Представяне на функции и операции с тях

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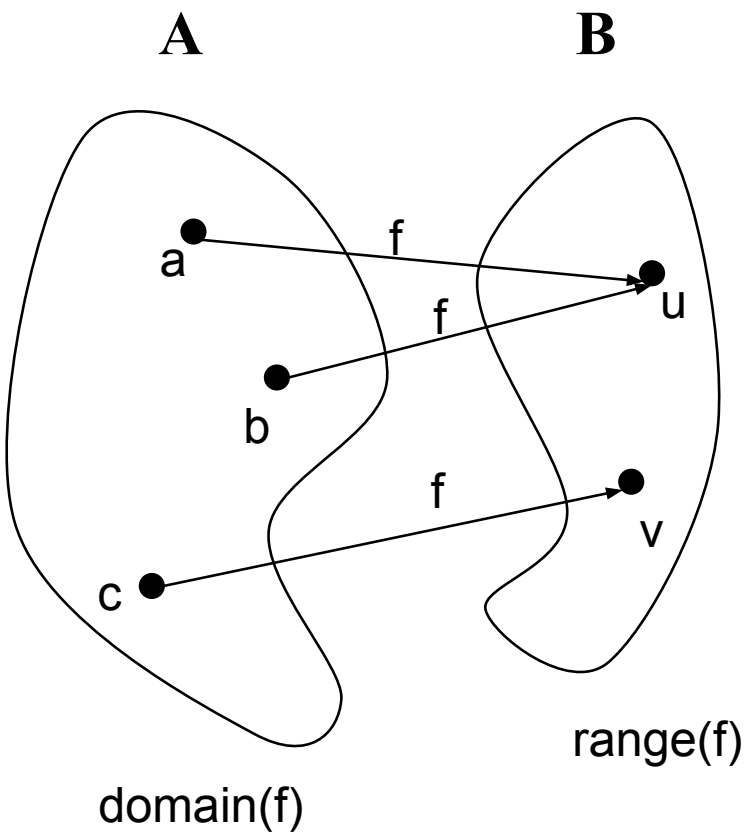
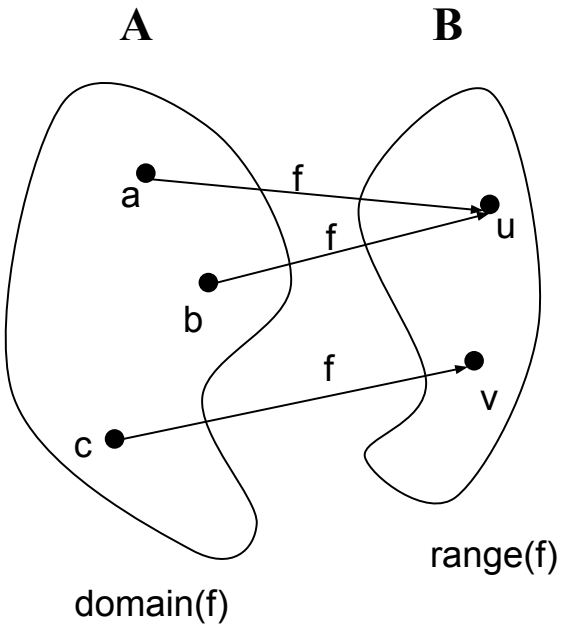
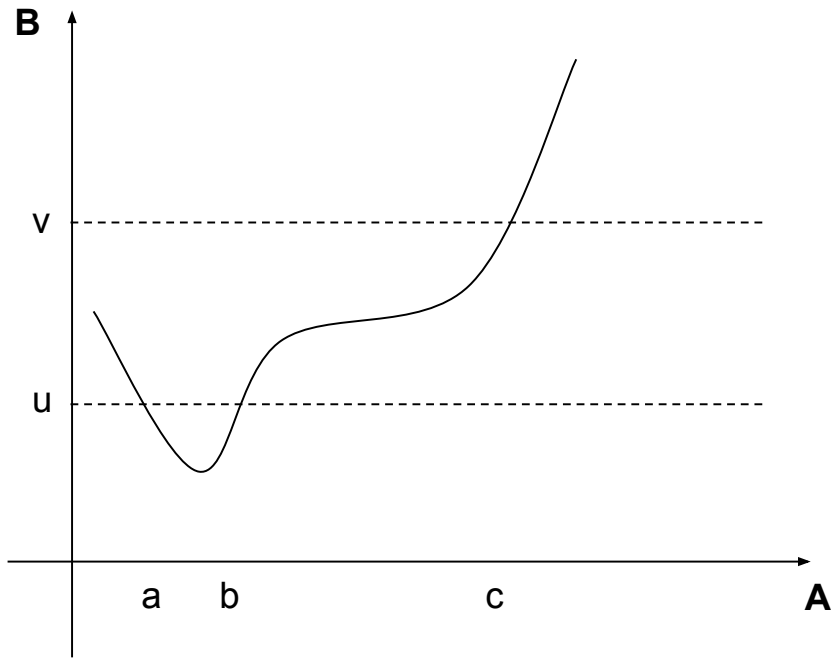
Какво е функция?

Изображение $f : A \to B$



Графика на функцията, $G(f) = \{(x, f(x))|x \in range(f)\}$





Как представяме функция компютърно?

Таблично представяне

x	f(x)
a	u
b	u
c	v

Представяне чрез програма

```
double f (double x)
{
  if (x == a) return u;
  if (x == b) return u;
  if (x == c) return v;
  return 0;
}
```

Представяне на функцията чрез “атрибути”,
които я дефинират еднозначно

Константна функция

$$f(x) = 5$$

```
class Constant
{
  private:
    double c;
  public:
    Constant (double val):c(val){}
}
```

Линейна функция

$$f(x) = 2x + 5$$

```
class Linear
{
  private:
    double coef;
    double constant;
  public:
    Linear (double a,double b):
        coef(a),constant(b){}
}
```

Полином

$$f(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_n$$

```
class Polynom
{
  private:
    vector<double> coefs;
  public:
    Polynom (vector<double>arr):
       coefs(arr){}
}
```

Кое е общото между всички
едноместни числови функции?

$$y = f(x)$$

```
class Function
{
  public:
  virtual double value (double x) = 0;
}
```

Константна функция

$$f(x) = 5$$

```
class Constant : public Funtion
{
  private:
    double c;
  public:
    Constant (double val):c(val){}

    double value (double x) {return c};
}
```

Линейна функция

$$f(x) = 2x + 5$$

```
class Linear : public Funtion
{
  private:
    double coef;
    double constant;
  public:
    Linear (double a,double b):
        coef(a),constant(b){}

    double value (double x)
    {
      retuen coef*x + constant;
    }
}
```

Полином

$$f(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_n$$

```
class Polynom : public Funtion
{/...
    double value (double x)
    {
      double sum = coefs[0];
      for (int i = 1; i < coefs.size()-1; i++)
        sum = sum * x + coefs[i];
      return sum;
    }
}
```

Оператори над функции

Функция vs. Оператор

- Функция: $f : A \to B$, $g : B \to C$
- Оператор: $\Gamma : (A \to B) \to (A \to B)$
- Оператор: $\Gamma : (A \to B) \times (B \to C) \to (A \to C)$

$\Gamma : (double \to double) \to (double \to double)$

$$\Gamma(f)(x) = \begin{cases} f(x) & f(x) \geq 0 \\ 0 & otherwise \end{cases}$$

```
class CutFunction : public Function
{
  private:
    Function* f;
  public:
    CutFunction (Function *_f):f(_f){}
    double value (double x){
      double y = f->value(x);
      if (y >= 0)
        return y;
      return 0;
    }
}
```

Използване на CutFunction

```
class CutFunction : public Function
{
  private:
    Function* f;
  public:
    CutFunction (Function *_f):f(_f){}
    double value (double x){
      double y = f->value(x);
      if (y >= 0)
        return y;
      return 0;
    }
}
```

```
int main ()
{
  Linear fn (-2,10);
  CutFunction cfn (&fn);

  cout << fn.value (10)
       << endl
       << cfn.value (10);
}
```

f и $CutFn(f)$ са функции

```
void printall (Function *functions[],
               int n,
               double x)
{
  for (int i = 0; i < n; i++)
    cout << functions[i]->value(x) << endl;
}
int main ()
{
  Linear fn (-2,10);
  CutFunction cfn (&fn);
  Function *functions[] = {&fn,&cfn};
  printall (functions,2,10);
}
```

$\Gamma : (double \to double) \times (double \to double) \to (double \to double)$

$$\Gamma(f, g)(x) = f(g(x))$$

```
class Composition : public Function
{
  private:
    Function* f;
    Function* g;
  public:
    Composition (Function *_f, Function *_g)
        :f(_f),g(_g){}
    double value (double x){
      return f->value (g->value (x));
    }
}
```

Използване на Composition

```
int main ()
{
  Linear fn (-2,10);
  CutFunction cfn (&fn);
  Composition comp (&fn,&cfn);

  cout << comp.value (10);

  Function *functions[] = {&fn,&cfn,&comp};
  printall (functions,3,10);

}
```

Програмите като оператори

IF като оператор

```
double G (double x)
{
  if (f(x) != 0)
    return g(x);
  return h(x);
}
```

$$\Gamma(f, g, h)(x) = \begin{cases} g(x) & f(x) \neq 0 \\ h(x) & otherwise \end{cases}$$

IF като оператор

$$\Gamma(f,g,h)(x) = \begin{cases} g(x) & f(x) \neq 0 \\ h(x) & otherwise \end{cases}$$

```
class IfOperator : public Function
{
  private:
    Function* condfn; //f
    Function* thenfn; //g
    Function* elsefn; //h
  public:
    IfOperator (Function *_c,
                Function *_t,
                Function *_e):condfn(_c),
                              thenfn(_t),
                              elsefn(_e){}
}
```

IF като оператор

$$\Gamma(f,g,h)(x) = \begin{cases} g(x) & f(x) \neq 0 \\ h(x) & otherwise \end{cases}$$

```
class IfOperator : public Function
{
  public:
    double value (double x){
      if (condfn->value(x) == 0)
        return elsefn->value (x);
      return thenfn->value (x);
    }
}
```

FOR като ператор

```
double G (double x)
{
  double z = x;
  for (int i = 0; i < N; i++)
    z = f(z);
  return z;
}
```

$$\Gamma(f, n)(x) = \underbrace{f(f \ldots (f}_{n}(x)..)$$

FOR като оператор

$$\Gamma(f, n)(x) = \underbrace{f(f...(f}_{n}(x)..)$$

```
class ForOperator : public Function
{
  private:
    Function* fn;
    int count;
  public:
    ForOperator (Function *_fn):fn(_fn){}
    double value (double x)
    {
      double z  = x;
      for (int i = 0; i < count; i++)
        z = fn->value(z);
      return z;
    }
}
```

Благодаря ви за вниманието!