COMP3630/6363 Practice Exam 1, 2020

The Australian National University

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1 General (3 credits each)

For each of the statements below, determine whether it is true or false, and justify your answer in at most two sentences.

1. For regular expressions r and s, we have that $L((r|s)^*) \subseteq L((r^*s^*)^*)$.
2. Let A be an NFA, and let B be the NFA that arises from A by swapping accepting and non-accepting states. Then the language of B is the complement of the language of A.
3. If a regular expression does not contain the symbol '$\emptyset$' then the language of this regular expression is never empty.
4. The language of the regular expression $(\emptyset^*)^*$ is empty.
5. The language $L = \{ww \mid w \in \{a\}^*\}$ is regular.
6. The language $L = \{ww \mid w \in \{a\}^*\}$ is context free.
7. If L is a language, and L^* is context free, then L is context free, too.
8. Let Σ be an alphabet with $* \in \Sigma$ and let STAR be the problem of determining whether a TM M ever writes $*$ onto the tape on input w. Then STAR is deciable.
9. Every language that is not recursive is infinite.
10. The problem 3CNFTAUT – given a boolean formula in 3-CNF, does it valuate to true for *all* truth value assignments – is in P.

2 Finite Automata and Regular Languages (15 credits)

If L is a language, let $D(L)$ be the set of strings that differ from a string in L at at most one position. Show that $D(L)$ is regular if L is regular.

3 Context Free Languages and Pushdown Automata (15 credits)

Show that the language consisting of all odd-length strings $w \in \{a, b\}^*$ where the first, middle, and last character are the same, is context free.

4 Turing Machines and Recursive Languages (20 credits)

Recall that we write $\langle M \rangle$ for the coding of a Turing machine M as a string. As this coding never contains '111' as a substring, we use '111' as a separator.

Let $L = \{\langle M_1 \rangle 111 \langle M_2 \rangle \mid L(M_1) \cap L(M_2) \neq \emptyset$. Show that L is recursively enumerable.

Let $L' = \{\langle M_1 \rangle 111 \langle M_2 \rangle \mid L(M_1) \cap L(M_2) = \emptyset\}$. Is L' also recursively enumerable? Justify your answer.

5 Complexity (20 credits)

For a DFA A, let $\langle A \rangle$ be a coding of the DFA as a string, analogously to the encoding of a Turing machine.

Show that $\{\langle A \rangle \mid L(A) = \Sigma^*\} \in P$, that is, the set of DFAs that accept all strings in the language is polytime decidable.