OSF-Material E

Mathematical Derivation: The Position of the Vertex of a Parabola

Consider the quadratic equation $Z = q_0 + q_1 X + q_2 X^2$, where $q_2 \neq 0$. To compute the position of the vertex, we understand this equation as a function f that maps X to the respective Z value:

$$Z = f(X) = q_0 + q_1 X + q_2 X^2. \tag{10}$$

The vertex of the parabola is the extremum of f (i.e., its minimum when the parabola is U-shaped, its maximum when it is an inverted U-shape). To determine its position, we need the first derivative f' of f:

$$f'(X) = q_1 + 2q_2 X. \tag{11}$$

Now compute the root of the first derivative to determine the X value of the vertex:

$$\begin{aligned} & f'(X) &&= 0 \\ \Leftrightarrow \quad & q_1 + 2q_2 X &&= 0 \\ \Leftrightarrow \quad & X &&= \tfrac{-q_1}{2q_2} \end{aligned} \tag{12}$$

That is, the extremum (i.e., vertex) of the quadratic equation is positioned at $X = \frac{-q_1}{2q_2}$.