Lecture 2: **Strong foundations**

Learning objectives

✓ Review the quantitative language of science

✓ Introduce programming and some best-practices

✓ Understand the need for clear communication in science

✓ Develop critical evaluation skills

Scientific examples

✓ Mars Climate Orbiter

✓ Widmark formula

Maths skills

✓ Use dimensional analysis to check if an equation is plausible

1.2 Units

- A unit of measurement is an agreed upon quantity of something; any other quantity of the same kind can be described by giving the ratio between that quantity and the unit. For example, when we write that the minimum length of a long jump pit is 2.75 metres, we are saying that 2.75 is the ratio between the minimum length of the long jump pit and the length that we have agreed to call a metre.

- The same quantity can be described by comparison to different reference quantities (units). It is important to use the same units for measuring the same kinds of quantities in any one calculation, and to communicate the units used clearly. The consequences of not doing so can be severe.

always

***Example* 1.2.1**

The Mars Climate Orbiter was launched in 1998 as part of a $USD330 million project, but in September 1999 it crashed into Mars. Here is an extract from the report into the accident[34]:

> *"During the 9-month journey from Earth to Mars, propulsion manoeuvres were periodically performed . . . coupled with the fact that the angular momentum (impulse) data was in English, rather than*

***Example** 1.2.1 (continued)*

metric, units, resulted in small errors being introduced in the trajectory estimate over the course of the 9-month journey. At the time of Mars insertion, the spacecraft trajectory was approximately 170 km lower than planned . . .

. . . it was discovered that the small forces ΔVs reported by the spacecraft engineers for use in orbit determination solutions was low by a factor of 4.45 (1 pound force = 4.45 Newtons) because the impulse bit data contained in the AMD file was delivered in lb-sec instead of the specified and expected units of Newton-sec."

1 N = 1 newton N s newton second

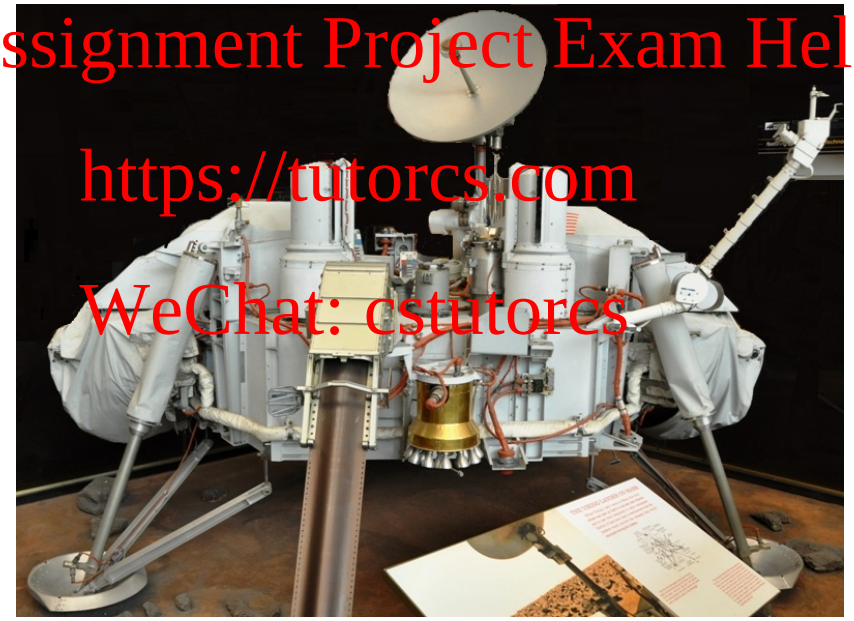

Photo 1.1: Mars Lander (proof test model) from the Viking program, launched 1975. (Source: PA.)

SI Units and prefixes

Australia adopted the **International System of units**, or **SI units**, in 1960 and this is the system still in use today. There are seven **SI base units**. The kinds of things they measure, their standard names and their symbols are shown in Table 1.1.

SI Units and prefixes *(continued)*

Base quantity	SI base unit	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Table 1.1: The seven SI base units.

Some lengths, like the charge radius of a proton, are much less than a metre; others, like the distance from the Earth to the centre of the Milky Way, are much more than a metre. To save ourselves writing numbers with many digits before or after the decimal place, we may write one of the SI prefixes, as shown in Table 1.2, in front of a unit of measurement to indicate a fraction or multiple of the unit.

formula sheet

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^{1}	deca	da	10^{-1}	deci	d
10^{2}	hecto	h	10^{-2}	centi	c
10^{3}	kilo	k	10^{-3}	milli	m
10^{6}	mega	M	10^{-6}	micro	μ
10^{9}	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

Table 1.2: The 20 SI prefixes.

- One of the SI base units, the kilogram, has a prefix built into its name and its symbol. When writing about mass you should use prefixes in front of grams, or g, rather than the base unit.

mKg Gg

Algebra for quantities and units

- If quantities have the same units then they can be added or subtracted and the result has the same units.
- If quantities are multiplied or divided then the corresponding units should be gathered on the right, then multiplied and divided (or cancelled) using the familiar algebra rules.
- We often write a dot, or leave a space, between units when they are to be multiplied, and use exponent notation to indicate "powers" of units. Negative exponents indicate quotients. So, for example, we write m^3 instead of $\mathrm{m} \cdot \mathrm{m} \cdot \mathrm{m}$, and $\mathrm{m} \cdot \mathrm{s}^{-1}$ or $\mathrm{m\,s}^{-1}$ instead of $\frac{\mathrm{m}}{\mathrm{s}}$.

Example 1.12.

If $A = 12.3$ kg and $B = 1.68\ \mathrm{m} \cdot \mathrm{s}^{-2}$ and $C = 4.62\ \mathrm{m}^2$, then

$$\frac{AB}{C} = \frac{(12.3\,\mathrm{kg})(1.68\ \mathrm{m} \cdot \mathrm{s}^{-2})}{4.62\ \mathrm{m}^2} = \frac{(12.3)(1.68)}{4.62}\ \frac{\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}}{\mathrm{m}^2}$$

$$= 4.47\ \mathrm{kg} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{-2}$$

Significant figures

The number of digits in a quoted number has implications about how precisely the associated quantity is known. This relates to the concept of uncertainty which we will not be covering in this course. Instead, we will generally work with two or three significant figures in the calculations that we perform.

Derived units

Many natural and scientific quantities require more complex units than SI base units. These **can always be defined** in terms of the seven base units, and are called **SI derived units.**

Example 1.2.3

Some frequently-used SI derived units have been given special names and symbols. Table 1.3 shows some well-known examples.

Quantity	Name	Symbol	In SI base units	In other SI Units
frequency	hertz	Hz	s^{-1}	-
force	newton	N	$m \cdot kg \cdot s^{-2}$	-
pressure, stress	pascal	Pa	$m^{-1} \cdot kg \cdot s^{-2}$	$N \cdot m^{-2}$
energy, work, quantity of heat	joule	J	$m^2 \cdot kg \cdot s^{-2}$	N · m
power, radiant flux	watt	W	$m^2 \cdot kg \cdot s^{-3}$	$J \cdot s^{-1}$
electric potential difference, electromotive force	volt	V	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$	$W \cdot A^{-1}$
volume	litre	L or l	$10^{-3}\,m^3$	-
time	day	d	86,400 s	24 h
time	hour	h	3,600 s	60 min
time	minute	min	60 s	-

Table 1.3: Some well-known derived units and their SI base units.

Converting between different units for the same quantity

The algebra rules for quantities and units provide a neat way to convert between different units for the same base quantity. For example, an atmosphere, abbreviated to atm, is a unit for pressure. By definition, 1 atm is equal to 101.325 kPa. It follows that $\frac{1\text{ atm}}{101.325\text{ kPa}}$ is like 1, and we may introduce it as a multiplicative factor whenever we like. For example, we can convert 233.05 kPa to the units of atm as follows:

$$P = 233.05\ \text{kPa} \times \frac{1\text{ atm}}{101.325\text{ kPa}} = \frac{233.05}{101.325}\ \text{atm}$$

$$= 2.3\ \text{atm}$$

Question 1.2.4

The *cardiac output* CO of a heart is the volume of blood ejected by the heart during a particular time period, and equals the *stroke volume* SV multiplied by the *heart rate* HR. If each beat has a volume of 70 mL and the heart beats 1.5 times each second, calculate CO in $\mathrm{L \cdot min^{-1}}$.

We're given $SV = 70\text{ mL}$, $HR = 1.5\ \mathrm{s^{-1}}$

Calculate

$$CO = SV \cdot HR$$
$$= 70\text{ mL} \times 1.5\ \mathrm{s^{-1}}$$
$$= 105\ \mathrm{mL/s}$$
$$= 105\ \frac{\mathrm{mL}}{\mathrm{s}} \times \frac{1\text{ L}}{1000\text{ mL}} \times \frac{60\text{ s}}{1\text{ min}}$$
$$= 6.3\ \mathrm{L/min}$$

Dimensional analysis

Dimensional analysis is an important technique in science. It involves applying the following principle: an equation describing a physical situation can be true *only* if it is **dimensionally homogeneous**; that is, both sides of the equation have the same units. Dimensional analysis allows a quick check of whether a calculation is 'plausible'. If the dimensions do not match, then there **must** be an error.

About units and counting things

When a number represents a count of something, we should record what it counts. For example, after observing a friend's backyard I may write that I observed 17 chickens. However, this does not mean that "chickens" is a unit. For something to be a unit, or at least a scientific unit, it should be something we can describe using only powers of SI base units and constants.

The number of chickens is 17

1.3 Huh?

Question 1.3.1

In 2005 a psychologist, Dr Cliff Arnall, proposed a function which models how depressing each day of the year is. By finding the date on which the function takes its maximum value, Arnall identified "Blue Monday", the most depressing day of the year. The "Blue Monday" model still receives significant press attention. The 'model' takes into account: weather (W), debt (D), monthly salary (d), time since Christmas (T), time since failing our new year's resolutions (Q), low motivational levels (M), and the feeling of the need to take action (N_a). His function has been variously reported as:

www.ft.com, en.wikipedia.org: $\dfrac{(W + (D - d)) \times T^Q}{M \times N_a}$

www.msnbc.msn.com: $\dfrac{[W + (D - d)] \times TQ}{M \times NA}$

www.scotsman.com: $([W + (D - d)] \times TQ)(M \times NA)$

www.peterboroughtoday.co.uk: $W + (D) \times T^Q / M \times Na$

news.bbc.co.uk, www.cbc.ca: $1/8W + (D - d)3/8 \times TQM \times NA$

Question 1.3.2

Write a short paragraph that explains to the general public in Australia why there is no underlying science behind "Blue Monday."

- Subjective
- Different versions
- Quantities that can't be measured
- Random
- nonsense
- units not correct

The Blue Monday formula makes no sense in Science. It attempts to combine parameters, some of which can't be measured, into an equation that is not quantitative in scientific terms. It was created for an advertising campaign not for science.

There are some excellent online articles that cover doubtful claims and errors in media reporting of quantitative science. The following links are very interesting, amusing, frightening and informative:

- "Behind the Headlines" [36]– provides a factual analysis of health-related claims, including the scientific background.
- "Bad Science" [21]– identifies and discusses mathematical and scientific errors in reported and published science.
- "Dodgy Boffins" [13]– discusses misuse of equations in the British media.
- "Helping Doctors and Patients Make Sense of Health Statistics" [20]– discusses the causes and impacts of errors in presenting and interpreting health statistics.

1.4 Programming

- Computation is important when formulating and applying models, particularly when dealing with complex phenomena. In SCIE1000 we will write programs to model some phenomena. Programming requires technical skill, experience and creativity.
- Refer to Appendix A for the Python programming instruction manual for this course.
- A *program* is a set of instructions that make a computer do something. Web browsers (for example, *Google Chrome*), word processors (*Microsoft Word*) and "apps" for a mobile phone are all familiar examples of programs. Even the Python programming language itself is a program.
- The first step in programming is to understand exactly what problem is being solved, and hence specify **exactly** what the program should do; specifications should be precise, accurate and complete. Once the problem has been understood, the programmer needs to write a sequence of commands that together solve the problem.

- Even the best and most experienced computer programmers will sometimes (even often) write programs with errors in them. The consequences of software errors (*bugs*) can be very serious.
- Types of programming error include: incomplete (or incorrect) problem description, design faults in the software, unanticipated 'special cases', coding errors, logic errors and miscommunication between programmers. Section A.4 in Appendix A provides some advice on how to test and check for coding errors.

In Lecture 1, we developed a very rough model for the blood alcohol of a person who had consumed alcohol. The model we developed is quite similar to a model which is often used in courts to determine blood alcohol levels; this is called the *Widmark formula* and it was developed in 1932. The equation is:

$$B = \frac{A}{rM} \times 100\% - 0.015t$$

where B is the blood alcohol concentration (as a %) at time t (where t is measured in hours since drinking commenced), A is the amount of alcohol consumed (in grams), M is the body mass (in grams) and r is an estimate of the proportion of body mass that is water.

***Question* 1.4.1**

Consider the Widmark formula. What are the units of the constant 0.015? What does this constant represent physically?

We can write a short Python program to calculate blood alcohol concentration based on the Widmark formula.

Program 1.1: Blood Alcohol Concentration

```
1  # Calculates blood alcohol concentration (BAC) for a person
2  from pylab import *
3
4  # Input values
5  mass_person = float(input("What is the person's mass (in kg?) "))
6  sex_person = int(input("Enter 1 for female, 2 for male: "))
7  no_drinks = int(input("Number of standard drinks? "))
8  drink_time = float(input("How many hours ago? "))
9
10 # Convert drinks to mass (10g alcohol per standard drink)
11 mass_alcohol = 10 * no_drinks
12
13 # Mass of water (in grams) for the person
14 if sex_person == 1:
15     mass_water = 1000 * mass_person * 0.6
16 else:
17     mass_water = 1000 * mass_person * 0.7
18
19 # Metabolic rate of removal (%/hour)
20 removal_rate = 0.015
21
22 # Apply Widmark formula
23 bac = (mass_alcohol / mass_water) * 100 - removal_rate * drink_time
24 print("The person's blood alcohol concentration is ", round(bac,3),"%")
```


Question 1.4.2

Briefly summarise in words the purpose of each line of code in the above program. If you're not sure, have a guess. Note that you are not expected to be familiar with all of this programming syntax yet!

Line 1 - comment describing program
Line 2 - imports standard functions
Lines 5-8 - input parameters from the user
Line 11 - convert no drinks to a mass of alcohol (1 standard drink contains 10g of alcohol)
Lines 14-17 - calculate mass of water in body depending on whether male or female
Line 20 - define rate at which alcohol is removed from the body by metabolism
Line 23 - calculate blood alcohol concentration
Line 24 - output the result

Testing and debugging

Most newly written programs include errors, and it is important to adopt a systematic approach to minimising the number of errors, then identifying and fixing any that occur. This process is called *testing* and *debugging*.

Some programming errors are relatively easy to find (such as missing brackets), some will result in runtime error messages (for example, trying to divide by zero), but in other cases a program may produce incorrect output without an error message. To find such errors, you will need to test your program with different input values, and check the output "by hand".

When writing programs, make sure that you:

- Think about the clearest and most logical way to solve the problem;
- Write your program in an organised, systematic manner.
- Include useful comments in the program.
- Test your programs on a range of data;
- Check some output carefully to make sure it is correct; and
- Pay attention to any error messages!