Learning objectives

✓ Understand what a mathematical function is and how they are used for modelling in science

✓ Develop linear models of real-world phenomenon

Scientific examples

✓ Temperature at different altitudes

Maths skills

✓ Use and interpret linear functions

✓ Find a linear function to fit given data

slope   y-intercept

$$y = mx + c$$

dep. variable   indep. variable

4.2 Temperature

Question 4.2.1

Although the SI unit for measuring temperature is the kelvin (K), when communicating to the public, the more commonly used unit is degrees Celsius (°C) (or degrees Fahrenheit (°F) in some countries).

(a) What facts do you know about temperatures in °C and °F? For example, at what temperature (for each of these units of measure) does water freeze? What is a body temperature (for each of these units of measure) associated with a fever?

	°C	°F
Freeze	0	32
Fever	38	100

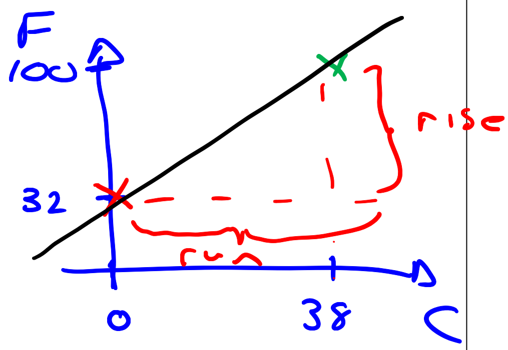
Question 4.2.1 *(continued)*

(b) Let C represent temperature measured in degrees Celsius and let F represent temperature measured in degrees Fahrenheit. Using part (a), develop a linear equation for F as a function of C.

Two points (C, F)

$(0, 32)$

$(38, 100)$



$F = mC + b$

Find m, b

slope $m = \frac{\text{rise}}{\text{run}} = \frac{100-32}{38-0} = 1.8\ {}^\circ F/{}^\circ C$

important!

intercept $b = 32\ {}^\circ F$

Hence, the relation is

$F = 1.8C + 32$

where C is in $^\circ C$ and F is in $^\circ F$

To convert $F \to {}^\circ C$ you should "subtract 32 and multiply by 5/9"

$$F - 32 = 1.8C = \frac{18}{10}C = \frac{9}{5}C$$

$$\Rightarrow C = \frac{5}{9}(F - 32)$$

Case Study 5: **Higher than a kite**

ISA = model

Photo 4.1: Jetliner cruising at an altitude of about 10,000 m. (Source: PA.)

- Scientists divide Earth's atmosphere into five primary regions: *troposphere*, *stratosphere*, *mesosphere*, *thermosphere* and *exosphere*.
- The *International Standard Atmosphere* (ISA) [27] is a model which further divides the atmosphere from the surface of Earth to the base of the thermosphere into eight layers. (Layer 0 is closest to the surface.)
- The ISA models various properties of each layer, including temperature, pressure and density.
- Layers in the ISA are defined as atmospheric regions in which *temperature is a linear function of altitude.*
- Table 4.1 and Figure 4.2 show various properties of the ISA temperature at different altitudes. (The *lapse rate* is the rate at which temperature changes as altitude increases.)

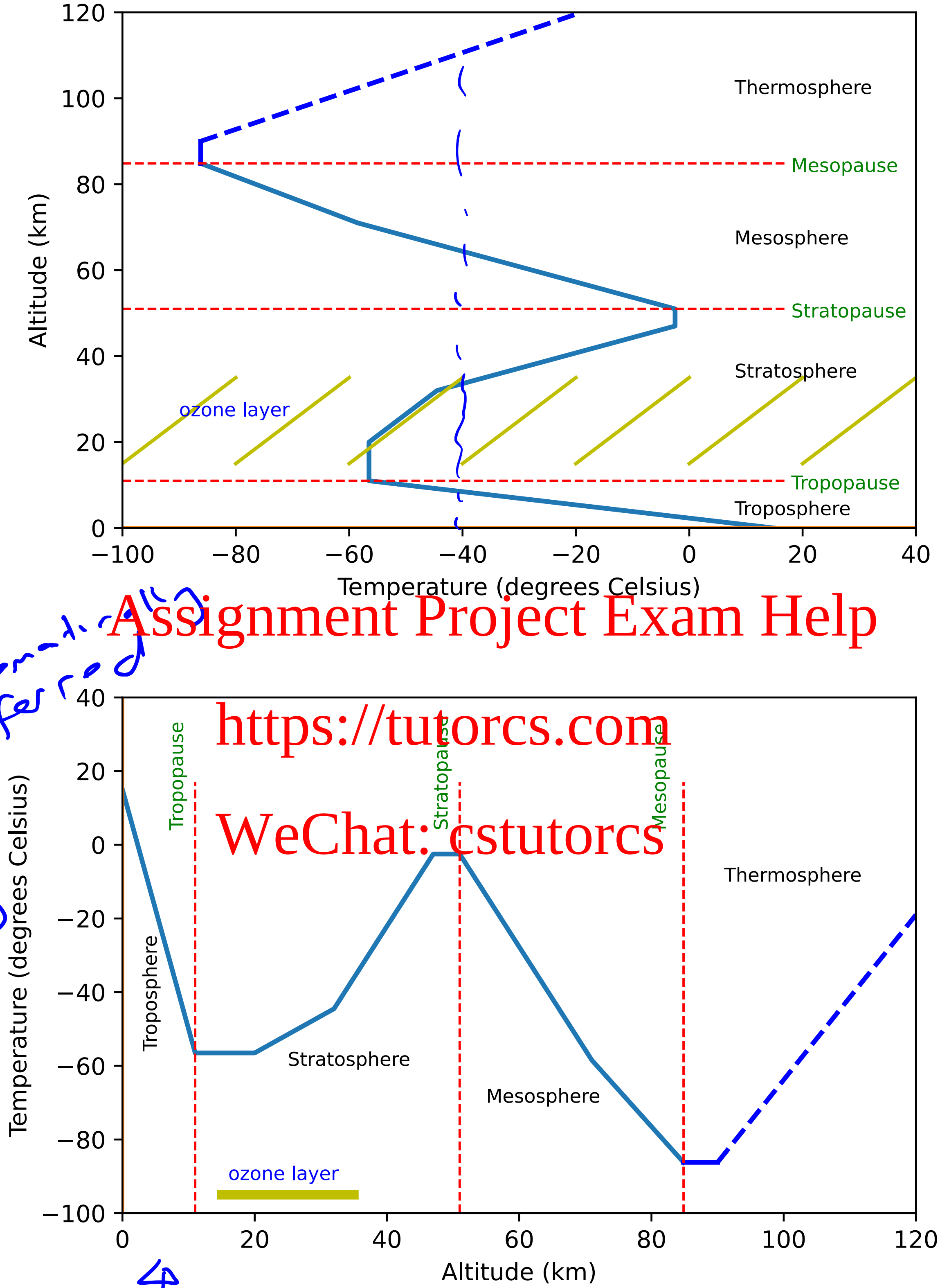
A m

Layer	Name	Height at base (km)	Lapse rate (°C/km)	Temperature at base (°C)
0	Troposphere	0.0	−6.5	+15.0
1	Tropopause	11.0	+0	−56.5
2	Stratosphere	20.0	+1.0	−56.5
3	Stratosphere	32.0	+2.8	−44.5
4	Stratopause	47.0	+0	−2.5
5	Mesosphere	51.0	−2.8	−2.5
6	Mesosphere	71.0	−2.0	−58.5
7	Mesopause	84.852	NA	−86.2

c (troposphere)

Table 4.1: Some properties of the layers within the International Standard Atmosphere.

T as a function of A



mathematically preferred

Figure 4.2: The relationships between temperature and altitude modelled by the ISA. The top graph shows altitude versus temperature, and the bottom graph shows temperature versus altitude. (The ISA does not model the thermosphere; temperature data displayed for the thermosphere are from other measurements.)

Question 4.2.2

Use the information about the ISA to answer the following.

(a) Write the troposphere temperature as a function of altitude.

Lapse rate = slope = m = $-6.5\,°C/km$

At an altitude A of 0 km, $T = +15\,°C$ (y-int, b)

$\Rightarrow b = 15\,°C$

$$T = mA + b$$

$$= -6.5A + 15$$

A is in km and T is in °C

(b) The Matterhorn is a mountain in the Swiss Alps, with a height of 4478 m above sea level. The summit air temperature can range from around 0 °C to −40 °C at different times of the year. Reconcile this with the temperature predicted by the ISA.

use our model: $T = -6.5A + 15$

$A = 4478\text{ m} = 4.478\text{ km}$

Thus $T = -6.5(4.478) + 15 \simeq -14\,°C$

ISA is a model. It doesn't account for seasons or changing weather.

Photo 4.2: The Matterhorn – Italian side (Source: PA.)

***Question** 4.2.2 (continued)*

(c) On an international flight, the following altitudes and external temperatures were reported on the in-flight information screen. The data are graphed in Figure 4.3.

$T = -6.5A + 15$

when $A = 2$ km, $T = +2°C$

when $A = 10$ km, $T = -50°C$

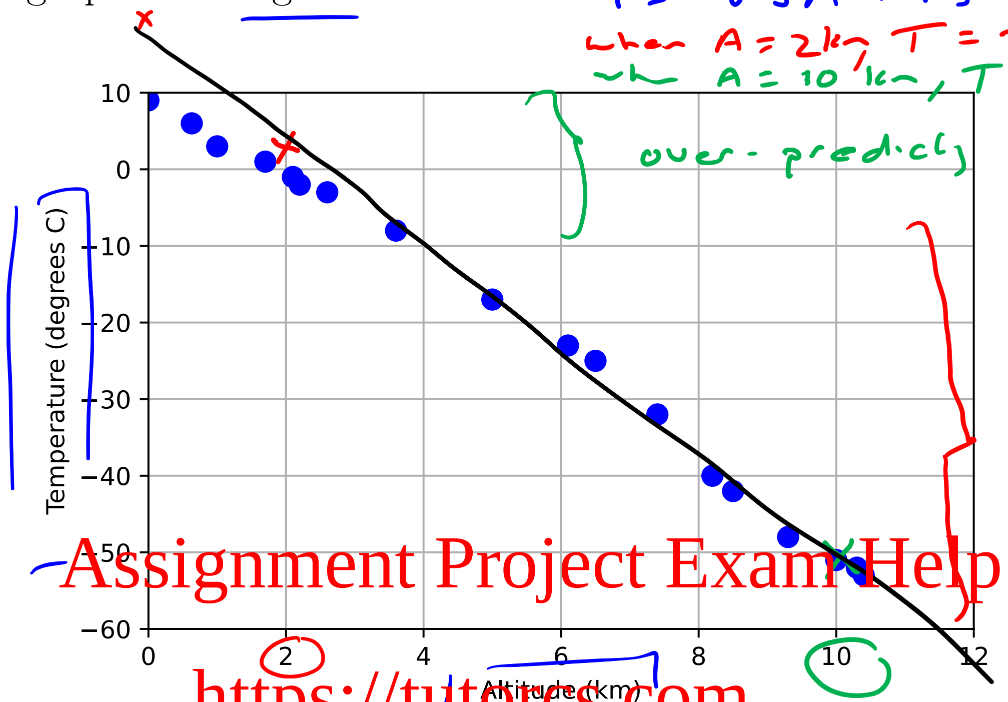


Figure 4.3: Measured external temperatures as a function of altitude.

Plot the function from Part (a) on the above graph and comment on the results.

Find two points & join with a straight line

Good fit above ~ 3km

Model over-predicts data < 3km (relates to weather)

(d) Write the temperature in ISA Layer 3 as a function of altitude.

Stratosphere At $A = 32$ km, $T = -44.5°C$

Lapse rate is $+2.8°C/km$

We can see that

$$T = +2.8A + b$$

Use $(32, -44.5) \Rightarrow -44.5 = +2.8(32) + b$

Hence $b = -44.5 - 2.8(32) \simeq -134°C$

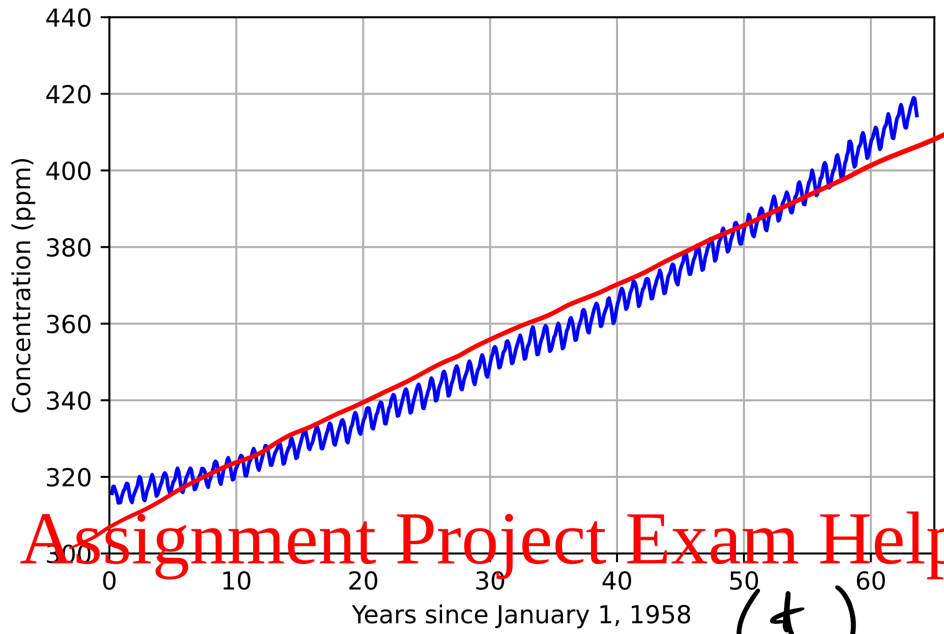
Thus $T = +2.8A - 134$ A in km, T in °C

End of Case Study 5: Higher than a kite.

Question 4.2.3

Keeling Model 1: Figure 4.4 shows a graph of the Keeling curve.

(C)



fit

(t)

Figure 4.4: The Keeling curve

(a) Find and plot a rough linear model of the Keeling curve.

use (0, 310), (60, 400)

y-intercept = 310 ppm

$$\text{slope} = \frac{400-310}{60} = \frac{90}{60} = 1.5\ \frac{\text{ppm}}{\text{year}}$$

$$C = 1.5t + 310$$

t is in years (since Jan 1, 1958)

C in ppm

(b) Discuss the limitations of your model.

No oscillations, not very accurate (doesn't reflect increase).