8.

Y \ X	1	2	3	4
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{4}$	0	0	0

The marginal distribution of X =

$$\left(\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{4},\ \frac{1}{16}+\frac{1}{8}+\frac{1}{16}+0,\ \frac{1}{32}+\frac{1}{32}+\frac{1}{16}+0,\ \frac{1}{32}+\frac{1}{32}+\frac{1}{16}+0\right)$$

$$=\left(\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{8}\right)$$

The marginal distribution of Y =

$$\left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}\right).$$

$$H(X) = \cancel{-10} - \sum_{\substack{\cancel{x \in f} \\ i=1}}^{4} P(x) \log P(x)$$

$$= -\frac{1}{2}\log\frac{1}{2} - \frac{1}{4}\log\frac{1}{4} - \frac{1}{8}\log\frac{1}{8} - \frac{1}{8}\log\frac{1}{8}$$

$$= -\frac{1}{2}(-1) - \frac{1}{4}(-2) - \frac{1}{8}(-3) - \frac{1}{8}(-3)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{3}{8}$$

$$= \frac{4+4+3+3}{8}$$

$$= \frac{14}{8}$$

$$= \frac{7}{4}$$

$$H(Y) = - \sum_{\substack{\cancel{x \in Y} \\ i=1}}^{4} P(x) \log P(x)$$

$$= -\frac{1}{4}\log\frac{1}{4} - \frac{1}{4}\log\frac{1}{4} - \frac{1}{4}\log\frac{1}{4} - \frac{1}{4}\log\frac{1}{4}$$

$$= -\frac{1}{4}(-2) - \frac{1}{4}(-2) - \frac{1}{4}(-2) - \frac{1}{4}(-2)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$= 2$$

Conditional Entropy:

$$H(X/Y) = \sum_{i=1}^{4} P(Y=i)\, H(X/Y=i)$$

$$= P(Y=1) \cdot H(X/Y=1) + P(Y=2) H(X/Y=2) + P(Y=3) H(X/Y=3) + P(Y=4) H(X/Y=4)$$

$$= \frac{1}{4} H\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right) + \frac{1}{4} H\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{8}\right) + \frac{1}{4} H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right) + \frac{1}{4} H(1,0,0,0)$$

$$= \frac{1}{4} \times \frac{7}{4} + \frac{1}{4} \times \frac{7}{4} + \frac{1}{4} \times 2 + \frac{1}{4} \times 0$$

$$= \frac{7}{16} + \frac{7}{16} + \frac{1}{2}$$

$$= \frac{7+7+8}{16}$$

$$= \frac{22}{16}$$

$$= \frac{11}{8}$$

$$H(Y/X) = \sum_{i=1}^{4} P(X=i)\, H(Y/X=i)$$

$$= P(X=1)\, H(Y/X=1) + P(X=2)\, H(Y/X=2)$$

$$+ P(X=3)\, H(Y/X=3) + P(X=4)\, H(Y/X=4)$$

$$= \frac{1}{2} H\left(\frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{2}\right) +$$

$$\frac{1}{4} H\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0\right) +$$

$$\frac{1}{8} H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0\right. +$$

$$\frac{1}{8} H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0\right)$$

$$= \frac{1}{2} \times \frac{7}{4} + \frac{1}{4} \times \frac{3}{2} + \frac{1}{8} \times \frac{3}{2} + \frac{1}{8} \times \frac{3}{2}$$

$$= \frac{7}{8} + \frac{3}{8} + \frac{3}{16} + \frac{3}{16}$$

$$= \frac{14+6+3+3}{16}$$

$$= \frac{26}{16}$$

$$= \frac{13}{8}$$

Joint Entropy:

$$H(x,y) = H(X) + H(Y/x)$$

$$= \frac{7}{4} + \frac{13}{8}$$

$$= \frac{14+13}{8}$$

$$= \frac{27}{8}$$

Mutual Information:

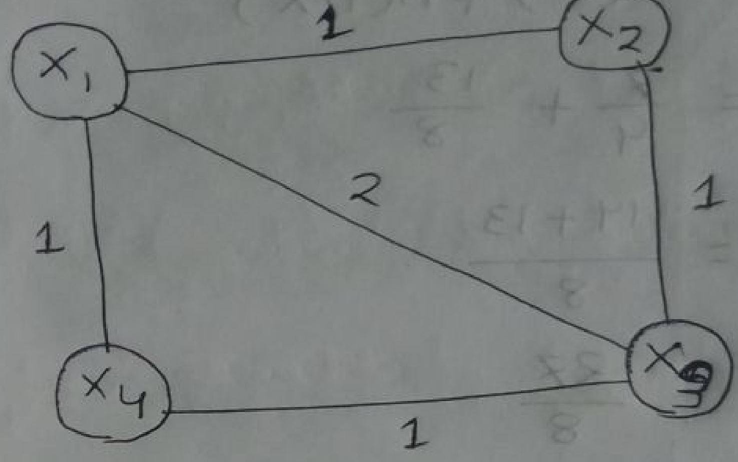
$$I(X;Y) = H(Y) - H(Y/x)$$

$$= 2 - \frac{13}{8}$$

$$= \frac{16-13}{8}$$

$$= \frac{3}{8}$$

7.



here, $i = x_1, x_2, x_3, x_4$

$W_i = W_1, W_2, W_3, W_4$

$$W_1 = \sum_j W_{ij}$$

$$= 1 + 1 + 2$$

$$= 4$$

$$W_2 = 1 + 1$$

$$= 2$$

$$W_3 = 1 + 1 + 2$$

$$= 4$$

$$W_4 = 1 + 1$$

$$= 2$$

$$\Sigma_i W_i = 2W$$

$$\Rightarrow W = \frac{\Sigma_i W_i}{2}$$

$$= \frac{W_1 + W_2 + W_3 + W_4}{2}$$

$$= \frac{4+2+4+2}{2}$$

$$\therefore W = 6$$

the stationary distribution is,

$$\mu_i = \frac{W_i}{2W}$$

$$= \frac{W_1, W_2, W_3, W_4}{2W}$$

$$= \frac{W_1}{2W}, \frac{W_2}{2W}, \frac{W_3}{2W}, \frac{W_4}{2W}$$

$$= \frac{4}{6\times 2}, \frac{2}{2\times 6}, \frac{4}{6\times 2}, \frac{2}{6\times 2}$$

$$= \frac{1}{3}, \frac{1}{6}, \frac{1}{3}, \frac{1}{6}$$

$$= \frac{1}{324} \quad 0.333, 0.1667, 0.333, 0.1667$$

$\cancel{= 0.00308}$

$$H\left(\frac{W_i}{2W}\right) = \mu_i$$

$$= (0.333, 0.16667, 0.333, 0.16667)$$

$$H\left(\frac{W_i}{2W}\right) = -0.333 \log_2 (0.333) + $$

$$-0.1667 \log_2 \cancel{(0.33} (0.1667) +$$

$$-0.333 \log_2 (0.333)$$

$$-0.1667 \log_2 (0.1667)$$

$$= 0.5282 + 0.43086 + 0.5282 + 0.43086$$

$$= 1.918$$

$$H\left(\frac{W_{ij}}{2W}\right) = \frac{W_1}{2W}, \frac{W_2}{2W}, \frac{W_3}{2W}, \frac{W_4}{2W}$$

$$= \left(\frac{0}{12}, \frac{1}{12}, \frac{2}{12}, \frac{1}{12}\right), \left(\frac{1}{12}, \frac{0}{12}, \frac{1}{12}, \frac{0}{12}\right.$$

$$\left(\frac{2}{12}, \frac{1}{12}, \frac{0}{12}, \frac{1}{12}\right), \left(\frac{1}{12}, \frac{0}{12}, \frac{1}{12}, \frac{0}{12}\right.$$

$= 0 \quad 0.0833 \quad 0.1667 \quad 0.0833 \quad 0.0833$

$0 \quad 0.0833 \quad 0 \quad 0.1667 \quad 0.0833$

$0 \quad 0.0833 \quad 0.0833 \quad 0 \quad 0.0833$

0

$= -0\log_2 0 - 0.0833\log_2(0.0833) - 0.1667\log_2(0.1667)$
$-0.0833\log_2(0.0833) - 0.0833\log_2(0.0833)$
$-0\log_2 0 - 0.0833\log_2(0.0833) - 0\log_2 0$
$-0.1667\log_2(0.1667) - 0.0833\log_2(0.0833)$
$-0\log_2 0 - 0.0833\log_2(0.0833) -$
$0.0833\log_2(0.0833) - 0\log_2 0 - 0.0833\log(0.0833)$
$-0\log_2 0$

$= 0.2986 + 0.43086 + 0.2986 + 0.2986 + 0.2986$
$+ 0.43086 + 0.2986 + 0.2986 + 0.2986 +$
0.2986

$= 3.251$

Entropy rate,

$$H(\chi) = H\left(\frac{W_{ij}}{2W}\right) - H\left(\frac{W_i}{2W}\right)$$

$$= 3.251 - 1.918$$

$$= 1.333$$