THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2019

SCIE1000 Theory and Practice in Science

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - specified materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

One A4 sheet of handwritten or typed notes double sided is permitted

One unmarked bilingual dictionary is permitted

Materials To Be Supplied To Students:

None

Instructions To Students:

Answer all questions on the exam paper in the spaces provided. You may remove the final page of this exam, which contains reference information and a formula sheet. Nothing on that page will be graded. It must remain in the examinations room when you leave.

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

For Examiner Use Only

Page	Mark
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
17	

Total ________/56

To answer each question you will need to use the information on Page 19. A formula sheet can be found on Page 20. Your solutions will be marked on the correctness and clarity of your explanations and communication. Include units in your answers wherever relevant.

1. Consider a plastic water bottle, with height h and radius r, both measured in cm. Modelling a water bottle as a cylinder, the surface area S of the bottle is: $S = 2\pi rh + 2\pi r^2$.

   (a) Suppose that a particular water bottle, with 565 mL capacity, has a 3 cm radius, a height of 20 cm, and the plastic of the bottle has a thickness of 0.05 cm. Show that the mass of the plastic in this bottle is approximately 20 g.

   (2 marks)

   (b) Approximately 8 million tonnes of plastic enter the ocean each year. Imagine that all of this plastic is in the form of water bottles with a size and shape as in part (a). Calculate the average number of water bottles that are discarded into the ocean *each year per person*. Assume a world population of 7.5 billion people (that is, 7.5×10^9 people).

   (2 marks)

(continued over)

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(c) Write a paragraph which communicates to the general public the urgent need for scientific study into the pollution of the oceans by plastic. You may wish to communicate the scale of the plastic pollution problem, why it is a problem, and why it is an urgent priority. Be precise, clear and concise! (3 marks)

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2. Suppose that you are part of a research team that is studying the *different types of plastic litter* found on ocean shorelines and beaches. An area of shoreline is divided into 10 metre by 5 metre cells and a volunteer scans each cell for plastic waste. Your research team tracks the cumulative number of different types of plastic found as a function of the number of cells searched. One researcher in your team proposes that the following model provides a good fit to the data:

$$T(c) = 4.2\, c^{0.31}$$

where c is the number of cells examined and T is the cumulative total number of different types of plastic waste observed in these cells.

(a) Consider a 200 km stretch of shoreline in a similar region to the one you've studied. Assuming that the shoreline has an average width of 5 m, determine the number of different types of plastic waste that would be found on the whole shoreline, using the given model.

(2 marks)

(b) According to the given model, how many 10 metre by 5 metre cells would need to be searched before your research team finds 50 different types of plastic waste?

(2 marks)

(continued over)

(c) Another researcher in your team says a linear model fits the data pretty well. In a few sentences, describe how you would convince your team member that a power function model, like the one given above, is likely to be a better fit in this situation. Be precise, clear and concise!

(3 marks)

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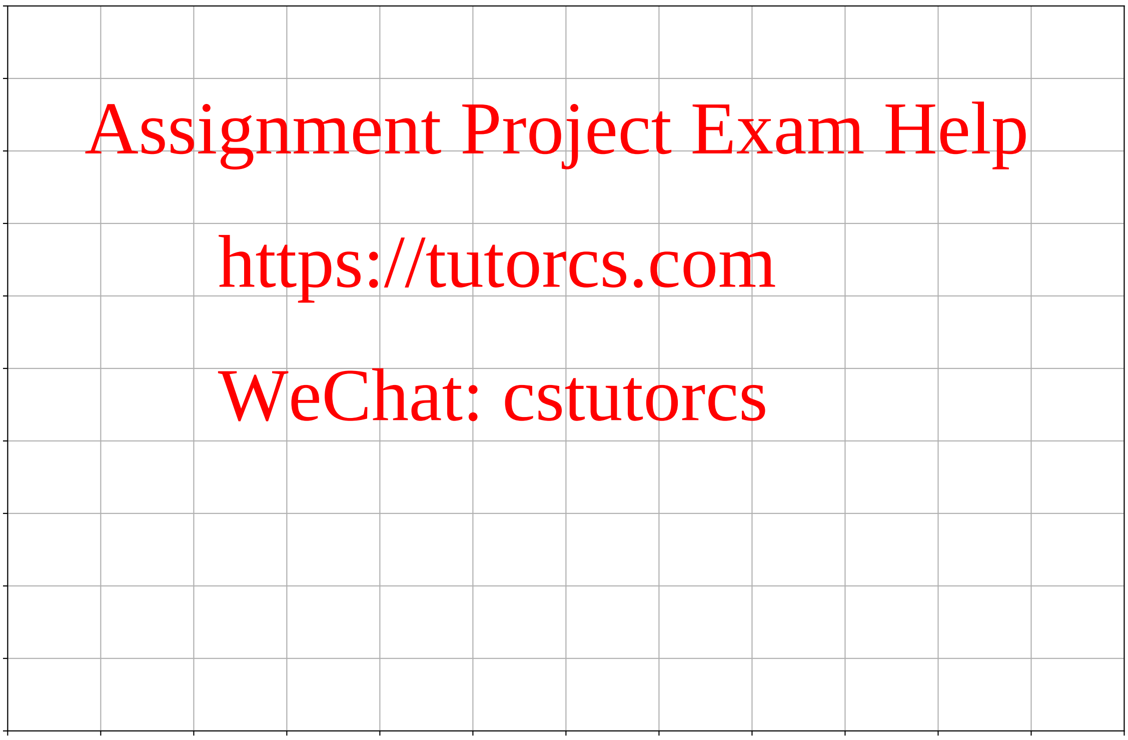
3. In an experiment, a set of plastic syringes is placed in an autoclave for steam sterilisation. A technician records the temperature of the autoclave at various times, as shown in the table below (where the time is measured in minutes since the syringes were placed into the autoclave).

Time (min)	0	5	10	20	30	35
Temperature (°C)	20	100	120	140	140	130

(a) Plot the data from the above table on the axes below. Use the horizontal axis to represent time and ensure that you communicate your graph clearly with labelling. Assume that the temperature changed linearly over time between each measured data point and the next, and indicate this by drawing appropriate lines on your graph.

(2 marks)



(b) Calculate the area under the curve for your graph in part (a) between 10 minutes and 35 minutes.

(2 marks)

(c) Calculate the plastic syringes' total exposure to temperatures *above a baseline temperature* of 120 °C during the entire experiment.

(2 marks)

(d) Why would a technician be more interested in the calculation done in part (c) than the calculation done in part (b)?

(1 mark)

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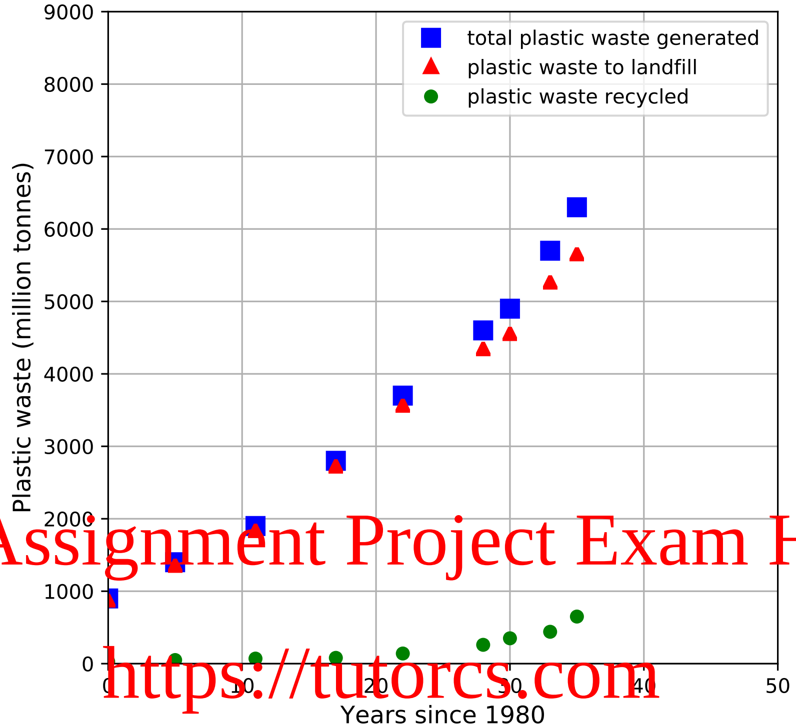
4. It is estimated that as of 2015, the total amount of plastic waste generated on Earth was about 6300 million tonnes. Below is some detailed data from 1980 to 2015 indicating the total plastic waste generated, the amount of this plastic waste that has been sent to landfill, as well as the amount of this plastic waste that has been recycled. Note that these values are cumulative totals.



(a) Develop a linear model for the total amount W of plastic waste generated (in millions of tonnes) as a function of time t (years since 1980).

(4 marks)

(continued over)

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(b) If a researcher were to use the model you developed in part (a), what assumption about the production of plastic waste are they making?

(1 mark)

(c) Use your model from part (a) to predict the year in which the Earth will have seen a total of 10,000 million tonnes of plastic waste generated.

(2 marks)

(continued over)

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(d) Suppose that the following model is used to predict the total amount of plastic waste generated W (in millions of tonnes) over time t (in years since 1980):

$$W(t) = 900e^{0.06t} - t^{0.4}$$

Note that the derivative of this function is $W'(t) = 54e^{0.06t} - 0.4t^{-0.6}$.

A researcher has estimated that it might take 50 years (since 1980) for the total amount of plastic waste generated to reach 10,000 million tonnes. Use one step of Newton's method and this revised model to improve upon this researcher's estimate.

(4 marks)

(continued over)

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(e) Explain any difference you see between your answers in part (c) and (d).

(1 mark)

(f) Consider the following statement:

The amount of plastic waste which has been recycled is growing exponentially, so the total amount of plastic waste which has been discarded into landfills may follow the shape of a logistic curve over then next few decades.

Is the above statement reasonable or not? Explain. [Hint: It may be useful to think about the data on Page 8 and the relationship between plastic that is recycled and sent to landfill.]

(2 marks)

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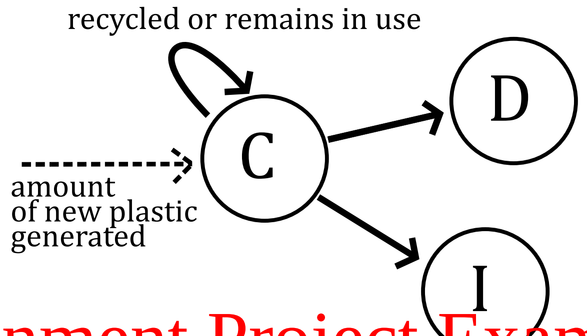
5. Sometimes the same mathematical model can describe a phenomenon at different scales, provided that the parameters are chosen appropriately. In this problem we will consider a model of plastic use that may be applied at a national or global scale. Often nations measure their performance by comparing national data to global data.

   Let $C(t)$ be the total amount of plastic in current use in some region, $D(t)$ be the total amount of plastic that has been discarded in that region, and let $I(t)$ be the total amount of plastic that has been incinerated in that region, where t is measured in years since 2015 and all quantities of plastic are measured in millions of tonnes. The following diagram represents a model of plastic use that is like a life-cycle.

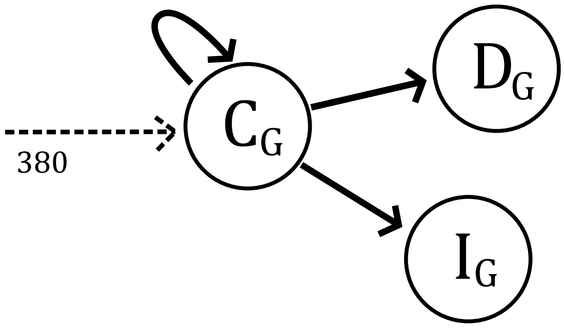


   (a) Let $C_G(t)$, $D_G(t)$ and $I_G(t)$ represent the global amounts of plastic (in millions of tonnes) in current use, that has been discarded, and that has been incinerated, respectively.

   Suppose that the global production of plastics is 380 million tonnes per year, and that each year, 9% of current-use plastics are incinerated, 59% of current-use plastics are discarded, 7% of current-use plastics are recycled, and 25% remain in use.

      (i) Below is a diagram to model plastic use at the global scale. Complete the diagram by labelling the three unlabelled arrows with the appropriate values.

(1 mark)



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(ii) An approximate model for how C_G, D_G and I_G change over time can be obtained by formulating a system of differential equations. Write down such a system of differential equations.

(4 marks)

(iii) Which of $C_G(t), D_G(t)$, or $I_G(t)$ could theoretically decrease? Explain mathematically.

(2 marks)

(continued over)

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(b) Now consider the model at a national scale, for Country X. Let $C_X(t)$, $D_X(t)$ and $I_X(t)$ represent the amounts of plastic (in millions of tonnes) in current use, that has been discarded, and that has been incinerated, respectively in Country X.

Suppose that the following system of differential equations describes how $C_X(t)$, $D_X(t)$ and $I_X(t)$ change over time:

$$\begin{aligned} C'_X(t) &= 12 - 0.26\,C_X(t) \\ D'_X(t) &= 0.26\,C_X(t) \\ I'_X(t) &= 0 \end{aligned}$$

Suppose that in 2015, the country had $C_X(0) = 7.5$, $D_X(0) = 9.2$ and $I_X(0) = 1.4$. Use an appropriate method with an increment step size of half a year to predict the values of $C_X(t)$, $D_X(t)$ and $I_X(t)$ in the year 2016.

(5 marks)

(continued over)

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(c) By comparing the systems of differential equations in part (a) and part (b), describe any differences in the management of plastic in Country X as compared to global management of plastic. Be precise, clear and concise!

(2 marks)

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6. Consider the following Python program.

```
from pylab import *

def mean(List):
    total = 0
    i = 0
    while i < size(List):
        total = total + List[i]
        i = i + 1
    ans = round(total/size(List), 1)
    return ans

def count_contaminated(List):
    total = 0
    i = 0
    while i < size(List):
        if List[i] > 0:
            total = total + 1
        i = i + 1
    return total

def printout_info(List):
    print(List)
    print(mean(List), " particles per litre on average")
    print("Percent of samples contaminated:")
    print(100*round(count_contaminated(List)/(size(List),2))

## data on microplastics found in samples of tap water and beer
TapWaterSamples = [2, 5, 20, 11, 4, 7, 4, 6, 0, 1]
BeerSamples = [1, 3, 2, 1, 0, 0, 10, 9, 12, 3, 1, 8]

t = int(input("Consider data on tap water? (1 for yes, 0 for no): "))
b = int(input("Consider data on beer? (1 for yes, 0 for no): "))
if t == 1:
    printout_info(TapWaterSamples)
else:
    print("skipping tap water data")

if b == 1:
    printout_info(BeerSamples)
else:
    print("skipping beer data")
```


(continued over)

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(a) Suppose a user has entered some input from the keyboard, and the screen reads as follows.

```
Consider data on tap water? (1 for yes, 0 for no): 0
Consider data on beer? (1 for yes, 0 for no): 1
```

In the box below, write down the output from this program. [Note: You do not need to re-write what is already written on the screen.]

(4 marks)

(b) Write a short, clear paragraph that explains the importance, role and use of writing new functions in programs. The intended audience is a UQ Science student who is learning to program and is just starting to learn about writing functions. Be precise, clear and concise!

(3 marks)

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Information on plastics

Plastic: Plastic is material composed of synthetic organic polymers (large synthetic molecules made up of long chains of shorter molecules) which are derived primarily from fossil fuels. The long unbroken molecular chains in plastics make them very malleable and durable; plastics have been used to make bags, bottles and packaging, as well as car parts and prosthetics. The first entirely synthetic plastic (called *bakelite*) was developed in 1907. Large-scale plastic production dates back to approximately the 1950's.

Microplastics: Microplastics are often recognised as plastic particles that are less than 5 mm at their longest dimension (this may be their diameter or length depending on the shape they form). Microplastics enter the environment either through products that originally contain microplastics (such as cosmetics and personal care products) or through the degradation of larger plastic items. Toxicity studies indicate that the ingestion of plastic particles poses risks to human health. Microplastics have been found in tap water, bottled water, beer and table salt (to name a few).

Microplastics ingested by sea creatures: In a 2018 paper, Smith et al. write: "Human activity has led to microplastic contamination throughout the marine environment. As a result of widespread contamination, microplastics are ingested by many species of wildlife including fish and shellfish. Because microplastics are associated with chemicals from manufacturing and that sorb from the surrounding environment, there is concern regarding physical and chemical toxicity. Evidence regarding microplastic toxicity and epidemiology is emerging. We characterize current knowledge and highlight gaps. We also recommend mitigation and adaptation strategies targeting the life cycle of microplastics and recommend future research to assess impacts of microplastics on humans. Addressing these research gaps is a critical priority due to the nutritional importance of seafood consumption."

Waste management of plastics: Plastics which are no longer needed for current use end up being recycled, or incinerated, or discarded into landfills or the natural environment. According to the 2016-17 Australian Plastics Recycling Survey, the term **'recycling'** is used in the plastics industry "to cover a range of activities including collection, sorting, reprocessing, export for reprocessing and manufacture of new products." In order to avoid double-counting materials, survey data is typically collected on the reprocessing stage, which is the stage where waste plastics are converted into a new product (may be finished or semi-finished).

Sterilising plastics: Sterilisation of plastics, such as plastics used for medical purposes, can be achieved by steam sterilisation in an autoclave (a strong container that can be heated to high temperatures and hold high pressures). According to the Centre of Disease Control, the minimum sterilisation temperatures are around 120 °C as high temperatures are required to kill micro-organisms. The time required for sterilisation depends on the type of autoclave as well as the type of material being sterilised.

Mass and density: One tonne is equivalent to 1000 kg. The density of common plastics, such as those used to make water bottles, is typically around $0.9\,\mathrm{g\,cm^{-3}}$.

Lengths, areas and volumes: A circle with radius r has area πr^2 and circumference $2\pi r$. The volume of a prism with height h and base area A is Ah. The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.

Some exponent & log laws: $a^x a^y = a^{x+y}$, $(a^x)^y = a^{xy}$, $\ln(xy) = \ln(x) + \ln(y)$, $\ln(x^n) = n\ln(x)$.

Miscellaneous: One million is 1×10^6 and one billion is 1×10^9.

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SCIE1000 / SCIE1100 Formula Sheet

Base quantity	SI unit name	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^1	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

Quantity	Name	Symbol	SI units	SI base units
frequency	hertz	Hz	-	s^{-1}
force	newton	N	-	$m \cdot kg \cdot s^{-2}$
pressure, stress	pascal	Pa	$N \cdot m^{-2}$	$m^{-1} \cdot kg \cdot s^{-2}$
energy, work, quantity of heat	joule	J	$N \cdot m$	$m^2 \cdot kg \cdot s^{-2}$
power, radiant flux	watt	W	$J \cdot s^{-1}$	$m^2 \cdot kg \cdot s^{-3}$
electric potential difference, electromotive force	volt	V	$W \cdot A^{-1}$	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
Celsius temperature	degree Celsius	$^\circ C$	-	K

function type	general form
linear	$y = mx + c$
quadratic	$y = ax^2 + bx + c$
power	$y = ax^p$
periodic	$y = A\sin(\frac{2\pi}{P}(t-S)) + E$
exponential	$y = Ce^{kt}$
surge	$y = at^p e^{-bt}$

Newton's method $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

Trapezoid rule $A_{trap} = (x_2 - x_1)(\frac{y_1+y_2}{2})$

Euler's method $t_{i+1} = t_i + h \qquad y_{i+1} = y_i + hy_i'$

True Status

	Yes	No
Test Positive	A	B
Test Negative	C	D

$$N = A + B + C + D$$

$$\text{accuracy} = \frac{A+D}{N}$$

$$\text{sensitivity} = \frac{A}{A+C}$$

$$\text{specificity} = \frac{D}{B+D}$$

Lotka-Volterra model

$$P' = -cP + dPQ$$
$$Q' = aP - bPQ$$

SIR model

$$S' = -a\frac{S}{N}I$$
$$I' = a\frac{S}{N}I - bI$$
$$R' = bI$$

END OF EXAMINATION

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