SUPPLEMENTARY MATERIAL: A SYSTEMATIC STUDY INTO THE FACTORS THAT AFFECT THE PREDICTIVE ACCURACY OF MULTILEVEL VAR(1) MODELS

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1. Study I: The effect of the number of measurement occasions, the number of persons, and the number of variables

The goal of the first simulation study was to investigate the effect of three data characteristics on the predictive accuracy of person-specific and multilevel AR(1) and VAR(1) models: the number of measurement occasions, the number of persons, and the number of variables. We predict that the number of measurement occasions and the number of persons are positively related to predictive accuracy, while the number of variables has a negative impact. The data were generated according to four models: a multilevel AR(1) model with fixed autoregressive effects (MAR.FE), a multilevel AR(1) model with random autoregressive effects (MAR.RE), a multilevel VAR(1) model with a fixed transition matrix (MVAR.FE), and a multilevel VAR(1) model with random transition matrices (MVAR.RE). Throughout the simulation study, we assumed that the person-specific intercepts equal zero. Note that we did not generate data with person-specific models in this first study, as questions regarding the distribution of individual differences were investigated in Study III.

1.1. Simulation Design

The data were generated according to a four-factorial design, including 10 replicates per design cell. Specifically, we manipulated: (i) the number of individuals: 20, 60, and 120; (ii) the number of measurement occasions per individual: 50, 100, and 200; and (iii) the number of

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variables: 2, 4, 6, and 8. The fourth factor pertained to the data generating model: MAR.FE, MAR.RE, MVAR.FE, MVAR.RE.

When the data generating process was a MAR.FE model, the autoregressive effects were sampled from a Uniform distribution on the interval $[.2, .6]$. The standard deviation of the within-individual errors σ_ϵ was set to one for all individuals. In case we simulated data with a MAR.RE model, the fixed autoregressive effects were sampled as in the MAR.FE setting. The random effects were drawn from a Gaussian distribution with mean given by the fixed autoregressive effects, and each of the random effect variances fixed to .1 (see e.g., Liu (2017)) and the covariances to zero. To ensure that the generated time series of each variable and each person conformed to the stationary assumption, we checked whether the person-specific autoregressive effect had an absolute value smaller than one. Therefore, the distribution of each random autoregressive effect approximates a truncated Gaussian distribution (Ernst et al., 2019).

In the MVAR.FE setting, the autoregressive effects were again drawn from a uniform distribution on the interval $[.2, .6]$. Half of the cross-regressive effects were sampled from a uniform distribution on the interval $[.05, .20]$, and the other half from a uniform distribution $[-.20, -.05]$. To guarantee that the generated time series were stationary, the regression weights were multiplied by $0.99/(|\lambda_p|)$, where $|\lambda_p|$ denoted the absolute value of the maximum eigenvalue of the transition matrix. Furthermore, the diagonal elements of the covariance matrix of the within-individual errors $\mathbf{\Sigma}_\epsilon$ were set to one, and the off-diagonal elements to .2. In the MVAR.RE scenario, the fixed autoregressive and cross-regressive effects were generated as was done in the MVAR.FE scenario. The random effects were generated from a multivariate Gaussian distribution with mean given by the fixed autoregressive and cross-regressive effects, and a diagonal covariance matrix in which the variances were set to .025 (see e.g., (Ernst et al., 2019)). We again ensured that the generated time series of each participant conformed to the stationary assumption by checking the eigenvalues of the individual transition matrices. To this end, the variance of the random effects was set to a lower value than in the MAR.RE setting.

For each of the 1440 resulting datasets, we computed the predictive accuracy of six different models, using 10-blocked CV: person-specific AR(1) (AR), person-specific VAR(1) (VAR), MAR.FE, MAR.RE, MVAR.FE, MVAR.RE. All analyses were performed in R version 3.6.3 (R

Core Team, 2013). Multilevel models were estimated using the lme4 package (Bates et al., 2015).

1.2. Results

1.2.1. Predictive Accuracy

The average obtained MSE values are shown in Table 1; in the supplementary material we present more detailed predictive performance results. The person-specific AR(1) and VAR(1) models very often exhibit large MSEs across all conditions, showing that person-specific models are in most settings too complex to adequately capture the regularities of the time series under study. Yet, as predicted, person-specific models perform slightly better when the number of measurement occasions increases and the number of variables decreases, whereas the number of persons seems to matter less.

For the MAR.FE case, we see that across all conditions the corresponding estimation model (MAR.FE) has the best overall predictive performance, which shows that the true model generalizes best to unseen data. Except for the settings with 50 measurement occasions only, the MAR.RE estimation model behaves similarly. The estimated MVAR.RE models have a similar predictive performance when the number of variables is 2, and when the number of persons is 120.

When the data generating process is a MAR.RE model, results show that estimating this model yields the smallest MSE across all conditions. Yet, the MSE slightly increases with the number of variables, and slightly decreases with the number of time points, and the number of persons, as predicted. A similar but stronger pattern is found for the MVAR.RE model. When fitting MAR.FE, predictive accuracy hardly changes across conditions.

Turning to the MVAR.FE scenario, results show that fitting the correct model again yields the best overall predictive performance. The MVAR.RE results follow the expected trends and increase with the number of variables and decrease with the number of persons and the number of measurement occasions. Both MAR models have a very similar performance and perform worse than the correct model.

Also for the MVAR.RE setting, the best predictions are obtained with the corresponding model. Interestingly, the person-specific VAR(1) model has the second-best predictive accuracy.

The FE models perform quite bad, since they cannot handle individual differences.

In sum, across all conditions, the best prediction results are obtained when the estimation model matches the true model. This pattern is more clear when the true model is an MVAR model, or when the true model allows for individual differences (i.e., RE model). Especially for the most complex data generating model, results are clearly affected by the number of variables and the number of measurement occasions, and to a lesser extent, by the number of persons.

1.2.2. Estimation Accuracy

To complement the predictive accuracy results presented above, we also evaluated the estimation accuracy of the different models, focusing on the recovery of the transition matrices $\mathbf{\Psi}_i$. Specifically, we computed the mean squared estimation error as follows:

$$\mathrm{MSE}_{\mathbf{\Psi}} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{P^2}\sum_{j=1}^{P}\sum_{k=1}^{P}(\hat{\psi}_{ijk} - \psi_{ijk})^2\right) \tag{1}$$

where $\mathbf{\Psi}_i = (\psi_{ijk})$ and $\hat{\mathbf{\Psi}}_i = (\hat{\psi}_{ijk})$ denote the true and estimated transition matrices.

The resulting averages are presented in the Appendix in Table 2. In general the best estimation performance is always obtained when the estimation model corresponds to the true model. For the MAR.FE case, we observe that all multilevel estimation procedures show rather good performance in estimating the true autoregressive effects, whereas the estimation accuracy of the person-specific AR(1) model is positively related to the number of measurement occasions. In the MVAR.FE case, the second-best performance is given by the MVAR.RE model. For the MVAR.RE scenario, the person-specific VAR(1) model reaches quite good estimation accuracy, especially when the number of measurement occasions is large.

References

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Tables

TABLE 1.

Simulation Results Study I. The prediction error estimates across simulation conditions.

Population Model	Method	Number of Variables				Number of Persons			Measurement Occasions		
		$P = 2$	$P = 4$	$P = 6$	$P = 8$	$N = 20$	$N = 60$	$N = 120$	$T = 50$	$T = 100$	$T = 200$
MAR.FE	AR	1.028	1.027	1.027	1.026	1.027	1.028	1.027	1.047	1.023	1.011
	MAR.FE	**1.002**	**1.001**	**1.001**	**1.000**	**1.001**	**1.001**	**1.000**	**1.001**	**1.001**	**1.000**
	MAR.RE	**1.002**	**1.001**	**1.001**	**1.000**	**1.001**	**1.001**	**1.000**	1.002	**1.001**	**1.000**
	MVAR.FE	**1.002**	1.002	1.002	1.002	1.003	1.002	**1.000**	1.004	1.002	1.001
	MVAR.RE	1.003	1.006	1.006	1.011	1.012	1.005	1.002	1.011	1.006	1.003
	VAR	1.045	1.103	1.103	1.165	1.103	1.104	1.103	1.192	1.081	1.037
MAR.RE	AR	1.027	1.028	1.028	1.028	1.029	1.028	1.027	1.048	1.024	1.012
	MAR.FE	1.014	1.014	1.014	1.015	1.015	1.014	1.014	1.015	1.015	1.014
	MAR.RE	**1.007**	**1.008**	**1.008**	**1.008**	**1.009**	**1.007**	**1.007**	**1.011**	**1.008**	**1.005**
	MVAR.FE	1.014	1.016	1.016	1.017	1.018	1.015	1.014	1.017	1.016	1.014
	MVAR.RE	1.008	1.014	1.014	1.020	1.021	1.012	1.009	1.021	1.013	1.007
	VAR	1.044	1.104	1.104	1.169	1.106	1.104	1.103	1.192	1.083	1.038
MVAR.FE	AR	1.050	1.122	1.122	1.204	1.122	1.122	1.122	1.144	1.119	1.103
	MAR.FE	1.023	1.093	1.093	1.174	1.095	1.093	1.092	1.095	1.095	1.091
	MAR.RE	1.024	1.094	1.094	1.174	1.096	1.093	1.092	1.095	1.095	1.091
	MVAR.FE	**1.003**	**1.002**	**1.002**	**1.004**	**1.005**	**1.001**	**1.000**	**1.003**	**1.002**	**1.001**
	MVAR.RE	1.004	1.007	1.007	1.013	1.014	1.004	1.002	1.011	1.006	1.003
	VAR	1.045	1.104	1.104	1.170	1.103	1.105	1.104	1.193	1.082	1.037
MVAR.RE	AR	1.082	1.358	1.358	1.708	1.355	1.361	1.359	1.362	1.357	1.356
	MAR.FE	1.095	1.408	1.408	1.800	1.399	1.412	1.413	1.383	1.409	1.432
	MAR.RE	1.065	1.341	1.341	1.690	1.339	1.342	1.341	1.329	1.343	1.349
	MVAR.FE	1.074	1.273	1.273	1.532	1.258	1.277	1.286	1.259	1.274	1.288
	MVAR.RE	**1.019**	**1.060**	**1.060**	**1.107**	**1.065**	**1.058**	**1.056**	**1.095**	**1.055**	**1.029**
	VAR	1.043	1.109	1.109	1.186	1.107	1.109	1.111	1.203	1.085	1.039

Appendix

TABLE 2.

Simulation Results Study I. The estimation accuracy (standard errors) in percentage across simulation conditions when $N = 60$.

Population		$P = 2$			$P = 4$			$P = 6$			$P = 8$		
Model	Method	$T = 50$	$T = 100$	$T = 200$	$T = 50$	$T = 100$	$T = 200$	$T = 50$	$T = 100$	$T = 200$	$T = 50$	$T = 100$	$T = 200$
MAR.FE	AR	0.994	0.422	0.219	0.522	0.227	0.111	0.328	0.151	0.074	0.224	0.111	0.053
		(0.022)	(0.016)	(0.009)	(0.014)	(0.010)	(0.002)	(0.012)	(0.004)	(0.002)	(0.005)	(0.002)	(0.001)
	MAR.FE	**0.009**	**0.008**	**0.003**	**0.007**	**0.003**	**0.001**	**0.005**	**0.002**	**0.001**	**0.004**	**0.002**	**0.001**
		(0.003)	(0.002)	(0.001)	(0.002)	(0.001)	(0.000)	(0.001)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)
	MAR.RE	0.021	0.015	0.006	0.015	0.008	0.003	0.009	0.005	0.003	0.006	0.003	0.002
		(0.005)	(0.003)	(0.002)	(0.004)	(0.002)	(0.000)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
	MVAR.FE	0.021	0.012	0.004	0.028	0.014	0.008	0.029	0.013	0.007	0.028	0.013	0.007
		(0.004)	(0.002)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
	MVAR.RE	0.057	0.026	0.013	0.089	0.049	0.024	0.106	0.053	0.025	0.127	0.060	0.031
		(0.007)	(0.004)	(0.003)	(0.007)	(0.004)	(0.001)	(0.005)	(0.002)	(0.001)	(0.006)	(0.001)	(0.001)
	VAR	2.079	0.862	0.444	2.271	0.971	0.465	2.387	0.993	0.458	2.552	1.042	0.460
		(0.036)	(0.031)	(0.016)	(0.042)	(0.024)	(0.008)	(0.030)	(0.014)	(0.007)	(0.021)	(0.006)	(0.005)
MAR.FE	AR	0.952	0.443	0.206	0.510	0.227	0.109	0.326	0.151	0.075	0.230	0.110	0.052
		(0.036)	(0.018)	(0.007)	(0.021)	(0.011)	(0.002)	(0.009)	(0.005)	(0.002)	(0.004)	(0.003)	(0.001)
	MAR.FE	0.514	0.514	0.474	0.237	0.268	0.233	0.170	0.165	0.170	0.130	0.124	0.126
		(0.022)	(0.017)	(0.019)	(0.005)	(0.015)	(0.007)	(0.005)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)
	MAR.RE	**0.328**	**0.244**	**0.142**	**0.168**	**0.122**	**0.072**	**0.114**	**0.077**	**0.053**	**0.086**	**0.056**	**0.037**
		(0.017)	(0.012)	(0.007)	(0.005)	(0.004)	(0.002)	(0.004)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)
	MVAR.FE	0.527	0.522	0.478	0.256	0.276	0.239	0.190	0.176	0.176	0.154	0.135	0.132
		(0.022)	(0.018)	(0.019)	(0.005)	(0.015)	(0.007)	(0.005)	(0.005)	(0.004)	(0.004)	(0.003)	(0.003)
	MVAR.RE	0.357	0.265	0.154	0.246	0.160	0.092	0.208	0.126	0.080	0.207	0.117	0.066
		(0.018)	(0.016)	(0.008)	(0.005)	(0.005)	(0.001)	(0.004)	(0.003)	(0.002)	(0.004)	(0.003)	(0.001)
	VAR	2.007	0.904	0.418	2.172	0.960	0.453	2.351	0.982	0.460	2.554	1.021	0.452
		(0.057)	(0.025)	(0.012)	(0.045)	(0.026)	(0.005)	(0.027)	(0.014)	(0.007)	(0.017)	(0.008)	(0.004)
MVAR.FE	AR	1.784	1.168	1.265	1.828	1.566	1.402	1.786	1.729	1.528	1.826	1.719	1.727
		(0.131)	(0.121)	(0.090)	(0.072)	(0.108)	(0.099)	(0.054)	(0.045)	(0.071)	(0.033)	(0.045)	(0.048)
	MAR.FE	0.854	0.757	1.069	1.401	1.368	1.312	1.554	1.610	1.477	1.657	1.662	1.704
		(0.127)	(0.128)	(0.098)	(0.079)	(0.115)	(0.099)	(0.064)	(0.050)	(0.072)	(0.033)	(0.052)	(0.049)
	MAR.RE	0.866	0.761	1.073	1.410	1.371	1.313	1.553	1.614	1.477	1.658	1.662	1.704
		(0.126)	(0.128)	(0.096)	(0.079)	(0.115)	(0.099)	(0.063)	(0.050)	(0.072)	(0.032)	(0.052)	(0.049)
	MVAR.FE	**0.034**	**0.015**	**0.007**	**0.030**	**0.015**	**0.009**	**0.028**	**0.016**	**0.007**	**0.027**	**0.014**	**0.007**
		(0.009)	(0.004)	(0.002)	(0.003)	(0.003)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)
	MVAR.RE	0.089	0.040	0.018	0.101	0.044	0.026	0.108	0.060	0.027	0.122	0.057	0.031
		(0.015)	(0.006)	(0.003)	(0.008)	(0.004)	(0.001)	(0.007)	(0.002)	(0.002)	(0.003)	(0.003)	(0.002)
	VAR	2.077	0.907	0.431	2.245	0.961	0.468	2.312	1.011	0.459	2.405	0.967	0.442
		(0.070)	(0.030)	(0.014)	(0.058)	(0.021)	(0.008)	(0.026)	(0.014)	(0.009)	(0.022)	(0.018)	(0.006)
MVAR.RE	AR	3.474	2.711	2.502	3.643	3.767	3.632	4.035	3.855	3.992	4.204	4.436	4.305
		(0.197)	(0.194)	(0.152)	(0.124)	(0.086)	(0.066)	(0.056)	(0.088)	(0.113)	(0.066)	(0.085)	(0.069)
	MAR.FE	3.871	3.525	3.612	3.950	4.456	4.439	4.650	4.451	4.752	4.940	5.248	5.314
		(0.242)	(0.162)	(0.217)	(0.132)	(0.129)	(0.134)	(0.151)	(0.142)	(0.185)	(0.163)	(0.128)	(0.213)
	MAR.RE	3.083	2.589	2.476	3.530	3.792	3.651	4.098	3.916	4.029	4.326	4.534	4.350
		(0.191)	(0.190)	(0.155)	(0.131)	(0.097)	(0.065)	(0.064)	(0.099)	(0.117)	(0.071)	(0.088)	(0.073)
	MVAR.FE	2.637	2.523	2.686	2.599	2.809	2.845	2.930	2.883	2.989	3.065	3.275	3.380
		(0.114)	(0.091)	(0.105)	(0.046)	(0.045)	(0.080)	(0.069)	(0.068)	(0.105)	(0.090)	(0.094)	(0.158)
	MVAR.RE	**1.084**	**0.628**	**0.371**	**1.054**	**0.580**	**0.348**	**1.055**	**0.604**	**0.323**	**1.021**	**0.583**	**0.323**
		(0.042)	(0.028)	(0.014)	(0.024)	(0.014)	(0.013)	(0.015)	(0.006)	(0.009)	(0.013)	(0.011)	(0.006)
	VAR	1.962	0.914	0.445	2.050	0.823	0.406	1.922	0.839	0.386	1.947	0.800	0.380
		(0.048)	(0.039)	(0.016)	(0.041)	(0.021)	(0.014)	(0.036)	(0.013)	(0.010)	(0.029)	(0.010)	(0.008)