The divisive normalization model provides an insufficient account of violations of independence

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SUPPLEMENTARY INFORMATION

Supplementary Methods

Procedures and settings for maximum likelihood estimation (MLE) and hierarchical Bayesian modeling (HBM) followed those of our original paper[1] as closely as possible. Therefore, we do not reiterate these procedures here but only mention the relevant deviations. For MLE, parameter σ was constrained to lie between $2.22*10^{-16}$ and 20, and a grid search with 10 equally spaced values from 0.01 to 18.1 was used before running the simplex minimization. Parameter ω was constrained to lie between 0 and 10 for the regular DN model and between -5 and 5 for the proposed extension, respectively. The grid search for this parameter used 10 equally spaced values from 0 to 1.81 for the regular DN model and from -0.9 to 0.9 for the proposed extension, respectively. Similar to our original paper[1], we used the best estimate of σ from the multinomial Logit to inform the parameter estimation of the DN model variants.

When implementing the HBM model, we followed the approach of Webb and colleagues[2] by drawing individual parameter estimates of σ and ω from gamma distributions. Note that this enforces positive estimates, which is a reasonable assumption for parameter σ but not necessarily for parameter ω (see main text). To allow negative estimates of ω and at the same time avoid a negative denominator in Equation 1 of Webb and colleagues[2], we subtracted the right-hand side of the following inequality from ω:

$$\omega_i > \frac{\sigma_i}{\max(\sum_t V_{i,t})}, \tag{1}$$

where $\max(\sum_t V_{i,t})$ refers to the largest sum of values that a participant i saw in anyone trial t, and added .000001 (for MLE, we ensured that this inequality would hold by using the linear inequality constraint feature of the minimization function *fminsearchcon.m*; note, however, that initial analyses without these precautions let to the same results). We used uniform priors ranging from 0 to 100 to draw the group-level shape and rate parameters of the gamma distributions. We also tried out different approaches, including i) drawing group-level means and standard deviations for the gamma distributions and then re-parameterizing these values to shape and rate parameters[3] and ii) drawing individual estimates from (half-)normal distributions. These variations did not change the results qualitatively. Our convergence criterion (i.e., R-hat < 1.01) was reached for all parameters of the multinomial Logit model. For the regular DN model, the group-level shape parameter had an R-hat value of 1.017, and for the extended DN model, the individual estimates of σ and ω for participant #3 (the participant who rated many snacks with 0, see our original paper[1]) had R-hat values of 1.143 and 1.223, respectively. All other R-hat values were below 1.01. For the dataset of the eye-tracking experiment, the convergence criterion was reached for all models and parameters.

To test for a correlation between parameter ω and the relationship of the height and variability of the BDM value ratings, we made use of the fact that each participant provided two BDM value ratings. We calculated the absolute difference of these ratings for every snack item. Then, we calculated the correlation between the sum of these absolute differences and the sum of the values of the options presented during the ternary choice task in every participant. These individual correlation coefficients were then correlated with parameter ω.

The custom code for these analyses are available on OSF.

Supplementary References

1. Gluth, S., Kern, N., Kortmann, M. & Vitali, C. L. Value-based attention but not divisive normalization influences decisions with multiple alternatives. *Nature Human Behaviour* (2020) doi:10.1038/s41562-020-0822-0.
2. Webb, R., Glimcher, Paul W., P. W. & Louie, K. Divisive normalization does influence decisions with multiple alternatives - an econometric analysis of Gluth et al. (2020). (submitted).
3. Kruschke, J. K. *Doing Bayesian Data Analysis*. (Academic Press, 2015).