# Lecture 14: **Tracking birds with sine**

**Learning objectives**

✓ Interpret sine function models of real-world phenomenon

**Scientific examples**

✓ Tracking migration of terns

$$y(t) = A\sin\left(\frac{2\pi}{P}(t-S)\right) + E$$

**Maths skills**

✓ Understand and interpret sine functions and their graphs

Photo 5.8: Migrating Canada Geese, *Branta canadensis*, New York State, USA. (Source: PA.)

Many animals undertake *migration*, during which they move from one area to another, and then return. This often happens on an annual basis, according to seasons or weather patterns. Migratory behaviour occurs in all major animal groups (birds, reptiles, mammals, amphibians, fish, insects and crustaceans); see [9].

humans ??



Examples of migration include: wildebeest and zebra on the Serengeti plains in Africa; geese "flying south for winter" in the northern hemisphere; humpback whales travelling north along the Queensland coast during winter; and sea turtles returning to beaches to lay eggs.

***Question*** 5.2.7

What are some of the reasons for, and benefits of, seasonal migration? How does this relate to daytimes?

- warmer weather
- food
- breeding
- better habitat

## *Question* 5.2.8

Recall that, if $t$ is the day number in the year (starting from $t = 0$ on January 1st) then the length of the daytime in hours at any point in the southern hemisphere is given by the function

amplitude

$$D(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right)$$

where $K$ is a constant determined by the latitude of the point.
In Brisbane, $K \approx 1.74$ hours, whereas $K \approx 1$ hour for Townsville, and $K \approx 3.3$ hours for Hobart. The graph for Brisbane is shown in Figure 5.4.

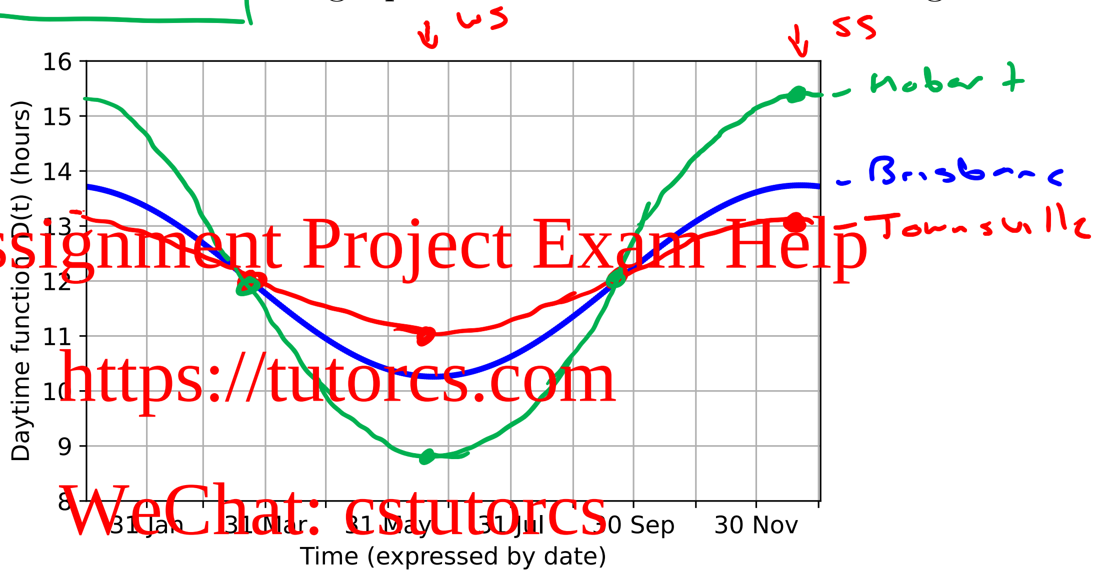





Figure 5.4: Daytimes in Brisbane over the year.

**(a)** Roughly sketch the graphs of $D(t)$ for Townsville and Hobart on the above graph.

**(b)** By how much is the daytime on the **summer** solstice in Hobart **longer** than in Townsville? By how much is the daytime on the **winter** solstice in Hobart **shorter** than in Townsville?

Reading off the graph

Summer Solstice
Daytime in hobart = 15.3 hours
Daytime in Townsville = 13 hours
2.3 hours longer in Hobart

Winter solstice
Daytime in hobart = 8.7 hours
Daytime in Townsville = 11 hours
2.3 hours shorter in hobart

***Question*** 5.2.8 *(continued)*

**(c)** Consider a yellow wattlebird in Hobart and an eastern whipbird in Townsville. Assuming that the birds stay in their respective local habitats all year, what is the **total** number of daylight hours they each experience for the entire year? Use the models for daylight hours given for Hobart and Townsville on the previous page, and justify your answer.

Average number of daylight hours in a day = 12

The total number of hours in a year's
= 365 days × 12 hours/day
= 4380 hours



**(d)** The eastern whipbird is sendentary and does not migrate. However, suppose that one brave whipbird flew to Hobart for the summertime, and then returned to Townsville. What effect would this have on the total number of daylight hours this bird would experience over that year?





The number of daylight hours experienced by the bird would increase!

**(e)** How might your answer to part (c) change if you were to consider the total number of "sunlight hours", as opposed to daylight hours?

No. of sunlight hours ≤ no. of daylight hours

depends on weather – clouds, fog

## Case Study 11: **To every thing, there is a season, tern, tern, tern**

The Arctic tern, *Sterna paradisaea*, is a seabird that migrates annually from its breeding grounds in the Arctic to the Antarctic and back.

mass of a tern = 150 g

limited to a device ≤ 4% of mass of bird

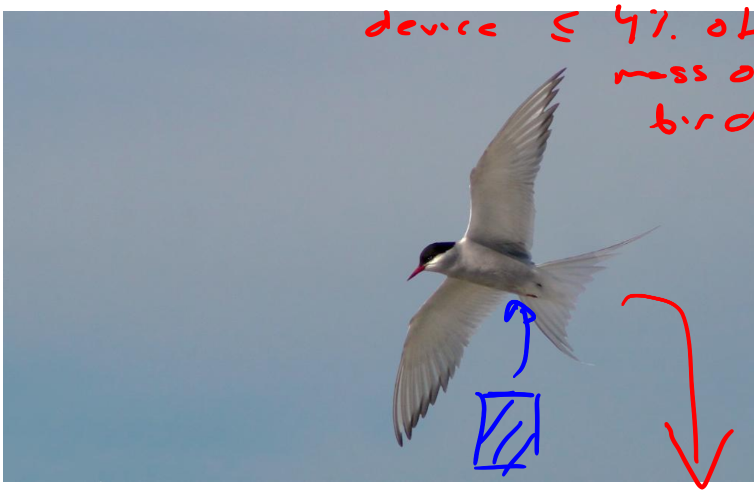


Image 5.2: Arctic tern in flight. (Source: en.wikipedia.org)

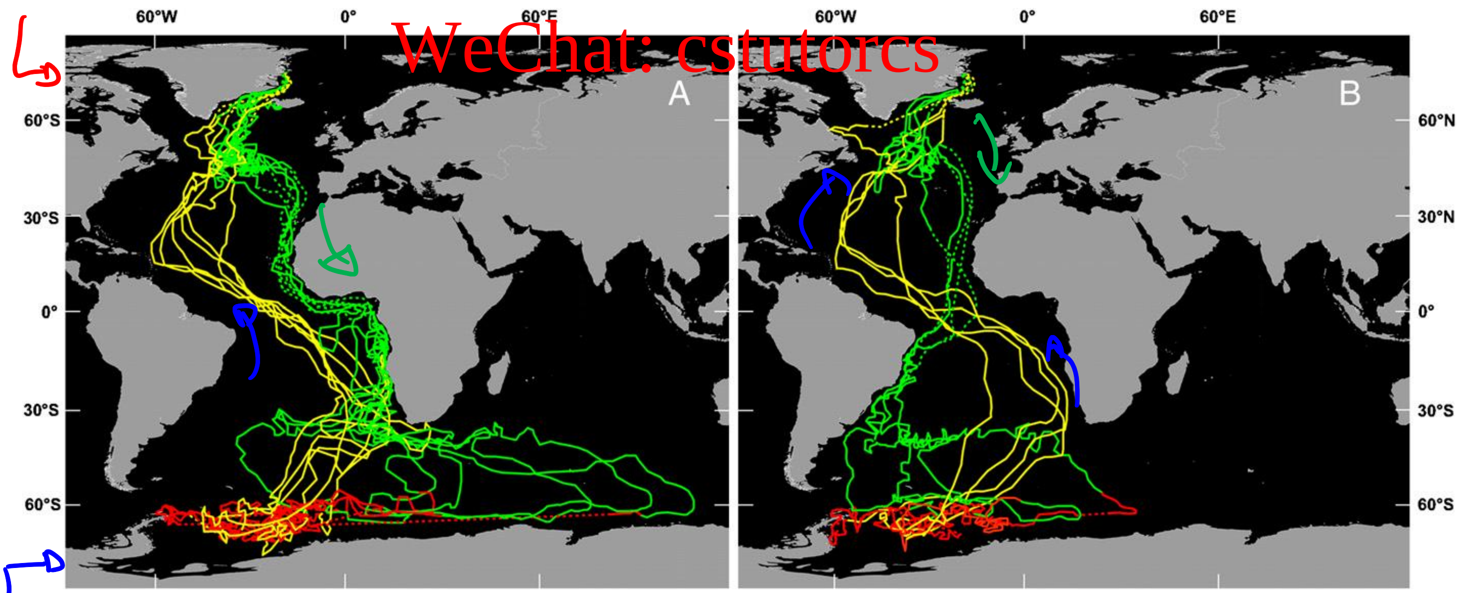
- Individual terns have been tracked travelling a distance of 400–700 km per day, and 80000 km in a year; this is the longest (known) migration of any animal.

2010

- Figure 5.5 shows tracked migration routes of 11 Arctic terns (see [11]).



Arctic




Antarctic

Figure 5.5: Interpolated geolocation tracks of 11 Arctic terns tracked from breeding colonies in Greenland ($n = 10$ birds) and Iceland ($n = 1$ bird). Green = autumn (postbreeding) migration (August/November), red = winter range (December/March), and yellow = spring (return) migration (April/May). Two southbound migration routes were adopted in the South Atlantic, either (A) West African coast (n = 7 birds) or (B) Brazilian coast. (This text is an extract from [11].)

measures intensity of light, time

***Question*** 5.2.9 ✓

Researchers in [11] attached miniature light loggers to individual birds in Iceland and in Greenland, and retrieved the data one year later. The light loggers have an internal clock and they record when they are exposed to light. For this question, assume the birds are in the southern hemisphere.

**(a)** Describe how researchers could have used this data to determine the latitude of a tern on any given day. (how far south)

we have our equation for daylight hours (S.H.)

$$D_S(t) = 12 + K \sin\left(\frac{2\pi}{365}(t - 264)\right)$$

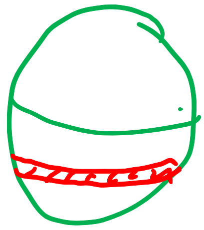
light logger

clock

hence we can solve for K

K depends on latitude!

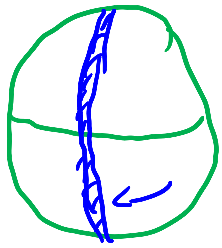
K ↓





**(b)** Describe how the researchers could have used the data from the light loggers to determine the longitude of a tern on any given day.

Use sunrise/sunset



hence can identify the location of the bird on the globe.

southern hemisphere

### *Question* 5.2.10

We have seen that the equation for the number of daylight hours in the southern hemisphere is $D_S(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right)$.

When the researchers track Arctic terns, they will need to use a different equation when the birds are in the northern hemisphere.

**(a)** Find a corresponding equation to model the number of daylight hours in the **northern** hemisphere, $D_N(t)$.

Change the shift, S

Take 264 days $-$ $\frac{1}{2}$ year

$= 264$ days $- \frac{365}{2}$ days $\simeq 81$ days

$D_N(t) = 12 + K \sin\left(\frac{2\pi}{365}(t - 81)\right)$



**(b)** A graph of $D_S(t)$ at the Antarctic Circle is shown below. Draw the corresponding graph for $D_N(t)$ at the Arctic Circle.

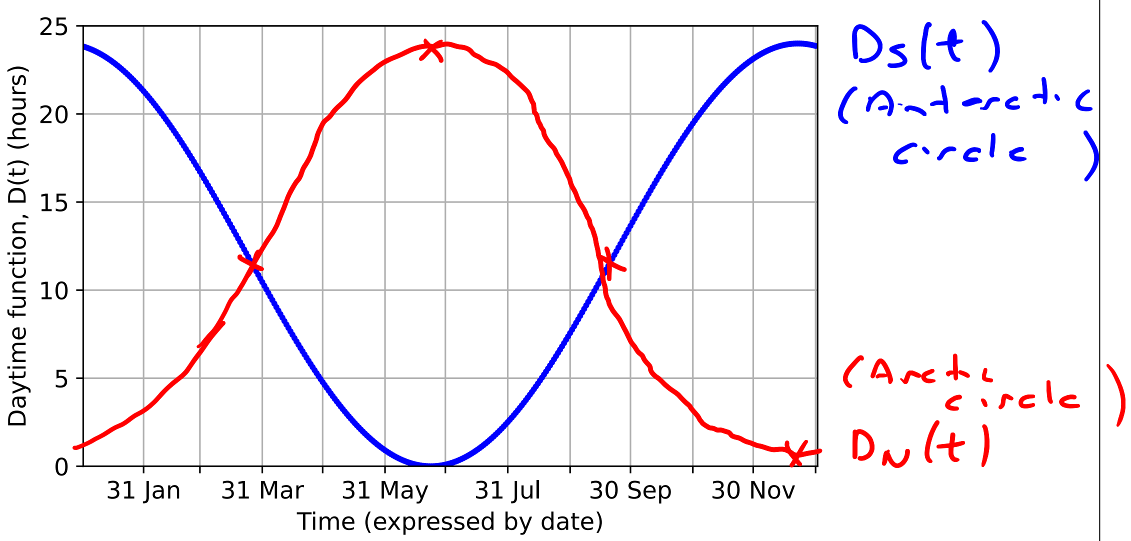



Figure 5.6: Daytimes at the Antarctic Circle.

- The paper [11] states that:

> *"Locations were unavailable at periods of the year when birds were at very high latitudes and experiencing 24 h daylight. In addition, only longitudes were available around equinoxes, when day length is similar throughout the world. Overall, after omitting periods with light level interference and periods around equinoxes, the filtered data sets contained between 166 and 242 days of locations for each individual."*

***Question** 5.2.11*

Why did the researchers in [11] need to comment on high latitudes and the equinoxes?

High latitude (in Summer)
=> 24 hours daylight — no sunrise (no longitude)
— length of day = 24 hours
(no K, no latitude)
model fails



Equinox (longitudes)
=> have sunrise => longitude

$$D_s(t) = 12 + K \underbrace{\sin\left(\frac{2\pi}{365}(t-264)\right)}_{\text{At equinox} \sim \text{zero}}$$

Use K to get latitude

End of Case Study 11: To every thing, there is a season, tern, tern, tern.