Heads & Tails: A study of the informational value of Greek coinage (650 BCE – 336 BCE)

results report

Barbara Pavlek & Olivier Morin

The aim of this project was to test three predictions on the dynamic of informative value of motifs on ancient Greek coins (see project preregistration document **coins-2017-11-20.pdf**).

0. Clarification to the registered research plan.

Our registered predictions were as follows:

(1) motifs become less informative about a coin's provenance;

(2) motifs become more informative about a coin's denomination;

(3) motifs are more informative about lower denominations than about higher ones.

These predictions could be interpreted in two different ways, depending on how we treat motifs. motifs can be considered in isolation, or they could be considered as sets of motifs, as they occur on coins. In the first case, our predictions should apply to single motifs; in the second, to motifs as sets.

We performed our analyses using the two interpretations, but this report insists on the second interpretation. As we soon figured out, motifs on their own are poorly informative. In hindsight, this makes sense: given a set of thousands of coins, simply knowing which of them bear an Owl and which do not cannot tell one much, relative to the size and complexity of the set of coin types one needs information about. Furthermore, what informational value there is among single motifs is quite unevenly distributed, and this skewness makes it very difficult to see definite trends.

With these caveats in mind, both types of results are presented here. For clarity's sake, we will be talking about "designs" when considering motifs as sets of motifs, reserving the term "single motifs" for motifs considered in isolation.

We'll be using the following abbreviations:

H(A): Entropy of authorities

H(A|M): Entropy of authorities given single motifs

H(A|d): Entropy of authorities given designs

H(D): Entropy of denominations

H(D|M): Entropy of denominations given single motifs

H(D|d): Entropy of denominations given designs

We present, therefore, two analyses for each prediction, one "designs-based" and one "single motifs-based". The exception is prediction 3. In the registration, we stated that "Prediction 3 will only be tested if the conditional entropy of denomination given motif is not trivially low, that is, if we can establish that some motifs reliably track some denominations." (see **coins-2017-11-20.pdf**, 4.). There is a mistake in this sentence ("low" should be "high": motifs do reliably track denominations if the conditional entropy of denomination given motif is low), but it clearly implies that prediction 3 shouldn't be tested on single motifs, as most of these are hardly informative at all.

1. Prediction 1: motifs become less informative about a coin's provenance

1.1. Designs-based analysis

In our study, we coded each coin type's provenance according to its issuing authority as "authority", a variable which included the information on the city-state that minted the coin (*polis*), or the mint, or the ruler. We used the information provided by the SNG dataset (see **coins-2017-11-20.pdf**, 3.2.3.).

Following our preregistered methods, we calculated the conditional entropy of authority given design, using the `infotheo` package (Meyer 2014) in R (version 3.4.2, R Core Team 2017). We computed the conditional entropy for each period between the coins' designs and the authorities which issued the coins. We found that the conditional entropy of authorities given designs H(A|d) does increase with time when we consider our preregistered three time periods (6th, 5th and 4th century BCE inside the time frame of 650 – 336 BCE; see **Figure 1a**).

We decided to subdivide our time frame into 13 time periods, in order to get a more precise idea on the informative value of designs about the coin's provenance (see **Figure 1b**). Since our dataset is skewed towards later periods, we created uneven time "bins" in order to spread the data more evenly among them: we joined the 7th and the 6th century (650-500 BCE) in one bin, split the first half of the 5th century in 20 years periods, the second half of the 5th century in 10 years periods, and the 4th century again in 20 years periods. We will mark the time periods on plots using the median year of the time period as a label.

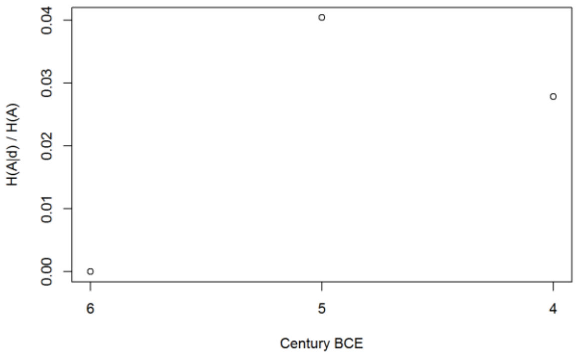

Figure 1a: Normalized conditional entropy of authorities given designs for 3 time periods corresponding to centuries BCE.

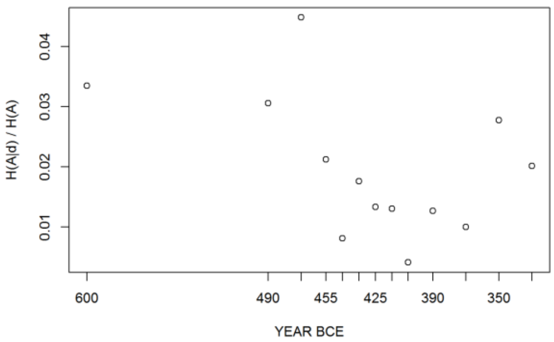

Figure 1b: Normalized conditional entropy of authorities given designs for 13 time periods. Each period is represented by the median year.

The re-analysis with more fine-grained time periods showed that the conditional entropy of authorities given designs tended to decrease with time (Spearman's rho = 0.44; p = 0.13; n = 13). This result remained after we normalized the values by dividing the conditional entropy of authorities given designs H(A|d) by the entropy of authorities H(A) (rho = 0.48; p = 0.09, n = 13).

Lower conditional entropy values indicate that the designs are becoming more informative about authorities with time, which is contrary to our prediction.

1.2. Single motifs-based analysis

In order to test the effect single motifs might have on the observed trend, we calculated the conditional entropy of authority given motifs separately for each motif, first on the 3 preregistered periods, and then on the fine-grained 13 periods.

To control for the basic increase in the entropy of authorities H(A), we normalized the conditional entropy values by dividing H(A|M) with H(A). When we consider the 3 preregistered periods, the normalized values vary between 0.80 and 1, with the mean values rising slightly from 0.992 in the 6th century BCE to 0.994 in the 4th, and the median values falling from 1 in the 6th century to 0.998 in the 4th (see **Figure 2a**). We performed a linear regression, controlling for the random effect of individual motifs. A null linear mixed effect model with normalized H(A|M)/H(A) as a dependent variable and the random effect for motifs was tested against a model which included the date (time period) as a predictor. We used `lme4` package (Bates et al. 2015) in R (version 3.4.2, R Core Team 2017). The model with predictor had slightly lower AIC score (-5805.0) than the null model (-5804.7). The date had a negative estimate (-8.005e-04, SE = 5.318e-04, t-value = -1.5), which means the conditional entropy of authorities given motifs increases with time (time in BCE being counted backwards), so the motifs become less informative about the authorities in time, as predicted. However, this result is probably influenced by the outliers with the comparatively low H(A|M)/H(A) in the earliest period (6^{th} century BCE).

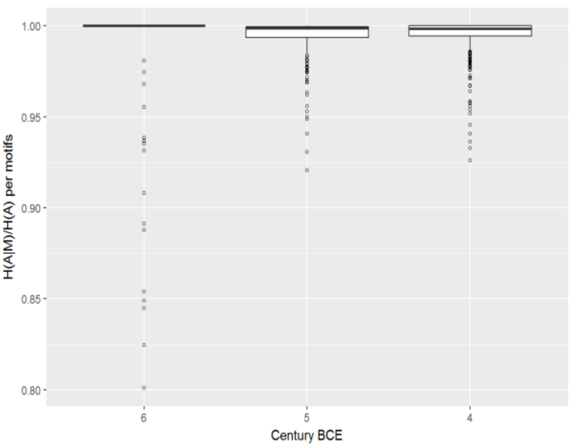

Figure 2a: Normalized conditional entropy of authorities given individual motifs for 3 time periods corresponding to centuries BCE.

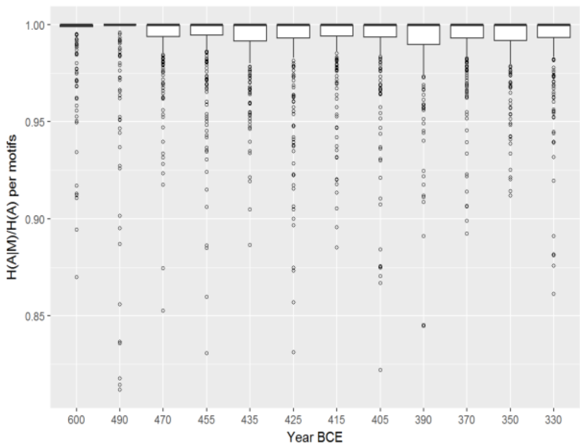

Figure 2b: Normalized conditional entropy of authorities given individual motifs for 13 time periods. Each period is represented by the median year.

For the 13 periods, the normalized values H(A|M)/H(A) vary between 0.81 and 1, with the constant median of 1 and the decrease in mean values from 0.994 in c. 600 BCE to 0.993 in c.330 BCE (see **Figure 2b**). The regression analysis yielded the opposite values than the one for 3 periods: the model with date as predictor had slightly lower AIC (-24250) than the null model containing only the random effect for motifs (AIC = -24249), but the estimate for date was in this case positive (5.330e-06, SE = 2.973e-06, t-value = 1.8), suggesting the decrease in the conditional entropy with time, which means that the motifs become more informative about authorities with time, opposite of what we predicted.

2. Prediction 2: motifs become more informative about a coin's denomination

2.1. Designs-based analysis

We tested the second prediction, which is similar to the first, but this time focusing on denominations (coin value). Using `infotheo` package in R, we computed the conditional entropy of denominations given designs H(D|d) for each preregistered period (6th, 5th and 4th century BCE inside the time frame of 650 – 336 BCE), and found that the conditional entropy decreases with time, meaning the designs become more informative about denominations, in line with our prediction (see **Figure 3a**).

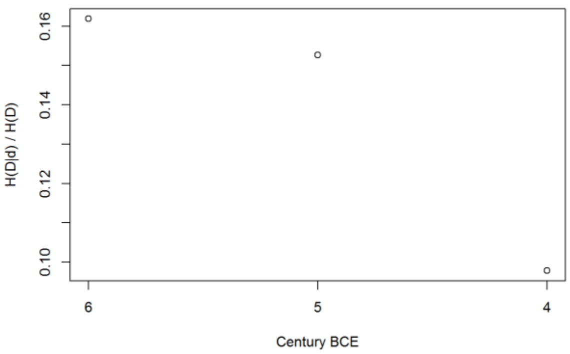

Figure 3a: Normalized conditional entropy of denominations given designs for 3 time periods corresponding to centuries BCE.

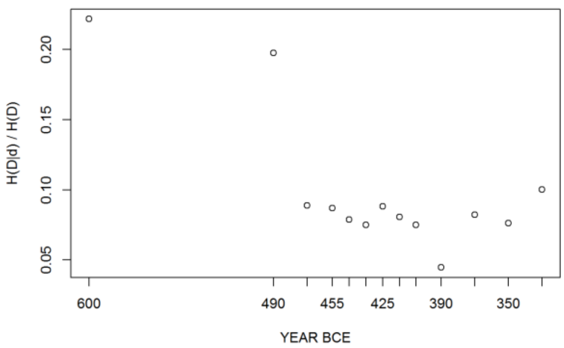

Figure 3b: Normalized conditional entropy of denominations given designs for 13 time periods. Each period is represented by the median year.

As we have done for the first prediction, we recalculated the conditional entropy values for 13 time periods (see **Figure 3b**) and again found that the informative value of designs decreased with time (Spearman's rho = 0.42, p = 0.1516, n = 13). This result remained when the values of H(D|d) were normalized by dividing them by the entropy of denominations H(D) (rho = 0.47, p = 0.1103, n = 13).

Because our dataset consists of coins issued by a number of different authorities, it is probable that the number of distinct authorities inside a particular period would affect the conditional entropy of denominations given designs, since two coin types from two different authorities could carry different designs and also differ in denomination. In order to control for this possible issue, we measured the normalized conditional entropy H(D|d)/H(D) inside each authority and period of interest (e.g. all coins of Syracuse inside the period 400-380 BCE). Dividing the conditional entropy measure by entropy of denominations ensured that the subsets of data inside each authority and period with more denominations would be comparable with those with fewer denominations, and excluded the subsets with only 1 or 0 denominations.

We proceeded to do a nested regression, nesting the data by authorities (each subset contained only coins minted by one authority) and date (the time period that delimits the subset). We applied this method twice, once with the time periods that we had registered (3 periods roughly corresponding to the 6th, 5th, and 4th century), and a second time with the more fine-grained 13 time periods (see **Figure 4a & b**). The regressions were performed using the `lme4` package for R. We compared the null mixed effects model with a random effect for authorities with a model using date as a predictor. Dependent variable was the normalized conditional entropy H(D|d)/H(D).

The test on 3 time periods showed that the model with date as a predictor has a much lower AIC (-15.5543) than the null model (1.6249). The estimate for date was positive (0.10910, SE = 0.02406, t-value = 4.535), which implies that the H(D|d) is decreasing with time (time BCE being counted backwards), as predicted. The test on the 13 time periods mirrors this result: the model with date has a lower AIC (-20.2463) than the null model (-9.7214), and the date has a positive estimate (8.247e-04, SE = 2.322e-04, t-value = 3.551). The substantial variation of H(D|d)/H(D) between different authorities diminishes with time, but still affects the overall trend.

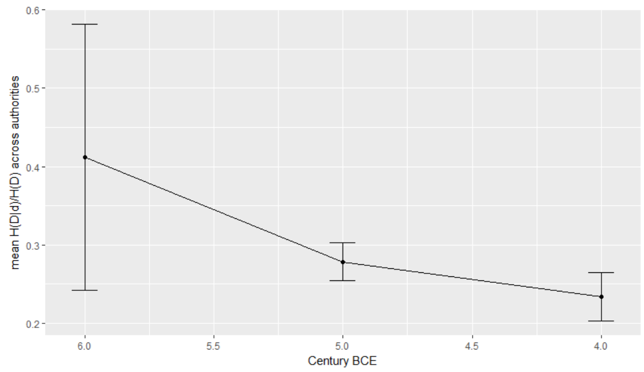

Figure 4a: Mean normalized conditional entropy of denominations given designs calculated across authorities for 3 time periods corresponding to centuries BCE. Error bars indicate the variation between authorities, and the dot stands for the mean value.

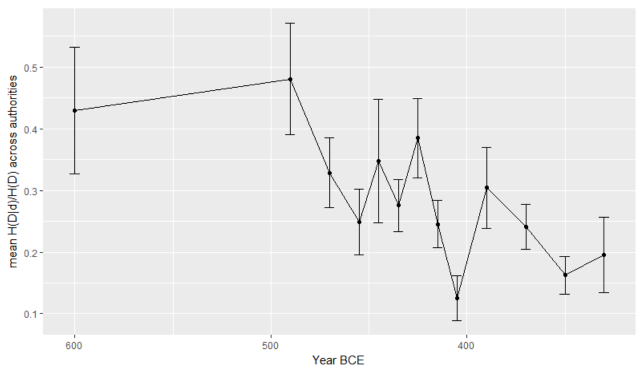

Figure 4b: Mean normalized conditional entropy of denominations given designs calculated across authorities for 13 time periods. Each period is represented by the median year. Error bars indicate the variation between authorities, and the dot stands for the mean value.

2.2. Single motifs-based analysis

To determine possible influence of single motifs on entropy measures, we calculated the normalized conditional entropy of denominations given motifs H(D|M)/H(D) separately for each motif.

The analysis on the preregistered 3 time periods (the 6th, 5th and 4th century BCE) shows a bigger variance in conditional entropy values in the 6th century (from 0.82 to 1), and smaller in the 5th and the 4th century (from 0.92 to 1). The mean H(D|M)/H(D) rises a little from 0.995 in the 6th century to

0.997 in the 5th, and falls to 0.996 in the 4th century. The median H(D|M)/H(D) drops from 1 in the 6th century to 0.998 in the 5th, and rises a bit to 0.999 in the 4th century (see **Figure 5a**).

We tested our hypothesis again with a linear regression, this time controlling for the random effect of individual motifs. A null linear mixed effects model with normalized conditional entropy H(D|M)/H(D) as a dependent variable and a random effect for motifs was tested against a linear mixed effects model which included date (3 time periods) as predictor, using `lme4` package in R. The model with date as a predictor had lower AIC (-6573.4) than the model with random effect only (-6570.0). The estimate for date was negative (-9.141e-04, SE = 3.906e-04, t-value = -2.3), which means that the conditional entropy is lower for the earlier centuries (time in BCE being counted backwards), which is opposite of what we predicted, but is probably due to the outliers with comparatively low entropy (as low as 0.82) in the earliest period (6th century BCE).

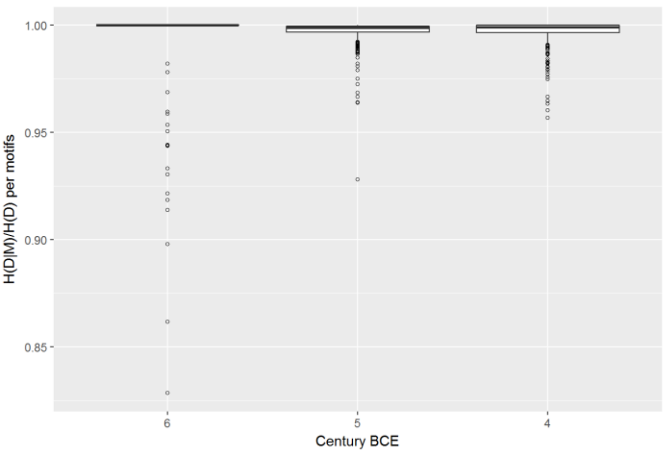

Figure 5a: Normalized conditional entropy of denominations given individual motifs for 3 time periods corresponding to centuries BCE.

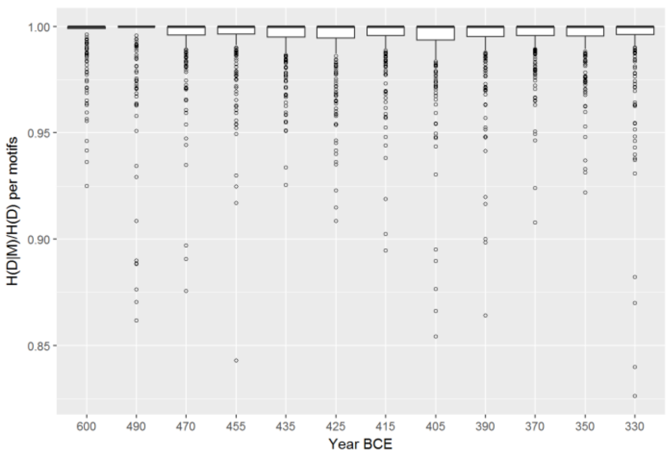

Figure 5b: Normalized conditional entropy of denominations given individual motifs for 13 time periods. Each period is represented by the median year.

We repeated the analysis on 13 time periods in order to get more fine-grained results. In the case of 13 periods, normalized values H(D|M)/H(D) vary between 0.82 and 1. The mean values of conditional entropy decrease slightly in later periods (from 0.996 in c. 600 BCE to 0.994 in c.330 BCE), but the median for all 13 periods is 1 (see **Figure 5b**).

We performed a linear regression test using `lme4` package in R on 13 periods, controlling for the random effect of individual motifs. Similar to the previous test, the model with date as a predictor had lower AIC (-26273) than the null model with random effect (-26271). The estimate for date was in this case positive (4.749e-06, SE = 2.391e-06, t-value = 2.0), which means that the conditional entropy is higher in the earlier time periods (time in BCE being counted backwards), as predicted.

The reason we got a very different result after dividing the timeline into smaller periods is due to the fact that we have more data for the later periods than for the earlier ones, and that the data shows much more variability in the earlier periods. In order to understand the change of informative value of motifs about denomination through time we need to get more data for the very early periods of Ancient Greek coin production and construct a more balanced dataset.

3. Prediction 3: motifs are more informative about lower denominations than about higher ones

As explained in section 0, this prediction is only tested on full designs, not single motifs.

3.1. Designs-based analysis

For this test, we first had to split the dataset in two, based on each coin type's denomination. We simplified complicated Greek denomination systems by identifying four base values: the Persian daric, the eastern Greek stater, the mainland Greek drachma and the south Italian litra, and we determined the value of other denominations in relation to the base values. The relation values are calculated based on VanHorn and Nelson (2009), Sayles (2007), and Daehn (2009). The base value was given a relational value of 1, and the dataset was split into a "higher denominations" dataset (relational values higher than 1, N=758 coin types) and a "lower denominations" dataset (relational values equal to or lower than 1, N=1198 coin types). We excluded 534 data points which lacked information on denomination.

The conditional entropy of denominations given designs H(D|d) was calculated for each dataset separately. The results are the opposite of what we predicted: the conditional entropy H(D|d) is higher for lower denomination coins (0.27) than for higher denomination coins (0.05), which means the designs are less informative about the lower denominations than about the higher ones.

We repeated the analysis separately for each authority, to control for a possibility that the differences in the number of coin types issued by different cities affect the structure of the dataset. This analysis confirmed that the effect we found was strong: a Wilcoxon rank sum test confirmed that the medians of conditional entropy values for higher and lower denomination datasets, taking authorities as data points, are significantly different ($p < .001$).

In order to verify if the difference we have observed between higher and lower denominations is indeed limited to the information that the designs carry about denominations, we repeated the test, this time looking at the conditional entropy of authorities given designs, H(A|d). The results show that the higher denominations subset has lower conditional entropy H(A|d) = 0.03 than the lower denominations subset (0.13), similar to the result for H(D|d) for the same subdivision. This suggests that the designs on higher denomination coins are generally more informative than the designs on lower denomination coins.

It is unlikely that the reason for this difference between the higher and lower denominations is due to the lack of motifs in the dataset necessary to compose informative designs. Both the entropy of denominations H(D) and authorities H(A) are lower than the entropy of motifs H(M), both in case of higher (HD=1.14; HA=3.65 < HM=5.86) and lower (HD=2.22; HA=4.36 < HM=6.27) denominations subsets. However, it was necessary to control for the possibility that lower H(D|d) and H(A|d) values for high denominations are simply due to the fact that higher denominations coins, being larger than lower denomination coins, can accommodate more motifs. We performed an additional analysis on the subsets of data subdivided according to the number of individual motifs on a coin type. Since the median number of motifs per coin type is 3, we split our dataset in order to get a subset of coins with more than 3 motifs (N = 968), and another with 3 or less motifs (N = 1572). The coins with more motifs display lower H(D|d) values (0.16) than the coins with less motifs (0.47), which means that the coins with more motifs contain designs that are more informative about denominations than the designs on coins with less motifs. However, if we group the coins by the number of motifs, we can observe that in all cases the higher denomination coins have lower H(D|d) than the lower denomination coins, except when they both have H(D|d) = 0 (see **Figure 6**). This supports the main result.

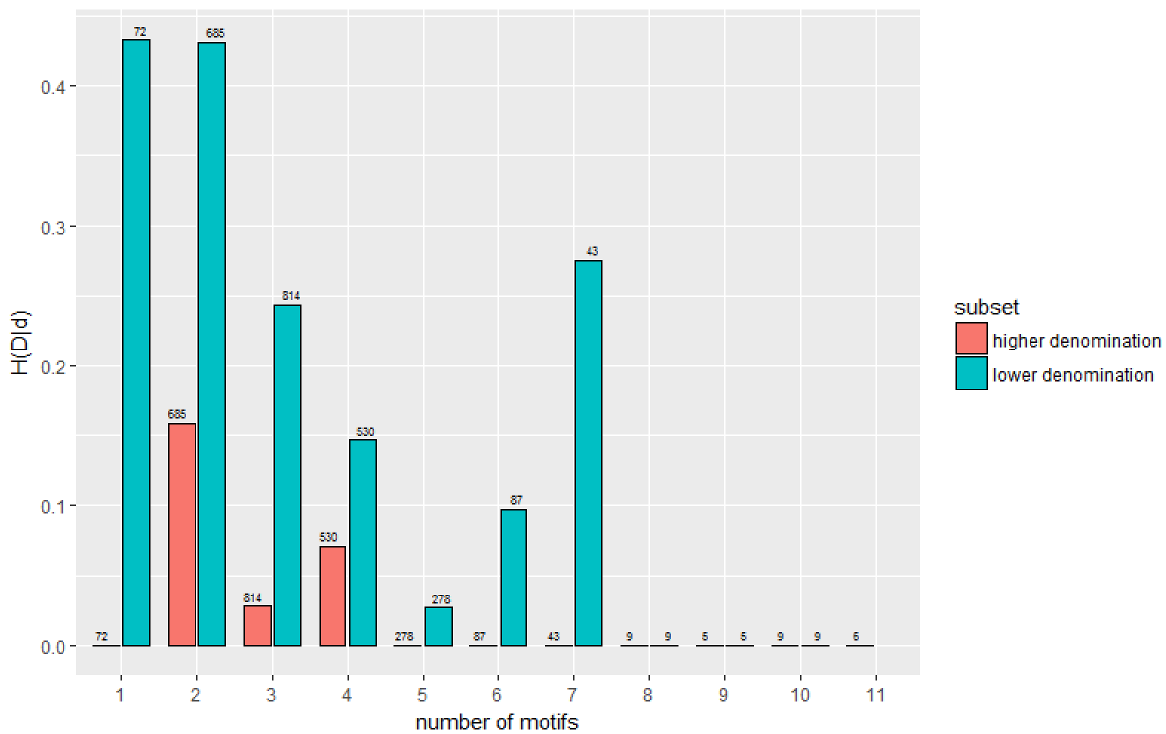

Figure 6: The conditional entropy of denominations given designs for various sets of coin types. The coin types are grouped according to the number of motifs they contain (x axis), and by their denomination into higher (red) or lower denomination subsets (blue). The number above each bar indicates the number of coin types in the subset.

4. Interpretation and follow-ups

4.1. Two evolutionary scenarios for the rise in denomination-related information

The apparent change in the informational value of motifs about coins' denomination could be due to two different phenomena.

Possibility A: Coin type differentiation only. In this scenario, coins become more informative as the coin types become more distinct: coin types with an identical set of motifs become less frequent.

This hypothesis leads to two predictions:

Prediction A1: If coin type motifs become more distinct, this makes them more informative about denominations (because two coins of identical denominations are less likely to bear the same motifs), but, for the same reason, it also makes them more informative about authorities. So whatever time-related increase in information about denomination is observed should be matched by a roughly equivalent increase in information about authority.

Prediction A2: The conditional entropy of motifs given denominations should not show a decrease. In other words, motif arrangements become more diverse but this does not make motifs better organized with respect to authorities or denominations.

Possibility B: motifs and designs acquire denomination-related meanings. In this scenario, coins become more informative about denominations because some motifs or sets of motifs acquire specific, denomination-related meanings. In other words, sets of coin types appear that share the same denomination and the same motifs or sets of motifs, but not necessarily the same authority.

Prediction B1: The opposite of prediction A1. The conditional entropy of denominations given designs may decrease without any decrease in the conditional entropy of authorities given designs (or with a more moderate decrease of it).

Prediction B2: The opposite of prediction A2. The conditional entropy of motifs given denominations should decrease. This means that denominations become a more reliable guide to predicting what motifs or sets of motifs appear on a coin.

Possibility B is meant to be more inclusive than Possibility A: in other words, possibility A admits that sheer coin types' differentiation may play a role. The data just presented already provide some support for A1 and against B1, without really allowing us to discriminate the two: While becoming more informative about denominations, coin types also tended to become more informative about authorities. We ran a follow-up analysis testing the second pair of predictions.

4.2. Do designs become organised according to denomination?

This test was carried out on the basis of the 13 new time "bins" we designed, rather than the three registered ones. We computed the conditional entropy of designs given denomination in two ways, the normal way H(d|D); and a measure normalised for the entropy of designs: H(d|D)/H(d).

To be rigorous, a real test of predictions B1 and B2 must, in our view, rely on the first, non-normalized measure. However, since the entropy of designs H(d) tends to increase with time (Spearman's rho = -0.65, p = 0.02, n = 13), we were curious to see whether in spite of this, designs may not in fact become better organised with respect to the coins' denomination.

We first ran two simple non-parametric correlations over all coin types without sorting them by their respective authorities. We found a tendency for H(d|D) to increase (rho = -0.59, p = 0.04, n = 13), suggesting the designs do not become more organised according to denomination. This tendency was less pronounced when we normalized this value to control for the basic increase in design entropy, with H(d|D)/H(d) (rho = -0.13, p = 0.67, n = 13).

In order to take into account the nested structure of the data, we ran a mixed effects model controlling for the random effect of authorities. The dependent variable was the conditional entropy of designs given denominations (normalised and non-normalised), measured for each of the 13 time periods and for each authority that yielded enough data for that time period (i.e. several coin types). This left 134 unique authorities, and 839 data points overall.

For both measures, H(d|D) and H(d|D)/H(d), the null model that includes only the nesting variable is less informative than a model that adds each coin type's time period as a predictor. For the test using the non-normalized conditional entropy, AIC is 1860.6 for the null model, 1857.7 for the model with the predictor (date, estimate = -4.319e-04, SE = 1.960e-04, t-value = -2.203). For the test using the normalized conditional entropy, AIC is 149.66 for the null model, and 146.72 for the model with the predictor (date, estimate = -5.174e-04, SE = 2.326e-04, t-value = -2.225).

The results of this analysis show that the conditional entropy of coin designs given denominations increases with time, suggesting that the designs did not become more organised according to denomination. This is in line with the prediction A2: we did not observe a decrease in H(d|D), but the opposite - the coin type differentiation seems to be the main reason for the observed trends of coin designs becoming more informative about both the denominations and authorities.

5. Summary and discussion

Of our three predictions, we successfully confirmed only the second one. The motifs arranged in designs appear to become more informative about denomination with time. This trend is more visible in earlier periods (6^{th} and beginning of 5^{th} century BCE) than in later periods, and it would be interesting to repeat the analysis on a bigger dataset expanded to include more coins from the earlier periods. The coin designs seem to become more informative about authority too, opposite of what we predicted. One possible cause for this trend is that coin types became more differentiated through time, irrespective of what they stood for, without designs becoming in any way more organized to reflect authorities or denominations. This mechanism is certainly a part of the explanation, since designs do tend to become more differentiated with time (resulting in an increase in design entropy). We were unable to prove that designs also became better organized to reflect denominations.

Testing the third prediction produced an interesting result, although opposite to what we predicted. It appears that higher denominations bore designs more informative than those on lower denominations. This effect cannot be due to number of motifs in the dataset, because entropy of denominations H(D) is both in case of higher and lower denominations subset much lower than the entropy of motifs H(M), meaning there were enough motifs in the dataset to use efficiently on any type of denomination. The size of the coin might have played the role, as one could fit more motifs on bigger coins of higher denominations. In order to test this, we subdivided the dataset according to the median number of motifs on coins (n = 3). The coins with more than 3 motifs display designs which are more informative about denomination (have lower H(D|d) values). However, when we observe the subsets of coins with the same number of motifs, the higher denomination coins still show lower H(D|d) values than the lower denomination coins, as we have observed in the main test. Therefore, it might be the case that the minters needed to put more information concerning denomination on higher denomination coins because it was more important to signal the differences between the coins of higher value. This possibility will be further tested on larger datasets representing different currency systems.

References

Daehn, W.E., 2009. *Greek Coin Weight Standards*. https://www.scribd.com/document/94991480/Weight-Standards-for-Greek-Coins

Meyer, Patrick E., 2014. *infotheo: Information-Theoretic Measures*. R package version 1.2.0. https://CRAN.R-project.org/package=infotheo

R Core Team, 2017. *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/

Sayles, W., 2007. *Ancient Coin Collecting II: Numismatic Art of the Greek World*. F+W Media, Inc., pp 12-19

VanHorn, D.S., Nelson, B.R., 2009. *Overview of The Handbook of Greek Coinage Series*, in: *Handbook of Syrian Coins - Royal and Civic Issues Fourth to First Centuries BC*, The Handbook of Greek Coinage Series. Classical Numismatic Group, Inc., Lancaster/London.

Bates, Douglas, Martin Maechler, Ben Bolker, Steve Walker. Fitting Linear Mixed-Effects Models Using lme4. Journal of Statistical Software, 67(1), 2015. 1-48. doi:10.18637/jss.v067.i01