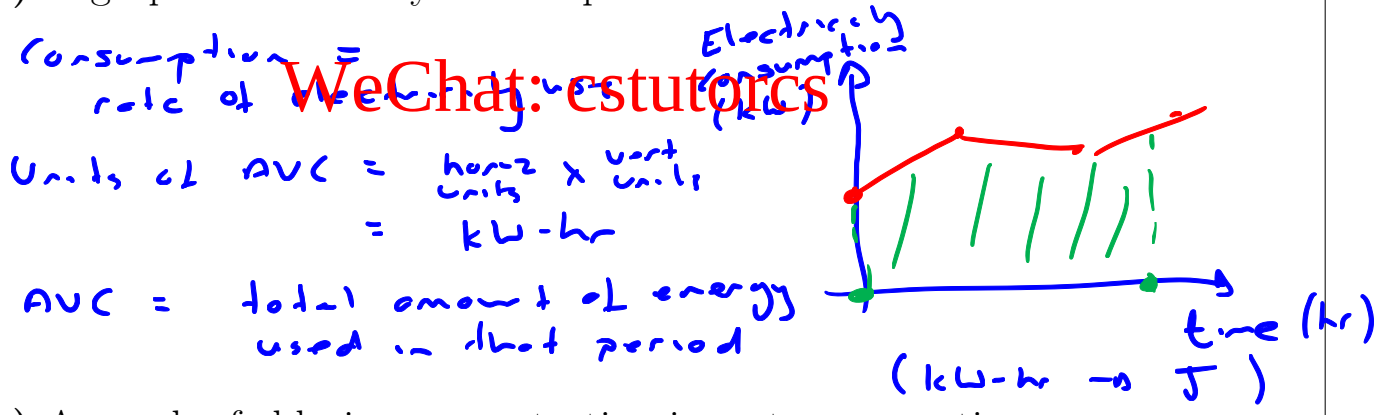
Question 10.1.1

What is meant by the AUC in each of the following.

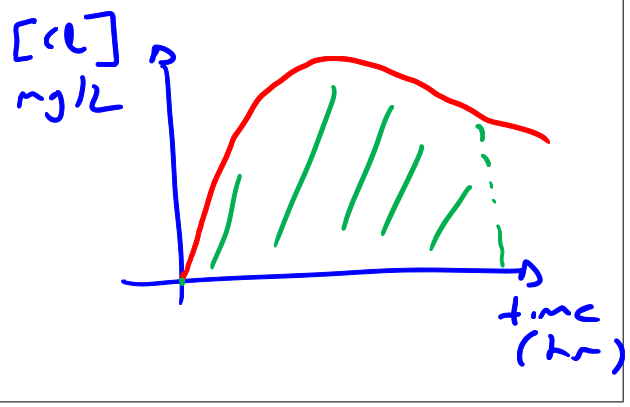
(a) A graph of velocity versus time.



(b) A graph of electricity consumption in a household versus time.



(c) A graph of chlorine concentration in water versus time.



Case Study 21: **Dying for a drink**

Photo 10.1: Left: mellow and yellow. Right: better red than dead. (Source: DM.)

Question 10.1.2

Figure 10.1 shows a graph with a line fitted to some measured blood alcohol concentrations (see [bb]).

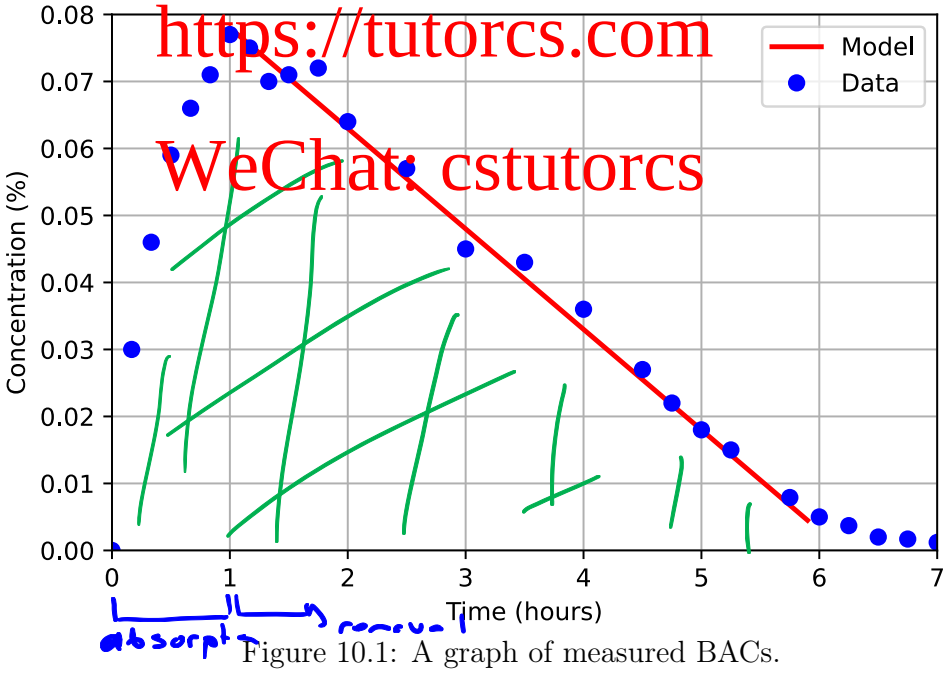


Figure 10.1: A graph of measured BACs.

(a) What are the units of the AUC in the graph?

% hr

***Question** 10.1.2 (continued)*

(b) What does the AUC represent and why is it significant?

AUC represents exposure to alcohol (poison) Potential health risks

- In addition to the immediate risks associated with alcohol consumption (such as accidents), the risk of many negative long-term health effects is increased by both the frequency and volume of consumption.
- Thus, long-term health risks are affected by the *exposure* to alcohol, (that is, the area under the alcohol concentration curve).
- Recall that the "standard" *Widmark formula* is:

$$B = \frac{A}{rM} \times 100\% - Vt.$$

- The "standard" Widmark formula, does not account for absorption, but it can be used to estimate the area under the alcohol concentration curve.
- Since each standard drink contains 10 grams of alcohol, and alcohol is removed from the blood at a rate of $0.015\%/h$, we have $A = 10n$

$$B = \frac{10n}{rM} \times 100\% - 0.015t,$$

where B is measured in %, n is the number of standard drinks, M is the person's mass in grams, and t is measured in hours.

- Recall that r is the proportion of the person's mass that is water. Typically, $r \approx 0.7$ for males (on average) and $r \approx 0.6$ for females (on average).

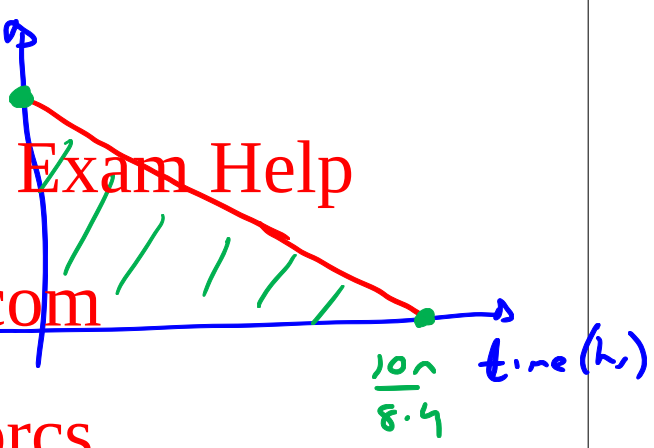
Question 10.1.3

The Widmark formula is used to estimate blood alcohol content (BAC); see Question 9.3.6. For a 'typical' 80 kg man drinking n standard drinks, his estimated % BAC at time t in hours since commencing drinking is

$$B = \frac{10n}{560} - 0.015t.$$

(a) Define the *total exposure to alcohol* E as the AUC of B from $t = 0$ until the BAC reaches 0 again. Find an expression for E for this man. [Hint: You will need to find an expression for the time at which his BAC returns to 0.]

AUC = $\frac{1}{2}$ base × height



height

Find y-intercept, B when $t = 0$

$B = \frac{10n}{560}$ (height)

$= \frac{10n}{560}$

width

Find x-intercept, t when $B = 0$

$0 = \frac{10n}{560} - 0.015t$

$\Rightarrow t = \frac{10n}{560 \times 0.015} = \frac{10n}{8.4}$ (base)

Combine

AUC = $\frac{1}{2}$ base × height

$= \frac{1}{2}\left(\frac{10n}{8.4}\right)\left(\frac{10n}{560}\right)$

$E \simeq 0.011n^2$

n - units (no. of drinks)

E - units % hours

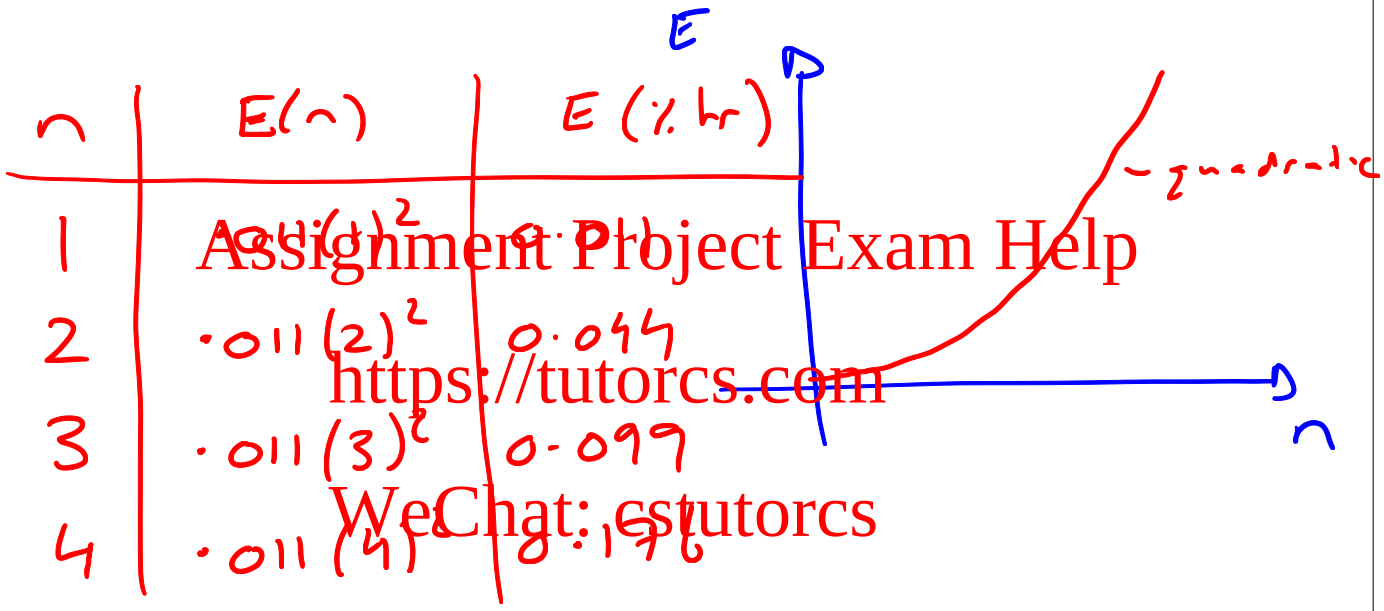
***Question** 10.1.3 (continued)*

(b) Assume that long-term damage to internal organs from consumption of alcohol is proportional to the total exposure to alcohol E (which is simplistic, but not unreasonable). Discuss the impact on E of "one extra drink for the road".

Exposure $E = 0.011 n^2$

!!

This is a quadratic function



n	$E(n)$	E (% hr)
1	$.011(1)^2$	0.011
2	$.011(2)^2$	0.044
3	$.011(3)^2$	0.099
4	$.011(4)^2$	0.176

"one for the road" can significantly increase exposure

e.g. 3 → 4 drinks almost double exposure

Question 10.1.3 (continued)

(c) A 'typical' man with mass 80 kg consumes two standard drinks every day. A second 'typical' man with the same mass consumes 14 standard drinks once a week, but does not drink at any other time. Estimate the weekly value of E for each man. What are some of the physical ramifications of your answer in relation to binge drinking?

"n"

"t"

$E = 0.011(n)^2$ (80 kg male)

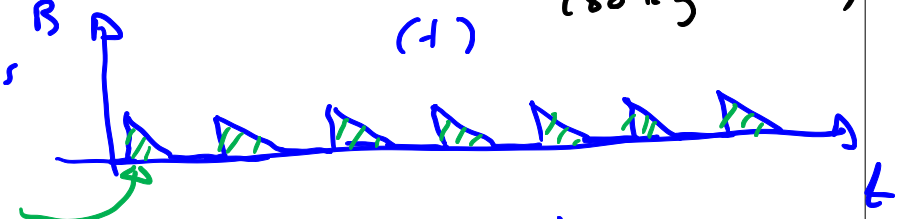
2 drinks / day for 7 days

Exposure for 2 drinks

$E_1 = 0.011(2)^2$

$= 0.044$ %. hrs

$E_{total} = 7(E_1) = 7 \times 0.044 \simeq 0.31$ %. hours

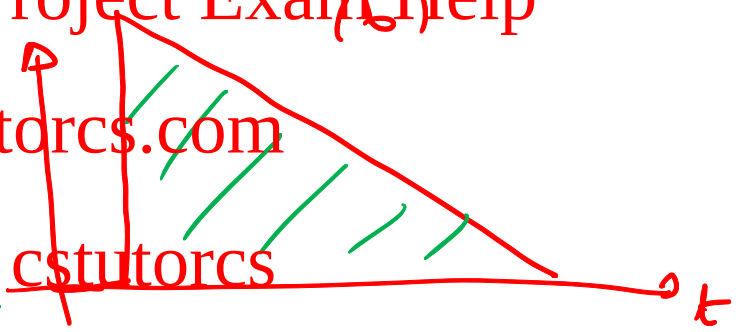


14 drinks (same) 7 days

$E_{total} = 0.011(14)^2$

$\simeq 2.16$ %. hrs

much higher exposure for "binge" drinking once (for some amount of alcohol)



(d) For a 'typical' woman of mass 60 kg, $E = 0.0257n^2$. Find the ratio of the values of E for the 'typical' woman and 'typical' man. What does this mean?

("typical"): $\dfrac{E_{woman}}{E_{man}} = \dfrac{0.0257n^2}{0.011n^2} \simeq 2.3$

More than twice the exposure for the "typical" woman compared to the "typical" man for the same number of drinks.

Program specifications: Write a Python program that uses the Widmark formula to graph the total exposure to alcohol for a person who consumes from zero to 14 standard drinks.

Program 10.1: Wilful exposure (to alcohol)

```
# Calculate exposure to alcohol.
from pylab import *

# Input
mass_person = float(input("What is the person's mass (in kg?) "))
sex_person = int(input("Enter 1 for female, 2 for male: "))

# Estimate exposure for each number of drinks
if sex_person == 1:
    mass_water = 1000 * mass_person * 0.7
else:
    mass_water = 1000 * mass_person * 0.6
no_drinks = arange(0,15)
peak_bac = 1000 * no_drinks / mass_water
time_bac_0 = peak_bac / 0.015
auc = time_bac_0 * peak_bac / 2.0

# Plot
plot(no_drinks, auc, "bo", markersize=6)
grid(True)
xlabel("Number of drinks")
ylabel("Total exposure (% hours)")
xlim(0,14)
ylim(0,3)
show()
```


The program was run with inputs of 80 kg and 1 (male), see Figure 10.2.

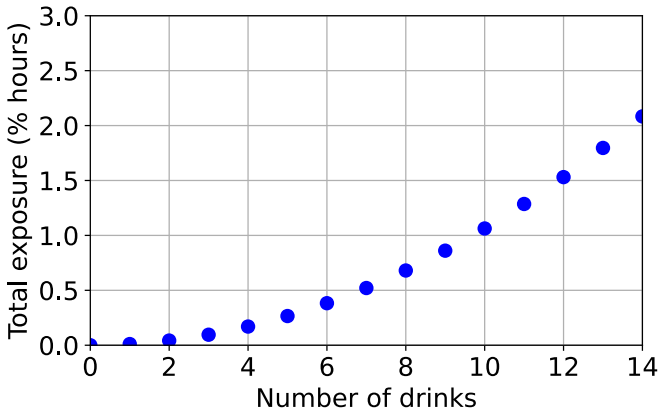


Figure 10.2: Program output showing total exposure to alcohol according to drinks consumed.

End of Case Study 21: Dying for a drink.