OSF-Material F

Mathematical Proof: Each Parabola is Axis Symmetric around the Vertical Line Through its Vertex

Consider again the quadratic function f (Equation 10) with $q_2 \neq 0$. To show that f is symmetric around the vertical axis through $X_{Vertex} = \frac{-q_1}{2q_2}$, we need to show that the value of f at "$X_{Vertex} + h$" equals the value of f at "$X_{Vertex} - h$" for any arbitrary number h:

$$\begin{aligned}
f(X_{Vertex}+h) &= f(\tfrac{-q_1}{2q_2}+h) \\
&= q_0 + q_1\left(\tfrac{-q_1}{2q_2}+h\right) + q_2\left(\tfrac{-q_1}{2q_2}+h\right)^2 \\
&= q_0 + q_1\left(\tfrac{-q_1}{2q_2}\right) + q_1h + q_2\left(\tfrac{-q_1}{2q_2}\right)^2 + 2q_2h\left(\tfrac{-q_1}{2q_2}\right) + q_2h^2 \\
&= q_0 + q_1\left(\tfrac{-q_1}{2q_2}\right) + q_1h + q_2\left(\tfrac{-q_1}{2q_2}\right)^2 - q_1h + q_2h^2 \\
&= q_0 + q_1\left(\tfrac{-q_1}{2q_2}\right) - q_1h + q_2\left(\tfrac{-q_1}{2q_2}\right)^2 + q_1h + q_2h^2 \\
&= q_0 + q_1\left(\tfrac{-q_1}{2q_2}\right) - q_1h + q_2\left(\tfrac{-q_1}{2q_2}\right)^2 + \tfrac{2q_2q_1h}{2q_2} + q_2h^2 \qquad (13)\\
&= q_0 + q_1\left(\tfrac{-q_1}{2q_2}\right) - q_1h + q_2\left(\left(\tfrac{-q_1}{2q_2}\right)^2 + \tfrac{2q_1h}{2q_2} + h^2\right) \\
&= q_0 + q_1\left(\tfrac{-q_1}{2q_2}\right) - q_1h + q_2\left(\left(\tfrac{-q_1}{2q_2}\right)^2 - 2h\left(\tfrac{-q_1}{2q_2}\right) + h^2\right) \\
&= q_0 + q_1\left(\tfrac{-q_1}{2q_2}-h\right) + q_2\left(\tfrac{-q_1}{2q_2}-h\right)^2 \\
&= f(\tfrac{-q_1}{2q_2}-h) \\
&= f(X_{Vertex}-h)
\end{aligned}$$