Additional OSF-Materials for

"Response Surface Analysis in Personality and Social Psychology: Checklist and Clarifications for the Case of Congruence Hypotheses"

OSF-Material A

Confidence Intervals and P-Values for the Example Analyses

Table 1
95% Confidence Intervals and P-Values for the Regression Coefficients of the Polynomial Model

	Estimated regression model $Z = b_0 + b_1X + b_2Y + b_3X^2 + b_4XY + b_5Y^2$					
	b_0	b_1	b_2	b_3	b_4	b_5
Figure 1a	[1.79, 1.8] (p=0)	[0, 0.01] (p=.34)	[-0.01, 0.01] (p=.85)	[-0.05, -0.04] (p=0)	[0.09, 0.1] (p=0)	[-0.06, -0.04] (p=0)
Figure 1b	[1.6, 1.62] (p=0)	[0.1, 0.12] (p=0)	[0.09, 0.11] (p=0)	[-0.06, -0.04] (p=0)	[0.09, 0.11] (p=0)	[-0.06, -0.04] (p=0)
Figure 1c	[1.59, 1.61] (p=0)	[0.09, 0.11] (p=0)	[0.1, 0.11] (p=0)	[-0.1, -0.09] (p=0)	[0.1, 0.11] (p=0)	[-0.11, -0.09] (p=0)
Figure 2a	[1.77, 1.8] (p=0)	[-0.19, -0.16] (p=0)	[0.16, 0.19] (p=0)	[-0.11, -0.1] (p=0)	[0.18, 0.21] (p=0)	[-0.1, -0.08] (p=0)
Figure 2b	[1.79, 1.81] (p=0)	[-0.01, 0.01] (p=.88)	[-0.01, 0] (p=.39)	[-0.1, -0.09] (p=0)	[0.1, 0.12] (p=0)	[-0.04, -0.02] (p=0)
Figure 2c	[15.09, 15.19] (p=0)	[0.05, 0.13] (p=0)	[1.51, 1.6] (p=0)	[-0.31, -0.25] (p=0)	[0.79, 0.9] (p=0)	[-0.6, -0.53] (p=0)
Figure 2d	[1.56, 1.64] (p=0)	[0.26, 0.32] (p=0)	[-0.04, 0.02] (p=.41)	[-0.1, -0.04] (p=0)	[-0.01, 0.08] (p=.09)	[-0.02, 0.04] (p=.49)
Figure 2e	[1.8, 1.81] (p=0)	[0, 0.01] (p=.74)	[0, 0.01] (p=.01)	[-0.01, 0] (p=.35)	[0.11, 0.12] (p=0)	[-0.12, -0.11] (p=0)
Figure 2f	[1, 1.01] (p=0)	[0, 0] (p=.66)	[0, 0] (p=.88)	[0.02, 0.03] (p=0)	[-0.05, -0.04] (p=0)	[0.02, 0.03] (p=0)

Table 2
95% Confidence Intervals and P-Values for the Coefficients of the First Principal Axis, LOC, and LOIC

	Position of first principal axis		Shape of surface along lines			
			LOC		LOIC	
	p_{10}	p_{11}	a_1	a_2	a_3	a_4
Figure 1a	[-0.07, 0.04] (p=.54)	[0.88, 1.03] (p=0)	[0, 0.01] (p=.56)	[-0.01, 0.01] (p=.96)	[-0.01, 0.01] (p=.53)	[-0.21, -0.18] (p=0)
Figure 1b	[-0.2, 0.04] (p=.19)	[0.89, 1.13] (p=0)	[0.2, 0.22] (p=0)	[-0.01, 0.01] (p=.98)	[0, 0.03] (p=.13)	[-0.22, -0.18] (p=0)
Figure 1c	[-0.03, 0.07] (p=.49)	[0.9, 1.08] (p=0)	[0.19, 0.21] (p=0)	[-0.1, -0.08] (p=0)	[-0.02, 0.01] (p=.52)	[-0.32, -0.29] (p=0)
Figure 2a	[0.79, 1.09] (p=0)	[1, 1.18] (p=0)	[-0.02, 0.02] (p=.87)	[-0.01, 0.01] (p=.95)	[-0.38, -0.32] (p=0)	[-0.42, -0.36] (p=0)
Figure 2b	[-0.09, 0.05] (p=.61)	[1.69, 1.97] (p=0)	[-0.01, 0.01] (p=.49)	[-0.03, -0.01] (p=0)	[-0.01, 0.02] (p=.53)	[-0.25, -0.22] (p=0)
Figure 2c	[0.79, 0.93] (p=0)	[0.68, 0.75] (p=0)	[1.61, 1.69] (p=0)	[-0.03, 0.04] (p=.74)	[-1.54, -1.39] (p=0)	[-1.79, -1.59] (p=0)
Figure 2d	[-21.45, 3.82] (p=.17)	[-0.62, 9.4] (p=.09)	[0.24, 0.31] (p=0)	[-0.06, 0.01] (p=.24)	[0.26, 0.35] (p=0)	[-0.18, -0.02] (p=.02)
Figure 2e	[0, 0.06] (p=.05)	[0.39, 0.44] (p=0)	[0, 0.02] (p=0)	[-0.01, 0] (p=.22)	[-0.02, 0] (p=.17)	[-0.24, -0.21] (p=0)
Figure 2f	(-)	(-)	[-0.01, 0] (p=.6)	[0, 0] (p=.7)	[-0.01, 0.01] (p=.88)	[0.08, 0.1] (p=0)

Note. The position of the first principal axis in the X-Y plane is given by $Y = p_{10} + p_{11}X$. The shape of the surface above the LOC is described by $Z = b_0 + a_1X + a_2X^2$, and the shape above the LOIC is $Z = b_0 + a_3X + a_4X^2$.
(-) For bowl-shaped surfaces, the first principal axis is of no interest when considering congruence effects.

OSF-Material B

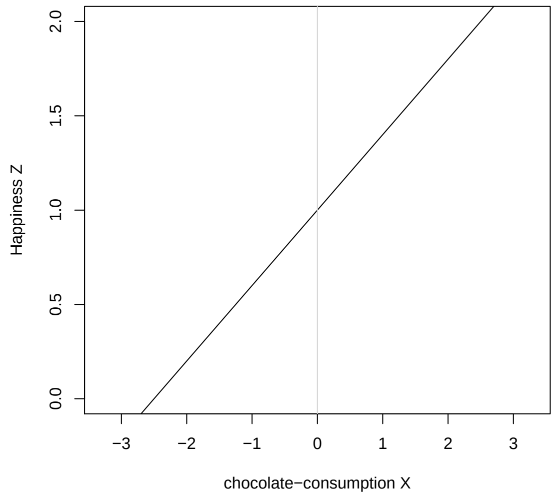
The Fundamentals of RSA Interpretation: Quadratic Equations

RSA interpretation includes several elements that require basic knowledge about quadratic equations. We therefore briefly recall these basics in this supplementary document. As a hypothetical example, imagine that we want to investigate the relation between people's chocolate-consumption and happiness. If we assume that chocolate-consumption X affects happiness Z in a quadratic way, we would assess the respective data and fit a quadratic regression model to the data:

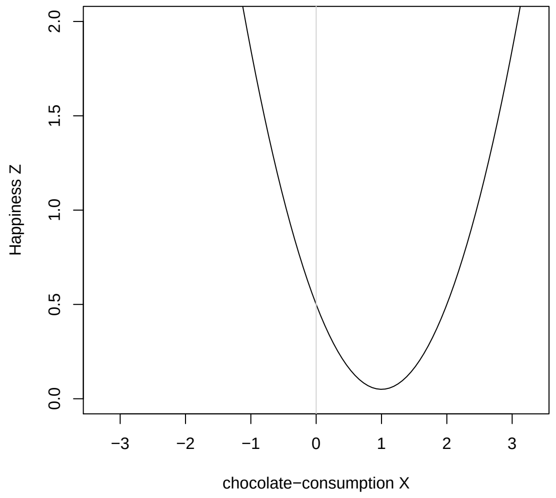
$$Z = q_0 + q_1 X + q_2 X^2, \tag{1}$$

where q_0, q_1, and q_2 are the estimated coefficients, and Z denotes the predicted happiness for a person with chocolate-consumption X. To see what the (estimated) coefficients of such a *quadratic equation* reveal about the association of X and Z, one should first consider the coefficient q_2 of the quadratic term. When q_2 is zero, Equation 1 simplifies to $Z = q_0 + q_1 X$, that is, the association of X and Z is linear. The corresponding graph is a straight line (see Figure 1a) with a slope of q_1 and an intercept of q_0.

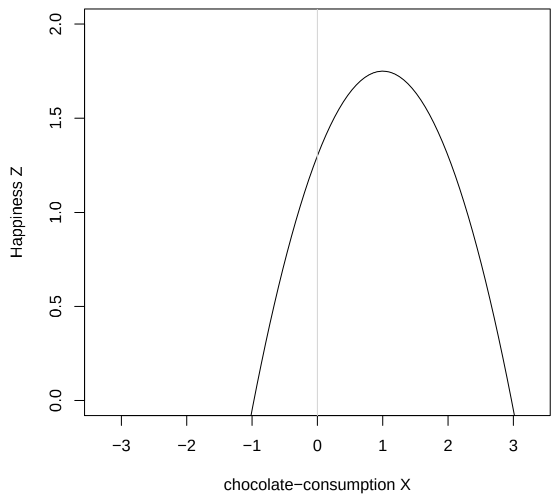
When q_2 is non-zero, the graph of Equation 1 is a *parabola*. When q_2 is positive, the graph has a U-shape (see Figure 1b), and when q_2 is negative, the graph has an inverted U-shape (see Figure 1c). The magnitude of q_2 reflects how "strong" the curvilinear effect of X on Z is: The closer q_2 is to zero, the wider the parabola is (see Figure 1d), and the larger q_2 is in magnitude, the steeper the parabola is (Figure 1c).



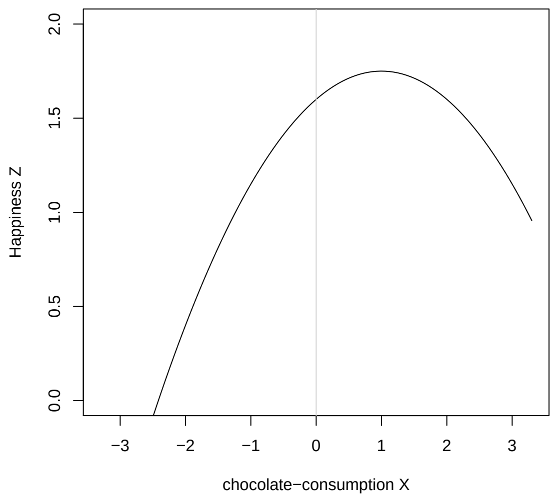
(a) Graph of the equation $Z = 1 + 0.4X + 0X^2$.



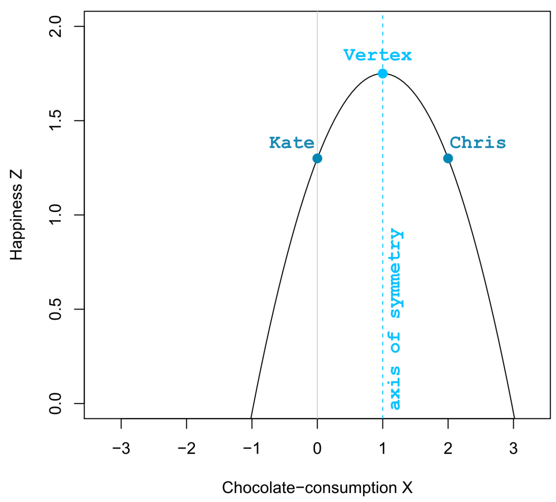
(b) Graph of the equation $Z = 0.5 - 0.9X + 0.45X^2$.



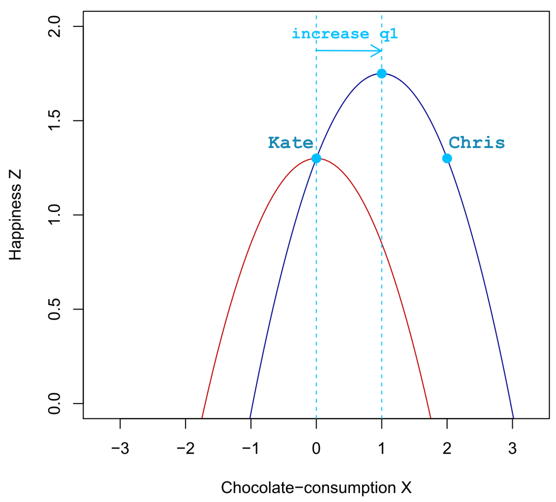
(c) Graph of the equation $Z = 1.3 + 0.9X - 0.45X^2$.



(d) Graph of the equation $Z = 1.6 + 0.3X - 0.15X^2$.



(e) Graph of the equation $Z = 1.3 + 0.9X - 0.45X^2$.



(f) Graph of the two equations $Z = 1.3 + 0X - 0.45X^2$ (left parabola) and $Z = 1.3 + 0.9X - 0.45X^2$ (right parabola).

Figure 1. Graphs of some quadratic functions $Z = q_0 + q_1X + q_2X^2$.

For the chocolate example, a negative value of q_2 would mean that there is some "optimal value" of chocolate-consumption: In Figure 1c, it looks as if people with a value of $X = 1$ are predicted to be the happiest, whereas happiness is lower the more chocolate-consumption deviates from 1. In technical terms, the parabola has a *vertex*, which is defined as its highest[1] point (see Figure 1e). To determine the exact amount of chocolate-consumption that leads to the highest happiness, we can determine the X value of the vertex via the formula (see OSF-Material E for the derivation)

$$X_{vertex} = \frac{-q_1}{2q_2} \tag{2}$$

In Figure 1c, the vertex is located at $X_{vertex} = \frac{-0.9}{2*(-0.45)} = 1$.

An important property of every parabola is its symmetry: When we draw a vertical line (i.e., a line that is perpendicular to the x-axis) through the vertex of a parabola, the parabola is symmetric around this *axis of symmetry* (see Figure 1e; see OSF-Material F for the mathematical proof). That is, the right-hand side of the axis of symmetry is an exact reflection of the left-hand side. For the chocolate example, this means that a person whose chocolate-consumption is higher than the optimal level to some extent (e.g., at "optimal level + 1" as for Chris) is predicted to be as happy as a person whose consumption is lower than the optimal level by the same amount (e.g., at "optimal level - 1" as for Kate).

When interpreting the results of quadratic equations, it is also crucial to understand the role of q_1 because this coefficient can substantially alter the interpretation in terms of how X is related to Z. Imagine that q_2 is negative, so we know that there is some optimal value of chocolate-consumption X, and we aim at finding out whether the optimal X value is zero. To this aim, we must consider the coefficient q_1. As a mathematical fact, q_1 equals the slope of the parabola at $X = 0$ (e.g., see Edwards, 2007). The vertex of a parabola is positioned at $X = 0$ only if q_1 is zero; in this case (assuming $q_2 < 0$), the parabola rises for negative X values, reverses at $X = 0$, and falls for positive X values (as in the left-hand side parabola in Figure 1f), predicting the highest happiness for people with a chocolate-consumption value of zero (e.g., Kate).

When we change q_1 from zero to a positive value (as in the right parabola in Figure 1f), the

[1] When $q_2 > 0$, the vertex is the *lowest* point of the parabola. Its position can also be computed via Equation 2.

parabola is shifted to the right,[2] such that the slope above $X = 0$ is now positive. In shifting the parabola, its shape and vertical orientation stay perfectly fixed: The axis of symmetry is shifted sideways along with the parabola and thereby remains perpendicular to the x-axis (see Figure 1f). When q_1 is non-zero, the parabola is no longer maximized at $X = 0$, such that Kate's chocolate-consumption value of zero is no longer optimal. Instead, when q_1 is positive, happiness rises when we approach $X = 0$ from negative consumption values, and it continues to rise at a value of zero consumption. To sum, the linear term coefficient q_1 in a quadratic equation (with $q_2 \neq 0$) equals the slope of the parabola at $X = 0$, and the vertex of the parabola is positioned at $X = 0$ only if q_1 is zero.

To conclude, a good strategy for interpreting the (estimated) coefficients of a quadratic equation consists of two steps (see Figure 2). First, consider the coefficient q_2 of the quadratic term, which indicates whether the graph is linear (if $q_2 = 0$), U-shaped (if $q_2 > 0$), or an inverted U-shape (if $q_2 < 0$). Second, consider the coefficient q_1. Its interpretation depends on what was found in the first step: When $q_2 = 0$, q_1 reflects the direction and strength of the linear association of X and Z. When $q_2 \neq 0$, q_1 indicates whether the vertex is positioned at $X = 0$. If this is not the case, the position of the vertex can be computed as $X_{vertex} = \frac{-q_1}{2q_2}$.

[2] and also up so that the intersection point with the y-axis (i.e., the intercept q_0) remains the same.

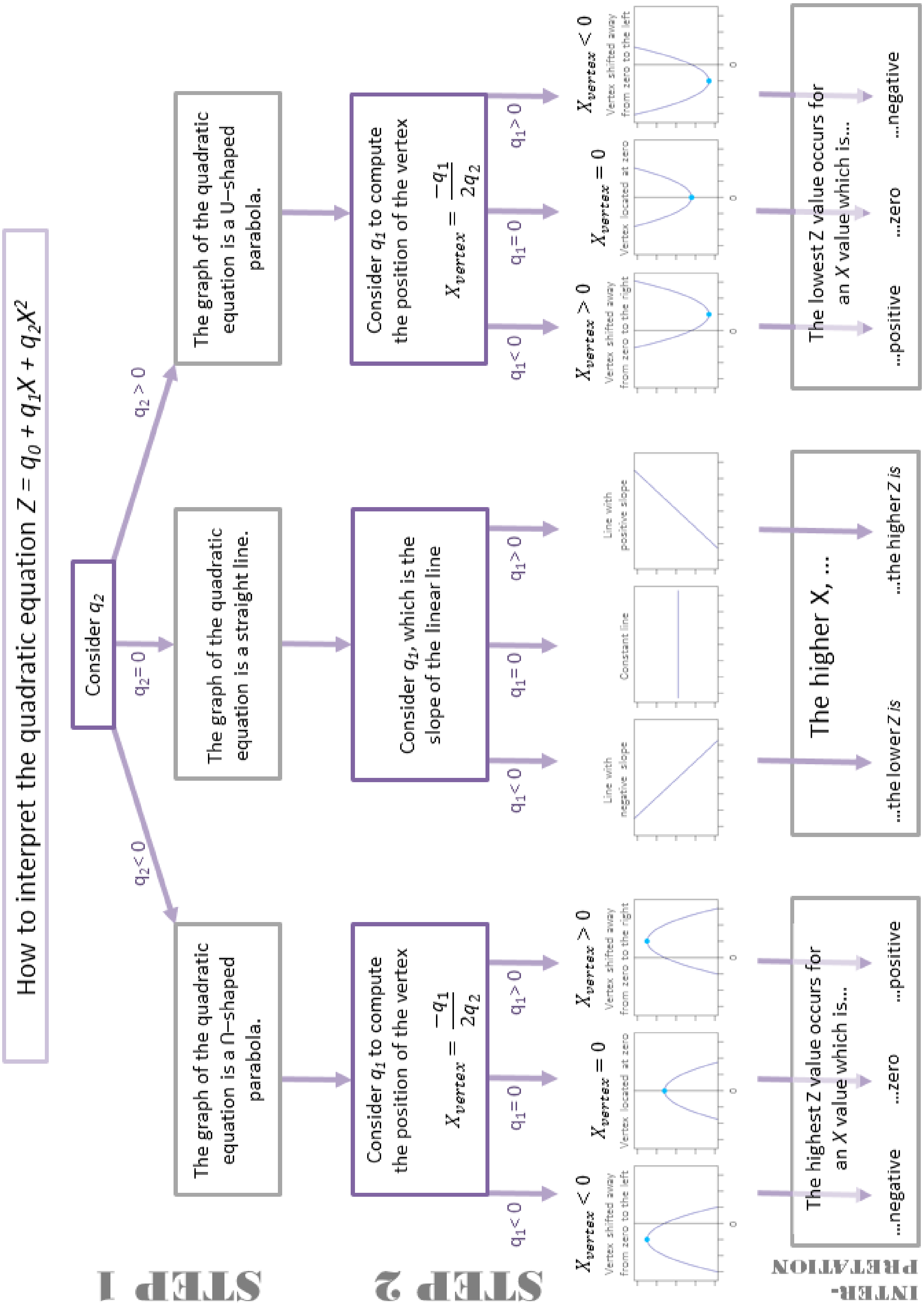


Figure 2. A recommended strategy for interpreting a quadratic equation.

OSF-Material C

Mathematical Proof: The Six Conditions Stated in Figure 3 are Sufficient for a Strict Congruence Effect

We can understand the second-degree polynomial model that builds the basis of RSA as a function g that maps X and Y to the respective Z value:

$$Z = g(X, Y) = b_0 + b_1X + b_2Y + b_3X^2 + b_4XY + b_5Y^2. \tag{3}$$

Now, let all six necessary conditions for a strict congruence effect hold, that is, let

1. $p_{10} = 0$
2. $p_{11} = 1$,
3. $a_2 = 0$,
4. $a_1 = 0$,
5. $a_4 < 0$,
6. $a_3 = 0$.

We will now proof that when all six conditions are satisfied, the model reflects a strict congruence effect in the sense that it predicts higher Z values the smaller the distance between X and Y. In mathematical terms, such an effect means that $g(X, Y) > g(X', Y')$ is equivalent to $|X - Y| < |X' - Y'|$. To proof that this equivalence holds when the six conditions are satisfied, we proceed in six steps.

Step 1: Show that $\mathbf{b_1 = b_2 = 0}$.

From the condition $a_1 = b_1 + b_2 = 0$, it follows that $b_1 = -b_2$. Moreover, because $a_3 = b_1 - b_2 = 0$, we know that $b_1 = b_2 = -b_1$. Consequently, $b_1 = 0$ and $b_2 = 0$.

Step 2: Show that $\mathbf{b_4 \neq 0}$.

We proof this claim by contradiction: Suppose $b_4 = 0$. Then, from $a_2 = b_3 + b_4 + b_5 = 0$, it follows that $b_3 + b_5 = 0$. Also, from $a_4 = b_3 - b_4 + b_5 < 0$, it follows that $b_3 + b_5 < 0$, which is a contradiction. Consequently, b_4 must differ from zero.

Step 3: Show that $\mathbf{b_5 = b_3}$.

From $p_{11} = \frac{b_5 - b_3 + \sqrt{(b_3-b_5)^2 + b_4^2}}{b_4} = 1$ (see Edwards, 2007, for the formula of p_{11}), it follows that

$$\begin{aligned} b_5 - b_3 + \sqrt{(b_3-b_5)^2 + b_4^2} &= b_4 \\ \Leftrightarrow \quad (b_3-b_5)^2 + b_4^2 &= ((b_3-b_5)+b_4)^2 \\ \Leftrightarrow \quad (b_3-b_5)^2 + b_4^2 &= (b_3-b_5)^2 + 2b_4(b_3-b_5) + b_4^2 \\ \Leftrightarrow \quad 0 &= 2b_4(b_3-b_5) \\ \Leftrightarrow \quad 0 &= b_3 - b_5 \\ \Leftrightarrow \quad b_5 &= b_3, \end{aligned} \tag{4}$$

where the second-last step is valid because $b_4 \neq 0$.

Step 4: Show that $\mathbf{b_4 = -2b_3}$.

From $a_2 = 0$ and $b_5 = b_3$, it follows that

$$\begin{aligned} a_2 = b_3 + b_4 + b_5 &= 0 \\ \Leftrightarrow \quad b_3 + b_4 + b_3 &= 0 \\ \Leftrightarrow \quad b_4 &= -2b_3 \end{aligned} \tag{5}$$

Step 5: Show that g simplifies to $\mathbf{g(X,Y) = b_0 + b_3(X-Y)^2}$ with $\mathbf{b_3 < 0}$.

Taken together, the function g (Equation 3) is given by

$$\begin{aligned} g(X,Y) &= b_0 + b_1X + b_2Y + b_3X^2 + b_4XY + b_5Y^2 \\ &= b_0 + 0X + 0Y + b_3X^2 - 2b_3XY + b_3Y^2 \\ &= b_0 + b_3(X-Y)^2. \end{aligned} \tag{6}$$

From $a_4 < 0$, $b_5 = b_3$, and $b_4 = -2b_3$, it follows that the coefficient b_3 is negative:

$$\begin{aligned} a_4 &< 0 \\ \Leftrightarrow \quad b_3 - b_4 + b_5 &< 0 \\ \Leftrightarrow \quad b_3 + 2b_3 + b_3 &< 0 \\ \Leftrightarrow \quad 4b_3 &< 0 \\ \Leftrightarrow \quad b_3 &< 0. \end{aligned} \tag{7}$$

Step 6: Show that $\mathbf{g(X, Y) > g(X', Y')}$ is equivalent to $\mathbf{|X - Y| < |X' - Y'|}$.

We can now use the simplified formula for g to show that there is a strict congruence effect:

$$\begin{aligned} g(X, Y) &> g(X', Y') \\ \Leftrightarrow \quad b_0 + b_3(X - Y)^2 &> b_0 + b_3(X' - Y')^2 \\ \Leftrightarrow \quad b_3(X - Y)^2 &> b_3(X' - Y')^2 \\ \Leftrightarrow \quad (X - Y)^2 &< (X' - Y')^2 \\ \Leftrightarrow \quad |X - Y| &< |X' - Y'|. \end{aligned} \tag{8}$$

where the third step is valid because $b_3 < 0$.

To sum up, we showed that the six conditions are sufficient for the existence of a strict congruence effect.

OSF-Material D

RSA Variants and More Advanced Response Surface Methodology

The scope of our manuscript was to comprehensively line out how to correctly identify congruence effects with RSA. Here, we provide a short overview about some variants, extensions and adaptions of RSA that go beyond this specific application.

Beyond Congruence Effects

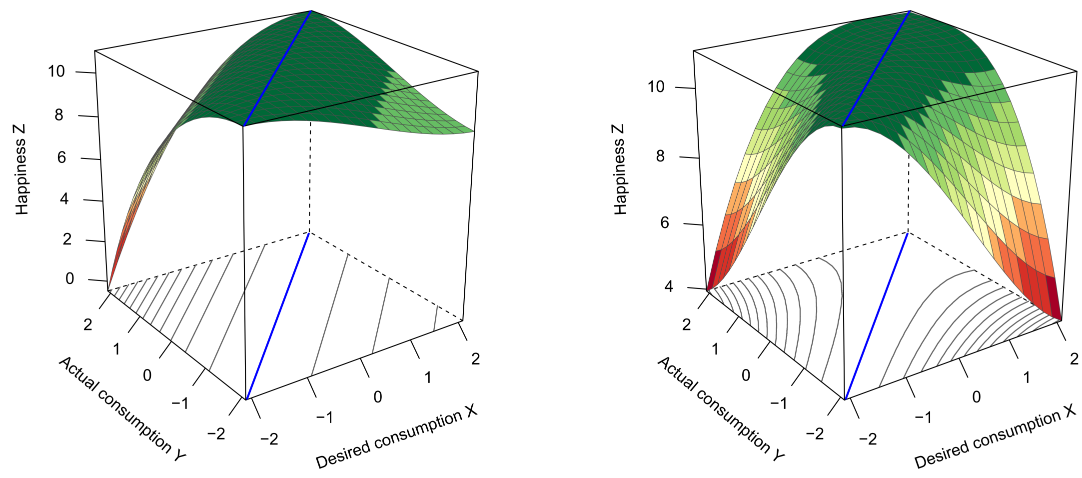
In case that a response surface does not reflect a congruence effect, further RSA tools should be used to derive working hypotheses on the effects that are present instead (e.g., see Cohen, Nahum-Shani, & Doveh, 2010; Edwards, 2002, 2007). These tools can for example reveal "optimal margin" effects (as in Figure 2a in the main part of the manuscript; see Baumeister, 1989). Furthermore, in case that the coefficients of the quadratic and interaction terms (i.e., b_3, b_4, and b_5) are non-significant, one can test for a linear effect of the discrepancy $X - Y$, which would require that $|a_3| > |a_1|$ (see Humberg et al., in press).

Beyond Second-Degree Effects

Because response surfaces based on Equation 3 are restricted to a "quadratic" shape, they cannot reflect arbitrarily complex effects. To test psychologically relevant hypotheses that are not detectable by quadratic surfaces (e.g., congruence effects where the direction of mismatch matters), one can extend the regression model in Equation 3 by "cubic" terms (i.e., X^3, X^2Y, XY^2, and Y^3):

$$Z = b_0 + b_1X + b_2Y + b_3X^2 + b_4XY + b_5Y^2 \quad + b_6X^3 + b_7X^2Y + b_8XY^2 + b_9Y^3 \tag{9}$$

This regression model can for example lead to a surface as in Figure 3a, where happiness is highest for persons whose chocolate-consumption matches their desire, while additionally, happiness falls faster when consumption exceeds desire than when desire exceeds consumption. Cubic RSA, in contrast to quadratic RSA, can also test whether exactly fulfilling one's chocolate-desire is more relevant for persons whose desire is low, compared to persons with high desire (Figure 3b).



(a) Response surface of the cubic model
$Z = 11 + 0X + 0Y - 0.4X^2 + 0.8XY - 0.4Y^2$
$+ 0.06X^3 - 0.18X^2Y + 0.18XY^2 - 0.06Y^3$.

(b) Response surface of the cubic model
$Z = 11 + 0X + 0Y - 0.4X^2 + 0.8XY - 0.4Y^2$
$+ 0.14X^3 - 0.14X^2Y - 0.14XY^2 + 0.14Y^3$.

Figure 3. Cubic response surfaces that reflect effects which cannot be detected with quadratic surfaces defined by Equation 3.

While cubic models can be estimated with the *RSA* package (Schönbrodt, 2016a), their interpretation is far more complex than the interpretation of quadratic RSA models. To ease a careful and correct application of cubic RSA, respective strategies are currently developed (Humberg, Nestler, Schönbrodt, & Back, 2017).

Beyond Multiple Parameter Tests

To test a congruence effect with RSA, instead of testing the four (or six) necessary conditions for a congruence effect one by one (Figure 3 in the main part of the manuscript), it can be convenient to test them simultaneously in a ”model testing” approach. The basic idea of the model testing approach is to fit a surface to the data that is restricted to perfectly reflect the expected effect (e.g., restricted to be shaped as in Figure 1a in the manuscript when testing strict congruence effects) and to test whether this *hypothesis-conform model* explains the data equally well as the freely estimated surface from Equation 1 in the manuscript (e.g., see Edwards, 2002; Schönbrodt, 2016b), by statistically comparing their R^2 values. This approach has the advantages that it (a) includes less significance tests than testing all six necessary conditions and (b) reduces the risk of unjustified interpretations for complex or equivocal surfaces.

Beyond Single Hypothesis Testing

In some disciplines, congruence hypotheses are challenged by theoretically grounded alternative expectations (e.g., optimal margin effects, see Baumeister, 1989). In these cases, the model testing approach can be extended to test all alternative hypotheses against each other. The idea is to derive respective hypothesis-conform models for all competing hypotheses and compare their empirical evidence, for example with an information-theoretic approach (e.g., see Burnham & Anderson, 2002; Humberg, Dufner, et al., 2017).

Beyond Simple Data Structures

In practice, we are often confronted with data structures that are more complex than the data described here, which require adapted versions of RSA. As some examples, one might want to apply RSA to multilevel data (e.g., students nested in classes Nestler, Humberg, & Schönbrodt, 2017), dyadic data (e.g., predicting both male and female relationship satisfaction from couples' similarity; see Nestler, Grimm, & Schönbrodt, 2015; Schönbrodt, Humberg, & Nestler, 2017). Moreover, one might want to investigate more complex effects, such as the longitudinal development of congruence effects (e.g., does friend similarity get more important over time), or congruence effects referring to profile similarity rather than to level congruence (e.g., social consequences of similar Big Five profiles; see also Edwards, 2007; Edwards & Parry, 1993). To allow a broad application of RSA, future research needs to adapt or extend RSA to such special research questions and data structures, while paying particular attention to the interpretation in terms of relevant hypotheses in the social and personality psychological literature.

OSF-Material E

Mathematical Derivation: The Position of the Vertex of a Parabola

Consider the quadratic equation $Z = q_0 + q_1 X + q_2 X^2$, where $q_2 \neq 0$. To compute the position of the vertex, we understand this equation as a function f that maps X to the respective Z value:

$$Z = f(X) = q_0 + q_1 X + q_2 X^2. \tag{10}$$

The vertex of the parabola is the extremum of f (i.e., its minimum when the parabola is U-shaped, its maximum when it is an inverted U-shape). To determine its position, we need the first derivative f' of f:

$$f'(X) = q_1 + 2q_2 X. \tag{11}$$

Now compute the root of the first derivative to determine the X value of the vertex:

$$\begin{aligned} f'(X) &= 0 \\ \Leftrightarrow \quad q_1 + 2q_2 X &= 0 \\ \Leftrightarrow \quad X &= \tfrac{-q_1}{2q_2} \end{aligned} \tag{12}$$

That is, the extremum (i.e., vertex) of the quadratic equation is positioned at $X = \frac{-q_1}{2q_2}$.

OSF-Material F

Mathematical Proof: Each Parabola is Axis Symmetric around the Vertical Line Through its Vertex

Consider again the quadratic function f (Equation 10) with $q_2 \neq 0$. To show that f is symmetric around the vertical axis through $X_{Vertex} = \frac{-q_1}{2q_2}$, we need to show that the value of f at "$X_{Vertex} + h$" equals the value of f at "$X_{Vertex} - h$" for any arbitrary number h:

$$
\begin{aligned}
f(X_{Vertex} + h) &= f(\tfrac{-q_1}{2q_2} + h) \\
&= q_0 + q_1 \left(\tfrac{-q_1}{2q_2} + h\right) + q_2 \left(\tfrac{-q_1}{2q_2} + h\right)^2 \\
&= q_0 + q_1 \left(\tfrac{-q_1}{2q_2}\right) + q_1 h + q_2 \left(\tfrac{-q_1}{2q_2}\right)^2 + 2q_2 h \left(\tfrac{-q_1}{2q_2}\right) + q_2 h^2 \\
&= q_0 + q_1 \left(\tfrac{-q_1}{2q_2}\right) + q_1 h + q_2 \left(\tfrac{-q_1}{2q_2}\right)^2 - q_1 h + q_2 h^2 \\
&= q_0 + q_1 \left(\tfrac{-q_1}{2q_2}\right) - q_1 h + q_2 \left(\tfrac{-q_1}{2q_2}\right)^2 + q_1 h + q_2 h^2 \\
&= q_0 + q_1 \left(\tfrac{-q_1}{2q_2}\right) - q_1 h + q_2 \left(\tfrac{-q_1}{2q_2}\right)^2 + \tfrac{2q_2 q_1 h}{2q_2} + q_2 h^2 \qquad (13) \\
&= q_0 + q_1 \left(\tfrac{-q_1}{2q_2}\right) - q_1 h + q_2 \left(\left(\tfrac{-q_1}{2q_2}\right)^2 + \tfrac{2q_1 h}{2q_2} + h^2\right) \\
&= q_0 + q_1 \left(\tfrac{-q_1}{2q_2}\right) - q_1 h + q_2 \left(\left(\tfrac{-q_1}{2q_2}\right)^2 - 2h \left(\tfrac{-q_1}{2q_2}\right) + h^2\right) \\
&= q_0 + q_1 \left(\tfrac{-q_1}{2q_2} - h\right) + q_2 \left(\tfrac{-q_1}{2q_2} - h\right)^2 \\
&= f(\tfrac{-q_1}{2q_2} - h) \\
&= f(X_{Vertex} - h)
\end{aligned}
$$

References

Baumeister, R. F. (1989). The optimal margin of illusion. *Journal of Social and Clinical Psychology*, *8*(2), 176–189. doi:10.1521/jscp.1989.8.2.176

Burnham, K. P. & Anderson, D. R. (2002). *Model selection and multimodel inference: A practical information-theoretic approach* (2nd ed.). New York, NY: Springer.

Cohen, A., Nahum-Shani, I., & Doveh, E. (2010). Further insight and additional inference methods for polynomial regression applied to the analysis of congruence. *Multivariate Behavioral Research*, *45*(5), 828–852. doi:10.1080/00273171.2010.519272

Edwards, J. R. (2002). Alternatives to difference scores: Polynomial regression analysis and response surface methodology. In F. Drasgow & N. W. Schmitt (Eds.), *Measuring and analyzing behavior in organizations: Advances in measurement and data analysis* (pp. 350–400). San Francisco, CA: Jossey-Bass.

Edwards, J. R. (2007). Polynomial regression and response surface methodology. In C. Ostroff & T. A. Judge (Eds.), *Perspectives on organizational fit* (pp. 361–372). San Francisco: Jossey-Bass.

Edwards, J. R. & Parry, M. E. (1993). On the use of polynomial regression equations as an alternative to difference scores in organizational research. *Academy of Management Journal*, *36*(6), 1577–1613. doi:10.2307/256822

Humberg, S., Dufner, M., Schönbrodt, F. D., Geukes, K., Hutteman, R., van Zalk, M. H. W., ... Back, M. D. (in press). Enhanced versus simply positive: A new condition-based regression analysis to disentangle effects of self-enhancement from effects of positivity of self-view. *Journal of Personality and Social Psychology*. doi:10.1037/pspp0000134

Humberg, S., Dufner, M., Schönbrodt, F. D., Geukes, K., Hutteman, R., van Zalk, M. H. W., ... Back, M. D. (2017). *Is accurate, positive, or inflated self-perception most advantageous for psychological adjustment? A competitive test of key hypotheses.* Manuscript submitted for publication.

Humberg, S., Nestler, S., Schönbrodt, F. D., & Back, M. D. (2017). *Testing advanced congruence effects with Response Surface Analysis of third degree*. Manuscript in preparation.

Nestler, S., Grimm, K. J., & Schönbrodt, F. D. (2015). The social consequences and mechanisms of personality: How to analyse longitudinal data from individual, dyadic, round-robin and network designs. *European Journal of Personality*, *29*, 272–295. doi:10.1002/per.1997

Nestler, S., Humberg, S., & Schönbrodt, F. D. (2017). *The study of congruence effects with Response Surface Analysis in case of multilevel data*. Manuscript in preparation.

Schönbrodt, F. D. (2016a). *RSA: An R package for response surface analysis (version 0.9.10)*. Retrieved from https://cran.r-project.org/package=RSA

Schönbrodt, F. D. (2016b). *Testing fit patterns with polynomial regression models*. Retrieved from osf.io/3889z

Schönbrodt, F. D., Humberg, S., & Nestler, S. (2017). *Testing similarity effects with dyadic response surface analysis*. Manuscript submitted for publication.