Supplementary Material I: PowerLAPIM: An Application to Conduct Power Analysis for Longitudinal Actor-Partner Interdependence Models that Include Quadratic Effects

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Population models of interest

The PowerLAPIM application includes 32 different longitudinal actor-partner interdependence models (L-APIM) that can be used to address different research questions (see Table 2 for an overview). The models can be expressed as multilevel regression models (see Kenny, Kashy, and Cook (2020)).

Table 1 shows a hypothetical longitudinal dyadic data set. For simplicity, we use the subscripts A and B to denote two distinguishable dyadic partners. Examples of dyadic partners include romantic partners, mother and child, therapist and client, and older and younger siblings. In the hypothetical dataset, both partners responded to momentary questionnaires at 4 equidistant time points. The first column contains the dyad identification numbers and the second column the observation numbers. The third and fourth columns include the outcomes (Y_{Ajt} and Y_{Bjt}) and predictors (X_{Ajt} and X_{Bjt}) of partner A and B in the jth dyad, which were measured on every occasion t[1]. We assume that the predictors X_{Ajt} and X_{Bjt} are bivariate normally distributed with mean μ_A and μ_B, and covariance matrix:

$$\mathbf{\Sigma}_X = \begin{bmatrix} \sigma^2_{X_A} & \sigma_{X_{AB}} \\ \sigma_{X_{AB}} & \sigma^2_{X_B} \end{bmatrix}$$

The fifth column includes a dichotomous dyad-level variable (Z_j) indicating group categories (e.g., dyads from different cultures or experimental conditions). The final two columns contain two time-varying covariates that are measured at each measurement occasion for each dyad t: C_{jt} refers to a continuous time-varying variable and D_{jt} to a dichotomous time-varying variable. We assume that C_{jt} is normally distributed with mean μ_W and variance σ^2_W.

[1] The indices reflect the realization of the variables for specific dyads.

The longitudinal actor–partner interdependence modeling framework

L-APIMs offer a popular framework to study interdependence between dyadic partners. Specifically, they allow investigating how the outcome of each dyad member is affected by their own predictor and their partner's predictor over time. The effect of a person's own predictor on their own outcome is denoted as the actor effect, and the effect of their partner's predictor on their own outcome is denoted as the partner effect. Model 1 describes a L-APIM in which both members' outcome variables Y_{Ajt} and Y_{Bjt} are predicted by X_{Ajt} and X_{Bjt} for the jth dyad at the tth observation:

$$Y_{Ajt} = \lambda_{Aj} + a_{AA}X_{Ajt} + p_{BA}X_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_{Bj} + a_{BB}X_{Bjt} + p_{AB}X_{Ajt} + \epsilon_{Bjt}$$

This model captures inter-dyadic differences by including a random intercept for partner A, λ_{Aj}, and a random intercept for partner B, λ_{Bj}. The model also includes a fixed slope for the actor effects a_{AA}, a_{BB} and the partner effects p_{BA}, p_{AB} for each dyad member respectively. To account for the dependencies between the dyadic partners, we allow for correlated errors. Therefore, the Level 1 errors ϵ_{Ajt} and ϵ_{Bjt} are bivariate normally distributed with mean zero and covariance matrix:

$$\boldsymbol{\Sigma}_{\epsilon} = \begin{bmatrix} \sigma^2_{\epsilon_A} & \sigma_{\epsilon_{AB}} \\ \sigma_{\epsilon_{AB}} & \sigma^2_{\epsilon_B} \end{bmatrix}$$

This model makes two assumptions about the distribution of the Level 1 errors: the within individual variances and covariance are stable over time, and adjacent time points are uncorrelated. It is worth mentioning that different structures for the covariance of the Level 1 errors have been proposed in the literature. For example, Gistelinck and Loeys (2019) propose a structure of autocorrelated Level 1 errors where the covariance matrix is the Kronecker product of a (2×2) covariance matrix for the Level 1 errors of each dyad member and a $(T \times T)$ autoregressive correlation matrix (for more information see

Gistelinck and Loeys (2019)). This structure allows modeling the correlation between adjacent measures. Unfortunately, the nlme version 3.1.149 package (Pinheiro, Bates, DebRoy, Sarkar, & Team, 2006) from the open-source software R version 4.0.3 (R Core Team, 2020) used to estimate the L-APIM excludes the possibility of estimating Kronecker products for the covariance structure of the Level 1 errors. Therefore, we account for autocorrelated outcomes through the inclusion of the lagged outcomes as predictors.

We now turn to the Level 2 regressions for the random intercepts, which are as follows:

$$\lambda_{Aj} = c_A + \nu_{Aj}$$

$$\lambda_{Bj} = c_B + \nu_{Bj}$$

These equations express the random intercept of each dyadic partner as a function of the fixed intercepts (c_A and c_B) and the Level 2 errors (nu_{Aj} and nu_{Bj}). The Level 2 errors express the deviation of each participant's intercept from the group-specific fixed intercept. The random intercepts are bivariate normally distributed with means given by the fixed effects and covariance matrix:

$$\mathbf{\Sigma}_\nu = \begin{bmatrix} \sigma^2_{\nu_A} & \sigma_{\nu_{AB}} \\ \sigma_{\nu_{AB}} & \sigma^2_{\nu_B} \end{bmatrix}$$

Model 1 yields 12 parameter estimates: the fixed intercepts for each dyadic partner (c_A and c_B, respectively), their fixed actor and partner effects (a_{AA}, p_{BA}, a_{BB}, p_{AB}, respectively), the within-individual variance and covariance estimates ($\sigma^2_{\epsilon_A}$, $\sigma^2_{\epsilon_B}$, $\sigma_{\epsilon_A B}$), and the variance and covariance estimates of the random effects ($\sigma^2_{\nu_A}$, $\sigma^2_{\nu_B}$, $\sigma_{\nu_{AB}}$) that capture the heterogeneity between random intercepts.

In this model, person-mean centering the predictors X_{Ajt} and X_{Bjt} is recommended because the fixed actor and partner effects provide an estimate that only reflects the (average) within-dyad association between the predictor and outcome (see Enders and Tofighi (2007), Raudenbush and Bryk (2002)).

Up to now, we assumed that the dyadic partners were distinguishable. However, we can also consider the L-APIM for indistinguishable partners. Olsen and Kenny (2006) consider different types of indistinguishability in the APIM: (i) equal residual variance for the outcomes ($\sigma^2_{\epsilon_A} = \sigma^2_{\epsilon_B}$); (ii) equal actor effects ($a_{AA} = a_{BB}$); (iii) equal partner effects ($p_{BA} = p_{AB}$); (iv) equal intercepts for the outcomes ($c_A = c_B$).

Model 2 considers indistinguishability of the actor and partner effects and intercepts. This corresponds to conditions (ii) to (iv), which is referred to as Y-mean indistinguishability (see Gistelinck, Loeys, Decuyper, and Dewitte (2018)). The corresponding Level 1 regression models for each dyadic partner are

$$Y_{Ajt} = \lambda_j + aX_{Ajt} + pX_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_j + aX_{Bjt} + pX_{Ajt} + \epsilon_{Bjt}$$

The model captures inter-dyadic differences by including one random intercept λ_i for both partners. The Level 2 regression for the random intercept is

$$\lambda_j = c + \nu_j$$

where c is the fixed intercept and ν_j the Level 2 error term. The random intercept is then normally distributed with mean c and variance σ^2_ν. As in Model 1, Model 2 accounts for the dependencies between members of a dyad by allowing for correlated Level 1 errors.

Model 2 estimates 7 parameters: the fixed intercept c, the fixed actor and partner effects a and p, the within-individual variance and covariance estimates ($\sigma^2_{\epsilon_A}$, $\sigma^2_{\epsilon_B}$, $\sigma_{\epsilon_{AB}}$), and the variance of the random intercept σ^2_ν.

For the next L-APIMs, unless explicitly mentioned otherwise, we assume the Level 1 errors follow the structure presented in Model 1. We assume the random intercepts follow the structure of Model 1 if partners are distinguishable and the structure of Model 2 if partners are indistinguishable.

Dichotomous time-invariant dyad-level moderator

Model 3 extends model 1 by allowing for differences in the actor and partner effects between two groups of dyads (e.g., dyads from different cultures or dyads in different experimental conditions) by including a dichotomous time-invariant dyad-level moderator Z_j that indicates to which group each dyad belongs. The Level 2 regressions for the random effects include the main effect of Z_j

$$\lambda_{Aj} = c_{A0} + c_{A1} Z_j + \nu_{Aj}$$

$$\lambda_{Bj} = c_{B0} + c_{B1} Z_j + \nu_{Bj}$$

The Level 1 regressions include cross-level interactions (see Raudenbush and Bryk (2002)) between the actor and partner effects and Z_j (i.e., Z_j moderates those effects). Within both groups, inter-dyadic differences are modeled by the random intercepts λ_{Aj} and λ_{Bj}. :

$$Y_{Ajt} = \lambda_{Aj} + a_{AA0} X_{Ajt} + a_{AA1} Z_j X_{Ajt} + a_{BA1} X_{Bjt} + p_{BA1} Z_j X_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_{Bj} + a_{BB0} X_{Bjt} + a_{BB1} Z_j X_{Bjt} + p_{AB0} X_{Ajt} + p_{AB1} Z_j X_{Ajt} + \epsilon_{Bjt}$$

where Level 1 errors ϵ_{Ajt} and ϵ_{Bjt} are again assumed to be normally distributed.

For dyads in the reference group (i.e., $Z_j = 0$), the fixed intercepts for the dyadic partners equal c_{A0} and c_{B0}, respectively. For individuals in the non-reference group (i.e., $Z_j = 1$), the fixed intercepts amount to: $c_{A0} + c_{A1}$ and $c_{B0} + c_{B1}$. Meanwhile, a_{AA0}, a_{BB0}, p_{BA0} and p_{AB0} represent the fixed actor and partner effects in the reference group and $a_{AA0} + a_{AA1}$, $a_{BB0} + a_{BB1}$, $p_{BA0} + p_{BA1}$ and $p_{AB0} + p_{AB1}$ represent the fixed actor and partner effects in the non-reference group, respectively. The effects of interest (i.e., the target effect for which power will be computed) are thus the differences in the fixed actor and partner effects between the two groups a_{AA1}, a_{BB1}, p_{BA1} and p_{AB1}.

Model 4 is a reduced version of model 2, by considering indistinguishable partners. The model equations are:

Level 2:

$$\lambda_j = c_0 + c_1 Z_j + \nu_j$$

Level 1:

$$Y_{Ajt} = \lambda_j + a_0 X_{Ajt} + a_1 Z_j X_{Ajt} + p_0 X_{Bjt} + p_1 Z_j X_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_j + a_0 X_{Bjt} + a_1 Z_j X_{Bjt} + p_0 X_{Ajt} + p_1 Z_j X_{Ajt} + \epsilon_{Bjt}$$

In this model, the fixed intercept in the reference group is c_0, while the fixed intercept in the non-reference group is $c_0 + c_1$. The actor and partner effects for dyads in the reference group are a_0 and p_0, respectively. In contrast, the actor and partner effects for dyads in the non-reference category are $a_0 + a_1$ and $p_0 + p_1$. The difference in the actor and partner effects in both groups are thus given by the fixed effects a_1 and p_1.

Continuous time-varying moderator

Next, we focus on a set of models that allow investigating moderation effects of a continuous time-varying variable C_{jt} on the actor and partner effects (see Garcia, Kenny, and Ledermann (2015)). In Model 5, the Level 1 regressions include interaction effects between the predictors X_{Ajt} and X_{Bjt} and the time-varying continuous moderator C_{jt}:

$$Y_{Ajt} = \lambda_{Aj} + a_{AA} X_{Ajt} + p_{BA} X_{Bjt} + b_A C_{jt} + b_{AA} C_{jt} X_{Ajt} + b_{BA} C_{jt} X_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_{Bj} + a_{BB} X_{Bjt} + p_{AB} X_{Ajt} + b_B C_{jt} + b_{BB} C_{jt} X_{Bjt} + b_{AB} C_{jt} X_{Ajt} + \epsilon_{Bjt}$$

The equations above include the fixed intercepts (c_A and c_B), the fixed actor effects (a_{AA}, a_{BB}), and the fixed partner effects (p_{BA}, p_{AB}). Moreover, the model includes fixed slopes for the effects of C_{jt} on the outcomes of partners A and B (b_A, b_B), the moderation effects on the actor effects (b_{AA}, b_{BB}), and the moderation effects on the partner effects (b_{BA}, b_{AB}).

Model 6 is again a simpler variant of Model 5 in that it assumes indistinguishability:

$$Y_{Ajt} = \lambda_j + aX_{Ajt} + pX_{Bjt} + bC_{jt} + b_a C_{jt} X_{Ajt} + b_p C_{jt} X_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_j + aX_{Bjt} + pX_{Ajt} + bC_{jt} + b_a C_{jt} X_{Bjt} + b_p C_{jt} X_{Ajt} + \epsilon_{Bjt}$$

Thus, this model estimates: the fixed intercept (c), the fixed actor and partner effects (a and b), the fixed effect of the continuous moderator (b), the fixed moderation effect on the actor effect denoted by b_a, and the fixed moderation effect on the partner effect denoted by b_p.

Dichotomous time-varying moderator

Models 7 and 8 are strongly related to models 5 and 6 but include a dichotomous (i.e., zero or one scores) rather than continuous time-varying moderator D_{jt}. The Level 1 regressions include interaction effects between the predictors X_{Ajt} and X_{Bjt} and D_{jt}:

$$Y_{Ajt} = \lambda_{Aj} + a_{AA} X_{Ajt} + p_{BA} X_{Bjt} + d_A D_{jt} + d_{AA} D_{jt} X_{Ajt} + d_{BA} D_{jt} X_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_{Bj} + a_{BB} X_{Bjt} + p_{AB} X_{Ajt} + d_B D_{jt} + d_{BB} D_{jt} X_{Bjt} + d_{AB} D_{jt} X_{Ajt} + \epsilon_{Bjt}$$

The model includes the effects of D_{jt} on the partners' outcomes (d_A, d_B), the moderation effect on the actor effects (d_{AA}, d_{BB}), and the moderation effects on the partner effects (d_{BA}, d_{AB}).

Model 8 again assumes that partners are indistinguishable and can therefore be described as:

$$Y_{Ajt} = \lambda_j + aX_{Ajt} + pX_{Bjt} + dD_{jt} + d_a D_{jt} X_{Ajt} + d_p D_{jt} X_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_j + aX_{Bjt} + pX_{Ajt} + dD_{jt} + d_a D_{jt} X_{Bjt} + d_p D_{jt} X_{Ajt} + \epsilon_{Bjt}$$

The model estimates the fixed intercept (c), the fixed actor and partner effects (a and b), the fixed effect of D_{jt} on the outcomes (d), and the fixed moderation effect on the actor effect denoted by d_a, and the moderation effect on the partner effect denoted by d_p.

The actor–partner interdependence model with quadratic effects

The longitudinal APIMs discussed so far assume that the actor and partner effects are linear. However, curvilinear dyadic processes have been receiving increasing attention. Such processes can be studied by including quadratic actor and partner effects in the longitudinal APIM.

Model 9 extends the L-APIM presented as Model 1 by including additional quadratic terms in the Level 1 regression equations

$$Y_{Ajt} = \lambda_{Aj} + a_{AA}X_{Ajt} + a_{AA2}X^2_{Ajt} + p_{BA}X_{Bjt} + p_{BA2}X^2_{Bjt} + \epsilon_{Bjt}$$

$$Y_{Bjt} = \lambda_{Bj} + a_{BB}X_{Bjt} + a_{BB2}X^2_{Bjt} + p_{AB}X_{Ajt} + p_{AB2}X^2_{Ajt} + \epsilon_{Bjt}$$

This model includes the linear and quadratic actor effects a_{AA}, a_{AA2}, a_{BB}, and a_{BB2}, as well as the linear and quadratic partner effects p_{BA}, p_{BA2}, p_{AB} and p_{AB2}. For example, in the quadratic model, the rate of change in Y_{Ajt}, conditional on the rest of the predictors being fixed, depends on the value of X_{Ajt}, rather than remaining constant. A positive quadratic effect indicates the expected rate of change is convex (U-shape), and a negative value indicates the expected rate of change is concave (inverse U shape). Again we recommend to person-mean center the predictors to obtain effect estimates that reflect only the (average) within-dyad association between the predictor and outcome.

In addition, we include a model version for indistinguishable partners (Model 10):

$$Y_{Ajt} = \lambda_j + aX_{Ajt} + a_2X^2_{Ajt} + pX_{Bjt} + p_2X^2_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_j + aX_{Bjt} + a_2X^2_{Bjt} + pX_{Ajt} + p_2X^2_{Ajt} + \epsilon_{Bjt}$$

This latter model estimates, amongst other parameters, indistinguishable linear and quadratic actor effects (a and a_2), and indistinguishable linear and quadratic partner effects (p and p_2).

Group differences in the actor and partner quadratic effects

Models 11 and 12 extend models 3 and 4 and allow estimating group differences in linear and quadratic actor and partner effects via a dichotomous time-invariant dyad-level moderator Z_j. The Level 1 regressions for partners A and B are:

$$Y_{Ajt} = \lambda_{Aj} + a_{AA0}X_{Ajt} + a_{AA1}Z_jX_{Ajt} + a_{AA02}X^2_{Ajt} + a_{AA12}Z_jX^2_{Ajt}+$$

$$p_{BA0}X_{Bjt} + p_{BA1}Z_jX_{Bjt} + p_{BA02}X^2_{Bjt} + p_{BA12}Z_jX^2_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_{Bj} + a_{BB0}X_{Bjt} + a_{BB1}Z_jX_{Bjt} + a_{BB02}X^2_{Bjt} + a_{BB12}Z_jX^2_{Bjt}+$$

$$p_{AB0}X_{Ajt} + p_{AB1}Z_jX_{Ajt} + p_{AB02}X^2_{Ajt} + p_{AB12}Z_jX^2_{Ajt} + \epsilon_{Bjt}$$

For dyads in the reference group (i.e., $Z_j = 0$), the linear and quadratic actor effects are a_{AA0} and a_{AA02} for partner A and a_{BB0} and a_{BB02} for partner B; the linear and quadratic partner effects are p_{BA0} and p_{BA02} for partner A and p_{AB0} and p_{AB02} for partner B. For dyads in the non-reference group (i.e., $Z_j = 1$), the linear and quadratic actor effects are $a_{AA0} + a_{AA1}$ and $a_{AA02} + a_{AA12}$ for partner A and $a_{BB0} + a_{BB1}$ and $a_{BB02} + a_{BB12}$ for partner B; the fixed linear and quadratic partner effects are $p_{BA0} + p_{BA1}$ and $p_{BA02} + p_{BA12}$ for partner A and $p_{AB0} + p_{AB1}$ and $p_{AB02} + p_{AB12}$ for partner B. Therefore, the effect of interest are the differences in the linear and quadratic actor and partner effects between the two groups: a_{AA1}, a_{AA12}, a_{BB1}, a_{BB12}, p_{BA1}, p_{BA12}, p_{AB1}, and p_{AB12}.

Model 12 assumes that partners are indistinguishable:

$$Y_{Ajt} = \lambda_j + a_0X_{Ajt} + a_1Z_jX_{Ajt} + a_{02}X^2_{Ajt} + a_{12}Z_jX^2_{Ajt}+$$

$$p_0X_{Bjt} + p_1Z_jX_{Bjt} + p_{02}X^2_{Bjt} + p_{12}Z_jX^2_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_j + a_0 X_{Bjt} + a_1 Z_j X_{Bjt} + a_{02} X_{Bjt}^2 + a_{12} Z_j X_{Bjt}^2 +$$

$$p_0 X_{Ajt} + p_1 Z_j X_{Ajt} + p_{02} X_{Ajt}^2 + p_{12} Z_j X_{Ajt}^2 + \epsilon_{Bjt}$$

In this model, the fixed linear and quadratic actor and partner effects for dyads in the reference group are a_0, a_{02}, p_0, and p_{02}. In contrast, the fixed linear and quadratic actor and partner effects for dyads in the non-reference group are $a_{10} + a_{11}$, $a_{20} + a_{21}$, $p_{10} + p_{11}$, and $p_{20} + p_{21}$. Therefore, the differences in the actor and partner effects in both groups are given by the fixed effects a_{11}, a_{21}, p_{11}, and p_{21}.

Continuous time-varying moderator of the quadratic effects

The next set of models adds moderation effects of a continuous time-varying moderator C_{jt} to models 9 and 10. The Level 1 regressions of Model 13 can be expressed as follows:

$$Y_{Ajt} = \lambda_{Aj} + a_{AA} X_{Ajt} + a_{AA2} X_{Ajt}^2 + p_{BA} X_{Bjt} + p_{BA2} X_{Bjt}^2 +$$

$$b_A C_{jt} + b_{AA} C_{jt} X_{Ajt} + b_{AA2} C_{jt} X_{Ajt}^2 + b_{BA} C_{jt} X_{Bjt} + b_{BA2} C_{jt} X_{Bjt}^2 + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_{Bj} + a_{BB} X_{Bjt} + a_{BB2} X_{Bjt}^2 + p_{AB} X_{Ajt} + p_{AB2} X_{Ajt}^2 +$$

$$b_A C_{jt} + b_{BB} C_{jt} X_{Bjt} + b_{BB2} C_{jt} X_{Bjt}^2 + b_{AB} C_{jt} X_{Ajt} + b_{AB2} C_{jt} X_{Ajt}^2 + \epsilon_{Bjt}$$

This model contains the same parameters as Model 9, but also includes the fixed effects of the continuous moderator, b_A and b_B, and the fixed moderation effects of the linear actor (b_{AA}, b_{BB}) and partner (b_{BA}, b_{AB}) effects, and of the quadratic actor (b_{AA2}, b_{BB2}) and partner (b_{BA2}, a_{AB2}) effects.

Model 14 is the counterpart of Model 13 assuming indistinguishable partners:

$$Y_{Ajt} = \lambda_j + a X_{Ajt} + a_2 X_{Ajt}^2 + p X_{Bjt} + p_2 X_{Bjt}^2 +$$

$$b C_{jt} + b_a C_{jt} X_{Ajt} + b_{a2} C_{jt} X_{Ajt}^2 + b_p C_{jt} X_{Bjt} + b_{p2} C_{jt} X_{Bjt}^2 + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_j + aX_{Bjt} + a_2X_{Bjt}^2 + pX_{Ajt} + p_2X_{Ajt}^2 +$$

$$bC_{jt} + b_aC_{jt}X_{Bjt} + b_{a2}C_{jt}X_{Bjt}^2 + b_pC_{jt}X_{Ajt} + b_{p2}C_{jt}X_{Ajt}^2 + \epsilon_{Bjt}$$

This model yields the following parameter estimates: the fixed intercept (c), the fixed linear actor effect (a), the fixed quadratic actor effect (a_2), the fixed linear partner effect (p), the fixed quadratic partner effect (p_2), the fixed effect of the continuous moderator (b), the fixed moderation effect of the linear actor (b_a) and partner (b_p) effects and the quadratic actor (b_{a2}) and partner (b_{p2}) effects, the variances and covariance of the Level 1 errors ϵ_{Ajt} and ϵ_{Bjt}, and the variance of the Level 2 errors (σ_ν^2).

Dichotomous time-varying moderator of the quadratic effects

Models 15 and 16's Level 1 and Level 2 equations resemble the ones from models 13 and 14, but the time-varying moderator is now a dichotomous variable D_{jt}:

$$Y_{Ajt} = \lambda_{Aj} + a_{AA}X_{Ajt} + a_{AA2}X_{Ajt}^2 + p_{MF1}X_{Bjt} + p_{BA2}X_{Bjt}^2 +$$

$$d_AD_{jt} + d_{AA}D_{jt}X_{Ajt} + d_{AA2}D_{jt}X_{Ajt}^2 + d_{BA}D_{jt}X_{Bjt} + d_{BA2}D_{jt}X_{Bjt}^2 + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_{Bj} + a_{BB}X_{Bjt} + a_{BB2}X_{Bjt}^2 + p_{AB}X_{Ajt} + p_{AB2}X_{Ajt}^2 +$$

$$d_BD_{jt} + d_{BB}D_{jt}X_{Bjt} + d_{BB2}D_{jt}X_{Bjt}^2 + d_{AB}D_{jt}X_{Ajt} + d_{AB2}D_{jt}X_{Ajt}^2 + \epsilon_{Bjt}$$

In addition to the linear and quadratic actor and partner effects, this model estimates the fixed effects of D_{jt}, d_A and d_B. We can interpret these effects as the changes in the fixed intercepts of partner A and B between occasions where D_{jt} is one and occasions when D_{jt} is zero. In other words, when D_{jt} is one the fixed intercepts are: $c_A + d_A$ and $c_B + d_B$; the linear actor effects are: $a_{AA} + d_{AA}$, $a_{BB} + d_{BB}$; the quadratic actor effects are: $a_{AA2} + d_{AA2}$, $a_{BB2} + d_{BB2}$; the linear partner effects are: $p_{BA} + d_{BA}$, $p_{AB} + d_{AB}$; and the quadratic partner effects are: $p_{BA2} + d_{BA2}$, $p_{AB2} + d_{AB2}$.

Model 16 again implements indistinguishable partners:

$$Y_{Ajt} = c + aX_{Ajt} + a_2X^2_{Ajt} + pX_{Bjt} + p_2X^2_{Bjt} +$$
$$dD_{jt} + d_aD_{jt}X_{Ajt} + d_{a2}D_{jt}X^2_{Ajt} + d_pD_{jt}X_{Bjt} + d_{p2}D_{jt}X^2_{Bjt} + \nu_j + \epsilon_{Ajt}$$
$$Y_{Bjt} = c + aX_{Bjt} + a_2X^2_{Bjt} + pX_{Ajt} + p_2X^2_{Ajt} +$$
$$dD_{jt} + d_aD_{jt}X_{Bjt} + d_{a2}D_{jt}X^2_{Bjt} + d_pD_{jt}X_{Ajt} + d_{p2}D_{jt}X^2_{Ajt} + \nu_j + \epsilon_{Bjt}$$

When D_{jt} is one, the fixed intercept is $c + d$, the linear actor effect is $a + d_a$, the quadratic actor effect is $a_2 + d_{a2}$, the linear partner effect is $p + d_p$, and the quadratic partner effect is $p_2 + d_{p2}$.

Accounting for temporal dependency through autoregressive effects

To account for the serial dependency that characterizes intensive longitudinal designs, we extended Models 1 to 16 to explicitly accommodate the temporal dependency in the partners' outcomes. Specifically, Models 17 to 32 allow estimating L-APIMs including lag-one autoregressive (AR(1)) effects. In these models, the lagged outcome variable (i.e., the observed outcome at the previous measurement occasion) is included as a predictor in the regression equations. For instance, Model 17, the extension of Model 1, is defined as

$$Y_{Ajt} = \lambda_{Aj} + \rho_{YA}Y_{Ajt-1} + a_{AA}X_{Ajt} + p_{BA}X_{Bjt} + \epsilon_{Ajt}$$
$$Y_{Bjt} = \lambda_{Bj} + \rho_{YB}Y_{Bjt-1} + a_{BB}X_{Bjt} + p_{AB}X_{Ajt} + \epsilon_{Bjt}$$

This model allows studying the mean autoregressive effect for each partner through ρ_{YA} and ρ_{YB}, respectively. This model assumes stationarity, and therefore, both effects have to fall into the interval $[-1, 1]$ (see e.g., Hannan (1976)). Given that temporal dependency is captured through the autoregressive effect, the residuals ϵ_{Ajt} and ϵ_{Bjt} remain serially uncorrelated and bivariate normally distributed with mean 0 and covariance matrix Σ_ϵ.

Note that this model differs from a model where the Level 1 errors follow an AR(1) process (see e.g., Gistelinck and Loeys (2019)). This is because the outcome at time $t-1$ affects the outcome at time t directly, whereas, in the model with AR(1) errors, the serial dependency is specified on the errors (see e.g., Asparouhov and Muthén (2020)). In Model 17, the outcomes at time $t-1$ affect the outcomes at time t directly; moreover, the predictor scores at time $t-1$, X_{Ajt-1} and X_{Bjt-1}, in turn affect the outcomes at time $t-1$ and thus have indirect effects on the outcomes at time t . In contrast, in the model with AR(1) errors, the effects of the predictors X_{Ajt-1} and X_{Bjt-1} are only contemporaneous, since X_{Ajt-1} and X_{Bjt-1} have no such indirect effect on Y_{Ajt} and Y_{Bjt}.

When modeling indistinguishable partners, we assume that the autoregressive effects are the same for each member of the dyad (i.e., $\rho_{YA} = \rho_{YB}$). Thus, Model 18, the extension of Model 2, is defined as:

$$Y_{Ajt} = \lambda_j + \rho_Y Y_{Ajt-1} + aX_{Ajt} + pX_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_j + \rho_Y Y_{Bjt-1} + aX_{Bjt} + pX_{Ajt} + \epsilon_{Bjt}$$

Finally, the AR(1) L-APIM that considers group differences in the actor and partner allows estimating differences in the autoregressive effect across two groups of dyads. The model for distinguishable partners is

$$Y_{Ajt} = \lambda_{Aj} + \rho_{YA0} Y_{Ajt-1} + \rho_{YA1} Z_j Y_{Ajt-1} + a_{AA0} X_{Ajt} + a_{AA1} Z_j X_{Ajt}$$

$$+ a_{BA1} X_{Bjt} + p_{MF1} Z_j X_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_{Bj} + \rho_{YB0} Y_{Bjt-1} + \rho_{YB1} Z_j Y_{Bjt-1} + a_{BB0} X_{Bjt} + a_{BB1} Z_j X_{Bjt} +$$

$$p_{AB0} X_{Ajt} + p_{AB1} Z_j X_{Ajt} + \epsilon_{Bjt}$$

The fixed autoregressive effect for partners in the reference group is ρ_{YA0} and ρ_{YB0}, and $\rho_{YA0} + \rho_{YA1}$ and $\rho_{YB0} + \rho_{YB1}$ for partners in the non-reference group. Note that moderation of the autoregressive effects is only included in this model and not in the models including time-varying moderators.

The AR(1) L-APIM for indistinguishable partners to estimate group differences between dyads is

$$Y_{Ajt} = \lambda_j + \rho_{Y0} Y_{Ajt-1} + \rho_{Y1} Z_j Y_{Ajt-1} + a_0 X_{Ajt} + a_1 Z_j X_{Ajt} + p_0 X_{Bjt} + p_1 Z_j X_{Bjt} + \epsilon_{Ajt}$$

$$Y_{Bjt} = \lambda_j + \rho_{Y0} Y_{Bjt-1} + \rho_{Y1} Z_j Y_{Bjt-1} + a_0 X_{Bjt} + a_1 Z_j X_{Bjt} + p_0 X_{Ajt} + p_1 Z_j X_{Ajt} + \epsilon_{Bjt}$$

The fixed autoregressive effect for participants in the reference group is ρ_{Y0}, and the fixed autoregressive effect for participants in the non-reference group is $\rho_{Y0} + \rho_{Y1}$.

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Table 1

Example rows of the hypothetical data set

Dyad ID	Observation	Y_A	Y_B	X_A	X_B	Z	C	D
1	1	2	3	3	5	1	1	1
1	2	3	4	5	4	1	2	1
1	3	3	2	4	7	1	3	0
1	4	4	3	7	7	1	4	0
2	1	5	4	3	1	0	1	0
2	2	4	3	2	2	0	2	1
2	3	4	5	1	2	0	3	1
2	4	3	6	3	1	0	4	0

Table 2

Overview of the effects of interest for the models available in the PowerLAPIM *application*

	Distinguishable	Quadratic	Time-invariant moderator	Time-variant moderator		AR(1)	
Model	Partners	Effects	Dichotomous variable	Continuous	Dichotomous	effects	Moderator
Model 1	X	-	-	-	-	-	-
Model 2	-	-	-	-	-	-	-
Model 3	X	-	X	-	-	-	X
Model 4	-	-	X	-	-	-	X
Model 5	X	-		X		-	X
Model 6	-	-		X		-	X
Model 7	X	-			X	-	X
Model 8	-	-			X	-	X
Model 9	X	X	-	-	-	-	-
Model 10	-	X	-	-	-	-	-
Model 11	X	X	X	-	-	-	X
Model 12	-	X	X	-	-	-	X
Model 13	X	X	-	X	-	-	X
Model 14	-	X	-	X	-	-	X
Model 15	X	X	-	-	X	-	X
Model 16	-	X	-	-	X	-	X
Model 17	X	-	-	-	-	X	-
Model 18	-	-	-	-	-	X	-
Model 19	X	-	X	-	-	X	X
Model 20	-	-	X	-	-	X	X
Model 21	X	-		X		X	X
Model 22	-	-		X		X	X
Model 23	X	-			X	X	X
Model 24	-	-			X	X	X
Model 25	X	X	-	-	-	X	-
Model 26	-	X	-	-	-	X	-
Model 27	X	X	X	-	-	X	X
Model 28	-	X	X	-	-	X	X
Model 29	X	X	-	X	-	X	X
Model 30	-	X	-	X	-	X	X
Model 31	X	X	-	-	X	X	X
Model 32	-	X	-	-	X	X	X