Auto Correlation

Auto correlation (Correlation with itself)

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$$E[u,v] = \sum_{x,y} w(x,y)\left[I(x+u,\, y+v) - I(x,y)\right]^2$$

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What are u and v?

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What are u and v?

Change / shift in x

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What are u and v?

u: Change / shift in x

v: Change / shift in y

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$$E[u,v] = \sum_{x,y} w(x,y)\left[I(x+u,\, y+v) - I(x,y)\right]^2$$

$u \rightarrow \Delta x, \quad v \rightarrow \Delta y$

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Quick Detour

Taylor Expansion

$$I(x+\Delta x, y+\Delta y)$$

$$\approx I(x,y) + I_x(x,y)\,\Delta x + I_y(x,y)\,\Delta y$$

$$+ \text{ higher order terms}$$

Quick Detour

Taylor Expansion

$$I(x+\Delta x, y+\Delta y)$$

$$\simeq I(x,y) + \underline{I_x}(x,y)\Delta x + \underline{I_y}(x,y)\Delta y$$

Partial Derivatives
in x and y directions.

Auto correlation (Correlation with itself)

$$E[\Delta x, \Delta y] = \sum_{x,y} w(x,y)\left[I(x+\Delta x,\, y+\Delta y) - I(x,y)\right]^2$$

Auto correlation (Correlation with itself)

$$E[\Delta x, \Delta y] = \sum_{x,y} w(x,y) \left[I(x+\Delta x, y+\Delta y) - I(x,y) \right]^2$$

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$$\simeq I(x,y) + I_x(x,y)\Delta x + I_y(x,y)\Delta y$$

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$$\approx \sum_{x,y} w(x,y)\left[I(x,y) + I_x(x,y)\Delta x + I_y(x,y)\Delta y - I(x,y)\right]^2$$

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$$= \sum_{x,y} w(x,y)\left[I_x(x,y)\Delta x + I_y(x,y)\Delta y\right]^2$$

$$= \sum_{x,y} w(x,y)\left[(\Delta x)^2 I_x^2(x,y) + (\Delta y)^2 I_y^2(x,y) + 2\Delta x \Delta y I_x(x,y) I_y(x,y)\right]$$

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$$= \sum_{x,y} w(x,y) \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \begin{bmatrix} I_x^2(x,y) & I_x(x,y)\, I_y(x,y) \\ I_x(x,y)\, I_y(x,y) & I_y^2(x,y) \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

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→ independent of x, y

Auto correlation (Correlation with itself)

$$E[\Delta x, \Delta y]$$

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Auto correlation (Correlation with itself)

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$$= [\Delta x \quad \Delta y] \underbrace{\sum_{x,y} w(x,y) \begin{bmatrix} I_x^2(x,y) & I_x(x,y) I_y(x,y) \\ I_x(x,y) I_y(x,y) & I_y^2(x,y) \end{bmatrix}}_{M} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$E[u,v] = [u \ v] \, M \begin{bmatrix} u \\ v \end{bmatrix}, \quad M = \sum_{x,y} \begin{bmatrix} I_x^2 W & I_x I_y W \\ I_x I_y W & I_x^2 W \end{bmatrix}$$

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Eigen Decomposition

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

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λ_1, λ_2 are eigen values

Rows of R are eigen vectors.

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Cornerness Value, $C = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$

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Cornerness Value, $C = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$

if this value is higher, λ_1 and λ_2 are large

$$E[u,v] = [u\ v]\, M \begin{bmatrix} u \\ v \end{bmatrix}$$

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$$E[u,v] = [u \; v] \, M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = R^{-1} \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}_{A} R$$

$$C = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

$$= \det(A) - \alpha \operatorname{trace}(A)$$

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We still have to compute eigen values.

$$E[u,v] = [u\ v]\, M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = R^{-1} \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}_{A} R$$

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We still have to compute eigen values.

$\hookrightarrow$ Costly operation.

How to reduce time complexity?

Again Detour.

$$M = R^{-1} A R$$

Again Detour.

$M = R^{-1} A R$

- $\det(M) = \det(R^{-1} A R)$

$= \det(R^{-1}) \det(A) \det(R)$

Again Detour.

$$M = R^{-1} A R$$

- $\det(M) = \det(R^{-1} A R)$

$$= \det(R^{-1}) \det(A) \det(R)$$

$$= \frac{1}{\det(R)} \det(A) \det(R)$$

Again Detour. $M = R^{-1} A R$

- $\det(M) = \det(R^{-1} A R)$

$$= \det(R^{-1}) \det(A) \det(R)$$

$$= \frac{1}{\det(R)} \det(A) \det(R)$$

$$= \det(A)$$

Again Detour. $M = R^{-1} A R$

- $\det(M) = \det(A)$
- $\operatorname{trace}(M) = ?$

Again Detour. $\quad M = R^{-1} A R$

- $\det(M) = \det(A)$
- $\text{trace}(M) = ?$

↳ Detour again (Detour Inception
↳ Deception)

B, C are two square matrices

$BC_{ij} = \sum_{k=1}^{n} B_{ik} C_{kj}$

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$CB_{ij} = \sum_{k=1}^{n} C_{ik} B_{kj}$

B, C are two square matrices

\# $BC_{ij} = \sum_{k=1}^{n} B_{ik} C_{kj}$

\# $CB_{ij} = \sum_{k=1}^{n} C_{ik} B_{kj}$

\# $\text{trace}(BC) = \sum_{i=1}^{n} BC_{ii}$

B, C are two square matrices

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$CB_{ij} = \sum_{k=1}^{n} C_{ik} B_{kj}$

$\text{trace}(BC) = \sum_{i=1}^{n} BC_{ii} = \sum_{i=1}^{n} \sum_{k=1}^{n} B_{ik} C_{ki}$

B, C are two square matrices

$BC_{ij} = \sum_{k=1}^{n} B_{ik} C_{kj}$

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$\text{trace}(BC) = \sum_{i=1}^{n} BC_{ii} = \sum_{i=1}^{n} \sum_{k=1}^{n} B_{ik} C_{ki}$

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$\text{trace}(CB) = \sum_{i=1}^{n} CB_{ii} = \sum_{i=1}^{n} \sum_{k=1}^{n} C_{ik} B_{ki}$

$\therefore \text{trace}(BC) = \text{trace}(CB)$

Again Detour.

$$M = R^{-1} A R$$

- $\det(M) = \det(A)$

- $$\begin{aligned} \operatorname{trace}(M) &= \operatorname{trace}(R^{-1} A R) \\ &= \operatorname{trace}(R^{-1}(AR)) \\ &= \operatorname{trace}((AR)R^{-1}) \\ &= \operatorname{trace}(A R R^{-1}) \\ &= \operatorname{trace}(A) \end{aligned}$$

$$E[u,v] = [u\ v]\, M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = R^{-1} \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}_{A} R$$

$$C = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

$$= \det(A) - \alpha \operatorname{trace}(A)$$

$$= \det(M) - \alpha \operatorname{trace}(M)$$

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$$= \det(M) - \alpha \ \mathrm{trace}(M)$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2(x,y) & I_x(x,y)\, I_y(x,y) \\ I_x(x,y)\, I_y(x,y) & I_y^2(x,y) \end{bmatrix}$$