Lecture 34: **Predator-prey systems**

Learning objectives

✓ Analyse the interaction between populations in which one species relies on another as their food source

✓ Critically evaluate real-world data compared with model predictions

Scientific examples

✓ Canadian lynx and snowshoe hare

Maths skills

✓ Interpret the meaning of terms in a system of DE's

12.3 Eat or be eaten

- In addition to modelling individual organisms with multiple life stages, systems of DEs can also model interactions between multiple species.

Assignment Project Exam Help

- For example, the classical *predator/prey* problem in ecology considers what happens to the populations of two species when one preys on the other.

https://tutorcs.com

- In laboratory situations there is control over these interactions. In nature, inter-species interactions are highly complex. We will first investigate a controlled example, then model a real interaction. The controlled example is very simple, but is not completely unreasonable.

WeChat: cstutorcs

Photo 12.4: Left: skeleton of *Tyrannosaurus rex*. Right: skeleton of *Triceratops horridus*. (Source: PA.)

Case Study 29: **It's just not cricket**

- One method of predicting what may happen in a real-world situation is to simulate it in a laboratory.
- Unpredictable phenomena complicate and impact predator/prey interactions in nature. However, controlled laboratory simulations can give valuable insight into real situations.
- Consider a controlled, time-compressed laboratory experiment simulating the effects of immigration, emigration, births and deaths on populations of frogs (predators) and crickets (prey).
- Initially the experiment comprises 60 frogs and 400 crickets. Each day:
  - 15 crickets are introduced into the experiment (modelling immigration and birth of crickets);
  - 25% of the frogs each eat a cricket (death of crickets);
  - 12 frogs are removed (modelling emigration and death of frogs); and
  - for each 25 crickets present, one new frog is introduced (modelling birth and immigration of frogs based on available food resources).

Assignment Project Exam Help
https://tutorcs.com
WeChat: cstutorcs

Photo 12.5: Left: Striped burrowing frog, *Litoria alboguttata*. Right: cricket. (Source: DM.)

Question 12.3.1

Let $F(t)$ and $C(t)$ be the populations of frogs and crickets at time t in days.

(a) Write DEs involving the rate of change of **each** of the populations.

$$C' = +15 - 0.25F$$

$$F' = -12 + \frac{C}{25}$$

can be solved (not in SCIE1000)

Assignment Project Exam Help

$A = 40$ ... $P = \frac{2\pi}{0.1}$ ... $P = 63$ days

(b) Using differentiation it can be shown that solutions to the DEs are:

https://tutorcs.com

$$F(t) = 40 \sin 0.1t + 60 \qquad C(t) = 100 \cos 0.1t + 300.$$

Figure 12.2 shows $C(t)$. On the graph, sketch $F(t) = 40 \sin 0.1t + 60$.

WeChat: cstutorcs

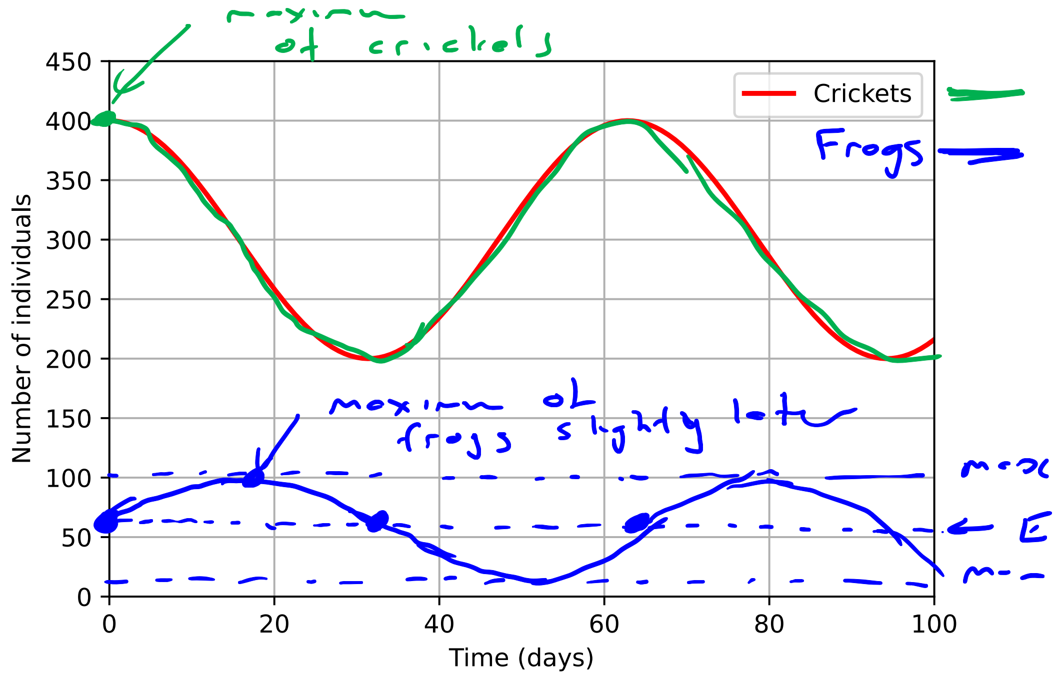


Figure 12.2: Population of crickets.

***Question** 12.3.1 (continued)*

(c) Interpret the population dynamics. One population is "leading" and the other "lagging". Identify which is which, and explain your answer.

Crickets "lead", frogs "lag"

Cricket population peaks before the frog population

Assignment Project Exam Help

https://tutorcs.com

WeChat: cstutorcs

Photo 12.6: Left: genuine KTF. Right: fossilised cricket. (Source: PA.)

End of Case Study 29: It's just not cricket.

- Now we will develop a more realistic predator/prey model. In general, such models are based on the assumptions that:
  - the prey has no other predators, and the predator no other prey;
  - there is no significant change to the environment or species' genetics;
  - the prey species is not resource limited and so grows rapidly; prey growth is regulated solely by predator consumption.
  - predator population growth depends on the availability of prey; the predator declines to extinction in the absence of prey.
  - there is no limit on the amount of prey the predators are able to consume.
- The best-known predator/prey model is the **Lotka-Volterra** model.

Lotka-Volterra model

predator prey

Let $P(t)$ and $Q(t)$ be the population sizes of a predator and prey species respectively, at any time t. The following system of DEs forms the *Lotka-Volterra model*:

Assignment Project Exam Help
https://tutorcs.com
WeChat: cstutorcs

prey
$$Q' = aQ - bPQ$$
predator
$$P' = -cP + dPQ$$

per unit time

where a, b, c and d are positive constants whose values depend on various characteristics of the species and their physical interactions.

Question *12.3.2*

Explain the meaning of each term in the Lotka-Volterra equations.

(Q') $+aQ$ — growth rate (breeding) of prey

(P') $-cP$ — competion between predators (die)

$$\left.\begin{array}{l} -bPQ \\ +dPQ \end{array}\right\}$$ interaction term — chance that a predator encounters a prey leading to predation

Question *12.3.3*

Consider the Lotka-Volterra equations for non-zero populations of both the predator and prey species.

(a) Under what conditions are the populations of each species:

(i) increasing?

$Q' > 0$ (Q increasing)

$a\cancel{Q} - bP\cancel{Q} > 0$

$a > bP$

or $\boxed{P < a/b}$

$P' > 0$ (P increasing)

$-c\cancel{P} + d\cancel{P}Q > 0$

$\boxed{Q > \frac{c}{d}}$

Assignment Project Exam Help

(ii) decreasing?

https://tutorcs.com

converse

$P > a/b$

WeChat: cstutorcs

$Q < c/d$

(b) What is the physical interpretation of this result?

$P < a/b \Rightarrow Q' > 0$

if number of predators is small, number of prey increases (threshold)

$Q > c/d \Rightarrow P' > 0$

if number of prey is large, number of predators increases (plenty of food)

Case Study 30: **Snowshoe hares and Canadian lynx**

Image 12.3: Canadian lynx chasing a snowshoe hare. (Source: www.animalspedia.com.)

- The Canadian lynx, *Lynx canadensis*, is a member of the feline family distributed predominantly in Canada and Alaska. Lynx are carnivorous, with individuals weighing 8 to 15 kg, and living for up to 15 years.
- The primary food source (up to 95%) of the Canadian lynx is the snowshoe hare, *Lepus americanus*. The hare has large hind feet (for moving on snow) and turns white in winter.

Assignment Project Exam Help
https://tutorcs.com

- People have hunted these lynx and hares for their fur for many years. Harvest records dating from the 1730s allow long-term population estimates.

WeChat: cstutorcs

- Figure 12.3 (from [30]) graphs these data over 90 years, and shows a series of reasonably regular fluctuations in the sizes of both populations. Note the similarity to the periodic population movements in the laboratory-controlled predator/prey relationship between frogs and crickets.

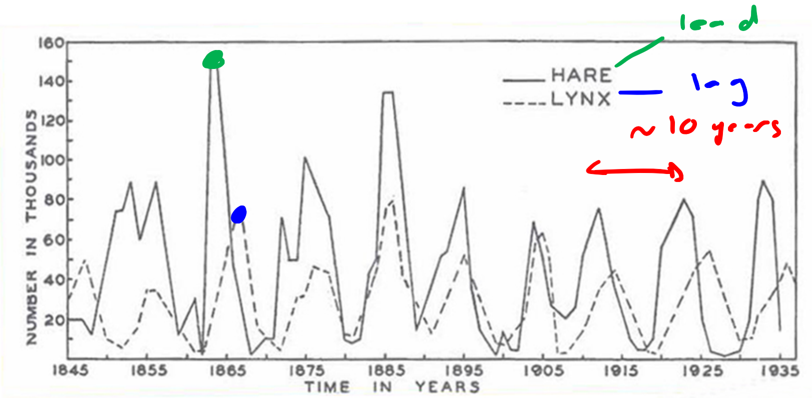


Figure 12.3: Numbers of Canadian lynx and snowshoe hares. (Source: [30].)

Question 12.3.4

Let $L(t)$ and $H(t)$ be the populations of lynx (predators) and hares (prey) respectively, **in thousands**. The Lotka-Volterra equations are:

$$H' = aH - bHL \qquad\qquad L' = -cL + dHL$$

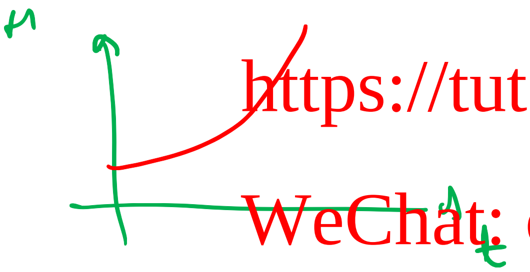
(a) If either population suddenly became extinct, what does the model predict will happen to the other population? Simplify the differential equations and then sketch rough graphs of the populations.

L=0

if L → 0

$\boxed{H' = aH} + 0$

Growth will be exponential



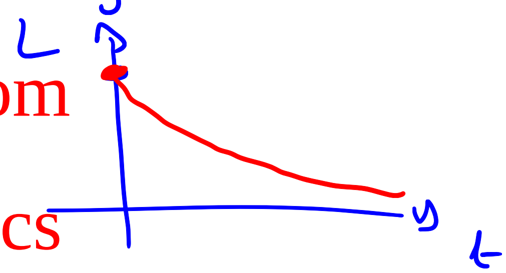
H=0

if H → 0

$\boxed{L' = -cL}$

Decay (exponential)

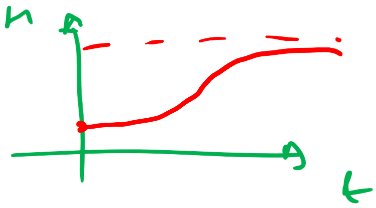
Lynx die out



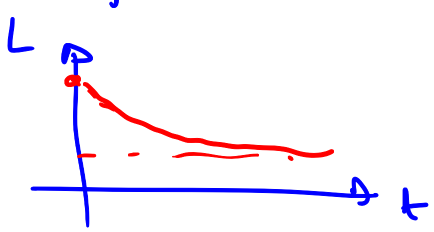
Assignment Project Exam Help
https://tutorcs.com
WeChat: cstutorcs

(b) What would you expect to happen in reality?

Hares – eventually resource limited



Lynx – find other food .... or die

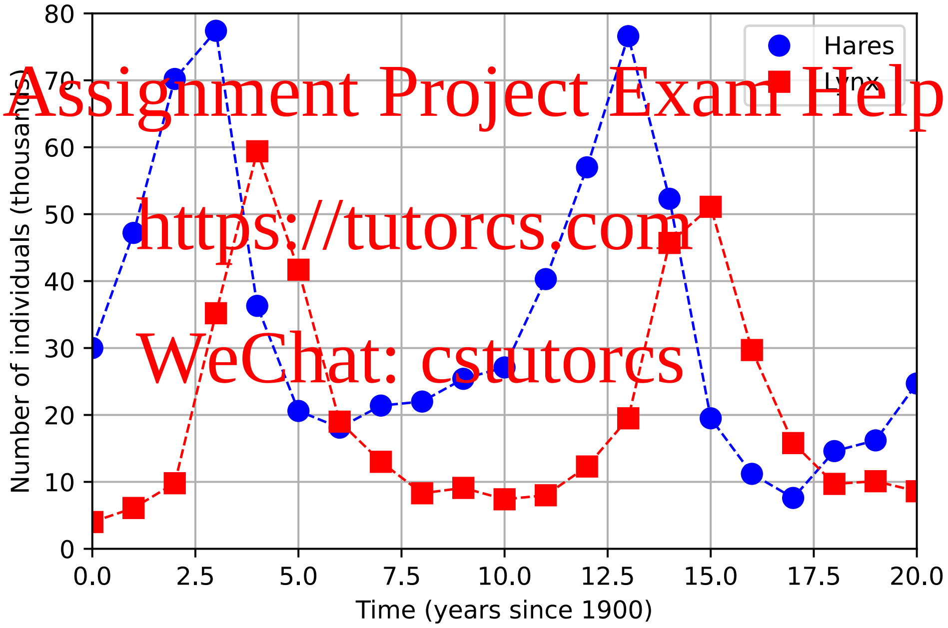


Example 12.3.5

Table 12.2 and Figure 12.4 show data from the Canadian Government and the Hudson's Bay Company, estimating the populations of hare and lynx in part of their range from 1900 to 1920. (All populations are in thousands.)

Year	Hares	Lynx	Year	Hares	Lynx	Year	Hares	Lynx	Year	Hares	Lynx
1900	30	4	1905	20.6	41.7	1910	27.1	7.4	1915	19.5	51.1
1901	47.2	6.1	1906	18.1	19	1911	40.3	8	1916	11.2	29.7
1902	70.2	9.8	1907	21.4	13	1912	57	12.3	1917	7.6	15.8
1903	77.4	35.2	1908	22	8.3	1913	76.6	19.5	1918	14.6	9.7
1904	36.3	59.4	1909	25.4	9.1	1914	52.3	45.7	1919	16.2	10.1
									1920	24.7	8.6

Table 12.2: Populations of lynx and hares (in thousands).



Assignment Project Exam Help
https://tutorcs.com
WeChat: cstutorcs

Figure 12.4: Graph of the populations of lynx and hares (in thousands).

- Experimentation and analysis show that for this time period, reasonable values for the constants a, b, c and d in the Lotka-Volterra equations are: $a = 0.484$, $b = 0.028$, $c = 1$ and $d = 0.032$ (in appropriate units). per year

Now we can use Euler's method to model the population sizes.

Program specifications: Develop a Python program that uses Euler's method with step size of 0.01 year to model the populations of lynx and hares.

Program 12.2: Lotka-Volterra model of hares and lynx.

```
# Uses Euler's method and Lotka–Volterra equations to model
# populations of lynx and hare from 1900 to 1920.
from pylab import *

# Initialise variables for Euler's method.
step = 0.01
time = arange(0,20+step,step)
a = 0.484
b = 0.028
c = 1
d = 0.032
hares = zeros(int(size(time)))
lynx= zeros(int(size(time)))
hares[0] = 30.0
lynx[0] = 4.0

# Step through Euler's method with stepsize step.
steps = size(time)
i = 0
while i < steps-1:
    dh = a * hares[i] - b * hares[i] * lynx[i]
    dl = -c * lynx[i] + d * hares[i] * lynx[i]
    hares[i+1] = hares[i] + step * dh
    lynx[i+1] = lynx[i] + step * dl
    i = i+1

# Output graphs.
plot(time, hares, "b-", linewidth=2, label='Hares')
plot(time, lynx, "r-", linewidth=2, label='Lynx')
xlabel("Time (years since 1900)")
ylabel("Number of individuals (thousands)")
xlim(0,20)
ylim(0,70)
grid(True)
legend()
show()
```

h

constants

initial

L-V DEs

Euler

output

Assignment Project Exam Help
https://tutorcs.com
WeChat: cstutorcs

Example *12.3.6*

Below is the output of the above program.

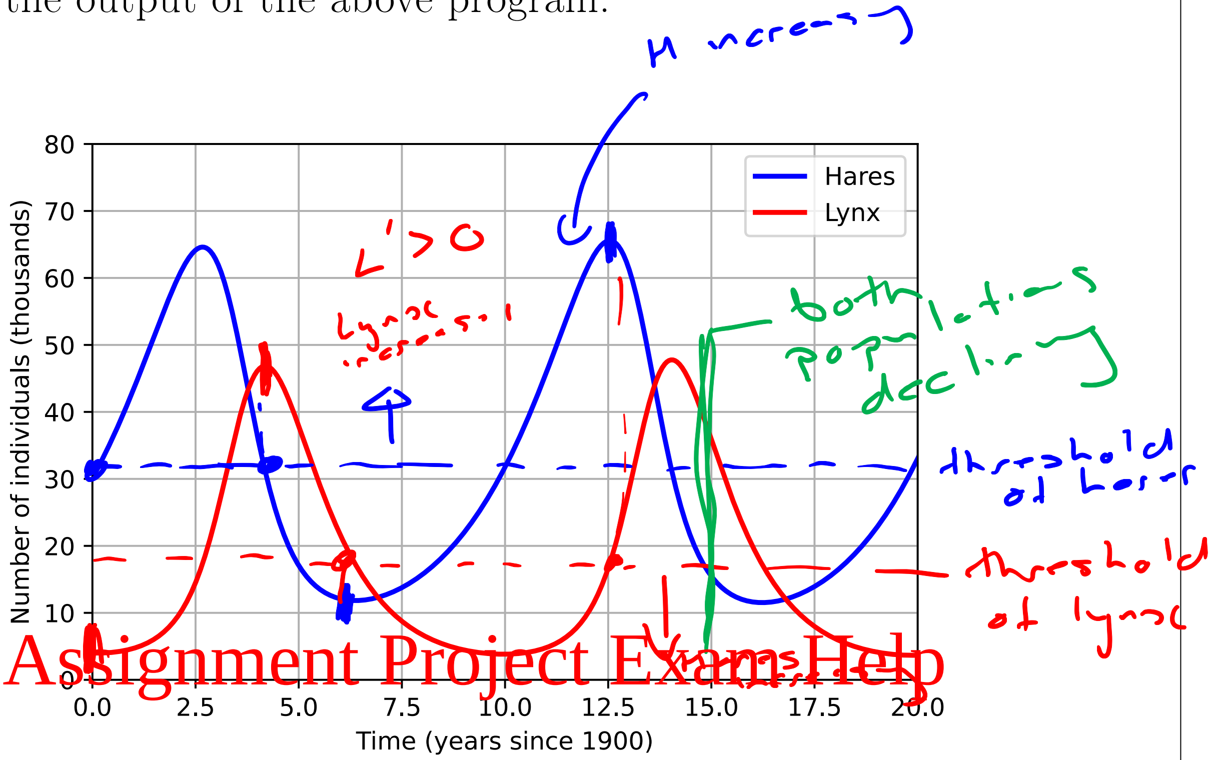


Figure 12.5: Modelled hare and lynx populations.

Assignment Project Exam Help
https://tutorcs.com
WeChat: cstutorcs

- At time $t = 0$ years (corresponding to year 1900), data show that there were 30 (thousand) hares and 4 (thousand) lynx in the monitored region.
- Figure 12.6 compares the modelled population sizes over 20 years with the real (measured) data for each population.

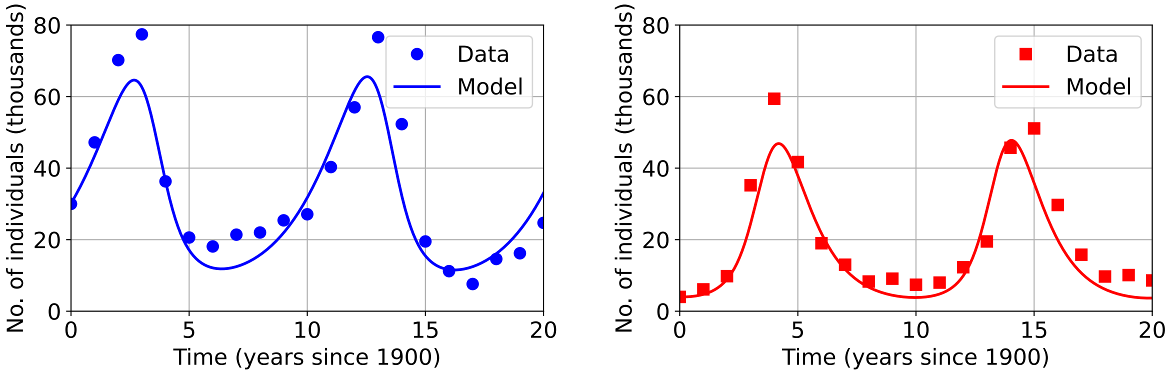


Figure 12.6: Real and modelled populations for hare (left) and lynx (right).

Question 12.3.7

(a) Comment on the results in Example 12.3.6.

Model is fairly consistent with data

- underestimate peaks
- consistent with period of oscillations

(b) Critically evaluate the following possible media statement:

> *A survey has shown that the populations of lynx and snowshoe hares are both in decline. We need to act promptly or else one or both species will become extinct.*

Assignment Project Exam Help

The system dynamics means that although both can be in decline, recovery is possible e.g. if lynx declines to near zero, hare population would recover

https://tutorcs.com

WeChat: cstutorcs

Photo 12.7: Three top predators. Left: polar bear, *Ursus maritimus*. Centre: Komodo dragon, *Varanus komodoensis*. Right: Siberian tiger, *Panthera tigris altaica*. (Source: PA.)

End of Case Study 30: Snowshoe hares and Canadian lynx.