

Solution to Problem 04: Orbital Resonances - Team - Blue Explorers

1. Derive the orbital period of the inner planet using Kepler's Third Law

Kepler's Third Law is given by:

$$T^2 = \frac{a^3}{M_{\odot}},$$

where:

- T is the orbital period in years,
- a is the semi-major axis in Astronomical Units (AU),
- M_{\odot} is the mass of the Sun (assumed to be 1 in solar mass units).

Given:

$$a = 0.8 \text{ AU}, M_{\odot} = 1,$$

we substitute into the equation:

$$T^2 = 0.8^3,$$

$$T^2 = 0.512,$$

$$T = \sqrt{0.512} \approx 0.716 \text{ years}.$$

Thus, the orbital period of the inner planet is approximately:

$$T_{\text{inner}} \approx 0.716 \text{ years}.$$

2. Calculate the semi-major axis and orbital period of the outer planet

The two planets are in a 3 : 2 resonance, meaning:

$$\frac{T_{\text{outer}}}{T_{\text{inner}}} = \frac{3}{2}.$$

The orbital period of the outer planet is:

$$T_{\text{outer}} = \frac{3}{2}T_{\text{inner}} = \frac{3}{2} \cdot 0.716 \approx 1.074 \text{ years}.$$

Using Kepler's Third Law to find the semi-major axis a_{outer} :

$$T_{\text{outer}}^2 = a_{\text{outer}}^3,$$

$$1.074^2 = a_{\text{outer}}^3,$$

$$1.153 = a_{\text{outer}}^3,$$

$$a_{\text{outer}} = \sqrt[3]{1.153} \approx 1.045 \text{ AU}.$$

Thus, the semi-major axis of the outer planet is approximately:

$$a_{\text{outer}} \approx 1.045 \text{ AU},$$

and its orbital period is approximately:

$$T_{\text{outer}} \approx 1.074 \text{ years}.$$

3. Discussion on the potential impact of resonance

- **Tidal Heating:** Similar to the Io-Europa-Ganymede system, a 3 : 2 resonance could lead to periodic gravitational interactions, causing tidal heating. This might enhance geological activity on the planets, potentially creating subsurface oceans or volcanic activity. However, excessive heating could render the environment uninhabitable.
- **Orbital Stability:** Resonances can stabilize orbits over long periods, but strong gravitational interactions might destabilize the system if additional perturbations occur, such as from a third planet.
- **Habitability:** Tidal heating and potential magnetic interactions could affect atmospheric retention, surface conditions, and the ability of the planets to support life. A balance between heating and cooling is crucial for habitability.

In summary, a 3 : 2 resonance introduces significant dynamics into the system, which might support or hinder habitability depending on the extent of tidal effects and orbital stability.