Lecture 33: **Stages of life**

Learning objectives

✓ Draw and interpret life cycle diagrams

✓ Understand how to get a system of DE's from a life cycle diagram

✓ Solve and interpret systems of DE's (Euler)

✓ Analyse different proposals for saving an endangered species

Scientific examples

✓ Endangered turtles

Maths skills

✓ Understand and be able to use Euler's method for systems of DE's

12.2 Going through a difficult stage

- We previously modelled populations using exponential and logistic DEs. In each case we assumed that populations were *homogeneous*; that is, every individual in the population had an identical impact on population growth.

- Many organisms have different life stages, each with substantial differences in typical survival rates and reproduction rates.

- For example, in many species, small juveniles have a low survival rate and do not reproduce, whereas mature individuals have a high survival rate and typically do reproduce.

- Hence, simple models based on single DEs are inaccurate for more advanced organisms, particularly those with long life spans. In such cases, systems of DEs give rise to better models.

- In one type of model, populations are classified into groups based on their *life stages*, such as *juvenile* or *breeding adult*.

- Rather than applying a constant growth rate to every individual in the population, a system of DEs includes:
  - the *distribution* of the population within the distinct groups;

- differing rates of *reproduction* and *death* within groups; and
- the *transitions* of individuals between groups.

- *Life-cycle diagrams* are useful aids to writing the equations in a system of DEs. These diagrams show the rates of *transition* between stages.

> **Life-cycle diagram**
>
> **Life-cycle diagrams** represent all possible transitions between stages in the life-cycle of an organism. Each stage is represented as a circle in the diagram, with a directed arrow joining Stage A to Stage B whenever it is possible for an individual to transition from Stage A to Stage B. Each arrow has an associated number, which is the *rate* of transition.

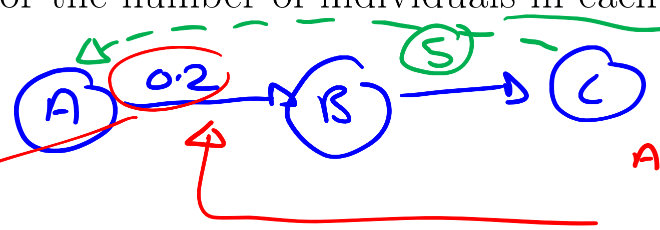
→ eg per year

- In a life cycle diagram, not all pairs of stages will have an arrow between them, as some particular transitions may not be possible.

- In order to draw the life-cycle diagram for an organism, we need to know the number of stages, all possible transitions to and from each stage, including reproduction, transitions due to the passage of time, and deaths, as well as the number or probability associated with each possible transition.

- We used a dashed arrow to represent reproduction rates to distinguish it from transitions between stages (an adult does not *become* the offspring).

- Once we have drawn a life-cycle diagram, it is usually "easy" to write a system of DEs for the number of individuals in each stage.



For every C, S of A are added per time step

A grows to become B

- 20% grow per unit time
- typically A's exist for 5 time steps => 20% $(\frac{1}{5})$ A → B

0.2 → proportion of A grows to B in a given time

"20% of A becomes B per unit time"

Question 12.2.1

Consider an idealised fish species with two distinct life stages: juvenile and adult. Each month, on average: time step

- J Juveniles do not breed, have a 50% probability of surviving to adulthood, and a 50% probability of dying.
- Adults produce 5 offspring (juveniles), and then die.

A

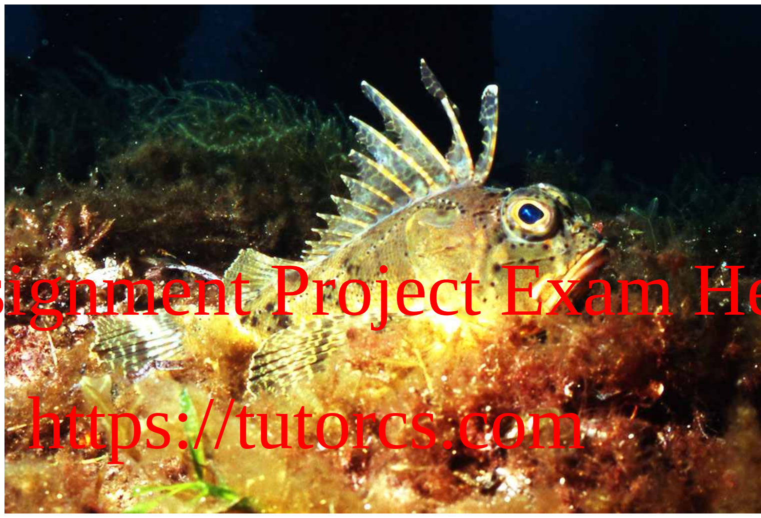
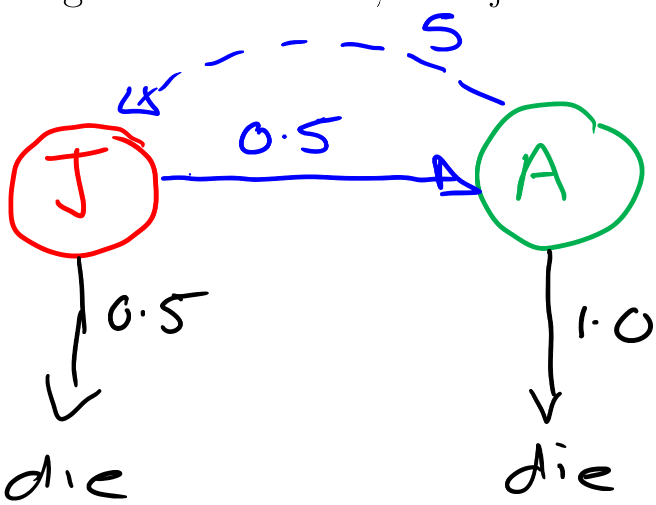


Photo 12.2: Bighead Gurnard Perch (*Neosebastes pandus*). (Source: DM.)

(a) Draw a life-cycle diagram for this fish, with juvenile and adult stages.



Question 12.2.1 (continued)

(b) Let the populations of juveniles and adults at any time be $J(t)$ and $A(t)$ respectively. Write a system of DEs for these populations.

(J→A) (death) (birth)

$$J' = -0.5J - 0.5J + 5A$$

$$\boxed{J' = -J + 5A}$$

$$\boxed{A' = +0.5J - 1A}$$

(J→A) (death)

initial

(c) Assume that a specific population comprises 20 juveniles and 3 adults at time $t = 0$ months. Use Euler's method and a step size of 1 month to estimate the number of fish in each stage at time $t = 2$ months.

we have $J(0) = 20$, $A(0) = 3$, $h = 1$ month

$$J(1) = J(0) + hJ'(0)$$
$$= J(0) + h[-J(0) + 5A(0)]$$
$$= 20 + 1[-20 + 5(3)] = 15$$

$$A(1) = A(0) + hA'(0)$$
$$= A(0) + h[0.5J(0) - A(0)]$$
$$= 3 + 1[0.5(20) - 3] = 10$$

$$J(2) = J(1) + hJ'(1)$$
$$= 15 + 1[-15 + 5(10)] = 50$$

$$A(2) = A(1) + hA'(1)$$
$$= 10 + 1[0.5(15) - 10] = 7.5$$

After 2 months we have 50 juveniles and 7.5 (8) adults.

Case Study 28: **Total turtle turmoil**

Image 12.2: Loggerhead sea turtle. (Source: en.wikipedia.org.)

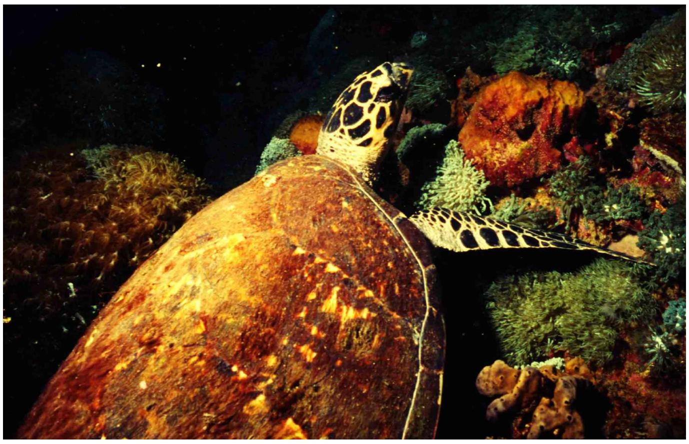


Photo 12.3: Sea turtle species.(Source: DM.)

- The loggerhead sea turtle (*Caretta caretta*) is a large marine turtle, reaching a length of around 1 m and a mass of more than 100 kg.
- The species is distributed throughout temperate, subtropical and tropical regions, and nests in a number of countries, including Australia.
- Individuals often live for more than 50 years.
- The species is listed as threatened, largely due to human activity, so is likely to become endangered within the foreseeable future.
- Ecologists have studied these turtles in detail, in order to better understand how populations change over time, to investigate possible management strategies and predict the impacts of further environmental change and human activity.
- Researchers in [4] and [6] found that these turtles move through seven distinct stages during their life cycle, and developed a population model based on these stages. = 7 DEs !
- (For interest, the researchers used a matrix model rather than a system of DEs. However, such models are equivalent to using a system of DEs and Euler's method with a step size of 1 time unit.)

- We will study a simplified version of their model, with the seven stages collapsed into three for ease of calculation.
- Table 12.1 shows the life stages used for the simplified model, along with the estimated proportion of the total turtle population, and the global number of individuals, in each stage.

Individual population

Stage	Description	Age (years)	Proportion	Global population
A	hatchlings	< 1	0.20651	1445570
B	youth	$1 - 23$	0.79097	5536790
C	breeding adult	$24 - 54$	0.00252	17640

Table 12.1: Loggerhead sea turtles classified into three life stages.

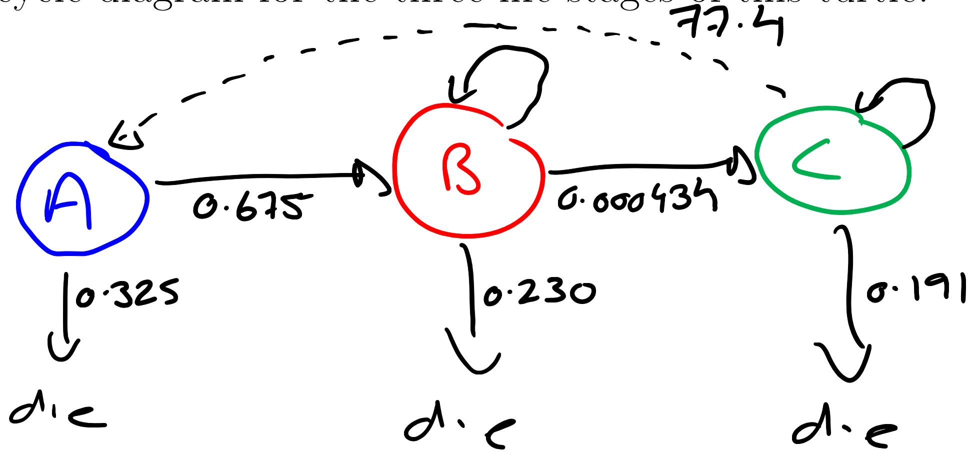
- Each year, turtles transition with the following (rounded) probabilities:
  - Hatchlings become youths with probability $p = 0.675$ or die with probability $p = 0.325$.
  - Youths become breeding adults with probability $p = 0.000434$ and youths die with probability $p = 0.230$.
  - Breeding adults produce new hatchlings (77.4 per adult), and die with probability $p = 0.191$.

time = years

- The estimated global population across all life stages was 7 million.

Question 12.2.2

(a) Draw a life-cycle diagram for the three life stages of this turtle.



Question 12.2.2 *(continued)*

(b) Write a system of DEs for the turtle population.

$A' = -0.675A - 0.325A + 77.4C$

$\boxed{A' = -A + 77.4C}$

$B' = +0.675A - (0.230 - 0.000434)B$

$\boxed{B' = 0.675A - 0.230434B}$

$\boxed{C' = +0.000434B - 0.191C}$

(c) Use Euler's method with a step size of 1 year to estimate the number of hatchlings after one year.

$A(0) = 1445570$
$C(0) = 17640$ } from table above

$h = 1$ year

$A(1) = A(0) + hA'(0)$

$= A(0) + h[-A(0) + 77.4C(0)]$

$= 1445570 + 1[-1445570 + 77.4(17640)]$

$= 1,365,336$

(d) Using Euler's method, after one year there are 5236685 turtles in the youth stage and 16673 breeding adults. What does this indicate about future turtle populations?

	Year 0	Year 1
Hatchlings (A)	1,445,570	1,365,336
Youths (B)	5,536,790	5,236,685
Adults (C)	17,640	16,673

All populations declining ⟹ bad!

This population can be modelled using a computer program.

Program specifications: Develop a Python program that uses Euler's method with a step size of 1 year to model the turtle population or 30 years.

Program 12.1: Turtles

```
# Uses Euler's method to model the turtle population.
from pylab import *

# Initialise variables.
maxt = 30
Apops = zeros(int(maxt+1))
Bpops = zeros(int(maxt+1))
Cpops = zeros(int(maxt+1))
Apops[0] = 1445570
Bpops[0] = 5536790
Cpops[0] = 17640
stepsize = 1

# Step through Euler's method for 30 years.
i = 1
while i < (maxt+1):
    dA = -Apops[i-1] + 77.1 * Cpops[i-1]
    dB = 0.675 * Apops[i-1] - 0.230434 * Bpops[i-1]
    dC = 0.000434 * Bpops[i-1]- 0.191 * Cpops[i-1]
    Apops[i] = Apops[i-1] + stepsize * dA
    Bpops[i] = Bpops[i-1] + stepsize * dB
    Cpops[i] = Cpops[i-1] + stepsize * dC
    i = i + 1

# Output the graph.
times = arange(0, maxt+1)
plot(times, Apops/1E6, "bx", markersize=8,label='Stage A')
plot(times, Bpops/1E6, "r+", markersize=8,label='Stage B')
plot(times, Cpops/1E6, "gs", markersize=6,label='Stage C')
plot(times, (Apops+Bpops+Cpops)/1E6, "ko", markersize=6,label='Total')
xlabel("Time (years)")
ylabel("Number of turtles (millions)")
grid(True)
legend()
show()
```

maximum time

arrays

initial populations

step size

DEs A', B', C'

Euler

Plot

Figure 12.1 shows the output from running the program.

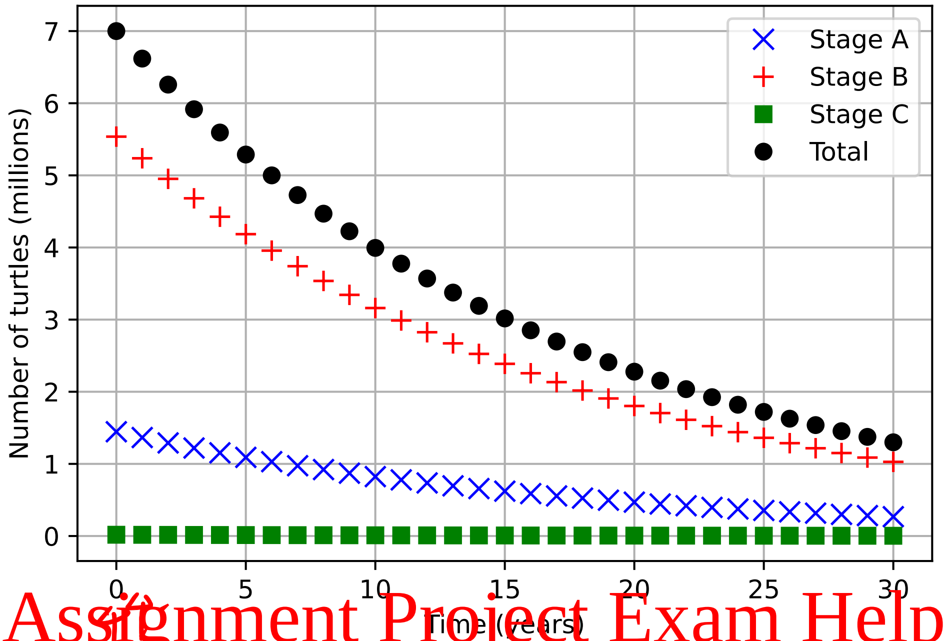


Figure 12.1: Turtle population modelled using Euler's method.

Question 12.2.3

Researchers and authorities have proposed various conservation strategies for the sea turtle; see [7]. Briefly discuss some possible strategies, and explain how the population model would change to reflect them.

- Enhance reproduction
- Protect habitat (hatchlings)
- Reduce pollution
- Fishing - TEDs "turtle exclusion devices"
  save youths, adults from being caught in fishing nets.

End of Case Study 28: Total turtle turmoil.