Lecture 13: **Sine and seasons**

✓ Interpret sine function models of real-world phenomenon

Scientific examples

✓ Seasons on Earth

✓ Daylight hours

Maths skills

✓ Understand and interpret sine functions and their graphs

$y = A \sin\left(\frac{2\pi}{p}(t-s)\right) + E$

5.2 Days, seasons, cycles


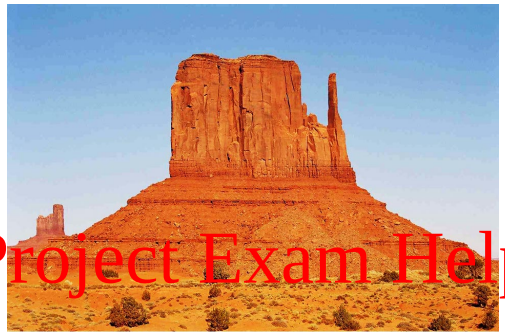

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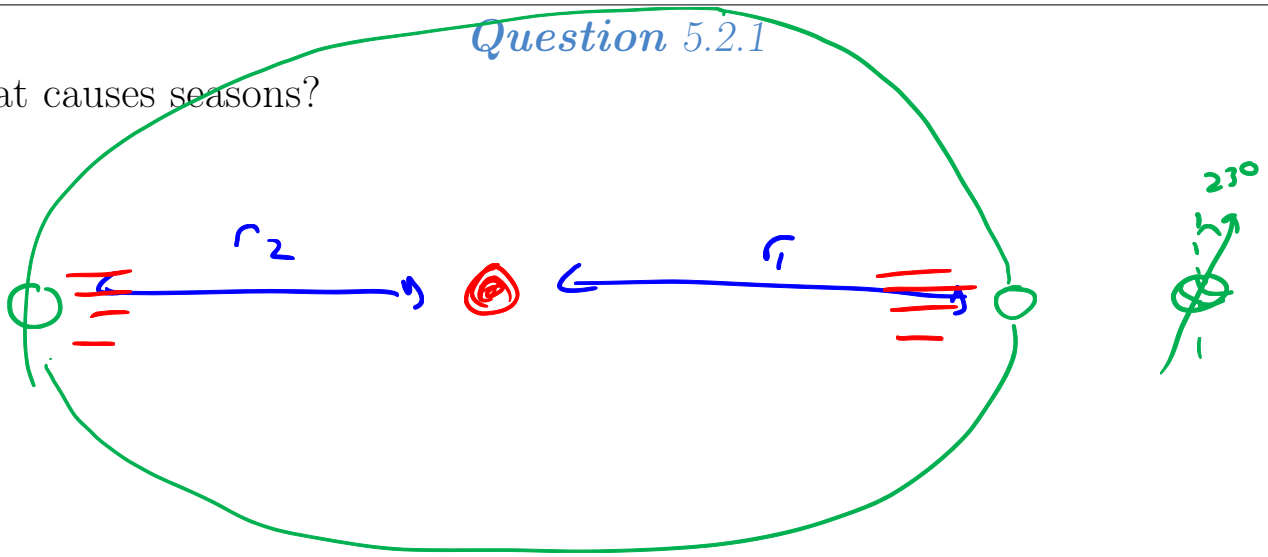
Photo 5.3: Spring – Lotus flower, *Nelumbo nucifera* (Tokyo, Japan); Summer – Monument Valley (Utah, USA); Autumn – sugar maple, *Acer saccharum* (Vermont, USA); Winter – Pine tree (Canyonlands, Utah, USA). (Source: PA.)

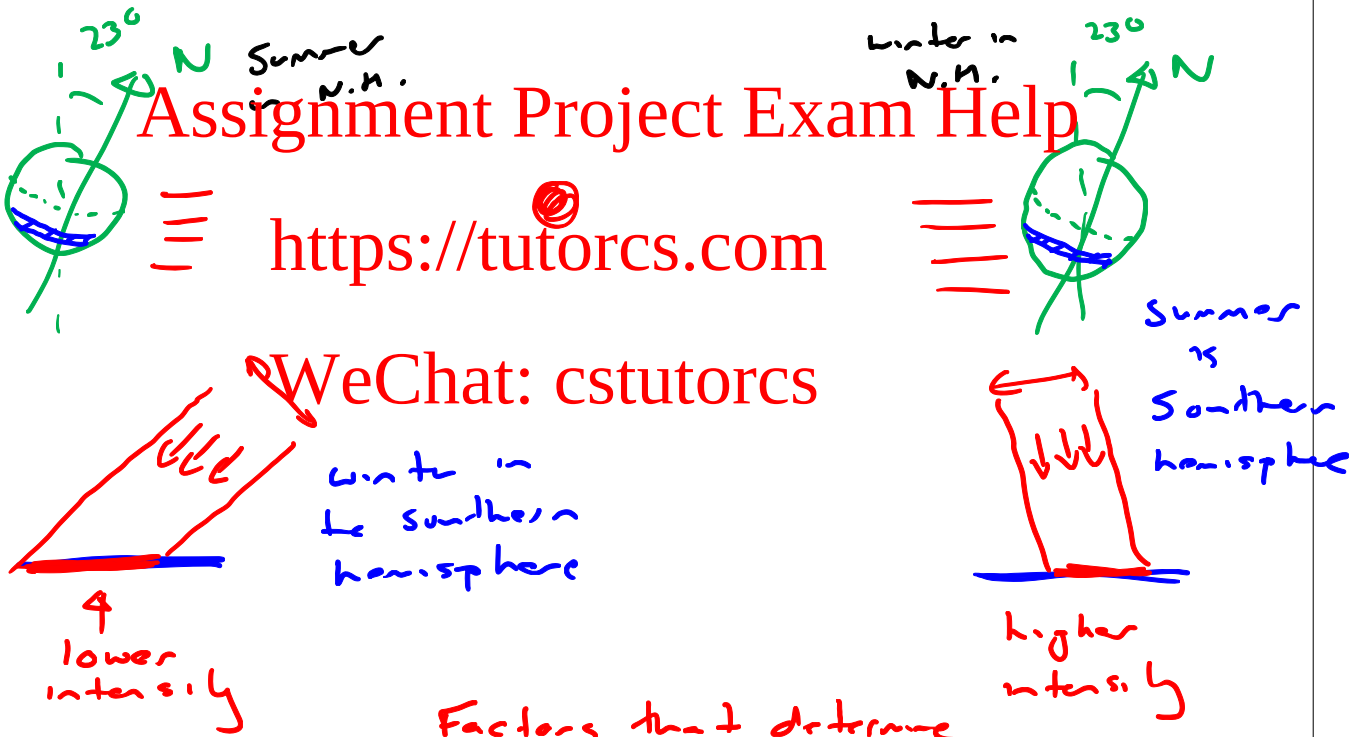
- The amount of sunlight available at a location on Earth on a given day can be modelled using *daytime*, defined as the time between sunrise and sunset. (This is independent of clouds or weather events.)
- Daytime lengths vary through the year. The **summer solstice** and **winter solstice** are the days with the longest and shortest daytimes (respectively). The **vernal equinox** and **autumnal equinox**, are the days in spring and autumn (respectively) with daytimes of exactly 12 hours.

spring equinox

Question 5.2.1

What causes seasons?





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Factors that determine length of the day

- latitude
- time of year
- length of the year

Question 5.2.2

Discuss the daytime lengths in midsummer and midwinter in each of:

(a) Brisbane;

Today : 4.48am ↓ 6.39pm

midsummer: ~14 hours (5am - 7pm)

midwinter: ~10 hours (7am - 5pm)

(b) Singapore (which is very close to the equator); and

midsummer: 12 hours

midwinter: 12 hours

(c) Santa Claus village, Rovaniemi, Finland (north of the Arctic Circle).

midsummer: 24 hours

midwinter: 0 hours

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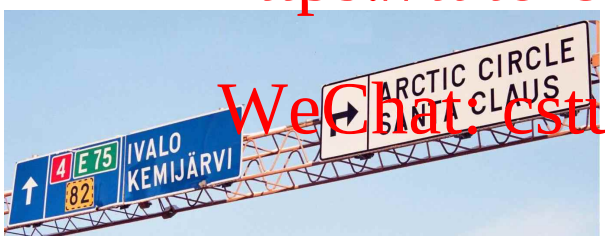

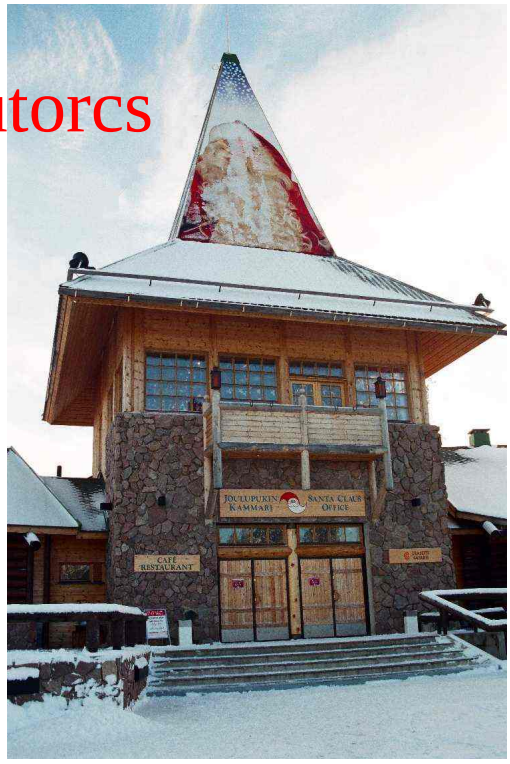

Photo 5.4: Top left: road sign to Santa (Rovaniemi, Finland). Right: the official home of Santa (Santa Claus Village, Finland). Bottom left: Singapore. (Source: PA.)

- At large distances from the equator, summer daytimes are very long; on some occasions there is no sunrise or sunset for a period greater than one day. For simplicity, in such cases we say that the daytime is 24 hours.
- Similarly, in midwinter we say that the daytime is 0 hours.
- Figure 5.2 shows the daytimes in Brisbane at weekly intervals throughout a calendar year. [1] The graph of daytime lengths in every year will be very similar; clearly, the graph resembles a sine wave!

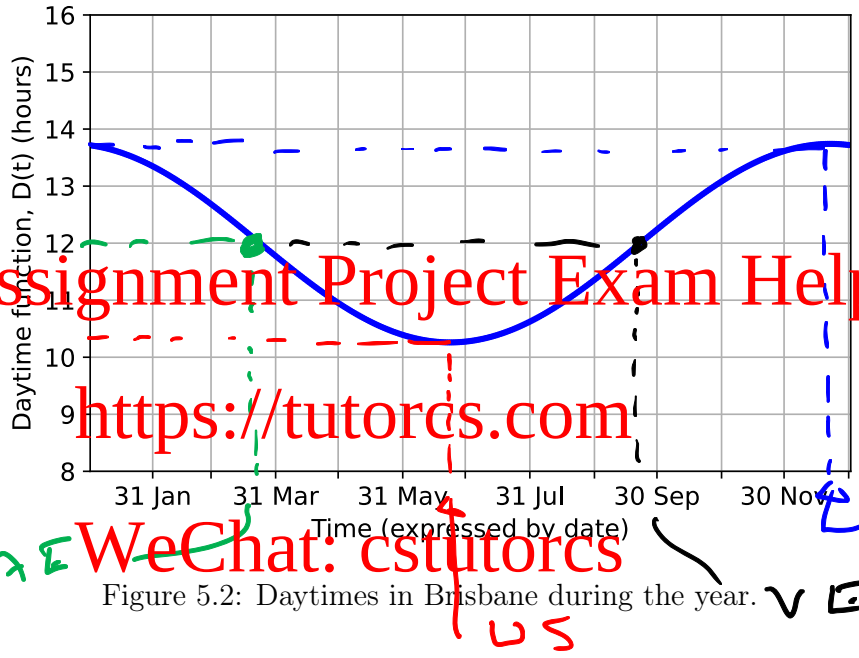

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Figure 5.2: Daytimes in Brisbane during the year.

Question 5.2.3

Use the graph in Figure 5.2 to answer the following questions.

(a) When are the solstices in Brisbane, and how long are the daytimes?

Summer solstice : 21 Dec , 13.7 hours

Winter solstice : 21 Jun , 10.3 hours

(b) When are the equinoxes in Brisbane?

Autumnal equinox : 21 Mar

Vernal equinox : 21 Sep

[1] Daytimes were found by subtracting the sunrise time from the sunset time. Sunrise time is defined as the time at which any part of the sun is first visible on a clear, cloudless day. Sunset time is defined as the time at which any part of the sun is first **not** visible on a clear, cloudless day. This definition of sunset differs slightly from standard usage.

Question 5.2.4

On some international flights, in-flight maps show areas of night and day superimposed on the surface of Earth; see Figure 5.5 for two examples.

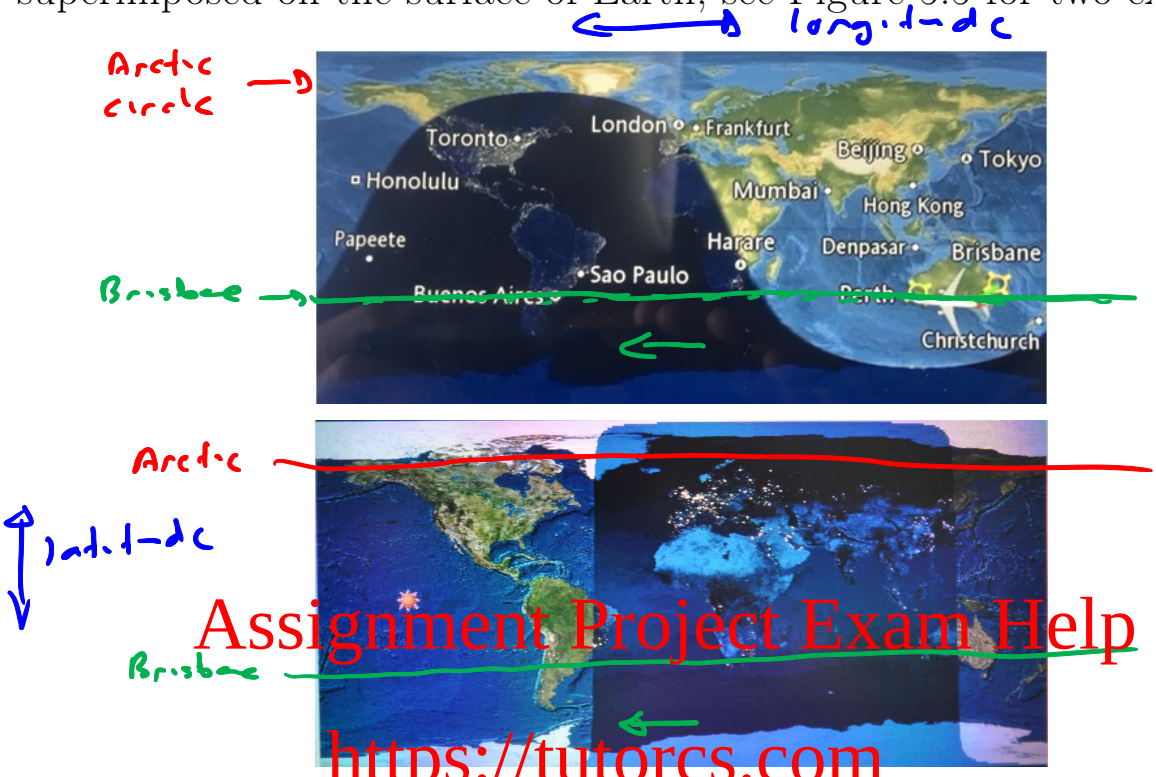

Photo 5.5: In-flight maps. (Source: PA.)

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(a) Describe everything you can about the date and time of day in Brisbane when the first photograph was taken. Justify your answer.

(b) Describe everything you can about the date and time of day in Brisbane when the second photograph was taken. Justify your answer.

Case Study 10: **Modelling daytimes**

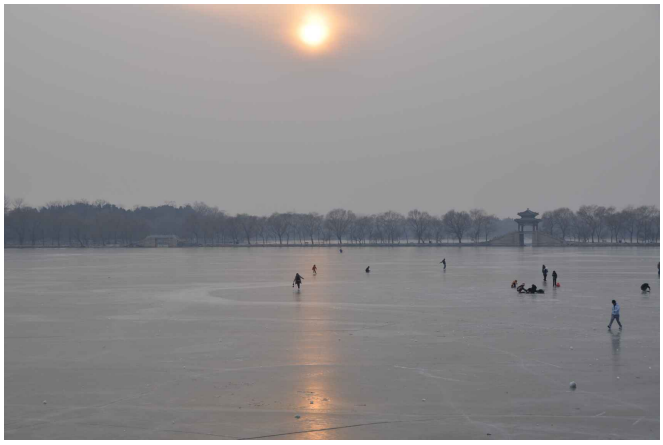

Photo 5.6: Sunrise over Kunming Lake in winter, Beijing, China. (Source: PA.)

Photo 5.7: Sunset over prison guard tower, near Krakow, Poland. (Source: PA.)

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- Every location on Earth has a *latitude*, which is a measure of its distance from the equator. On any given day, **every location with the same latitude has the same daytime length.**

- At each location on Earth, the daytimes repeat in a yearly pattern. Therefore, they can be modelled using sin, as a function of the day of the year. (In reality, daytimes will vary slightly from these functions as days are discrete time steps whereas the Sun and Earth move continuously.)

Question 5.2.5

If t is the day number in the year (starting from $t = 0$ on January 1st), then the length of the daytime in hours at any point in the southern hemisphere is given by the function

$$D(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right)$$

where K is a constant determined by the latitude of the point. Near the equator $K \approx 0$ hours, and its value increases for more southerly locations.

***Question** 5.2.5 (continued)*

$D(t)$ for Brisbane is plotted in Figure 5.3, where $K \approx 1.74$ hours.

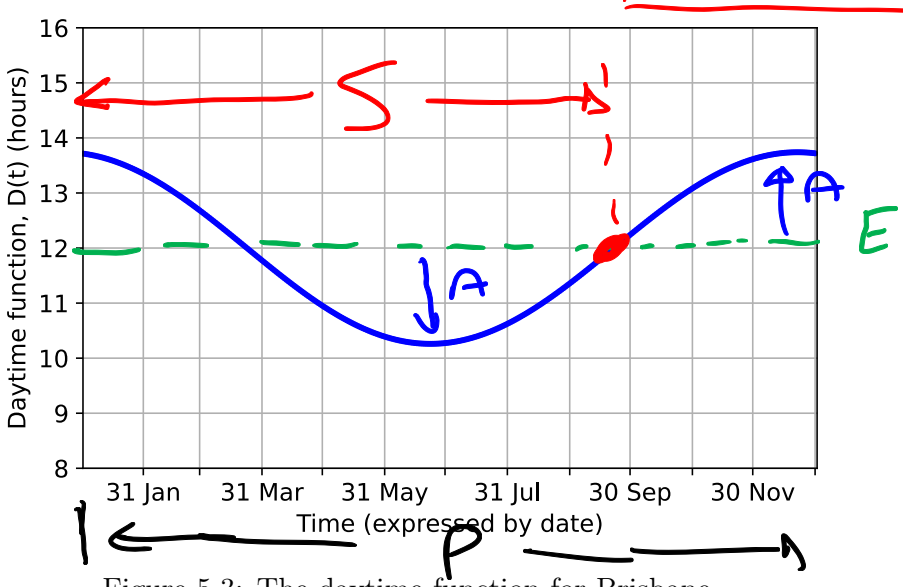

Figure 5.3: The daytime function for Brisbane.

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Fill in the missing entries in the following table, describing the physical and mathematical significance of each term in $D(t)$ for Brisbane, and indicate these on the graph.

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Value	Mathematical meaning	Physical meaning
12 hours	Equilibrium of the sine wave	Average number of daylight hours
1.74 hours	Amplitude of the sine wave	maximum increase or decrease in day length from equilibrium
365 days	Period of the sine wave	Number of days in a year, as the phenomenon follows a yearly cycle
264 days	Shift of the sine wave (to the right) horizontally	Time of year of the vernal equinox

Question 5.2.6

Briefly explain how to mathematically find when the solstices occur in Brisbane, using the function $D(t) = 12 + 1.74 \sin\left(\frac{2\pi}{365}(t - 264)\right)$ (as opposed to reading off the graph).

Summer solstice – longest day

Need $D(t)$ to be a maximum

This is when $\sin(\ ) = +1$

$\frac{2\pi}{365}(t - 264) = \pi/2$ $\left[\sin 0 = 0,\ \sin\frac{\pi}{2} = +1\right.$

$t - 264 = \frac{365}{4}$

$t = 264 + \frac{365}{4} \simeq 355$ days

(or 21 Dec)

with solstice, $\sin(\ ) = -1$

$\frac{2\pi}{365}(t - 264) = 3\pi/2$

$t = 264 + \frac{3 \times 365}{4} \simeq 538$ days

or $538 - 365 \simeq 173$ days

(21 June)

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Note that we modelled the number of daylight hours for the southern hemisphere. We will consider a model for the northern hemisphere in the next lecture.

End of Case Study 10: Modelling daytimes.