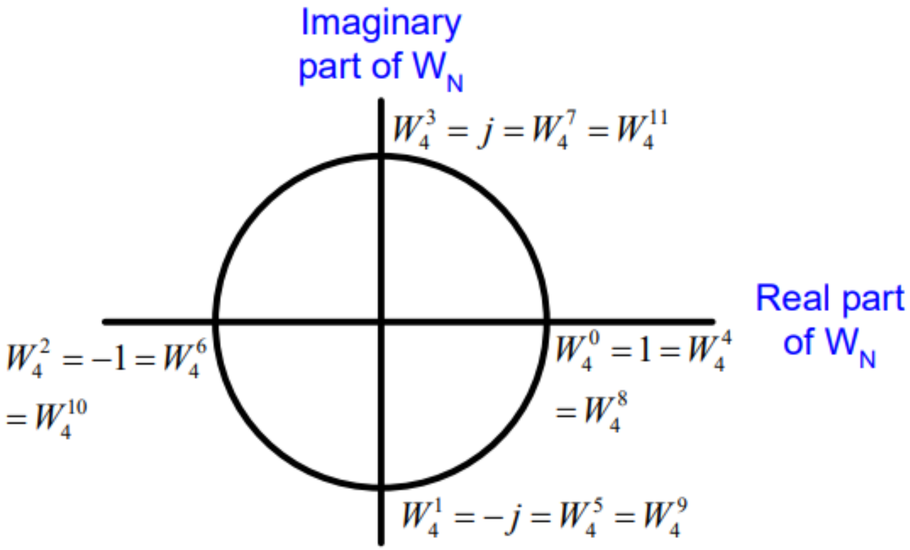
Examples of DFT

Numerical Example

Find DFT for a given a sequence x[0]=1, x[1]=2, x[2]=2, x[3]=1, x[n]=0 otherwise: x = [1,2,2,1]
Solution: x(n) = [1 2 2 1] for k=0,1,2,3
The DFT of the sequence x(n) = [1 2 2 1] is [6, − 1 − j1, 0, − 1 + j1]



The FT of discrete function, f(x) with N value is

$$F(u) = \sum_{x=0,1,\ldots,N-1} f(x) e^{-j2\pi u \frac{x}{N}}$$

$$X(0) = \sum_{n=0}^{3} x(n)e^{0} = 1e^{0} + 2e^{0} + 2e^{0} + 1e^{0} = [1+2+2+1] = 6$$

$$X(1) = \sum_{n=0}^{3} x(n)e^{-j2\pi n/4} = 1e^{0} + 2e^{-j\pi/2} + 2e^{-j\pi} + 1e^{-j3\pi/2}$$

$$\Rightarrow X(1) = [1 - j2 - 2 + j1] = [-1 - j1]$$

$$X(2) = \sum_{n=0}^{3} x(n)e^{-j4\pi n/4} = 1e^{0} + 2e^{-j\pi} + 2e^{-j2\pi} + 1e^{-j3\pi}$$

$$\Rightarrow X(2) = [1 - 2 + 2 - 1] = [0] = 0$$

$$X(3) = \sum_{n=0}^{3} x(n)e^{-j6\pi n/4} = 1e^{0} + 2e^{-j3\pi/2} + 2e^{-j3\pi} + 1e^{-j9\pi/2}$$

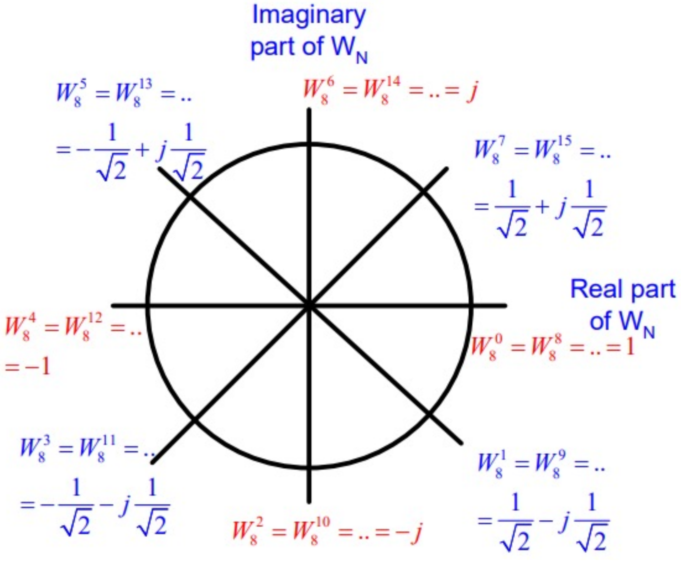
$$\Rightarrow X(3) = [1 + 2j - 2 - 1j] = [-1 + j1]$$

How to compute IDFT?

$$f(x) = \frac{1}{N} \sum_{u=0,1,\ldots,N-1} F(u) e^{j2\pi u \frac{x}{N}}$$

Numerical Example

kn	$W_8^{kn} = e^{-\frac{\pi}{4}kn}$	Result
0	$W_8^0 = e^0$	Magnitude 1 Phase 0
1	$W_8^1 = e^{-j\frac{\pi}{4}1} = e^{-j\frac{\pi}{4}}$	Magnitude 1 Phase $-\pi/4$
2	$W_8^2 = e^{-j\frac{\pi}{4}2} = e^{-j\frac{\pi}{2}}$	Magnitude 1 Phase $-\pi/2$
3	$W_8^3 = e^{-j\frac{\pi}{4}3} = e^{-j3\frac{\pi}{4}}$	Magnitude 1 Phase $-3\frac{\pi}{4}$
4	$W_8^4 = e^{-j\frac{\pi}{4}4} = e^{-j\pi}$	Magnitude 1 Phase $-\pi$
5	$W_8^5 = e^{-j\frac{\pi}{4}5} = e^{-j3\frac{\pi}{5}}$	Magnitude 1 Phase $-5\pi/4$
6	$W_8^6 = e^{-j\frac{\pi}{4}6} = e^{-j3\frac{\pi}{2}}$	Magnitude 1 Phase $-3\pi/2$
7	$W_8^7 = e^{-j\frac{\pi}{4}7} = e^{-j7\frac{\pi}{4}}$	Magnitude 1 Phase $-7\pi/4$
8	$W_8^8 = e^{-j\frac{\pi}{4}8} = e^{-j2\pi}$	Magnitude 1 Phase -2π $W_8^8 = W_8^0$
9	$W_8^9 = e^{-j\frac{\pi}{4}9} = e^{-j(2\pi+\frac{\pi}{4})}$	Magnitude 1 Phase $(-2\pi+\pi/4)$ $W_8^9 = W_8^1$
10	$W_8^{10} = e^{-j\frac{\pi}{4}10} = e^{-j(2\pi\frac{\pi}{2})}$	Magnitude 1 Phase $(-2\pi+\pi/2)$ $W_8^{10} = W_8^2$
11	$W_8^{11} = e^{-j\frac{\pi}{4}11} = e^{-j2\pi+\frac{3\pi}{4}}$	$W_8^{11} = W_8^3$



Numerical Example

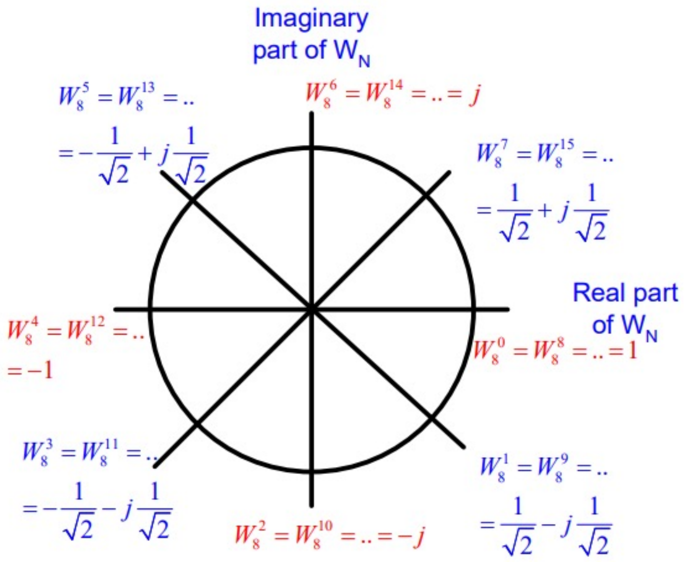
$The\ FT\ of\ discrete\ function,\ f(x)\ with\ N\ value\ is$

$$F(u) = \sum_{x=0,1,\ldots,N-1} f(x)e^{-i2\pi u\frac{x}{N}}$$

$Find\,8\,point\ DFT\ for\ x(n) = [1,1,1,1].\ Also\,calculate\,magnitude\,and\,phase$

$Solution:\ The\ 8\ point\ DFT\ is\ of\ length\ 8.\ Append\ zeros\ at\ the\ end$

$$\Rightarrow x(n) = [1\ 1\ 1\ 1\ 0\ 0\ 0\ 0]$$

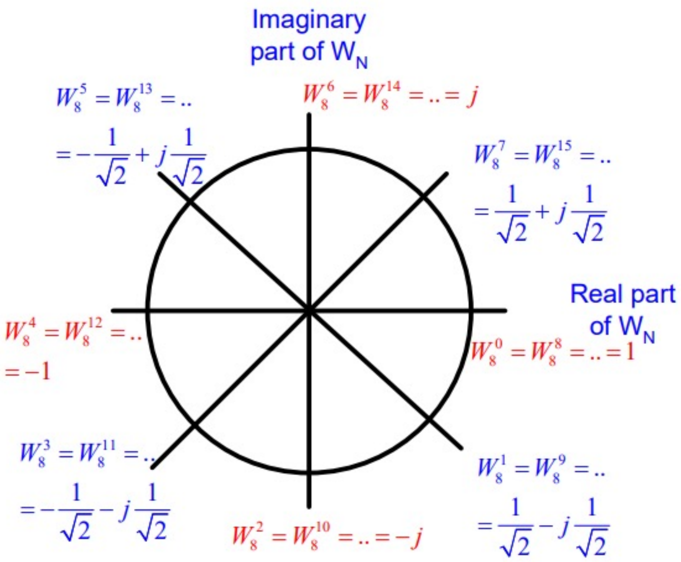


Numerical Example

$$
\begin{aligned}
W_8^0 &= W_8^8 = W_8^{16} = W_8^{24} = W_8^{40} \ldots = 1\\
W_8^1 &= W_8^9 = W_8^{17} = W_8^{25} = W_8^{33} \ldots = \tfrac{1}{\sqrt{2}} - j\tfrac{1}{\sqrt{2}}\\
W_8^2 &= W_8^{10} = W_8^{18} = W_8^{26} = W_8^{34} \ldots = -j\\
W_8^3 &= W_8^{11} = W_8^{19} = W_8^{27} = W_8^{35} \ldots = -\tfrac{1}{\sqrt{2}} - j\tfrac{1}{\sqrt{2}}\\
W_8^4 &= W_8^{12} = W_8^{20} = W_8^{28} = W_8^{36} \ldots = -1\\
W_8^5 &= W_8^{13} = W_8^{21} = W_8^{29} = W_8^{37} \ldots = -\tfrac{1}{\sqrt{2}} + j\tfrac{1}{\sqrt{2}}\\
W_8^6 &= W_8^{14} = W_8^{22} = W_8^{30} = W_8^{38} \ldots = j\\
W_8^7 &= W_8^{15} = W_8^{23} = W_8^{31} = W_8^{39} \ldots = \tfrac{1}{\sqrt{2}} + j\tfrac{1}{\sqrt{2}}
\end{aligned}
$$

$$
\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5)\\ X(6)\\ X(7) \end{bmatrix}
=
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\\
1 & -j & -1 & j & 1 & -j & -1 & j\\
1 & -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & j & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1\\
1 & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & j & -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\\
1 & j & -1 & -j & 1 & j & -1 & -j\\
1 & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & j & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -j & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}
\end{bmatrix}
=
\begin{bmatrix} 1\\ 1\\ 1\\ 1\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} 4\\ 1 - j(1+\sqrt{2})\\ 0\\ 1 + j(1-\sqrt{2})\\ 0\\ 1 - j(1-\sqrt{2})\\ 0\\ 1 + j(1+\sqrt{2}) \end{bmatrix}
=
\begin{bmatrix} X_R(0)\\ X_R(1)\\ X_R(2)\\ X_R(3)\\ X_R(4)\\ X_R(5)\\ X_R(6)\\ X_R(7) \end{bmatrix}
\begin{bmatrix} 4\\ 1\\ 0\\ 1\\ 0\\ 1\\ 0\\ 1 \end{bmatrix}
\ \textit{and}\
\begin{bmatrix} X_I(0)\\ X_I(1)\\ X_I(2)\\ X_I(3)\\ X_I(4)\\ X_I(5)\\ X_I(6)\\ X_I(7) \end{bmatrix}
\begin{bmatrix} 0\\ -(1+\sqrt{2})\\ 0\\ (1-\sqrt{2})\\ 0\\ -(1-\sqrt{2})\\ 0\\ (1+\sqrt{2}) \end{bmatrix}
$$



Numerical Example

$Find\ 4\ point\ DFT\ for\ continuous\ function,$
$f(t) = 5 + 2cos(2\pi t - 90°) + 3cos(4\pi t)$