Feedforward Neural Networks

Deep Learning
CS 435/635

Course Instructor: Chandresh
AI Lab Coordinator @IIT Indore

Course Content

Module I: History of Deep Learning, Sigmoid Neurons, Perceptrons, and learning algorithms. Multilayer Perceptrons (MLPs), Representation Power of MLPs,.

Module II: **Feedforward Neural Networks**. Backpropagation. first and second-order training methods. NN Training tricks, Regularization

Module III: Introduction to Autoencoders and their characteristics, relation to PCA, Regularization in autoencoders, and Types of autoencoders.

Module IV: Architecture of Convolutional Neural Networks (CNN), types of CNNs.

Module V: Architecture of Recurrent Neural Networks (RNN), Backpropagation through time. Encoder-Decoder Models, Attention Mechanism. Advanced Topics: Transformers and BERT.

Acknowledgement

- Neural Networks and Deep Learning course by Danna Gurari University of Colorado Boulder

Today’s Topics

- Motivation for neural networks: need non-linear models
- Neural network architecture: hidden layers
- Neural network architecture: activation functions
- Neural network architecture: output units
- Programming tutorial

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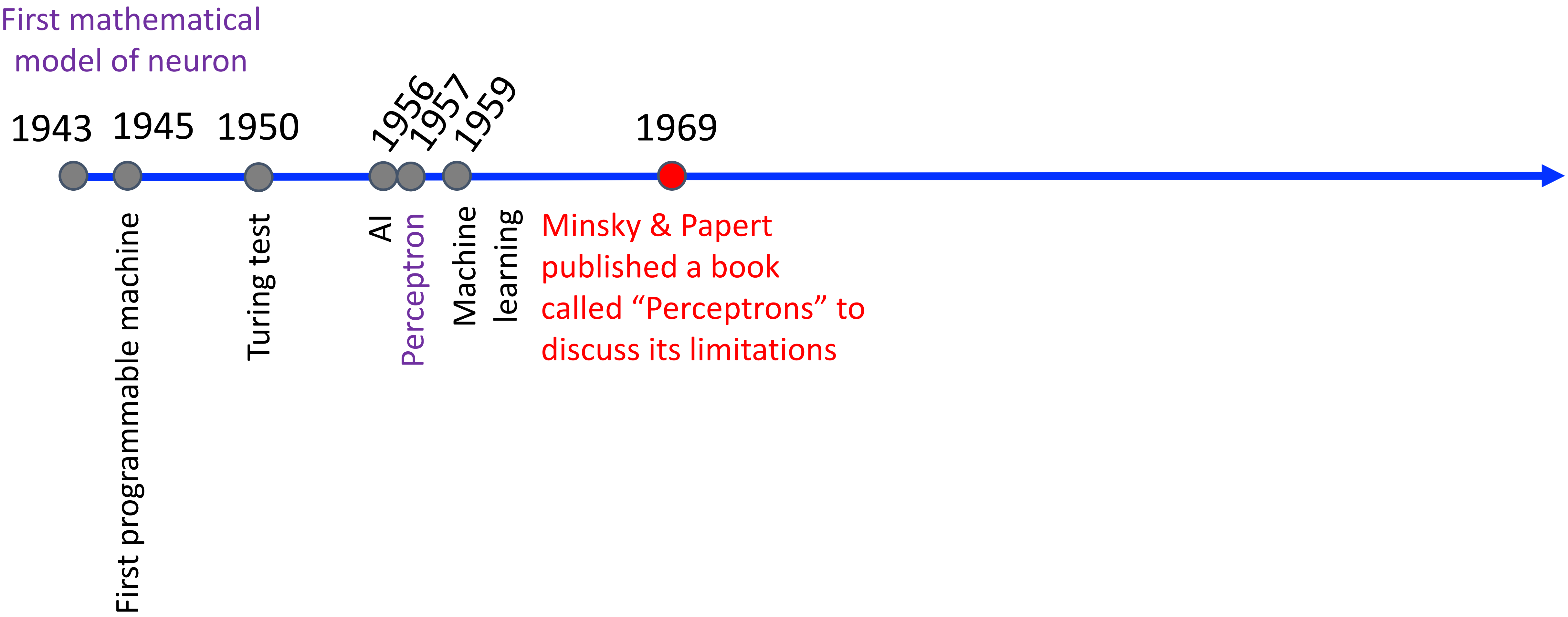
Historical Context: Artificial Neurons



Recall: Vision for Perceptron

Frank Rosenblatt
(Psychologist)

"[The perceptron is] the embryo of an electronic computer that [the Navy] expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence…. [It] is expected to be finished in about a year at a cost of $100,000."

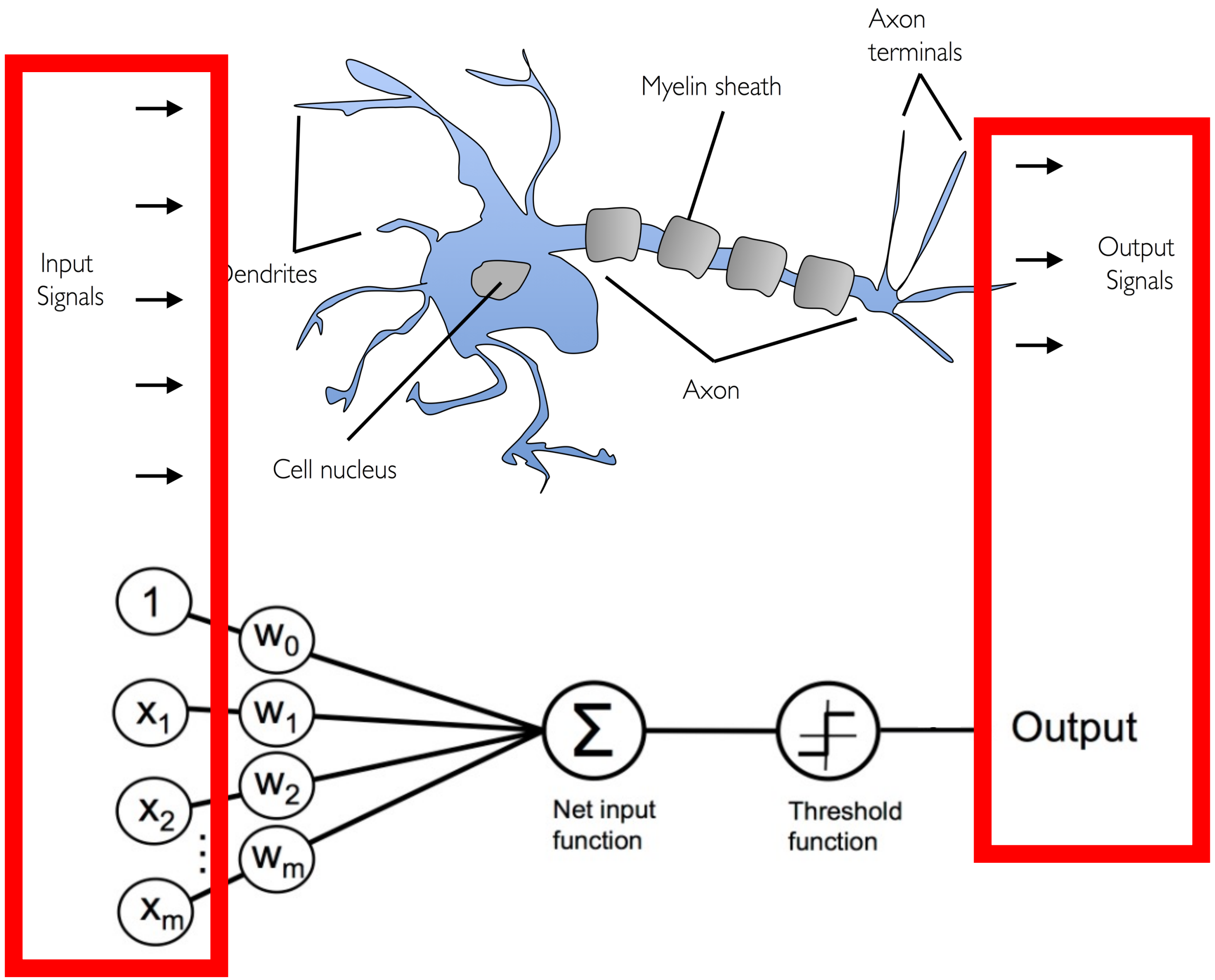
1958 New York Times article: https://www.nytimes.com/1958/07/08/archives/new-navy-device-learns-by-doing-psychologist-shows-embryo-of.html

https://en.wikipedia.org/wiki/Frank_Rosenblatt

Recall: Perceptron

Biological Neuron:

Artificial Neurons
(e.g., Perceptron):



Python Machine Learning; Raschka & Mirjalili
https://github.com/rasbt/python-machine-learning-book-2nd-edition/blob/master/code/ch02/ch02.ipynb

Perceptron Limitation: XOR Problem

XOR = "Exclusive Or"

- Input: two binary values x_1 and x_2
- Output:
  - 1, when exactly one input equals 1
  - 0, otherwise

x_1	x_2	x_1 XOR x_2
0	0	?
0	1	?
1	0	?
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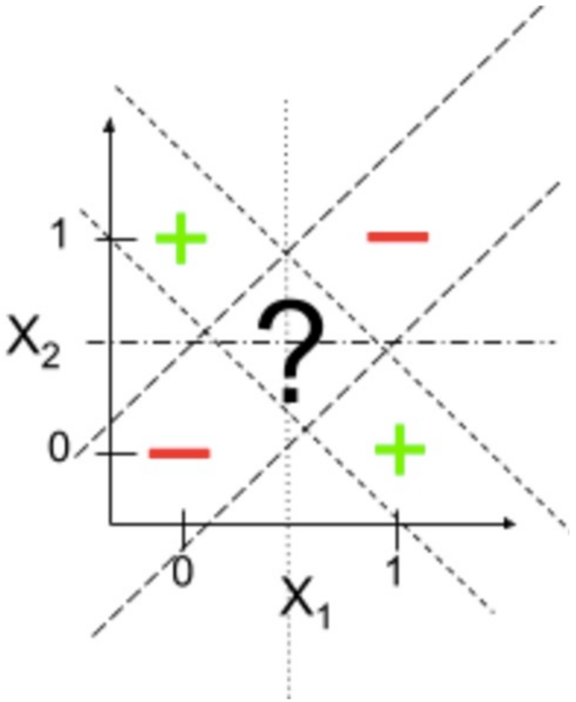
Perceptron Limitation: XOR Problem

Cannot solve XOR problem and so separate 1s from 0s with a perceptron (linear function):



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How can a machine be "conscious" when it can't solve the XOR problem?

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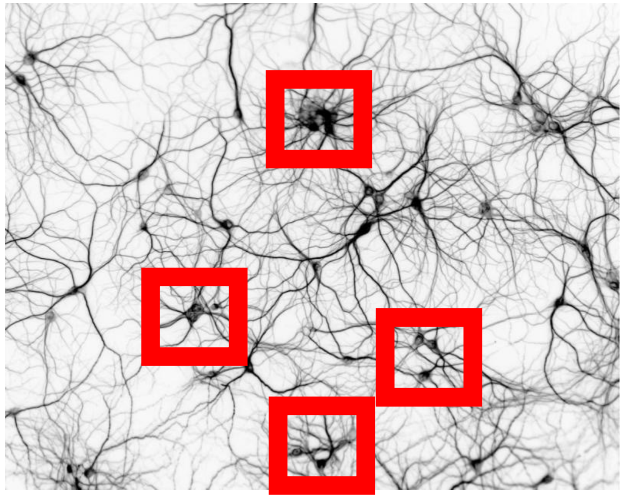
- Neural network architecture: hidden layers
- Neural network architecture: activation functions
- Neural network architecture: output units
- Programming tutorial

Neural Networks: Connected Neurons

Biological Neural Network:



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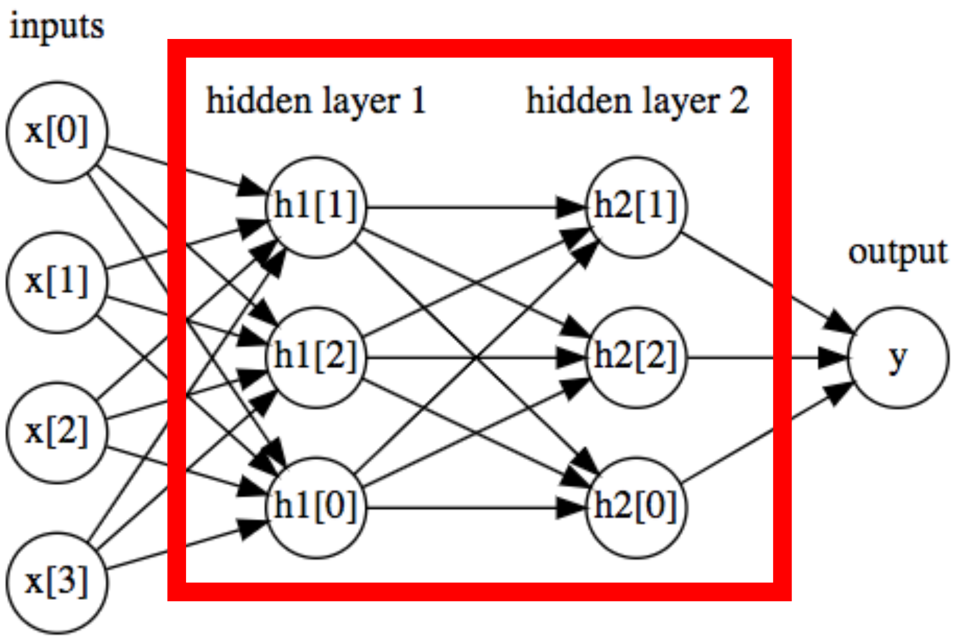
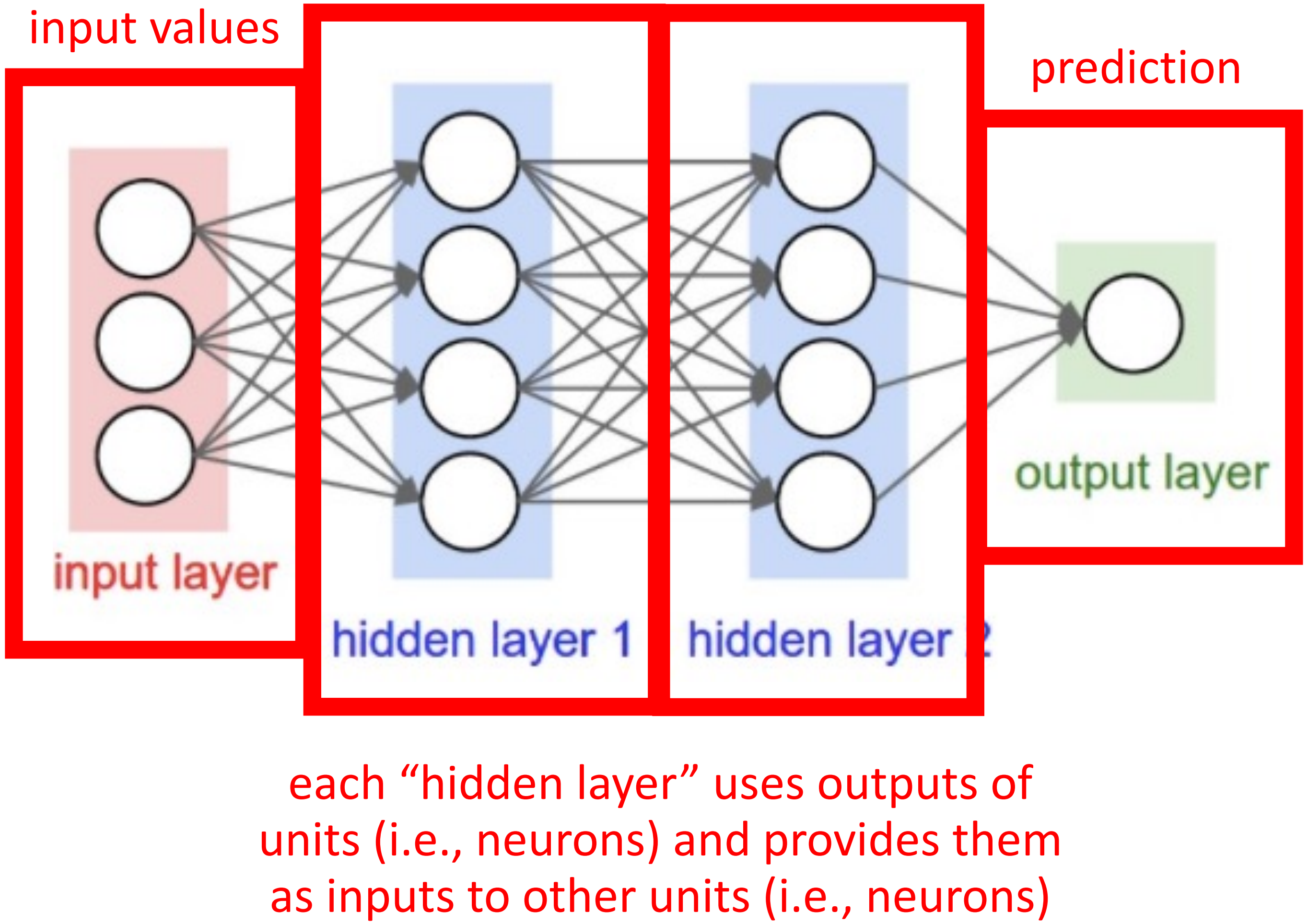


https://github.com/amueller/introduction_to_ml_with_python/blob/master/02-supervised-learning.ipynb

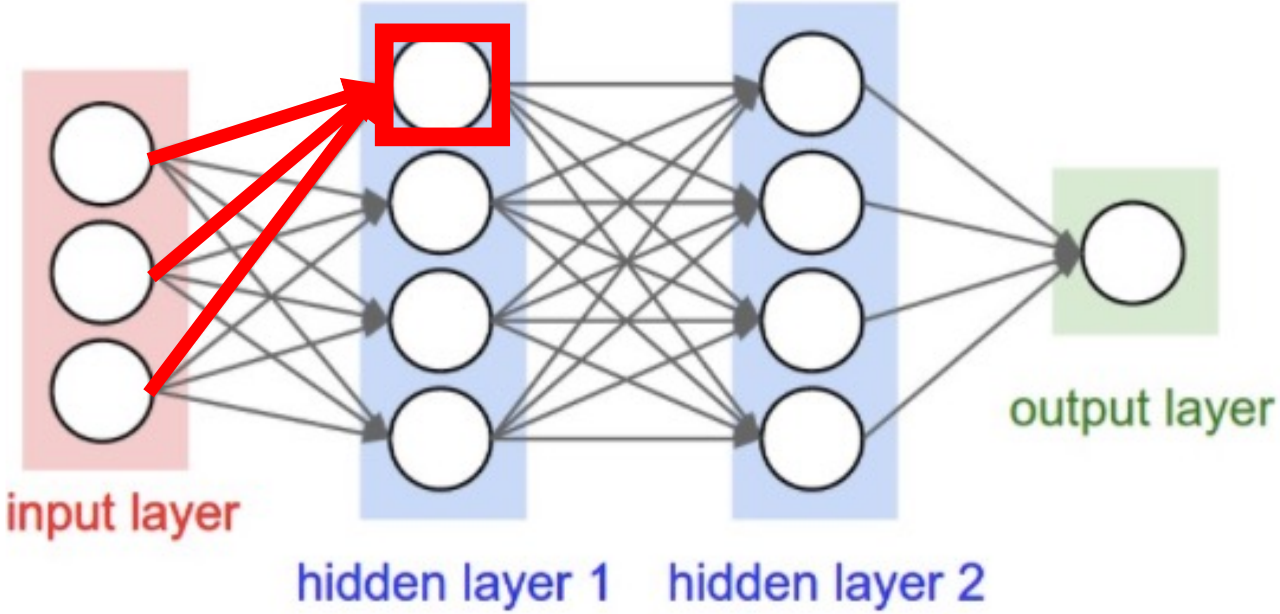
Neural Network



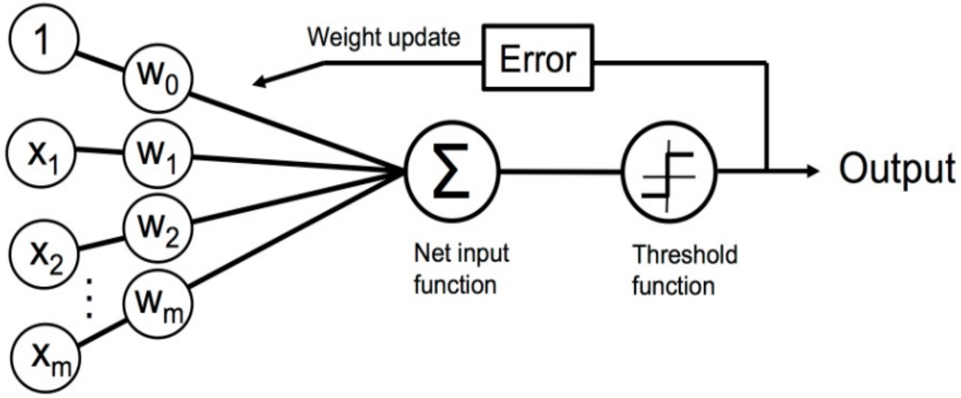
This is a 3-layer neural network (i.e., count number of hidden layers plus output layer)

http://cs231n.github.io/neural-networks-1/

Neural Network



- How does this relate to a perceptron?



- Unit: takes as input a weighted sum and applies an activation function

Python Machine Learning; Raschka & Mirjalili

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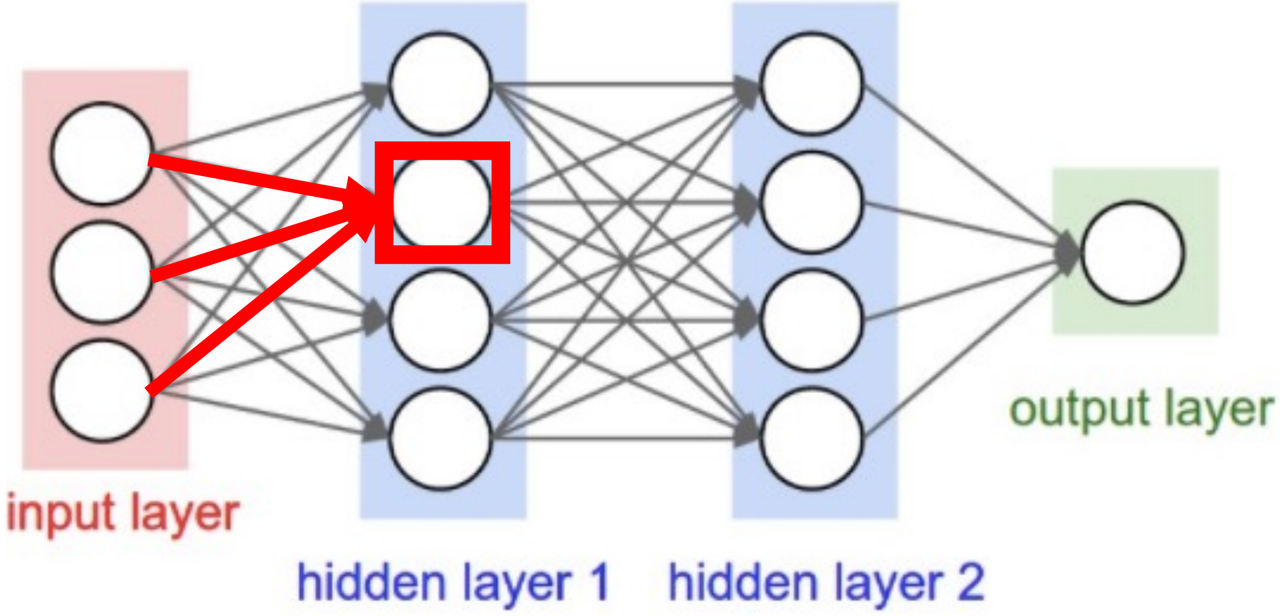

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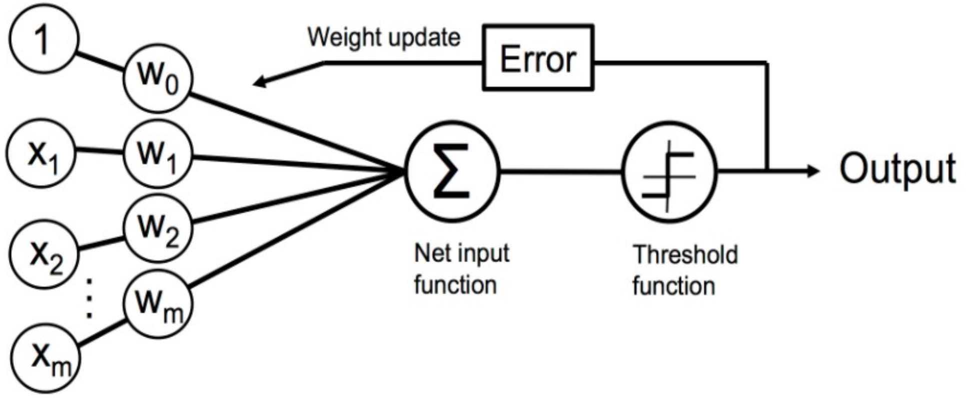

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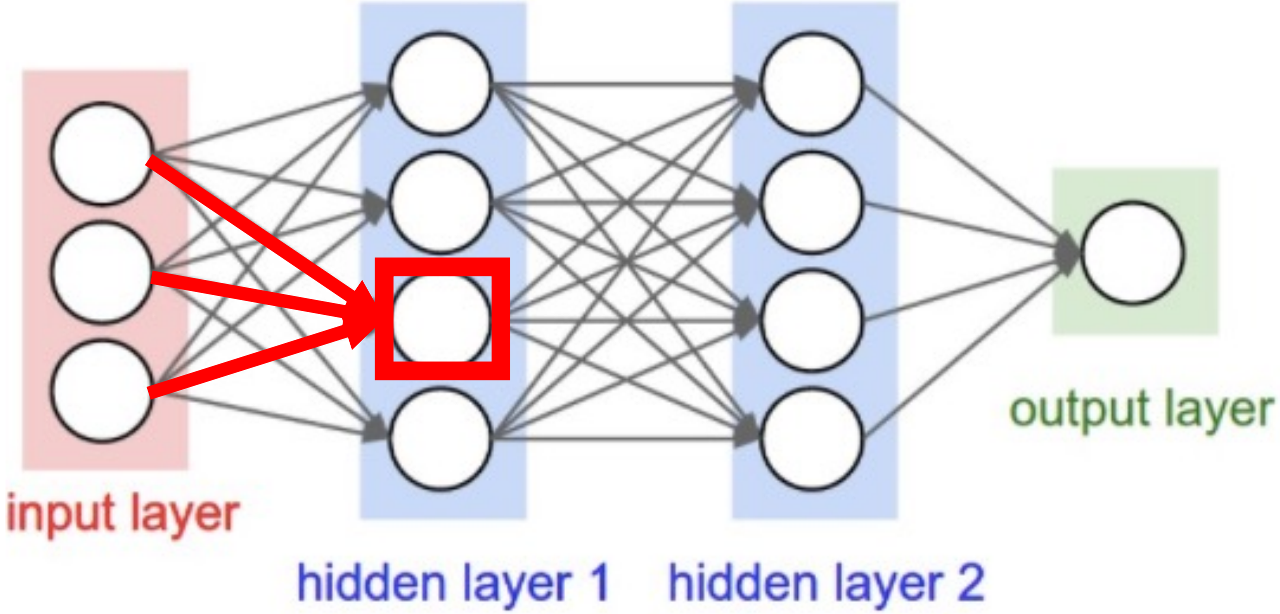

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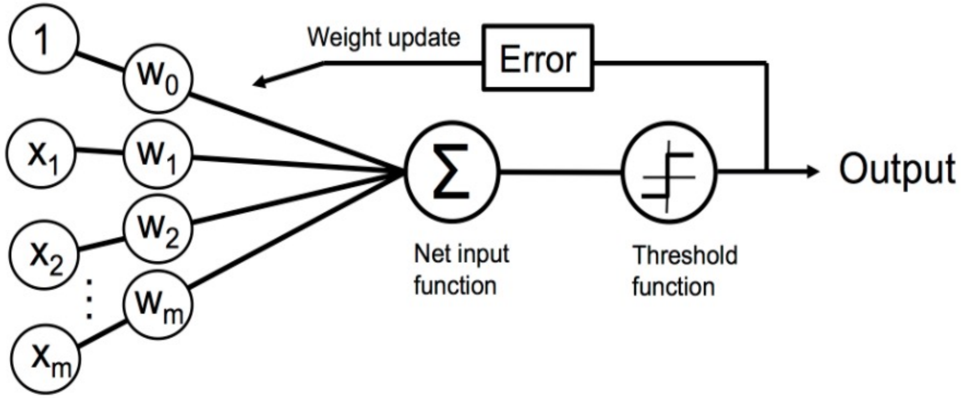

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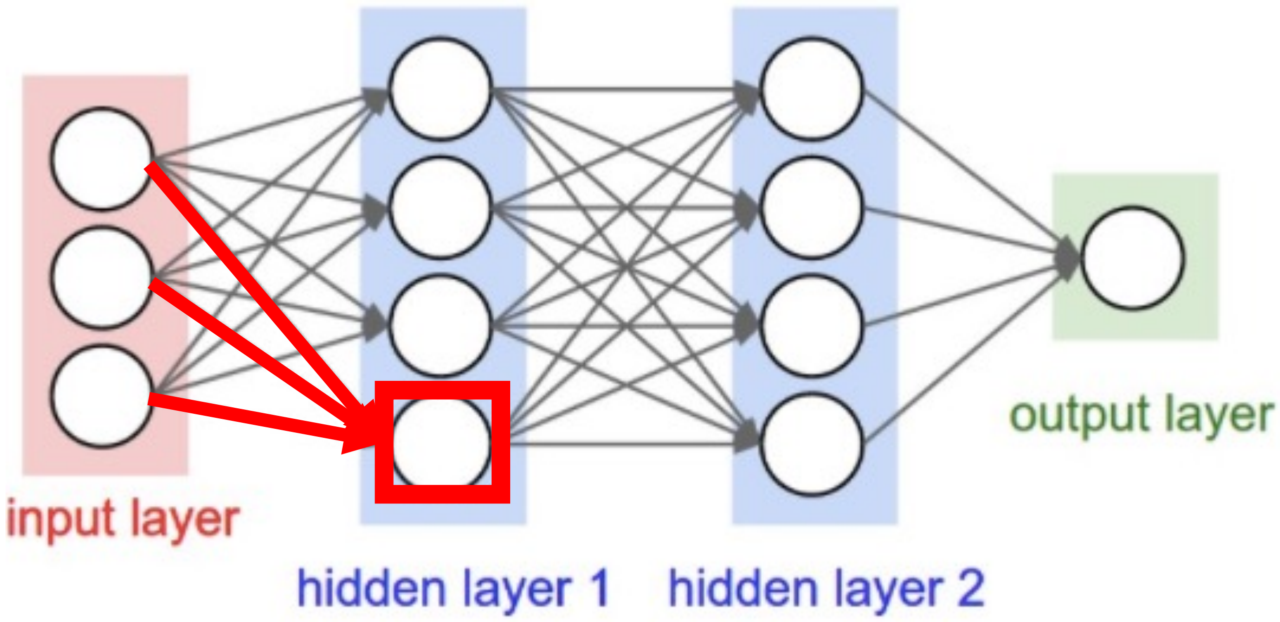

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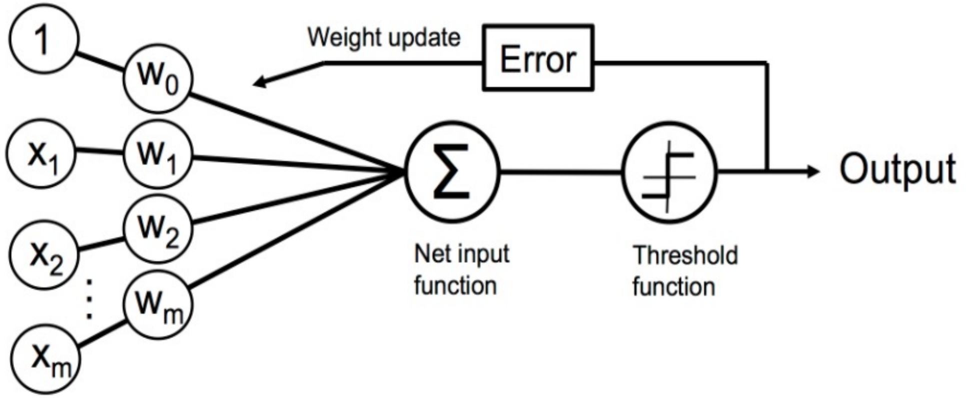

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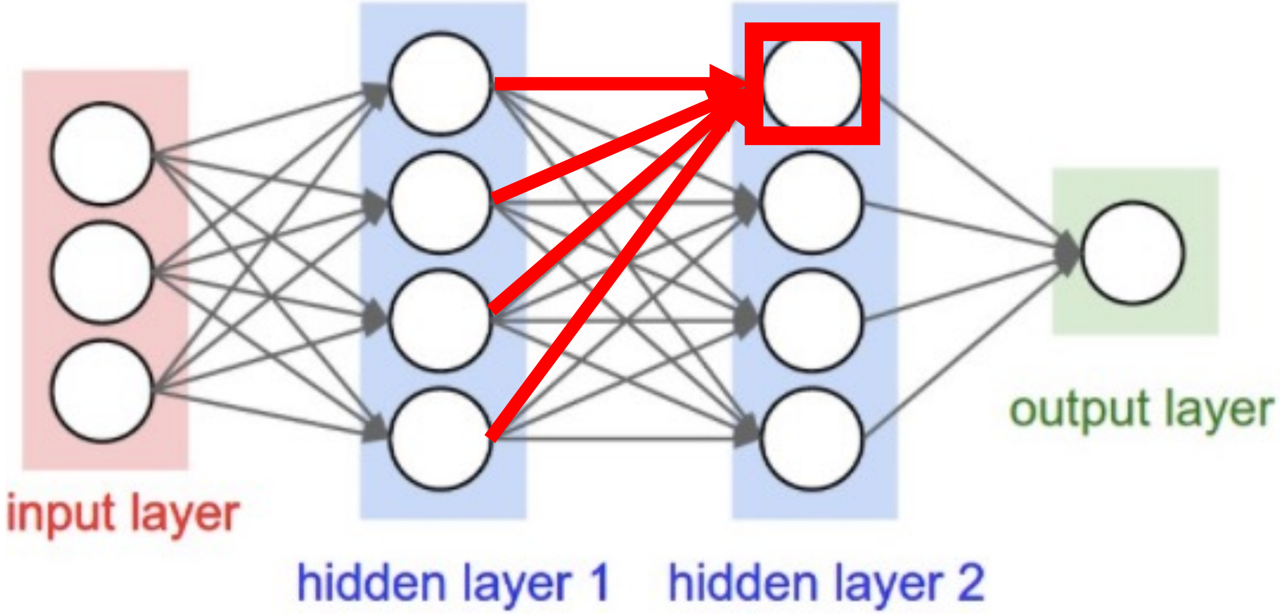

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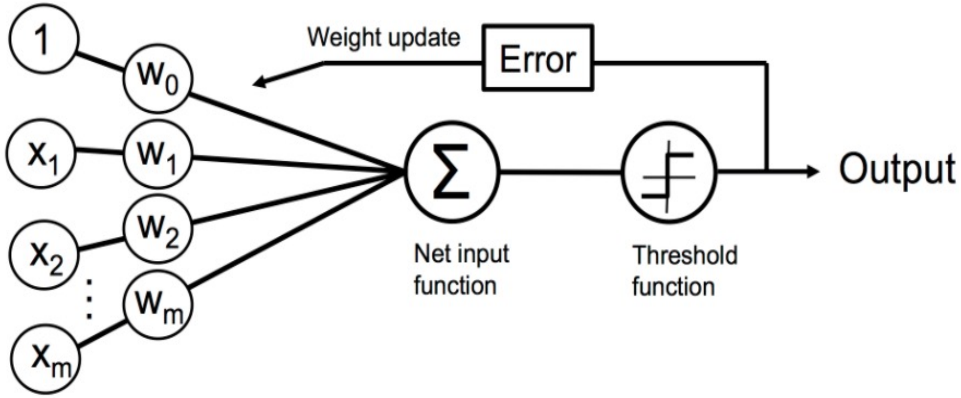

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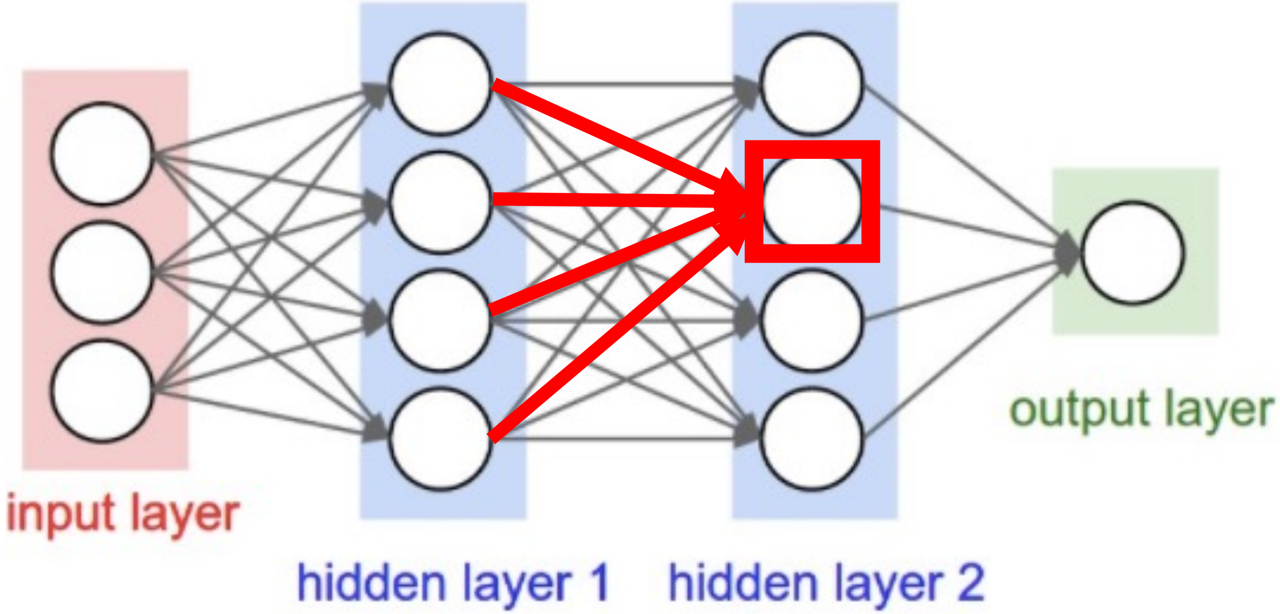

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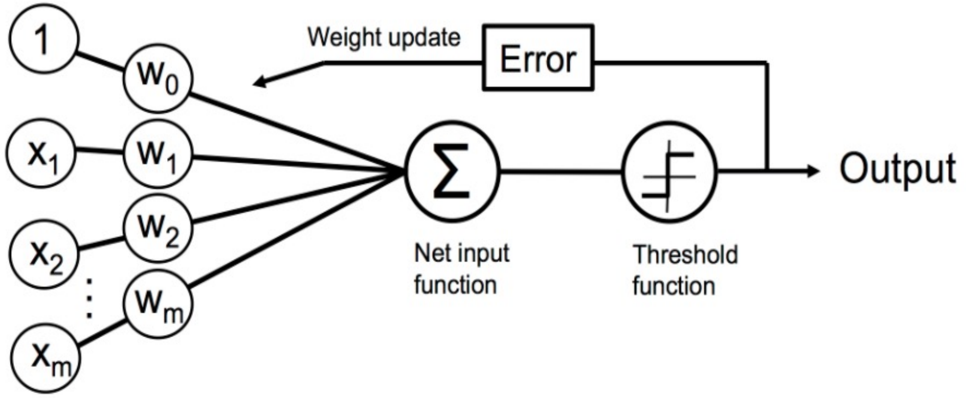

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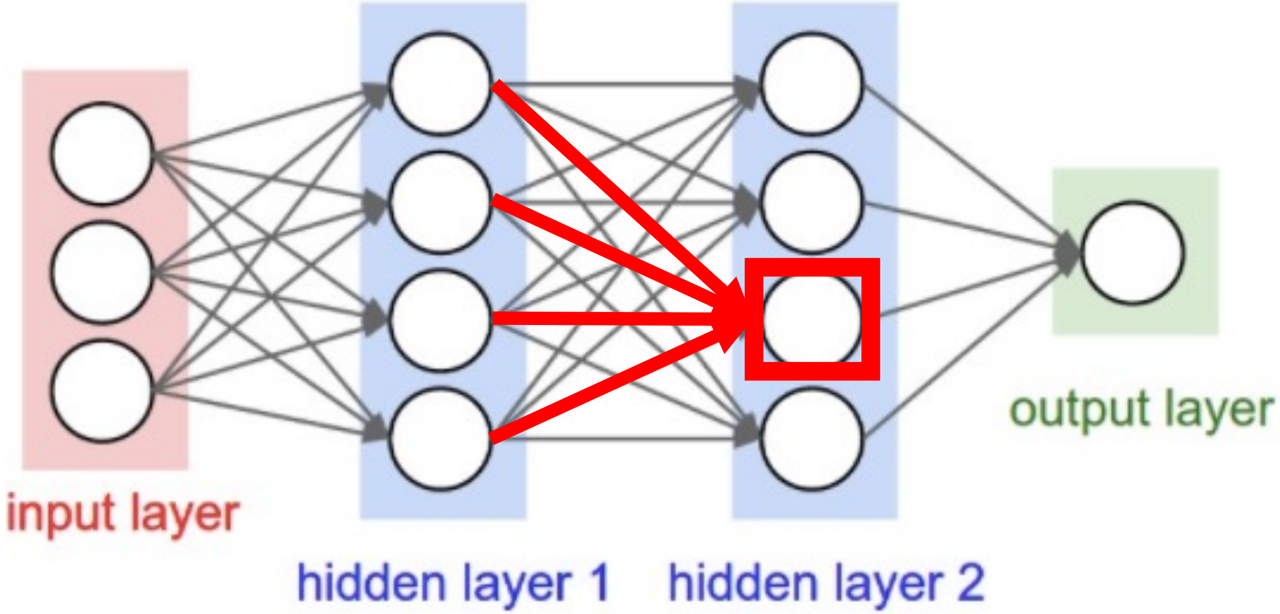

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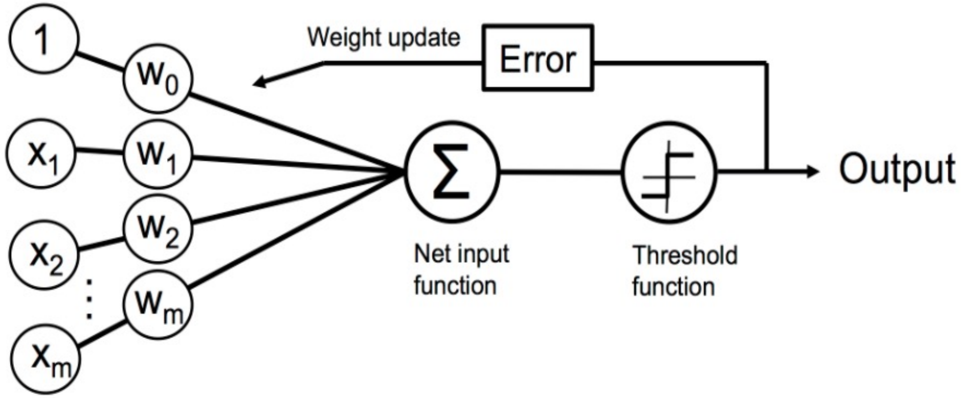

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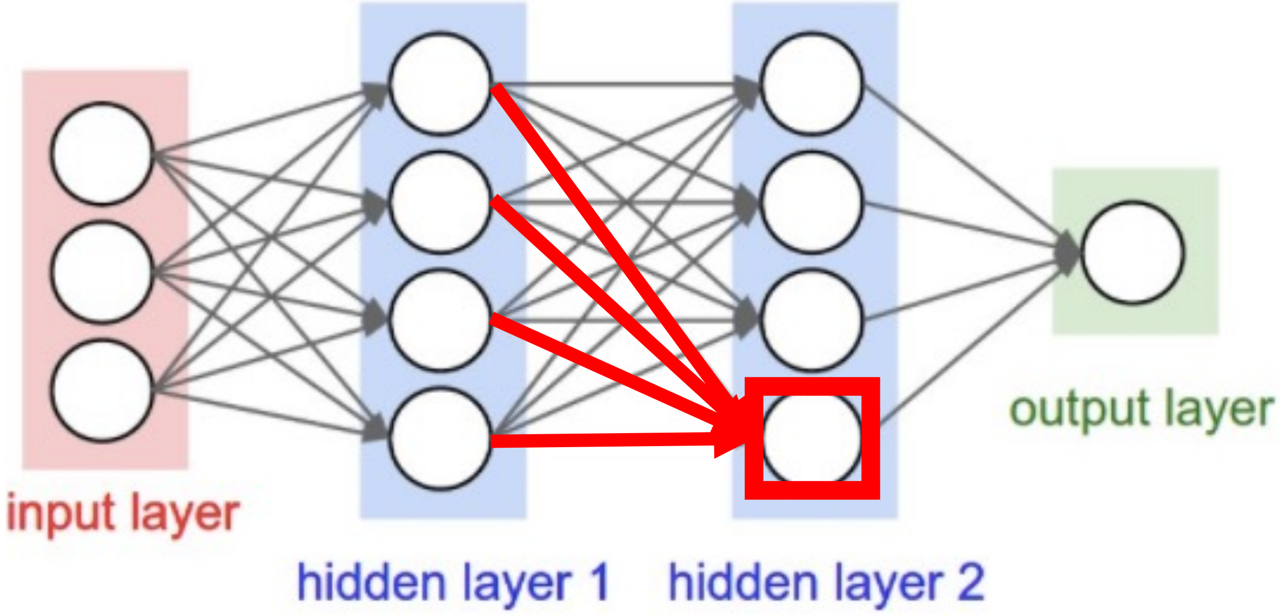

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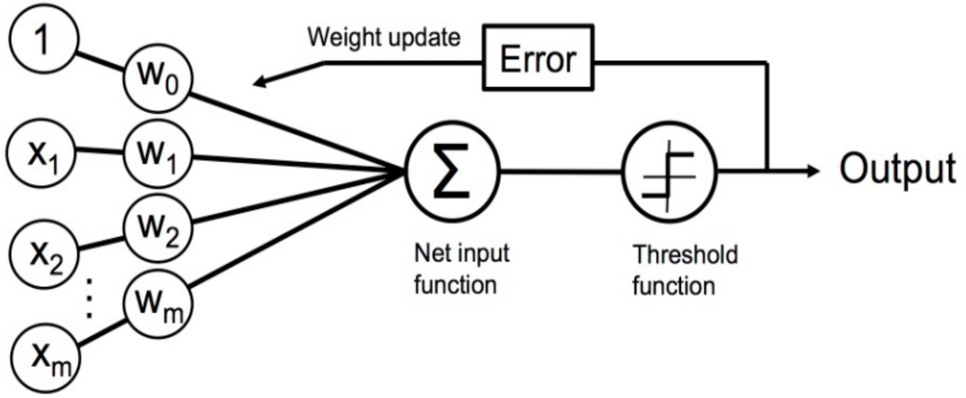

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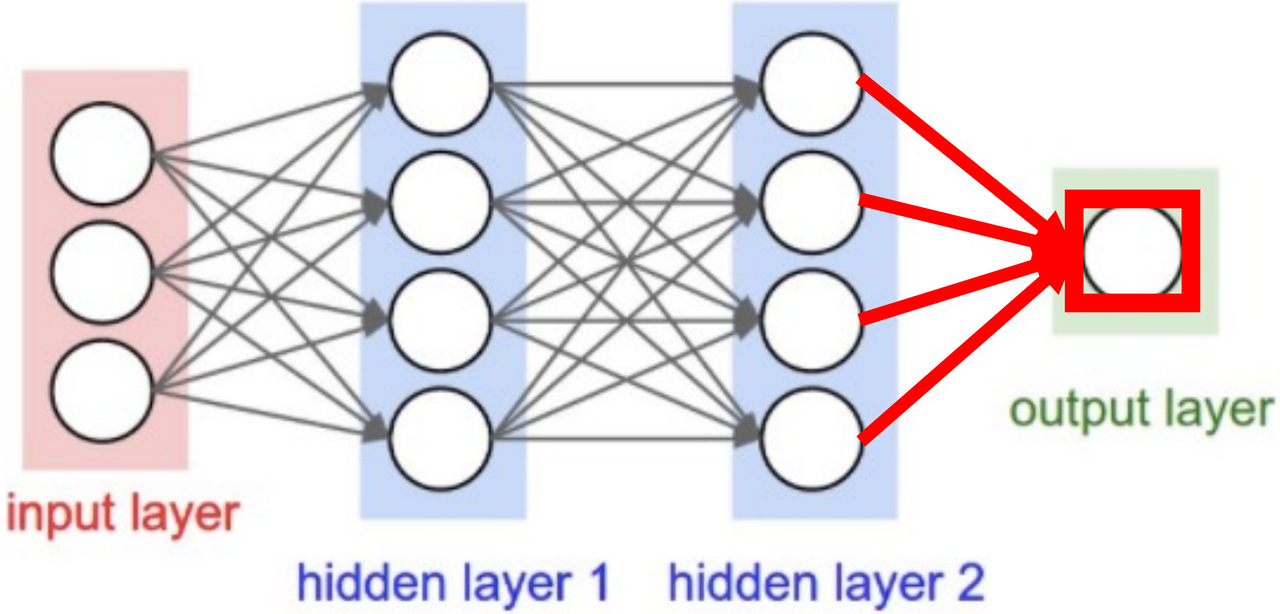

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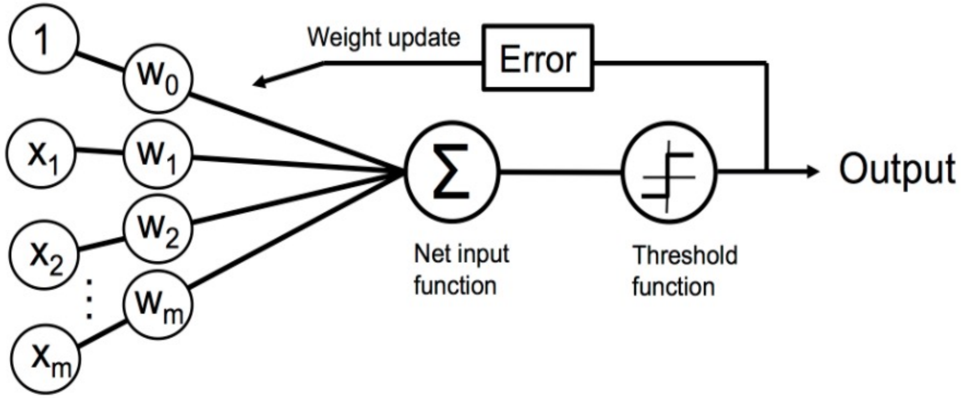

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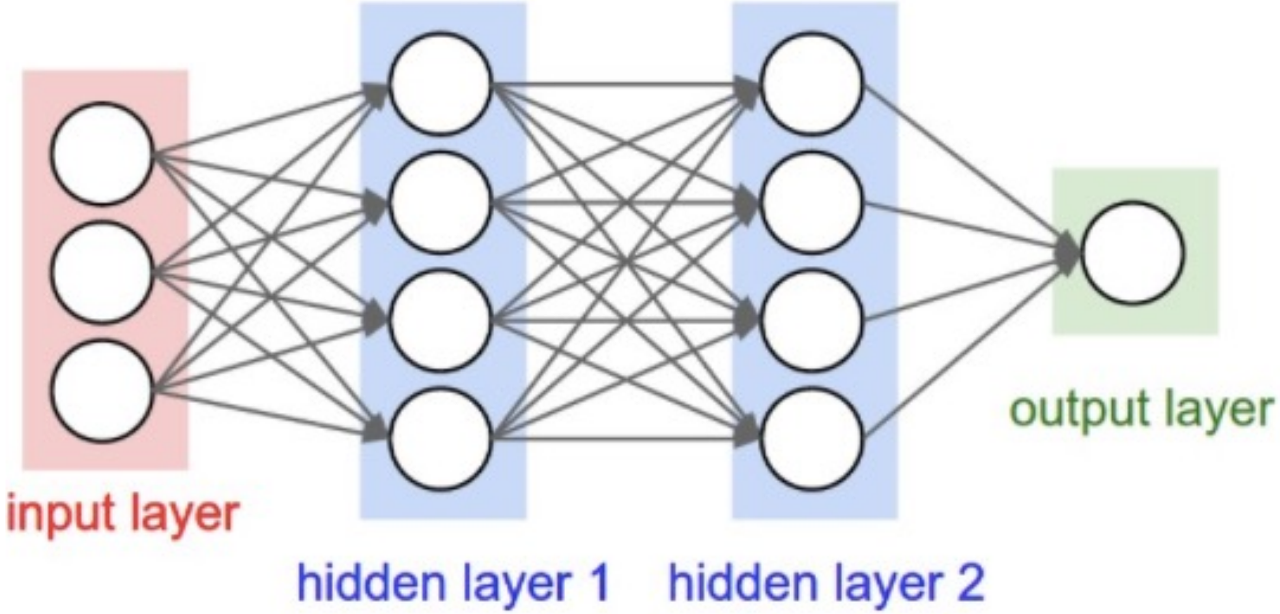
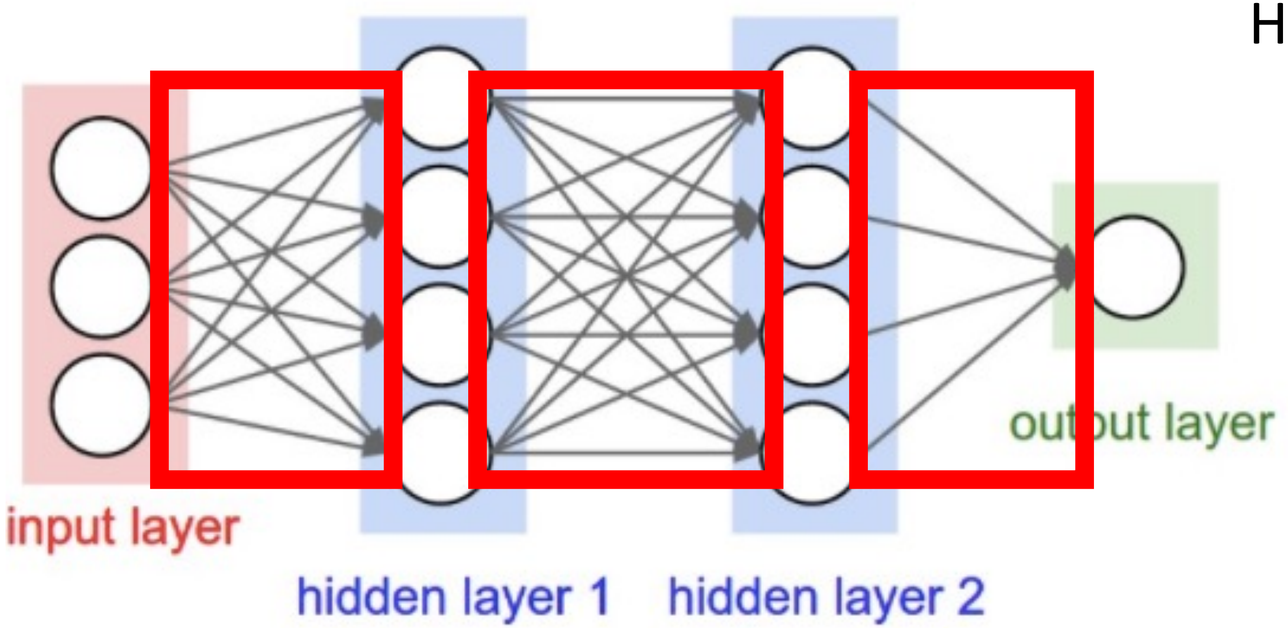


- **Training goal: learn model parameters**
- Layers are called “hidden” because algorithm decides how to use each layer to produce its output

http://cs231n.github.io/neural-networks-1/

Neural Network

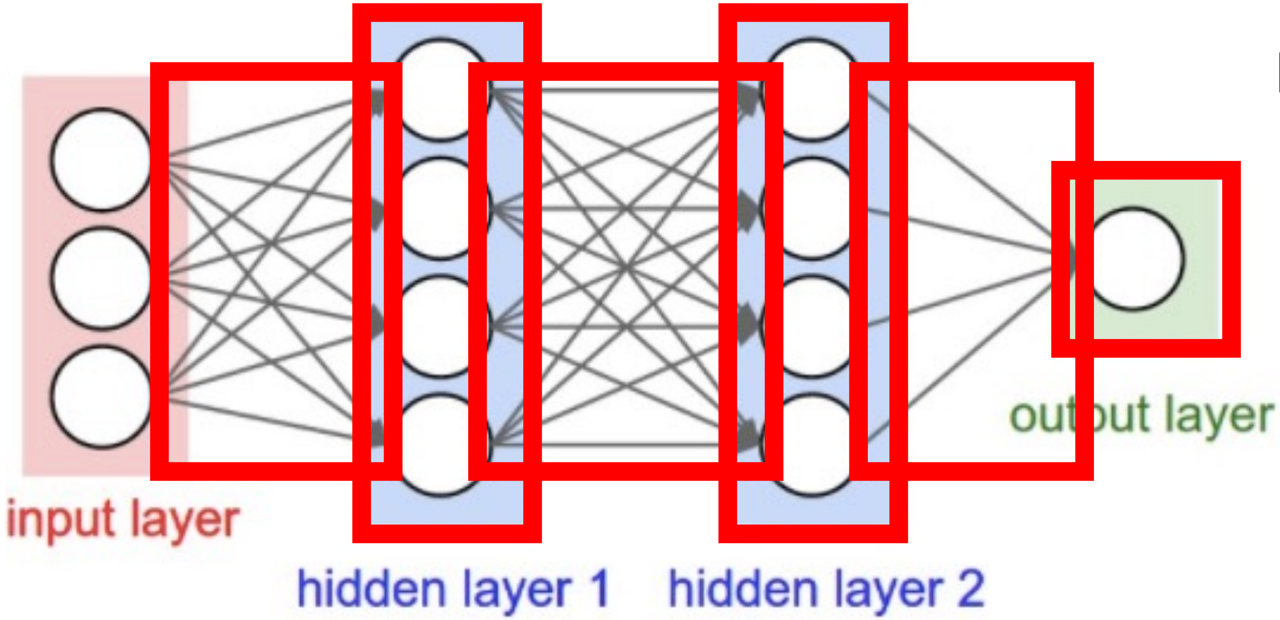


How many weights are in this model?

- Input to Hidden Layer 1:
  - 3x4 = 12
- Hidden Layer 1 to Hidden Layer 2:
  - 4x4 = 16
- Hidden Layer 2 to Output Layer
  - 4x1 = 4
- Total:
  - 12 + 16 + 4 = 32

http://cs231n.github.io/neural-networks-1/

Neural Network

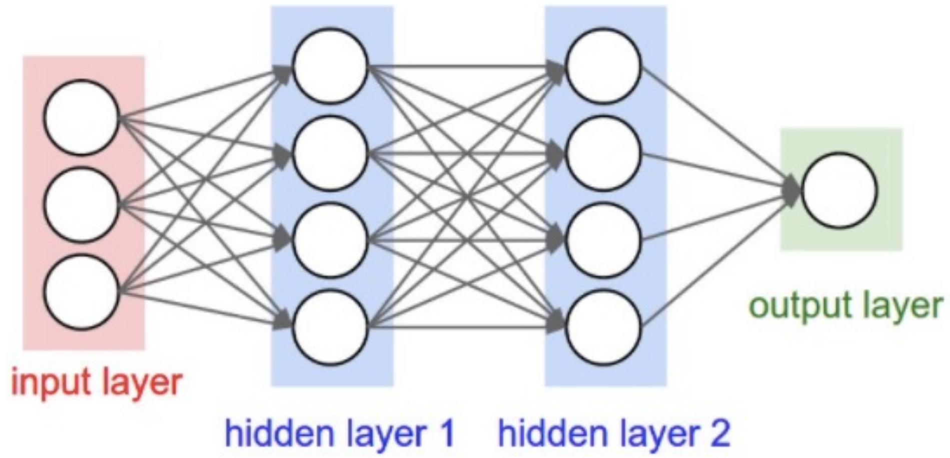


How many parameters are there to learn?

- Number of weights:
  - 32
- Number of biases:
  - 4 + 4 + 1 = 9
- Total
  - 41

http://cs231n.github.io/neural-networks-1/

Fully Connected, Feedforward Neural Networks



- What does it mean for a model to be fully connected?
  - Each unit provides input to each unit in the next layer

- What does it mean for a model to be feedforward?
  - Each layer serves as input to the next layer with no loops

http://cs231n.github.io/neural-networks-1/

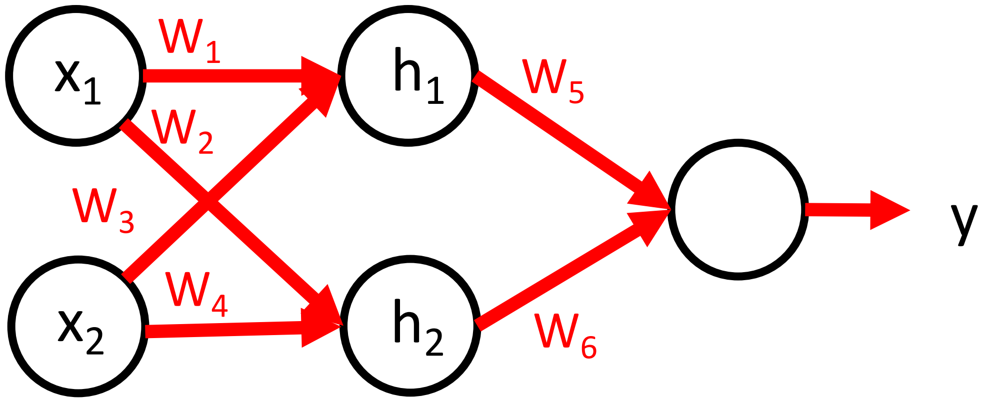
How Many Layers and Units Should be Used?

To be explored more in lab assignment set 1 and this course

Hidden Layers Alone Are NOT Enough to Model Non-Linear Functions

Key Observation: feedforward networks are just functions chained together

e.g.,



- What is function for h_1?
  - $h_1 = w_1x_1 + w_3x_2 + b_1$
- What is function for h_2?
  - $h_2 = w_2x_1 + w_4x_2 + b_2$
- What is function for y?
  - $y = h_1w_5 + h_2w_6 + b_3$
  - $y = (w_1x_1 + w_3x_2 + b_1)w_5 + (w_2x_1 + w_4x_2 + b_2)w_6 + b_3$
  - $\boxed{y = w_1w_5x_1 + w_3w_5x_2 + w_5b_1 + w_2w_6x_1 + w_4w_6x_2 + w_6b_2 + b_3}$

A chain of LINEAR functions at any depth is still a LINEAR function!

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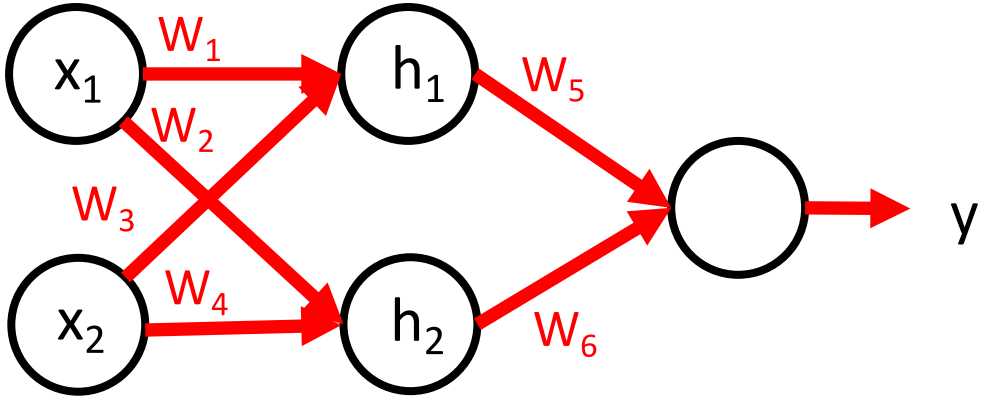

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- What is function for y?
  - $y = \underbrace{h_1w_5} + \underbrace{h_2w_6} + b_3$

Constant x linear function = linear function

A chain of LINEAR functions at any depth is still a LINEAR function!

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Key Idea: Use Connected Neurons to Non-linearly Transform Input into Useful Features for Predictions

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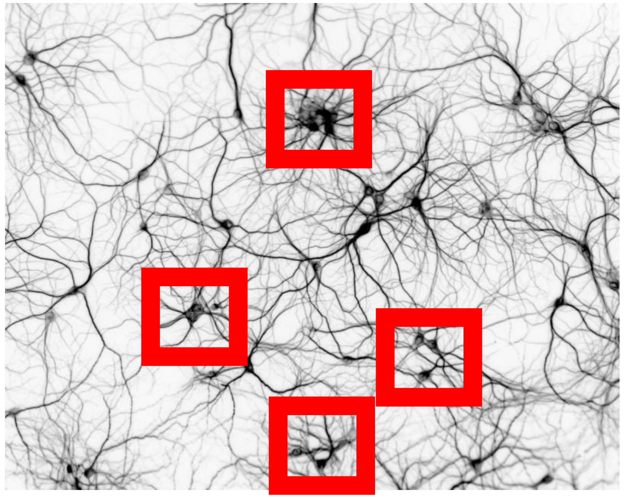

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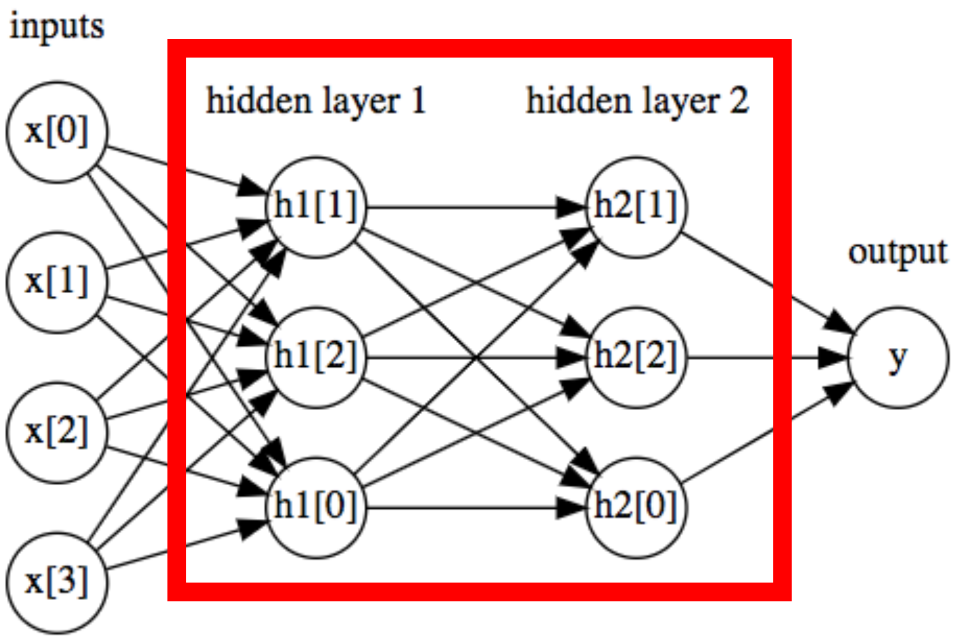

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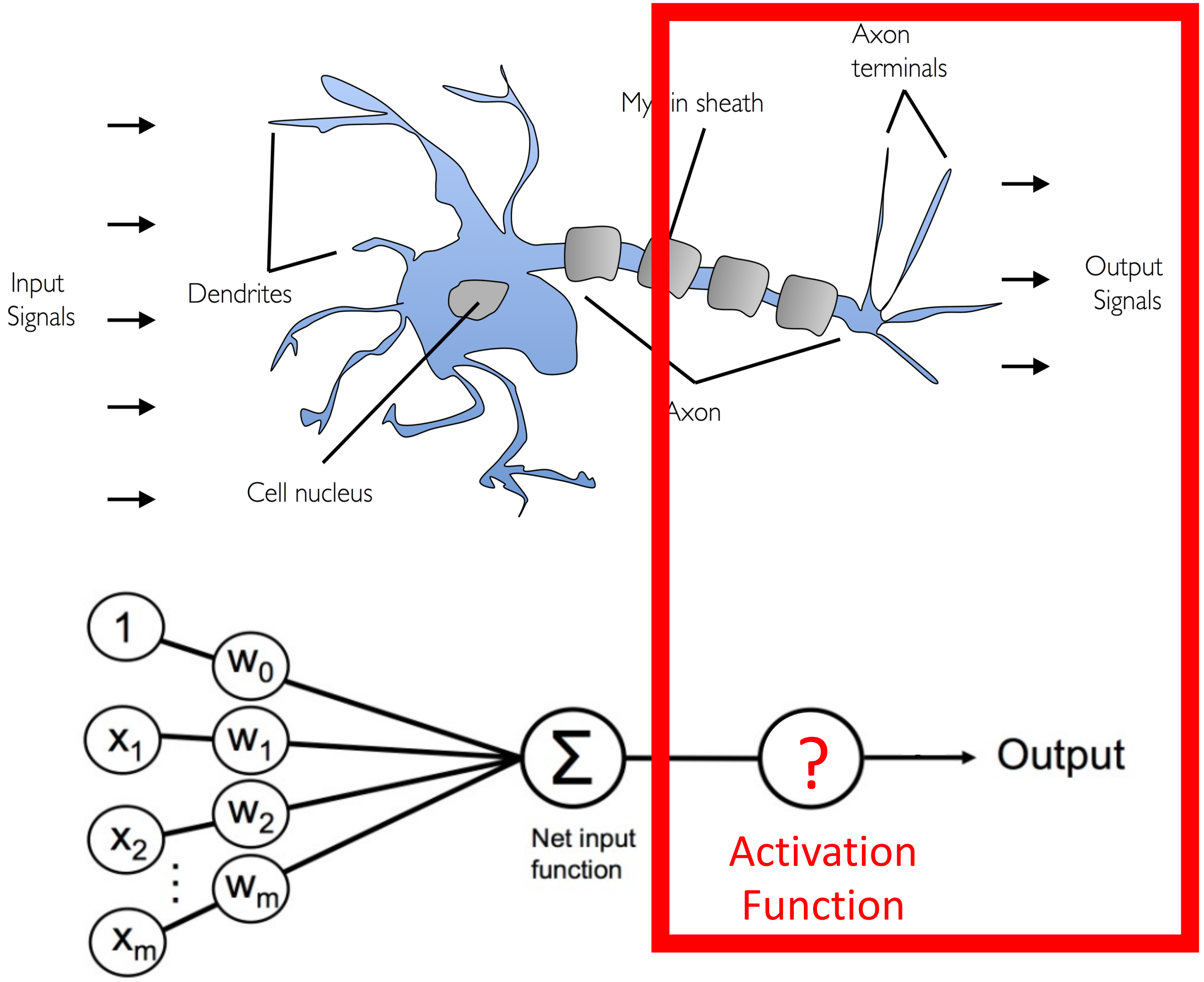
Biological Neuron:

Artificial Neurons (e.g., Perceptron):



Mimic a neuron firing, by having each unit apply a non-linear “activation” function to the weighted input

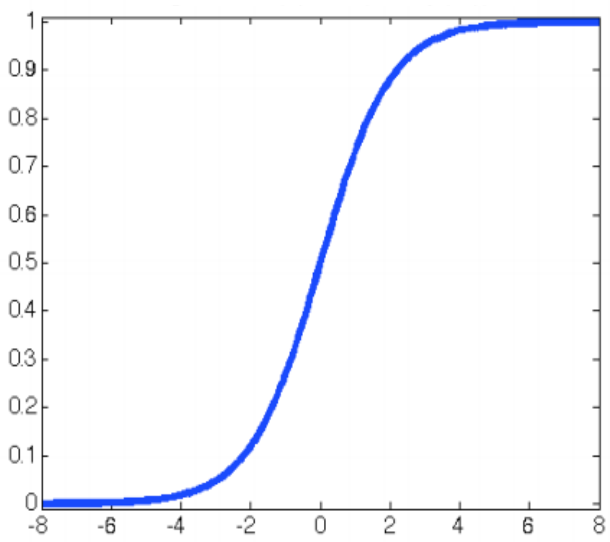
Python Machine Learning; Raschka & Mirjalili

Non-Linear Activation Functions

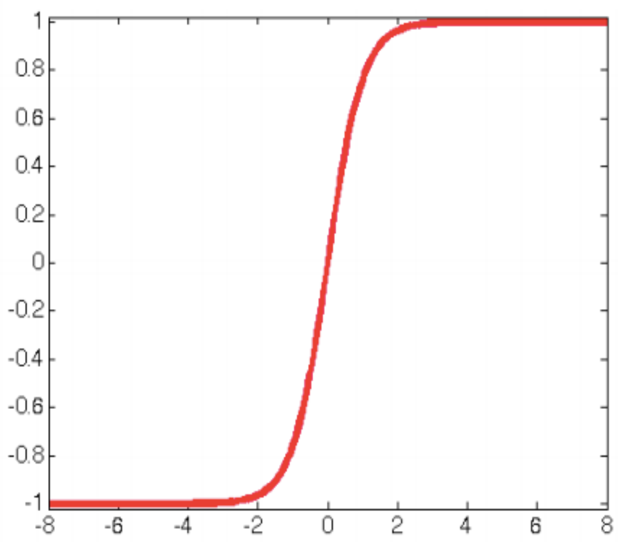
- Each unit applies a non-linear “activation” function to the weighted input to mimic a neuron firing

Computationally faster

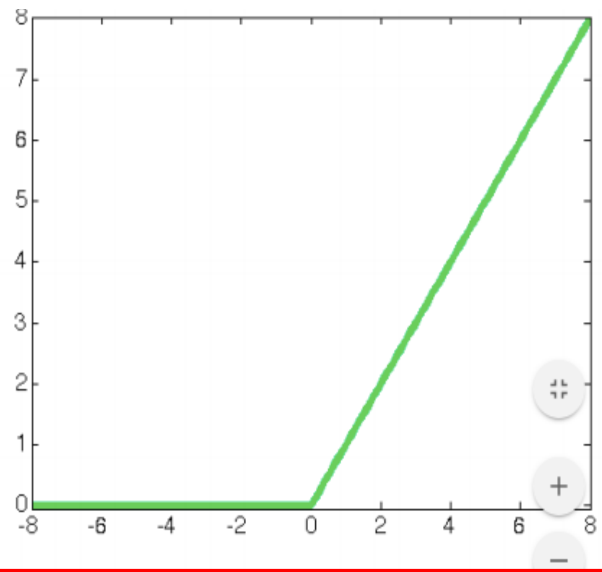
Sigmoid



Tanh



ReLU

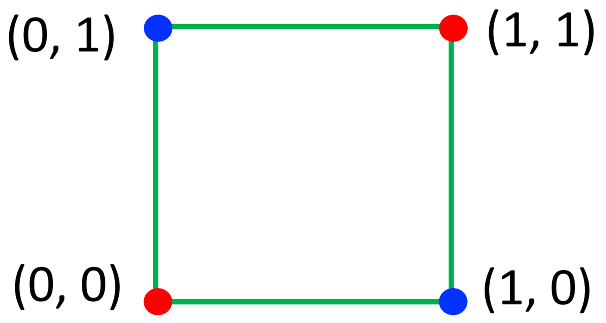


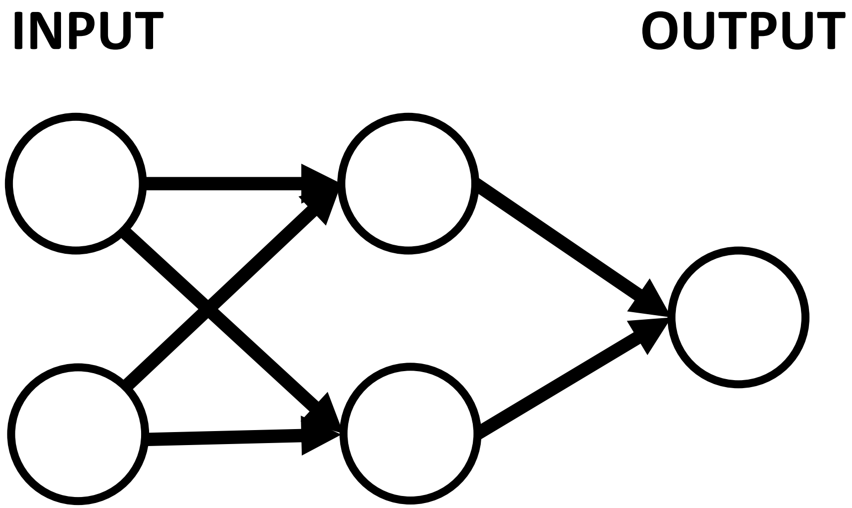
$$\sigma(z) = \frac{1}{1+\exp(-z)} \qquad \tanh(z) = \frac{\exp(z)-\exp(-z)}{\exp(z)+\exp(-z)} \qquad \mathrm{ReLU}(z) = \max(0, z)$$

http://www.cs.utoronto.ca/~fidler/teaching/2015/slides/CSC411/10_nn1.pdf

Non-Linear Example: Revisiting XOR problem

- Non-linear function: separate 1s from 0s:





INPUT		OUTPUT
A	B	A XOR B
0	0	0
0	1	1
1	0	1
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Non-Linear Example: Revisiting XOR problem

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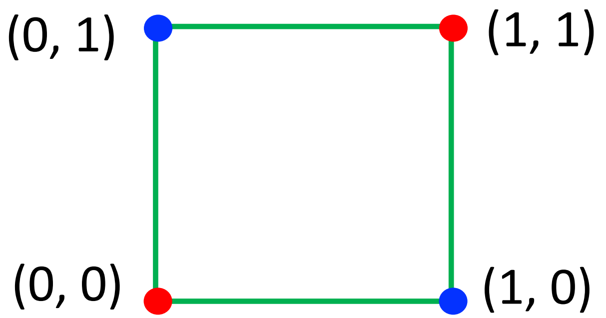
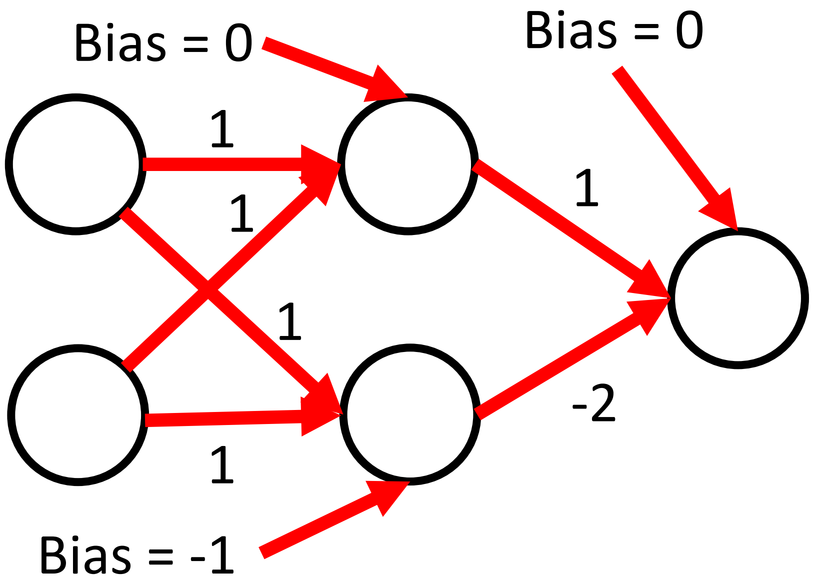


- Approach: ReLU activation function ($\mathrm{ReLU}(z) = \max(0, z)$) with these parameters:



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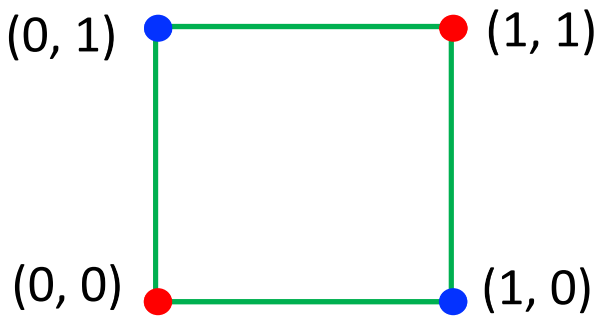

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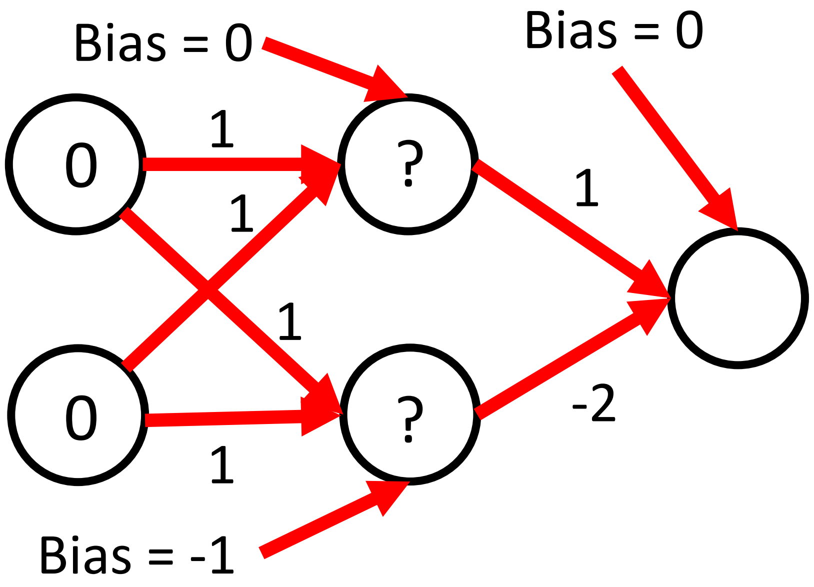

INPUT		OUTPUT
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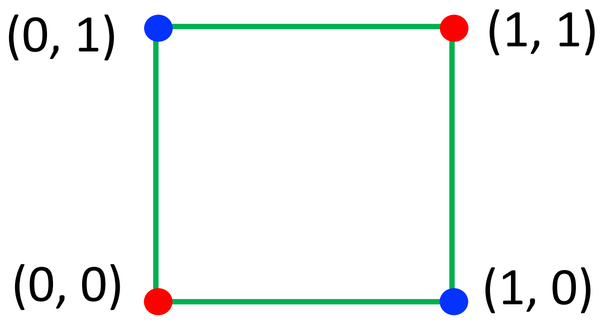
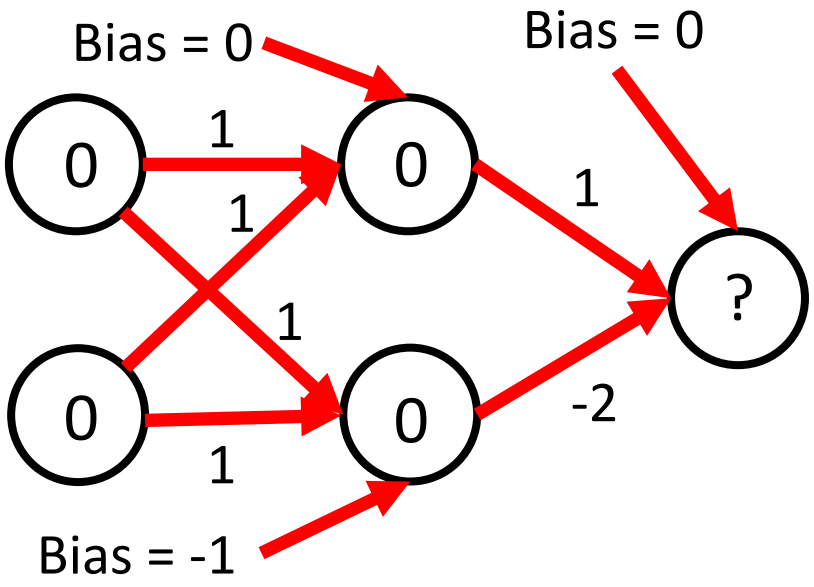
Non-Linear Example: Revisiting XOR problem

- Non-linear function: separate 1s from 0s:



- Approach: ReLU activation function ($\mathrm{ReLU}(z) = \max(0, z)$) with these parameters:



INPUT		OUTPUT
A	B	A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

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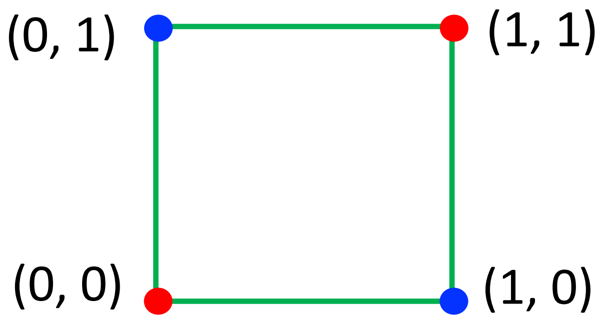

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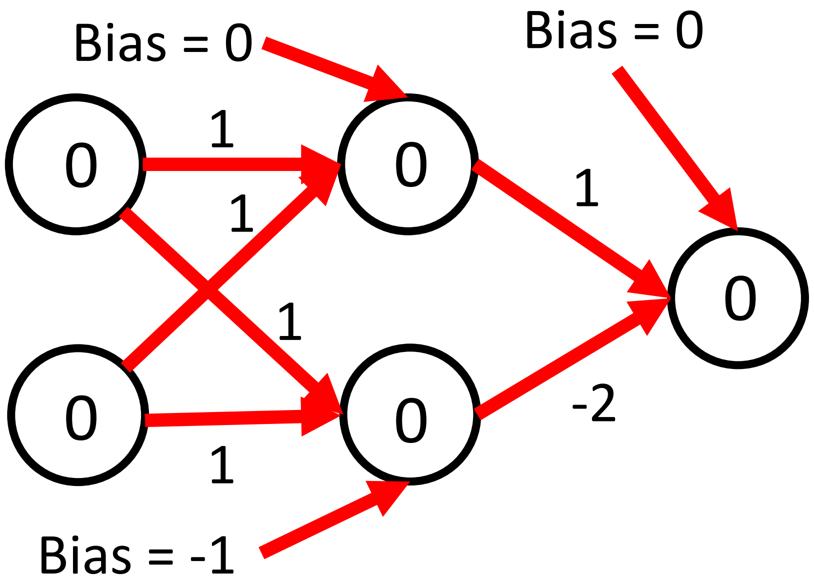

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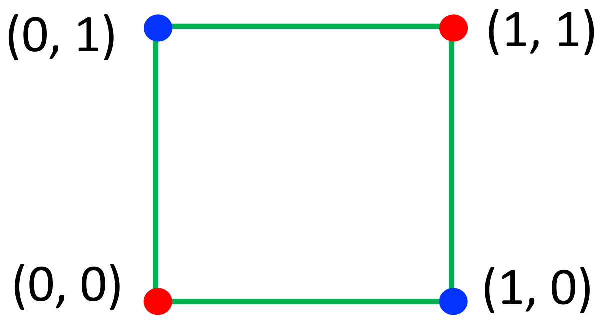

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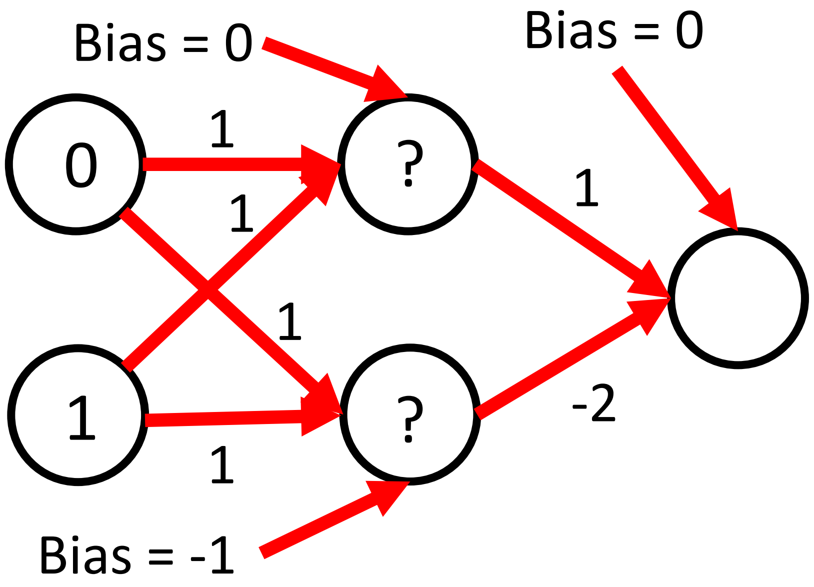

INPUT		OUTPUT
A	B	A XOR B
0	0	0
$\boxed{0}$	$\boxed{1}$	1
1	0	1
1	1	0

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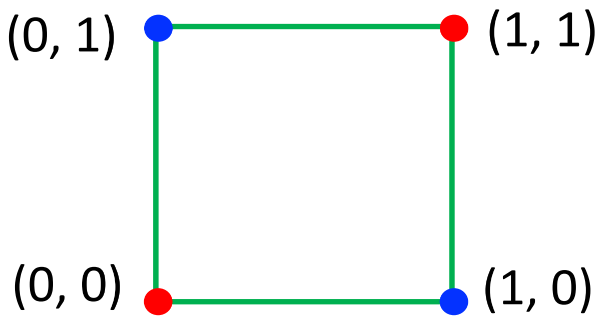

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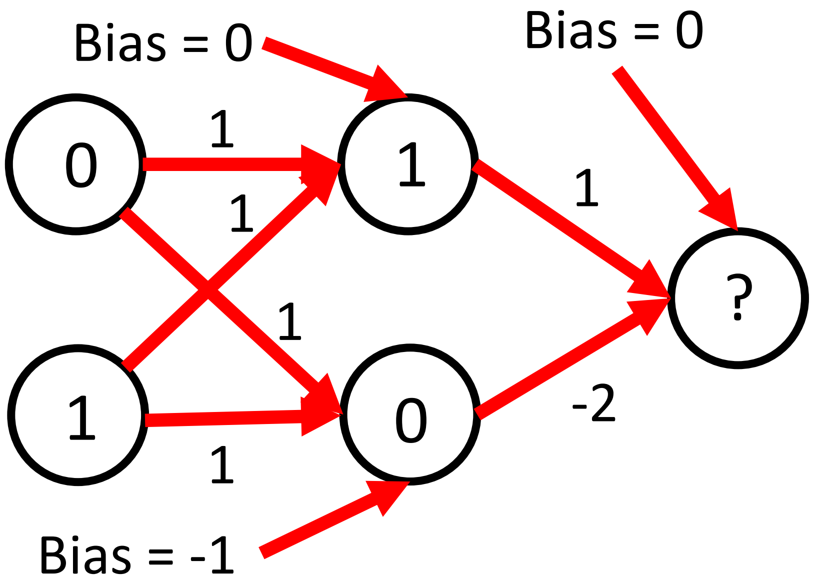

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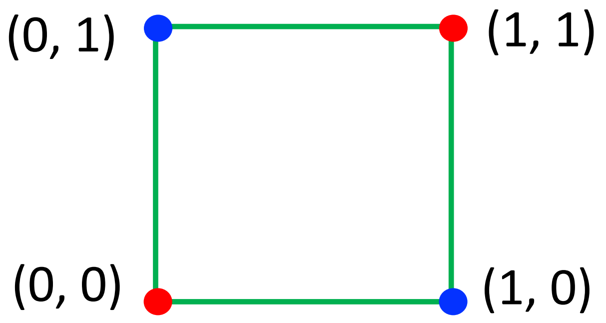

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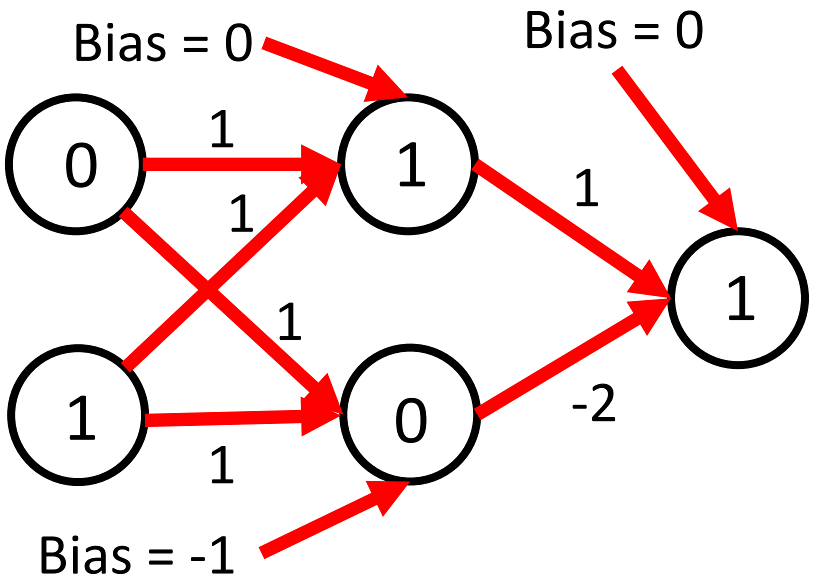

INPUT		OUTPUT
A	B	A XOR B
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1	0	1
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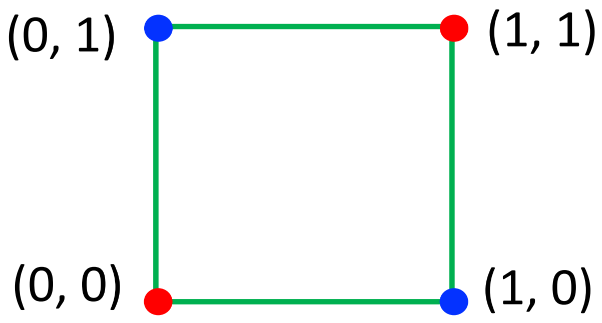

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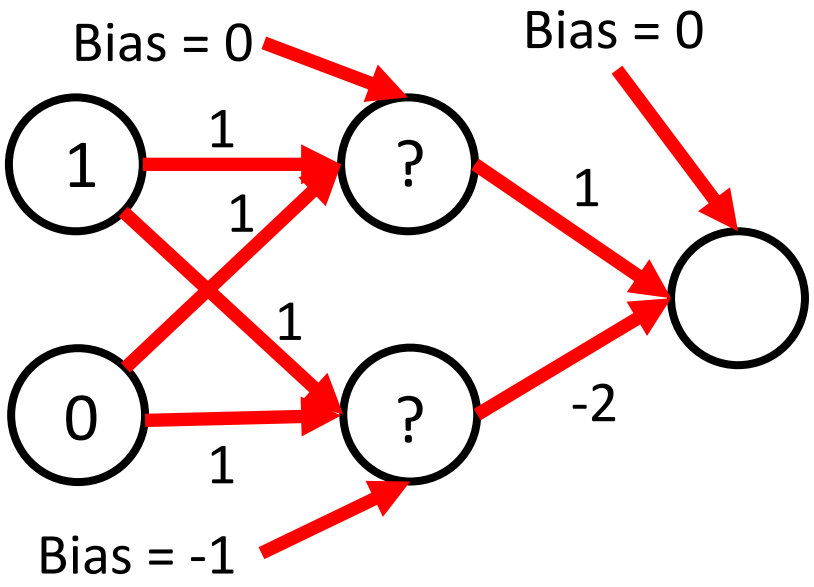

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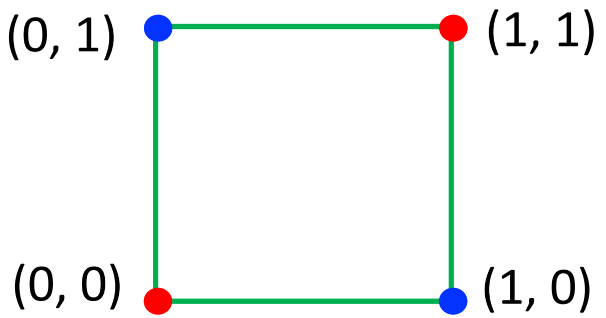

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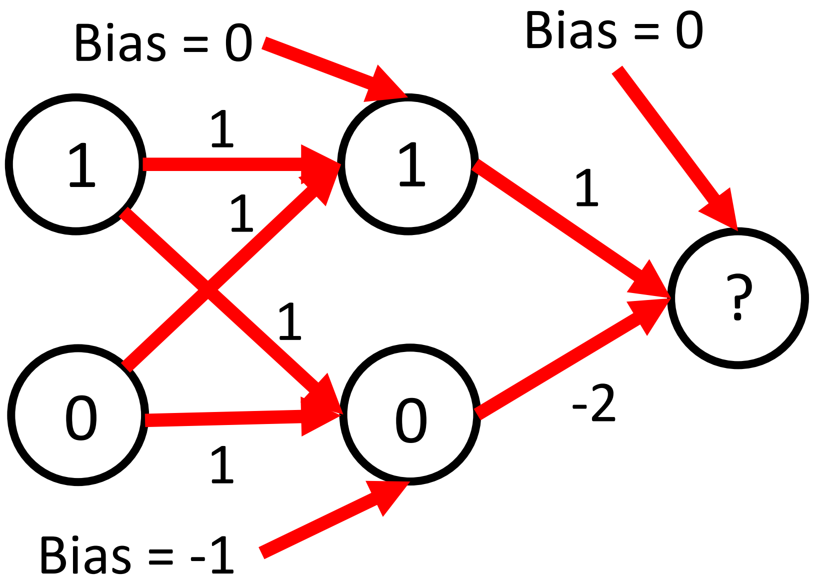

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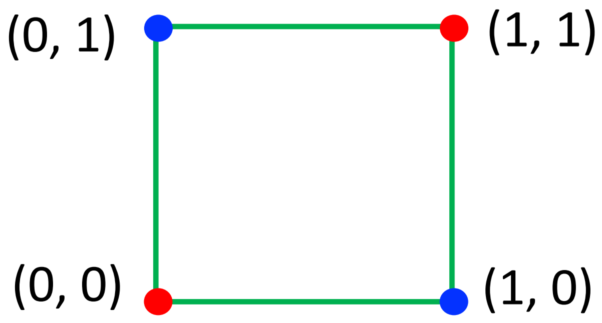

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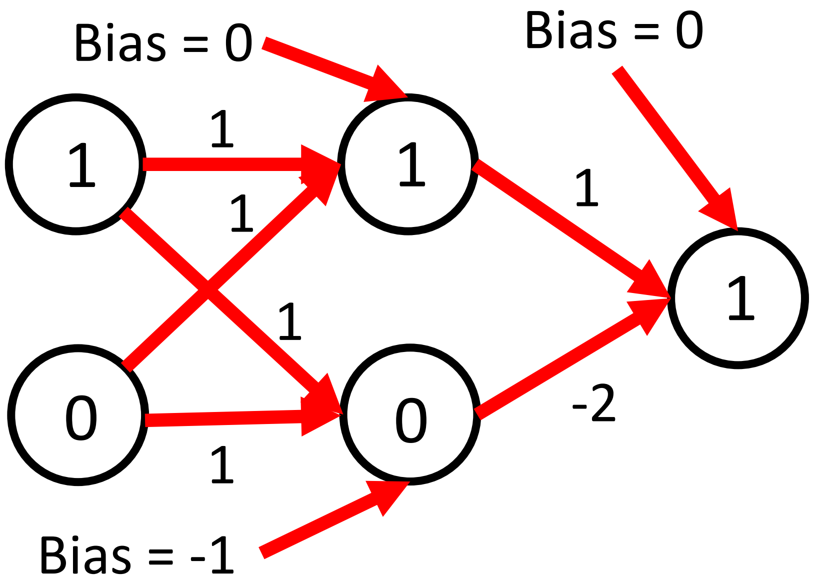

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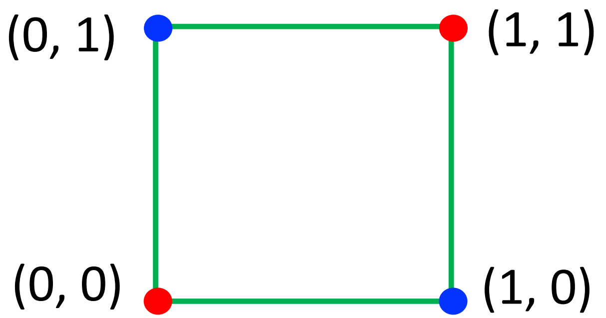

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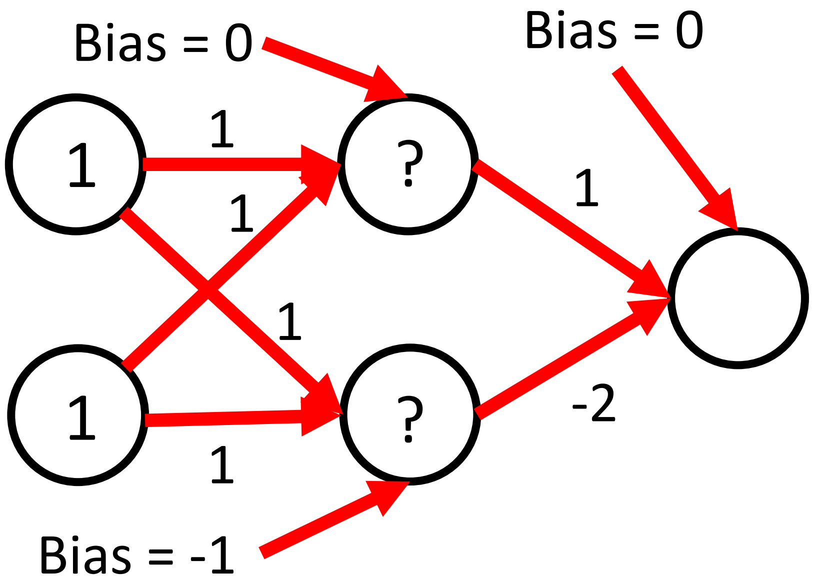

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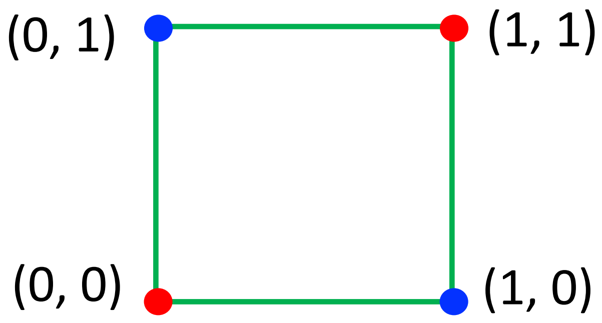

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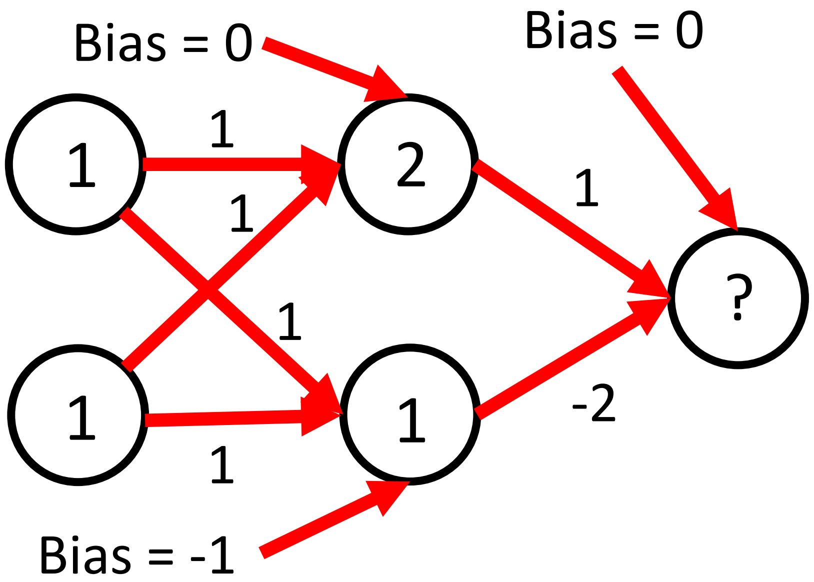

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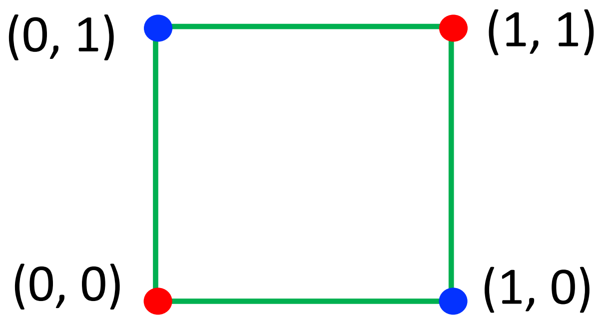

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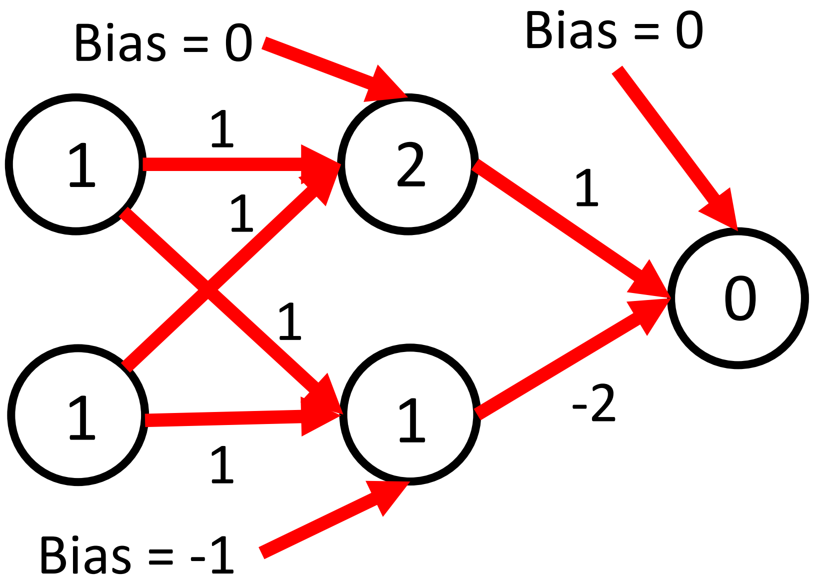

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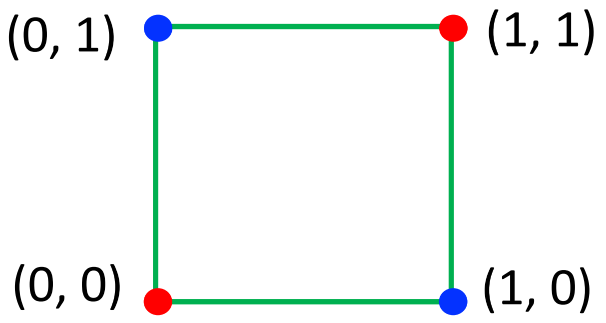

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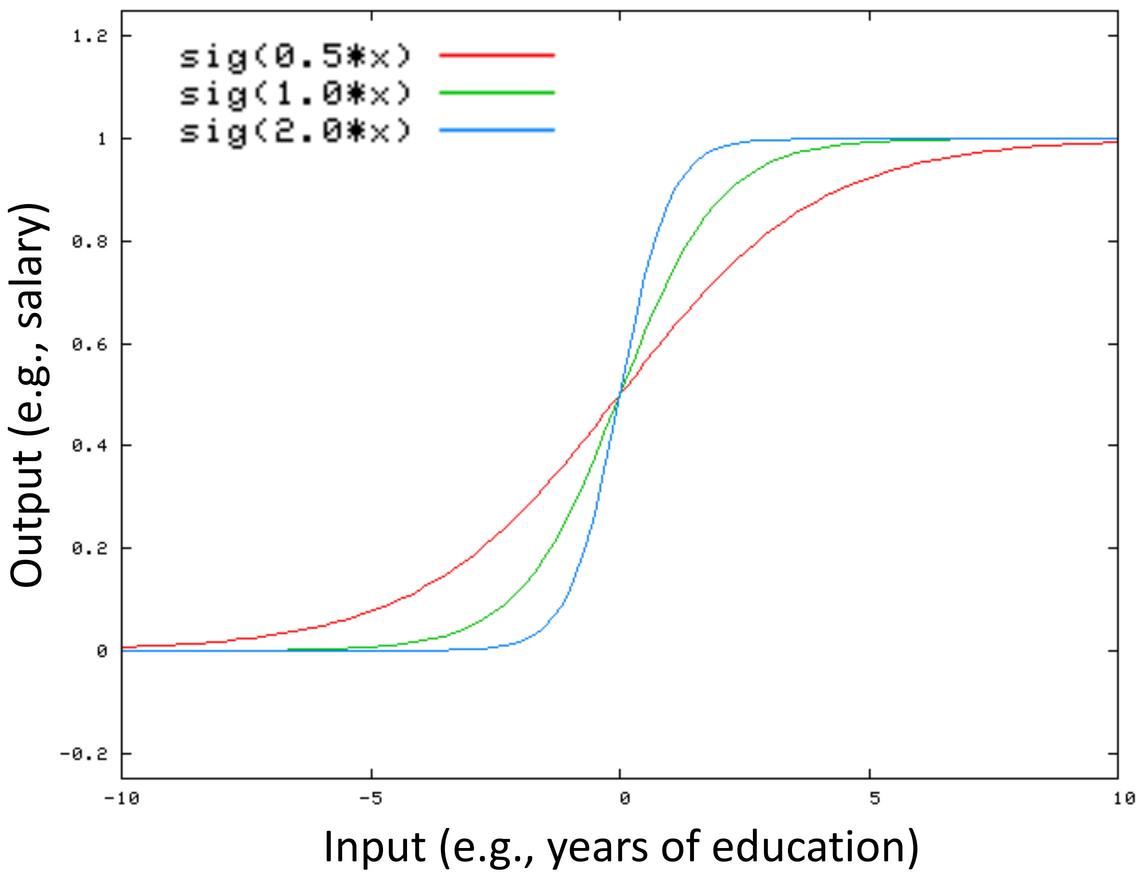
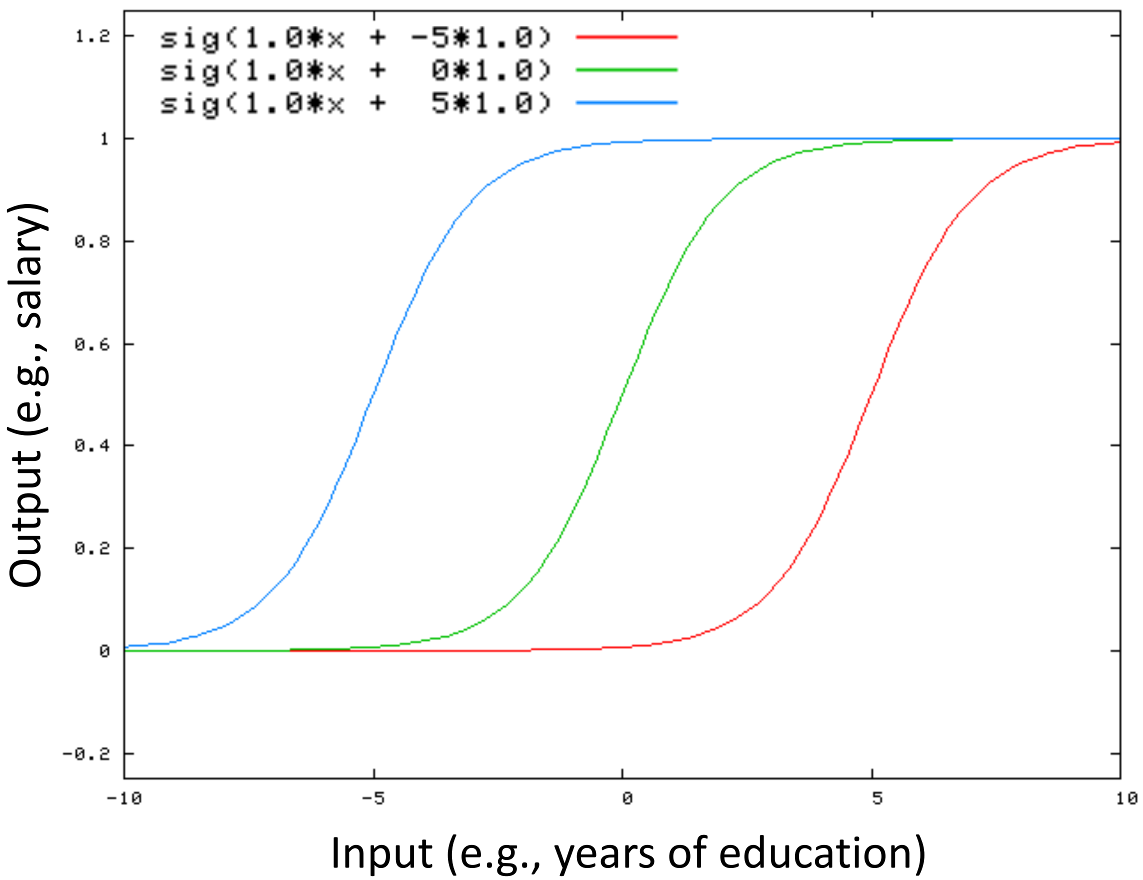
Neural networks can solve XOR problem...
and so model non-linear functions!

Activation Functions and Model Parameters (e.g., Sigmoid)

Weights determine curvature:



Biases determine shifted position:



https://stackoverflow.com/questions/2480650/what-is-the-role-of-the-bias-in-neural-networks

Which Activation Functions Should be Used?

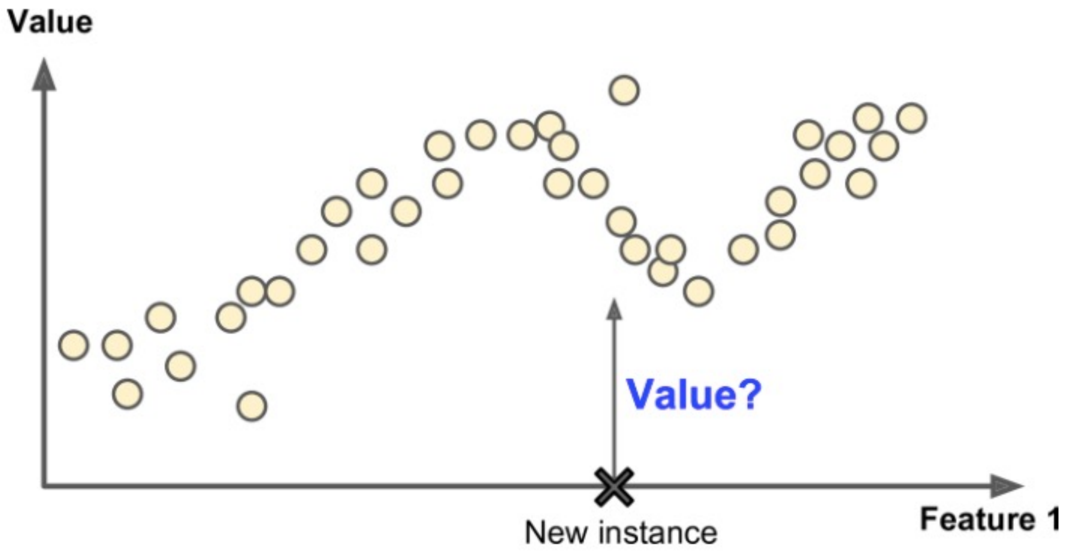
To be explored more in lab assignment set 1 and this course

Today’s Topics

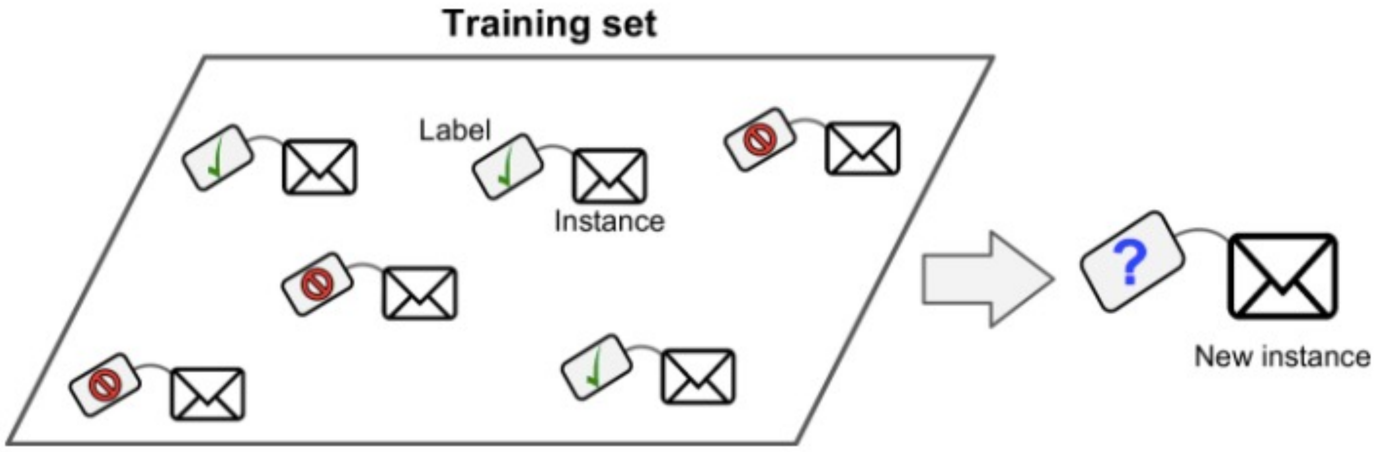
- Motivation for neural networks: need non-linear models
- Neural network architecture: hidden layers
- Neural network architecture: activation functions
- Neural network architecture: output units
- Programming tutorial

Desired Output Driven by Task

Regression
(predict **continuous** value)

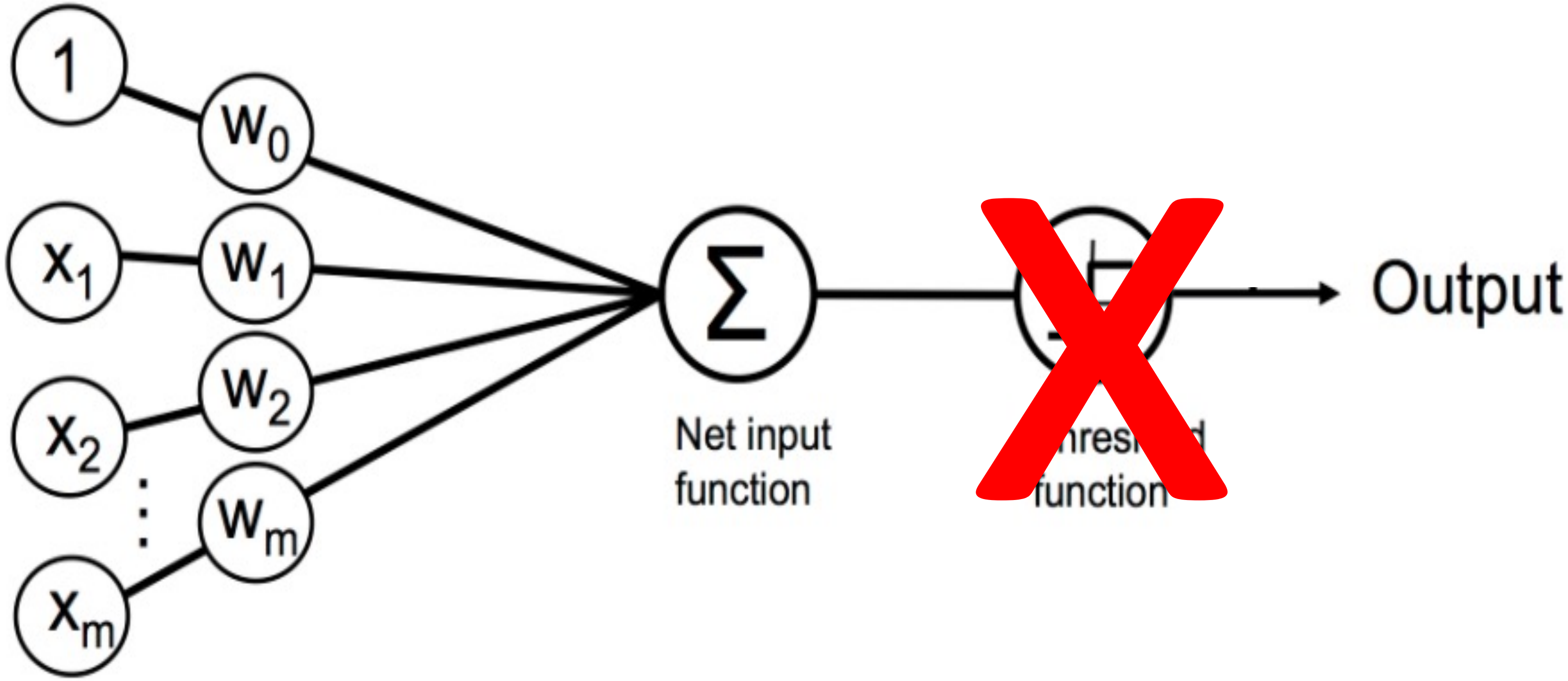


Classification
(predict **discrete** value)



Hands-on Machine Learning with Scikit-Learn & TensorFlow, Aurelien Geron

Linear (No Activation Function)



Python Machine Learning; Raschka & Mirjalili

Desired Output Driven by Task

Regression
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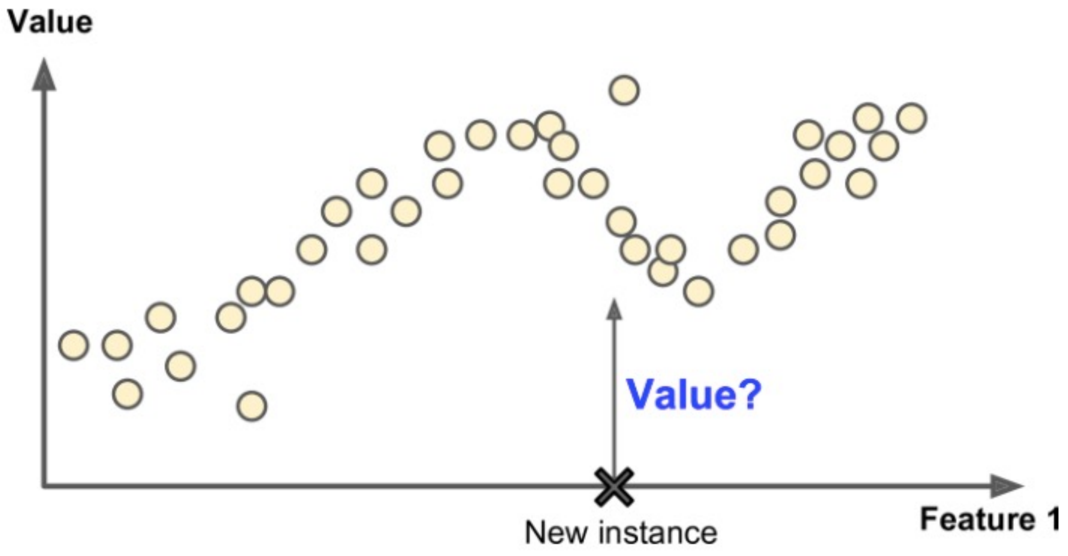

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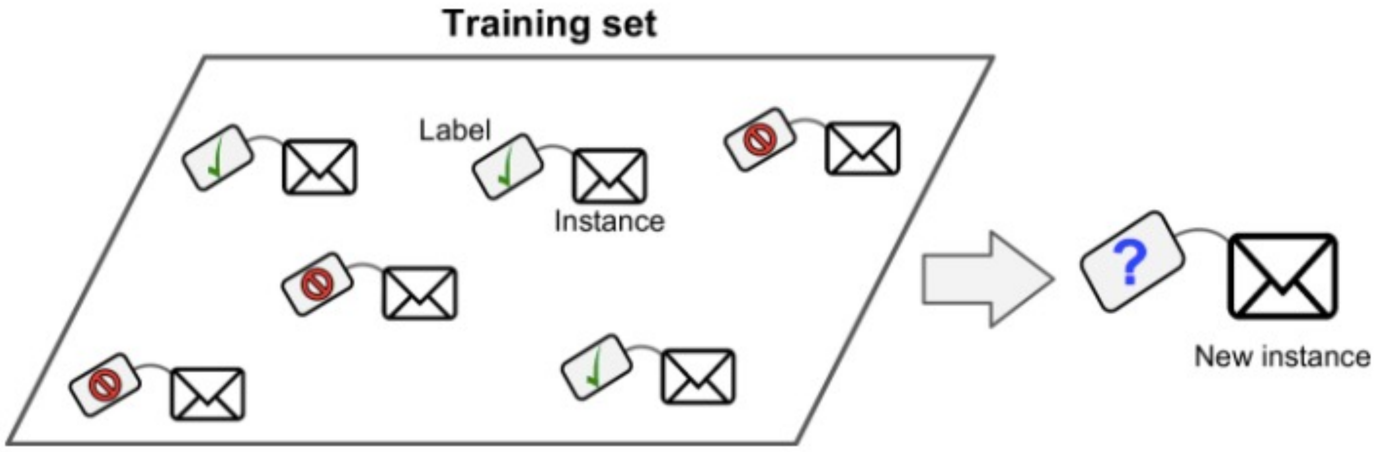

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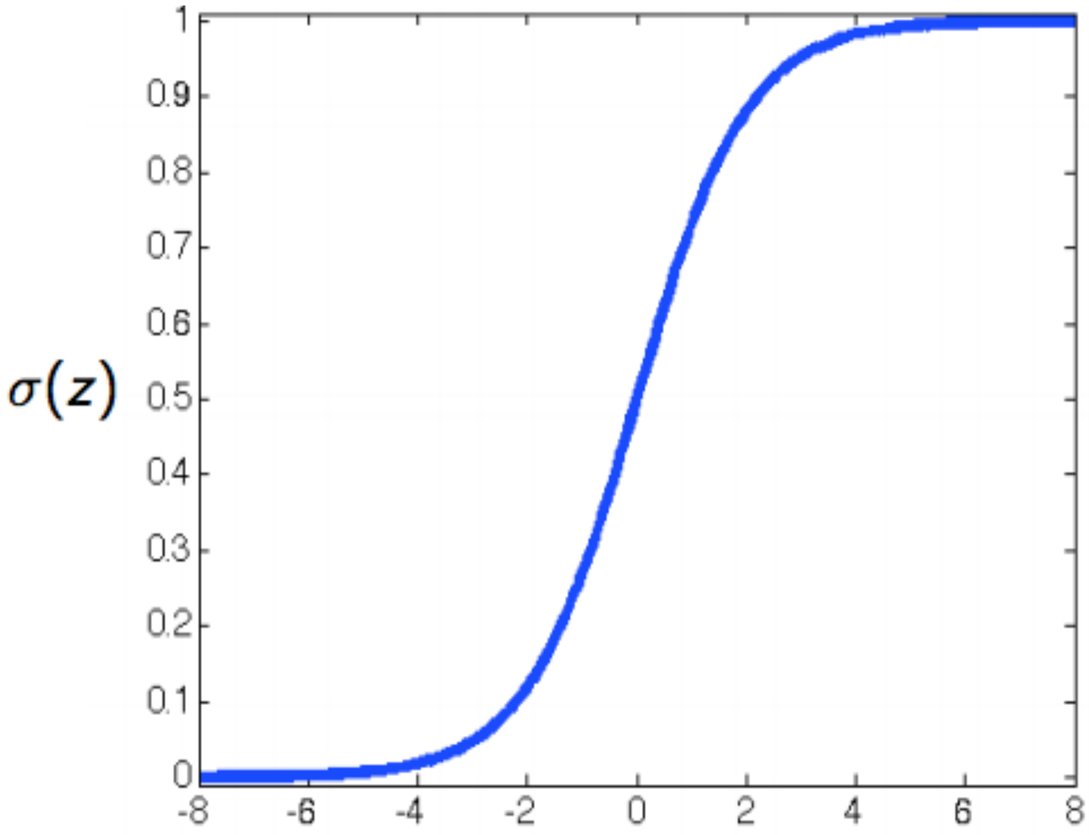
Sigmoid (for Binary Classification); aka – Logistic Regression

$$\sigma(z) = \frac{1}{1+\exp(-z)}$$



z (weighted sum of inputs at final node)

If z >= 0.5, output 1; Else, output 0

Why not use z instead of $\sigma(z)$?

- We want a probability in [0, 1]

What happens to the output as z becomes more positive?

- e^-z approaches 0 so value approaches 1

What happens to the output as z becomes more negative?

- e^-z becomes larger so value approaches 0

Softmax (for Multiclass Classification)

Converts vector of scores into a probability distribution that sums to 1; e.g.,

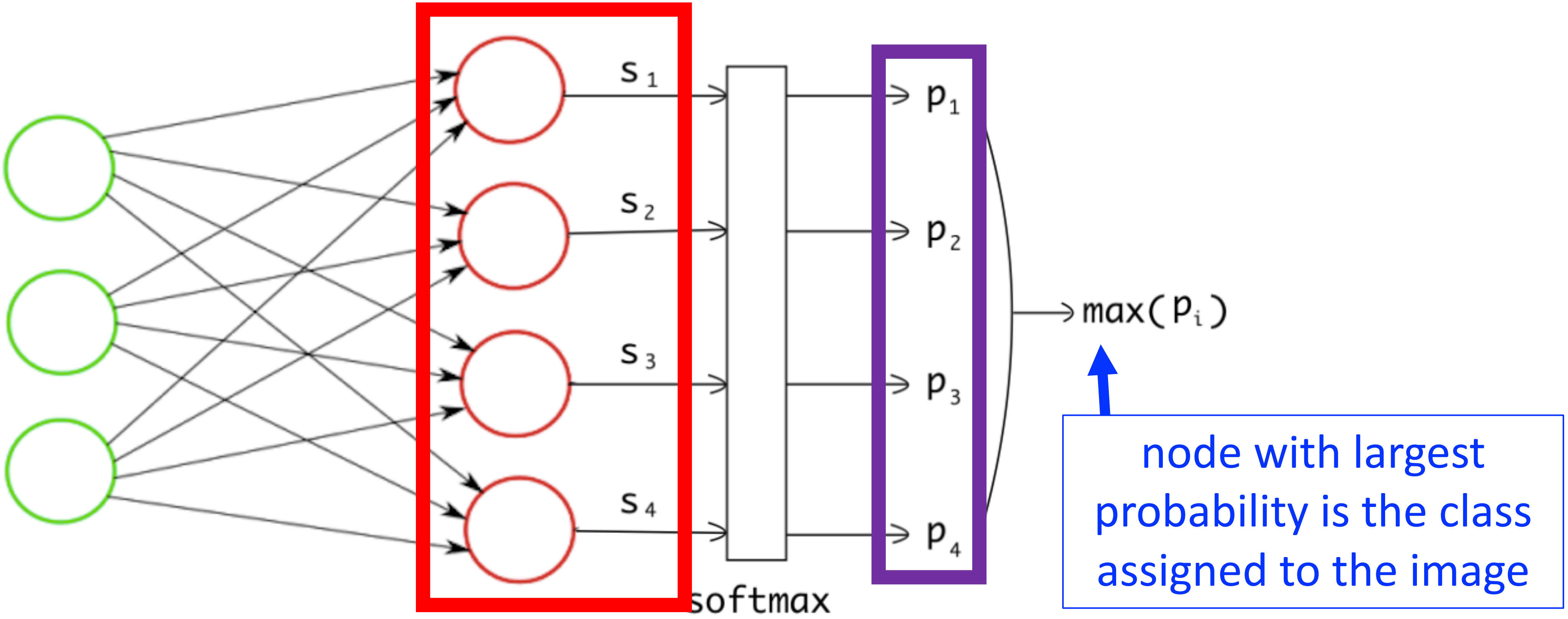


Figure Source: https://towardsdatascience.com/multi-label-image-classification-with-neural-network-keras-ddc1ab1afede

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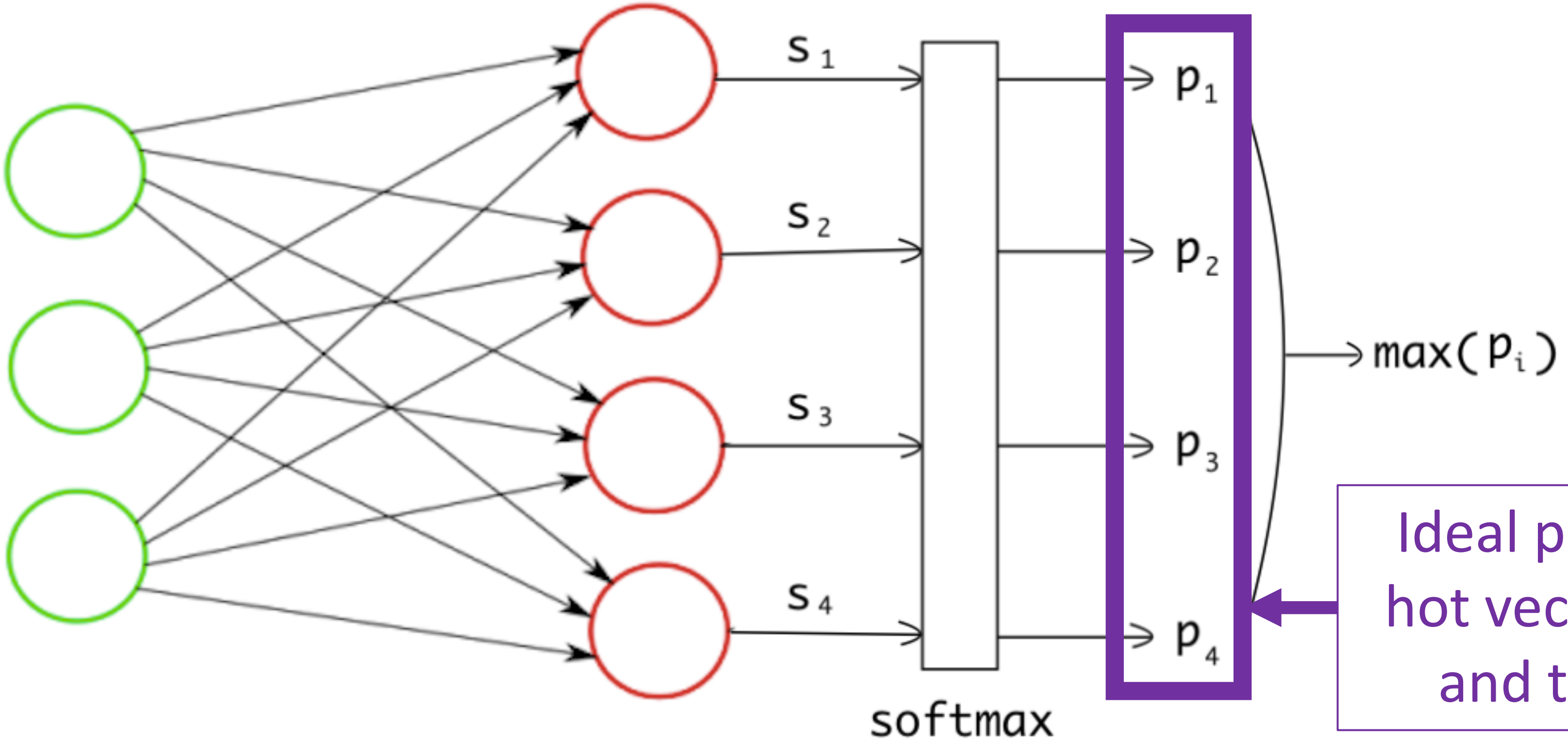

Ideal prediction; 1 hot vector of one 1 and the rest 0s

Figure Source: https://towardsdatascience.com/multi-label-image-classification-with-neural-network-keras-ddc1ab1afede

Softmax (for Multiclass Classification)

Converts vector of scores into a probability distribution that sums to 1

Removes negative values while preserving original order of scores: e causes negative scores to become slightly larger than 0 while positive values grow exponentially; choosing *e* rather than another exponent base simplifies math during training

$$\sigma(\mathbf{z})_i = \frac{\boxed{e}^{\boxed{z_i}}}{\boxed{\sum_{j=1}^{K} \boxed{e}^{z_j}}}$$

i = 1, ..., K

Number of classes

Want to divide each node's score by sum of all entries to make them sum to 1 (normalization)

Useful tutorial: https://towardsdatascience.com/exploring-the-softmax-function-578c8b0fb15

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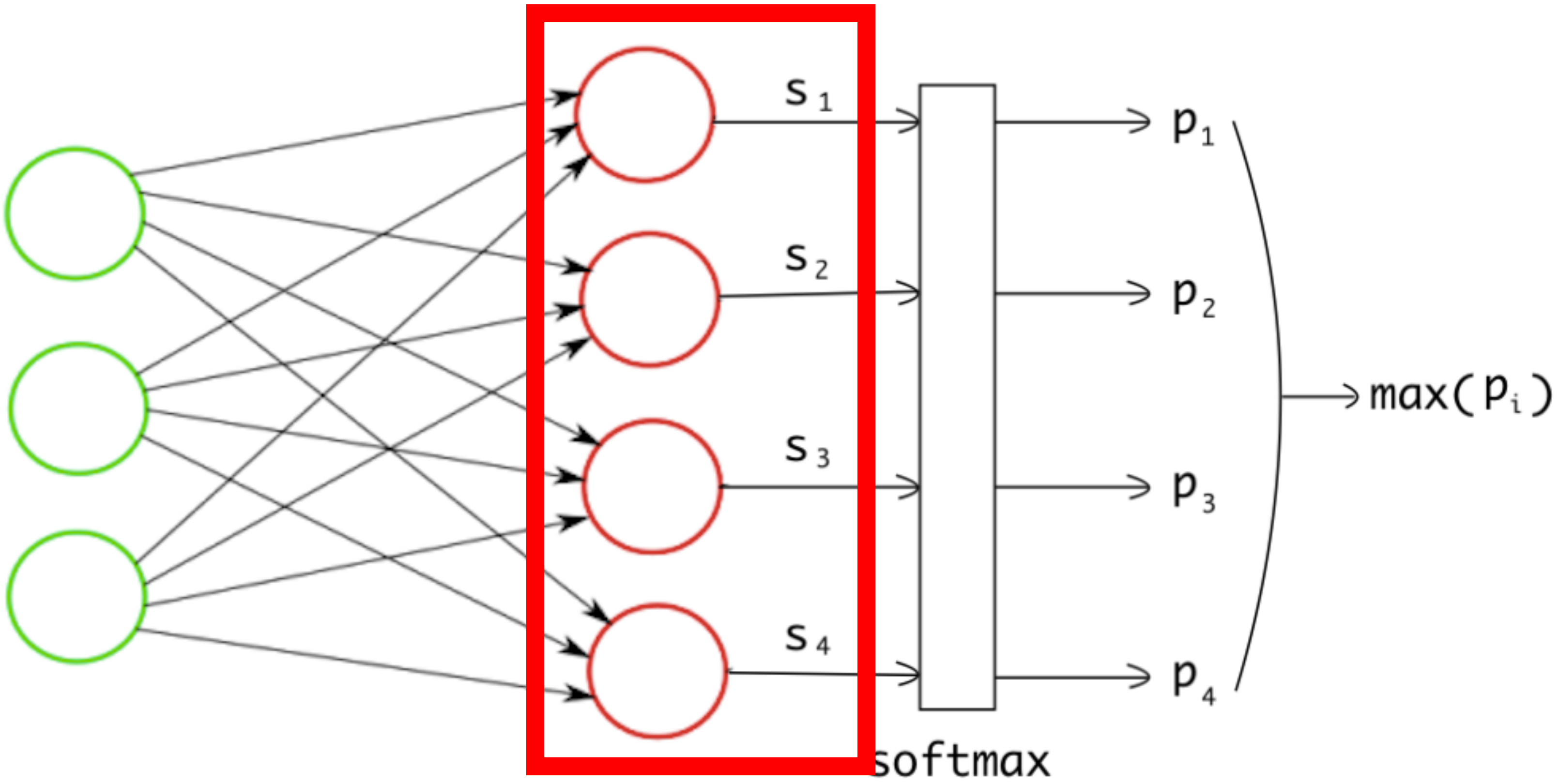

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	Scoring Function
Dog	-3.44
Cat	1.16
Boat	-0.81
Airplane	3.91

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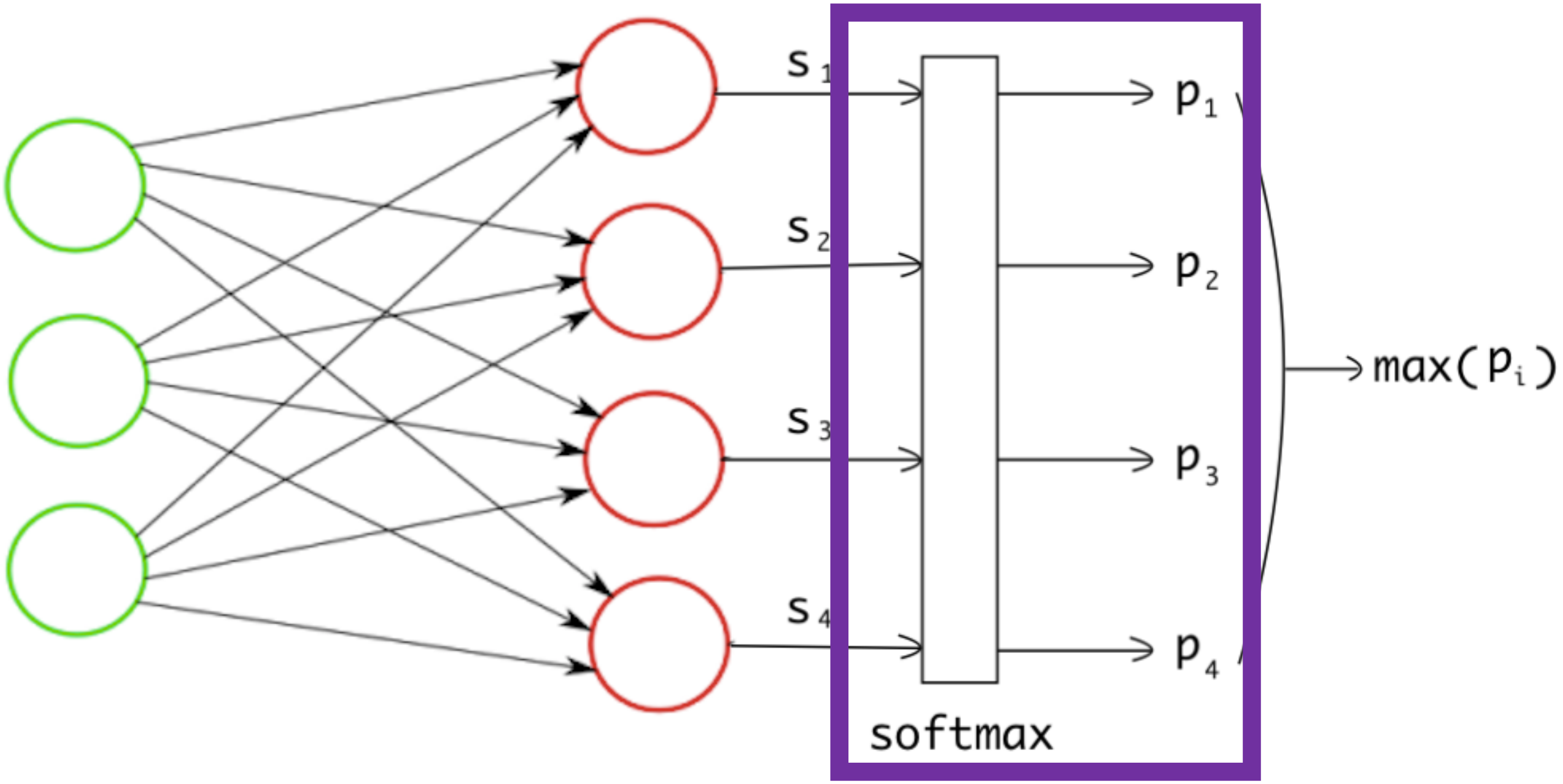

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	Scoring Function	Unnormalized Probabilities e^{z_i}	Normalized Probabilities $\frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}}$
Dog	-3.44	0.0321	0.0006
Cat	1.16	3.1899	0.0596
Boat	-0.81	0.4449	0.0083
Airplane	3.91	49.8990	0.9315

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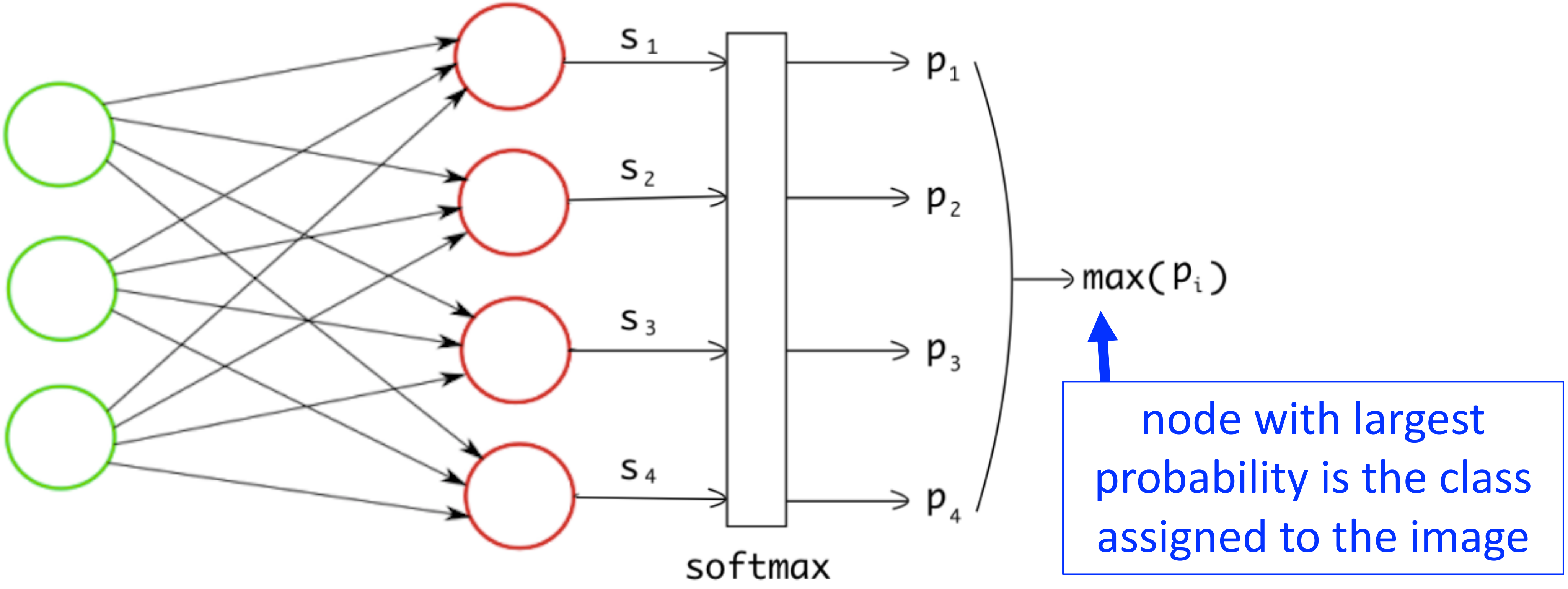

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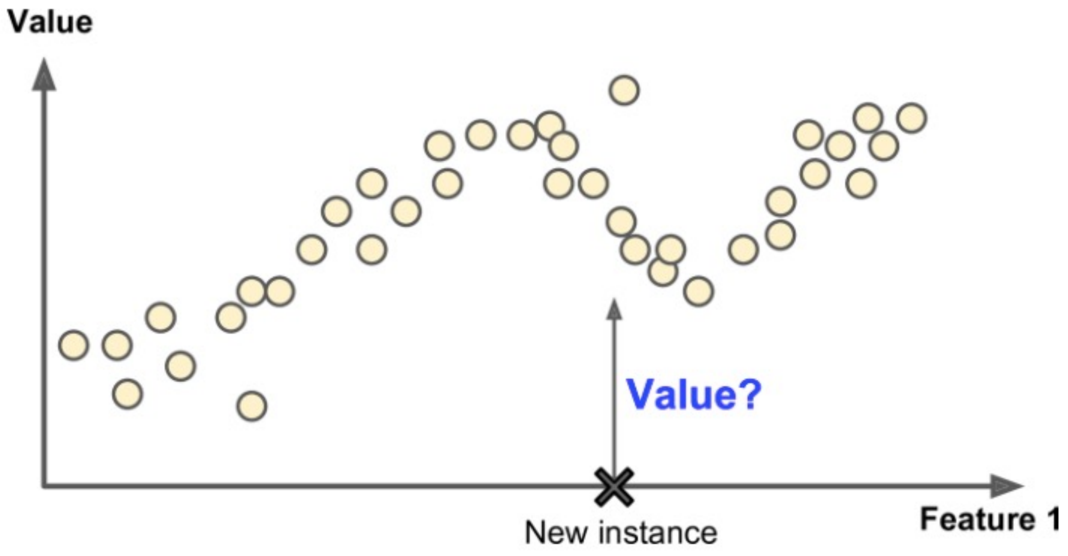

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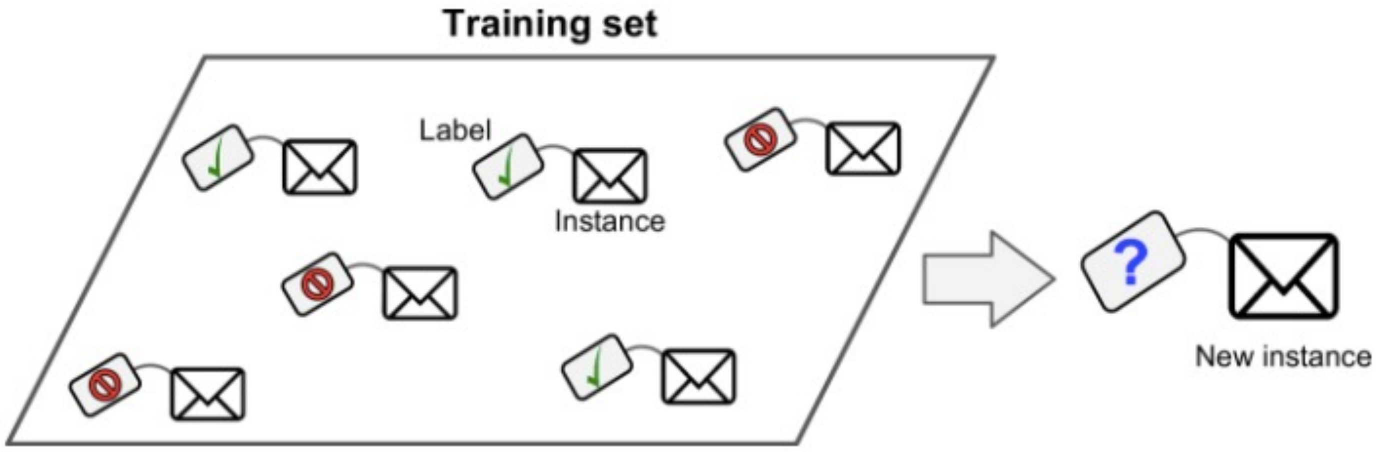

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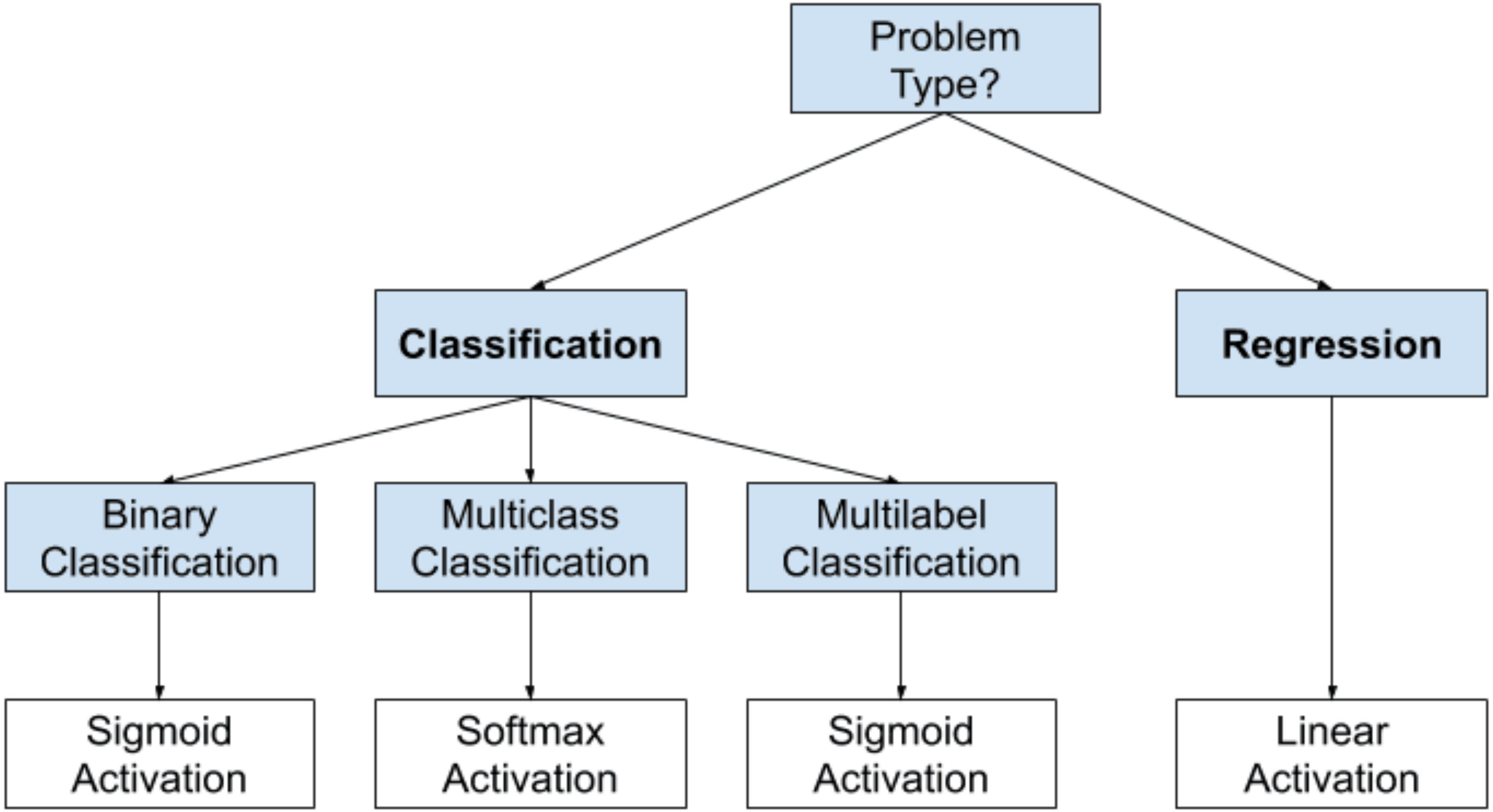
Desired Output Driven by Task



https://machinelearningmastery.com/choose-an-activation-function-for-deep-learning/

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The End