Lecture 36: **Modelling for public health**

SIR model

Learning objectives

✓ Analyse predictions of outbreaks and pandemics

✓ Understand and interpret the threshold vaccination levels required to avoid epidemics

✓ Understand and interpret how deaths due to disease impact transmission dynamics

Scientific examples

✓ Vaccination targets

✓ Catastrophe planning

Maths skills

✓ Interpret differential equations

Case Study 32: **Vaccinations**

IP - infectious period

Question *12.4.7*

Find an expression for R_0 (the basic reproduction number of a disease) in terms of the infection rate a and the recovery rate b. What does this expression indicate about the relative values of a and b in order for an epidemic to begin?

we had

infection rate $a = \frac{R_0}{IP} \Rightarrow IP = \frac{R_0}{a}$

recovery rate $b = \frac{1}{IP} \Rightarrow IP = \frac{1}{b}$

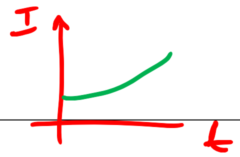
$\frac{R_0}{a} = \frac{1}{b} \Rightarrow \boxed{R_0 = \frac{a}{b}}$

If $a > b$ epidemic occurs

Need $R_0 > 1$

if $R_0 < 1$

if $R_0 > 1$



- An epidemic occurs if introducing a group of infected people into a population causes an increase in the number of infectives in the population (that is, $\boxed{I' > 0}$).

Question 12.4.8

If a is the infection rate and b is the recovery rate then the DE for I' is:

$$\boxed{I' = a \times \frac{S}{N} \times I - bI.}$$

(a) Show that for an epidemic to occur, the proportion of susceptibles in the population must be more than $1/R_0$ (according to the SIR model).

An epidemic occurs if $I' > 0$

Hence $a\frac{S}{N}\cancel{I} - b\cancel{I} > 0$

$$a\frac{S}{N} > b$$

Recall $R_0 = \frac{a}{b}$

$$\frac{S}{N} > \frac{b}{a}$$

$$\frac{S}{N} > \frac{b}{a} = \frac{1}{R_0}$$

An epidemic occurs if $\boxed{\frac{S}{N} > \frac{1}{R_0}}$

Rubella – $R_0 = 5$

Thus $\frac{S}{N} > \frac{1}{5} = 0.2$

If there are than 20% susceptibles in the population an epidemic will occur

Measles – $R_0 = 18$

Thus $\frac{S}{N} > \frac{1}{18} \approx 0.05$

If there are more than 5% susceptible then an epidemic occurs

***Question** 12.4.8 (continued)*

(b) With reference to the effective reproduction number, R_e, explain why an epidemic will occur if a fraction of more than $1/R_0$ of a population is susceptible.

Defined $R_e = R_0 \frac{S}{N}$

Epidemic continues if $R_e > 1$

$R_0 \frac{S}{N} > 1$

$\frac{S}{N} > \frac{1}{R_0}$ as required

(c) If the susceptible proportion of a population is smaller than $1/R_0$, then the population is said to have *herd immunity* to the disease. Why is herd immunity to, say, Rubella, desirable?

- slows / stops the spread of the disease
- Babies younger than 12 months can't be vaccinated ⇒ herd immunity protects those who can't be vaccinated

(d) Mass public vaccination aims to vaccinate a certain proportion of people. What level of coverage do authorities typically aim for? Why?

want to have sufficient numbers of the population vaccinated so that $I' < 0$ and the disease doesn't spread

want to reduce S so that $\frac{S}{N} < \frac{1}{R_0}$

Introduce V - no. vaccinated.

in a population either susceptible or vaccinated

we have $S + V = N \Rightarrow S = N - V$

if $\frac{S}{N} < \frac{1}{R_0} \Rightarrow \frac{N-V}{N} < \frac{1}{R_0} \Rightarrow 1 - \frac{V}{N} < \frac{1}{R_0}$

Hence $\frac{V}{N} > 1 - \frac{1}{R_0}$

Question 12.4.9

Explain why the target vaccination rate for measles is (at least) 95%. What is the figure for rubella?

Need $\frac{V}{N} > 1 - \frac{1}{R_0}$

measles $R_0 = 18 \Rightarrow \frac{V}{N} > 1 - \frac{1}{18} \simeq 0.944$

need at least 95% vax

rubella $R_0 = 5 \Rightarrow \frac{V}{N} > 1 - \frac{1}{5} \simeq 0.8$

Need 80% or more vaccinated

Covid $R_0 = 3.7$

$\frac{V}{N} > 0.73$ or 73%.

Question 12.4.10

Should vaccinations be compulsorily enforced? Why or why not?

Yes: protect those who can't be vaccinated

protect the vulnerable

No: civil liberties

medical reasons - not everybody can tolerate a vaccine.

!!

Example 12.4.11

In 1998, a paper published in the Lancet (with lead author Dr Andrew Wakefield) claimed to identify a link between the MMR (Measles, Mumps and Rubella) vaccine and autism in children. The findings had a large impact on the public perception of the vaccination. As a result, more than 3 million young people in the UK were not fully vaccinated.

In recent years, the study linking MMR vaccines with autism has been completely discredited. Wakefield was found to have committed scientific fraud by falsifying data, to have acted dishonestly and irresponsibly, and to have a conflict of interest. A number of his research papers were retracted by the journals that had previously published them, and in 2010 he was struck off the UK medical register.

Question 12.4.12

In 1995/6, around 91% of 2 year old children in the UK had received the MMR vaccine. The figure then dropped steadily to 79.9% in 2003/4, and then rose to 92.3% in 2012/13. Figure 12.8 shows the number of laboratory confirmed cases of measles and rubella in England over recent years.

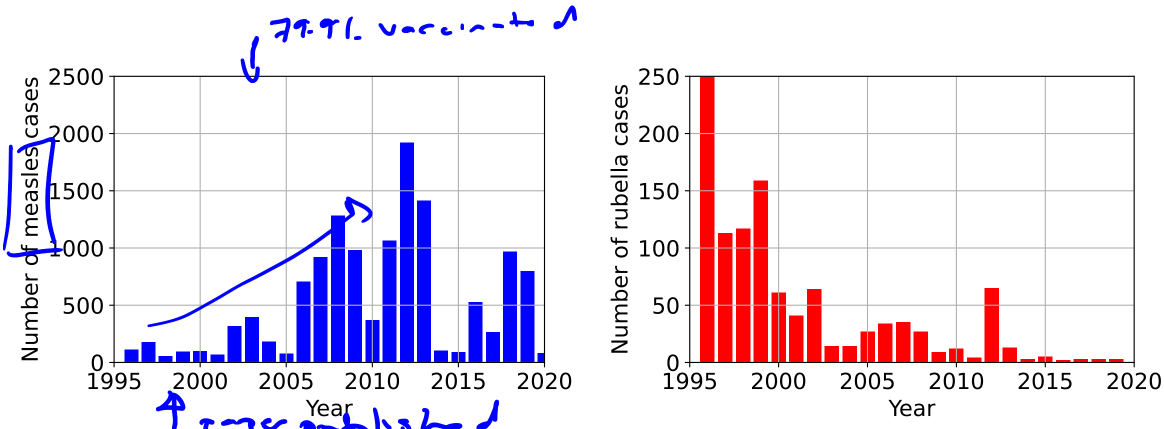


Figure 12.8: Number of laboratory confirmed cases of measles (left) and rubella (right) in England.

(a) Why might the measles graph in Figure 12.8 have that shape?

Vaccination rates dropped ~ 80%.
Herd immunity for measles ~ 95%.
Vaccination levels dropped below herd immunity so epidemics occurred.

(b) Why might the rubella and measles graphs have different shapes?

herd immunity for rubella ~ 80%.
Still sufficient vaccination levels for herd immunity
=> no spread

Question 12.4.13

How could the SIR model be adapted to account for:

(a) a *preventative* vaccination strategy (i.e. vaccinations occur before disease has a chance to spread)?

vaccinated population is removed from S and placed in R before running the model

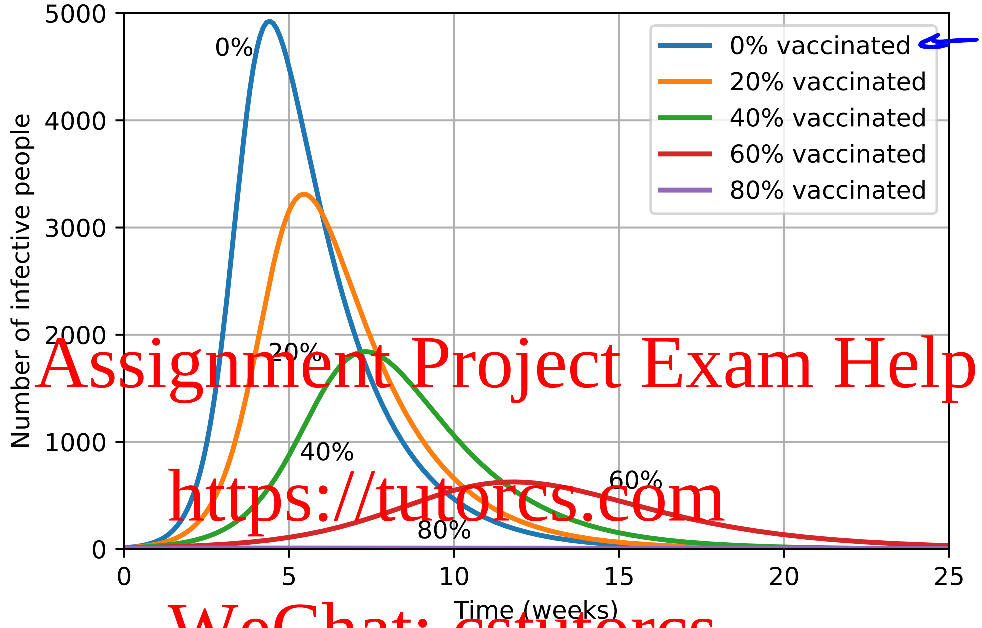
(b) a *continuous* vaccination strategy (i.e. vaccinations occur as disease spreads)?

v, add transition (vaccination)

$$\begin{tikzcd} S \arrow[r] \arrow[rr, bend left, "v"] & I \arrow[r] & R \end{tikzcd}$$

Question 12.4.14

Earlier we modelled a rubella epidemic in a city with 9990 susceptible people and 10 infective people. Figure 12.9 shows the predicted numbers of infective people $I(t)$ under five scenarios, with *preventative* vaccination rates of 0%, 20%, 40%, 60% and 80%.



← as before

Figure 12.9: The effect of different vaccination rates on a rubella epidemic.

Interpret and explain the graphs. What are the benefits of increased vaccination rates?

- lower peak : reduces strain on the medical system
- delayed peak : more time to prepare
- AUC smaller : fewer infected
- > 80% no epidemic

End of Case Study 32: Vaccinations.

12.5 Catastrophe planning

- Many governments conduct *catastrophe planning*, modelling the potential impacts of disastrous events, such as nuclear explosions, terrorist strikes, tsunamis, earthquakes and pandemics.

Example 12.5.1 recorded

In terms of numbers of fatalities, five of the worst catastrophes in (European) Australian history are:

- Spanish influenza in 1918–19, causing more than 12000 deaths;
- a polio epidemic in 1946–55, causing more than 1000 deaths;
- the COVID-19 pandemic starting in 2020, causing more than ~~900~~ 2800 deaths;
- a naval battle in the Second World War, causing 727 deaths; and
- a bubonic plague epidemic in 1900–1910, causing 550 deaths.

In addition, thousands of indigenous Australians died from communicable diseases introduced by European settlement.

Example 12.5.2

In the 1300s, the bubonic plague or *Black Death* killed around 20 million Europeans in six years, which was about one third of the population. In the worst affected urban areas, around half of the population died. The plague returned regularly, with around 100 epidemics occurring in the next 400 years.

Photo 12.9: Plague monuments. Left: Brno, Czech Republic. Centre: Vienna, Austria. Right: Olomouc, Czech Republic. (Source: PA.)

Example 12.5.3

A Spanish influenza pandemic occurred in 1918–1919. Within six months the global death toll was 25 million (more than the number who died from combat in the First World War). The flu was so virulent and deadly that it 'burnt itself out', disappearing completely within 18 months.

- Clearly there are many instances where we may need to extend the SIR model to account for deaths due to disease.
- The **SIRD model** divides the population into **four** distinct groups: Susceptible, $S(t)$; Infective, $I(t)$; Recovered, $R(t)$; and Dead, $D(t)$.
- In this instance we assume that the only possible movements of people *between* compartments are: susceptible people can become infective; infective people can recover or die.
- We again neglect natural births and deaths, and assume that deaths due to disease only occur while a person is infective.
- Different diseases have different transmission and recovery dynamics, so there are many possible alternative models. The important thing is to be clear about the assumptions that underpin the model.

The equations for the SIRD model

If a population of N people is divided into four groups, susceptible $S(t)$, infective $I(t)$, recovered $R(t)$, and dead $D(t)$ then the SIRD model is:

$$
\begin{aligned}
S' &= -a \times \frac{S}{N} \times I \\
I' &= a \times \frac{S}{N} \times I - bI - cI \\
R' &= b\,I \\
D' &= c\,I
\end{aligned}
$$

where a is the infection rate, b is the recovery rate, and c is the rate of death due to disease.

Question 12.5.4

Write equations for the total population size, $N(t)$, and the rate of change in population, $N'(t)$, for the SIRD model.

$N(t) = S + I + R$

(D not included) — N reduces

$N' = S' + I' + R' = -cI$

Population decreases

Case Study 33: **Avian influenza**

- The World Health Organisation (WHO) has warned that:
  - the risk of an influenza pandemic is high;
  - H5N1 (avian) influenza is endemic in many bird populations;
  - bird-to-human transmission has already caused fatalities; and
  - there is a serious risk that the virus could mutate and become human-to-human transmissible.

Photo 12.10: Left: Marabou Stork, *Leptoptilos crumeniferus*. Right: Painted Desert, USA. (Source: PA.)

- We will use the **SIRD** model to investigate the potential impact of a catastrophe caused by human-transmissible avian influenza.

- There has never been a verified case of human-to-human transmission of avian influenza, but by choosing reasonable values for all parameters, we can model a hypothetical avian flu epidemic.

- Researchers estimate the following values for the Spanish Flu pandemic in 1918–1919. We will use these values in our SIRD model of a hypothetical avian flu outbreak. $R_0 = \frac{a}{b} \approx 1.4$, $IP = \frac{1}{b} = 0.7$ week $= 5$ days

$$a = \text{infection rate} = 1.9 \text{ week}^{-1}; \quad b = \text{recovery rate} = 1.4 \text{ week}^{-1}; \quad c = \text{mortality rate} = 0.065 \text{ week}^{-1}.$$

≈ 5% infected

- Now we can use Euler's method and the SIRD model to investigate various scenarios in a city such as Brisbane with $N = 2 \times 10^6$.

Example 12.5.5

There is one infective person in a city in which $N = 2 \times 10^6$ and everyone else is susceptible. Figure 12.10 shows the results. The model predicts that the disease outbreak will last for about 45 weeks, around 870000 people will become ill, the largest number of infective people at any time will be about 59500, and that approximately 38500 people will die.

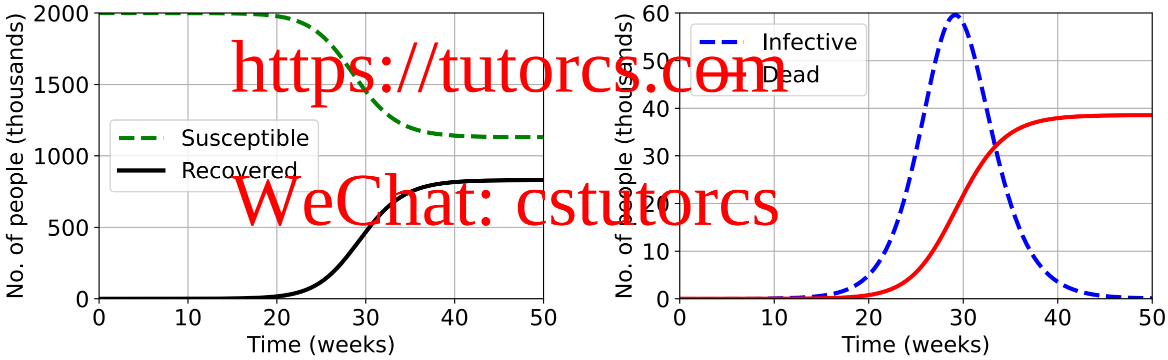


Figure 12.10: The impact of a possible human-transmissible avian influenza epidemic on a city of two million people, as modelled by Euler's method.

(6 months

End of Case Study 33: Avian influenza.

Monday - review + mega kahoot SCIE1000 "bragging rights"
(not a catastrophe)