cs2001
Introduction to the Theory of Computation

Lecture 1

Ruth Urner

January 9, 2023

Organization of the class

Basic Information

- **Instructor:** Ruth U @eecs.yorku.ca)
- **Emails:** start your subject line with [eecs2001].
- **Office Hours:** I will stay available for questions on zoom after each class and tutorial.
- **Website:** available on eClass.

Lecture times Monda[illegible]seday, 1:00-2:20pm.

Tutorial times Friday, 1:00-2:20pm.

Meeting place Zoom, meeting ID available on eclass.

First lecture Today :) January 9.

Last lecture Wedneday, April 5.

Last tutorial Monday, April 10 (make-up Friday for Good Friday on April 7).

Reading week February 18-24 (week 7).

Evaluation (to be confirmed before January 23)

4 or 5 Assignments

- Spaced throughout the term
- Exact dates will be announced soon

In-persron Midterm exam (30%)

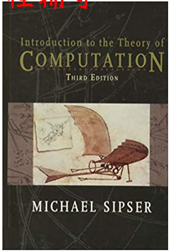
- Expect in the week after reading week
- Date to be announced once we have a room

In-person Final exam (40%)

- Date to be scheduled by the university
- Examination period: April 12-27

Introduction to the Theory of Computation (ITC)
by Michael Sipser (third edition)

- Available on Amazon.
- Available from York U bookstores.



Please get the text book. While I will aim to provide slides with a self-contained presentation of the material, it can be useful to be able to consult an additional resource. I will aim to keep presentation and notation consistent with this textbook. I would discourage consulting additional resources, since encountering deviating notation or terminology can be confusing when first learning a subject. Rather, take time and patience to learn the material from the text book, lectures, slides, class videos and always feel free to ask questions when something is unclear!

Classroom/lecture format

- I will hold lectures nounced times on zoom.

- Preliminary versions of the lecture slides will be posted to eclass before the lectures. I will update these during the 24 hours after the lectures.

- Tutorial meetings will be in person in DB 0016 and also on the same zoom meeting. Practice questions for each tutorial will be posted/announced during the week and solutions will be presented during the tutorial.

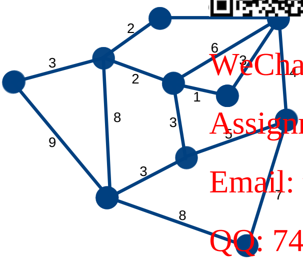
Encouragement

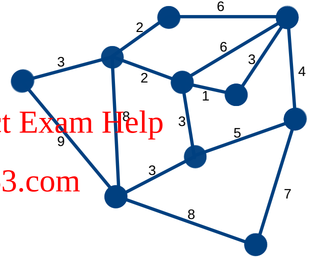
Please always feel free to ask questions if anything is unclear!

Theory of Computation–Motivation

Reading: ITC Section 0.1

Some problems are i[illegible] more difficult than others.





Hamiltonian cycle

Minimum spanning tree

Complexity theory

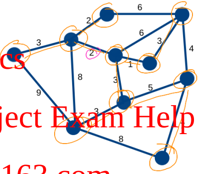
Some problem[...] more difficult than others.

doesnot have a Hamiltonian cycle

has a Hamiltonian cycle

Hamiltonian cycle

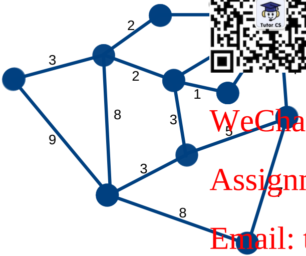
(Traveling Salesman problem)

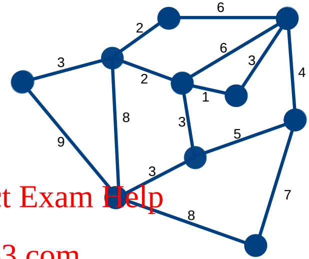


Minimum spanning tree

Talking: Ruth Urner

Some problems are inherently more difficult than others. They have different computational complexity





Hamiltonian cycle
Very difficult (NP-hard) problem

Minimum spanning tree
Easy problem,
can be solved in O(m log(n)) steps

→ In this course, we will learn how to formalize computational problems and analyze and compare their computational difficulty (complexity).

Why is it important t[illegible] and the computational complexity of a problem?

- If you have a solution (algorithm) for a problem, you may want to know if your solution is optimal, or whether it could be improved.
- Many safety critical applications rely on computational hardness for security and privacy:
  - In cryptography we want a proof that some encryption scheme is safe!

Some problems are s[illegible] that no algorithm can solve them...

Question:
Can one write a program HALT(P, i) that, when it gets the code of another program P checks whether program P will halt or loop forever when run on input i?

Answer:
No. One can prove that such a program can not exist. The halting problem is uncomputable.

Complexity theory ar[illegible]ability theory require precise definitions of problem[illegible]mputers.

Automata theory provides models of computation and the theory of formal languages provides ways to formalize what computational problems are.

→ We will use finite automata as a starting point for understanding how computation can be studied and understood in a mathematically sound way.

→ We start by reviewing some basic mathematical concepts, notation and terminology.

Basic mathematical notation and terminology

Reading: ITC Section 0.2

Sets

Set-theory is the foundation of mathematics.

We will start with the set of natural numbers as a basic given set to start with:

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

Further, we will assume the existence of the empty set: $\emptyset$

Sets

Set-theory is of mathematics.

We will start with the set of natural numbers as a basic given set to start with:

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

Further, we will assume the existence of the empty set: $\emptyset$

$\emptyset$ does not contain any elements

$\emptyset$ is a subset of every set

Elements of sets

A set consists of elements, denoted by the $\in$-relation:

$$a \in A$$

means a is an element of set A.

To state that a is not an element of set A, we use the notation

$$a \notin A.$$

Important properties:

- Sets are not ordered and contain each element only once!
- The empty set has no elements.

Elements of sets

A set consists … enoted by the $\in$-relation:

$$a \in A$$

means a is an … A.

To state that a is not an element of set A, we use the notation

$$a \notin A.$$

Important properties:

- Sets are not ordered and contain each element only once!

$\{2, 4, 6, 4\} = \{2, 4, 6\} = \{4, 6, 2\} = \{6, 4, 2, 2\}$

Subsets

Using the element re can define what subsets are:

- $A \subseteq B$

- $A \subsetneq B$

- $A = B$

Subsets

Using the ele[…]e can define what subsets are:

- $A \subseteq B$ "[…] set of B"
  If $c \in A$ then $c \in B$.
  $\{1,2,4\} \subseteq \mathbb{N}$, $\{1,2,4\} \subseteq \{1,2,4\}$
- $A \subsetneq B$ "A is a proper subset of B"
  $A \subseteq B$ and […] exists $b \notin A$.
  $\{1,2,4\} \subsetneq \mathbb{N}$, $\{n \in \mathbb{N} \mid n \text{ even}\} \subsetneq \mathbb{N}$
- $A = B$ "A and B are equal as sets"
  $A \subseteq B$ and $B \subseteq A$.
  $\{1,2,4,2\} = \{4,2,1\}$

How to define sets

Here are some tools [illegible]g sets:

1. Make a list of th[illegible]ts
   - Set of all students in this class
   - Set of odd natural numbers smaller than 10: $\{1, 3, 5, 7, 9\}$

   Problem: this technique fails for large or infinite sets

2. Identify by a common characteristic
   - Odd natural numbers $\{n \in \mathbb{N} \mid n \text{ is not divisible by } 2\}$

   Problem: sometimes we don't know a precise defining characteristic

3. Inductive definition
   - We'll see how to do this later

How to define sets – a warning

Warning:
When we define sets, [illegible]s need to specify from which universe (that is a possibly much larger ground set, for example the natural numbers) the elements of our set should be taken from!

Example:

- Odd natural numbers $\{n \in \mathbb{N} \mid n \text{ is not divisible by } 2\}$
- Interval on the real line $\{x \in \mathbb{R} \mid 2 \leq x \leq 4\}$

Otherwise we can fall into Russell's paradox...!

Russel's paradox

Consider the followin[illegible]on of a set:

$$\{r|r \notin r\}$$

That is, the set R contains all those sets that do not contain themselves as an element.

Question: Is R an element of R?

Now, $R \in R$ implies that $R \notin R$, and vice versa ($R \notin R$ implies that $R \in R$)– a contradiction.

Russel's paradox

Consider the ion of a set:

$$= \{r | r \notin r\}$$

That is, the set R contains all those sets that do not contain themselves as an element.

Question: Is R an element of R?

Now, $R \in R$ implies that $R \notin R$, and vice versa ($R \notin R$ implies that $R \in R$)– a contradiction.

Set operations

By default, we will fo[illegible]ume that all our sets are subsets of the natural numbers. [illegible]ve assume a universe $U = \mathbb{N}$. Then we can define:

- The set-difference $A \setminus B$ of two sets A and B

- The intersection $A \cap B$ of two sets A and B

- The union $A \cup B$ of two sets A and B

Set operations

By default, w… ssume that all our sets are subsets of the natural n… we assume a universe $U = \mathbb{N}$. Then we can define

- The set-difference $A \setminus B$ of two sets A and B

$A \setminus B = \{a \in A \mid a \notin B\}$

$A = \{1, 3, 5, 7\}$, $B = \{1, 2, 3\}$

$A \setminus B = \{5, 7\}$

always: $A \setminus B \subseteq A$