# Solution to Problem 05: Escape Velocity from a White Dwarf

## 1. Derive the escape velocity formula using conservation of energy

The escape velocity is derived from the principle of conservation of energy. The total energy of an object escaping from the gravitational field of a white dwarf is:

$$E_{\text{total}} = K + U = 0,$$

where:

- $K = \frac{1}{2}mv^2$ is the kinetic energy,
- $U = -\frac{GMm}{r}$ is the gravitational potential energy,
- $v$ is the escape velocity,
- $G$ is the gravitational constant,
- $M$ is the mass of the white dwarf,
- $r$ is the radius of the white dwarf,
- $m$ is the mass of the escaping object.

Setting the total energy to zero:

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0.$$

Solving for $v$:

$$v = \sqrt{\frac{2GM}{r}}.$$

Thus, the escape velocity is:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}}.$$

## 2. Calculate the escape velocity as a fraction of the speed of light

Given:

- $M = 1.4M_\odot$, where $M_\odot = 1.989 \times 10^{30}$ kg,
- $r = 0.008R_\odot$, where $R_\odot = 6.957 \times 10^{8}$ m,
- $G = 6.674 \times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$,

- $c = 3.0 \times 10^8\,\mathrm{m/s}$.

First, calculate the mass and radius of the white dwarf:

$$M = 1.4 \times 1.989 \times 10^{30}\,\mathrm{kg} \approx 2.785 \times 10^{30}\,\mathrm{kg},$$

$$r = 0.008 \times 6.957 \times 10^{8}\,\mathrm{m} \approx 5.566 \times 10^{6}\,\mathrm{m}.$$

Now substitute into the escape velocity formula:

$$v_{\mathrm{escape}} = \sqrt{\frac{2GM}{r}},$$

$$v_{\mathrm{escape}} = \sqrt{\frac{2 \cdot 6.674 \times 10^{-11} \cdot 2.785 \times 10^{30}}{5.566 \times 10^{6}}}.$$

Simplify:

$$v_{\mathrm{escape}} \approx \sqrt{\frac{3.716 \times 10^{20}}{5.566 \times 10^{6}}} \approx \sqrt{6.678 \times 10^{13}} \approx 8.17 \times 10^{6}\,\mathrm{m/s}.$$

Express $v_{\mathrm{escape}}$ as a fraction of the speed of light:

$$\frac{v_{\mathrm{escape}}}{c} = \frac{8.17 \times 10^{6}}{3.0 \times 10^{8}} \approx 0.027\,(2.7\%\,\mathrm{of}\,c).$$

Thus, the escape velocity is approximately 2.7% of the speed of light.

## 3. Discussion on the impact of high escape velocity

- **Electron Degeneracy:** The high escape velocity results from the intense gravitational field of the white dwarf. This field compresses matter to extremely high densities, supported by electron degeneracy pressure. This pressure balances gravity and prevents the white dwarf from collapsing further.

- **Accretion Disks:** Matter falling onto the white dwarf from a companion star forms an accretion disk. The high escape velocity means that the infalling material must lose significant gravitational potential energy, which is converted into heat and radiation, leading to bright emissions.

- **Type Ia Supernovae:** If the mass of the white dwarf approaches the Chandrasekhar limit ($\approx 1.4 M_{\odot}$), runaway nuclear fusion can occur, leading to a Type Ia supernova.

In summary, the high escape velocity profoundly affects the physics near the white dwarf, influencing both the behavior of accreting matter and the stability of the white dwarf itself.